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Competition for Procurement Shares

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Competition for Procurement Shares[*]

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July 25, 2011

Abstract

We propose a new procurement procedure which allocates shares of the total amount to be procured depending on the bids of suppliers. Among the properties of the mechanism are: (i) Bidders have an incentive to participate in the procurement procedure, as equilibrium payoffs are strictly positive. (ii) The mechanism allows to vary the extent to which affirmative action objectives, like promoting local industries, are pursued. (iii) Surprisingly, even accomplishing affirmative action goals, procurement expenditures might be lower than under a classical auction format.

Keywords: Procurement Auction, Affirmative Action.
JEL: C72, D44, H57

1 Introduction

Procurement through government contracts is an important part of economic activity. The OECD (2002) estimates that the ratio of government procurement markets to Gross Domestic Product is 19.96% for OECD countries and 14.48% for non-OECD countries. Public procurement has three main objectives[1]:

[1]In the private sector the value of procurement transactions is estimated to be even larger than in the public sector (Dimitri et al., 2006). Private firms find it sometimes desirable to have more than one provider (multiple sourcing). This introduces a similar trade-off to the one we explain in what follows for the case of public procurement. We focus, however, for simplicity of the exposition throughout the paper on public procurement.

(*a*) minimization of public expenditures;

(*b*) provision of population needs; and

(*c*) promotion of affirmative action policies.

While objective (a) is uncontroversial, it is worth to discuss briefly the importance of aims (b) and (c) as well as to point out some of their implications.

Effective provision of the needs of the population might require more than one provider, when there are advantages from multiple sourcing. A multiple sourcing strategy avoids that the buyer is "locked in" with one provider and experiences shortage in the case that this supplier cannot fulfill his obligations. For example, in the autumn of 2004, the U.S. experienced a severe influenza vaccine shortage because one of two suppliers (Chiron) failed to produce the expected half of the necessary vaccines. In order to prevent similar problems in the future the supply of influenza vaccine is procured through multiple suppliers.[2]

In the U.S. government contracts have long been an important area of affirmative action policies at the local, state, and federal level (see e.g. the Small Business Act from 1953). Targets of these policies are women-owned businesses, minority-owned businesses, small businesses, disabled-owned businesses, veteran-owned businesses and others. There are also policies that favor domestic producers: a legal preference for in-state bidders is provided by twenty-seven states; twenty-one states have "Buy American" laws that affect public procurement; and at the federal level the 2009 American Recovery and Reinvestment Act includes a "Buy American" provision.[3] Many affirmative action policies accept that the targeted group is less efficient and establish certain market shares for these groups as explicit affirmative action goals.[4]

This implies that multiple sourcing and affirmative action might require to forgo economies of scale or to procure from several providers with different efficiency levels. For this reason conventional wisdom holds that there is a conflict between objective (a) on one hand, and aims (b) and (c), on the other. Following the language of the affirmative action literature we will refer to this in the sequel as the trade-off between expenditure minimization and minority representation. The main contribution of this paper is to propose a procurement mechanism that is able to reconcile these objectives.

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[2]For instance, California supplies for this reason influenza vaccine through multiple suppliers, see Department of General Services (2009). For an overview of the 2004 influenza vaccine shortage, see for example the editorial "An Influenza Vaccine Debacle" in the New York Times, October 20, 2004.

[3]The Act is available at http://publicservice.evendon.com/RecoveryBill1M.htm accessed on 03/03/2010. For an exposition and assessment of different affirmative action policies see Holzer and Neumark (2000). See also NASPO (2009).

[4]In public procurement bid preferences (or bid credits) are the most popular policies in order to favor disadvantaged bidders, besides quotas, set-asides or subsidies. Bid preferences assume implicitly that the targeted group is less efficient. Although any firm that is awarded the contract is paid the full amount of its bid, targeted firms are treated special. In order to calculate the winning bid their bids are lowered by a specified percentage amount. Explicit market shares are, for example, established in California's Disabled Veteran Business Enterprise and Small Business Certification Programs.

As we will discuss later, other studies have identified particular situations of affirmative action policies for which the conventional wisdom is not true either (see e.g. the assessment in Holzer and Neumark, 2000). It is, however, important to understand the strategic incentives that can be induced through different bidding mechanisms in such a context. Moreover, as procurement markets are divers, it is important to identify a wide range of procurement mechanisms that can reconcile the objectives and assess their performance in different situations. Avoiding trade-offs between the objectives is important because it influences the political support for and the prevalence of affirmative action policies. Ayres and Cramton (1996), for example, report that various California ballot initiatives tried to end state-sponsored affirmative action because of the concern that eliminating affirmative action could help to solve budget problems.

Many procurement procedures employed by public entities resemble roughly a first-price reverse auction: the buyer announces what kind of items she wishes to buy and how much can be spent (the budget constraint or reserve price); potential providers propose prices at which they are willing to provide the items (make their bids); and the provider proposing the lowest price is chosen.[5] In this article we modify this procedure in one crucial aspect: potential providers are assigned shares depending endogenously on the bids submitted.

We consider a simple model of two providers with heterogeneous costs. Having in mind affirmative action policies that favor domestic firms, the more efficient firm is called the foreign firm and the less efficient the domestic provider. This model is introduced in Section 3, where we also introduce a general formulation of procurement share auctions and discuss how different procurement outcomes are evaluated.

Section 4 considers the simplest possible framework and serves as a benchmark: The buyer's budget constraint is truthfully revealed, each seller is completely informed about the cost structure of his rival, and a specific procurement auction, which we call *contested procurement auction*, is analyzed. Variations of this benchmark in each of these dimensions are considered successively in later sections. We show the existence of a unique pure strategy Nash equilibrium in which the domestic high-cost supplier obtains a positive market share, providing him a strong incentive to participate in the auction.[6] The section concludes with the important result that when competition between suppliers is weak because of a large asymmetry in costs, then the contested procurement auction can increase competition and procure at lower costs than a standard auction. Competition is enhanced because the share auction introduces a new trade-off compared to a standard auction: increasing the price increases the

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[5]Carpineti et al. (2006) report, based on a survey conducted in 2004 among a group of European and American Public Procurement Entities, that public procurement usually awards contracts based on a first-price sealed bid auction. Krasnokutskaya (2011) reports that the Michigan Department of Transportation conducts procurement auctions for construction and maintenance of most roads within Michigan through a first-price sealed-bid auction with reserve price for every project.

[6]Stimulating participation is important because a large number of participants is considered to result in tough competition and therefore in economically advantageous conditions of procurement, see Albano et al. (2006).

mark-up over costs but it decreases the share.

In Section 5 we explore the following modification of the benchmark Section 4. The buyer might have the possibility of hiring an *informed auctioneer* who knows the relevant market well and can determine the budget constraint strategically. We show that if all agents (the auctioneer, as well as the two providers) are completely informed about the cost structure of both sellers, there is a unique equilibrium and, thus, the buyer's optimal budget constraint is also unique. Moreover, the result of the benchmark is strengthened: for any cost structure the buyer can determine the budget constraint in such a way that the procurement share auction allows to purchase at lower costs than a standard auction.

Section 6 extends the benchmark Section 4 in a different dimension, returning to the assumption that the buyer's budget constraint is truthfully revealed. We generalize the contested procurement auction to a class of mechanisms parameterized by a single parameter, determining the extent to which affirmative action objectives are pursued. This parameter can be interpreted as the price elasticity of a seller's market share and adjusting it affects the trade-off between mark-up and share that providers face when they decide on their bids. The procurement auction generalizes the first-price reverse auction because the extreme values of zero and infinity for the elasticity yield the special cases of equal shares independently of bids and the first-price auction, respectively; the simple mechanism of Section 4 corresponds to the case in which the elasticity is one. Our strategic analysis reveals that for any elasticity there is a unique Nash equilibrium in pure strategies. Moreover, we show that this equilibrium is robust by relating the original game to a slightly modified game which can be solved by an iterative process of sequential elimination of dominated strategies (Moulin, 1979). Lastly, we investigate the optimal choice of the elasticity. We show that for any combination of supplier costs there exists always a mechanism under which the domestic firm has a positive market share and total procurement costs are lower than under a standard auction.

The analysis so far has considered the polar case in which providers are completely informed about each other's characteristics. The complete information setting is considered appropriate for situations in which sellers know each other well (Moldovanu and Sela, 2003), like the case of construction contracting (Bernheim and Whinston, 1986) or in "environments with stable technology" (Anton and Yao, 1992, p. 691). Our last extension in Section 7 relaxes this assumption supposing that at the beginning of the auction each provider has only information about his own cost structure. We propose a *continuous-time descending price auction* in order to assign procurement shares. During the course of the auction all the relevant information is revealed so that at the unique equilibrium the providers' actions coincide with those in the complete information environment of Section 6.

Finally, Section 8 discusses our main assumptions and outlines future research questions. For convenience of the exposition, all proofs are relegated to the Appendix. We turn now to placing our paper into the relevant literature.

4

2 Literature

The present paper relates and contributes to several different strands of literature.

2.1 Share auctions, contests and split award auctions

In his seminal paper on auctions for shares, Wilson (1979) studies a different bidding mechanism and symmetric bidders and finds the opposite result to ours: a share auction can yield a significantly lower sale price than a standard auction. Kremer and Nyborg (2004) have shown that this conclusion depends on the allocation rule (see also Bernheim and Whinston, 1986). Our setting differs from Wilson's because we focus on asymmetric bidders.[7]

In a contest contenders exert effort in order to win an indivisible rent; the purpose of these models is thus different form ours. From a mathematical point of view, however, the central element of a contest, the so-called contest success function, is closely related to the concept of a procurement share auction proposed in the present paper. From this perspective, the present paper makes the following contribution to contest theory. Contest games are either specified as all-pay auctions (see Konrad, 2009) or as winner-pay contests (see Yates, 2011). The latter corresponds to the present paper. Epstein et al. (2011) consider an all-pay auction setting and a contest organizer whose payoff function is a weighted sum of contestants welfare and total effort. They establish that the organizer chooses the most deterministic contest success function which yields important support for the (deterministic) all-pay auction. Our results, however, indicate that in a winner-pay contest this result might be reversed (at least when the weight associated to total effort is high enough).[8]

A split-award auction divides procurement between two suppliers. In a seminal paper Anton and Yao (1989) have shown that sole-source procurement (or single sourcing) is more advantageous than a split-award (or multiple sourcing). Much of the subsequent literature has focused on conditions under which this conclusion is true (Anton and Yao, 1992; Perry and Sákovics, 2003; Inderst, 2008; Anton et al., 2010; Gong et al., 2011). Given that in a split-award auction the split is exogenous, while in our proposal the shares depend endogenously on bids, our results suggests that Anton and Yao's conclusion might be sensitive to the assumption of an exogenous split.

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[7]Our model can easily be reformulated as a standard auction in which the organizer is a seller and participants are buyers. This would not affect most of our results.

[8]We are investigating this issue in ongoing work. The generalized contested procurement auction proposed in the present paper is very closely related to the serial contest success function in Alcalde and Dahm (2007). The latter paper, however, analyzes an all-pay auction. Yates (2011) is closely related to our paper because he applies the serial contest success function in a winner-pay contest. He establishes an equilibrium in a setting that corresponds to our Theorem 1.

2.2 Affirmative action in procurement

An important implication of our paper is that pursuing affirmative action and minimizing purchasing costs are not necessarily in conflict. This is an important finding because it implies that affirmative action policies are not necessarily costly and thus do not need to be justified (solely) on fairness grounds. Other affirmative action policies also have the potential to reconcile both objectives (see also the assessment in Holzer and Neumark, 2000).

Rothkopf et al. (2003) offer a model of asymmetric providers in a common value environment in order to analyze the effect of subsidies to a high-cost supplier. They show that subsidies can reduce procurement costs because they induce low-cost suppliers to bid more aggressively. While it is difficult to compare the performance of our mechanism to subsidies, our share auction seems to be more tractable. Rothkopf et al. (2003) restrict each bidder to bid a multiple of his cost because the analysis of this asymmetric auction setting is challenging. In contrast, under complete information our mechanism is dominance solvable, while with private information the equilibrium is in dominant strategies.

Bid preferences also have the potential to foster competition between suppliers and reconcile both objectives. This has been shown in theoretical models of bid preferences for the Federal Communications Commission (FCC) auctions (Ayres and Cramton, 1996), government procurement from domestic and foreign firms (McAfee and McMillan, 1989), and entry in procurement auctions (Hubbard and Paarsch, 2009).[9] The share auction of the present paper offers some advantages over bid preferences because the success of affirmative action does not depend on whether the cost difference between providers exceeds a threshold or not. To see this, suppose a bid preference of five percent and a large cost difference between two suppliers such that the equilibrium bid difference is six percent. Here the market share of the "weak" bidder is zero. In contrast, our mechanism always assigns a positive share to the "weak" bidder.

2.3 Optimal auctions with asymmetric bidders

The bidding mechanism of the present paper assigns a positive share to the "weak" bidder. As a result competition increases and revenue is raised compared to a second price auction. This parallels findings in the literature on auctions for an indivisible object with asymmetric bidders, where competition and revenue can also be increased when the item is not always awarded to a bidder with the highest valuation.[10] In Myerson (1981) the optimal auction may require

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[9]The empirical literature on bid preferences, however, yields mixed results. Support comes from experimental evidence (Corns and Schotter, 1999) and snow removal contracts in Montreal (Flambard and Perrigne, 2006), while studies of road construction contracts (Marion, 2007; Krasnokutskaya and Seim, 2011) do not find such an effect.

[10]Notice that the auction theoretical literature usually considers models of incomplete information. We extend our findings to such a setting in Section 7. In addition, the comparison of our model to this literature is meaningful when the shares assigned by the bidding mechanism are interpreted as win probabilities of an indivisible object. This requires to assume that bidders are risk neutral.

to give the object to a bidder whose valuation is not the highest. Maskin and Riley (2000) compare the sealed high-bid and "English" auctions. They show that, even when the asymmetry between bidders is not so large that the "strong" bidder always preempts the "weak" bidder and the latter sometimes wins, expected revenue in the sealed high-bid auction may be higher than in the "English" auction. In that literature, however, the auction choice depends on the asymmetry. In contrast, we show that for any degree of asymmetry the auction proposed can be modified such that it performs better than a standard auction.

2.4   Allocation of a divisible commodity

Bidding mechanisms have been proposed for the allocation of a variety of divisible goods for which our mechanism might be interesting because revenue and non-revenue objectives might be in conflict. For instance, Morgan (1995) analyzes the management of fisheries and proposes to allocate fishing quotas through an auction in order to obtain revenues and to take into account other objectives, like protecting traditional, native users of the resource or meeting conservation goals. In the assignment of takeoff and landing rights at airports non-revenue considerations might include favoring domestic or small airlines and Grether et al. (1981) propose to auction off these rights.

Back and Zender (1993) and Swierzbinski and Börgers (2004) analyze Treasury bond auctions in which there is a concern that the bond market can be manipulated. This leads in some markets (including the U.S.) to the establishment of a limit on the amount a single buyer can purchase. Cramton (2002) discusses the virtues of spectrum auctions and reports a trade-off between generating revenue for the Treasury and avoiding a collusive industry structure, which explains why regulators usually limit the amount of spectrum that a firm can hold in a geographic area.

Boycko et al. (1993) provide an account of the Russian privatization program which used voucher auctions for shares of firms. They report that political support for privatization required that "weak" bidders (like workers in enterprises being privatized) obtained an important share of the firm. Dana and Spier (1994) characterize the optimal auction for production rights and stress that the choice between monopoly, duopoly or government production should be endogenous as a function of bids. They also discuss the trade-off between revenue of privatization and assigning several firms a positive market share which induces competition between firms.

3   Preliminaries

3.1   Agents

A buyer, which we denote with subindex 0, wishes to buy a certain quantity of a good. This good is perfectly divisible and the total amount to be procured is

normalized to 1. The buyer faces a budget constraint B_0, which is considered fixed throughout the paper.

There are two sellers (or providers) who are able to provide the buyer's needs entirely or in part. Let $\mathcal{I} = \{d, \ f\}$ denote the set of potential providers, where d refers to the *domestic* provider and f to the *foreign* one. When providing the whole amount seller $i \in \mathcal{I}$ incurs (total) cost c_i. We assume throughout the paper that

$$0 \leq c_f < c_d < B_0. \tag{1}$$

Both firms are able to provide the whole amount at constant marginal costs without exceeding the buyer's budget; and–given that we are interested in affirmative action–the domestic firm is less efficient than the foreign provider.

3.2 Procurement Share Auctions

We start by proposing a general notion of *procurement share auctions*, that assigns procurement shares endogenously depending on a vector of bids $P = (p_d, p_f)$. We interpret p_i as the price of providing the procurer's demand completely. Notice that the following definition *a priori* does not require positive minority representation.

Definition 1 *A procurement share auction is a function*

$$\Psi : \mathbb{R}_+^3 \to \mathbb{R}_+^2$$

such that for each budget constraint[11] $B \geq 0$, and any $P \in \mathbb{R}_+^2$,

$$\Psi_d (B, P) + \Psi_f (B, P) \in \{0, 1\} \, .$$

We consider procurement auctions satisfying two requirements. On one hand, a feasibility condition: any provider proposing a price higher than the buyer's budget constraint will not be a supplier. On the other hand, a monotonicity condition: the sellers' shares are responsive to the prices announced. Formally,

(i) for each $i \in \mathcal{I}$, if $B < p_i$ then $\Psi_i (B, P) = 0$; and

(ii) for each $P \in \mathbb{R}_+^2$, any $i \in \mathcal{I}$, and $j \neq i$, if $p_i \leq p_j$ then $\Psi_i (B, P) \geq \Psi_j (B, P)$.

When seller i provides a procurement share Ψ_i of the total quantity, his profits are

$$U_i (B, P) = \Psi_i (B, P) [p_i - c_i] \, . \tag{2}$$

Given a procurement auction Ψ, and the budget constraint B_0, we can describe the normal form game $\Gamma = \{\mathcal{I}, S, U, \Psi\}$ in which the set of agents is $\mathcal{I}$, each agent's strategy space is the interval $[0, B_0]$ and provider i's utility follows expression (2) with $B = B_0$.

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[11] In most of our analysis the budget constraint B will be assumed to coincide with the real constraint B_0.

3.3 Procurement Objectives

As explained in the Introduction, conventional wisdom holds that there exists a trade-off between expenditure minimization, on one hand, and minority representation, on the other. A main result of this paper is to show that such a trade-off does not need to exist. We derive this result under the assumption that the buyer only values expenditure minimization and show that this can justify assigning the domestic firm a positive share. Minority representation is realized as a byproduct. This assumption makes it more difficult to reconcile the objectives, and so makes our results more striking. We discuss different ways to model the benefits from minority representation in the concluding section.

In order to measure expenditure minimization consider a procurement share vector $\psi = (\psi_d, \psi_f) \geq 0$, with $\psi_d + \psi_f = 1$; and a price vector $P = (p_f, p_d)$. The savings function associates to each pair (ψ, P) and budget constraint B, the value[12]

$$S_0 (B, \psi, P) = B - \psi \cdot P = \psi_d (B - p_d) + \psi_f (B - p_f). \tag{3}$$

As already mentioned, procurement auctions are usually organized as first-price auctions. Therefore, we will compare the procurement costs of our mechanism to the cost of the domestic firm, which is also the cost of a Vickrey auction.[13] Throughout the paper we measure minority representation by the share of the domestic provider.

4 Contested Procurement Auction

In this section we propose a procurement auction inspired in a classical proposal for bankruptcy situations: the *Contested Garment Principle*. (See Dagan, 1996 for an analysis of this bankruptcy solution).[14] From a normative point of view, this auction fulfills not only the monotonicity condition proposed in the previous section, but is also continuous.[15] From a positive point of view, we will see, that it admits a unique Nash equilibrium.

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[12]To be fully precise, as equation (3) is maximized when there is no procurement, that is $\psi_d + \psi_f = 0$, we postulate in this case $S_0 (B, \psi, P) = 0$.

[13]Given that we start by considering a complete information environment, there is the technical issue that the first-price sealed-bid auction mechanism under complete information does not possess a pure strategy Nash equilibrium. The focus on the cost of the domestic firm, however, can be justified as the limiting equilibrium of a modified game with smallest monetary unit when the grid size goes to zero and ties are randomly broken, see Alcalde and Dahm (2011a).

[14]Following the contested garment principle a creditor (provider) receives half the estate E plus half of the difference between his claim c_i and the opponent's claim c_j: $(E/2 + (c_1 - c_2)/2, E/2 + (c_2 - c_1)/2)$. To see the relationship to our procurement auction set the estate equal to one and define a claim as the savings that each provider offers $c_i = B_0 - p_i$. Supposing $c_1 > c_2$, procurement shares are $(1/2 + (c_1 - c_2)/(2c_1), 1/2 + (c_2 - c_1)/(2c_1))$. So the difference is that our procurement auction depends on the percentage mark-up $(c_1 - c_2)/c_1$ rather than the absolute mark-up $c_1 - c_2$. This implies homogeneity of degree zero in claims which seems to be a desirable property in the context of procurement auctions.

[15]The auction is continuous everywhere except when $p_f = p_d = B_0$.

Definition 2 *Given B_0 and $P = (p_d, p_f)$, the contested procurement auction Ψ^{cp} associates to provider i the share*

$$\Psi_i^{cp}(B_0, P) = \frac{B_0 - 2p_i + \max\{p_d, p_f\}}{2[B_0 - \min\{p_d, p_f\}]}.$$

To be fully precise, when $p_d = p_f = B_0$ the auction assigns equal shares, as required by the monotonicity condition. Remember also that the feasibility condition restricts providers to propose prices that do not exceed the budget B_0.

We refer to the game associated to Ψ^{cp}, namely $\Gamma^{cp} = \{\mathcal{I}, S, U, \Psi^{cp}\}$, as contested procurement. Our first result determines the unique Nash equilibrium of this game. Since this result is a corollary of Theorem 3, its proof is omitted.

Theorem 1 *Game Γ^{cp} has a unique Nash equilibrium, P^*, described by*

$$
\begin{aligned}
p_d^* &= \frac{B_0 + c_d}{2}, \text{ and} \\
p_f^* &= B_0 - \frac{(B_0 - c_f)^{\frac{1}{2}}(B_0 - c_d)^{\frac{1}{2}}}{2}.
\end{aligned}
$$

Let us observe that, at equilibrium, the domestic provider obtains a positive profit. Moreover, the buyer's expected savings are[16]

$$S_0(B_0, P^*) = \frac{B_0 - c_d}{4}\left[2\left(\frac{B_0 - c_f}{B_0 - c_d}\right)^{\frac{1}{2}} + \left(\frac{B_0 - c_d}{B_0 - c_f}\right)^{\frac{1}{2}} - 1\right].$$

Note that, in particular, if

$$\frac{B_0 - c_f}{B_0 - c_d} > \left(\frac{21}{8} + \frac{5}{8}\sqrt{17}\right) \approx 5.2019 \tag{4}$$

it holds that $S_0(B_0, P^*) > B_0 - c_d$. This implies the following result.

Corollary 1 *When the cost difference between providers is large enough, the buyer prefers contested procurement to a second-price reverse auction.*

The intuition for this result is as follows. The domestic provider has a strong incentive to participate and propose a price p_d below the budget B_0, since this guarantees strictly positive profits. As a result competition is enhanced because the foreign supplier faces a new trade-off compared to a standard auction: increasing the price increases the mark-up over costs but it decreases the share. When competition is weak because the cost difference of providers is large, increasing the share is an important consideration for the foreign supplier. Consequently contested procurement performs better than a second-price reverse auction.

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[16]For simplicity, given a procurement auction Ψ, we denote throughout the paper the savings by $S_0(B_0, P)$ instead of $S_0(B_0, \Psi(B_0, P), P)$.
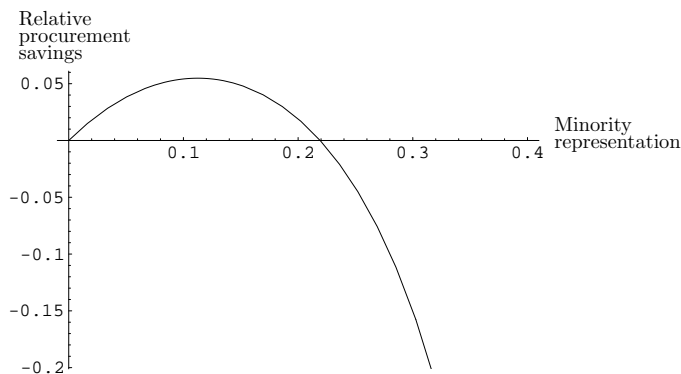
Figure 1: Cost savings relative to standard auction.

5 Contested Procurement with an Informed Auctioneer

As we have seen in the previous section contested procurement has the potential to reconcile the aims of expenditures minimization and minority representation. This result hinges, however, on the cost difference between providers being large enough. In this section we investigate how the buyer can take advantage of an informed auctioneer who knows the relevant market well. Such an auctioneer has more information about the cost structure of the providers and can determine the budget constraint strategically.

We will assume the extreme case in which the auctioneer is completely informed about the providers' cost. Notice, however, that even under the assumption that the auctioneer does not know the costs of suppliers, a reserve price can be determined. For instance, Krasnokutskaya (2011) reports that the Michigan Department of Transportation conducts procurement auctions for construction and maintenance of most roads within Michigan through a first-price sealed-bid auction. It also derives a reserve price for every project which is based on an engineer's assessment of the work required to perform each task, prices derived from the winning bids for similar projects, adjustment through a price deflator and the requirement that the winning bid should be lower than 110% of the engineer's estimate.

Before formalizing the strategic analysis of the auctioneer's choice of the budget constraint, it is useful to describe the trade-off between expenditures minimization and minority representation further. Notice that when the declared budget constraint does not coincide with B_0, expression (3) does not measure the savings correctly. For this reason we measure expenditures minimization by total procurement cost of the contested auction.

Figure 1 represents the trade-off between expenditures minimization and

minority representation. The latter is measured on the abscissa, while the ordinate indicates the cost difference between a standard auction and the contested procurement auction as a proportion of the cost difference of the providers. The figure shows that the contested procurement mechanism does not perform worse than a standard auction until a minority representation of roughly 22% is reached. Given that we assume that the buyer wishes to minimize procurement costs, she should choose the reserve price B in such a way that minority representation is roughly 11% and procurement cost savings with respect to the sealed-bid first-price auction is roughly 5,5%.

It is, however, worth pointing out that in reality buyers with affirmative action goals accept to trade-off some procurement costs for minority representation. The figure shows that higher minority representations, exceeding 11%, can only be reached when cost savings are lower (and eventually negative). How this trade-off is resolved depends on the preferences of the buyer and will vary with the context. For example, California's Disabled Veteran Business Enterprise and Small Business Certification Programs have with three percent and twenty-five percent, respectively, different aims for minority representation. Notice that these values are close to those for which the contested procurement auction performs better than a standard auction.

We formalize now the auctioneer's choice of the budget constraint. Consider the following sequential game:

(1) **Budget Stage:** The auctioneer selects $B \in [0, B_0]$,[17] which becomes known to the suppliers.

(2) **Contest Stage:** Providers choose simultaneously prices $p_i \in [0, B) \cup \{\emptyset\}$, $i \in \mathcal{I}$. Choosing $p_i = \emptyset$ allows providers to reject supplying the buyer when the budget is very low.[18]

Given the actions selected by all the players $(B; p_d, p_f)$, the mechanism proceeds as follows:

(A) If there is a provider $i \in \mathcal{I}$ such that $p_i = \emptyset$, then there is no procurement and providers and auctioneer obtain zero payoffs.[19]

(B) Otherwise, i.e. if both providers propose a price not exceeding B, then the payoffs are:

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[17] We suppose for simplicity that $B \leq B_0$. This makes sense when the auctioneer is required to document that the buyer is able to procure at this price. When B_0 is very low, there could be an incentive to choose $B > B_0$ but it is straightforward to take that into account in the proof of Theorem 2.

[18] We exclude for simplicity that $p_i = B$. This assures that the domestic firm has a unique best reply when $B = c_d$ and does not reduce the set of equilibria.

[19] This assumption is a simple way of focusing on the range for B that allows minority representation. By assuming that the buyer is only interested in cost minimization we have made our results very strong. Assuming that when $p_i = \emptyset$ the other firm is the sole provider opens the door for the uninteresting choice $B = c_f$.
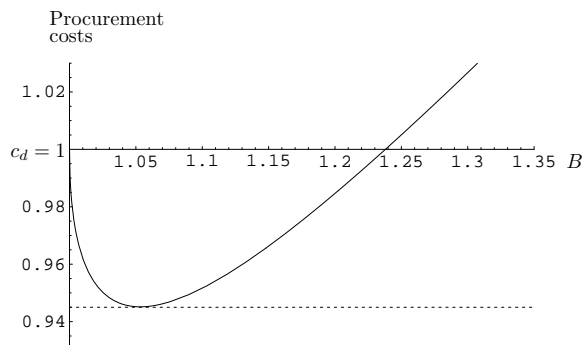
Figure 2: Optimal B when $c_f = 0$ and $c_d = 1$.

(a) *For the auctioneer*

$$S_0\left(B_0; \Psi^{cp}\left(B; P\right); P\right) = B_0 - p_d \Psi_d^{cp}\left(B; p_d, p_f\right) - p_f \Psi_f^{cp}\left(B; p_d, p_f\right).$$

(b) *For each provider $i \in \mathcal{I}$*

$$\Pi_i\left(B_0; \Psi^{cp}\left(B; P\right); P\right) = \Psi_i^{cp}\left(B; p_d, p_f\right)\left[p_i - c_i\right].$$

Let us denote by Γ^{2Scp} the game previously described. Our solution concept is Subgame Perfect Nash Equilibrium, SPNE henceforth. We also focus on undominated prices (in pure strategies) at the Contest Stage.

Theorem 2 *Game Γ^{2Scp} has a unique[20] SPNE.*

The proof of Theorem 2 is relegated to Appendix A but an intuition for this result can be gained with the help of Figure 2, which indicates procurement costs as a function of different budget choices, assuming that $c_d = 1$ and $c_f = 0$. Notice first that in order to obtain positive profits the auctioneer must propose a budget that is so high that both providers can submit a profitable bid. That is, the budget constraint must exceed c_d. This implies by Theorem 1 that for each budget the second stage has a unique Nash equilibrium. Moreover, procurement costs are a strictly convex function of the budget, and approach c_d (from below) as B tends to c_d, see Figure 2. Therefore, at the first stage there is an optimal value $\hat{B}$ different from c_d.

This has two implications. On the one hand, whenever $B_0 > \hat{B}$, the cost of employing the contested procurement auction and revealing the budget constrained truthfully exceeds the cost of the same auction when the budget is strategically chosen.

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[20]To be fully precise, there are several payoff-equivalent SPNE. The multiplicity arises because the foreign firm might choose any undominated bid when the auctioneer chooses a budget that lies between the costs of providers. On the equilibrium path such a budget is not chosen. Thus, there is a unique $\left(B^*; p_d^*, p_f^*\right)$ selected at equilibrium.

Corollary 2 *Let $\left(p_d^*, \, p_f^*\right)$ be the unique Nash equilibrium for Γ^{cp}. Similarly, let $\left(\hat{B}; \, \hat{p}_d, \, \hat{p}_f\right)$ be selected at a SPNE for Γ^{2Scp}. Then*

$$\hat{p}_d \Psi_d^{cp}\left(\hat{B}; \hat{p}_d, \hat{p}_f\right) + \hat{p}_f \Psi_f^{cp}\left(\hat{B}; \hat{p}_d, \hat{p}_f\right) \le p_d^* \Psi_d^{cp}\left(B_0; p_d^*, p_f^*\right) + p_f^* \Psi_f^{cp}\left(B_0; p_d^*, p_f^*\right).$$

On the other hand, the cost of a standard auction exceeds the cost of the contested procurement auction when the budget is strategically chosen. In the example of Figure 2 a standard auction implies procurement costs of 1. The contested procurement auction with a strategically chosen budget has costs of 0.945; the cost reduction is 5.5%, as mentioned before.

Corollary 3 *The buyer prefers to employ Ψ^{2Scp} and choose the budget strategically, rather than to have a second-price reverse auction.*

6 Generalized Contested Procurement

For the remainder of the paper we return to the assumption that the buyer's budget constraint is truthfully revealed. In this section we introduce a generalized version of contested procurement. This defines a family of procurement auctions characterized by a single parameter. We offer a strategic analysis of this family and investigate the buyer's optimal choice of a member of this family.

6.1 Definition and Properties

Consider the following generalization of Definition 2.[21]

Definition 3 *Let α be a positive real number. Given B_0 and $P = (p_d, \, p_f)$, the generalized contested procurement auction $\Psi^{cp\alpha}$ is defined as*

$$\Psi^{cp\alpha}\left(B_0, \, P\right) = \begin{cases} \left(\frac{2(B_0-p_d)^\alpha - (B_0-p_f)^\alpha}{2(B_0-p_d)^\alpha}, \, \frac{(B_0-p_f)^\alpha}{2(B_0-p_d)^\alpha}\right) & \text{if } p_d \le p_f < B_0 \\[2ex] \left(\frac{(B_0-p_d)^\alpha}{2(B_0-p_f)^\alpha}, \, \frac{2(B_0-p_f)^\alpha - (B_0-p_d)^\alpha}{2(B_0-p_f)^\alpha}\right) & \text{if } p_f \le p_d < B_0 \end{cases}$$

where the first component refers to the domestic and the second to the foreign provider.

Following Subsection 3.2, generalized contested procurement is defined as the game associated to $\Psi^{cp\alpha}$, namely $\Gamma^{cp\alpha} = \{\mathcal{I}, \, S, \, U, \, \Psi^{cp\alpha}\}$. Notice that this generalized mechanism fulfills several interesting properties.

On one hand, it defines a family of auctions characterized by a single parameter. It is straightforward to verify that this parameter measures the elasticity of a supplier's procurement share with regard to his price. Generalized contested

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[21]As before, when $p_d = p_f = B_0$, the auction assigns equal shares. Remember also that the feasibility condition restricts providers to propose prices that do not exceed the budget B_0.

procurement generalizes the first-price reverse auction, because it encompasses it as a polar case (for $\alpha \to \infty$; the other polar case is egalitarian assignment irrespective of bids for $\alpha \to 0$). Contested procurement investigated in Section 4 is obtained for $\alpha = 1$.

On the other hand, it can easily be verified that the auction fulfills the following desirable properties. First, it is anonymous so that shares are independent of providers' labels and depend only on prices. Second, it is continuous.[22] Third, the shares are monotonic in bids. Fourth, the assignment is homogeneous, that is, it is independent of the numéraire employed.

6.2   Strategic Analysis

We establish now that the game $\Gamma^{cp\alpha} = \{\mathcal{I}, S, U, \Psi^{cp\alpha}\}$ has a unique Nash equilibrium. A proof of this result is relegated to Appendix B but it is instructive to notice that the equilibrium prices of providers can be computed in a simple sequential way. First, the domestic provider computes his optimal price conditional on not being lower than the price of the foreign supplier

$$p'_d = \left. \begin{array}{cc} \arg\max & U_d\left(B_0; p_d, p_f\right) \\ \text{s.t.} & p_f \le p_d. \end{array} \right\} \tag{5}$$

It is important that it turns out that p'_d is independent of p_f, except for the fact that the latter must not be higher than p'_d. Once the optimal price of the domestic firm is anticipated, the foreign provider computes his optimal price, under the assumption not to exceed p'_d

$$p'_f = \left. \begin{array}{cc} \arg\max & U_f\left(B_0; p'_d, p_f\right) \\ \text{s.t.} & p_f \le p'_d. \end{array} \right\} \tag{6}$$

We have the following result.

Theorem 3 *For each $\alpha > 0$, game $\Gamma^{cp\alpha} = \{\mathcal{I}, S, U, \Psi^{cp\alpha}\}$ has a unique Nash equilibrium.*

6.3   Robustness

We next offer a robustness analysis of the equilibrium. We show that an iterative procedure of deleting dominated strategies converges to a unique strategy for each player. This strategy is the one employed at the unique equilibrium in Theorem 3. Therefore, in the terminology of Moulin (1979) the game is dominance solvable.

This result requires an additional assumption. The reason for this assumption is technical; to avoid an open set problem at the starting point of the iterative procedure. This assumption is a mild strengthening of the feasibility condition in Subsection 3.2 and can be motivated as follows. Suppose there is a

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[22]To be fully precise, it is continuous everywhere except when $p_f = p_d = B_0$.

smallest monetary unit and the convention that providers have to bid strictly below the budget constrained B_0. To fix ideas, imagine that $B_0 = \$1,000,000.01$, and that no agent will ask for more than $1 million.

Formally, any price above some threshold B', close to B_0 implies that the provider's share is zero.[23] Strengthening the feasibility condition in Subsection 3.2 in this way implies the following slight change to the generalized contested procurement auction Ψ^{cp_α} from Definition 3. Let $\Psi^{cp_\alpha B'}$ be the function that to each B_0, $P = (p_d, p_f)$ and $B' < B_0$ associates the vector

$$\Psi^{cp_\alpha B'}(B_0; p_d, p_f) = \begin{cases} \left(1 - \frac{(B_0 - p_f)^\alpha}{2(B_0 - p_d)^\alpha}, \frac{(B_0 - p_f)^\alpha}{2(B_0 - p_d)^\alpha}\right) & \text{if } p_d \leq p_f \leq B' \\ \left(\frac{(B_0 - p_d)^\alpha}{2(B_0 - p_f)^\alpha}, 1 - \frac{(B_0 - p_d)^\alpha}{2(B_0 - p_f)^\alpha}\right) & \text{if } p_f \leq p_d \leq B' \\ (1, 0) & \text{if } p_d \leq B' < p_f \\ (0, 1) & \text{if } p_f \leq B' < p_d \\ (0, 0) & \text{if } B' < \min\{p_d, p_f\}. \end{cases}$$

As a consequence, the induced game $\Gamma^{cp_\alpha B'} = \left\{\mathcal{I}, S, U, \Psi^{cp_\alpha B'}\right\}$ is a slight modification of the original game Γ^{cp_α}. Although the games Γ^{cp_α} and $\Gamma^{cp_\alpha B'}$ are not the same, provided that B' is arbitrarily close to B_0, the equilibrium predictions for $\Gamma^{cp_\alpha B'}$ should also be reasonable for the game Γ^{cp_α}. With this in mind the implication of the next result, whose proof can be found in Appendix B, is that Theorem 3 is robust.

Theorem 4 *For any $\alpha > 0$ and each $B' < B_0$, the game $\Gamma^{cp_\alpha B'}$ is dominance solvable.*

6.4 Optimal Choice of the Elasticity

In this subsection we investigate the buyer's optimal choice of the elasticity of the procurement auction. To do so we relate the buyer's savings to α.

To gain an intuition notice that the solution of problem (5) is[24]

$$p'_d = \frac{B_0 + \alpha c_d}{\alpha + 1} \tag{7}$$

and that the solution of problem (6) must satisfy

$$\left(B_0 - p'_f\right) = \frac{1}{2}\left(\frac{B_0 - p'_d}{B_0 - p'_f}\right)^\alpha \left[B_0 + (\alpha - 1)p'_f - \alpha c_f\right]. \tag{8}$$

Analyzing the polar cases yields the following intuitive facts:

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[23] Even though Theorem 4 is true for any $B' < B_0$, for interpretative purposes we will consider that B' is very close to B_0.

[24] Equations (7) and (8) are obtained from the standard first order conditions relative to programs (5) and (6) respectively.

(i) When $\alpha \to 0$ the allocation procedure is perfectly inelastic and assigns half of the demand to each supplier independent of prices. The equilibrium converges to the one in a *fair lottery with an indivisible prize* and each procurer asks for the whole budget, i.e.

$$\lim_{\alpha \to 0} p'_f(\alpha) = \lim_{\alpha \to 0} p'_d(\alpha) = B_0.$$

(ii) When $\alpha \to \infty$ the allocation procedure is perfectly elastic and converges to a *first-price reverse auction*, i.e.[25]

$$\lim_{\alpha \to \infty} p'_f(\alpha) = \lim_{\alpha \to \infty} p'_d(\alpha) = c_d; \text{ and}$$
$$\lim_{\alpha \to \infty} \Psi_f^{cp\alpha}(B_0; P') = 1, \ \lim_{\alpha \to \infty} \Psi_d^{cp\alpha}(B_0; P') = 0.$$

These two facts seem to indicate that any generalized contested procurement induces an average price higher than the second lowest cost. This intuition is reinforced by inspection of equation (7), because–although p'_d is a strictly decreasing function of α–the domestic provider's price is always higher than his costs. The foreign supplier could, thus, choose a price between the domestic provider's price and costs. Nevertheless, as the next theorem shows, this intuition is not true. The reason is that for high α equation (8) introduces a concave relationship between p'_f and α. Taking into account the extreme values of p'_f – i.e. B_0 for $\alpha \to 0$, and c_d for $\alpha \to \infty$ – this concavity implies the existence of a value $\hat{\alpha}$ whose associated p'_f is a minimum, and thus is lower than c_d. It turns out that the foreign supplier might be induced to make a very competitive bid. As a result, the following statement is true.

Theorem 5 *There is $\alpha > 0$ such that, at equilibrium, $S_0(B_0, P') > B_0 - c_2$.*

A formal proof of Theorem 5 can be found in Appendix B. The theorem indicates a trade-off in the selection of the elasticity. Given a pair of bids, lowering the elasticity raises the market share of the domestic supplier. This implies, on one hand, to pay for the total share of the domestic firm a relatively high price. But on the other hand, it forces the foreign supplier to bid lower than he otherwise would, driving down the payment for this market share. For moderate choices of the elasticity, the overall effect is to induce very competitive procurement. But for extreme values this is not true. With very egalitarian policies the share of the domestic firm is large and procurement becomes costly. When the elasticity is high, the foreign supplier wins a large share and does not need to bid very competitive, as in a standard auction.

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[25]First-price auctions have no Nash equilibrium in pure strategies under complete information. Nevertheless, the following strategies can be considered the "intuitive solution" and motivated by imposing the existence of a smallest monetary unit; see Alcalde and Dahm (2011a) for more details. Given that our assignment process admits a probabilistic interpretation (assuming that bidders are risk neutral), a by-product of our model is to provide a different way to restore the equilibrium in the first-price auction under complete information: As the noise in the assignment process goes to zero, the intuitive solution is obtained.

7 Generalized Contested Procurement with Private Information

The analysis so far has considered the polar case in which providers are completely informed about each other's characteristics. We relax this assumption now and focus on the other polar case in which each supplier only has (private) information about his own costs, but does not know the costs of his rival. We propose to implement generalized contested procurement by means of a (continuous-time) sequential mechanism. During the course of the auction information about rivals is not needed because all the relevant information is revealed. As a result at the unique equilibrium the providers' actions coincide with those in the complete information environment of Section 6.

For α given, the mechanism proceeds as follows. At any $t \in [0, 1)$ each provider i knows his own costs c_i and the messages selected by both sellers at any $t' < t$. This information structure is common knowledge. Both providers simultaneously select their messages $m_i(t) \in \{0, 1\}$, where 0 is interpreted as 'continuing' and 1 as 'stopping'. At $t = 1$ the auction is over and we interpret this as both sellers choosing message $m_i(1) = 1$. Given the providers' message functions $m_i : [0, 1] \to \{0, 1\}$, let $\tilde{t}_i$ be the first time in which i's message is 1.[26] With this the price paid for seller i's share is

$$\tilde{p}_i = \left(1 - \tilde{t}_i\right) B_0.$$

Given the sellers' prices $\tilde{P} = (\tilde{p}_d, \tilde{p}_f)$, procurement shares are assigned according to Ψ^{cp_α}, and thus provider i's utility is

$$\Pi_i = (\tilde{p}_i - c_i) \Psi_i^{cp_\alpha} \left(B_0; \tilde{P}\right).$$

Let Γ^{ctcp_α} denote the continuous time generalized contested procurement game above described. Let us observe that a strategy for provider i, say s_i, must prescribe, for each $t \in [0, 1)$ a message m_i that should depend not only on t, but also on the messages set by both providers at any $t' < t$.

We explore now the behavior of suppliers in this bidding game for a given α. Consider seller i and suppose that his opponent's message is 0 for any t lower than some $\bar{t}$. This implies that seller i also selects, at time $\bar{t}$ a message $m_i(\bar{t}) = 0$, except if

$$(1 - \bar{t}) B_0 = \frac{B_0 + \alpha c_i}{\alpha + 1}. \tag{9}$$

The reason is that, given that his opponent has not yet fixed a price, by setting a message equal to 1, supplier i becomes the seller asking for the highest price. As we have seen in Section 6, when provider i asks for the highest price he selects p_i as established in equation (9).

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[26]I.e. $\tilde{t}_i$ is such that $m(\tilde{t}_i) = 1$, and $m(t) = 0$ for all $t < \tilde{t}_i$.

For $\hat{t}$ given, let us assume that both agents have chosen message $m_d(t) = m_f(t) = 0$ for each $t < \hat{t}$, and that provider j selects $m_j(\hat{t}) = 1$. In such a case, any optimal decision for provider $i \neq j$ involves that $m_i(t) = 0$, for any $t < \breve{t}_i$ and $m_i(\breve{t}_i) = 1$, where[27]

$$\left. \begin{array}{rl} \breve{t}_i = & \arg\max \quad [(1-t)B_0 - c_i]\,\Psi_f^{cp\alpha}\left(B_0; (1-\hat{t})B_0, (1-t)B_0\right) \\ & \text{s.t.} \quad \hat{t} \leq t \leq 1 \end{array} \right\} \quad (10)$$

Note that these arguments allow us to say that the strategies used by providers to select their messages are dominant strategies. The next theorem, whose proof is omitted,[28] establishes the *equivalence* of the equilibrium procurement shares under $\Gamma^{cp\alpha}$ and $\Gamma^{ctcp\alpha}$.

Theorem 6 *Given a profile of strategies $\left(s'_d, s'_f\right)$ for $\Gamma^{ctcp\alpha}$, let m'_i denote the seller i's message function induced by $\left(s'_d, s'_f\right)$. Then, $\left(s'_d, s'_f\right)$ is an equilibrium for $\Gamma^{ctcp\alpha}$, in dominant strategies, if, and only if,*

(a) $m'_d(t) = 0$ for each $t < t_d$, and $m'_d(t_d) = 1$; and

(b) $m'_f(t) = 0$ for each $t < t_f$, and $m'_f(t_f) = 1$, where

$$\left. \begin{array}{l} t_d = \frac{\alpha}{\alpha+1}\frac{B_0 - c_d}{B_0}, \text{ and} \\ t_f \text{ maximizes } [(1-t)B_0 - c_f]\,\Psi_f^{cp\alpha}\left(B_0; (1-t_d)B_0, (1-t)B_0\right). \end{array} \right\} \quad (11)$$

Note that, as a consequence of Theorem 6, we have the following.

Corollary 4 *Let $\left(p'_d, p'_f\right)$ be the equilibrium prices when agents play the game $\Gamma^{cp\alpha}$, and $(\hat{p}_d, \hat{p}_f)$ those established when playing $\Gamma^{ctcp\alpha}$. Then, $\left(p'_d, p'_f\right) = (\hat{p}_d, \hat{p}_f)$.*

8 Conclusion

This paper is a step toward understanding under what conditions procurement can reconcile the conflicting aims of expenditures minimization and minority representation. Rather than reviewing our results, in this concluding section we discuss some of our assumptions most of which indicate questions for future research.

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[27] Note that, since $\Psi^{cp\alpha}$ is symmetric, i.e $\Psi_d^{cp\alpha}\left(B_0, p_d, p_f\right) = \Psi_f^{cp\alpha}\left(B_0, p_f, p_d\right)$, there is no loss of generality in considering that the supplier described in Expression (10) below is the foreign seller.

[28] A proof which is not intended for publication can be found in Appendix C.

Practicability of an improvement over a standard auction

We have seen that the conflicting procurement objectives can be reconciled when the buyer or her auctioneer chooses either the budget constraint strategically (Section 5) or the elasticity of the procurement auction (Subsection 6). In both cases the optimal choice requires information on providers' costs.

It is not unreasonable that some information can be obtained. We have already discussed that the Michigan Department of Transportation derives cost estimates for construction and maintenance of most roads within Michigan (see Krasnokutskaya, 2011 and Section 5). Also when procurement auctions take place repeatedly the buyer might have experience with the suppliers and choose a mechanism that performs well. In other instances, however, the buyer might have less reliable estimates of the bidders' costs. But our findings are robust.

In the context of Section 5, imprecise estimates of providers' costs combined with the desire "to be on the safe side" could lead to a budget choice that is higher than the optimal budget. It turns out, however, that procurement costs are lower than under a second price auction provided $(3.2725c_d - c_f)/2.2725 \geq B_0$ holds. This threshold is increasing in the cost difference that justifies affirmative action. In the extreme when $c_f = 0$, the mechanism outperforms a second-price reverse auction until the budget is 44% higher than the domestic firm's cost.

In Subsection 6 when the elasticity is selected,—rather than searching for the optimal value of the elasticity—the buyer might choose among a small set of elasticities and still improve upon a standard auction. Consider the following example in which again $c_f = 0$ and the budget is normalized to 1. To make affirmative action meaningful suppose that c_d is larger than 0.25. It can be shown that it is enough to choose among a low, intermediate and high elasticity. More precisely, it is enough to choose (roughly) for values between 0.25 and 0.65 an elasticity of 15, for values between 0.65 and 0.9 an elasticity of 5, and for values between 0.9 and 1 an elasticity of 1. Figure 3 shows on the abscissa the costs of the domestic provider. The ordinate displays the savings associated to a standard auction (dotted line) and to the contested procurement auction depending on the choice among these elasticities.

The multi-provider case

Our analysis restricts to two providers and a natural extension is to consider the multi-provider case. Alcalde and Dahm (2011b) explore such a setting and propose an extension of the procurement share auction which maintains the philosophy underlying the *contested procurement auction*. Alcalde and Dahm (2011b) show that when the sellers' provision costs are relatively homogeneous but different the equilibrium might not be unique. As a consequence equilibrium prices might not be not accurately correlated with the cost of providers; i.e. it might be the case that $c_1 < c_2 < c_3$ and, at some equilibrium, $\hat{p}_3 < \hat{p}_1 < \hat{p}_2$. For these reasons Alcalde and Dahm (2011b) pursue a different route and propose to reduce the original multi-provider problem to the two-provider case. Roughly
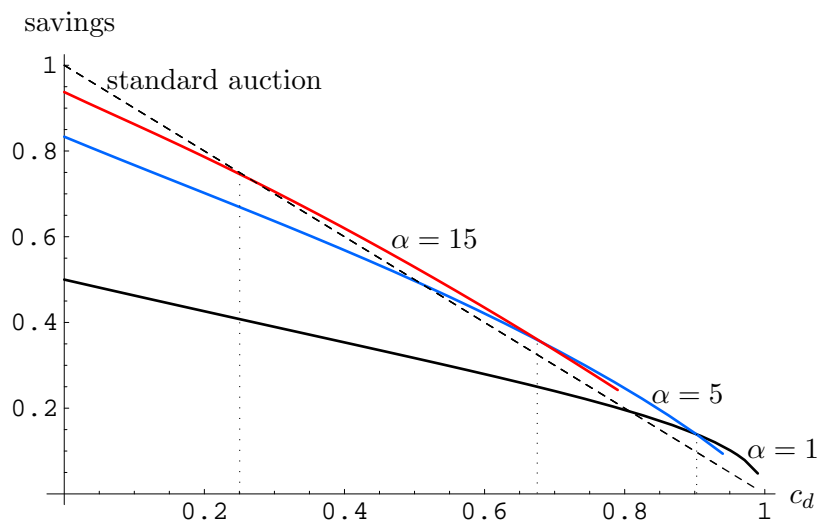
savings

Figure 3: Example of savings for $c_f = 0$ and $B_0 = 1$.

speaking the procedure is as follows. The *potential providers* fix prices and the two lowest bidders compete in a contested procurement auction in which the relevant budget constraint is endogenously determined through the prices of the remaining high bidders. From the perspective of the present paper the extension to multiple providers does not seem to yield new insights into the trade-off between expenditure minimization and minority representation.

More general cost structures

An interesting question for future research considers a setting in which the technologies of providers are not (necessarily) linear. On one hand, the introduction of economies of scale should make it more difficult to reconcile expenditure minimization and minority representation. This does, however, not necessarily mean that minority representation is undesirable. For example, for influenza vaccines Scherer (2007) analyzes the trade-off between economies of scale and protection against stochastic shortage risk through multiple sourcing. He concludes that for plausible scenarios multiple sourcing yields more benefits than costs. On the other hand, it seems that our results rely on the hypothesis of *constant average cost*. Therefore, an interesting question is to consider a richer structure of the technologies of suppliers and to explore whether the contested procurement auction can be redefined in such a way that in equilibrium comparable results are obtained.

The optimal procurement share auction

We have proposed a particular procurement share auction and shown that it has remarkable properties. We have not derived an "optimal" procurement share auction. An interesting question is whether there are other procurement share auctions that have similar properties to the one analyzed here. From the outset it is clear that the basic trade-off that we introduce—increasing the price increases the mark-up over costs but it decreases the share—does not depend on the particular functional form of contested procurement. For a related class of forward auctions Yates (2011) establishes the existence of a unique equilibrium. His analysis, however, also seems to indicate that for other functional forms of share auctions the remarkable properties of tractability and robustness of contested procurement are lost.

Valuing minority representation

The present paper reaches stark results because contested procurement outperforms standard auctions although the buyer does not put any value on minority representation. In reality, however, the seller will be willing to trade-off some savings for minority representation. For example, California's Certification Programs establish minimum market share goals for designated bidders, who are the target of the affirmative action.

A very simple way to extend our analysis to such a setting is to generalize equation (3) to a CES utility function, as in

$$U_0\left(B_0, \Psi, P\right) = \left[\sum_{i=1}^{n} \left(\Psi_i\left(B_0 - p_i\right)\right)^\rho\right]^{\frac{1}{\rho}}, \text{ where } 0 < \rho < 1.$$

Here we define $\rho = 1 - \frac{1}{\sigma}$, where $\sigma > 1$ is the (constant) elasticity of substitution. Notice that when $\sigma \to \infty$, $U_0\left(B_0, \pi, P\right)$ tends to $S_0\left(B_0, \pi, P\right)$. On the other hand, when σ is smaller, indifference curves are strictly convex. Therefore, for the same per capita saving, two providers with the same market share are preferred to only one. It can be shown that in the context of the example displayed in Figure 3 $\sigma = 2$ pushes the utility associated to $\alpha = 1$ so much upwards that for any costs of the domestic provider contested procurement is preferred to a standard auction. Of course, this does not mean that this is the best the buyer could do and the optimal choice of the elasticity should depend on the cost structure of sellers.

When the buyer has no information on this cost structure, generalized contested procurement offers another practical way to trade-off the conflicting objectives. As different values of the price elasticity of a suppliers market share solve the trade-off in different ways, the buyer might simply choose one value once and for all.

Other approaches to putting a value on minority representation might build on a foundation for the benefits derived from it. In the Introduction we motivated these benefits as reducing the risk incurred that a provider cannot fulfill

his obligations or the effects of successful affirmative action policies. Concerning the former, one might want to distinguish between situations in which one seller is not able to provide his share and the other is or is not able to increase his share on short notice. Also, it seems reasonable that the risk depends on the firm's efficiency level, and it might be increasing in the firm's market share. Concerning the latter, often the underlying idea is that a positive market share allows the domestic firm to learn by doing and to become more efficient over time. One might thus want to evaluate the success of affirmative action policies with a dynamic model of the evolution of the domestic firm's efficiency. Denoting by c_d^t the cost of the domestic firm at time t one possibility is the following. Assume that the foreign firm's efficiency level is stable, $c_f^t = c_f^{t-1}$, and that for each t, $c_d^t \geq c_f$ implies $\Psi_d^t \leq \Psi_f^t$. Learning by doing might be captured by

$$c_d^t = \left(1 - \frac{\Psi_d^{t-1}}{\Psi_f^{t-1}} \right) c_d^{t-1} + \frac{\Psi_d^{t-1}}{\Psi_f^{t-1}} \, c_f.$$

This means that although the seller assigns a higher share to the most efficient firm, the cost difference diminishes as shares approach. Further work tackling these issues are challenging questions for future research.

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Appendix

A   Strategic Analysis of Γ^{2Scp}

The aim of this appendix is to provide a formal analysis of the sequential game Γ^{2Scp} in Section 5. We prove Theorem 2 applying backward induction arguments. Note that Corollary 2 follows from the fact that the informed auctioneer is minimizing the buyer's provision cost. Corollary 3 follows from equation (13) below.

Proof of Theorem 2

Denote by B the budget constraint selected by the auctioneer at the Budget Stage. Note that at the Contest Stage supplier i's price p_i is undominated if, and only if $p_i \in (c_i, B)$ when $c_i < B$ and $p_i = \emptyset$ otherwise.

Let us consider the following cases:

Case 1 $B \leq c_d$. Let us observe that the unique undominated price for the domestic firm is $p_d = \emptyset$, implying that there is no procurement and the auctioneer obtains zero payoffs.

Case 2 $B > c_d$. By Theorem 1 we know that for each B each seller optimally chooses

$$
\left.
\begin{array}{rcl}
p_d & = & \frac{B+c_d}{2}, \text{ and} \\[2mm]
p_f & = & B - \frac{(B-c_f)^{\frac{1}{2}}(B-c_d)^{\frac{1}{2}}}{2}.
\end{array}
\right\}
\tag{12}
$$

Moreover, we have that

$$
\begin{aligned}
S_0\left(B_0; \Psi^{cp}(B;P); P\right) & = & B_0 - p_d \Psi_d^{cp}(B; p_d, p_f) - p_f \Psi_f^{cp}(B; p_d, p_f) \\[2mm]
& \geq & B_0 - \frac{1}{2}(p_d + p_f) \geq B_0 - \frac{1}{2}(B + c_d) \\[2mm]
& \geq & \frac{1}{2}(B_0 - c_d) > 0, \text{ as } B_0 > c_d.
\end{aligned}
$$

Therefore, at the Budget Stage the auctioneer, anticipating the sellers' optimal decisions, selects a budget $B^* > c_d$ minimizing procurement cost

$$
C(B;P) = \frac{B - p_d}{2(B - p_f)} \, p_d + \left[1 - \frac{B - p_d}{2(B - p_f)}\right] p_f.
$$

Taking into account (12), this becomes

$$
\hat{C}(B) = B - \frac{B - c_d}{4}\left[2\left(\frac{B - c_f}{B - c_d}\right)^{\frac{1}{2}} + \left(\frac{B - c_d}{B - c_f}\right)^{\frac{1}{2}} - 1\right].
\tag{13}
$$

Note that its derivative, with respect to B is

$$\frac{d\hat{C}}{dB}(B) = \frac{1}{4}\left[5 - \frac{5}{2}\left(\frac{B-c_d}{B-c_f}\right)^{\frac{1}{2}} - \left(\frac{B-c_f}{B-c_d}\right)^{\frac{1}{2}} + \frac{1}{2}\left(\frac{B-c_d}{B-c_f}\right)^{\frac{3}{2}}\right].$$

The unique real root is reached at

$$B^* \simeq \frac{c_d - 0.050657c_f}{0.949343}$$

or, equivalently, when

$$\frac{B^* - c_f}{B^* - c_d} \simeq 19.741.$$

Moreover, since

$$\frac{d\hat{C}}{dB}(B) < 0 \text{ for any } B < B^*,$$

and

$$\frac{d\hat{C}}{dB}(B) > 0 \text{ for any } B > B^*,$$

we obtain that $min\{B^*, B_0\}$ is the unique minimizer for $\hat{C}(B)$. $\qquad\square$

B Analysis of Γ^{cp_α}

Proof of Theorem 3

Throughout this proof we consider a fixed parameter $\alpha > 0$. We proceed as follows. We first show in Lemma 1 that the generalized contested procurement function is continuously differentiable. Second, Lemma 2 shows that each seller's utility function is strictly quasi-concave on the bidding space. Finally, we use a result by Moulin and Shenker (1992) to construct an equilibrium and to show that this equilibrium is unique.

Auxiliary Lemmas

Lemma 1 *Let k be such that $0 < k < B_0$. The function $\phi : [0, B_0] \to \mathbb{R}_+$ described by*

$$\phi(x) = \begin{cases} \frac{1}{2}\left(\frac{B_0 - x}{B_0 - k}\right)^\alpha & \text{if } k < x \leq B_0 \\ \frac{2(B_0 - x)^\alpha - (B_0 - k)^\alpha}{2(B_0 - x)^\alpha} & \text{if } 0 \leq x \leq k \end{cases}$$

is continuously differentiable in $(0, B_0)$.

Proof. Note that, since ϕ is polynomial for $x \neq k$, we only need to check that

$$\lim_{x_0 \to k^-} \frac{\partial \phi}{\partial x}(x_0) = \lim_{x_0 \to k^+} \frac{\partial \phi}{\partial x}(x_0).$$

Now, taking into consideration that

$$\frac{\partial \phi}{\partial x}(x_0) = \begin{cases} -\frac{\alpha}{2} \frac{(B_0 - x_0)^{\alpha - 1}}{(B_0 - k)^\alpha} & \text{if } k < x_0 < B_0 \\ -\frac{\alpha}{2} \frac{(B_0 - k)^\alpha}{(B_0 - x_0)^{\alpha + 1}} & \text{if } 0 < x_0 < k \end{cases}$$

we conclude that

$$\lim_{x_0 \to k^-} \frac{\partial \phi}{\partial x}(x_0) = -\frac{\alpha}{2} \frac{(B_0 - k)^\alpha}{(B_0 - k)^{\alpha + 1}} = -\frac{\alpha}{2(B_0 - k)}, \text{ and}$$

$$\lim_{x_0 \to k^+} \frac{\partial \phi}{\partial x}(x_0) = -\frac{\alpha}{2} \frac{(B_0 - k)^{\alpha - 1}}{(B_0 - k)^\alpha} = -\frac{\alpha}{2}(B_0 - k)^{-1}.$$

$\square$

Lemma 2 *For each k, $0 < k < B_0$ and any h in $[0, B_0]$ the function $\varphi : [0, B_0] \to \mathbb{R}$ described by*

$$\varphi(x) = \begin{cases} \frac{1}{2}\left(\frac{B_0 - x}{B_0 - k}\right)^\alpha (x - h) & \text{if } k < x \leq B_0 \\ \frac{2(B_0 - x)^\alpha - (B_0 - k)^\alpha}{2(B_0 - x)^\alpha}(x - h) & \text{if } 0 \leq x \leq k \end{cases}$$

is strictly quasi-concave in (h, B_0).

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Proof.   Let ϕ be the function explored in Lemma 1. Note that, given h, φ can be described as

$$\varphi(x) = (x - h) f(x).$$

This function is twice continuous differentiable for $x \neq k$, and it is continuously differentiable for $x = k$. Let us consider the following cases:

(a) $h < x_0 < k$. Then

$$\frac{\partial^2 \varphi}{\partial x^2}(x_0) = \frac{1}{2} \alpha \frac{(B_0 - k)^\alpha}{(B_0 - x_0)^{2+\alpha}} (h + x_0 - 2B_0 + h\alpha - x_0\alpha).$$

Therefore,

$$sign \, \frac{\partial^2 \varphi}{\partial x^2}(x_0) = sign \, (h + x_0 - 2B_0 + h\alpha - x_0\alpha). \tag{14}$$

Since $h \leq x_0 < B_0$,

$$h + x_0 - 2B_0 + h\alpha - x_0\alpha =$$
$$(1 + \alpha) h + (1 - \alpha) x_0 - 2B_0 \leq 2(x_0 - B_0) < 0.$$

(b) $k < x_0 < B_0$. Then

$$\frac{\partial^2 \varphi}{\partial x^2}(x_0) = \frac{1}{2} \frac{\alpha}{(B_0 - x_0)^2} \left(\frac{B_0 - x_0}{B_0 - k} \right)^\alpha (h + x_0 - 2B_0 - \alpha h + \alpha x_0).$$

By (14) we have that $\frac{\partial^2 \varphi}{\partial x^2}(x_0) \geq 0$ only if $h - B_0 \geq B_0 - x_0 + \alpha(h - x_0)$. But, since

$$\frac{\partial \varphi}{\partial x}(x_0) = \frac{(B_0 - x_0)^\alpha}{2(B_0 - k)^\alpha} \left(1 - \alpha \frac{x_0 - h}{B_0 - x_0} \right) =$$
$$\frac{(B_0 - x_0)^\alpha}{2(B_0 - k)^\alpha} \left(\frac{B_0 - x_0 + \alpha(h - x_0)}{B_0 - x_0} \right),$$

we have that φ is decreasing at x_0 if it is not strictly concave.

Summing up, we have that φ is strictly concave at x_0 whenever

$$x_0 < \frac{2B_0 + (\alpha - 1) h}{\alpha + 1},$$

and for x_0 higher or equal the above, if φ is not strictly concave, it is strictly decreasing. Thus φ must be strictly quasi-concave. $\square$

The following lemma is equivalent to Lemma 2 in Moulin and Shenker (1992).

Lemma 3 *Let $u_1(x)$ and $u_2(x)$ be two strictly quasi-concave functions from $[a, b]$ to $\mathbb{R}$ that coincide from $c \in (a, b)$:*

$$u_1(x) = u_2(x) \,\, for \,\, all \,\, x, \,\, c \leq x \leq b.$$

Then the (unique) maximizers of $u_i(x)$, denoted x_i, are on the same side of c:

$$x_1 \geq c \Leftrightarrow x_2 \geq c; \,\, x_1 = c \Leftrightarrow x_2 = c.$$

We can now proceed to prove Theorem 3.

Proof of Theorem 3

We first show that an equilibrium (in pure strategies) exists. Let $\hat{p}_d$ and $\hat{p}_f$ be the solutions to

$$\max U_d\left(B_0; \hat{p}_f, p_d\right) \text{ s.t. } \hat{p}_f \leq p_d \leq B_0, \text{ and}$$
$$\max U_f\left(B_0; p_f, \hat{p}_d\right) \text{ s.t. } c_f \leq p_f \leq \hat{p}_d.$$

Note that the sellers' strategies can be computed using the following sequential procedure:

(a) Calculate $\hat{p}_d$ by maximizing the function

$$m_d\left(p_d\right) = \left(B_0 - p_d\right)^{\alpha}\left(p_d - c_d\right)$$

on $[c_d, B_0]$. Note that this function is strictly quasi-concave on this segment. Moreover, any solution of the problem above also maximizes the function

$$\frac{\left(B_0 - p_d\right)^{\alpha}}{2\left(B_0 - \tilde{p}_f\right)^{\alpha}}\left(p_d - c_d\right)$$

which coincides with $U_d\left(B_0; \tilde{p}_f, p_d\right)$ when $\tilde{p}_f \leq p_d$.

(b) Compute $\hat{p}_f$ by maximizing the function

$$m_f\left(p_f\right) = \frac{2\left(B_0 - p_f\right)^{\alpha} - \left(B_0 - \hat{p}_d\right)^{\alpha}}{2\left(B_0 - p_f\right)^{\alpha}}\left(p_f - c_f\right)$$

on $[c_f, \hat{p}_d]$. Note that, since m_f is strictly quasi-concave, it must have a unique maximizer.

Let us remark that $(\hat{p}_f, \hat{p}_d)$ is characterized by

$$\hat{p}_d = \frac{B_0 + \alpha c_d}{\alpha + 1}, \text{ and}$$

$\hat{p}_f$ satisfies

$$\left(B_0 - \hat{p}_f\right) = \left(B_0 - \alpha c_f + (\alpha - 1)\hat{p}_f\right)\frac{\left(B_0 - \hat{p}_d\right)^{\alpha}}{2\left(B_0 - \hat{p}_f\right)^{\alpha}}.$$

Now we show that $(\hat{p}_f, \hat{p}_d)$ described above constitutes an equilibrium for the game induced by $\Psi^{cp\alpha}$. First, note that $\tilde{p}_f$, the unique maximizer of

$$u_f\left(p_f\right) = \frac{\left(B_0 - p_f\right)^{\alpha}}{2\left(B_0 - k\right)^{\alpha}}\left(p_f - c_f\right)$$

is, for any $k < B_0$,

$$\tilde{p}_f = \frac{B_0 + \alpha c_f}{\alpha + 1} \leq \hat{p}_d. \tag{15}$$

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Therefore, since for $k = \hat{p}_d$, the function u_f and the foreign seller's utility

$$U_f\left(B_0; p_f, \hat{p}_d\right) = \begin{cases} \frac{(B_0 - p_f)^\alpha}{2(B_0 - \hat{p}_d)^\alpha}\left(p_f - c_f\right) & \text{if } \hat{p}_d < p_f \leq B_0 \\[2ex] \frac{2(B_0 - p_f)^\alpha - (B_0 - \hat{p}_d)^\alpha}{2(B_0 - p_f)^\alpha}\left(p_f - c_f\right) & \text{if } 0 \leq p_f \leq \hat{p}_d \end{cases}$$

coincide on $[\hat{p}_d, B_0]$, by Lemma 3 we have that $\hat{p}_f$ is the foreign seller's best-response to the domestic seller's strategy. Applying a similar reasoning to agent d we see that $\hat{p}_d$ is his best-response to $\hat{p}_f$.

We now deal with uniqueness of equilibrium. Proceeding by contradiction, let us assume that there is an equilibrium $(\tilde{p}_f, \tilde{p}_d) \neq (\hat{p}_f, \hat{p}_d)$. Note that, by the above arguments, we can ensure that $\tilde{p}_f > \tilde{p}_d$. Otherwise, we could argue that $(\tilde{p}_f, \tilde{p}_d)$ is not an equilibrium.

Now, since $\tilde{p}_f > \tilde{p}_d$, agent 1's strategy should be a maximizer of

$$\frac{(B_0 - p_f)^\alpha}{2\left(B_0 - \tilde{p}_d\right)^\alpha}\left(p_f - c_f\right),$$

i.e. $\tilde{p}_f$ should follow expression (15) above. Since the functions

$$u_d\left(p_d\right) = \frac{(B_0 - p_d)^\alpha}{2\left(B_0 - \tilde{p}_f\right)^\alpha}\left(p_d - c_d\right), \text{ and}$$

$$U_d\left(B_0; \tilde{p}_f, p_d\right) = \begin{cases} \frac{(B_0 - p_d)^\alpha}{2(B_0 - \tilde{p}_f)^\alpha}\left(p_d - c_d\right) & \text{if } \tilde{p}_f < p_d \leq B_0 \\[2ex] \frac{2(B_0 - p_d)^\alpha - (B_0 - \tilde{p}_f)^\alpha}{2(B_0 - p_d)^\alpha}\left(p_d - c_d\right) & \text{if } 0 \leq p_d \leq \tilde{p}_f \end{cases}$$

coincide on $[\tilde{p}_f, B_0]$, and $\hat{p}_d \geq \tilde{p}_f$ is the unique maximizer of u_d, by Lemma 3 we know that $\tilde{p}_d \geq \tilde{p}_f$. A contradiction. $\qquad\square$

Proof of Theorem 4

Again, throughout this proof we consider a fixed parameter $\alpha > 0$.

First, note that when

$$B' \leq \frac{B_0 + \alpha c_d}{1 + \alpha} \tag{16}$$

the proof is straightforward. This is because B' is a dominant strategy for the domestic provider d.

Therefore, we can conclude that

(a) If $(1 + \alpha)B' \leq B_0 + \alpha c_f$, both agents have a dominant strategy, $\hat{p}_i = B'$, and the result is proven; or

(b) If

$$c_f < \frac{(1 + \alpha)B' - B_0}{\alpha} \leq c_d,$$

then the *domestic* provider has a dominant strategy, which is $\hat{p}_d = B'$ and, therefore the *foreign* seller must have a unique undominated strategy, provided that his opponent selects B' as his proposed price. This strategy is the unique maximizer of $U_f (B_0; p_f, B')$.

Therefore, we assume in the rest of the proof that

$$\frac{B_0 + \alpha c_d}{1 + \alpha} < B' < B_0.$$

We proceed as follows. We propose an iterative procedure to sequentially reduce each seller's set of undominated strategies. Then we see that this process is convergent and that the limit set of (sequentially) undominated strategies is a singleton.

First, note that it is straightforward to see that, for each agent the set of undominated strategies is (a subset of)

$$S_i^0 = (c_i, B'].$$

The reason is that an agent selecting a strategy outside S_i^0 will obtain a non-positive utility, whereas any strategy inside S_i^0 will report a positive utility to that agent.

Now, consider the intervals $S_i^1 = [\ell_i^1, v_i^1]$, where

$$v_i^1 = \arg\max U_i (B_0, (p_i, B')) \tag{17}$$

and ℓ_i^1 is the minimal price on S_i^0 such that[29]

$$U_i \left(B_0, \left(v_i^1, c_j\right)\right) \geq U_i \left(B_0, (p_i, c_j)\right). \tag{18}$$

Note that, since U_i is continuous, strictly quasi-concave (for p_j given), and also $U_i \left(B_0, \left(v_i^1, c_j\right)\right) > 0$, we have the existence of $\ell_i^1 > c_i$ being the minimal solution for inequality (18).

Let us observe that v_i^1 dominates any strategy outside S_i^1 when the other seller is selecting a strategy on S_j^0. The reason is the following. Let us assume that provider j selects p_j, and consider the following cases:

Case 1. $p_j \leq \frac{B_0 + \alpha c_i}{1 + \alpha}$. Note that, in such a case, the best strategy for provider i is $\hat{p}_i = \frac{B_0 + \alpha c_i}{1 + \alpha}$, and his utility function, for p_j fixed, is strictly increasing on $\left[c_i, \frac{B_0 + \alpha c_i}{1 + \alpha}\right]$. Therefore, for each $p_i < \ell_i^1$

$$U_i (B_0, (p_i, p_j)) < U_i \left(B_0, \left(\ell_i^1, p_j\right)\right) = U_i \left(B_0, \left(v_i^1, p_j\right)\right).$$

Case 2. $p_j > \frac{B_0 + \alpha c_i}{1 + \alpha}$. Note that, in such a case, the best reply for seller i is a price $\hat{p}_i$ that belongs to $\left(\frac{B_0 + \alpha c_i}{1 + \alpha}, v_i^1\right]$. Moreover, for p_j given, i's utility function is decreasing on $[p_j, d]$. Therefore, for each $p_i > v_i^1$

$$U_i \left(B_0, \left(v_i^1, p_j\right)\right) > U_i (B_0, (p_i, p_j)).$$

---

[29] As usual, when we are analyzing provider i, sub-index j refers to the other supplier.

So, v_i^1 dominates any strategy outside S_i^1.

Now, let us construct, for each provider i, the sequence $S_i^k = \left[\ell_i^k, v_i^k\right]$, where

$$v_i^k = \arg\max U_i\left(B_0, \left(p_i, v_j^{k-1}\right)\right) \tag{19}$$

and ℓ_i^k is the minimal price on S_i^{k-1} such that

$$U_i\left(B_0, \left(v_i^1, v_j^{k-1}\right)\right) \geq U_i\left(B_0, \left(p_i, v_j^{k-1}\right)\right).$$

Let us observe that, for each agent i, and iteration k, $S_i^{k+1} \subseteq S_i^k$, and this inclusion is strict whenever $v_i^k > \frac{B_0 + \alpha c_i}{1+\alpha}$. The reason is that each agent's utility function is strictly quasi-concave, and that v_i^k is strictly increasing in v_j^{k-1}, which is decreasing on k whenever S_j^k is not a singleton.

Therefore, the sequence of intervals $\left\{S_i^k\right\}_{k=1}^{\infty}$ is decreasing (in length), and thus convergent.

Moreover, since

$$
\begin{aligned}
\left.\frac{dv_i}{dv_j}\right|_{v_i < v_j} &= -\frac{\dfrac{\partial\left(1 - \frac{1}{2}(B_0 - v_j)^\alpha\left(\frac{B_0 - v_i + \alpha(v_i - c_i)}{(B_0 - v_i)^{\alpha+1}}\right)\right)}{\partial v_j}}{\dfrac{\partial\left(1 - \frac{1}{2}(B_0 - v_j)^\alpha\left(\frac{B_0 - v_i + \alpha(v_i - c_i)}{(B_0 - v_i)^{\alpha+1}}\right)\right)}{\partial v_i}} = \\
&= \frac{(B_0 - v_i)}{(B_0 - v_j)}\frac{B_0 - v_i + (v_i - c_i)\alpha}{B_0 - v_i + (v_i - c_i)\alpha + B_0 - c_i} > \\
&> \frac{(B_0 - v_i)}{(B_0 - v_i) + (B_0 - c_i)} > \frac{1}{2},
\end{aligned}
$$

we have that, whenever $v_i^k < v_j^{k-1}$, $v_i^k - v_i^{k+1}$ is always positive and does not converge to zero. This implies that the sequence $\left\{S_i^k\right\}_{k=1}^{\infty}$ converges to a singleton. $\qquad\square$

Proof of Theorem 5

Let us observe that, at equilibrium,[30] $\hat{P} = (\hat{p}_f, \hat{p}_d)$ must satisfy

$$\hat{p}_d = \frac{B_0 + \alpha c_d}{\alpha + 1}, \text{ and} \tag{20}$$

$$(B_0 - \hat{p}_f) = \frac{1}{2}\left(\frac{B_0 - \hat{p}_d}{B_0 - \hat{p}_f}\right)^\alpha \left[B_0 + (\alpha - 1)\hat{p}_f - \alpha c_f\right]. \tag{21}$$

By equation (20) we know that, by selecting α high enough, we can guarantee that $\hat{p}_d$ is very close to c_d. Moreover, as the following Lemma 4 establishes, we can select α such that $\hat{p}_f < c_d$. Theorem 5 is a direct consequence of these facts.

---

[30]Note that $\hat{P}$ depends on α. Slightly abusing notation we use $\hat{P}$ instead of $\hat{P}(\alpha)$.

Lemma 4 *There is $\hat{\alpha} > 1$ such that, at equilibrium*

$$\hat{p}_f = \frac{\hat{\alpha}}{\hat{\alpha} - 1} c_d - \frac{1}{\hat{\alpha} - 1} B_0.$$

Proof.   Let us construct the function $g : \mathbb{R} \to \mathbb{R}$ as

$$g(\alpha) = \frac{\alpha}{\alpha - 1} - \frac{\alpha(c_d - c_f)}{2(B_0 - c_d)} \left(\frac{\alpha - 1}{\alpha + 1} \right)^\alpha.$$

Let us observe that g is continuous for $\alpha > 1$. Moreover,

(i) $\lim_{\alpha \to 1^+} g(\alpha) = +\infty$, and

(ii) $\lim_{\alpha \to +\infty} g(\alpha) = -\infty$.

Thus, by applying Bolzano's Theorem, we obtain the existence of $\hat{\alpha}$ such that $g(\hat{\alpha}) = 0$. Taking into account that g is strictly decreasing, we also can guarantee that $\hat{\alpha}$ is unique.

Let us observe that $g(\hat{\alpha}) = 0$ implies that

$$\hat{p}_f = \frac{\hat{\alpha}}{\hat{\alpha} - 1} c_d - \frac{1}{\hat{\alpha} - 1} B_0$$

is a solution of

$$(B_0 - \hat{p}_f) = \frac{1}{2} \left(\frac{B_0 - \hat{p}_d}{B_0 - \hat{p}_f} \right)^{\hat{\alpha}} [B_0 + (\hat{\alpha} - 1) \hat{p}_f - \hat{\alpha} c_f],$$

which is the desired result.   □

We next show that, by selecting the parameter α appropriately, the buyer can guarantee that her savings exceed $B_0 - c_d$. The statement of Theorem 5 is a direct consequence of our next result.

Lemma 5 *There exist $\tilde{\alpha}$ such that, at equilibrium,*

$$S_0 \left(B_0, \hat{P} \right) > B_0 - c_d.$$

Proof.   Given sellers' costs, let $\tilde{\alpha} > 1$ be such that

$$B_0 - \hat{p}_f = \frac{\tilde{\alpha}}{\tilde{\alpha} - 1} (B_0 - c_d),$$

whose existence was proved in Lemma 4 above. Let us observe that

$$S_0 \left(B_0, \hat{P} \right) = \frac{1}{2} \left(\frac{B_0 - \hat{p}_d}{B_0 - \hat{p}_f} \right)^{\tilde{\alpha}} (B_0 - \hat{p}_d) + \left[1 - \frac{1}{2} \left(\frac{B_0 - \hat{p}_d}{B_0 - \hat{p}_f} \right)^{\tilde{\alpha}} \right] (B_0 - \hat{p}_f).$$

Taking into account that

$$B_0 - \hat{p}_f = \frac{\tilde{\alpha}}{\tilde{\alpha}-1}(B_0 - c_d), \text{ and}$$

$$B_0 - \hat{p}_d = \frac{\tilde{\alpha}}{\tilde{\alpha}+1}(B_0 - c_d),$$

we have that

$$\frac{B_0 - \hat{p}_d}{B_0 - \hat{p}_f} = \left(\frac{\frac{\tilde{\alpha}}{\tilde{\alpha}+1}}{\frac{\tilde{\alpha}}{\tilde{\alpha}-1}}\right) = \frac{\tilde{\alpha}-1}{\tilde{\alpha}+1} < 1,$$

and thus

$$\Psi_d^{cp_{\tilde{\alpha}}}\left(B_0, \hat{P}\right) < \frac{1}{2}, \text{ and } \Psi_f^{cp_{\tilde{\alpha}}}\left(B_0, \hat{P}\right) > \frac{1}{2}.$$

Therefore,

$$S_0\left(B_0, \hat{P}\right) > \frac{1}{2}\left[(B_0 - \hat{p}_f) + (B_0 - \hat{p}_d)\right] = \frac{\tilde{\alpha}^2}{\tilde{\alpha}^2 - 1}(B_0 - c_d) > (B_0 - c_d),$$

which is the desired result. $\qquad\square$

C   Analysis of Γ^{ctcp_α}

For the convenience of the referees, this Appendix provides a proof of Theorem 6. It is, however, not intended for publication.

When mechanism Γ^{ctcp_α} is applied, let us remember that at each $t \in [0,1)$ both providers simultaneously select an action $a_i(t) \in \{0,1\}$. This action depends on the history of the game, namely the actions taken by both agents at any $t' < t$. Notice that the only relevant information that a provider extracts at time t from the history is:

(i) If he selected *stop* at an earlier point in time, i.e. if there is $t' < t$ such that $a_i(t') = 1$. In such a case his action at t does not affect the final outcome.

(ii) If his rival selected *stop* at an earlier point in time, i.e. if there is $t' < t$ such that $a_j(t') = 1$.

(iii) If $a_j(t') = 1$ for some $t' < t$, and $a_i(t') = 0$ for each $t' < t$, the minimum value of $\hat{t}$ for which $a_j(\hat{t}) = 1$, i.e. $\tilde{t}_j$.

For our purposes the history of the game can thus be represented as

$$\mathcal{H} : [0,1) \to [0,1]^2 \tag{22}$$

such that, for each $t \in (0,1)$, and any i, $\mathcal{H}_i(t) \in [0,t) \cup \{1\}$. In such a case, $\mathcal{H}_i(t) = t' < t$ refers to the case in which provider i selects *stop* at t' after *continuing* at any $t'' < t'$. Similarly, $\mathcal{H}_i(t) = 1$ is interpreted as *continuing* or $a_i(t') = 0$ for all $t' < t$.

Let us observe that, given the history of the game at time t, $\mathcal{H}(t)$, provider i's optimal action is:

(1) Any action if $\mathcal{H}_i(t) < t$. This is so because he selected *stop* at an earlier point in time and is thus unable to affect the final outcome.

(2) If $\mathcal{H}_i(t) = 1$, we consider the following two scenarios:

(a) $\mathcal{H}_j(t) = 1$. Then, if $a_i(t) = 1$, it should be the case that $p_i \geq p_j$. This is the optimal action for provider i whenever

$$p_i \geq \frac{B_0 + \alpha c_i}{1 + \alpha},$$

or equivalently in terms of time when

$$t \geq \frac{\alpha}{1+\alpha} \frac{B_0 - c_i}{B_0}.$$

Therefore, the optimal action for agent i is

$$a_i(t) = \begin{cases} 0 & \text{if } t < \frac{\alpha}{1+\alpha} \frac{B_0 - c_i}{B_0} \\ 1 & \text{if } t \geq \frac{\alpha}{1+\alpha} \frac{B_0 - c_i}{B_0}. \end{cases}$$

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(b) $\mathcal{H}_j(t) = t' < t$. Note that, in such a case $p_i < p_j$. Provider i computes his optimal decision provided that $p_i \leq (1 - t) B_0$; i.e. he solves the problem

$$\left. \begin{array}{ll} max_{t^+} & \left(1 - \left(\frac{t'}{t^+}\right)^\alpha\right)\left((1 - t^+) B_0 - c_i\right) \\ \text{s.t.} & t^+ \geq t \end{array} \right\} \tag{23}$$

Note that the above description characterizes dominant strategies for the agents, and that the actions of the providers implied by these strategies satisfy:

(a) For the domestic provider

$$a_d(t) = \left\{ \begin{array}{ll} 0 & \text{if } t < \frac{\alpha}{1+\alpha}\frac{B_0 - c_d}{B_0} \\ 1 & \text{if } t = \frac{\alpha}{1+\alpha}\frac{B_0 - c_d}{B_0}. \end{array} \right.$$

(b) For the foreign provider

$$a_f(t) = \left\{ \begin{array}{ll} 0 & \text{if } t < \hat{t} \\ 1 & \text{if } t = \hat{t}, \end{array} \right.$$

where $\hat{t}$ is the unique maximizer of

$$\left(1 - \left(\frac{\alpha}{1+\alpha}\frac{B_0 - c_d}{tB_0}\right)^\alpha\right)\left((1 - t) B_0 - c_f\right)$$

in the interval $\left[\frac{\alpha}{1+\alpha}\frac{B_0 - c_d}{B_0}, 1\right]$.