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Affirmative Action through Extra Prizes*

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Abstract

Some affirmative action policies establish that a set of disadvantaged competitors has access to an extra prize. Examples are gender quotas or a prize for national competitors in an international competition. We analyse the effects of creating an extra prize by reducing the prize in the main competition. Contestants differ in ability and agents with relatively low ability belong to a disadvantaged minority. All contestants compete for the main prize, but only disadvantaged agents can win the extra prize. We show that an extra prize is a powerful tool to ensure participation of disadvantaged agents. Moreover, for intermediate levels of the disadvantage of the minority, introducing an extra prize increases total equilibrium effort compared to a standard contest. Thus, even a contest designer not interested in affirmative action might establish an extra prize in order to enhance competition.

Keywords: Asymmetric contest, equality of opportunity, affirmative action, discrimination, prize structure, exclusion principle

JEL: C72, D72, I38, J78

1 Introduction

Some affirmative action policies establish that a set of disadvantaged competitors has access to an extra prize. Examples are gender quotas, prizes for national competitors in an

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international competition, like in film or fireworks awards, or a prize for the best paper by a young scientist. The purpose of the present paper is to investigate the incentive effects of this affirmative action instrument. Our main result is to show that this policy is not only appealing from a normative point of view but can also enhance competition. It can thus be desirable on efficiency grounds, fostering thereby the social acceptance of the policy.

We analyse the effects of extra prizes in a contest model. These models have been insightful in a variety of competitive situations, including rent-seeking, promotional competition, labour market tournaments, sports competitions or conflict. Following Stein (2002) or Franke et al. (2013), we investigate an asymmetric contest in which contestants differ in ability. Agents with relatively low ability belong to a 'disadvantaged minority'.[1]

A standard result in contest theory says that the most inefficient (or least able) agents might not actively participate in the competition (Stein 2002). And indeed, 'minority representation' is an important concern in real competitions. For instance, in California the Disabled Veteran Business Enterprise and Small Business Certification Programs establish explicit target market shares for these disadvantaged groups. Similarly, the European Union has target shares for female representation on firms' boards. The challenge is then to design affirmative action policies that can reconcile the conflicting aims of reaching both (i) a sufficient level of minority representation and (ii) a sufficient level of competition. Avoiding trade-offs between these objectives is important because it influences the political support for and the prevalence of affirmative action policies. Ayres and Cramton (1996), for example, report that various California ballot initiatives tried to end state-sponsored affirmative action because of the belief that eliminating affirmative action could help to solve budget problems.

In our model the contest designer can create an extra prize at the cost of reducing the prize in the main competition. All contestants compete for the main prize, but only disadvantaged agents can win the extra prize. This fits, for example, quotas for disadvantaged minorities, like gender quotas, in which the establishment of the quota reduces the budget available in the main competition. Disadvantaged agents thus should have an incentive to exert higher effort but it is far from obvious that the overall level of competition will be strengthened, as advantaged agents have lower incentives to invest.

[1] For a survey see Konrad (2009). See also Cornes and Hartley (2005) and Ryvkin (2013) for general models of asymmetric contests. Throughout the paper we follow the language of the affirmative action literature and use for example the term 'disadvantaged minority' for the agents favoured through affirmative action.

We show that disadvantaged agents indeed do have an incentive to exert higher effort and that we can think of the effects of extra prizes 'as raising the ability of disadvantaged agents'. In this sense extra prizes create a 'level playing field', as the abilities of contestants become more homogeneous. This leads to our first major result that an extra prize is a powerful tool to ensure participation of disadvantaged agents. With an extra prize both groups of agents are active; using the language of the affirmative action literature, there is diversity.

Our main result is to show that extra prizes have the potential to strengthen the overall level of competition. The reason is that, as the disadvantaged minority competes stronger, advantaged agents might exert more effort than they otherwise would, resulting in a higher overall level of competition. More precisely, we show that for intermediate levels of the disadvantage of the minority, introducing an extra prize increases total equilibrium effort compared to a standard contest (for example in Stein 2002). We show that the magnitude of the increase of total effort due to the extra prize might potentially be quite important and that this effect might arise with or without an increase in minority representation. Thus, even a contest designer not interested in affirmative action might establish an extra prize in order to enhance competition.[2]

A distinctive feature of our model is that some agents might win more than one prize with a sole effort choice. This fits quotas, provided that at the time of investment (for example in education) minority members might still choose between participating in the main competition or as a minority member. But there are further situations in which this feature is realistic. For instance, in 2011 the Catalan film *Black Bread* won both the (Spanish) Goya Award and the (Catalan) Gaudí Award in the category of Best Film. Another example is the fireworks contest yearly organized by the City Council of Tarragona. In 2009 a local firm won both the main (international) prize and the prize for Catalan competitors. Also, in chess the World Championship does not exclude women, juniors or seniors, but each of these groups have in addition their separate championship. Currently, in Germany a prominent firm organizes a photo competition that awards a main annual prize and a secondary

[2]We are not aware of an empirical study that fits exactly our model. The predictions of our model are, however, in line with empirical evidence. Brown (2011) shows that large differences in ability might reduce effort. Balafoutas and Sutter (2012) provide experimental evidence that related (but different) affirmative action policies can have an important impact on minority participation, while not harming the efficiency of the competition, as predicted by our model. See also Schotter and Weigelt (1992), Franke (2012b) and Calsamiglia et al. (2013) for further evidence of performance enhancing incentive effects of affirmative action.

monthly prize that both can be won with the same photo.[3]

There might, however, be further situations that do not fit our model exactly but for which our model might serve as a benchmark. Consider the Spanish research programme "Proyectos Europa Excelencia". In order to compete in this programme a proposal must have competed unsuccessfully for a "Starting Grant" of the European Research Council. Consider promotional competition. Firms invest effort in building up brands. These brand names affect the market shares of the firms' products in several markets but not all brands have products in all markets. Consider for instance rent-seeking. Interest groups expend effort in activities including setting up offices close to political decision makers, building up personal networks to legislators, or developing a reputation for competence on specific issues. Lobbies are often affected by divers legislative issues but not all groups have stakes in all issues. A broader implication of our model is that in these situations the model of a standard contest might underestimate the incentives to participate of agents with low stakes, and sometimes even underestimate total effort.[4]

Our paper contributes to two strands of literature. The first analyses the incentive effects of affirmative action policies in competitive situations and the second investigates the prize structure in contests.

Our paper relates to the first strand, because we show that extra prizes create a 'level playing field' which may lead to more intense competition. A growing literature has determined other policies that affect competition in a similar way, including subsidies to high-cost suppliers (Ewerhart and Fieseler 2003; Rothkopf et al. 2003), bid preferences and other biases in the selection of the winner (Ayres and Cramton 1996; Fu 2006; Franke 2012a; Franke et al. 2013; Lee 2013), share auctions (Alcalde and Dahm 2013), and the handicap or even exclusion of the most efficient participant (Baye et al. 1993; Che and Gale 2003; Kirkegaard 2012).[5] To the best of our knowledge the affirmative action policy considered

[3]See www.olympus.de/omd, accessed on 02/08/2013.

[4]There are also situations which might be interpreted as being the opposite to affirmative action, because the most efficient agents have access to extra prizes. Consider (European) soccer teams and their investment in players. All teams compete in the national leagues. In addition, however, the best teams compete in European competitions, like the Champions League. Or consider elite colleges that have a bias in favour of applicants who are children of alumni.

[5]We comment further on the relationship to Baye et al. (1993) in the concluding section. The creation of a 'level playing field' does not always result in the most intense competition. Pérez-Castrillo and Wettstein (2012) analyse innovation contests under asymmetric information. They provide conditions under which in a symmetric setting discrimination among contestants is optimal. Such a discrimination (where the size of

here has not been analysed before.

Our paper also contributes to the literature on the optimal prize structure in contests (e.g. Glazer and Hassin 1988; Clark and Riis 1998b; Moldovanu and Sela 2001; Szymanski and Valletti 2005; Moldovanu and Sela 2006; Azmat and Möller 2009; Fu and Lu 2009; Möller 2012), because the introduction of an extra prize establishes a specific prize structure. Our model, however, differs from this literature by allowing for some contestants to win more than one prize with a sole effort choice.[6]

The paper is organized as follows. The next section collects our assumptions and fixes notation. We conduct our strategic analysis in Section 3. The last section contains concluding remarks. All proofs are relegated to an Appendix.

2 The Model

A set of risk-neutral contestants $N = \{1, 2, ..., n\}$ competes for a budget B.[7] An agent i's share of the budget depends on his effort exerted, which is denoted by e_i. Expenditures are not recovered. Players have different abilities $\alpha_i > 0$ that are reflected in heterogeneous effort costs $c_i(e_i) = e_i/\alpha_i$. Without loss of generality assume that lower indexed agents have higher ability, so that $\alpha_i \geq \alpha_{i+1}$ for all $i \in \{1, 2, ..., n-1\}$.

There is an observable characteristic that distinguishes agents in such a way that they can be partitioned into two groups, $N = M \cup D$. We interpret D as the disadvantaged group that is the objective of affirmative action. For this reason we assume that agents in

the prize depends on the identity of the winner) makes the contest asymmetric and can be thought of as 'the opposite' of the creation of a 'level playing field'. The creation of a 'level playing field' can also have unintended consequences and increase sabotage, see Brown and Chowdhury (2012).

[6]There are a few models in which a contestant can win multiple prizes. But this requires allocating resources to different contests, as in Gradstein and Nitzan (1989), or choosing effort twice, as in Sela (2012). There are also models in which the contest success function is biased towards some participant, see Franke et al. (2013) for a general model, Franke (2012a) for an affirmative action context and Farmer and Pecorino (1999) or Dahm and Porteiro (2008) for other environments. This is analytically different from creating an extra prize. The contest success function considered in the present paper is unbiased.

[7]In the literature, the outcome of contests has been interpreted to capture either win probabilities or shares of a prize, see Corchón and Dahm (2010). Since we assume that agents are risk neutral, we do not distinguish between both interpretations. Our model thus allows for contestants winning prizes with some probability, as in the case of the aforementioned film awards, or for agents winning shares of a overall budget, as in the case of quotas.

$M = \{1, 2, ..., m\}$ have higher ability than agents in $D = \{m+1, ..., n\}$.[8] To distinguish our setting from a standard contest we suppose $1 \leq m \leq n-2$.

The contest designer aims to maximize total effort by choosing $\beta \in [0, 1]$. The parameter β divides the budget into two prizes: $B_1 = (1-\beta)B$ and $B_2 = \beta B$. Members of group M only compete for prize B_1, while members of group D compete for both prizes. In this sense prize B_2 is an extra prize for group D. Notice that although an agent in D exerts effort only once, he might win both prizes. Note also that when $\beta = 0$ or $\beta = 1$ we have a standard contest without extra prize.[9] In order to focus on the effects of an extra prize we follow most of the literature and consider for each prize an imperfectly discriminating contest in which an agent i's share of the budget is proportional to his effort expended, see Tullock (1980).[10]

We introduce the following notation. A vector of individual efforts is denoted by $e = (e_1, e_2, \ldots, e_n)$ and total effort of a group G of agents is $E_G = \sum_{k\in G} e_k$. Similarly, the vector of abilities is denoted by $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)$. Given a group of agents G with cardinality $|G|$, the harmonic mean of abilities is given by

$$\Gamma_G \equiv \frac{|G|}{\sum_{k\in G} \frac{1}{\alpha_k}}.$$

In the contest with extra prize the expected payoff of player i is

$$EU_i(e) = \frac{e_i B_1}{E_N} + \frac{e_i B_2 z_i}{E_D} - \frac{e_i}{\alpha_i}, \tag{1}$$

where $z_i \in \{0, 1\}$ takes value 1 if and only if $i \in D$, and value 0 otherwise. Notice that in this formulation the win probabilities of the two prizes are independent. It is interesting to observe that our model also captures a situation in which a disadvantaged agent who wins

[8] We take it as given that minority agents are (weakly) disadvantaged. See Holzer and Neumark (2000), Fang and Moro (2010) and Niederle and Vesterlund (2011) for an assessment of the disadvantage in different contexts.

[9] With the term standard contest we refer to a situation in which (given the value of β or the behaviour of other agents) the objective function of active agents reduces to the one in Stein (2002). For simplicity of the exposition we exclude the case $\beta = 1$ from most of our derivations. Sometimes, however, it is convenient to include this case. The statements referring to $\beta = 1$ follow from Stein (2002).

[10] We follow most of the literature and assume that when in the competition for a prize none of k agent exerts effort, each agent wins the prize with probability $1/k$. Skaperdas (1996) and Clark and Riis (1998a) provide axiomatic characterizations of the contest success function employed, while Corchón and Dahm (2010) give a micro-foundation for the interpretation that the outcome is the choice of a designer.

the main prize also wins the extra prize. In such a situation the expected payoff of player i is

$$EU_i(e) = \frac{e_i B_1}{E_N} + \frac{e_i B_2 z_i}{E_N} + \frac{E_M}{E_N}\frac{e_i B_2 z_i}{E_D} - \frac{e_i}{\alpha_i},$$

which is equivalent to equation (1).

3 Strategic analysis

Our first result establishes that contests with extra prizes are a powerful tool to make sure that there will be minority representation, since the extra prize will not be uncontested.

Lemma 1. *For any (α, B) at least two contestants participate in the contest. Moreover, if $\beta > 0$, at least one agent $i \in D$ is active.*

For later reference we observe that Lemma 1 implies that $E_D > 0$, if $\beta > 0$. In order to analyse participation in the contest further, we take the derivative of equation (1) and obtain

$$\frac{\partial EU_i(e)}{\partial e_i} = \frac{E_N - e_i}{(E_N)^2}B_1 + \frac{E_D - e_i}{(E_D)^2}B_2 z_i - \frac{1}{\alpha_i}. \tag{2}$$

Given that (1) is concave in e_i, the first-order conditions require that $\partial E_i(e)/\partial e_i = 0$ if $e_i > 0$ and $\partial E_i(e)/\partial e_i \leq 0$ if $e_i = 0$. The former implies that

$$\frac{E_N - e_i}{(E_N)^2}B_1 + \frac{E_D - e_i}{(E_D)^2}B_2 z_i = \frac{1}{\alpha_i}. \tag{3}$$

Consider the agents in set M. For these agents condition (3) reduces to the familiar expression

$$e_i = E_N\left(1 - \frac{E_N}{B_1 \alpha_i}\right), \tag{4}$$

see Stein (2002). This implies that the higher the ability of an agent, the higher his equilibrium effort. If the ability of an agent is low enough, he does not participate. We denote by $m^* \in M$ the agent with $e_{m^*} > 0$ such that for all $i < m^*$, $e_i > 0$ and for all $i \in M$ with $i > m^*$, $e_i = 0$. We denote by $M_{m^*} \subseteq M$ or $M^* \subseteq M$ the set of active advantaged agents, depending on whether we wish to stress the identity of the active agents. If no advantaged agent is active, we set $M^* = \emptyset$ and $m^* = 0$.

Consider the agents in set D. Here condition (3) becomes

$$e_i = E_N E_D \frac{B_2 E_N + B_1 E_D\left(1 - \frac{E_N}{B_1 \alpha_i}\right)}{B_1 (E_D)^2 + B_2 (E_N)^2}. \tag{5}$$

Again, equilibrium effort is ordered by ability. We define $D_{d^*}, D^* \subseteq D$, and d^* analogously to M_{m^*}, M^* and m^*.[11] If no disadvantaged agent is active, we set $D^* = \emptyset$ and $d^* = 0$.

The aim of our strategic analysis is to show that contests with extra prize admit a unique equilibrium (a formal statement will be provided in Proposition 4), and to investigate the effects of an extra prize on participation (in Subsection 3.3) and on total effort (in Subsections 3.4, 3.5 and 3.6). Doing so, however, requires looking first at the two different types of equilibria that might arise. In the first type of equilibrium only one group is active, and behaviour is similar to that in a standard contest. In the second there is diversity and complex effects emerge. We start by analysing each type of equilibrium successively in Subsections 3.1 and 3.2.

3.1 Standard equilibria

There are two situations in which equilibria that appear in a contest with extra prize are similar to those in a standard contest. The first is the trivial case when $\beta = 0$; when there is no extra prize. In the second members of the advantaged group are discouraged from participating because the extra prize is sufficiently large. With a large extra prize the prize in the main competition is very small and thus it might happen that no advantaged agent is active. In both cases our model reduces to a standard contest that has been analysed by Stein (2002). For our purpose it is sufficient to summarize his results as follows.[12]

Lemma 2. *[Stein, 2002] In a standard contest in which a group of agents $P = \{1, 2, \ldots, p\}$ competes for a prize B, the number of active players $|P^*|$ is larger than two and total equilibrium effort is given by*

$$E_N = \frac{|P^*| - 1}{|P^*|} B \Gamma_{P^*}. \tag{6}$$

In order to describe when standard equilibria appear, it is useful to start with a definition. To do so denote the set of active disadvantaged agents when no advantaged agent exerts effort by $D^*_{M^*=\emptyset}$.

[11] Notice that d^* does not indicate the cardinality of the set of active agents but the index of the most disadvantaged active agent: $|D^*| = d^* - m$. Note also that if $\beta = 0$ or no advantaged agent is active equation (5) reduces to equation (4). The contest with extra prize becomes a standard contest in which only the agents of the disadvantaged group participate.

[12] For the exact expressions of the equilibrium number of active players, individual efforts, win probabilities and expected utilities, see Stein (2002).

Definition 1. *Let*

$$\overline{\beta} \equiv 1 - \frac{|D^*_{M^*=\emptyset}| - 1}{|D^*_{M^*=\emptyset}|} \frac{\Gamma_{D^*_{M^*=\emptyset}}}{\Gamma_{\{1\}}}.$$

Notice that $\Gamma_{\{1\}} = \alpha_1 > 0$ and $|D^*_{M^*=\emptyset}| \geq 2$, implying that $\overline{\beta}$ is well defined. The following result establishes that in any contest, in addition to the trivial case when $\beta = 0$, there are situations in which standard equilibria emerge.

Proposition 1. *For any (α, B) we have that $\overline{\beta} < 1$. Moreover, for any $\beta \in [\overline{\beta}, 1]$, it is an equilibrium that the set of active agents is $D^*_{M^*=\emptyset}$ and equilibrium behaviour is as in a standard contest for a prize of size B.*

Intuitively, in the equilibria described in Proposition 1 the extra prize is too large. The extra prize reaches the aim of inducing participation of the disadvantaged group but it does so by discouraging the members of the advantaged group. We turn now to more moderate extra prizes which generate equilibria in which members of both groups are active.

3.2 Equilibria with diversity

We start by defining a measure of minority representation, as the percentage of total effort that is expended by disadvantaged agents

$$\Omega \equiv \frac{E_D}{E_N}.$$

We say that there is diversity if $\Omega \notin \{0, 1\}$. The following result establishes that an extra prize of intermediate size guarantees diversity.

Proposition 2. *For any (α, B) we have that $0 < \overline{\beta}$. Moreover, for any $\beta \in (0, \overline{\beta})$, in equilibrium, there is diversity.*

For later reference, we observe that Proposition 2 implies that $E_N > E_D > 0$. In order to describe total equilibrium effort in an equilibrium with diversity, denote the number of active agents in the contest by $|N^*| = |M^*| + |D^*|$.

Proposition 3. *Let $\beta \in (0, \overline{\beta})$. For any (α, B),*

$$E_N = \Upsilon + \sqrt{\Upsilon^2 - \Phi}, \tag{7}$$

where

$$\begin{aligned}\Upsilon &\equiv \frac{B_1}{2}\left(\frac{|M^*|-1}{|M^*|}\Gamma_{M^*}+\frac{|N^*|-1}{|N^*|}\Gamma_{N^*}\right) \textit{ and}\\ \Phi &\equiv \frac{B_1\Gamma_{M^*}\Gamma_{N^*}}{|M^*||N^*|}\left((|N^*|-1)(|M^*|-1)B_1-(|D^*|-1)B_2\right).\end{aligned}$$

Notice that equation (7) includes the case in which only one disadvantaged agent is active. In this case increasing the extra prize establishes a transfer to the active disadvantaged agent and reduces the main prize in some proportion. Total effort, however, is like in a standard contest for a prize of size B_1.[13]

We complete now the description of the candidate equilibrium for $\beta \in (0,\overline{\beta})$. We will later confirm that this is indeed an equilibrium. A formal statement of existence and uniqueness of equilibrium will be provided in Proposition 4. Summing up equation (4) over all advantaged agents and rearranging, we determine

$$\begin{aligned}\Omega &= |M^*|\frac{\Upsilon+\sqrt{\Upsilon^2-\Phi}}{B_1\Gamma_{M^*}}-(|M^*|-1)\in(0,1),\\ E_M &= \left(\Upsilon+\sqrt{\Upsilon^2-\Phi}\right)(1-\Omega) \quad \text{and}\\ E_D &= \left(\Upsilon+\sqrt{\Upsilon^2-\Phi}\right)\Omega.\end{aligned}$$

The expressions for individual efforts of the active agents are obtained as follows.[14] First introducing equation (7) in equation (4), yielding for $i \in M^*$

$$e_i=\left(\Upsilon+\sqrt{\Upsilon^2-\Phi}\right)\left(1-\frac{\Upsilon+\sqrt{\Upsilon^2-\Phi}}{\alpha_i B_1}\right). \tag{8}$$

For $i \in D$ we use equations (7) and (17) in equation (5) obtaining

$$e_i=\left(\Upsilon+\sqrt{\Upsilon^2-\Phi}\right)\Omega\frac{B_1\Omega\left(1-\frac{\Upsilon+\sqrt{\Upsilon^2-\Phi}}{\alpha_i B_1}\right)+B_2}{B_1\Omega^2+B_2}. \tag{9}$$

Since agents of the advantaged group only compete for one prize, only the win probability for prize B_1 is of interest. This is immediately determined. For $i \in M^*$ we have

$$p_i = 1-\frac{\Upsilon+\sqrt{\Upsilon^2-\Phi}}{\alpha_i B_1}.$$

[13] See Claim 1 in Appendix A.1.5.

[14] When there is no extra prize ($\beta = 0$) or when $|D^*| = 1$ both expressions coincide and reduce to the one in Stein (2002). The first observation follows from the fact that for $\beta = 0$ equation (7) reduces to equation (6), see the proof of Proposition 5. For the second observation, notice that $\Omega = 1-\left(\Upsilon+\sqrt{\Upsilon^2-\Phi}\right)/(B_1\Gamma_{D^*})$ when $|D^*| = 1$.

Members of the disadvantaged group, however, have the chance to obtain two prizes, and thus two win probabilities. The win probability of agent $i \in D^*$ for prize B_1 is

$$p_i = \Omega \frac{B_1 \Omega \left(1 - \frac{\Upsilon + \sqrt{\Upsilon^2 - \Phi}}{\alpha_i B_1}\right) + B_2}{B_1 \Omega^2 + B_2},$$

while the win probability of agent $i \in D^*$ for prize B_2 is

$$q_i = \frac{B_1 \Omega \left(1 - \frac{\Upsilon + \sqrt{\Upsilon^2 - \Phi}}{\alpha_i B_1}\right) + B_2}{B_1 \Omega^2 + B_2}.$$

Lastly, we state the expected equilibrium utilities of the active agents. For $i \in M^*$ we have

$$EU_i = B_1 \left(1 - \frac{\Upsilon + \sqrt{\Upsilon^2 - \Phi}}{\alpha_i B_1}\right)^2,$$

and for $i \in D^*$ one obtains

$$EU_i = \frac{\left[B_1 \Omega \left(1 - \frac{\Upsilon + \sqrt{\Upsilon^2 - \Phi}}{\alpha_i B_1}\right) + B_2\right]^2}{B_1 \Omega^2 + B_2}.$$

3.3 The effects of the extra prize on participation

We are now in a position to investigate the effects of the extra prize on participation. Remember that with the help of condition (4) we have already established that $e_i > 0$ for $i \in M$ requires sufficient ability

$$\alpha_i > \frac{\Upsilon + \sqrt{\Upsilon^2 - \Phi}}{B_1}, \tag{10}$$

and thus $e_i > 0$ for $i \in M$ implies $e_j > 0$ for $j < i$. Moreover, for disadvantaged agents a similar property holds; $e_i > 0$ for $i \in D$ requires

$$\alpha_i > \frac{\Upsilon + \sqrt{\Upsilon^2 - \Phi}}{B_1} \frac{B_1 \Omega}{B_1 \Omega + B_2}. \tag{11}$$

Again $e_i > 0$ for $i \in D$ implies $e_j > 0$ for $m < j < i$. Hence it suffices to characterize candidate sets of active agents by the highest index of the agents in the sets: $M_{m^*} \subseteq M$ and $D_{d^*} \subseteq D$. The overall set of active agents can then be characterized with the help of these two indexes: $N_{d^*}^{m^*} \equiv M_{m^*} \cup D_{d^*}$.[15]

[15]If one of these sets is empty, say $M^* = \emptyset$, we write $N_{d^*}^0$.

Example 1. *Consider $M = \{1,2\}$ and $D = \{3,4\}$. Since at least two contestants are active, the candidate sets of active agents are $N_0^2 = \{1,2\}$, $N_3^1 = \{1,3\}$, $N_4^0 = \{3,4\}$, $N_3^2 = \{1,2,3\}$, $N_4^1 = \{1,3,4\}$, and $N_4^2 = \{1,2,3,4\}$. When $\beta = 0$, it can not be that agent 3 is active when agent 2 is not. Hence we exclude N_4^0, N_3^1, and N_4^1. When $\beta > 0$, Proposition 2 implies that N_0^2 will not be relevant.*

Lastly, building on equation (11), we define for the disadvantaged agents $i \in D$ the effective ability $\hat{\alpha}_i$ as follows

$$\hat{\alpha}_i \equiv \alpha_i \left(1 + \frac{B_2}{B_1 \Omega}\right) \geq \alpha_i.$$

So we can think of the extra prize as boosting the ability of the disadvantaged agents–the left hand side of equation (11)–, while the threshold for being active remains as in a standard contest–the right hand side of equation (10). We summarize the preceding formally as

Corollary 1. *For any (α, B), the set of active contestants $N_{d^*}^{m^*}$ is found as the largest index $m^* = \{0,1,2,\ldots,m\}$ such that, given d^*,*

$$\alpha_{m^*} > \frac{\Upsilon + \sqrt{\Upsilon^2 - \Phi}}{B_1} \tag{12}$$

holds and the largest index $d^* = \{0, m+1, m+2, \ldots, n\}$ such that, given m^*,

$$\hat{\alpha}_{d^*} > \frac{\Upsilon + \sqrt{\Upsilon^2 - \Phi}}{B_1} \tag{13}$$

is true.

Corollary 1 complements Lemma 1 and Proposition 2. It shows from a different angle that the extra prize might have a strong effect on participation. When $\beta = 0$ and there is no extra prize, effective ability $\hat{\alpha}_i$ is equal to α_i. Disadvantaged agents have the lowest incentives among all contestants to participate, as their ability to compete is lowest. Introducing the extra prize, however, affects the participation conditions of both types of agents.

To see this it is instructive to consider the derivative of the right hand side of both participation conditions (12) and (13) with respect to β. Since $\partial\left(E_N/B_1\right)/\partial\beta \geq 0$, the extra prize discourages participation of advantaged agents. For disadvantaged agents, however, there is a countervailing effect, because their effective ability $\hat{\alpha}_i$ is also raised, as

$\partial\left(B_2/B_1\Omega\right)/\partial\beta > 0$.[16]

These two countervailing effects imply that extra prizes have the potential to induce participation of disadvantaged agents who are not active without such a prize and to discourage participation of advantaged agents who are active without such a prize. We will observe these forces in later sections in more specific settings.

We are now in a position to state formally existence and uniqueness of equilibrium. For $\beta \notin (0,\overline{\beta})$ the result follows from Stein (2002) or Fang (2002). For $\beta \in (0,\overline{\beta})$ we provide in Appendix A.1.6 a proof proceeding in three steps. First we confirm that the candidate strategies in equations (8) and (9) are indeed an equilibrium. Second, we prove uniqueness of the equilibrium in pure strategies by showing that the set of active contestants $N_{d^*}^{m^*}$ is unique and, lastly, we show that there is no equilibrium in mixed strategies.

Proposition 4. *For any (α, B), there is a unique equilibrium in which the set of active contestants $N_{d^*}^{m^*}$ employ the pure strategies described in equations (8) and (9), while the other agents exert no effort.*

3.4 The effects of the extra prize on total effort

We establish first the existence of an optimal size for the extra prize.

Proposition 5. *For any (α, B), there is $\beta^* \in [0,\overline{\beta})$ such that $E_N(\beta)$ attains a maximum at β^*.*

The previous proposition establishes that a maximum is well defined. Concerning the optimal size of the extra prize, it uses the well known result that it is never optimal to exclude all advantaged agents (Fang, 2002). In the general model, determining the optimal size of the extra prize is complex because, as we will see for example in Example 5, total effort is not differentiable with respect to β. In light of this problem, we derive now a necessary condition for $\beta^* > 0$.

Given the participation condition (4), we can always find $\beta = \epsilon$ with $\epsilon > 0$ but sufficiently close to zero such that the set of active agents will consist of the same set of agents

[16] For completeness we mention that $\partial\left(E_N/B_1\right)/\partial\beta > 0$ requires $|D^*| > 1$, see Appendix A.1.5. When only one disadvantaged agent is active, increasing the extra prize does not affect participation of active agents. The reason is that increasing the extra prize establishes a transfer to the active disadvantaged agent and reduces the main prize in some proportion. This does not affect participation of active agents, because in a Tullock contest participation is not affected when valuations are multiplied by a constant. Note also that when $E_N/B_1 = \alpha_1$ and the most efficient agent ceases to be active, condition (11) becomes condition (10), where B_1 is replaced by B.

as for $\beta = 0$, except when there was no minority representation (Proposition 2). In the latter case, at least the most efficient disadvantaged agent also becomes active. Formally, we define the following sets of agents. Let $M^*_{\beta=0} \subseteq M$ and $D^*_{\beta=0} \subseteq D$ be the sets of advantaged and disadvantaged agents that are active for $\beta = 0$, respectively. Let

$$D^*_{\alpha_{m+1}} = \{i \in D : \alpha_i = \alpha_{m+1}\}$$

be the most able of the disadvantaged agents. Notice that this set has at least cardinality one. The cardinality is higher when there is more than one agent with the highest ability. In the following Proposition, M^*_ϵ, D^*_ϵ, and N^*_ϵ refer to $M^*_{\beta=0}$, $D^*_{\beta=0} \cup D^*_{\alpha_{m+1}}$, and $M^*_\epsilon \cup D^*_\epsilon$, respectively. Lastly we define the following numbers

$$\gamma \equiv \frac{|M^*_\epsilon| - 1}{|M^*_\epsilon|}\frac{|N^*_\epsilon|}{|N^*_\epsilon| - 1}, \quad \delta \equiv \frac{|M^*_\epsilon| - 1}{|M^*_\epsilon|}\frac{|N^*_\epsilon| - 1}{|N^*_\epsilon| - 2} \quad \text{and } \zeta \equiv \frac{|N^*_\epsilon|\big(|N^*_\epsilon| - 2\big)}{\big(|N^*_\epsilon| - 1\big)^2}.$$

Notice that when $|M^*_\epsilon| > 1$, $\zeta = \gamma/\delta$.

Proposition 6. *Let $|D^*_\epsilon| > 1$. For any (α, B),*

$$\beta^* > 0 \quad \text{if} \quad \gamma\delta < \frac{\Gamma_{N^*_\epsilon}}{\Gamma_{M^*_\epsilon}} < \zeta. \tag{14}$$

The previous Proposition requires $|D^*_\epsilon| > 1$. This is a weaker condition than asking to have more than one agent with exactly ability α_{m+1}. It rules out situations in which only one disadvantaged agent is active. If $|D^*_\epsilon| = 1$, increasing the extra prize establishes a transfer to the active disadvantaged agent and reduces the main prize in some proportion. Thus the contested rent is reduced and total effort declines.[17]

Proposition 6 provides a condition assuring that a revenue maximizing contest organizer finds it in his interest to establish an extra prize. Why does an extra prize have the potential to increase total effort? The reason is that an extra prize increases the effective ability $\hat{\alpha}_i$ of disadvantaged agents. This balances the competition and results in higher total effort, provided condition (14) is fulfilled. The latter condition requires that the ratio of harmonic means is strictly smaller than than one, because γ, δ and ζ are all strictly smaller than one when $|D^*_\epsilon| > 1$. Thus, the minority must be disadvantaged, as it can be shown that $\Gamma_{N^*_\epsilon}/\Gamma_{M^*_\epsilon} < 1$ if and only if $\Gamma_{D^*_\epsilon} < \Gamma_{M^*_\epsilon}$. On the other hand, there is also a lower bound on the ratio of harmonic means, so that the disadvantage of the minority must be of an intermediate level. We will gain further intuition into condition (14) in what follows.

[17] In Claim 1 in Appendix A.1.5 we show that in this case $E_N = (|N^*_\epsilon| - 1)B(1 - \beta)/|N^*_\epsilon|$.

3.5 Homogeneity within groups

We consider now the case in which all agents in a given group have the same ability. Such a situation represents the benchmark of minimal heterogeneity in our model. We will see that condition (14) is also a sufficient condition under this assumption.

Suppose that $\alpha_i = \overline{\alpha}$ for all $i \in M$ and $\alpha_i = \underline{\alpha}$ for all $i \in D$. We denote $\underline{\alpha}/\overline{\alpha} \equiv \alpha \in (0,1]$.

Because of the symmetry within groups conditions (4) and (5) imply that either all group members are active, or none is. So for $\beta = 0$ the set of active agents is either M or N. For $\beta \in (0, \overline{\beta})$, the set of active agents is N, while for larger values for β only group D is active. Moreover, $\overline{\beta}$ simplifies to $1-(n-m-1)\alpha/(n-m)$.

Consider now Proposition 6. Notice that by assumption if D is active at least two disadvantaged agents are expending effort; thus $|D^*_\epsilon| > 1$ is always fulfilled. Straightforward algebra yields

$$\frac{\Gamma_{N^*_\epsilon}}{\Gamma_{M^*_\epsilon}} = \frac{\alpha}{1-\frac{m}{n}(1-\alpha)}.$$

The right hand side of this expression is strictly increasing in α. Moreover, it tends to zero when α goes to zero, and goes to one when α approaches one. Thus condition (14) is fulfilled for intermediate values of α. The next result states the precise condition.

Proposition 7. *For any (α, B), $\beta^* > 0$ if and only if*

$$\frac{(m-1)^2(n-m)}{m(m(n-m)-1)} < \alpha < \frac{(n-m)(n-2)}{1+(n-m)(n-2)}. \tag{15}$$

Moreover, for any n and m, there exists α such that $\beta^* > 0$.

The assumption of homogeneity within groups allows to strengthen Proposition 6 considerably. On one hand, condition (14) becomes also a sufficient condition. On the other, for any configuration of groups there exist intermediate levels of disadvantage such that a revenue maximizing contest organizer finds it in his interest to establish an extra prize.

We provide now two examples that provide further intuition into condition (15) and illustrate the magnitude by which an extra prize might increase total effort. In both examples we set $B = 1$. We also normalize $\overline{\alpha} = 1$, so that $\underline{\alpha} = \alpha$.

Example 2. *Let $M = \{1\}$ and $D = \{2,3\}$. Consider a standard contest with $\beta = 0$. Since at least two agents are active, all agents are active, and total effort is given by $E_N = 2\alpha/(2+\alpha)$. Introducing a sufficiently small extra prize, $\beta < \overline{\beta} = 1-\alpha/2$, all agents stay active. Condition*

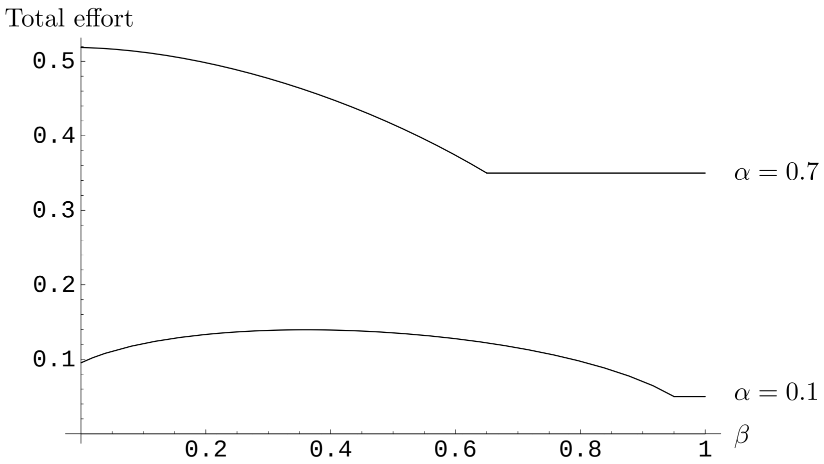

Figure 1: Total effort in Example 2.

(15) becomes $0 < \alpha < 2/3$. *Once the extra prize becomes sufficiently large,* $\beta > \overline{\beta}$, *only disadvantaged agents are active and total effort is given by* $E_D = \alpha/2$.

Figure 1 shows the effect of the extra prize (horizontal axis) on total effort (vertical axis). The lower curve assumes $\alpha = 0.1$, *for which condition (15) holds. Total effort reaches a maximum for* $\beta^* = 0.36$, *where total effort is equal to 0.1395. This implies a percentage increase of 46% compared to the standard contest. The upper curve assumes* $\alpha = 0.7$, *for which condition (15) does not hold. Total effort is weakly decreasing in the size of the extra prize. Hence* $\beta^* = 0$.

Notice that in the previous example the left hand side of condition (15) is zero. The next example considers two advantaged agents and shows that abilities can be both too homogeneous and too heterogeneous for an extra prize to have an effect. In the latter case, however, the extra prize stimulates participation.

Example 3. *Let* $M = \{1, 2\}$ *and* $D = \{3, 4\}$. *Consider a standard contest with* $\beta = 0$. *If all agents are active total effort is given by* $E_N = 3\alpha/(2+2\alpha)$. *If only advantaged agents are active total effort is given by* $E_M = 1/2$. *In both cases follows from condition (4) that disadvantaged agents are active if* $\alpha > 1/2$, *and total effort is continuous in* α. *Introducing a sufficiently small extra prize,* $\beta < \overline{\beta} = 1 - \alpha/2$, *assures that all agents are active. Condition (15) becomes* $1/3 < \alpha < 4/5$. *Once the extra prize becomes sufficiently large,* $\beta > \overline{\beta}$, *only disadvantaged agents are active and total effort is given by* $E_D = \alpha/2$.

Again, we display in Figure 2 several cases. The highest curve assumes $\alpha = 0.9$, *while*

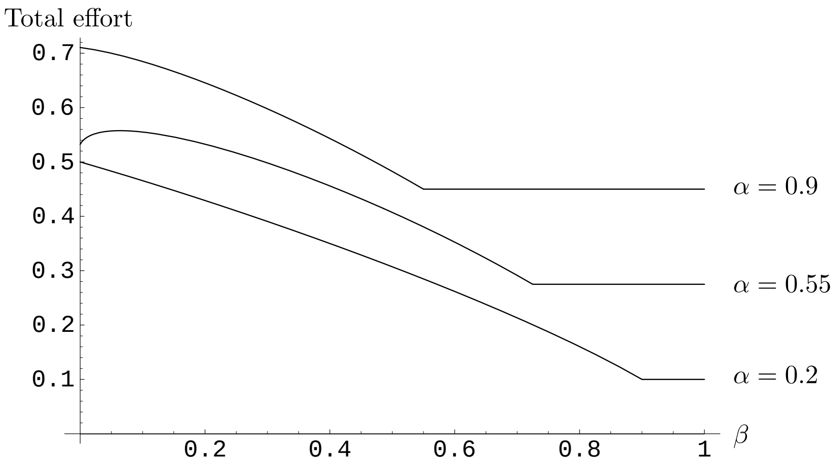

Figure 2: Total effort in Example 3.

the lowest supposes $\alpha = 0.2$*. In both condition (15) does not hold. The curve in the middle displays* $\alpha = 0.55$*, for which the condition holds. In the case of* $\alpha = 0.2$ *minority participation has increased.*[18]

3.6 Heterogeneity within groups

We consider now very briefly the case in which agents in a group have different abilities. We look at the simplest three agent setting in order to make the following two points. Example 4 proves that Condition (14) can be informative in such a setting, while Example 5 shows that Condition (14) is no longer a sufficient condition under this assumption. In both examples we set $B = 1$.

Example 4. *Let* $M = \{1\}$ *and* $D = \{2, 3\}$*. Consider the following vector of abilities* $\alpha = (1, 0.105, 0.1)$*. Here disadvantaged agents are strong enough and always active. Straightforward calculation reveals that Condition (14) holds. Indeed the optimal extra prize is* $\beta^* = 0.35$*, inducing an almost 45% increase of total effort with respect to a standard contest. Once the extra prize is large enough (*$\beta = 0.948$*) agent 1 ceases to be active. The upper curve in Figure 3 displays this case.*

[18] Unlike the biased contest model in Franke et al. (2013) the optimal contest in case of $\alpha = 0.2$ has only two active agents.

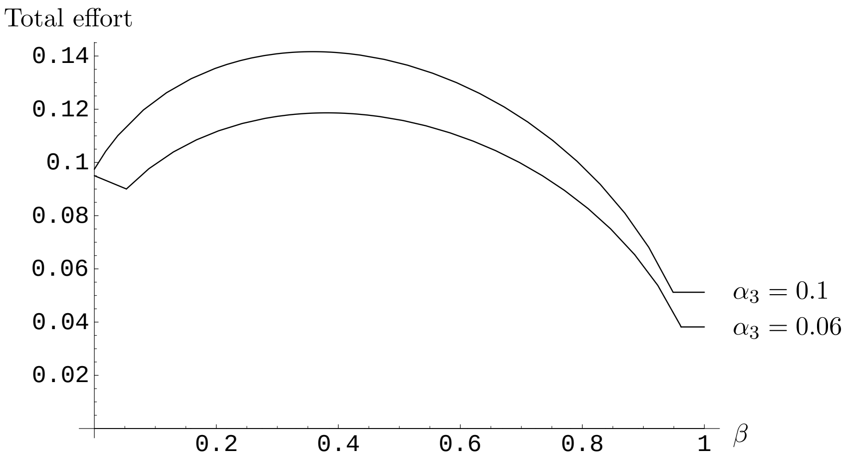

Figure 3: Total effort in Examples 4 and 5.

Example 5. *Let $M = \{1\}$ and $D = \{2, 3\}$. Consider the same abilities as in Example 4 but set $\alpha_3 = 0.06$. This weakens agent 3 so that he is not active without an extra prize. Notice that Condition (14) does not hold. Once the extra prize becomes large enough, however, agent 3 becomes active ($\beta = 0.0526$). This fosters competition and increases total effort. The optimal extra prize is $\beta^* = 0.38$, inducing an almost 45% increase of total effort with respect to a standard contest. Once the extra prize is large enough ($\beta = 0.96$) agent 1 ceases to be active. The lower curve in Figure 3 visualizes this case.*

Notice that Examples 3 and 5 imply that the increase in total effort triggered by the introduction of an extra prize might arise with or without an increase in minority representation.

4 Conclusions

This paper analysed the effects of establishing an extra prize for disadvantaged agents in a contest model. Examples of this affirmative action policy are gender quotas or prizes for national competitors in an international competition. We have shown that even very small extra prizes are very effective in making sure that there is minority representation in the competition. Moreover, for intermediate levels of the disadvantage of the minority, establishing an extra prize increases total equilibrium effort compared to a standard contest. Extra prizes might therefore be designed purely on efficiency grounds, which should facilitate the social acceptance of this affirmative action policy.

But in the real world affirmative action is not designed purely on efficiency grounds. In so far our assumption that the designer does not value minority representation at all–which is captured supposing that he is only interested in total effort–is very conservative and even unrealistic. In reality, as in the aforementioned examples of California's Disabled Veteran Business Enterprise and Small Business Certification Programs or the European Union's target shares for female representation on firms' boards, he will be willing to trade-off some effort for minority representation and thus might find it desirable to establish an extra prize even at the cost of some reduction in total effort.[19]

An important result in contest theory is the exclusion principle, see Baye et al. (1993). This principle applies when the contest success function is responsive enough to effort, as in the all-pay auction (see Baye et al. 1993; Alcalde and Dahm 2007; and Alcalde and Dahm 2010). It says that a contest designer might sometimes strengthen competition and increase total effort by excluding the contestant with the highest valuation from participating in the competition. For the contest success function employed in this paper, however, Fang (2002) has shown that the exclusion principle does not apply. Our analysis allows a deeper understanding of the forces underlying the exclusion principle. The reason is that establishing an extra prize reduces the main prize and *partially* excludes the most efficient competitor(s). As Fang (2002) has shown, complete exclusion is never beneficial. *Partial* exclusion, however, might foster competition and increase total effort. In this sense, a *partial* exclusion principle applies to Tullock contests.

Our analysis suggests several avenues for future research. A first generalizes the contest structure to several extra prizes. Think of researchers competing with very similar research proposals for funding from local governments (the extra prizes) and from the central government (the main prize). What is the optimal degree of decentralization of research funds? Another avenue endows the contest designer not only with the opportunity to create an extra prize but also with the power to choose the contestants that qualify for it. What is the optimal set of agents competing for the extra prize?

[19]Notice that $1-\Omega$ measures the win probability of advantaged agents, which declines as the extra prize increases (as $\partial\left(E_N/B_1\right)/\partial\beta > 0$ provided at least two disadvantaged agents are active). Thus the designer can easily balance the trade-off between total effort and minority representation.

A Appendix

A.1 Proofs

In this Appendix we provide a proof for the results stated in the main text. In addition to the notation introduced there, we simplify mathematical expressions using

$$\Sigma \equiv \frac{1}{4}\left(\frac{|M^*|-1}{|M^*|}\Gamma^*_M - \frac{|N^*|-1}{|N^*|}\Gamma^*_N\right)^2 + \frac{\Gamma_{M^*}\Gamma_{N^*}}{|M^*||N^*|}(|D^*|-1)\frac{\beta}{1-\beta},$$

and, given a group $G \in \{N, M, D\}$, it will also prove useful to define a weighted harmonic mean of abilities as

$$\Lambda_{G^*} \equiv \frac{|G^*|-1}{|G^*|}\Gamma_{G^*}.$$

In addition we simplify vectors of individual efforts when we focus on an agent i using the shorter notation $e = (e_i, e_{-i})$, where $e_{-i} = (\ldots, e_{i-1}, e_{i+1}, \ldots)$.

A.1.1 Proof of Lemma 1

For $\beta = 0$, the statement follows from Stein (2002). By way of contradiction let $\beta > 0$ and suppose that there is an equilibrium in which $e_i = 0$ for all agents $i \in D$. Suppose some advantaged agent exerts effort (otherwise a standard argument applies and proves the lemma). Notice that $EU_i(e_i, e_{-i}) = B_2/(n-m)$. Consider the alternative effort $\tilde{e}_i = \epsilon > 0$. This deviation yields

$$EU_i(\tilde{e}_i, e_{-i}) > B_2 - \frac{\epsilon}{\alpha_i}.$$

Since $B_2 - \epsilon/\alpha_i$ is larger than $B_2/(n-m)$ for ϵ small enough, an equilibrium cannot have $e_i = 0$ for all agents $i \in D$. Q.E.D.

A.1.2 Proof of Proposition 1

Consider the first statement. Since $|D^*_{M^*=\emptyset}| \geq 2$, we have that $\Gamma_{D^*_{M^*=\emptyset}} > 0$. Thus we have $\overline{\beta} < 1$.

Consider now the second statement. It follows from Stein (2002) that if $M^* = \emptyset$ no agent $i \in D$ can profitably deviate from the strategies described in the statement. So let the agents $i \in D$ use these strategies and assume that $j \in M$ deviates to $e_j > 0$. Since the payoffs are concave, $\partial E_j(e)/\partial e_j|_{e_j=0} > 0$ must hold. This implies $\alpha_j B_1 > E_D$, where $D = D^*_{M^*=\emptyset}$, or equivalently

$$1-\beta > \frac{|D^*_{M^*=\emptyset}|-1}{|D^*_{M^*=\emptyset}|}\frac{\Gamma_{D^*_{M^*=\emptyset}}}{\Gamma_{\{j\}}}.$$

But since $\alpha_1 \geq \alpha_j$ and $\beta \in [\overline{\beta}, 1]$, we have

$$\frac{|D^*_{M^*=\emptyset}|-1}{|D^*_{M^*=\emptyset}|}\frac{\Gamma_{D^*_{M^*=\emptyset}}}{\Gamma_{\{j\}}} \geq \frac{|D^*_{M^*=\emptyset}|-1}{|D^*_{M^*=\emptyset}|}\frac{\Gamma_{D^*_{M^*=\emptyset}}}{\Gamma_{\{1\}}} \geq 1-\beta,$$

a contradiction. Q.E.D.

A.1.3 Proof of Proposition 2

Consider the first statement. We have that $\overline{\beta} > 0$ if and only if

$$\alpha_1 > \frac{|D^*_{M^*=\emptyset}|-1}{|D^*_{M^*=\emptyset}|}\Gamma_{D^*_{M^*=\emptyset}}.$$

This inequality holds, because

$$\alpha_1 > \Xi_{D^*_{M^*=\emptyset}} \geq \Gamma_{D^*_{M^*=\emptyset}} > \frac{|D^*_{M^*=\emptyset}|-1}{|D^*_{M^*=\emptyset}|}\Gamma_{D^*_{M^*=\emptyset}},$$

where $\Xi_{D^*_{M^*=\emptyset}}$ is the arithmetic mean of abilities of the agents in $D^*_{M^*=\emptyset}$.[20]

Now consider the second statement and suppose $\beta \in (0, \overline{\beta})$. From Lemma 1 we know that $D^* \neq \emptyset$. By way of contradiction suppose that $M^* = \emptyset$. Then $e_1 = 0$, implying that $\partial E_1(e)/\partial e_1 \leq 0$ must hold. This implies $\alpha_1 B_1 \leq E_N$. On the other hand, $M^* = \emptyset$ implies that $E_N = E_D$, where $D = D^*_{M^*=\emptyset}$. Therefore the following must hold

$$1-\beta \leq \frac{|D^*_{M^*=\emptyset}|-1}{|D^*_{M^*=\emptyset}|}\frac{\Gamma_{D^*_{M^*=\emptyset}}}{\Gamma_{\{1\}}}.$$

Since, however $\beta < \overline{\beta}$ we obtain

$$\frac{|D^*_{M^*=\emptyset}|-1}{|D^*_{M^*=\emptyset}|}\frac{\Gamma_{D^*_{M^*=\emptyset}}}{\Gamma_{\{1\}}} < 1-\beta,$$

a contradiction. Q.E.D.

[20] It is well known that the the arithmetic mean is not smaller than the harmonic mean and a proof is thus omitted.

A.1.4 Proof of Proposition 3

We prove the statement with the help of two lemmatas.

Lemma 3. *For any* (α, B)*, if* $\beta \in (0, \overline{\beta})$*, then*

$$E_N \in \{\Upsilon - \sqrt{\Upsilon^2 - \Phi}, \Upsilon + \sqrt{\Upsilon^2 - \Phi}\}.$$

Proof: First notice that the two candidate expressions for E_N are well defined, because we can write

$$\begin{aligned}\Upsilon^2 - \Phi &= \frac{(B_1)^2}{4}\left(\frac{|M^*|-1}{|M^*|}\Gamma_{M^*} - \frac{|N^*|-1}{|N^*|}\Gamma_{N^*}\right)^2 \\ &\quad + \frac{B_1 \Gamma_{M^*}\Gamma_{N^*}}{|M^*||N^*|}(|D^*|-1)B_2 \geq 0. \end{aligned} \tag{16}$$

Summing up equation (3) over all $i \in M$ and rearranging yields

$$E_D = \frac{(E_N)^2}{B_1}\frac{|M^*|}{\Gamma_{M^*}} - (|M^*|-1)E_N. \tag{17}$$

Summing up equation (3) over all $i \in D$, inserting equation (17) and rearranging, yields the following quadratic equation

$$\begin{aligned} 0 &= (E_N)^2 \frac{\left(\sum_{i \in N^*}\frac{1}{\alpha_i}\right)\left(\sum_{i \in M^*}\frac{1}{\alpha_i}\right)}{B_1} \\ &\quad - E_N\left(\left(\sum_{i \in N^*}\frac{1}{\alpha_i}\right)(|M^*|-1) + \left(\sum_{i \in M^*}\frac{1}{\alpha_i}\right)(|N^*|-1)\right) \\ &\quad + (|N^*|-1)(|M^*|-1)B_1 - (|D^*|-1)B_2. \end{aligned} \tag{18}$$

From here we obtain

$$(E_N)^2 - E_N 2\Upsilon + \Phi = 0,$$

implying the statement. Q.E.D.

Lemma 4. *For any* (α, B)*, if* $\beta \in (0, \overline{\beta})$*, then*

$$E_N \neq \Upsilon - \sqrt{\Upsilon^2 - \Phi}.$$

Proof: Suppose $M^* \neq \emptyset$ and $E_N = \Upsilon - \sqrt{\Upsilon^2 - \Phi}$. Following equation (16), we can write $E_N = \Upsilon - \sqrt{X^2 + Y}$, where

$$\begin{aligned} X &\equiv \frac{B_1}{2}\left(\frac{|M^*|-1}{|M^*|}\Gamma_{M^*} - \frac{|N^*|-1}{|N^*|}\Gamma_{N^*}\right) \\ Y &\equiv \frac{B_1 \Gamma_{M^*}\Gamma_{N^*}}{|M^*||N^*|}(|D^*|-1)B_2. \end{aligned}$$

Since the function $f(x) = \sqrt{x}$ is increasing in its argument and $Y \geq 0$, we have $E_N \leq \Upsilon - \sqrt{X^2}$, where

$$\sqrt{X^2} = \begin{cases} \frac{B_1}{2}(\Lambda_{M^*} - \Lambda_{N^*}) & \text{if} \quad \Lambda_{M^*} \geq \Lambda_{N^*} \\ \\ \frac{B_1}{2}(\Lambda_{N^*} - \Lambda_{M^*}) & \text{if} \quad \Lambda_{M^*} < \Lambda_{N^*} \end{cases}. \tag{19}$$

The remainder of the proof distinguishes these two cases and shows that each of them leads to $E_D \leq 0$, contradicting Lemma 1.

Suppose $\Lambda_{M^*} \geq \Lambda_{N^*}$, which implies that $E_N \leq \Lambda_{N^*} B_1$. Using equation (17) we obtain

$$E_D \quad \leq \quad E_N \left(\Lambda_{N^*} \frac{|M^*|}{\Gamma_{M^*}} - (|M^*| - 1) \right) \leq 0,$$

where the last inequality comes from the fact that

$$\Lambda_{M^*} \geq \Lambda_{N^*} \Leftrightarrow \Lambda_{N^*} \frac{|M^*|}{\Gamma_{M^*}} \leq |M^*| - 1.$$

Suppose $\Lambda_{M^*} > \Lambda_{N^*}$, which implies that $E_N \leq \Lambda_{M^*} B_1$. Using equation (17) we obtain

$$E_D \quad = \quad E_N \left(\frac{E_N}{B_1} \frac{|M^*|}{\Gamma_{M^*}} - (|M^*| - 1) \right) \leq 0,$$

where the last inequality comes from the fact that

$$E_N \leq \Lambda_{M^*} B_1 \Leftrightarrow \frac{E_N}{B_1} \frac{|M^*|}{\Gamma_{M^*}} \leq |M^*| - 1.$$

Q.E.D.

Proposition 3 follows directly from Lemmatas 3 and 4. Q.E.D.

A.1.5 The derivatives mentioned in Subsection 3.3

Remember that by Proposition 2, $|D^*| \geq 1$ holds.

Claim 1. $\frac{\partial(E_N/B_1)}{\partial \beta} = 0$ *if* $|D^*| = 1$ *and* $\frac{\partial(E_N/B_1)}{\partial \beta} > 0$ *if* $|D^*| > 1$.

Proof: Let $\beta > 0$. Suppose $|D^*| = 1$. From equation (16), we have $E_N = \Upsilon + \sqrt{X^2}$, where

$$\sqrt{X^2} = \begin{cases} \frac{B_1}{2}(\Lambda_{M^*} - \Lambda_{N^*}) & \text{if} \quad \Lambda_{M^*} \geq \Lambda_{N^*} \\ \\ \frac{B_1}{2}(\Lambda_{N^*} - \Lambda_{M^*}) & \text{if} \quad \Lambda_{M^*} < \Lambda_{N^*} \end{cases}, \tag{20}$$

implying

$$E_N = \begin{cases} \Lambda_{M^*} B_1 & \text{if} \quad \Lambda_{M^*} \geq \Lambda_{N^*} \\ \\ \Lambda_{N^*} B_1 & \text{if} \quad \Lambda_{M^*} < \Lambda_{N^*} \end{cases} . \tag{21}$$

Assume $E_N = \Lambda_{M^*} B_1$. Equation (17) and Proposition 2 imply $E_N > \Lambda_{M^*} B_1$, a contradiction. Thus $E_N = \Lambda_{N^*} B_1$ and $\partial\left(E_N/B_1\right)/\partial\beta = 0$.

Suppose $|D^*| > 1$. We have that

$$\frac{E_N}{B_1} \quad = \quad \frac{1}{2}\left(\frac{|M^*|-1}{|M^*|}\Gamma_M^* + \frac{|N^*|-1}{|N^*|}\Gamma_N^*\right) + \sqrt{\Sigma},$$

with $\sqrt{\Sigma} > 0$. Taking the derivative we obtain

$$\frac{\partial\left(E_N/B_1\right)}{\partial\beta} = \frac{\Gamma_{M^*}\Gamma_{N^*}(|D^*|-1)}{2|M^*||N^*|(1-\beta)^2\sqrt{\Sigma}} > 0.$$

Q.E.D.

Claim 2. $\frac{\partial(B_2/(B_1\Omega)}{\partial\beta} > 0.$

Proof: Let $\beta > 0$. We have that

$$\frac{\partial\left(\frac{B_2}{B_1\Omega}\right)}{\partial\beta} = \frac{\partial\left(\frac{\beta}{1-\beta}\right)}{\partial\beta}\frac{1}{\Omega} + \frac{\beta}{1-\beta}\frac{\partial\left(\frac{1}{\Omega}\right)}{\partial\beta}.$$

Notice that

$$\frac{\partial\left(\frac{\beta}{1-\beta}\right)}{\partial\beta}\frac{1}{\Omega} = \frac{1}{(1-\beta)^2\Omega} > 0,$$

as $\Omega \in (0,1)$. Suppose $|D^*| = 1$. From Claim 1, we know that $E_N = \Lambda_{N^*} B_1$. This implies that $\Omega = 1 - \Lambda_{N^*}/\Gamma_{D^*}$. Therefore $\frac{\partial(1/(\Omega)}{\partial\beta} = 0$ and $\frac{\partial(B_2/(B_1\Omega)}{\partial\beta} > 0$.

Suppose $|D^*| > 1$ and remember that this implies that $\sqrt{\Sigma} > 0$. Using Claim 1, we obtain

$$\frac{\beta}{1-\beta}\frac{\partial\left(\frac{1}{\Omega}\right)}{\partial\beta} = -\frac{\beta}{(1-\beta)^3\Omega^2}\frac{\Gamma_{N^*}(|D^*|-1)}{2|N^*|\sqrt{\Sigma}}.$$

Hence we have that $\frac{\partial(B_2/(B_1\Omega)}{\partial\beta} > 0$ if and only if

$$1 - |M^*| + \frac{E_N}{B_1}\frac{|M^*|}{\Gamma_{M^*}} > \frac{\beta}{(1-\beta)}\frac{\Gamma_{N^*}(|D^*|-1)}{2|N^*|\sqrt{\Sigma}}.$$

Introducing E_N/B_1 from Claim 1 and rearranging yields

$$\frac{1}{2}\left(\frac{|N^*|-1}{|N^*|}\Gamma_N^* - \frac{|M^*|-1}{|M^*|}\Gamma_M^*\right) + \sqrt{\Sigma} > \frac{\beta}{(1-\beta)}\frac{|D^*|-1}{2\sqrt{\Sigma}}\frac{\Gamma_{N^*}}{|N^*|}\frac{\Gamma_{M^*}}{|M^*|}.$$

Multiplying by $2\sqrt{\Sigma}$ and collecting terms we obtain

$$\begin{aligned} & \frac{1}{2}\left(\frac{|M^*|-1}{|M^*|}\Gamma_M^* - \frac{|N^*|-1}{|N^*|}\Gamma_N^*\right)^2 + \frac{\beta(|D^*|-1)}{(1-\beta)}\frac{\Gamma_{N^*}}{|N^*|}\frac{\Gamma_{M^*}}{|M^*|} \\ > \; & \left(\frac{|M^*|-1}{|M^*|}\Gamma_M^* - \frac{|N^*|-1}{|N^*|}\Gamma_N^*\right)\sqrt{\Sigma}. \end{aligned}$$

Squaring and cancelling terms yields finally that $\frac{\partial(B_2/(B_1\Omega)}{\partial\beta} > 0$ if and only if

$$\frac{\beta(|D^*|-1)}{(1-\beta)}\frac{\Gamma_{N^*}}{|N^*|}\frac{\Gamma_{M^*}}{|M^*|} > 0,$$

which is true for $\beta > 0$. Q.E.D.

A.1.6 Proof of Proposition 4

We prove Proposition 4 in three steps. In Claim 3 we confirm that the candidate strategies in equations (8) and (9) are indeed an equilibrium. Second, we prove in Claim 4 uniqueness of the equilibrium in pure strategies. Lastly, we show in Claim 5 that there is no equilibrium in mixed strategies.

Claim 3. *The candidate efforts described in Proposition 4 constitute an equilibrium.*

Proof: Denote the vector of individual candidate equilibrium efforts by e^*. Consider an agent i and assume that all other agents exert the candidate equilibrium effort e_j^* for any $j \neq i$. Agent i chooses $e_i \geq 0$ in order to maximize equation (1). Since we already know that active agents do not have an incentive to deviate from the candidate equilibrium effort, consider inactive agents.

Consider first agent $i \in \{m^*+1, \ldots, m\}$. The first order condition evaluated at $e_i^* = 0$ is

$$\frac{1}{E_N(e^*)}B_1 - \frac{1}{\alpha_i} \leq 0,$$

which by equation (10) is negative. Hence $e_i^* = 0$ is indeed a best response.

Consider next agent $i \in \{d^*+1, \ldots, n\}$. The first order condition evaluated at $e_i^* = 0$ is

$$\frac{1}{E_N(e^*)}B_1 + \frac{1}{E_D(e^*)}B_2 - \frac{1}{\alpha_i} \leq 0,$$

which using the definition of Ω and equation (11) can be shown to be negative. Thus $e_i^* = 0$ is indeed a best response. Q.E.D.

Claim 4. *There is no other pure strategy equilibrium.*

Proof: Proceeding by contradiction, suppose that for a given β there are two different sets of active contestants $H \equiv N^m_{d-j}$ and $J \equiv N^{m-k}_d$. Notice that j and k must both be strictly larger than zero.[21] Moreover, Lemma 1 implies that we can focus on $1 \leq k \leq m$ and $1 \leq j < d$. In each equilibrium we indicate total effort by E_H and E_J; and distinguish similarly Ω_H and Ω_J.

In equilibrium H the following participation conditions must hold

$$\alpha_m > \frac{E_H}{B_1}, \quad \alpha_d \leq \frac{E_H}{B_1}\frac{B_1\Omega_H}{B_1\Omega_H + B_2},$$

while in equilibrium J we must have, when $k < m$,

$$\alpha_m \leq \frac{E_J}{B_1}, \quad \alpha_d > \frac{E_J}{B_1}\frac{B_1\Omega_J}{B_1\Omega_J + B_2},$$

and for $k = m$,

$$\alpha_d > \frac{E_J}{B_1}.$$

Notice that the conditions referring to agent m imply that $E_J > E_H$. We establish now that $\Omega_J > \Omega_H$ holds.

$$\begin{aligned}
\Omega_J - \Omega_H &= m\left(1 - \frac{E_H}{B_1\Gamma_{N^m_{d-j}\cap M}}\right) - (m-k)\left(1 - \frac{E_J}{B_1\Gamma_{N^{m-k}_d\cap M}}\right) \\
&= k - \frac{mE_H}{B_1\Gamma_{N^m_{d-j}\cap M}} + \frac{(m-k)E_J}{B_1\Gamma_{N^{m-k}_d\cap M}} \\
&> k - \frac{mE_H}{B_1\Gamma_{N^m_{d-j}\cap M}} + \frac{(m-k)E_H}{B_1\Gamma_{N^{m-k}_d\cap M}}.
\end{aligned}$$

This last expression is strictly larger than zero if and only if

$$\frac{B_1}{E_H}k - \sum_{i\leq m}\frac{1}{\alpha_i} + \sum_{i\leq m-k}\frac{1}{\alpha_i} = \frac{B_1}{E_H}k - \sum_{i=m-k+1}^{m}\frac{1}{\alpha_i} > 0.$$

Since

$$\sum_{i=m-k+1}^{m}\frac{1}{\alpha_i} \leq k\frac{1}{\alpha_m},$$

the last expression is implied by the participation condition of agent m in equilibrium H.

[21] If both are zero, the sets are the same. By construction of the sets, if $j = 0$ and $k > 0$, then, given d^*, $m - k$ is not the largest index, as $m > m - k$; and similarly for $j > 0$ and $k = 0$.

Lastly, notice that $E_J > E_H$ and $\Omega_J > \Omega_H$, on one hand, and the participation conditions of agent d in both equilibria, on the other, imply the following. For $k < m$,

$$\alpha_d > \frac{E_J}{B_1}\frac{B_1\Omega_J}{B_1\Omega_J + B_2} > \frac{E_H}{B_1}\frac{B_1\Omega_H}{B_1\Omega_H + B_2} \geq \alpha_d,$$

and for $k = m$,

$$\alpha_d > \frac{E_J}{B_1} > \frac{E_H}{B_1}\frac{B_1\Omega_H}{B_1\Omega_H + B_2} \geq \alpha_d.$$

In both cases we reach the desired contradiction. Q.E.D.

Claim 5. *There is no mixed strategy equilibrium.*

Proof: Given that equation (1) is strictly concave for $z_i = 1$, the assertion can be proved adapting Claim 3 in Fang (2002). Q.E.D.

The above three claims complete the proof of Proposition 4. Q.E.D.

A.1.7 Proof of Proposition 5

We start proving that E_N is continuous on $[0,1]$.

Lemma 5. *$E_N(\beta)$ is continuous on $[0,1]$.*

Proof: The statement follows from four claims.

Claim 6. *For $\beta = 0$, equation (7) reduces to equation (6).*

Proof: Let $\beta = 0$. As in the proof of Claim 1, from equation (16), we have $E_N = \Upsilon + \sqrt{X^2}$, where $\sqrt{X^2}$ is defined in equation (20). Again total effort is given in equation (21). Assume $E_N = \Lambda_{M^*}B_1$. Equation (17) implies that $E_D = 0$ and thus $\Lambda_{M^*}B_1 = \Lambda_{N^*}B_1$. Summarizing, for $\beta = 0$ equation (7) reduces to $E_N = \Lambda_{N^*}B_1$, equation (6). Q.E.D.

Claim 7. *For $\beta = \overline{\beta}$, equation (7) reduces to equation (6), which is constant on $[\overline{\beta}, 1]$.*

Proof: Let $\beta = \overline{\beta} - \epsilon$, for $\epsilon > 0$ arbitrarily close to zero. If $i \in M^*$ then $\alpha_i = \alpha_1$. This implies that $\Gamma_{M^*} = \alpha_1$. Moreover, when β goes to $\overline{\beta}$ we have that $e_i = 0$ for all $i \in M$ and $|N^*| = |D^*|$ (and $\Gamma_{N^*} = \Gamma_{D^*}$). Using these simplifications and the definition of $\overline{\beta}$ in equation (16), we have

$$\begin{aligned}
\Upsilon^2 - \Phi &= \left(B\frac{\Lambda_{D^*}}{\alpha_1}\right)^2 \left(\frac{1}{4}\Lambda_{M^*}^2 + \frac{\alpha_1^2}{|M^*|} + \frac{1}{4}\Lambda_{D^*}^2 + \frac{1}{2}\Lambda_{M^*}\Lambda_{D^*} - \frac{\alpha_1}{|M^*|}\Lambda_{D^*}\right) \\
&= \left(B\frac{\Lambda_{D^*}}{\alpha_1}\right)^2 \frac{1}{4}\left(\frac{(|M^*|+1)}{|M^*|}\alpha_1 - \Lambda_{D^*}\right)^2.
\end{aligned}$$

Since $(|M^*|+1)\alpha_1/|M^*| > \Lambda_{D^*}$,

$$\begin{aligned} E_N &= \frac{B}{2}\frac{\Lambda_{D^*}}{\alpha_1}\left(\frac{|M^*|-1}{|M^*|}\alpha_1 + \Lambda_{D^*} + \frac{|M^*|+1}{|M^*|}\alpha_1 - \Lambda_{D^*}\right) \\ &= \frac{|D^*|-1}{|D^*|}\Gamma_{D^*}B. \end{aligned}$$

Q.E.D.

We next show continuity of total effort when, given a set of active agents, the most inefficient advantaged agent (different from agent 1) ceases to be active.

Claim 8. *Let $m^* > 1$ and consider $E_N(\beta)$ given by equation (7). When β is such that $1/\alpha_{m^*} = B_1/E_N$, $E_N(\beta)$ is continuous.*

Proof: From the proof of Lemma 3, we know that equation (7) is the root of equation (18). We show that at the value for β in question, equation (18) is the same as a similar expression defined for the case in which agent m^* has exited. Thus the root must be the same, too. Using $1/\alpha_{m^*} = B_1/E_N$ in equation (18) we obtain

$$\begin{aligned} &\frac{(E_N)^2}{B_1}\left(\left(\sum_{i\in N^*\setminus m^*}\frac{1}{\alpha_i}\right)+\frac{B_1}{E_N}\right)\left(\left(\sum_{i\in M^*\setminus m^*}\frac{1}{\alpha_i}\right)+E_N\right) \\ &-E_N\left(\left(\left(\sum_{i\in N^*\setminus m^*}\frac{1}{\alpha_i}\right)+\frac{B_1}{E_N}\right)(|M^*|-1)+\left(\left(\sum_{i\in M^*\setminus m^*}\frac{1}{\alpha_i}\right)+\frac{B_1}{E_N}\right)(|N^*|-1)\right) \\ &+(|N^*|-1)(|M^*|-1)B_1-(|D^*|-1)B_2. \end{aligned}$$

Rearranging yields

$$\begin{aligned} &\frac{(E_N)^2}{B_1}\left(\sum_{i\in N^*\setminus m^*}\frac{1}{\alpha_i}\right)\left(\sum_{i\in M^*\setminus m^*}\frac{1}{\alpha_i}\right) \\ &-E_N\left(\left(\sum_{i\in N^*\setminus m^*}\frac{1}{\alpha_i}\right)(|M^*|-2)+\left(\sum_{i\in M^*\setminus m^*}\frac{1}{\alpha_i}\right)(|N^*|-2)\right) \\ &+(|N^*|-2)(|M^*|-2)B_1-(|D^*|-1)B_2, \end{aligned}$$

that is, equation (18) for m^*-1 active agents. Q.E.D.

We turn now to show continuity of total effort when, given a set of active agents, a disadvantaged agent becomes active or ceases to be active.

Claim 9. *Let $j \in D$ and consider $E_N(\beta)$ given by equation (7). When β is such that $1/\alpha_j = B_1/E_N + B_2/E_D$, $E_N(\beta)$ is continuous.*

Proof: We proceed as in the proof of Claim 8. Notice that when $1/\alpha_j = B_1/E_N + B_2/E_D$, agent i's optimal effort choice is zero. Using the equilibrium values for E_D and Ω this condition can be written as

$$B_2 = \frac{(E_N)^2}{B_1} \frac{\left(\sum_{i\in M^*} \frac{1}{\alpha_i}\right)}{\alpha_j} - E_N \left(\sum_{i\in M^*} \frac{1}{\alpha_i}\right) - \frac{(|M^*|-1)E_N}{\alpha_j} + (|M^*|-1)B_1.$$

Suppose agent $i \in D$ becomes active. From equation (18) we obtain

$$\begin{aligned}
&\frac{(E_N)^2}{B_1}\left(\sum_{i\in M^*}\frac{1}{\alpha_i}\right)\left(\left(\sum_{i\in N^*\cup j}\frac{1}{\alpha_i}\right) - \frac{1}{\alpha_j}\right) \\
&-E_N\left(\left(\sum_{i\in N^*\cup j}\frac{1}{\alpha_i}\right)(|M^*|-1) + \left(\sum_{i\in M^*}\frac{1}{\alpha_i}\right)|N^*|\right) + E_N\left(\frac{|M^*|-1}{\alpha_j} + \left(\sum_{i\in M^*}\frac{1}{\alpha_i}\right)\right) \\
&+|N^*|(|M^*|-1)B_1 - (|M^*|-1)B_1 - |D^*|B_2 + B_2.
\end{aligned}$$

Rearranging and using the above expression for B_2 yields

$$\begin{aligned}
&\frac{(E_N)^2}{B_1}\left(\sum_{i\in M^*}\frac{1}{\alpha_i}\right)\left(\sum_{i\in N^*\cup j}\frac{1}{\alpha_i}\right) - E_N\left(\left(\sum_{i\in N^*\cup j}\frac{1}{\alpha_i}\right)(|M^*|-1) + \left(\sum_{i\in M^*}\frac{1}{\alpha_i}\right)|N^*|\right) \\
&+|N^*|(|M^*|-1)B_1 - |D^*|B_2.
\end{aligned}$$

The proof of the case in which agent $i \in D$ ceases to be active proceeds along the same lines. Q.E.D.

We are now in a position to apply Weierstrass' extreme value theorem, which guarantees that for any (α, B), there is $\beta^* \in [0,1]$ such that $E_N(\beta)$ attains a maximum at β^*. Moreover, given that $E_N(\beta)$ is constant on $[\overline{\beta}, 1]$ and Proposition 1 in Fang (2002), which says that with the contest success function assumed the exclusion principle does not apply, we conclude that $\beta^* \in [0, \overline{\beta})$. Q.E.D.

A.1.8 Proof of Proposition 6

For simplicity, in this proof we drop the subindex ϵ. Consider $\beta \in (0, \bar{\beta}]$ for which total effort is given by equation (7). In order to compute the derivative with respect to β rewrite equation (7) as follows

$$E_N = B_1\left[\frac{1}{2}(\Lambda_{M^*} + \Lambda_{N^*}) + \sqrt{\Sigma}\right].$$

The derivative of total effort with respect to β can then be expressed as

$$\frac{\partial E_N}{\partial \beta} = \frac{\partial B_1}{\partial \beta}\left[\frac{1}{2}(\Lambda_{M^*} + \Lambda_{N^*}) + \sqrt{\Sigma}\right] + B_1\frac{\partial\left[\frac{1}{2}(\Lambda_{M^*} + \Lambda_{N^*}) + \sqrt{\Sigma}\right]}{\partial \beta}.$$

Deriving each term and rearranging, we obtain

$$\frac{\partial E_N}{\partial \beta} = \frac{B\Gamma_{N^*}\Gamma_{M^*}(|D^*|-1)}{2(1-\beta)\,|N^*|\,|M^*|\,\sqrt{\Sigma}} - \frac{B}{2}\,(\Lambda_{M^*}+\Lambda_{N^*}) - B\sqrt{\Sigma}.$$

This expression is well defined, because $|D^*| > 1$ implies that $\sqrt{\Sigma} > 0$. On the other hand when β goes to zero, $\sqrt{\Sigma}$ goes to

$$\begin{cases} 0 & \text{if} \quad \Lambda_{M^*} = \Lambda_{N^*} \Leftrightarrow \frac{\Gamma_{N^*}}{\Gamma_{M^*}} = \gamma \\ \frac{\Lambda_{M^*}-\Lambda_{N^*}}{2} & \text{if} \quad \Lambda_{M^*} > \Lambda_{N^*} \Leftrightarrow \frac{\Gamma_{N^*}}{\Gamma_{M^*}} < \gamma \\ \frac{\Lambda_{N^*}-\Lambda_{M^*}}{2} & \text{if} \quad \Lambda_{M^*} < \Lambda_{N^*} \Leftrightarrow \frac{\Gamma_{N^*}}{\Gamma_{M^*}} > \gamma \end{cases}.$$

Hence in the first case $\Lambda_{M^*} = \Lambda_{N^*}$, when β goes to zero, $\partial E_N/\partial\beta > 0$.

Assume the second case $\Lambda_{M^*} > \Lambda_{N^*}$. We have that $\partial E_N/\partial\beta|_{\beta=0} > 0$ if and only if

$$\frac{\Gamma_{N^*}\Gamma_{M^*}}{|N^*|\,|M^*|}\,(|D^*|-1) > \Lambda_{M^*}\,(\Lambda_{M^*}-\Lambda_{N^*})\,.$$

Because of Proposition 2 and $|D^*| > 1$, $|N^*| = |M^*| + |D^*| > 2$,

$$(|D^*|-1) + (|M^*|-1)(|N^*|-1) = |M^*|\,(|N^*|-2) > 0,$$

the previous expression can be rewritten as,

$$\gamma\delta < \frac{\Gamma_{N^*}}{\Gamma_{M^*}}. \tag{22}$$

Assume the third case $\Lambda_{M^*} < \Lambda_{N^*}$. We have that $\partial E_N/\partial\beta|_{\beta=0} > 0$ if and only if

$$\frac{\Gamma_{N^*}\Gamma_{M^*}}{|N^*|\,|M^*|}\,(|D^*|-1) > \Lambda_{N^*}\,(\Lambda_{N^*}-\Lambda_{M^*})\,,$$

yielding through a similar transformation

$$\frac{\Gamma_{N^*}}{\Gamma_{M^*}} < \zeta. \tag{23}$$

Summarizing, we have that $\beta^* > 0$ if one of the following holds:

$$\gamma\delta < \frac{\Gamma_{N^*}}{\Gamma_{M^*}} < \gamma; \qquad \frac{\Gamma_{N^*}}{\Gamma_{M^*}} = \gamma; \qquad \gamma < \frac{\Gamma_{N^*}}{\Gamma_{M^*}} < \zeta. \tag{24}$$

The fact that $\gamma\delta \leq \gamma < \zeta$ with the first inequality being strict whenever $|M^*| > 1$ implies the statement. Q.E.D.

A.1.9 Proof of Proposition 7

The fact that condition (15) implies that $\beta^* > 0$ follows from straightforward algebraic simplification of condition (14).

Suppose $\beta^* > 0$. Notice that $E_N(\beta = 0) > E_N(\beta = \overline{\beta})$ by Proposition 5. Note also that in between these values for β the function $E_N(\beta)$ is twice differentiable. The fact that when β goes to zero, $\partial E_N / \partial \beta > 0$ follows then from noticing that in between these values for β the function $E_N(\beta)$ is strictly concave and that the condition on the derivative implies condition (15).[22]

The last statement follows from observing that the upper bound of condition (15) is strictly larger than the lower bound. Q.E.D.

[22]The second derivative of total effort is negative,

$$\frac{\partial^2 E_N}{\partial \beta^2} = -\frac{B}{(1-\beta)\sqrt{\Sigma}} \left(\frac{\bar{\alpha}\Gamma_N(|D^*|-1)}{2(1-\beta)|M^*||N^*|\sqrt{\Sigma}} \right)^2 .$$

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