An information theory perspective on the informational efficiency of gold price

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Abstract

This paper studies the informational efficiency of the gold market, and its variability due to economic distress situations. The period under study goes from 1968 until 2017. We use quantifiers derived from Information Theory in order to analize the stochastic dynamics of gold price. In particular, we use permutation entropy, permutation statistical complexity and Fisher In-formation Measure, to assess the time varying dynamics of price time series. We find that the stochastic regime in the time series exhibits three distinct dynamics, roughly divided in years 1968-1981, 1981-2003, 2003-2017. Addi-tionally, informational efficiency is affected by major economic and political events. Finally, we detect a strong persistence in volatility.

Keywords:
gold, permutation entropy, statistical complexity, Fisher Information Measure, economic crisis
JEL: G01, G14

1. Introduction

During centuries, gold has been a representation of wealth. Several ancient communities around the world minted coins in this precious metal, in order to use them as an exchange good. In recent times, nation states issued money backed by gold reserves in their central banks. However, in the last quarter of the 20th century, fiat currencies abandoned the gold standard. Nowadays, gold is not (commonly) used for daily domestic transactions. Nevertheless, collective memory of societies keeps gold as a safe haven in situations of economic, financial or political distress. Therefore, gold has the particular characteristic of being, in addition to a popular traded commodity, a strategic investment asset. The importance of this market is reflected in the value of ounces transferred, with a daily average worth 24.8 billion as of March 2018 (London Bullion Market Association, 2018). In addition to gold itself, there are several derivatives, such as futures and options based on daily gold price. Consequently, the study of this commodity's price dynamics is of great relevance for investors.

The aim of this paper is to study the stochastic features of the time series of daily gold price and volatility, in order to identify if important economic events affect the underlying dynamics of price and volatility. Therefore, this paper

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deals, mainly, with testing the weak form of the Efficient Market Hypothesis (EMH) in this particular market. The EMH is a key element of financial economics, and broadly of economic theory. We use a set of powerful information-theory-derived quantifiers, which are able to discriminate different stochastic and chaotic dynamics in time series.

The informational efficiency of the gold market has been extensively analyzed, mainly by means of econometric techniques. The research by Tschoegl (1980) could be considered one of the first empirical studies on the EMH in the gold market, using serial correlation. Monroe and Cohn (1986) measure the forecasting success in order to assess the informational efficiency. More re-cently, Baur et al. (2014) use Bayesian analysis. Methodologies have evolved across time, and an overview could be consulted in Section 3. The development of techniques that are alternative to traditional econometrics, could shed light on some aspects. In particular, economic uncertainty can be tackled with elements borrowed from information theory and symbolic analysis (Ahn et al., 2019). Our proposal considers three information-theory-derived quantifiers, namely permu-tation entropy, permutation complexity and permutation Fisher Information Measure. These three measures allow to describe the degree of accomplishment of the EMH. In other words, unlike the traditional approach of testing if a mar-ket is efficient or not, our proposal innovates by identifying different degrees of gold market informational efficiency.

Our paper contributes to the existing literature in three main aspects: (i) it updates and enhances previous studies by considering the EMH as a dynamic process using sliding windows, (ii) it studies the persistence in price volatility, and (iii) it introduces a methodology based on information theory quantifiers to the gold market. This methodology is a suitable alternative to standard econometric tests.

The paper is organized as follows. Section 2 briefly review representative literature on the EMH. Section 3 focuses on EMH studies on the gold market. Section 4 introduces the methodology used in the paper. Section 5 describes data and discusses the main findings. Finally, Section 6 outlines the conclusions of our analysis.

2. The Efficient Market Hypothesis

Even though the first mathematical model of security prices is found in the doctoral dissertation of Bachelier (1900), its economic formalization remained latent until its theoretical development by Samuelson (1965). Shortly afterwards, Fama (1970) set the standard definition and classification of what is now known as the EMH: a market is informationally efficient if prices incorporate all relevant information. Roughly speaking, the EMH requires that returns of financial assets follow a memoryless stochastic process with respect to the underlying information set. Fama (1970), based on Roberts (1959), classify the informational efficiency into three categories, depending on the benchmark information set. The first category is the weak efficiency, where prices reflect all the information contained in the history of past prices. The second category is semi-strong efficiency, where prices embed all public information. Finally, the third category is strong efficiency, where prices are the result of processing all kind of information, public and private. The EMH is a necessary condition for the existence of equilibrium in a competitive market, in which arbitrage opportunities cannot exist. According to Ross (2005), prices are the result of individual decisions based on an information set. As a corollary, with the same information set it is not possible to obtain superior returns. Therefore, an investor, whose information set is the same of or inferior to the market information set, cannot beat the market. In addition, investors cannot control the flow of their informative endowment towards the market, since their own transactions (according to its direction and volume) act as signals to the market, tending, thus, to an equalization of the informative sets of the different participants. This produces that, on average, participants cannot beat the market on a regular basis. In its weak form, the EMH says that price time series of an speculative asset should be totally random. Consequently, the EMH precludes the possibility of prices forecasts based on past prices. As a consequence, experts have no advantage in developing investment strategies. In this line, Sharpe (1991) finds that active managed funds do not get (on average) better returns than passive managed funds. Even more, after considering operating costs the return of the former category is lower than the return of the latter category. Brock et al. (1992) reinforce the previous analysis, discarding the predictive power of technical analysis. Similarly, Metcalf and Malkiel (1994) conclude that, once controlled by risk, experts cannot beat the market, except by chance. Buguk and Brorsen (2003) detect that the verification of the EMH is sensitive to the applied methodology. They find that, while parametric tests backs the EMH, nonparametric tests find evidence against it. Most studies focus on the random walk properties.

Regarding used methodologies, and without the aim of being exhaustive, the majority of the papers use slight variations of one of the following methodologies: (a) runs tests, (b) unit root tests, (c) variance ratio tests, (d) long memory tests (e.g. Hurst exponent or fractional integration coefficient). One of the first studies, Cowles and Jones (1937), performs a runs test to assess the random distribution of sign changes in returns. Other studies centered into finding unit roots in the time series of prices (Kawakatsu and Morey (1999)). Lo and MacKinlay (1988) use variance ratio tests, in order to test the property, that, in a random walk, volatility growths with the square root of the holding period. Variance ratio tests are also available in non-parametric setting (Wright (2000)). Finally, long memory tests are based either in directly measuring the Hurst exponent (Di Matteo et al. (2005)) or fitting returns time series using Autoregressive Fractionally Integrated Moving Average (ARFIMA) models (Rodríguez (2017)).

Previous studies on the EMH in stock and bond markets found that informational efficiency varies through time (Ito and Sugiyama, 2009). There are various explanations regarding this changing memory component. Bariviera (2011) finds that time-varying long-range dependence in the Thai Stock Market is weakly influenced by the liquidity level and market size. Cajueiro et al. (2009)

find that financial market liberalization increases the informational efficiency in the Greek Stock Market. Kim et al. (2011) find that return predictability is altered by political and economic crises but not during market crashes.

Majumder (2012) concludes that efficiency is central to any market, but there are phases of specific inefficiency due to agents' behavior. One example is the so-called "herd effect", which causes occasional imbalances in the fair value of securities.

Cheung and Chinn (2001) find that macroeconomic news are quickly embedded into exchange rates. In addition, they find that short term price movements are more affected by speculative behavior rather than by economic fundamentals. However, speculation enhances market efficiency and liquidity, affecting positively to the market.

In the following section, we focus the literature review about EMH in the gold market.

3. Gold market informational efficiency

The literature on the informational efficiency of the gold market is very rich and varied. It covers different aspects, such as the role of gold as inflation hedge, the relationship between gold price and stock markets or macroeconomic variables, the degree of informational efficiency, etc. This section provides an overview on the current state of the art.

Bretton Woods agreements of 1944 forced to deliver, to the central banks of the 44 signatory countries, gold at USD 35 per ounce. The United States had to keep this price and change dollars for gold to the governments at any time. With an absolute freedom of movement of the gold, the gold standard was established as the most suitable mechanism to maintain the internal and external balance of the economies with a minimum intervention by the public powers. The Bretton Woods agreements favored the great economic stability of the 1950s and 1960s, given that there were no asset bubbles or major financial crises.

The association of the US dollar and gold lasted until 1971, when dollar convertibility to gold was cancelled. It was partially due to the large expenditures caused by the Vietnam War and to the fact that in 1971 the country had a trade deficit for the first time in the 20th century. According to Sjaastad (2008), the fact that exchange rates become floating is the biggest source of instability for the gold market, in the sense that strong appreciations or depreciations of the dollar against the euro or the yen, cause strong effects on the gold price.

O'Connor and Lucey (2012), and later Gilroy (2014), sustain that there is an inverse relationship between dollar and gold. A fall in the US dollar increases the value of other currencies and, therefore, their capacity to buy gold that quotes in dollars. On the other hand, a fall in the US dollar causes investors to look for other sources of value such as gold.

In broad sense, gold is considered a hedge against inflation, due to its reaction to changes in the consumer price index of any country. Several authors have studied this relationship. One of the first works on this topic is Chua and Woodward (1982). They study the period between 1975 and 1980 in 6 industrial countries, concluding that gold is only an effective hedge in the US, and for an investment period that ranges between 1 and 6 months. Ghosh et al. (2004) reaffirm that gold is a good hedge only for certain circumstances. The rationale for such behavior is that changes in interest rates and default risk (among others), do not affect directly gold volatility in the short term. Consequently, it conveys gold the capability to hedge against inflation in the long term. According to Worthington and Pahlavani (2007) (using monthly data between 1945 and 2006), gold has been an effective hedging asset. Beckmann and Czudaj (2013) were not so optimistic, affirming that gold can hedge partially inflation in the long term, in the US and UK, and to a lesser extent in Japan and the Euro Zone for the period between 1971 and 2011. Hoang et al. (2016), instead, argue that gold is useful as a hedge in countries such as US, UK and India but only for the short term. Aye et al. (2016) carry out one of the most extensive works, using yearly data from 1833 to 2013. They note a mixed result, where gold only performs as a hedge in some periods of history.

Measuring the impact of macroeconomic news on gold is very important for most investors, policy makers and academics. Gold is usually considered an important asset due to its role as an investment asset, a store of value, or a safe haven. In that sense, Lawrence (2003), analyzing quarterly data from 1975 to 2001 of gold price, macroeconomic data of the US and its financial markets, argues that there is no statistically significant correlation among gold profitability and the changes in the following macroeconomic variables: Gross Domestic Product, inflation and the interest rates.

According to Barnhart (1989), only two macroeconomic announcements influences (with statistical significance) gold price: the US M1 monetary aggregate, and the Federal Reserve unexpected increase in the interest rate. However Ghura (1990) upholds that between 1985 and 1989 only an unexpected employment report has an statistically significant effect on the gold price, but not the inflation. On the other hand, Hess et al. (2008) point out that gold price reaction to macroeconomic news between 1989 and 2005 depends on the phase of the economy. Roache and Rossi (2010) stand out that gold behaves differently than the other commodities, strengthening its role as a store of value or a safe haven. Specifically, bad macroeconomic news affects gold price much more than the good ones, increasing its price and developing a countercyclical role.

Regarding gold price volatility, Christie-David et al. (2000) verify that, between 1992 and 1995, volatility is higher around monthly macroeconomic news announcements, and especially when focused on inflation, employment rate, GDP and industrial production. In that sense, Cai et al. (2001) add personal income to the former study, albeit in his opinion, what causes the biggest falls in the gold price is the massive gold reserves sales by Central Banks.

Some authors study the importance of gold as a safe haven, through the relationship between gold and stock markets. Baur and McDermott (2010) show, through an econometric and descriptive analysis for the period between 1979 and 2009, that gold behaves a safe haven especially for the US and Europe's stock markets, but not for the Australian, Canadian and Japaneses ones. Contradicting previous works, Coudert and Raymond (2010) affirm that for the American, British, French, and German stock markets the correlation is close to zero in the short term, but not for the long term. The reason for such behavior is attributed to the existence of a negative correlation in several episodes of the crisis, which does not allow investors to construct a hedging portfolio immune to all crises. Smith (2001), nine years earlier, had already verified that in the short term, the relationship between gold and the main US's indices is very low and insignificant, and depends basically on the studied period. Baur and Lucey (2010) expand the study framework to the relationship to the bond markets, asserting that gold is not a safe haven for them, and for stock markets only at most for 15 trading days.

Tests carried out by Gallais-Hamonno et al. (2015) at the official gold market between 1948 and 1955 in Paris, reject the hypothesis of the random walk. The weak informational efficiency hypothesis cannot also be accepted, even, for the clandestine market between 1941 and 1948. They conclude then, that changes in the legal status of the market, going from clandestine to legal, does not affect the informational inefficiency. Hoang (2014) corroborates the same conclusions of informational inefficiency for Paris and London markets between 1948 and 2008.

Booth and Kaen (1979) studied the behavior of the daily gold and silver prices in the London Market during 1972 to 1979 in order to verify the EMH, concluding that the London gold market is inefficient. Solt and Swanson (1981) conclude that investors can neither before nor after transaction costs generate superior returns through technical analysis, that is to say, they can not improve the "buy and hold" strategy, due to the informational efficiency of the market. Similarly, Aggarwal and Soenen (1988) note that the market is efficient in its weak form, and investors can not generate extraordinary returns. In the same line, Charles et al. (2015) affirm that gold market has the highest degree of weak efficiency and, this has gradually improved over time due to the fact that the forecast capacity of the price of this metal has been showing a downward trend. Baur and Glover (2015) insist on that the gold price throughout history has shown an explosive bubble behavior in certain periods between 2002 and 2012, and that rather than responding to its real demand, it responds to chartist behaviors of price speculation, affecting its credibility as a refuge value and as an efficient market. Białkowski et al. (2015) also study the presence of an asset bubble in the gold market, using a Markov regime-switching Augmented Dickey-Fuller test. Nicolau and Palomba (2015) analyze the dynamic relationship between futures and spot prices of gold, in order to search for a relationship between them. They conclude that it is not possible to predict future prices based on spot prices. Finally, Wendt et al. (2018) test many different gold trading strategies *vis-à-vis* buy-and-hold strategy, claiming that active trading does not offer superior returns than a passive strategy. In other words, these findings support the EMH. As it can be observed, previous empirical studies are inconclusive, being results highly dependent on the period under study and the econometric techniques used. In order to overcome these drawbacks, we propose an alternative methodology derived from information theory, which is suitable for studying financial time series. The use of information theory quantifiers, such as permutation entropy, allows to characterize the degree of unpredictability in the future evolution of financial variables, such as the gold price and volatility. The proposed methodology is robust to the presence of (observational) noise, is computationally efficient, and has been successfully tested in other markets, such as commodities (Zunino et al., 2011a), LIBOR rates (Bariviera et al., 2016), and cryptocurrencies (Bariviera et al., 2018b; Stosic et al., 2019), among others.

4. Information theory quantifiers

As mentioned in Section 2, there is a variety of econometric methodologies used in the verification of the random walk hypothesis in stock, bond and gold markets. However, those methodologies have two major drawbacks. The first one is that they assume *a priori* a distribution for returns. The second one is that they give a yes-or-no answer: the market is either efficient or inefficient. We believe, following Farmer and Lo (1999), that informational efficiency is a matter of degree. Precisely, this two problems are solved using information theory quantifiers. On one hand, the proposed methodology in this paper does not require to assume any specific distribution for prices or returns. On the other hand, it is possible to assess the degree of efficiency/inefficiency of a given time series. This is specially relevant when comparing different time series or different periods, as in our paper.

The realizations of different physical, natural or economic processes generate data in form of time series (TS). They are usually the departing point for many empirical studies, given the extremely value of information they convey.

Methods based upon information theory constitute a research strand aimed at detecting determinism in TS. In this section we describe the information theory quantifiers that help to characterize the underlying dynamics of the time series.

4.1. Permutation entropy

Without loss of generality, let consider a discrete random variable x. We are interested in measuring the quantity of information received when we observe this variable. This measure will depend on the probability distribution $p(x)$. Entropy arises a natural measure of information for dynamical systems. It is related to the disorder of the system. In other words, it is related with the probability of occurrence of the different accessible states. In the case of a certain event, the information we receive is minimal. Given that the event was certain, we already knew it was going to occur, reaching a Shannon entropy equal to zero. However, in case of a very unlikely event, the knowledge on the occurrence of such event gives us a great amount of information. In fact, the uncertainty regarding the event will be maximal for the uniform distribution. This rationale inspired Shannon and Weaver (1949) to define the celebrated

Shannon's Entropy as:

$$S[P] = -\sum_{j=1}^{N} p_j \ln(p_j) \quad (1)$$

where $P = \{p_j; j = 1, \ldots, N\}$ is a given probability distribution. This is a "global" measure, not very sensitive to strong local changes in the distribution. $S[P]$ can be normalized, $0 \leq \mathcal{H} \leq 1$, dividing it by the maximum attainable value[1]

$$\mathcal{H}[P] = \frac{S[P]}{S_{\max}} = \frac{-\sum_{j=1}^{N} p_j \ln(p_j)}{\ln N} \quad (2)$$

Entropy, as a measure of disorder, is closely related to the idea of an ef-ficient market. In this sense, when $\mathcal{H}$ is close to 0 the stochastic process has a strong deterministic component, meaning a low degree of informational effi-ciency. Contrary, when $\mathcal{H} \approx 0.92$ the stochastic process follows approximately a random walk, fulfilling the EMH.

4.2. Statistical complexity

Even though Shannon's entropy is a widespread measure of the uncertainty associated to a physical process (proxied by its probability density function), according to Feldman and Crutchfield (1998) and Feldman et al. (2008), entropic measures are not sufficient to fully describe the dynamics of a complex signal. A measure of complexity is necessary to appreciate its degree of physical structure. There is not a strict consensus on the definition of complexity. However, we can agree that there are two special instances, that can be regarded as non complex: (i) perfect order (i.e. an arithmetic sequence), and (ii) complete randomness (i.e. a fair coin toss). These two cases are simple and lack of complex physical structure. In between, there is a wide range of processes, whose correlation structure is non trivial.

López-Ruiz et al. (1995) proposed a statistical measure of complexity, consisting in the interaction between an entropic measure and a "disequilibrium" between two probability density functions (one from the system under study and the other from a benchmark system):

$$\mathcal{C}[P] = \mathcal{H}_S[P] \cdot \mathcal{Q}[P, P_e] \quad (3)$$

where $\mathcal{H}_S[P]$ is the normalized Shannon entropy, and $\mathcal{Q}[P, P_e]$ is the so-called *disequilibrium*, between our system distribution (P) and the uniform distribution (P_e). The original proposal used the Euclidean distance as a measure of

[1] (Permutation) entropy yields a maximum when the knowledge of the system is minimal. This is tantamount to say that every accessible state has the same probability. In other words, permutation entropy is maximal in case of a uniform probability density function. In fact, let consider a uniform distribution, $P_e = \{p_i = 1/N, \forall i = 1 \cdots N\}$. In this case $S[P_e] = -\sum \frac{1}{N} \ln \frac{1}{N} = -N \frac{1}{N}(-\ln N) = \ln N = S_{\max}$

disequilibrium. In a series of papers, Rosso and coauthors, proposed first the Wootters statistical distance, and later the Jensen-Shannon divergence in order to quantify the disequilibrium (Martin et al., 2003; Lamberti et al., 2004; Martín et al., 2006; Rosso et al., 2007), obtaining better results regarding the discrimination of the systems dynamics.

The statistical complexity used in this paper computes the disequilibrium using the Jensen-Shannon divergence, as it was shown in several econophysics and biophysics applications to successfully distinguish between Gaussian and non-Gaussian processes (Zunino et al., 2010, 2011b, 2012; Fernández Bariviera et al., 2013).

4.3. The Fisher's Information Measure

The *Fisher's Information Measure* (FIM), $\mathcal{F}$, constitutes a measure of the gradient content of the distribution $f(x)$ (Fisher, 1922; Frieden, 2004). As a consequence, it is sensitive even to tiny localized perturbations. It can be written as:

$$\mathcal{F}[f] \;=\; \int_{\Delta} \frac{1}{f(x)} \left[\frac{df(x)}{dx}\right]^2 \, dx \;=\; 4 \int_{\Delta} \left[\frac{d\psi(x)}{dx}\right]^2 \; . \tag{4}$$

FIM is subject to interpretation from several viewpoints. It can be thought as the ability to estimate a parameter, as the amount of information that can be extracted from a set of measurements, or as a measure of the state of disorder of a system (Frieden, 2004). In equation 4, the division by $f(x)$ is not convenient if $f(x) \to 0$ at certain $x-$values. We avoid this if we work with real probability amplitudes $f(x) = \psi^2(x)$ (Fisher, 1922; Frieden, 2004), which is a simpler form (no divisors) and shows that $\mathcal{F}$ simply measures the gradient content in $\psi(x)$. The gradient operator significantly influences the contribution of tiny local $f-$variations to FIM's value. Accordingly, this quantifier can be regarded as a "local" one (Frieden, 2004).

When working with discrete probability density functions (as in our empirical case), a problem of information-loss arises due to discretization. This issue has been studied previously (Olivares et al., 2012a,b), and entails no problem for our present purposes. For the FIM we take the expression in terms of real probability amplitudes as starting point, then a discrete normalized FIM, $0 \le \mathcal{F} \le 1$, convenient for our present purposes, is given by

$$\mathcal{F}[P] \;=\; F_0 \sum_{j=1}^{N-1} [(p_{j+1})^{1/2} - (p_j)^{1/2}]^2 \; . \tag{5}$$

where p_{j+1}, p_j are two consecutive probabilities. It has been extensively discussed that this discretization is the best behaved in a discrete environment (Sánchez-Moreno et al., 2009).

Here the normalization constant F_0 reads

$$F_0 \;=\; \begin{cases} 1 & \text{if } p_{j^*} = 1 \text{ for } j^* = 1 \text{ or } j^* = N \\ & \text{ and } p_j = 0 \;\forall j \neq j^* \\ 1/2 & \text{otherwise} \end{cases} \; . \tag{6}$$

Olivares et al. (2012a,b) showed that this FIM discrete version behaves in opposite direction to that of Shannon entropy, except for periodic motions. The local sensitivity of FIM is reflected in the ordering of the different probability states p_j. Thus, finding a "proper" order is relevant. Different ordering of the summands in Equation 5, would lead to a different FIM value. The reason for this is that summands can be interpreted as a sort of distance between two juxtaposed probabilities. In this work, following Bariviera et al. (2015), we follow the lexicographic order described in Schwarz (2014) in the generation of Bandt-Pompe probability density function (see subsection 4.5).

The simultaneous analysis of local FIM and global Shannon entropy conforms the Shannon–Fisher plane, $\mathcal{H} \times \mathcal{F}$, introduced by Vignat and Bercher (2003). These authors revealed the usefulness of this plane in order to characterize the non-stationary behavior of complex signals.

4.4. Permutation Efficiency Index

Considering that the aim of this paper is to study the variations on the degree of informational efficiency along the period under study, we will use the index proposed by Bariviera et al. (2015), based on the permutation entropy and Fisher Information Measure of the time series. The aforementioned index, $\mathcal{E}$, is:

$$\mathcal{E}[P] = \mathcal{H}[P] - \mathcal{F}[P] \tag{7}$$

Both quantifiers, $\mathcal{H}$ and $\mathcal{F}$, are bounded between 0 and 1, and their behavior is opposite. Consequently, $\mathcal{E}$ is also bounded in the interval $[-1, 1]$. The most informational efficient behavior (*i.e.* the most random behavior) is when Shannon entropy is maximized and Fisher information minimized.

4.5. Probability density function estimation: the Bandt-Pompe method.

The previous subsections made clear that the key element in computing the information-theory-based quantifiers, is the generation of an appropriate probability density function (PDF) of the time series. Two popular extraction procedures are based on (i) histograms, and (ii) symbolic analysis.

Histogram-based methodology has the drawback that the temporal ordering is destroyed. Since its focus is to extract a PDF via amplitude, elements in the time series are reordered, breaking thus the chronological order. In addition, the selection of the number of bins in the partition is a nontrivial issue (De Micco et al., 2008).

A promising alternative is offered by the symbolic analysis of time series. In this line, Bandt and Pompe (2002) developed a methodology (hereinafter BP) to evaluate the PDF based on the ordinal structure of the time series. A deeper discussion and comparison between these two PDF extraction procedures can be found in Kowalski et al. (2012).

Given a time series $\mathcal{S}(t) = \{x_t; t = 1, \ldots, M\}$, one must select an embedding dimension D, and a time delay τ. The embedding dimension generates partitions of the pertinent $D-$dimensional space, and the delay the sampling frequency. For example, for $D = 4$ and $\tau = 1$, we take 4 consecutive observations of the time series; then we move forward 1 position, and take the following 4 consecutive observations; this process continues until the end of the time series. Clearly, the greater the embedding dimension D, the more temporal information in our vectors. BP methodology is a symbolic technique that transforms the partitioned time series into a finite number of symbols, based on the ordinal position of the observations within each vector. Without lost of generality, let consider an embedding dimension $D > 1$, and $\tau = 1$. Vector s is given by:

$$(s) \mapsto (x_{s-(D-1)}, x_{s-(D-2)}, \dots, x_{s-1}, x_s) \tag{8}$$

By "ordinal pattern" related to (s) we mean the permutation $\pi = (r_0, r_1, \dots, r_{D-1})$ of the symbols $(0, 1, \dots, D-1)$ defined by

$$x_{s-r_{(D-1)}} \leq x_{s-r_{(D-2)}} \leq \cdots \leq x_{s-r_1}, x_{s-r_0} \tag{9}$$

For continuous distributions, equal values are very unusual, so we set $r_i < r_{i-1}$ if $x_{s-r_i} = x_{s-r_{i-1}}$. For example, let consider the first seven values of our actual time series

$$\{x_i\} = \{38.00, 37.60, 37.70, 36.70, 37.20, 37.00, 37.25\} \tag{10}$$

The BP symbolization for $D = 4$ and $\tau = 1$ is:

$$\begin{aligned}
(38.00, 37.60, 37.70, 36.70) &\mapsto (3\ 1\ 2\ 0) \\
(37.60, 37.70, 36.70, 37.20) &\mapsto (2\ 3\ 0\ 1) \\
(37.70, 36.70, 37.20, 37.00) &\mapsto (1\ 3\ 2\ 0) \\
(36.70, 37.20, 37.00, 37.25) &\mapsto (0\ 2\ 1\ 3) \\
&\dots
\end{aligned} \tag{11}$$

The generation of ordinal patterns is straightforward, since it comes for the comparison of juxtaposed values of the time series. As a consequence, each vector is converted into a unique symbol $\hat{x}_i$. Then, a PDF function can be constructed using the frequency of such symbols: $P_{\hat{x}_i} = \{p(\hat{x}_i, i = 1, \dots, D!)\}$. Taking a sufficiently long time series, all ordinal patterns will appear (Carpi et al., 2010). For an ordinary white noise, BP pattern distribution should be uniform. It was recently found in Bariviera et al. (2018a) that correlated noises, specially those with a Hurst exponent greater than 0.5, generate PDF distributions that moves away from the uniform distribution, generating preferred patterns. In addition, Bariviera et al. (2018a) showed that the sampling frequency τ gives insight on the underlying dynamics of the process.

The advantages of BP method are, beyond its simplicity, its robustness and its invariance to nonlinear monotonous transformations. Daily time series in traded and liquid markets such as the one under examination in this paper minimize the presence of tied consecutive values. Therefore, there is a low risk of detecting spurious correlations. However, a previous examination of the time series is required, in order to determine if additional techniques should be used to "break" the presence of tied values. For a discussion on different options to

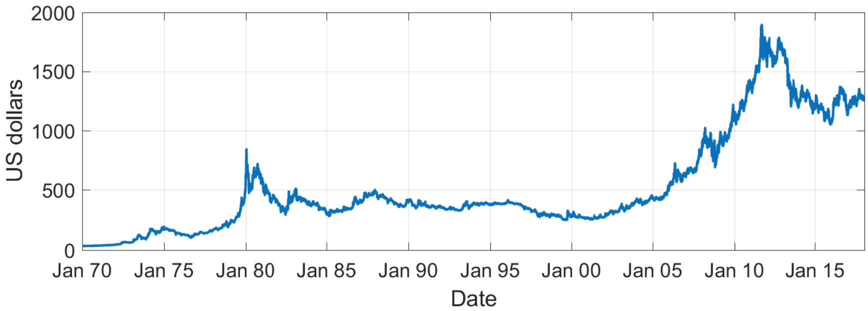


Figure 1: Daily price of Gold Bullion LBM U /Troy Ounce-A.M. official

deal with time series with several tied values, see Zunino et al. (2017); Traversaro et al. (2018). Finally, and given that we work with local windows of 250 data, non-stationarity of the whole time series is not an issue. In fact, BP methodology only requires windows with stationary increments, which is fulfilled in our case.

5. Data and results

We use daily price data of gold (Gold Bullion LBM U /Troy Ounce-A.M. Official). The period under examination goes from 01/04/1968 to 08/12/2017, for a total of 12965 observations. All data used in this paper was retrieved from Thomson Reuters Eikon. A graphical representation of the time series is shown in Figure 1. Additionally, we compute descriptive statistics of the whole time series and several subperiods, as displayed in Table 1.

We also use the highest and lowest daily price, in order to compute daily volatility. These data are only available from 02/10/1989 until 08/12/2017, constituting 7355 observations. Thus, daily volatility is measured as the logarithmic return of the daily extreme prices:

$$\text{vol} = \ln P_t^{\max} - \ln P_t^{\min} \tag{12}$$

where $P_t^{\max}$ and $P_t^{\min}$ are the maximum and minimum prices on the same day, respectively. The graphical representation of daily volatility is presented in Figure 2.

In order to observe the varying informational efficiency, we compute the quantifiers detailed in Section 4 using sliding windows. The sliding window approach works as follows: we compute the quantifiers for the first 250 datapoints (roughly one trading year), then move forward by 20 datapoints (roughly one trading month), select the following 250 observations and compute the quantifiers. We continue in this way until the end of the data. Each window represents, roughly, one trading year; and the step forward is approximately one trading month. As a consequence of this procedure we obtain 635 (344) windows using price (volatility) time series. We compute the PDF according to the BP method,

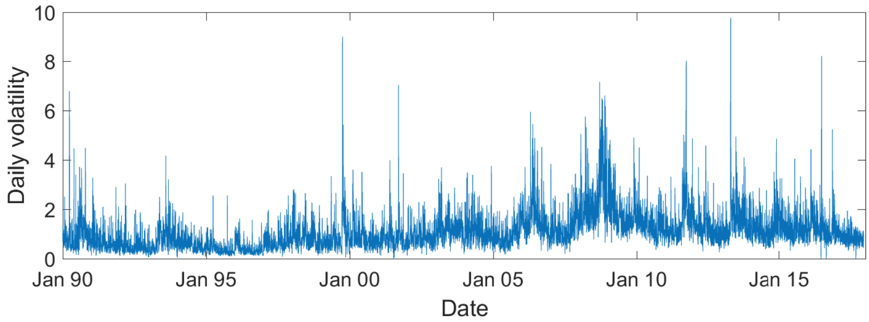


Figure 2: Daily price volatility as defined by Equation 12.

Table 1: Descriptive statistics of gold prices, computed for the whole sample and for subsamples between two economics events detailed in Table 2

	01/04/68 - 08/12/17	A: 01/04/68- 31/08/71	B: 01/09/71- 31/10/73	C: 01/11/73 31/01/79	D: 01/02/79 31/08/82	E: 01/09/82 30/11/89	F: 01/12/89- 31/12/90
Observations	12465	660	483	1278	906	1843	281
Mean	518.902	39.584	72.266	157.183	448.043	392.071	385.458
Median	379.900	39.850	65.350	155.425	428.250	392.600	383.400
1st Quartile	283.425	37.750	48.750	135.888	342.800	345.700	370.850
3rd Quartile	616.425	41.438	91.550	175.563	551.750	431.823	403.250
Minimum	34.950	34.950	41.450	90.300	229.600	284.200	345.300
Maximum	1898.990	44.250	126.450	242.750	835.000	509.200	422.700
Range	1864.040	9.300	85.000	152.450	605.400	225.000	77.400
Interq. range	333.000	3.688	42.800	39.675	208.950	86.123	32.400
St. Dev.	421.594	2.562	25.048	28.835	131.749	49.779	19.590
Kurtosis	0.740	-1.019	-0.886	0.013	-0.862	-0.924	-0.999
Skewness	1.319	-0.191	0.615	0.420	0.295	0.007	0.120

	G: 02/01/91- 29/08/97	H: 01/09/97- 28/09/01	I: 01/10/01- 30/09/08	J: 01/10/08- 31/01/12	K: 01/02/12- 30/06/16	L: 01/07/16- 30/11/16	End: 01/12/16- 08/12/17
Observations	1735	1062	1822	870	1150	109	266
Mean	367.288	283.237	510.267	1232.035	1352.809	1300.871	1248.973
Median	372.500	283.800	428.800	1196.100	1285.075	1319.840	1255.205
1st Quartile	350.300	270.600	361.500	953.975	1205.135	1266.205	1225.740
3rd Quartile	384.500	293.200	647.075	1436.288	1572.903	1337.505	1280.350
Minimum	317.600	252.300	272.500	710.300	1051.360	1172.700	1128.340
Maximum	415.100	334.600	1002.300	1898.990	1788.550	1366.400	1348.800
Range	97.500	82.300	729.800	1188.690	737.190	193.700	220.460
Interq. range	34.200	22.600	285.575	482.313	367.768	71.300	54.610
St. Dev.	20.978	16.590	190.411	292.441	203.203	48.680	44.784
Kurtosis	-0.985	0.091	-0.428	-0.925	-0.860	-0.095	0.400
Skewness	-0.386	0.456	0.810	0.297	0.694	-0.902	-0.689

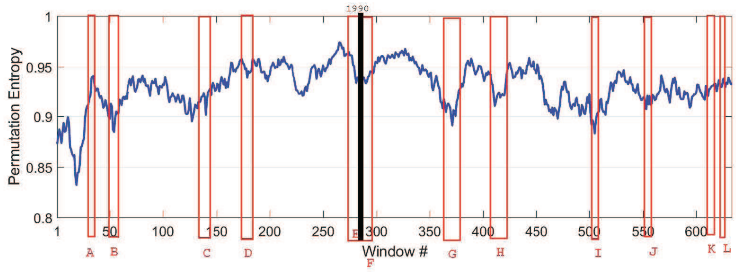


Figure 3: Permutation entropy evolution across the different sliding windows, computed with parameters $D = 4, \tau = 1, \delta = 20$. Letters in red refer to the events detailed in Table 2. The bold black vertical line indicates the starting point of data for volatility analysis.

using $D = 4$ (pattern length), $\tau = 1$ (daily sampling), and $\delta = 20$ (window jump of 20 datapoints).

The economic and political events considered in this paper are detailed in Table 2.

Figure 3 clearly shows that the "disorder" or permutation randomness in the price time series varies throughout the period under study. We use a normalized permutation entropy. It means that $\mathcal{H}$ is bounded in the interval $[0, 1]$, where $\mathcal{H} = 0$ means a fully deterministic process, $\mathcal{H} \approx 0.92$ represents a random behavior, and $\mathcal{H} \approx 1$ means an antipersistent fractional Brownian process.

In the inception of the market, there is a remarkably inefficiency ($\mathcal{H} < 0.9$). We can observe that different economic and political events are contemporary of periods with changes in the degree of randomness. In particular, most global events overlaps to periods of diminishing permutation entropy, i.e. diminishing informational efficiency. These results could indicate that investors could exploit active trading strategies during periods of reduced informational efficiency. That is to say, when $\mathcal{H}$ diverges from the 0.92 value. The more far away from this value, the greater probability to beat the market.

In order to give a better understanding of the behavior of the time series of gold prices, considered as realizations of a dynamical system, we introduce in Figure 4 the Complexity Entropy Causality Plane (CECP). This planar representation, consisting in the permutation entropy in the $x-$axis, and the permutation statistical complexity in the $y-$axis, allows for a better discrimination of

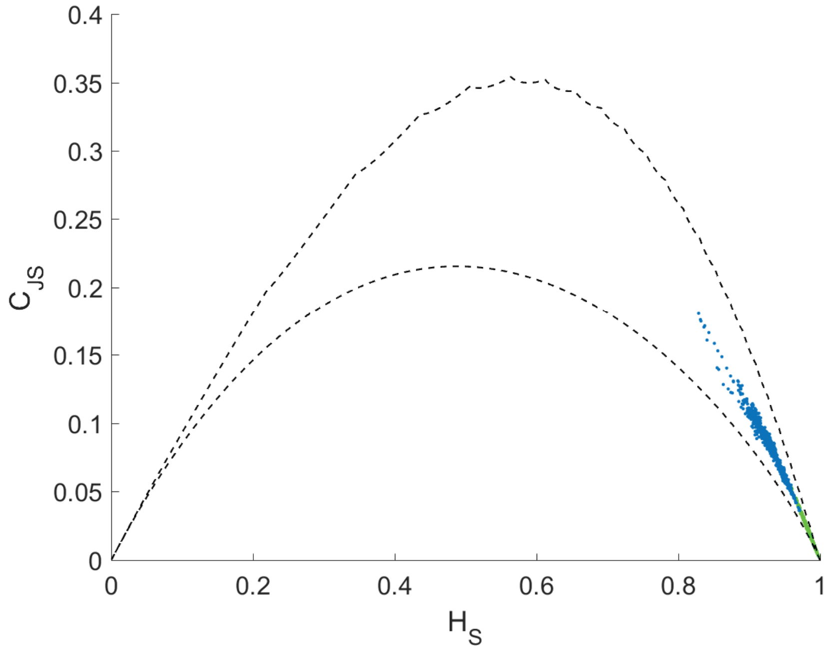
Table 2: Economic and political events

Graphic reference	Orientative date	Event	Window #
A	Aug-1971	Abandonment of the dollar's convertibility into gold, and later collapse of the Bretton Woods Agreements.	35
B	Oct-1973	Oil crisis of 1973: OPEC halted oil production and established an embargo for oil shipments to western countries.	50
C	Jan-1979	Oil crisis of 1979 due to the Iranian revolution and the Iran-Iraq War.	140
D	Aug-1982	Latin America default on foreign debt, causes severe crisis in the region.	175
E	Nov-1989	Fall of the Berlin Wall.	271
F	Dec-1990	Collapse of the USSR in 1990/91, and The Gulf War.	294
G	Aug-1997	Asian crisis and its contagion.	360
H	Sep-2001	Terrorist Attack in the World Trade Center.	426
I	Sep-2008	Bankruptcy of Lehman Brothers.	516
J	Jan-2012	Strong demand for investment in gold due to the European debt crisis, especially the Greek crisis.	560
K	Jun-2016	United Kingdom decision to leave the European Union (Brexit).	616
L	Nov-2016	Donald Trump's victory gave rise to speculation that his commitment to spending on infrastructure will boost growth.	630

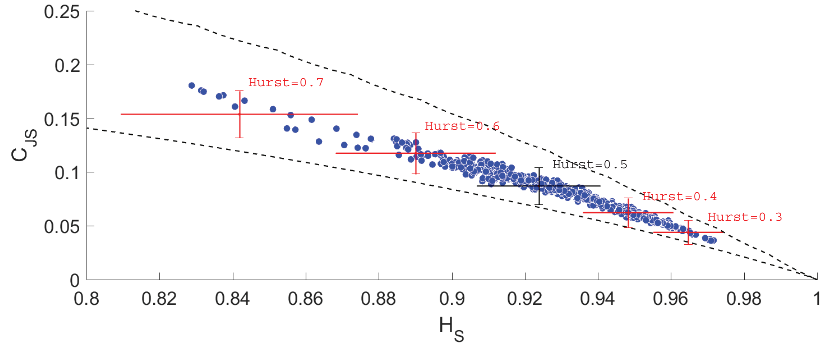
the system's behavior. Figure 4a shows the localization of each sliding window in the plane. Not every value of $\mathcal{H}[P]$ and $\mathcal{C}[P]$ are allowed in the CECP. The dashed black lines are the theoretical upper and lower bounds determined by Martín et al. (2006). The localization of the windows in the CECP is not produced by chance. In fact, each region of the CECP represents different stochastic or chaotic dynamics of the underlying system. In order to show this, we shuffled the time series and generate the same number of windows as in the original time series. As a consequence of the randomization, all windows are located close to $\mathcal{H} \approx 1, \mathcal{C} \approx 0$, corresponding to a memoryless process (see the green dots in Figure 4a).

A detailed view of the CECP area, where our results actually fall is in Figure 4b. There, we can observe that along our period under study, the time series follows different stochastic behavior. In fact, the different locations correspond to stochastic processes characterized by distinct Hurst exponents. The Hurst exponent characterizes the scaling behavior of the range of cumulative departures of a time series from its mean. The study of long range dependence can be traced back to seminal paper by Hurst (1951), whose original methodology was applied to detect long memory in hydrological time series. This method was also explored by Mandelbrot and Wallis (1968) and later introduced in the study of economic time series by Mandelbrot (1972). An standard Brownian motion is characterized by a Hurst exponent equal to 0.5. When the Hurst exponent is different from this value, we are in presence of a fractional Brownian motion (fBm). For Hurst exponent greater than 0.5, the stochastic process is persistent, meaning that positive (negative) changes are more likely to be followed by positive (negative) changes. On contrary, Hurst exponents less than 0.5 reflect an antipersistent process, meaning that positive (negative) returns are more likely to be followed by negative (positive) ones. We performed 1000 independent simulations of fBm of the same length of the sliding windows, with Hurst exponents from 0.2 until 0.7. The average and standard deviation of these simulations are displayed in Figure 4b (red crosses). It can be observed there, that many sliding windows show persistent (Hurst greater than 0.5) or antipersistent (Hurst less than 0.5) behavior. This is a sign of time-varying informational inefficiency, which could be relevant for investors in order to exploit arbitrage opportunities.

Figure 5 represents a colormap of the formula (7). We can identify three main dynamics in the period under examination. The first one goes between windows 1 to 160 (years 1960-1981), with a relative low informational efficiency (mean $\mathcal{E} = 0.8391$). The second period goes between window 161 to 455 (years 1981-2003), with a greater informational efficiency (mean $\mathcal{E} = 0.8949$). Finally the last period goes from window 456 until 635 (years 2003-2017), with lower informational efficiency *vis-à-vis* the second period (mean $\mathcal{E} = 0.8566$). The selected events detailed in Table 2 are contemporary to reductions in the informational efficiency of the gold market, with the exception of the 2016 US presidential campaign outcome. This could reflect a special sensitivity of this market to general economic and political conditions, and to some extent also the visualization of gold as a safe haven in case of political and economic turmoil.



(a) Complexity Entropy Causality Plane. Dashed black lines represent the upper and lower bounds of the quantifiers as computed by Martín et al. (2006). Blue dots are the positions of the different windows in the CECP. Green dots are the positions of windows from the shuffled time series.



(b) Detail of the Complexity Entropy Causality Plane. Red lines indicate the planar location of fBm with different Hurst exponents, obtained by averaging 1000 independent simulations of the same length of original windows, using Matlab wfbm function.

Figure 4: Gold price informational efficiency displayed in the Complexity Entropy Causality Plane, with parameters $D = 4, \tau = 1, \delta = 20$. Each point represents a sliding window of 250 observations.

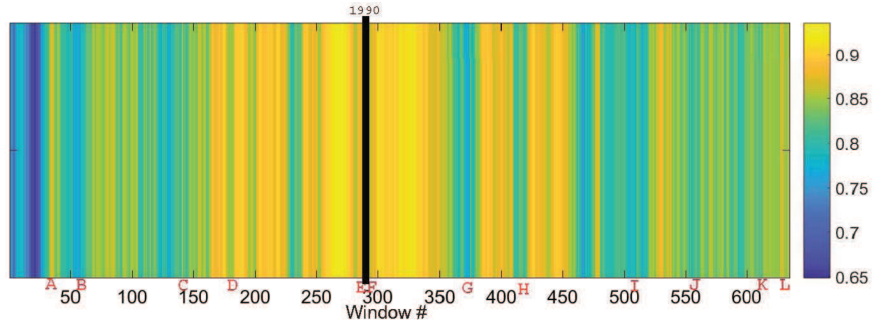


Figure 5: Inefficiency index $\mathcal{E}[P]$ according to Equation 7, computed with parameters $D = 4, \tau = 1, \delta = 20$. Color corresponds to the value of the index on the picture's legend. Letters in red refer to the events detailed in Table 2. The bold black vertical line indicates the starting point of data for volatility analysis.

Volatility analysis is carried out during the period 1989-2017. As can be observed in Figure 6, volatility behaves as a fractional Gaussian noise (fGn), with strong persistence. In fact, many of the windows estimates fall in areas corresponding to fractional stochastic processes with Hurst exponent greater than 0.7. This result is compatible with volatility clustering, one of the stylized facts of several financial time series (Cont, 2001). One of the practical consequences of the presence of persistence volatility is to guide modelization of daily returns under frameworks compatible with autoregresive conditional volatility, within the broad family of ARCH models. Our results are in line with previous findings of GARCH effects in the gold market. Nevertheless, our methodology has the advantage of not assuming any distribution for gold returns, making our results more robust.

Readers interested in obtaining details on the value of the quantifiers in each window, can contact the corresponding author.

6. Conclusions

This paper investigates the time-varying character of the informational efficiency of gold prices. We study price dynamics during the last 50 years. In addition, we study daily price volatility since the last quarter of 1989. The analysis was carried out using a powerful technique based on information theory quantifiers. These quantifiers (permutation entropy, permutation statistical complexity, and Fisher Information Measure) are able to discriminate different stochastic and chaotic dynamics, with very few prior assumptions. colorblue In particular, our methodology has the advantage of not assuming any distribution for gold returns, making our results more robust. Additionally, and unlike traditional 'yes-or-no' answer to the EMH, the joint analysis of these quantifiers allows to distinguish different levels or degrees of informational efficiency.

We find that gold market has been varying the degree of informational efficiency, measured according to the Efficiency index developed in Bariviera et al.

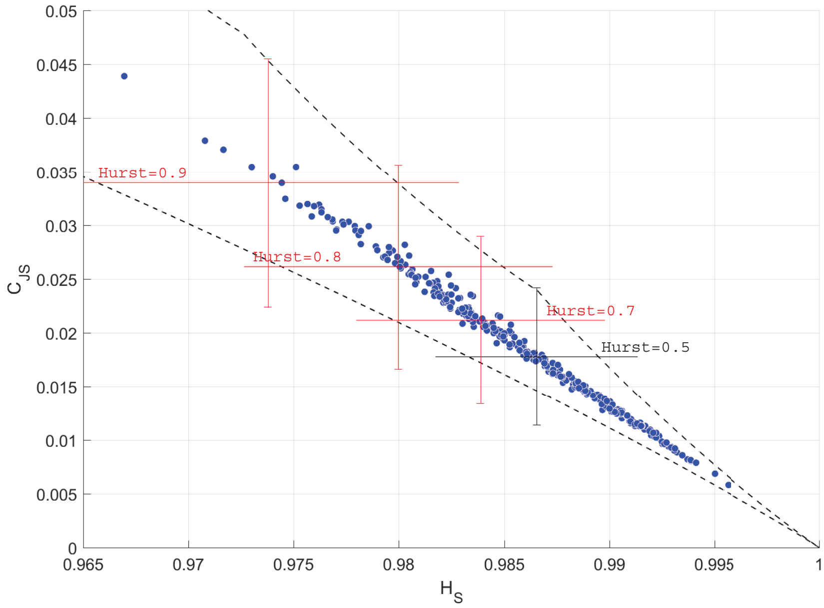


Figure 6: Statistical dynamics of gold price volatility revealed by the Complexity Entropy Causality Plane, with parameters $D = 4, \tau = 1, \delta = 20$. Each point represents a sliding window of 250 observations. Red lines indicate the planar location of fGn with different Hurst exponents, obtained by averaging 1000 independent simulations of the same length of original windows, using Matlab wfbm function. Dashed black lines represent the upper and lower bounds of the quantifiers as computed by Martín et al. (2006)

(2015). Their changes are contemporary to some important economic and political events.

We identify three main dynamics in the period under examination: (i) from 1960 to 1981, with a relative low informational efficiency; (ii) from 1981 to 2003, with a greater informational efficiency; and (iii) from 2003 to 2017, with lower informational efficiency vis-à-vis the second period. These results have important implications for investors and portfolio managers, considering that less informational efficiency implies a better chance of return forecasts.

Another important finding is that persistence in volatility is higher than persistence in prices, with dynamics compatible with Hurst exponent much greater than 0.5. This fact could allow to exploit some arbitrage possibilities, specially when persistence is concomitant in prices and volatility. Finally, this gold market feature makes ARCH and dynamic correlation models suitable for finding profitable trading strategies in this market.

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