Conflict and Competition over Multi-issues*

António Osório
Universitat Rovira i Virgili and CREIP†

November 2, 2017

Abstract

Real life disputes, negotiations and competitive situations involve multi-issue considerations in which the final outcome depends on the aggregated effort over several dimensions. We consider two allocation systems, the I-system, in which each issue is disputed and award independently, and the A-system, in which all issues are aggregate in a single prize award. In the A-system, we propose a contest success function that aggregates the individuals' multi-issue efforts in a single outcome. Among other results, we found that the A-system tends to induce higher total effort than the I-system. The model is also able to reproduce a large set of strategic behaviors. For instance, under decreasing returns to effort, individuals maximize their payoffs by distributing effort over all issues, while under increasing returns to effort, individuals focus on a single issue. Hybrid equilibria, in which one individual focus in a single issue while the other individual diversifies effort over all issues, may also emerge when individuals hold different returns to effort. Strategic behavior is simultaneously influenced by the weight of each issue on the final outcome and by comparative advantages. Throughout the manuscript, we link our results with strategic behavior observed in electoral competition, i.e., "issue ownership", "issue divergence/convergence" and "common value issues". We expect that our findings will help researchers and practitioners to better understand the process of endogenous selection of issues in competitive contexts and to provide guidance in the implementation of the optimal allocation mechanism.

JEL: C72, D72, D74, D81.

KEYWORDS: Contest success function, multi-issue competition, effort maximization, electoral competition.

*Financial support from the Barcelona Graduate School of Economics and Univ. Rovira i Virgili are gratefully acknowledged. I would like to thank Matthias Dahm, Sabine Flamand, David Pérez-Castrillo, Ricardo Ribeiro and Santiago Sanchez-Pages as well as several seminars and congresses participants for helpful comments and discussions. All remaining errors are mine.

†Department of Economics; Reus (Barcelona); E-mail: superacosta@hotmail.com.

1 Introduction

Most real life disputes, negotiations and competitive situations involve multi-issue considerations, in which the involved parties argue, claim or compete for some resource or outcome by providing effort in several dimensions. On the other hand, social planners and decision-makers must decide how to allocate the resource under dispute based on the parties' efforts over multi-issues. For instance, social planners and decision-makers may consider the effort in each issue independently and take several independent decisions (which we will denote as I-system), or alternatively, they may aggregate the effort in all issues into a single decision (which we will denote as A-system). Therefore, the final outcome of multi-issue disputes, negotiation and competitive situations depends crucially on how issues are settled or aggregated (Schelling, 1956, 1960).

In this context, three research questions emerge naturally:

(Q.1) From a theoretical perspective, how should we aggregate the efforts of different individuals over multi-issues into a single decision?

(Q.2) From the social planner or decision-maker perspective, what allocation system leads to higher equilibrium total effort?

(Q.3) From the involved parties' perspective, what is the optimal strategy, i.e., the distribution of effort over multi-issues that maximizes the winning prospects at minimum cost?

These questions assume enormous relevance because disputes, negotiations and competitive situations over multi-issues tend to be the rule rather than the exception. The final outcome depends on the aggregate effort over several dimensions. Actually, unidimensional treatments are justified mainly for their simplicity and analytical convenience, rather than for their realism.

For instance, in electoral competition, candidates compete by spending costly campaign efforts on several issues (e.g., education, health, distribution, security, among others) to influence the voters' decision (Baron, 1994; Skaperdas and Grofman, 1995).

In the academic tenure track system, candidates are evaluated over multiple dimensions: research output, teaching quality and other activities. The review follows a holistic evaluation of all the parts into a single decision.

In the vast majority of markets for goods and services, firms compete for consumers over multiple dimensions: quality, design, delivery time, among others. The more effort a producer places in each dimension, the more likely is to gain the consumers' preferences. Other examples include competing interest groups that may allocate lobbying effort over several politicians with the objective of influencing some decision (Tullock, 1980; Nitzan, 1994), or firms that can allocate research funds over several projects or R&D units with the objective of achieving some technological discovery (Loury, 1979; Nalebuff and Stiglitz, 1983).

The objective of this paper is to answer the above questions. We start by noticing that in a competitive process, the more effort an individual spends, the more likely she is to influence the final outcome in her favor. This observation establishes the link between competition over multi-issues and the Tullock's (1980) Contest Success Function (CSF) (Corchón (2007) and Konrad (2009) survey the rent-seeking literature). However, the literature has mostly focused on single-issue competition ignoring multi-issue considerations (with some exceptions that are discussed below). In the present paper, we extend the Tullock's single-issue approach to multi-issue problems. In particular, we are interested in situations in which issues are not equally important and individuals hold different abilities (i.e., qualities, merits, advantages or strengths) on each issue.

Regarding *(Q.1),* we propose a CSF that aggregates the individuals' efforts over multi-issues into a single decision. The additive separability of the different issues allows the strategic distinction of the effects associated with different abilities among the individuals, the importance or the value of each issue, returns to effort, among other considerations, in a comprehensive manner. This is a crucial aspect because as pointed out by Berliant and Konishi (2005) tractability is easily lost when dealing with multi-issue problems. In addition, the proposed CSF also satisfies a set of desirable properties that are considered as fundamental in the literature (Clark and Riis, 1998; Münster, 2009; Skaperdas, 1996).

Regarding *(Q.2),* we found that the A-system tends (but not always) to induce more effort than the I-system because of the extra competition intensity associated with the aggregation of several prizes into a single prize - the effort for the full prize is higher than the sum of the efforts for each component prize.

The exception occurs in situations in which there is a strong individual, but with highest ability not in the most valued issue. In this case, under the A-system, the stronger individual attempts to win the full prize by spending relatively more effort in the issue in which she has highest ability. However, the large asymmetry in the individuals' abilities relaxes competition in the A-system relatively more than in the I-system. Under the I-system, the competition for the most valued issue remains relatively high in comparative terms.

Therefore, if the objective is to maximize total effort the A-system tends to be more effective. The I-system is justified only in contexts in which there are relevant asymmetries between competitors. However, if the objective is to minimize the total effort the argument is reversed - issue-by-issue negotiation (I-system) is likely to be more effective.

Regarding *(Q.3)*, we found a clear difference in terms of strategic behavior between the I-system and the A-system. Under the A-system, strategic behavior is mainly driven by comparative advantages, i.e., the individuals' abilities, while under the I-system, strategic behavior is mainly driven by the value of each issue.

Under the A-system with decreasing returns to effort individuals distribute effort over all issues with a bias towards the ones in which they hold higher abilities, while with increasing returns to effort, individuals concentrate effort in a single issue - the one in which they hold the highest ability.

These results are compatible with observed real life strategic behavior. For instance, in some elections, the political debate is diversified over multiple issues (Berliant and Konishi, 2005), as in our model under decreasing returns to effort. In other elections, candidates attempt to drive the political debate to the issue in which they have a comparative advantage. The latter scenario relates to the idea of "issue ownership" (Aragonès et al., 2015; Denter, 2016; Petrocik, 1996; Riker, 1996), "divisive issues" (Dragu and Fan, 2016; Ash et al., 2015) and "issue divergence" (Amorós and Socorro Puy, 2007; Colomer and Llavador, 2011; Egorov, 2015), as in our model under increasing returns to effort.

We also found that if issues have different weight in the final decision, strategic behavior becomes simultaneously driven by comparative advantages and by the value attached to each issue. In this context, we may observe "issue convergence" (Amorós and Socorro Puy, 2007; Colomer and Llavador, 2011; Egorov, 2015) towards "common value issues" (Ash et al., 2015). For instance, in the academia, tenure candidates tend to direct their efforts to research, not necessarily because they hold a comparative advantage in that issue, but because it is the issue with largest weight in the final outcome.

In addition, we also found that if individuals have different returns to effort, we may observe one individual (the one with increasing returns to effort) directing all effort to a single issue, while the other (the one with decreasing returns to effort) distributing effort over several issues.

These results are discussed in more detail throughout the manuscript. Table 2 provides a brief schematic of our results for varying returns to effort.

The remainder of the paper is organized as follows: Section 2 reviews the literature, Section 3 introduces some notation and assumptions, Sections 4 and 5 consider the multi-issue I-system and A-system, respectively, Section 6 compares both systems, Section 7 discusses extensions, and Section 8 concludes.

2 Literature review

This section briefly reviews several branches of the multi-issue literature.

A few papers have studied multi-issue competition in rent-seeking contests. The exceptions are Arbatskaya and Mialon (2010, 2012) and Epstein and Hefeker (2003). In this context, a crucial question is how to extend the Tullock's (1980) CSF into multi-issue situations. Arbatskaya and Mialon (2010) axiomatize a multi-issue CSF and study the implications in terms of effort from varying the number of issues. Their multiplicative formulation is particularly suitable to study situations in which effort over multi-issues shows some degree of complementarity. In the present paper, we propose an alternative CSF that satisfies a set of axioms that are considered fundamental in the literature (Clark and Riis, 1998; Münster, 2009; Skaperdas, 1996). Our additive formulation is particularly suitable to study situations in which effort over multi-issues shows some degree of substitution. In this context, the present paper complements the work of Arbatskaya and Mialon (2010, 2012) and Epstein and Hefeker (2003).

The present paper is also related with the literature in sabotage in rent-seeking contests (Chen, 2003; Konrad, 2000; Lazear, 1989; among others). In sabotage, contestants have two types of effort: effort used to improve the own performance and effort used to reduce the rival performance. Translated into our context, this setting is similar to a two-issue contest.

The I-system in the present paper is related with the literature on the Colonel Blotto game in which two budget constrained individuals allocate resources over a finite number of issues (see Kovenock and Roberson (2012) for a survey of the multiple variations of the original problem). The main difference between our approach and the Colonel Blotto game is that individuals are not budget constrained and the outcome of each issue is modeled with a Tullock's (1980) type CSF (Friedman, 1958; Snyder, 1989; Osório, 2013; Robson, 2005).

Our results are related with the recent and growing literature in multi-issue electoral competition. As in our setting, the main observation is that candidates tend to direct efforts either towards the issues in which they hold some advantage or towards the issues that are more valued by the voters (Amorós and Socorro Puy, 2007; Aragonès et al., 2015; Colomer and Llavador, 2011; Denter, 2016; Dragu and Fan, 2016; Egorov, 2015). In particular, Denter (2016) follows a contest approach similar to our I-system, but in which the candidates' efforts affect their winning chances but also the importance that voters give to each issue. Consequently, a candidate may be forced to spend a large amount of resources to defend an issue in which she has no comparative advantage. This behavior diverts political focus from "common values" towards "divisive issues" (Ash et al., 2015). On the contrary, Berliant and Konishi (2005) argue that candidates prefer to campaign on all issues. The multi-issue approach in this paper is particularly suitable to address the questions raised in this literature. For instance, translated into our setting, "divisive" issues are the issues in which individuals hold a comparative advantage, while "common values" correspond to the most valued issues. Issue convergence towards "common values" emerges when some issue is highly valued by voters.

The present paper is also related with the multi-issue bargaining literature (Keeney and Raiffa, 1976; Raiffa, 1982). Multi-issue negotiations consider three main procedures; the package deal procedure (i.e., issues are bundled together, as in the A-system), the simultaneous procedure (i.e., issues are discussed independently, as in the I-system) and the sequential procedure (Bac and Raff, 1996; Fatima et al. 2006; Inderst, 2000; Lang and Rosenthal 2001). These procedures have different properties and yield different outcomes (Fershtman, 2000). In this paper, we analyze the first two procedures. However, in our setting negotiations are costly - individuals must invest in costly efforts in order to have chances of reaching some agreement.

The present paper is also related with the combinatorial auctions literature, which considers the sale of a variety of distinct items (Cramton et al., 2006; de Vries and Vohra, 2003). For instance, in estate auctions it is common to auction individual or small packages of items (i.e., as in the I-system), but also to accept bids for the full package (i.e., as in the A-system). Then, the seller chooses the procedure that maximizes the revenue. Analogously, our interest is on the effort maximizing system is similar to the combinatorial auctions revenue maximizing objective.

Finally, the present paper is also related with multi-issue allocation problems (Calleja et al., 2005), in which individuals hold claims in different and independent issues (Giménez-Gómez and Osório, 2015). The straightforward aggregation of these issues leads to well-known bankruptcy problems (O'Neill, 1982; Thomson, 2003). See Corchón and Dahm (2010) for the connection between the "rent-seeking" to the bankruptcy literature. In the present paper, we also aggregate multiple issues, but with conceptually different implications. In our setting, instead of "type of claimant" (Young, 1994), individuals are differentiated by their abilities on each issue. In addition, individuals must actively produce costly efforts in order to hold a claim that is decided through a CSF.

3 Model, notation and assumptions

In order to simplify the analysis and to provide better intuition, we consider a competitive situation involving two individuals and two issues.[1]

Let v_j denotes the prize or the value of issue $j = 1, 2$. Let $x_j(s)$ and $y_j(s)$ denote the effort of the individuals 1 and 2 in issue j, respectively, under the system $s \in \{I, A\}$, where I denotes the *independent prize system* and A denotes the *aggregate prize system*. The unit cost of the effort is assumed linear and equal to one. When required the subscripts x and y are used to distinguish between the individuals 1 and 2, respectively.

In the *independent prize system* (I-system) there are several independent award decisions ($m = 2$ in our case), one for each issue. In the *aggregate prize system* (A-system) all prizes are aggregated into a single prize $v = v_1 + v_2$ and a single award decision. Each of these two systems is discussed in detail below.

Let $\lambda_j \in (0, 1)$ and $1 - \lambda_j$ denote the ability of the individuals 1 and 2 in issue j, respectively.[2] The assumptions $\lambda_j \in (0, 1)$ and $1 - \lambda_j$ are not needed in general. However, they are convenient because they bound and link the individuals' ability in a consistent way, both in relative and absolute terms. For instance, as λ_j approaches 1, in relative terms, the individual 1 ability in issue j becomes indisputable because individual 2 ability in issue j, i.e., $1 - \lambda_j$, approaches 0. The difference between λ_j and $1 - \lambda_j$ increases. Therefore, taking $\lambda_j \uparrow 1$ in our context in which $\lambda_j \in (0, 1)$ is equivalent to take $\lambda_j \uparrow \infty$ in the unrestricted case in which $\lambda_j \in (0, \infty)$. Similarly, if $\lambda_j = 1/2$, then individuals 1 and 2 have the same ability in issue j.

In addition, in order to simply the exposition, we assume that the individual 1 strongest issue is issue 1, which implies that the individual 2 strongest issue is issue 2, i.e.,

$$\lambda_1 \geq \lambda_2 \text{ and } 1 - \lambda_2 \geq 1 - \lambda_1. \tag{1}$$

This assumption links the individual 1 with issue 1 and the individual 2 with issue 2.

In single-issue conflict and allocation problems, the final outcome depends on the indi-

[1] The model is going to depend on $2m+1$ parameters, where m denotes the number of issues. Consequently, the interpretation of the results becomes cumbersome. In order to deal with this difficulty we consider $m = 2$ from the beginning. This simplification is without loss of generality. The general case with $m > 2$ issues is discussed in more detail in the Appendix. In order to have a sufficiently tractable model, we have also considered two individuals. Similarly, the consideration of a larger number of individuals adds complexity, but does not bring new insights in terms of strategic behavior.

[2] We follow Stein (2002) in assuming that λ_j and $1 - \lambda_j$ capture the individuals' relative ability. However, depending on the context and the specific application, this parameter can have other meanings (e.g., quality, merit, strength, advantage or bias, among others).

viduals' strategies, preferences and information (Harsanyi, 1977), in multi-issue conflict and allocation problems, the final outcome also depends on how issues are settled and aggregated (Schelling, 1956, 1960).

Since each of the allocation systems in this paper is characterized by a particular prize distribution, in the following two sections, we consider how effort is transformed into winning prospects for each of these two systems.

4 Multi-issue (I)ndependent prizes system

In the I-system, each issue is associated with a prize, which is independent of the other issues and prizes. What is relevant to the resolution of a given issue is the effort associated with that issue only.

In technical terms, the I-system is simply the sum on several independent Tullock (1980) contests. For that reason, the objective of this section is to present a set of results that are needed for the comparison between the I-system and the A-system, which is done below in Section 6.

Hence, the probability that individual 1 wins the issue j is given by:

$$p_j(I) = \lambda_j x_j^{\alpha}(I)/(\lambda_j x_j^{\alpha}(I) + (1-\lambda_j)y_j^{\alpha}(I)), \tag{2}$$

with $p_j(I) = \lambda_j$ if $x_j(I) = y_j(I) = 0$.[3] The probability that individual 2 wins the issue j is simply $1 - p_j(I)$.

The parameter α measures the returns to effort and is related to the effort producing technology. The higher the value of α, the higher the efficiency of effort.

Note also that under the I-system an individual may win any number of prizes, ranging from none to all. In our context, there are no equity considerations.

The general problem For each issue j, the individuals 1 and 2 simultaneously choose the efforts $x_j(I) \geq 0$ and $y_j(I) \geq 0$ that maximizes the respective expected payoff net of the cost of effort:

$$\pi_x(I) = \textstyle\sum_{j=1}^{2} p_j(I)v_j - \sum_{j=1}^{2} x_j(I) \text{ and } \pi_y(I) = \sum_{j=1}^{2}(1-p_j(I))v_j - \sum_{j=1}^{2} y_j(I), \tag{3}$$

[3] This functional form have been frequently applied in economics and litigation (Dahm and Porteiro, 2008; De Mot et al., 2015; Hirshleifer and Osborne, 2001; Leininger, 1993; Robson and Skaperdas, 2008; Skaperdas and Vaidya, 2012; Stein, 2002). Franke et al. (2013) study the general case with asymmetric costs of effort but, with $\alpha = 1$. In the litigation context, Osório (2015) introduces a different functional form, but with intuitive similarities.

subject to the respective participation constraint, i.e., $\pi_x(I) \geq 0$ and $\pi_y(I) \geq 0$.

Results

The following result characterizes the equilibrium effort under the I-system.

Proposition 1 *For $0 < \alpha \leq \min\{1/\lambda_j, 1/(1-\lambda_j)\}$, the individuals equilibrium efforts in each issue $j = 1, 2$ are given by:*

$$x_j(I) = x_j(I) = \alpha\lambda_j(1-\lambda_j)v_j. \tag{4}$$

Since the I-system is simply the sum of several independent Tullock (1980) contests, the proof of this result is standard and is for that reason omitted.

In equilibrium, both individuals spend the same symmetric and strictly positive effort independently of their abilities.

The participation condition in issue j is satisfied (i.e., the expected payoff in each issue j is non-negative) for any $\alpha \leq 1$. Since $\min\{1/\lambda_j, 1/(1-\lambda_j)\} \in (1, 2]$, the participation constraint must fail for some $\alpha > 1$ (see the comment on equilibrium existence in Section 5.3).

Since the solution (4) is symmetric the total effort is just the double of the sum of the individual efforts. The following result formalizes this observation.

Proposition 2 *For $0 < \alpha \leq \min\{1/\lambda_j, 1/(1-\lambda_j)\}$ and $j = 1, 2$, the sum of both individuals equilibrium efforts is given by:*

$$T(I) = 2\alpha \textstyle\sum_{j=1}^{2} \lambda_j(1-\lambda_j)v_j, \tag{5}$$

where:

- *$T(I)$ increases with λ_j if $\lambda_j < 1/2$ for $j = 1, 2$, and the opposite otherwise.*
- *$T(I)$ has a maximum equal to $\alpha v/2$ at $\lambda_j = 1/2$ for $j = 1, 2$.*

The proof is simple and is for that reason omitted.

Note that the highest effort in a given issue j is obtained when the individuals are similar in ability terms, i.e., at $\lambda_j = 1/2$ for $j = 1, 2$.

5 Multi-issue (A)ggregate prize system

In the A-system each issue is independent, but only one prize is awarded. The total prize is the sum of the individual prizes, i.e., $v = (v_1 + v_2)$. The final outcome depends on the aggregate effort over all issues. A social planner or decision-maker gathers the individual efforts in each issue into a single decision. In this context, the crucial question is how to aggregate these efforts.

In order to deal with this type of problems, we propose a contest success function that transforms aggregate efforts into winning prospects. The approach is a generalization of the Tullock's (1980) type CSF (2) to situations in which individuals compete in several dimensions for a single prize. Hence, under the A-system, the probability that individual 1 wins, by providing effort in all (or some) of the issues is given by:

$$p(A) = \textstyle\sum_{j=1}^{2} \lambda_j x_j^{\alpha}(A) / (\sum_{j=1}^{2} \lambda_j x_j^{\alpha}(A) + \sum_{j=1}^{2} (1 - \lambda_j) y_j^{\alpha}(A)), \tag{6}$$

with $p(A) = \sum_{j=1}^{2} \lambda_j / 2$ if $x_j(A) = y_j(A) = 0$ for all $j = 1, 2$. Therefore, the probability that individual 2 wins is simply $1 - p(A)$. Note that in the single issue case, expression (6) reduces to expression (2).

In expression (6) the outputs associated with the individual efforts in each issue are summed. For instance, the sum of the individual 1 outputs in each issue is $\sum_{j=1}^{2} \lambda_j x_j^{\alpha}(A)$. The proposed additive separable formulation is particularly suitable to deal with situations in which the effort over multi-issues shows some degree of substitution.[4] This approach allows us to study strategic behavior relatively to the individuals' ability, returns to effort and the value of each issue in a comprehensive manner. The proposed specification is particularly tractable, which is a crucial aspect because, as pointed out by Berliant and Konishi (2005), in multi-issue problems analytical tractability is easily lost.

Properties The CSF (6) satisfies a set of desirable properties that are considered fundamental in the rent-seeking literature (Clark and Riis, 1998; Skaperdas, 1996). These properties are imperfect discrimination, monotonicity, consistency, independence of the irrelevant alternatives and homogeneity of degree zero.

In technical terms the multi-issue CSF (6) is an extension of the "group CSF" characterized in Münster (2009). In the multi-issue CSF (6) the effort in each issue impacts

[4] This approach complements the multiplicative formulation in Arbatskaya and Mialon (2010, 2012) and Epstein and Hefeker (2003), which is more adequate to deal with situations in which effort over multi-issues shows some degree of complementarity.

differently in the individuals winning probability, while in Münster (2009) the effort of each group member has the same impact in the group output, and consequently in the group winning probability. However, the crucial difference between multi-issue and group contests is that the distribution of efforts over multi-issues depends on the preferences of a single individual, while the distribution of efforts inside each group depends on the preferences of (potentially) different individuals. These considerations lead to two strategically different problems, but that can have in common the same CSF.

In this context, the Axiom 8 in Münster (2009) can be reinterpreted by an analogous axiom that imposes that the total output remains unchanged if the output (and not the effort) in one issue increases while the output in another issue decreases by the same amount. Along these lines and under the assumption that the efforts in each issue are non-negative and continuously differentiable, we can apply the Euler's homogeneous function theorem to uniquely characterize the CSF (6) (Osório, 2016).

The general problem For each issue j, the individuals 1 and 2 simultaneously choose the efforts $x_j(A) \geq 0$ and $y_j(A) \geq 0$ that maximizes the respective expected payoff net of the cost of effort:

$$\pi_x(A) = p(A)v - \textstyle\sum_{j=1}^{2} x_j(A), \text{ and } \pi_y(I) = (1 - p(A))v - \textstyle\sum_{j=1}^{2} y_j(A), \tag{7}$$

subject to the respective participation constraint, i.e., $\pi_x(A) \geq 0$ and $\pi_y(A) \geq 0$.

5.1 Intuition and discussion

In the A-system, we must distinguish between $\alpha < 1$ and $\alpha \geq 1$, because they lead to different strategic behaviors in equilibrium. The following discussion provides the intuition.

Intuition: A-system case $0 < \alpha < 1$ In this case, the output $\sum_{j=1}^{2} \lambda_j x_j^{\alpha}(A)$ is concave in the effort $x_j(A)$. Consequently, the marginal benefit from effort decreases when effort increases. Therefore, the efficient use of costly effort is achieved by distributing effort between all issues in such a way that the marginal utility from each issue is the same in equilibrium. For instance, since $\lambda_1 \geq \lambda_2 > 0$, we must expect individual 1 to distribute effort between both issues, but with a bias towards the issue 1, i.e., $x_1(A) \geq x_2(A) > 0$, because this is the issue in which individual 1 has the highest ability. Similar reasoning applies to individual 2.

Intuition: A-system case $\alpha \geq 1$ In this case, the output $\sum_{j=1}^{2} \lambda_j x_j^{\alpha}(A)$ is convex in effort $x_j(A)$. Consequently, since the marginal utility from effort increases, the more effort is devoted to a given issue, the larger is the impact in the winning probability. Therefore, in equilibrium, each individual places all the effort in a single issue - the one in which the individual has the highest ability. For instance, since we assume $\lambda_1 \geq \lambda_2$, we must expect individual 1 to place all effort in issue 1 and no effort in issue 2, i.e., $x_1(A) > 0$ but $x_2(A) = 0$. This is the most efficient way that individuals have to benefit from the increasing returns to effort. Similar reasoning applies to individual 2 with issue 2.

5.2 Results: A-system case $0 < \alpha < 1$

We start by presenting the equilibrium efforts under the multi-issue A-system for the case that $0 < \alpha < 1$.

Proposition 3 *For $0 < \alpha < 1$, the individuals equilibrium efforts in each issue $j = 1, 2$ are given by:*

$$x_j(A) = \alpha\phi\gamma_{xj}v, \text{ and } y_j(A) = \alpha\phi\gamma_{yj}v, \tag{8}$$

where $\phi = s_x^{\alpha-1}s_y^{\alpha-1}/(s_x^{\alpha-1} + s_y^{\alpha-1})^2$, $\gamma_{xj} = \lambda_j^{1/(1-\alpha)}/s_x$ and $\gamma_{yj} = (1-\lambda_j)^{1/(1-\alpha)}/s_y$ with $s_x = \sum_{j=1}^{2} \lambda_j^{1/(1-\alpha)}$ and $s_y = \sum_{j=1}^{2}(1-\lambda_j)^{1/(1-\alpha)}$.

The proof is shown in the Appendix.

The power sums $s_x \in (0, m)$ and $s_y \in (0, m)$ are aggregated measures of the individuals 1 and 2 *overall ability*, respectively. For instance, if $s_x > s_y$, then individual 1 has higher overall ability than individual 2, and the opposite otherwise.

Note that since $\gamma_{x1} + \gamma_{x2} = 1$ and $\gamma_{y1} + \gamma_{y2} = 1$, the ratios γ_{xj} and γ_{yj} measure the *relative ability in the issue j* of the individuals 1 and 2, respectively. For instance, since we assume $\lambda_1 \geq \lambda_2$, the individual 1 has higher relative ability in issue 1 than in issue 2, i.e., $\gamma_{x1} \geq \gamma_{x2}$. The opposite happens with individual 2.

The parameter $\phi \in (0, 1)$ captures the *overall ability asymmetry* between the individuals.[5] The larger the asymmetry between the individuals, smaller the value of ϕ, and consequently, lower the effort in expression (8).

Contrary to the equilibrium found in Proposition 1, the equilibrium in Proposition 3 is not symmetric across individuals. The individual with the largest relative ability in issue j spends more effort in that issue than the opponent. For instance, if $\gamma_{xj} > \gamma_{yj}$ then $x_j(A) > y_j(A)$, and the opposite otherwise.

[5] Note that ϕ is the product of two ratios that sum to one, i.e., $s_x^{\alpha-1}/(s_x^{\alpha-1}+s_y^{\alpha-1})$ and $s_y^{\alpha-1}/(s_x^{\alpha-1}+s_y^{\alpha-1})$. It is similar to the variance of a Bernoulli random variable. Therefore, in our context, ϕ captures the individuals' degree of asymmetry or heterogeneity in terms of overall ability.

The equilibrium is also not symmetric across issues. Each individual spends more effort on the issue in which has the highest ability. For instance, since we assume $\lambda_1 \geq \lambda_2$, we must have $x_1(A) \geq x_2(A)$ and $y_1(A) \leq y_2(A)$, and the opposite otherwise.

Proposition 3 rationalizes strategic behavior in situations in which individuals compete by providing effort on all issues, but with a natural bias towards the issues in which each individual hold higher relative ability.

Since the usual objective of the social planners and decision-makers is to implement the system that maximizes or minimizes total effort, we formally state the following result.

Proposition 4 *For $0 < \alpha < 1$, the sum of both individuals equilibrium efforts is given by:*

$$T(A) = 2\alpha\phi v, \tag{9}$$

where $\phi = s_x^{\alpha-1} s_y^{\alpha-1}/(s_x^{\alpha-1} + s_y^{\alpha-1})^2$ with $s_x = \sum_{j=1}^{2} \lambda_j^{1/(1-\alpha)}$ and $s_y = \sum_{j=1}^{2} (1-\lambda_j)^{1/(1-\alpha)}$.

- $T(A)$ increases with λ_j for $j = 1, 2$ if $s_x < s_y$, and decreases otherwise.

- $T(A)$ has a maximum equal to $\alpha v/2$ at $s_x = s_y$.

The proof is shown in the Appendix.

Even though that the equilibrium found in Proposition 3 is asymmetric, the individuals total effort in Proposition 4 is symmetric (i.e., $T_x(A) = T_y(A)$), because individuals are symmetric in terms of valuations and costs of effort and are not effort constrained. Therefore, each individual spends the same total effort, but distributes this effort differently between each issue according to their relative abilities in each issue.

The aggregate effort depends crucially on the overall ability asymmetry ϕ. Intuitively, the larger the overall asymmetry between the individuals, the smaller the value of ϕ, and consequently lower the total effort. For instance, if $s_x < s_y$, an increase in λ_j for $j = 1, 2$ cause an increase in s_x in the direction of s_y, which reduces the asymmetry and increases total effort $T(A)$. Consequently, the maximum aggregate effort occurs when individuals are symmetric in the overall ability, i.e., at $s_x = s_y$.

In this context, many distributions of individual ability lead to overall ability symmetry. This observation is important because the condition that guarantees that $T(A)$ in Proposition 4 is maximal is not the same condition that guarantees that $T(I)$ in Proposition 2 is maximal. For instance, the distribution $\lambda_1 = 1/2$ and $\lambda_2 = 1/2$ with $\alpha = 0.5$ leads to $s_x = s_y = 0.5$, which implies that $T(I)$ and $T(A)$ are maximal according to Propositions 2 and 4, respectively. Similarly, the distribution $\lambda_1 = 0.6$ and $\lambda_2 = 0.4$ with $\alpha = 0.5$ also leads to overall ability symmetry, i.e., $s_x = s_y = 0.52$, which implies that $T(A)$ in Proposition 4 is maximal, but not $T(I)$ in Proposition 2. Therefore, the maximum $T(A)$ is obtained under weaker conditions than the maximum $T(I)$. This aspect will help us to understand why the A-system tends to be superior to the I-system in terms of effort (see Section 6).[6]

5.3 Results: A-system case $\alpha \geq 1$

In the case that $\alpha \geq 1$, individuals place all effort in a single issue - the one in which they hold the highest ability. Before proceeding recall from assumption (1) that individual 1 highest ability is in issue 1 and individual 2 highest ability is in issue 2.

Proposition 5 *For*

$$1 \leq \alpha \leq \min\left\{(\lambda_1 + 1 - \lambda_2)/\lambda_1, (\lambda_1 + 1 - \lambda_2)/(1 - \lambda_2)\right\}, \tag{10}$$

the individuals equilibrium efforts in each issue $j = 1, 2$ are given by:

$$x_1(A) = y_2(A) = \alpha\lambda_1(1 - \lambda_2)v/(\lambda_1 + 1 - \lambda_2)^2, \tag{11}$$

and $x_2(A) = y_1(A) = 0$.

The proof is shown in the Appendix.

Similarly to the equilibrium found in Proposition 1, the equilibrium found in Proposition 5 is symmetric. Moreover, the equilibrium expressions (4) and (11) have similar structures.[7]

The result states that each individual provides positive effort in the issue with highest ability and ignores all other issues. Intuitively, under increasing returns to effort, individuals focus on their strongest ability. This type of equilibrium configuration in which individuals provide effort in a single-issue and ignore all the other issues is new to the "rent-seeking" literature (see Corchón (2007) and Konrad (2009) for comprehensive surveys of this literature).

The result obtained in Proposition 5 rationalizes the theory of comparative advantages as part of a well-defined strategic behavior. For instance, strategic behavior of this type is observed in political competition and is compatible with the idea of "issue ownership"

[6] A well-known result in the "rent-seeking" literature is that increasing the returns to effort (α) or the value of the dispute (v) increases the effort intensity. These results are also always true in multi-issue.

[7] The equilibrium obtained in Proposition 5 evaluated at $\alpha = 1$, can be obtained as the limit $\alpha \uparrow 1$ of the equilibrium obtained in Proposition 3 for $0 < \alpha < 1$. This observation shows the existence of continuity in the equilibrium at the transition between decreasing to increasing returns to scale.

(Petrocik, 1996; Riker, 1996). According to Proposition 5, electoral competition under increasing returns to effort will be centered on a small set of issues, which are not necessarily the most valued by the voters, but the ones in which candidates have stronger arguments. This type of strategic behavior is in line with the results obtained by several authors in multi-issue electoral competition in which candidates tend to direct their campaign efforts towards the issues in which they have some comparative advantage (Amorós and Socorro Puy, 2007; Aragonès et al., 2015; Colomer and Llavador, 2011; Denter, 2016; Dragu and Fan, 2016; Egorov, 2015). In equilibrium, we observe "issue divergence" because the candidates hold strongest arguments on different issues. These results are in line with Aragonès et al. (2015) and Ash et al. (2015).[8]

Later, in Section 7, we will see that the multi-issue model is also able to capture "issue convergence", in which different candidates direct the political focus to the same issue. Recall also that in Section 5.2, the model was able to rationalize strategic behavior that motivates simultaneous competition in several issues. In this context, the flexibility to rationalize different forms of strategic behavior is one of the multi-issue model strongest features. This aspect will become even more evident in Section 7 when we consider extensions to the baseline model.

A Comment on existence: *In this paper, we focus in pure strategy equilibria. If the inequality (10) fails, the expected payoff of at least one of the individuals becomes strictly negative and an equilibrium in pure strategies fails to exist in Proposition 5. The same happens when the inequality in Proposition 1 fails. In our context, we have equilibrium existence problems similar to the ones that occur with the Tullock's (1980) contest success function for* $\alpha > 2$ *(Pérez-Castrillo and Verdier, 1992; Tullock, 1980), but under even more restrictive conditions because* $\min\{1/\lambda_j, 1/(1-\lambda_j)\} < 2$ *for* $j = 1, 2$ *and* $\min\{(\lambda_1 + 1 - \lambda_2)/\lambda_1, (\lambda_1 + 1 - \lambda_2)/(1 - \lambda_2)\} < 2$. *Baye et al. (1994) discuss a set of solutions that have been offered in the literature to deal with existence problems (see also, Alcalde and Dahm, 2010; Ewerhart, 2015). The same set of solutions can be applied in our context. These same references discuss mixed strategy equilibria.*

The following result considers the aggregate effort.

[8] Issue divergence may have some undesirable implications. Ash et al. (2015) point out that the winning motives distort the political focus from "common values" toward "divisive issues" which creates a misalignment between the issues that are subject to political debate and the issues that are actually relevant for the voters and the society.

Proposition 6 *Under condition (10), the sum of both individuals equilibrium efforts is given by:*

$$T(A) = 2\alpha\lambda_1(1-\lambda_2)v/(\lambda_1+1-\lambda_2)^2. \tag{12}$$

- *$T(A)$ increases with λ_1 and λ_2 if $\lambda_1 < 1-\lambda_2$, and the opposite otherwise.*
- *$T(A)$ has a maximum equal to $\alpha v/2$ at $\lambda_1 = 1-\lambda_2$.*

The proof is simple and for that reason omitted.

Since the equilibrium found in Proposition 5 is symmetric, the aggregate effort in Proposition 6 is simply the double of the individual effort.

Total effort increases with an individual highest ability if that individual highest ability is below the opponent's highest ability and the opposite otherwise. Consequently, the maximum total effort symmetry conditions obtained in Propositions 2 and 4, issue-by-issue symmetry (i.e., at $\lambda_j = 1/2$ for all $j = 1, 2$) and overall symmetry (i.e., at $s_x = s_y$ for $0 < \alpha < 1$), respectively, have an analogous symmetry condition in Proposition 6 - the highest total effort occurs when individuals are symmetric in their highest ability (i.e., at $\lambda_1 = 1-\lambda_2$ for $\alpha \geq 1$).

6 Comparison between multi-issue allocation systems

In general, social planners and decision-makers are interested in implementing the system that maximizes total effort. This is a crucial question in the single-issue "rent-seeking" literature, but also in multi-issue. For instance, in litigation, the more evidence is presented by both parties, the more likely is that a judge takes the correct decision. In electoral competition, the higher the campaign effort the more qualified and informed are the voters. Similarly, in lobbying situations, the identification of the system that is more costly to bribe is of general interest for the society.

In this section, we discuss the differences between the I-system and the A-system regarding effort incentives.

6.1 General case

The following two results establish the conditions under which the A-system is effort superior to the I-system.

Proposition 7 *For $0 < \alpha < 1$, the A-system leads to higher total effort than the I-system iff:*

$$\phi v \geq \lambda_1(1-\lambda_1)v_1 + \lambda_2(1-\lambda_2)v_2, \tag{13}$$

where $\phi = s_x^{\alpha-1}s_y^{\alpha-1}/(s_x^{\alpha-1}+s_y^{\alpha-1})^2$ *with* $s_x = \sum_{j=1}^{2}\lambda_j^{1/(1-\alpha)}$ *and* $s_y = \sum_{j=1}^{2}(1-\lambda_j)^{1/(1-\alpha)}$.

Inequality (13) is obtained after some algebra on the inequality $T(A) \geq T(I)$, where $T(I)$ and $T(A)$ for $0 < \alpha < 1$ are given by expressions (5) and (9), respectively.

Proposition 8 *For* $1 \leq \alpha \leq \min\{1/\lambda_1, 1/(1-\lambda_2)\}$,[9] *the* A*-system leads to higher total effort than the* I*-system iff:*

$$\lambda_1(1-\lambda_2)v/(\lambda_1+1-\lambda_2)^2 \geq \lambda_1(1-\lambda_1)v_1 + \lambda_2(1-\lambda_2)v_2. \tag{14}$$

Inequality (14) is obtained after some algebra on the inequality $T(A) \geq T(I)$, where $T(I)$ and $T(A)$ are given by expressions (5) and (12), respectively.

In terms of intuition, there is no great difference between the inequalities (13) and (14). However, they are difficult to interpret. In what follows, we discuss some particular cases in more detail. We start with the symmetric case. Then, we consider asymmetric situations in which the A-system is not always effort superior.

6.2 Symmetric individuals

In the case in which individuals have symmetric distributions of abilities, the A-system always leads to higher effort than the I-system. The result is independent of the value attached to each issue.

Corollary 9 *For* $0 < \alpha \leq 1/\lambda$ *with* $\lambda_1 = 1-\lambda_2 = \lambda$, *the* A*-system leads to higher total effort than the* I*-system.*

The proof is simple. Since $\lambda_1 = \lambda \geq \lambda_2 = 1-\lambda$, we must have $\lambda \geq 1/2$. Then, expressions (9) and (12) become equal to $T(A) = \alpha v/2$, and expression (5) becomes equal to $T(I) = 2\alpha(1-\lambda)\lambda(v_1+v_2)$. After some algebra, we obtain that $T(A) > T(I)$ for $\lambda > 1/2$ and $T(A) = T(I)$ for $\lambda = 1/2$. Corollary 9 extends to any number of issues and symmetric distributions of abilities.

[9] If existence fails in a given system, then the comparison between systems makes no sense. The existence condition considers the existence conditions in Propositions 1 and 5, and applies the one that is most difficult to satisfy. In order to see how the existence condition $1 \leq \alpha \leq \min\{1/\lambda_1, 1/(1-\lambda_2)\}$ in Proposition 8 is obtained, note that the inequality assumption (1) implies that $1/\lambda_1 \leq 1/\lambda_2$ and $1/(1-\lambda_2) \leq 1/(1-\lambda_1)$, and that $1/\lambda_1 \leq (\lambda_1+1-\lambda_2)/\lambda_1$ and $1/(1-\lambda_2) \leq (\lambda_1+1-\lambda_2)/(1-\lambda_2)$.

6.3 Asymmetric individuals

However, the superiority of the A-system over the I-system is not true in general because individuals are different in terms of ability and issues have different values. In order to get a better intuition, consider the illustration in Figure 1 and the related numerical example in Table 1.

Discussion and intuition In Figure 1, we plot $T(I)$ and $T(A)$ on the vertical axis and λ_1 on the horizontal axis with $\alpha = 0.5$, $v = 1$, $v_2 = 1 - v_1$, and for $1 - \lambda_2 = 0.25$ and $1 - \lambda_2 = 0.75$. The total effort in the I-system $T(I)$ is the same for $1 - \lambda_2 = 0.25$ and $1 - \lambda_2 = 0.75$, but changes with v_1, e.g., $v_1 = 0.75$ (light green line), $v_1 = 0.5$ (medium green line) and $v_1 = 0.25$ (dark green line). The total effort in the A-system $T(A)$ is independent of v_1 because v remains constant when v_1 changes, but varies with $1 - \lambda_2$, e.g., $1 - \lambda_2 = 0.25$ (grey line) and $1 - \lambda_2 = 0.75$ (black line).

Since we assume that $\lambda_1 \geq \lambda_2$, the case $1 - \lambda_2 = 0.25$ only makes sense for $\lambda_1 \geq 0.75$, while the case $1 - \lambda_2 = 0.75$ only makes sense for $\lambda_1 \geq 0.25$ (the two vertical dashed lines establish the frontier). By symmetry, every statement regarding one individual is reversely true for the other individual.

Proposition 2 states that under the I-system the highest effort is obtained when individuals are similar. This result is shown in Figure 1 at $\lambda_1 = 0.5$ for the three green lines. Proposition 4 states that under the A-system the highest effort is obtained when individuals are similar in terms of overall ability $s_x = s_y$. This result is shown in Figure 1 at $\lambda_1 = 0.75$ with $1 - \lambda_2 = 0.75$ (black line).

Figure 1 shows that if $v_1 = 0.5$ (medium green line), we always have $T(A) \geq T(I)$. In other words, if all issues are equally important, the A-system induces higher effort than the I-system. In this context and in line with Corollary 9, the inequality $T(A) \geq T(I)$ tends to be true in general. However, there are two exceptions in Figure 1:

Case (A) - In the interval $\lambda_1 \in [0.25, 0.484]$ the light green line is above the black line in Figure 1, which means that $T(A) < T(I)$ in this interval. In this case, the individual 2 is stronger in both issues, but with highest ability in issue 2 (i.e., in this case $1 - \lambda_1 \in [0.516, 0.75]$ and $1 - \lambda_2 = 0.75$), while the individual 1 is weaker in both issues, but with highest ability in issue 1 (i.e., $\lambda_1 \in [0.25, 0.484]$ and $\lambda_2 = 0.25$). Since issue 1 has higher value (i.e., $v_1 = 0.75$), the relatively high competition in issue 1 under the I-system explains why $T(A) < T(I)$. This observation can be seen in the numerical example in Table 1 for

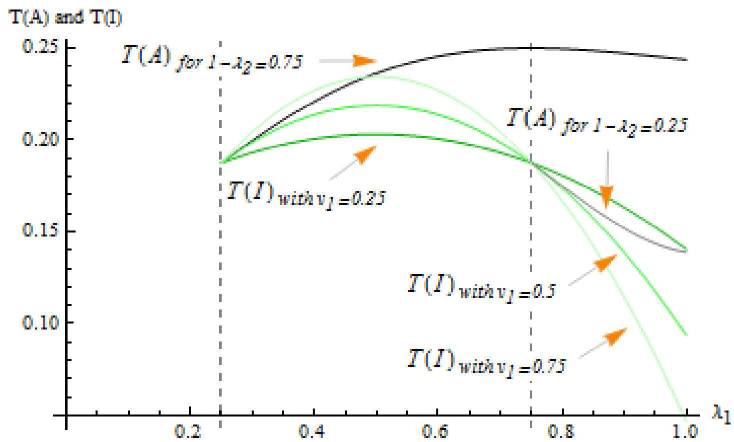

Figure 1: **Total effort in both systems** ($\alpha = 0.5$, $v_2 = v - v_1$ and $v = 1$). $T(A)$ constant in v_1, but varies for $1 - \lambda_2 = 0.75$ (black line) and $1 - \lambda_2 = 0.25$ (gray line), $T(I)$ is the same for $1 - \lambda_2 = 0.25$ and $1 - \lambda_2 = 0.75$, but varies for $v_1 = 0.75$ (light green line), $v_1 = 0.50$ (medium green line) and $v_1 = 0.25$ (dark green line).

$\lambda_1 = 0.3$ and $\lambda_2 = 0.25$. The total effort in issue 1 under the I-system (i.e., $T_1(I) = 0.158$) is significantly above the total effort in issue 1 under the A-system (i.e., $T_1(A) = 0.105$).

Case (B) - In the interval $\lambda_1 \in [0.75, 1)$ the light green line is above the grey line in Figure 1, which means that $T(A) < T(I)$ in this interval. In this case, the individual 1 is stronger in both issues, but with highest ability in issue 1 (i.e., in this case $\lambda_1 \in [0.75, 1)$ and $\lambda_2 = 0.75$), while the individual 2 is weaker in both issues, but with highest ability in issue 2 (i.e., $1 - \lambda_1 \in (0, 0.25]$ and $1 - \lambda_2 = 0.25$). Since issue 2 has higher value (i.e., $v_2 = v - v_1 = 0.75$), the relatively high competition in issue 2 under the I-system explains why $T(A) < T(I)$. This observation can be seen in the numerical example in Table 1 for $\lambda_1 = 0.8$ and $\lambda_2 = 0.75$. The total effort in issue 2 under the I-system (i.e., $T_2(I) = 0.141$) is significantly above the total effort in issue 2 under the A-system (i.e., $T_2(A) = 0.094$).

In connection with Corollary 9, these observations allow us to conclude that the A-system induces higher effort than the I-system, except in situations in which there is a strong individual with highest ability not in the most valued issue. Intuitively, under the A-system the stronger individual attempts to win the full prize by spending relatively more effort in the issue in which she has highest ability. However, the large asymmetry between individuals relaxes competition in the A-system relatively more than in the I-system, because under the I-system the competition for the most valued issue remains relatively high in comparative terms. The exceptions to $T(A) \geq T(I)$ discussed in Cases (A) and (B) have in common these facts. They explain why the total effort under the I-system is larger than under the

	(A)	(I)	(A)	(I)
	Case (A)		Case (B)	
$x_1(s)$	0.059	0.079	0.047	0.020
$y_1(s)$	0.046	0.079	0.034	0.020
$T_1(s)$	0.105	0.158	0.081	0.040
$x_2(s)$	0.041	0.023	0.041	0.070
$y_2(s)$	0.053	0.023	0.053	0.070
$T_2(s)$	0.094	0.047	0.094	0.141
$T(s)$	0.200	0.204	0.175	0.181

Table 1: **Particular cases in which** $T(A) < T(I)$ **(**$\alpha = 0.5$, $v_2 = v - v_1$ **and** $v = 1$**). Case (A)**: $\lambda_1 = 0.3$, $\lambda_2 = 0.25$ and $v_1 = 0.75$. **Case (B)**: $\lambda_1 = 0.8$, $\lambda_2 = 0.75$ and $v_1 = 0.25$.

A-system in Cases (A) and (B).

For instance, consider a strong politician with strong ability in the "education issue" and in the "health issue" (but stronger in the "education issue" than in the "health issue") competing against a weaker politician. If voters value the "health issue" much more than the "education issue", then under the I-system, total effort would be higher because of the higher campaign efforts on the decisive "health issue". Under the A-system the total effort is lower because of the large asymmetry between the candidates and because the strong politician would try to gain the election by influencing voters [mostly] through her strongest "education issue", instead of the most decisive "health issue". In this context, since effort results in better informed voters and in the election of the best candidate, the desired election mechanism that induces larger effort would be the democratic I-system in which voters would cast a vote on each issue independently.[10]

A Comment on increasing returns Figure 1 and the numerical example in Table 1 refer to decreasing returns to effort. However, under increasing returns to effort the intuition remains the same. In fact, increasing returns to effort increase the superiority of the A-system over the I-system. The inequality $T(A) \geq T(I)$ becomes even easier to satisfy. For instance, in Case (B) of Table 1, but with $\alpha = 1$, the total effort under the I-system moves below the total effort under the A-system ($T(I) = 0.361 < T(A) = 0.363$). In this case, we pass from $T(A) < T(I)$ for $\alpha = 0.5$ to $T(A) > T(I)$ for $\alpha = 1$. In Case (A) of Table 1, but with $\alpha = 1$, the total effort under the I-system remains, but only slightly, above the total

[10] In order to motivate the existence of an electoral I-system, we can think that each issue results in an independent outcome. For instance, the winning party in the "education issue" nominates the minister and the cabinet of education; the winning party in the "health issue" nominates the minister and the cabinet of health, and so on. This form of democratic election does not exist yet in reality, but we can question this possibility. This seems an interesting topic for further research.

effort under the A-system ($T(I) = 0.409 > T(A) = 0.408$).

7 Extensions

In this section we discuss some extensions to the multi-issue baseline model.

7.1 Unequally relevant issues

Until now, we have considered that social planners and decision-makers weight equally the effort in each issue. One implication of this assumption is that under the A-system, there is no issue convergence in the baseline model because candidates have comparative advantages in different issues. However, some issues can be more relevant than others for the final decision. In other words, we can think that each issue has a different weight, $w_j \in (0,1)$ with $w_1 + w_2 = 1$, in the final decision.

The distinction between issues in terms of value is a feature of the I-system that can be incorporated into the A-system CSF (6) by giving different weights to the effort in different issues as follows:

$$p^w(A) = \sum_{j=1}^2 w_j\lambda_j x_j^\alpha(A)/(\sum_{j=1}^2 w_j\lambda_j x_j^\alpha(A) + \sum_{j=1}^2 w_j\lambda_j y_j^\alpha(A)).$$

In this context, the resulting equilibrium would be similar to the one found in Propositions 3 and 5. In Propositions 3 and 5 it would be enough to multiply the individuals ability in each issue by the respective weight.[11]

The crucial difference between weights and abilities is that weights are specific to each issue while abilities are specific to each individual. For instance, an individual might be strong in some issue, but the weight given to that issue almost irrelevant. Consequently, the distribution of efforts takes into consideration not only the individuals' ability, but also the importance of each issue in the final decision.

The introduction of weights strengthens (even more) the tendency found in Section 6 for the A-system to be effort superior to the I-system, because it reduces the degree of effort substitution between issues and concentrate effort competition on similar issues.

Competition intensity on the same issue becomes more likely because there is a strategic

[11] In the case of decreasing returns to effort, the equilibrium efforts given in expression (8) would become $x_j^w(A) = \alpha\phi\gamma_{xj}^w v$ and $y_j^w(A) = \alpha\phi\gamma_{yj}^w v$, where $\gamma_{xj}^w = (w_j\lambda_j)^{1/(1-\alpha)}/s_x^w$ and $\gamma_{yj}^w = (w_j(1-\lambda_j))^{1/(1-\alpha)}/s_y^w$ with $s_x^w = \sum_{j=1}^2 (w_j\lambda_j)^{1/(1-\alpha)}$ and $s_y^w = \sum_{j=1}^2 (w_j(1-\lambda_j))^{1/(1-\alpha)}$. In the case of increasing returns to effort and that $w_1\lambda_1 \geq w_2\lambda_2$ and $w_2(1-\lambda_2) \geq w_1(1-\lambda_1)$, the equilibrium efforts given in expression (11) would become $x_1^w(A) = y_2^w(A) = \alpha w_1\lambda_1 w_2(1-\lambda_2)v/(w_1\lambda_1 + w_2(1-\lambda_2))^2$ and $x_2^w(A) = y_1^w(A) = 0$.

bias towards the most valued issue, i.e., the "common value" issues. This behavior is referred as "issue convergence" in the electoral competition literature (Amorós and Socorro Puy, 2007; Aragonès et al., 2015; Ash et al. 2015; Colomer and Llavador, 2011; Denter, 2016; Dragu and Fan, 2016; Egorov, 2015). Nonetheless, "issue divergence" does not disappear - all depends on the relative balance between comparative advantages (i.e., the candidates abilities) and the weighted value (i.e., the voters' preferences) in the decision process.

Another example comes from the academia, where tenure candidates are evaluated over multiple dimensions: research output, teaching quality and other academic merits. The review follows a holistic evaluation of all the parts into a single decision. However, in reality we observe that candidates direct their efforts to the research output because this is often the most relevant issue, i.e., the issue with highest weight in the final decision.

7.2 Different returns to effort

The parameter α denotes the efficiency of the effort technology. For instance, in litigation, an individual with a high value of α can be seen has being represented by a better lawyer than other with a low value of α. This aspect is distinct from the individual merits in the dispute. Similarly, some electoral candidates may be more efficient producing arguments than others.

However, in order to have a sufficiently tractable model, it is commonly assumed in the literature that the returns to effort are constant for all individuals. From the mathematical point of view, the relaxation of this assumption requires the use of numerical methods. However, there are interesting insights in terms of model predictions. In this context, we can consider two different cases:

(i) The case in which the returns to effort vary among issues. In this case, in equilibrium the issues with larger returns to effort will receive higher effort, in line with the comments made in Sections 5 and 6.

(ii) The case in which the returns to effort vary among individuals. In this context, one particular case is of special interest because it leads to a different equilibrium structure. Suppose that one individual, say individual 1, has increasing returns to effort $\alpha_x \geq 1$, while individual 2 has decreasing returns to effort $\alpha_y < 1$. In this case, we have a new type of equilibrium that is a mixture of the equilibria found in Propositions 3 and 5. In this new equilibrium, individual 1 directs all the effort to a single issue in order to benefit the most from the increasing returns to effort, while individual 2 distributes the effort among all issues.[12]

[12]In equilibrium, we obtain the implicit solution: $x_1^+(A) = \alpha_x p(A)(1-p(A))v$ and $x_2^+(A) = 0$, and

Table 2 provides a brief schematic of individuals' strategic behavior for different returns to scale.

This type of equilibrium is not so uncommon in real life. For instance, in the 2008 American presidential election, John McCain strategy intended to drive the focus of the electoral debate to the "presidential skills issue". Initially, John McCain thought to have relatively more ability and higher returns to scale than Barack Obama on that issue, by invoking Vietnam War and political experience. On the other hand, Barack Obama strategy covered a wider range of issues (e.g., the Bush's and the Iraq War unpopularity, general economic issues, the crises and the health care system, among others), in line with the decreasing returns to effort argument. Later, John McCain realized that he has no increasing returns in the "presidential skills issue" and shifted his strategy by diversifying his campaign efforts towards other issues.

Finally, note that we can simultaneously incorporate weights as discussed in Section 7.1 and different returns to effort. In this case, the individuals' effort becomes simultaneously affected by the returns to effort and by the weight of each issue.

8 Conclusion

This paper studies multi-issue competition, i.e., situations in which the individuals compete for some prize by providing costly effort in several independent issues. We propose a novel contest success function that aggregates the individuals' efforts over multi-issues (denoted as the A-system). Then, we compare the total effort in this situation with the case in which each issue is contested independently (denoted as the I-system).

In the A-system, the baseline model shows two different types of strategic behavior. Under decreasing returns to effort, individuals distribute effort over all issues with a natural bias towards the ones in which they have higher ability, while under increasing returns to effort, individuals direct their efforts to a single issue, the one with highest ability. In both cases, strategic behavior is driven by comparative advantages (i.e., considerations regarding the individuals' abilities).

The baseline model is extended to accommodate more realistic situations. We consider the case in which issues have different weights in the final outcome. In this case, strategic

$y_j^+(A) = (\alpha_y p^2(A)(1-\lambda_j)v/\lambda_j)^{1/(1-\alpha_y)}/(x_1^+(A))^{\alpha_x/(1-\alpha_y)}$, for $j = 1, 2$, where $p(A)$ is given by the CSF (6). In this equilibrium, there is no symmetry in terms of total effort, as in Propositions 2, 4 and 6, but the equality $x_1^+(A)/\alpha_x = (y_1^+(A) + y_2^+(A))/\alpha_y$, which implies that individual 1, the one with higher returns to effort, has higher incentives to provide effort.

behavior becomes simultaneously driven by comparative advantages and value considerations. In addition, if individuals have different returns to effort, the model is able to capture strategic behavior in which one individual directs all effort to a single issue, while the other individual distributes effort among several issues. Table 2 provides a brief schematic of these results.

	decreasing returns $0 < \alpha_y < 1$	increasing returns $\alpha_y \geq 1$
decreasing returns $0 < \alpha_x < 1$	both individuals 1 and 2 provide effort in all issues	ind. 1 provides effort in all issues ind. 2 provides effort in a single issue
increasing returns $\alpha_x \geq 1$	ind. 1 provides effort in a single issue ind. 2 provides effort in all issues	both individuals 1 and 2 provide effort in a single issue

Table 2: **Results schematic (returns to effort):** the individuals' strategic distribution of effort and the returns to effort.

We found that the A-system tends (but not always) to induce more effort because of the extra competition intensity associated with the aggregation of several prizes in a single prize - the effort for the full prize is higher than the sum of the efforts for each component prize. The superiority of the A-system becomes even stronger with increasing returns to effort or unequal weights. However, under the I-system the several independent prizes are likely to be more evenly distributed among individuals, which corresponds to an equity advantage over the A-system. The usual trade-off between equity and efficiency seems to emerge in our setting. Considerations of this kind should be the object of further research.

Our approach can also be extended to consider asymmetric valuations and non-linear costs of effort. In addition, we ignore the existence of complementarity between issues. The contest success function proposed in this paper is not exclusive and we can consider other ways to model multi-issue competition (Arbatskaya and Mialon, 2010, 2012; Epstein and Hefeker, 2003).

Other extensions to the original model are possible. For instance, we can consider that in the A-system the full prize is awarded to the individual that won more issues. These considerations extend the majority and the plurality rules into multi-issue contests (Kovenock and Roberson, 2012; Snyder, 1989), and the possibility of alliance formation in multi-issue contests (Skaperdas, 1998).

Finally, our approach and results can be applied to disputes, negotiations and other competitive situations involving multi-issue considerations such as litigation, advertising or electoral competition, among others. In this context, we expect that our results will help researchers and practitioners to better understand the process of endogenous selection of issues in competitive contexts, and consequently to provide guidance in the implementation of the optimal allocation and decision mechanisms in real life situations.

Appendix

In the main text of this paper, in order to simplify the analysis and to provide better intuition, we have focused in the two individuals and two issues case. In this Appendix, we briefly consider the general model with an arbitrary number of issues. In this context, we should introduce extra notation to distinguish between individuals and issues.

The general model

Let $x_{ij}(s)$ denotes the effort of individual $i = 1, ..., n$ in issue $j = 1, ..., m$ under the system $s \in \{I, A\}$. Let $\lambda_{ij} > 0$ denote the ability of individual $i = 1, ..., n$ in issue $j = 1, ..., m$. Recall that v_j denotes the prize of issue $j = 1, ..., m$ and $v = \sum_{j=1}^{m} v_j$ denotes the aggregated prize.

In this context, under the multi-issue A-system, the probability that individual $i = 1, ..., n$ wins by providing effort in all (or some) issues $j = 1, ..., m$, is given by:

$$p_i(A) = (\textstyle\sum_{j=1}^{m} \lambda_{ij} x_{ij}^{\alpha_{ij}}(A))/(\sum_{k=1}^{n} \sum_{j=1}^{m} \lambda_{kj} x_{kj}^{\alpha_{ij}}(A)), \tag{15}$$

with $p_i(A) = \sum_{j=1}^{m} \lambda_{ij}/n$ if $x_{ij}(A) = 0$ for all $i = 1, ..., n$ and $j = 1, ..., m$. Expression (15) is the generalized version of expression (6).

Under the multi-issue A-system, each individual $i = 1, ..., n$ simultaneously chooses a profile of efforts, $x_{ij}(A) \geq 0$ for $j = 1, ..., m$, that maximizes the expected payoff net of the cost of effort, which is given by:

$$\pi_i(A) = p_i(A)v - \textstyle\sum_{j=1}^{m} x_{ij}(A), \tag{16}$$

subject to the participation constraint $\pi_i(A) \geq 0$. Expression (16) is the generalized version of expression (7).

In order to have a sufficiently tractable A-system model, we do the following simplifying assumptions: $\alpha_{ij} = \alpha$ and $\lambda_{ij} \in (0,1)$ with $\sum_{i=1}^{n} \lambda_{ij} = 1$ for all $i = 1, ..., n$ and $j = 1, ..., m$ and $n = 2$.

General results and proofs

In the case $0 < \alpha < 1$, the solution to the general problem (16) is framed in the following result.

Proposition 10 *For $0 < \alpha < 1$, the individuals $i = 1, 2$ equilibrium efforts in each issue $j = 1, ..., m$ are given by:*

$$x_{ij}(A) = \alpha\phi\gamma_{ij}v, \tag{17}$$

where $\phi = s_i^{\alpha-1}s_{-i}^{\alpha-1}/(s_i^{\alpha-1} + s_{-i}^{\alpha-1})^2$ and $\gamma_{ij} = \lambda_{ij}^{1/(1-\alpha)}/s_i$ with $s_i = \sum_{j=1}^{m} \lambda_{ij}^{1/(1-\alpha)}$ for $i = 1, 2$ and $j = 1, ..., m$.

Proof of Proposition 10 (and Propositions 3 and 4) . The proof of Proposition 3 is just a particular case of this proof. From the problem 16, the associated set of $n \times m$ first order conditions is given by:

$$\alpha_{ij}\lambda_{ij}x_{ij}^{\alpha_{ij}-1}(A)(\textstyle\sum_{k=1}^{m} \lambda_{-ik}x_{-ik}^{\alpha_{-ik}}(A))v/(\sum_{l=1}^{n}\sum_{k=1}^{m} \lambda_{lk}x_{lk}^{\alpha_{lk}}(A))^2 = 1, \tag{18}$$

for $i = 1, ..., n$ and $j = 1, ..., m$. Under $\alpha_{ij} = \alpha$ for all $i = 1, ..., n$ and $j = 1, ..., m$ and $n = 2$, after some algebra on the system of $2m$ first order conditions we obtain that:

$$\lambda_{i1}x_{i1}^{\alpha-1}(A) = \lambda_{i2}x_{i2}^{\alpha-1}(A) = ... = \lambda_{im}x_{im}^{\alpha-1}(A), \tag{19}$$

for $i = 1, 2$, which corresponds to $2m - 2$ independent equations. Consequently, we need two additional equations in order to solve the system. One such equation is obtained by noticing that since individuals are not budget constrained, and the total prize and the unit cost of effort are the same, in equilibrium we must have: $\sum_{j=1}^{m} x_{ij} = \sum_{j=1}^{m} x_{-ij}$. This equality relation can be easily shown by rewriting the system of first order conditions (18) in the form:

$$x_{ij}(A) = \alpha\lambda_{ij}x_{ij}^{\alpha}(A)(\textstyle\sum_{k=1}^{m} \lambda_{-ik}x_{-ik}^{\alpha}(A))v/(\sum_{l=1}^{n}\sum_{k=1}^{m} \lambda_{lk}x_{lk}^{\alpha}(A))^2,$$

and, then summing over all issues $j = 1, ..., m$ for each $i = 1, 2$. The last independent equation is obtained by any of the first order conditions in the system (18). After some algebra, on this system of $2m$ equations, we obtain the unique solution given by expression (17), from where expression (8) is a particular case. Now, we need to verify under which conditions such solution corresponds to a Nash equilibrium (Pérez-Castrillo and Verdier, 1992; Szidarovszky and Okuguchi, 1997). Since the second derivative of each first order condition is strictly negative, i.e.,:

$$-\frac{(1+\alpha)\lambda_{ij}x_{ij}^{\alpha}(A)+(1-\alpha)(\sum_{l=1}^{n}\sum_{k=1}^{m}\lambda_{lk}x_{lk}^{\alpha}(A)-\lambda_{ij}x_{ij}^{\alpha}(A))}{\left(\sum_{l=1}^{n}\sum_{k=1}^{m}\lambda_{lk}x_{lk}^{\alpha}(A)\right)^{3}/(\alpha\lambda_{ij}x_{ij}^{\alpha-1}(A)(\sum_{k=1}^{m}\lambda_{-ik}x_{-ik}^{\alpha}(A)))}v<0,$$

then, $\pi_i(A)$ is strictly concave for $\alpha \in [0,1)$. In addition, since the vector $(x_{i1}(A),...,x_{im}(A)) \subset \mathbb{R}_{+}^{m}$ is defined on a convex space, then the first order condition is simultaneously necessary and sufficient for a maximum. Finally, since the effort in expression (17) is positive we are left to show that in equilibrium $\pi_i(A) \geq 0$. Note that the winning probability of individual i is given by: $p_i(A) = s_{-i}^{\alpha-1}/(s_{-i}^{\alpha-1}+s_{i}^{\alpha-1})$, and the individual i total effort is given by: $\alpha s_i^{\alpha+1}s_{-i}^{\alpha+1}v/(s_i^{\alpha}s_{-i}+s_i s_{-i}^{\alpha})^2$ (i.e., $T(A)/2$). Consequently, after replacing these expressions into $\pi_i(A) = p_i(A)v - T(A)/2$ we obtain the expected payoff:

$$\pi_i(A) = \left(s_{-i}^{\alpha-1}/(s_{-i}^{\alpha-1}+s_{i}^{\alpha-1}) - \alpha s_i^{\alpha-1}s_{-i}^{\alpha-1}/(s_i^{\alpha-1}+s_{-i}^{\alpha-1})^2\right)v$$

for $i = 1, 2$. Since effort does not create value, the minimum payoff is obtained when effort is maximal, i.e., at $s_i = s_{-i}$ (see Proposition 4). Then, it is easy to show that participation is guaranteed for $\alpha < 2$, which is always true for $\alpha < 1$.

The proof of Proposition 4 is just a particular case of the following more general proof. Since in equilibrium each individual provides the same aggregate effort, i.e., $\sum_{j=1}^{m} x_{ij} = \sum_{j=1}^{m} x_{-ij}$, and $T(A) = 2\sum_{j=1}^{m} x_{ij}(A)$. Then, after some algebra we obtain $2\alpha s_i^{\alpha-1}s_{-i}^{\alpha-1}v/(s_i^{\alpha-1}+s_{-i}^{\alpha-1})^2$ which is equivalent to expression (9). In order to study whether the aggregate effort $T(A)$ increases with the ability λ_{ij} simply differentiate $T(A)$ with respect to λ_{ij} to obtain that:

$$\partial T(A)/\partial\lambda_{ij} = 2\alpha\lambda_{ij}^{\alpha/(1-\alpha)}\phi(s_i^{\alpha-1}-s_{-i}^{\alpha-1})v/(s_i(s_i^{\alpha-1}+s_{-i}^{\alpha-1})),$$

is strictly positive if $s_i < s_{-i}$ because $\alpha < 1$ for all i and j, and the opposite otherwise. Since $T(A)$ is twice continuously differentiable, the maximal effort occurs at $s_{-i} = s_i$. At this point, the Hermitian matrix of i is negative-semidefinite, i.e., with non-positive eigenvalues, which are either 0 or $-\alpha v\sum_{j=1}^{m}\lambda_{ij}^{2a/(1-\alpha)}/(4s_i^2)$. ■

In the case $\alpha \geq 1$, individuals place all effort in a single issue - the one in which they have highest ability. Let $j[i] = \arg\max_j\{\lambda_{i1},...,\lambda_{im}\}$ denotes the issue in which individual i has highest ability.

Proposition 11 *For*

$$1 \leq \alpha \leq \min \left\{ (\lambda_{1j[1]} + \lambda_{2j[2]})/\lambda_{1j[1]}, (\lambda_{1j[1]} + \lambda_{2j[2]})/\lambda_{2j[2]} \right\}, \tag{20}$$

the individuals equilibrium efforts in each issue $j = 1, 2$ are given by:

$$x_{ij[i]}(A) = \alpha \lambda_{1j[1]} \lambda_{2j[2]} v/(\lambda_{1j[1]} + \lambda_{2j[2]})^2, \tag{21}$$

where $j[i] = \arg\max_j \{\lambda_{i1}, ..., \lambda_{im}\}$, and $x_{ij}(A) = 0$ for $j \neq j[i]$.

Proof of Proposition 11 (and Proposition 5). The proof of Proposition 5 is just a particular case of this proof. Note that for $\alpha \geq 1$, in order for the sequence of equalities (19) obtained from the first order condition (18) to be satisfied, the issues with larger ability λ_{ij} must receive lower effort. However, such behavior cannot be an equilibrium because since $\alpha \geq 1$ the effort output $\sum_{j=1}^{m} \lambda_{ij} x_{ij}^{\alpha}(A)$ is convex in $x_{ij}(A)$, and the marginal utility from effort increases. Consequently, optimal behavior requires that each individual places effort in a single issue, i.e., the one with largest abilities (see the discussion in Section 5.1). Let $j[i] = \arg\max_j \{\lambda_{i1}, ..., \lambda_{im}\}$ denotes the issue in which individual i has the highest ability. Under $\alpha_{ij} = \alpha$ for all $i = 1, ..., n$ and $j = 1, ..., m$ and $n = 2$, individual $i = 1, 2$ chooses $x_{ij[i]}(A) > 0$ to maximize (16) and set $x_{ij}(A) = 0$ for all $j \neq j[i]$. The associated system of two first order conditions is given by:

$$\alpha \lambda_{ij[i]} x_{ij[i]}^{\alpha-1}(A) \lambda_{-ij[-i]} x_{-ij[-i]}^{\alpha}(A) v/(\lambda_{ij[i]} x_{ij[i]}^{\alpha}(A) + \lambda_{-ij[-i]} x_{-ij[-i]}^{\alpha}(A))^2 = 1,$$

for $i = 1, 2$. The solution returns the unique, symmetric and strictly positive equilibrium effort given in (21), from where expression (11) is a particular case. The expected payoff is non-negative if $\alpha \leq (\lambda_{ij[i]} + \lambda_{-ij[-i]})/\lambda_{-ij[-i]}$ for $i = 1, 2$.[13] The individual $i = 1, 2$ second order condition for a maximum is given by:

$$-(\lambda_{ij[i]} + \lambda_{-ij[-i]})((1 + \alpha)\lambda_{ij[i]} - (\alpha - 1)\lambda_{-ij[-i]})/(\alpha \lambda_{ij[i]} \lambda_{-ij[-i]} v),$$

which is negative if $\alpha < ((1+\alpha)\lambda_{ij[i]} + \lambda_{-ij[-i]})/\lambda_{-ij[-i]}$. This condition is implied by the more restrictive non-negative expected payoff condition $\alpha \leq (\lambda_{ij[i]} + \lambda_{-ij[-i]})/\lambda_{-ij[-i]}$ for $i = 1, 2$. This inequality corresponds to the existence condition (20), from where the condition (10) is a particular case. ■

[13]For completeness, the expected payoff (7) and the winning probability (6) are: $\pi_i(A) = \lambda_{ij[i]}(\lambda_{ij[i]} - (\alpha - 1)\lambda_{-ij[-i]})v/(\lambda_{ij[i]} + \lambda_{-ij[-i]})^2$ and $p_i(A) = \lambda_{ij[i]}/(\lambda_{ij[i]} + \lambda_{-ij[-i]})$ for $i = 1, 2$, respectively.

References

[1] Alcalde, J., Dahm, M. (2010). "Rent seeking and rent dissipation: A neutrality result." Journal of Public Economics, 94, 1-7.

[2] Amorós, P., and Socorro Puy, M. (2013). "Issue convergence or issue divergence in a political campaign." Public Choice, 155, 355-371.

[3] Aragonès, E., M. Castanheira, and M. Giani (2015). "Electoral Competition through Issue Selection." American Journal of Political Science, 59, 71-90.

[4] Arbatskaya, M. and Mialon, H. (2010). "Multi-activity contests." Economic Theory, 43, 23-43.

[5] Arbatskaya, M. and Mialon, H. (2012). "Dynamic Multi-Activity Contests." The Scandinavian Journal of Economics, 114, 520-538.

[6] Ash, E., Morelli, M., and Van Weelden, R. (2015). "Elections and Divisiveness: Theory and Evidence." NBER Working Paper No. 21422.

[7] Bac, M. and H. Raff (1996). "Issue-by-issue negotiations: The role of information and time preferences." Games and Economic Behavior, 13, 125-134.

[8] Baron, D. (1994). "Electoral competition with informed and uninformed voters." American Political Science Review, 88, 33-47.

[9] Baye, M., Kovenock, D., and de Vries, C. (1994). "The solution to the Tullock rent-seeking game when R>2: Mixed-strategy equilibria and mean dissipation rates", Public Choice, 81(3-4), 363-380.

[10] Berliant, M., and H. Konishi (2005). "Salience: Agenda Choices by Competing Candidates." Public Choice, 125, 129-149.

[11] Calleja, P., Borm, P., Hendrickx, R., (2005). "Multi-issue allocation situations". European Journal Operation Research, 164, 730-747.

[12] Chen, K. (2003). "Sabotage in Promotion Tournaments." Journal of Law, Economics, and Organization, 19, 119-140.

[13] Clark, D. and Riis, C. (1998). "Contest Success Functions: an Extension." Economic Theory, 11, 201-204.

[14] Cramton, P., Y. Shoham, and R. Steinberg (2006). "Introduction to combinatorial auctions", in Combinatorial Auctions, P. Cramton, Y. Shoham, and R. Steinberg, Eds. Cambridge, MA: MIT Press, 1-13.

[15] Colomer, J. and H. Llavador (2011). "An Agenda-Setting Model of Electoral Competition." Journal of the Spanish Economic Association, 3, 73-93.

[16] Corchón, L. (2007). "The theory of contests: a survey." Review of Economic Design, 11(2), 69-100.

[17] Corchón, L., and M. Dahm (2010). "Foundations for contest success functions." Economic Theory, 43(1), 81-98.

[18] Dahm, M., and Porteiro, N. (2008). "Biased contests." Public Choice, 136(1-2), 55-67.

[19] De Mot, J., and B. Luppi and F. Parisi (2012) "Are Sequential Trials Really Better than Unitary Trials?" mimeo.

[20] Denter, P. (2016). "Campaigning Equilibria." mimeo.

[21] de Vries, S., and R. Vohra (2003). "Combinatorial auctions: A survey". INFORMS Journal on computing, 15(3), 284-309.

[22] Dragu, T., and X. Fan (2016). "An Agenda-Setting Theory of Electoral Competition. Journal of Politics, Forthcoming.

[23] Egorov, G. (2015). "Single-Issue Campaigns and Multidimensional Politics." NBER Working Paper No. 21265.

[24] Epstein, G., and Hefeker, C. (2003). "Lobbying contests with alternative instruments." Economics of Governance, 4, 81-89.

[25] Ewerhart, C. (2015). "Mixed equilibria in Tullock contests." Economic Theory, 60, 59-71.

[26] Fatima, S., Wooldridge, M. and N. Jennings (2006). "Multi-issue negotiation with deadlines." Journal of Artificial Intelligence Research, 27, 381-417.

[27] Fershtman, C. (2000). "A note on multi-issue two-sided bargaining: bilateral procedures." Games and Economic Behavior, 30, 216-227.

[28] Franke, J., Kanzow, C., Leininger, W. and Schwartz, A. (2013). "Effort maximization in asymmetric contest games with heterogeneous contestants." Economic Theory, 52, 589-630.

[29] Friedman, L. (1958). "Game-theory models in the allocation of advertising expenditures." Operations Research, 6, 699-709.

[30] Giménez-Gómez, J. and Osório, A. (2015). "Why and How to Differentiate in Claims Problems? An axiomatic approach." European Journal of Operations Research, 241, 842-850.

[31] Harsanyi, J. (1977). Rational behavior and bargaining equilibrium in games and social situations. Cambridge University Press.

[32] Hirshleifer, J. and Osborne, E. (2001). "Truth, effort and the legal battle." Public Choice, 108, 169-195.

[33] Inderst, R. (2000). "Multi-issue bargaining with endogenous agenda." Games and Economic Behavior, 30, 64-82.

[34] Keeney, R., and Raiffa, H. (1976). Decisions with Multiple Objectives: Preferences and Value Tradeoffs. New York: John Wiley.

[35] Konrad, K. (2000). "Sabotage in Rent-Seeking Contests." Journal of Law, Economics, and Organization, 16, 155-165.

[36] Konrad, K. (2009). "Strategy and Dynamics in Contests." Oxford University Press.

[37] Kovenock, D. and Roberson, B. (2012). "Conflicts with Multiple Battlefields." Oxford Handbook of the Economics of Peace and Conflict (edited by M. Garfinkel and S. Skaperdas).

[38] Lang, K. and R. Rosenthal (2001). "Bargaining piecemeal or all at once." Economic Journal, 111, 526-540.

[39] Lazear, E. (1989). "Pay equality and industrial politics." Journal of Political Economy, 97, 561-580.

[40] Leininger, W. (1993). "More efficient rent-seeking—a Münchhausen solution." Public Choice, 75(1), 43-62.

[41] Loury, G., (1979). "Market structure and innovation." Quarterly Journal of Economics, 93, 395-410.

[42] Münster, J. (2009). "Group contest success functions." Economic Theory, 41, 345-57.

[43] Nalebuff, B., and J. Stiglitz, (1983). "Prizes and incentives: towards a general theory of compensation and competition." Bell Journal of Economics, 14, 21-43.

[44] Nitzan, S., (1994). "Modelling rent-seeking contests." European Journal of Political Economy." 10, 41-60.

[45] O'Neill, B., (1982). "A problem of rights arbitration from the Talmud". Mathematical Social Sciences, 2, 345-371.

[46] Osório, A. (2013). "The Lottery Blotto Game." Economics Letters, 120, 164-166.

[47] Osório, A. (2015). "Argumentation Quantity and Quality: A Litigation Success Function." mimeo.

[48] Osório, A. (2016). "Group contest success function: heterogeneous individuals." mimeo.

[49] Pérez-Castrillo, D., and Verdier T. (1992). "A general analysis of rent-seeking games", Public Choice, 71, 351-361.

[50] Petrocik, J. (1996). "Issue Ownership in Presidential Elections, with a 1980 Case Study." American Journal of Political Science, 40, 825-850.

[51] Raiffa H. (1982). The Art and Science of Negotiation. Harvard University Press, Cambridge, MA.

[52] Riker, W. (1996). "The Strategy of Rhetoric: Campaigning for the American Constitution." Yale University Press: New Haven.

[53] Robson, A. (2005). "Multi-item contests." Australian National University, Working Paper No. 446.

[54] Robson, A. and Skaperdas, S. (2008). "Costly enforcement of property rights and the coase theorem." Economic Theory, 36, 109-128.

[55] Schelling, T. (1956). "An essay on bargaining." American Economic Review, 46, 281-306.

[56] Schelling, T. (1960). The strategy of conflict. Oxford University Press.

[57] Skaderpas, S. (1996). "Contest Success Functions." Economic Theory, 7, 283-290.

[58] Skaperdas, S. (1998). "On the formation of alliances in conflict and contests." Public Choice, 96(1), 25-42.

[59] Skaperdas, S., and B. Grofman (1995). "Modeling negative campaigning." American Political Science Review, 89, 49-61.

[60] Snyder, J. (1989). "Election goals and the allocation of campaign resources." Econometrica, 57, 637-660.

[61] Stein, W. (2002). "Asymmetric rent-seeking with more than two contestants." Public Choice, 113 (3-4), 325-336.

[62] Szidarovszky, F., Okuguchi, K. (1997). "On the existence and uniqueness of pure Nash equilibrium in rent-seeking games", Games and Economic Behavior, 18, 135-140.

[63] Thomson, W. (2003). "Axiomatic and game-theoretic analysis of bankruptcy and taxation problems: a survey", Mathematical Social Sciences, 45, 249-297.

[64] Tullock, G. (1980). "Efficient rent seeking", In J. Buchanan, R. Tollison and G. Tullock (eds), Towards a Theory of the Rent-Seeking Society, 97–112. College Station, TX: Texas A&M University Press.

[65] Young, H., (1994). Equity, in Theory and Practice. Princeton University Press, Princeton, NJ.