

# Disordered Tm<sup>3+</sup>,Ho<sup>3+</sup>-codoped CNGG garnet crystal: Towards efficient laser materials for ultrashort pulse generation at ~2 μm

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**ABSTRACT.** We report on the growth, structure refinement, optical spectroscopy, continuous wave and femtosecond mode-locked laser operation of a  $\text{Tm}^{3+}, \text{Ho}^{3+}$ -codoped disordered calcium niobium gallium garnet (CNGG) crystal. The 2.64 at.% Tm, 0.55 at.% Ho:CNGG is grown by the Czochralski method. Its cubic structure, sp. gr.  $Ia\bar{3}d - O_{h}^{10}$ ,  $a = 12.4952(1) \text{ \AA}$ , is refined by the Rietveld method revealing a random distribution of  $\text{Ga}^{3+}$  and  $\text{Nb}^{5+}$  cations over octahedral and tetrahedral sites. The  $\text{Ho}^{3+}$  transition probabilities are determined within the Judd-Ofelt theory accounting for an intermediate configuration interaction (ICI). For the  ${}^5\text{I}_7 \rightarrow {}^5\text{I}_8$   $\text{Ho}^{3+}$  transition, the maximum stimulated-emission cross-section  $\sigma_{\text{SE}}$  is  $0.47 \times 10^{-20} \text{ cm}^2$  at 2080.7 nm. The gain bandwidth of Tm,Ho:CNGG at  $\sim 2 \text{ \mu m}$  is  $>150 \text{ nm}$  and the thermal equilibrium decay time - 6.80 ms. The  $\text{Tm}^{3+} \leftrightarrow \text{Ho}^{3+}$  energy transfer parameters are determined. A diode-pumped Tm,Ho:CNGG microchip laser generated 413 mW at 2088.4 nm with a slope efficiency of 15.9%. A continuous wavelength tuning between 1940.3 and 2144.6 nm is demonstrated. Ultrashort pulses as short as 73 fs are achieved at 2061 nm from a Tm,Ho:CNGG laser mode-locked by a GaSb semiconductor saturable absorber mirror at a repetition rate of 89.3 MHz.

**Keywords:** garnets; crystal structure; optical spectroscopy; holmium ions; energy transfer; laser operation.

## 1. Introduction

The disordered calcium niobium gallium garnet (shortly CNGG) is a well-known laser host crystal for doping with rare-earth ions ( $\text{RE}^{3+}$ ) [1,2]. It belongs to the cubic crystal class (sp. gr.  $Ia\bar{3}d$ ). The structure disorder originates from a random distribution of the  $\text{Ga}^{3+}$  and  $\text{Nb}^{5+}$  cations over the same lattice sites (octahedral and tetrahedral) [3]. The composition of CNGG typically deviates from the stoichiometric one,  $\text{Ca}_3\text{Nb}_{1.5}\text{Ga}_{3.5}\text{O}_{12}$ , so that the structure may accommodate cationic vacancies serving to ensure charge neutrality. A great variety of structural elements in CNGG leads to splitting and inhomogeneous broadening of the spectral bands of the  $\text{RE}^{3+}$  dopants [3,4], resulting in a glassy-like behavior [1]. CNGG crystals melt congruently at relatively low temperatures around 1430-1470 °C (depending on the composition), so that they can be easily grown by the conventional Czochralski (Cz) method [2]. As a host matrix, CNGG also exhibits attractive thermo-mechanical properties, e.g., moderate thermal conductivity ( $\kappa = 4.3 \text{ Wm}^{-1}\text{K}^{-1}$ ), weak thermal expansion ( $\alpha = 7.8 \times 10^{-6} \text{ K}^{-1}$ ) and a positive thermo-optic coefficient ( $dn/dT = 7.8 \times 10^{-6} \text{ K}^{-1}$  at  $\sim 1 \mu\text{m}$ ) [3]. CNGG shows a broad transparency range of 0.28 – 8  $\mu\text{m}$  [3] (for oxide crystals) and, thus, it is suitable for short-wave infrared emission at 2 – 3  $\mu\text{m}$ .

The broadband emission properties of  $\text{RE}^{3+}$  ions in CNGG determine its applications in mode-locked (ML) lasers. The first studies focused on crystals doped with  $\text{Nd}^{3+}$  and  $\text{Yb}^{3+}$  ions for laser emission at  $\sim 1 \mu\text{m}$  and, recently, interesting results were achieved in the mode-locked operation regime [5-8]. Ma *et al.* demonstrated sub-50 fs soliton pulses (47 fs, with external compression) at 1061 nm from a  $\text{Yb}^{3+}, \text{Na}^+:\text{CNGG}$  laser ML by a Semiconductor Saturable Absorber Mirror (SESAM) [8].

Furthermore, the interest shifted to the spectral range of  $\sim 2 \mu\text{m}$ . Such emission is eye-safe and it spectrally overlaps with absorption lines of atmospheric and bio-species such as  $\text{H}_2\text{O}$  or  $\text{CO}_2$ . Ultrafast lasers at  $\sim 2 \mu\text{m}$  are of practical importance for highly localized surgery (medicine), soft material processing, frequency comb generation, molecular spectroscopy and supercontinuum generation. They are used as seed sources for frequency down-conversion into mid-infrared and terahertz (THz) spectral ranges [9]. It is common to achieve laser emission at  $\sim 2 \mu\text{m}$  with  $\text{Tm}^{3+}$  and  $\text{Ho}^{3+}$  dopant ions.

$\text{Tm}^{3+}$ -doped CNGG crystals were first studied by Voronko *et al.* [10,11] and interesting advances have been published recently. Pan *et al.* reported on the Cz crystal growth, spectroscopy and continuous-wave (CW) laser operation of CNGG codoped with active  $\text{Tm}^{3+}$  and passive  $\text{Na}^+(\text{Li}^+)$  cations serving for charge compensation [12]: a diode-pumped  $\text{Tm}^{3+}, \text{Na}^+:\text{CNGG}$  laser generated 1.05 W at 2007.7 nm with a slope efficiency of 35%.

Broadband wavelength tuning in a  $\text{Tm}^{3+}, \text{Li}^+:\text{CNGG}$  laser (tuning range: 224 nm) was realized [12]. Wang *et al.* reported on a  $\text{Tm}^{3+}, \text{Li}^+:\text{CNGG}$  laser ML by a single-walled carbon nanotube (SWCNT) saturable absorber (SA) delivering pulses as short as 78 fs without external compression at 2017 nm, at a pulse repetition rate of 86 MHz [13]. Slightly longer pulses (84 fs) were generated in a similar laser based on a  $\text{Tm}^{3+}, \text{Na}^+:\text{CNGG}$  crystal [14].

The Holmium ion ( $\text{Ho}^{3+}$ , electronic configuration:  $[\text{Xe}]4f^{10}$ ) is attractive for laser emission at wavelengths slightly above 2  $\mu\text{m}$  owing to the  $^5\text{I}_7 \rightarrow ^5\text{I}_8$  electronic transition. This emission is red-shifted with respect to that of the  $\text{Tm}^{3+}$  ion and is thus more attractive for ML lasers because it avoids the unwanted overlap with the structured water vapor absorption in the atmosphere [15]. Two routes to excitation of  $\text{Ho}^{3+}$  ions are known. The first one is the so-called in-band pumping, exciting directly to the  $\text{Ho}^{3+}$  upper laser level ( $^5\text{I}_7$ ) which benefits from a high laser efficiency [16] being suitable for power scaling [17]. However, the required pump sources (e.g., powerful Tm fiber lasers or GaSb laser diodes) are expensive and still under development. The second pump scheme is relying on  $\text{Tm}^{3+}, \text{Ho}^{3+}$  codoping [18].  $\text{Tm}^{3+}$  ions can be pumped by commercial AlGaAs laser diodes emitting at  $\sim 0.8 \mu\text{m}$  transferring part of the energy of the electronic excitation to the  $\text{Ho}^{3+}$  ions. Despite the limited power scaling of Tm, Ho lasers owing to thermal effects, the codoping scheme may greatly increase the gain bandwidth of the material by combining the individual emissions from both  $\text{Tm}^{3+}$  and  $\text{Ho}^{3+}$  ions. This is an attractive feature for ML laser development [19].

To date, very limited information exists about  $\text{Ho}^{3+}$ -doped CNGG crystals. Ryabochkina *et al.* reported on the Cz growth, spectroscopy and in-band pumped laser performance of Ho:CNGG delivering 2.1 W at 2095 nm with a slope efficiency of 37% [20]. Xue *et al.* have grown such crystals by the  $\mu$ -pulling down method [21]. Very recently, we demonstrated a Tm, Ho:CNGG laser ML by a SWCNT-SA delivering 83-fs pulses (without external compression) at 2081 nm at a repetition rate of 102 MHz [22].

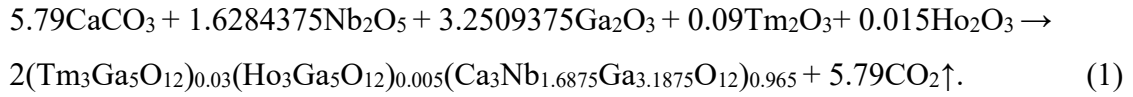
In the present paper, we study in detail the structure, spectroscopic and laser properties of the disordered Tm, Ho:CNGG garnet with the goal of developing an efficient laser material for sub-100 fs ML lasers at  $\sim 2 \mu\text{m}$ .

## 2. Experimental section

### 2.1. Crystal growth

A 3.0 at.%  $\text{Tm}^{3+}$ , 0.5 at.%  $\text{Ho}^{3+}$  (in the melt) doped CNGG single crystal was grown by the Cz method using Ar atmosphere in an Ir crucible. The starting materials,  $\text{CaCO}_3$  (purity: 4N),  $\text{Nb}_2\text{O}_5$  (4N),  $\text{Ga}_2\text{O}_3$  (5N),  $\text{Tm}_2\text{O}_3$  (5N) and  $\text{Ho}_2\text{O}_3$  (5N), were first weighed according to

the chemical formula  $(\text{Tm}_3\text{Ga}_5\text{O}_{12})_{0.03}(\text{Ho}_3\text{Ga}_5\text{O}_{12})_{0.005}\text{Ca}_3\text{Nb}_{1.6875}\text{Ga}_{3.1875}\text{O}_{12})_{0.965}$ . To compensate the volatilization of  $\text{Ga}_2\text{O}_3$  during the synthesis of the polycrystalline material and the crystal growth, an excess of 1.0 wt%  $\text{Ga}_2\text{O}_3$  was added. The equation of the chemical reaction reads:



The components of the growth charge were first mixed, ground and heated at 1173 K in a Pt crucible for 10 h to decompose  $\text{CaCO}_3$ . After the crucible was cooled down to room temperature (RT, 293 K), the mixture was pressed into pellets and reheated at 1373 K for 15 h to synthesize the  $\text{Tm,Ho:CNGG}$  polycrystalline material through solid-state reaction. The latter was placed into an Ir crucible and melted by an intermediate-frequency heater. As a seed for crystal growth, an oriented along the [111] direction  $\text{Yb}_3\text{CNGG}$  was used. During the crystal growth process, the pulling rate varied from 0.5 to 1.0 mm/h and the crystal rotation speed was kept at 8 to 15 revolutions per minute (rpm). After the growth was completed, the crystal was cooled down to RT at a stepped rate of 15 to 25 °K/h.

For spectroscopic studies, we also grew a singly  $\text{Ho}^{3+}$ -doped CNGG crystal using a similar methodology.

## 2.2. Characterization

The actual concentration of the dopant ions ( $\text{Tm}^{3+}$  and  $\text{Ho}^{3+}$ ) was determined by inductively-coupled plasma mass spectrometry (ICP-MS, Agilent). The X-ray powder diffraction pattern was measured at room temperature using a Bruker D2 Phaser diffractometer for diffraction angles  $2\theta$  in the range of 10 - 80° with  $\text{Cu K}\alpha_1$  radiation (1.54051 Å).

The thermogravimetric (TG) analysis and differential scanning calorimetry (DSC) were carried out on a simultaneous Mettler-Toledo TGA/DSC 2/1600 thermal analyzer in a flowing  $\text{N}_2$  atmosphere. The sample was enclosed in a corundum crucible and heated from room temperature to 1600°C at a heating rate of 1.5°C/min.

The Raman spectrum was measured using a confocal Raman microscope (Renishaw inVia) equipped with an  $\times 50$  objective and an  $\text{Ar}^+$  ion laser ( $\lambda = 514$  nm).

For the spectroscopic and laser studies, we cut rectangular elements with a thickness ( $t$ ) of 3...6 mm and aperture of  $3 \times 3$  mm<sup>2</sup> using the annealed crystal boule. They were oriented for light propagation along the [111] crystallographic direction.

All the spectroscopic studies were performed at room temperature (RT, 293 K). The absorption spectra in the range of 300-2200 nm were measured using a CARY 5000 (Varian)

spectrophotometer; the spectral bandwidth (SBW) was 0.2 nm. The luminescence spectrum was measured using an optical spectrum analyzer (OSA, AQ6375B, Yokogawa, SBW = 0.1 nm) whose spectral response was calibrated using a Hg lamp. The excitation was from a Ti:Sapphire laser tuned to 802 nm. For the luminescence decay studies, we used a nanosecond optical parametric oscillator (OPO, Horizon, Continuum) as the excitation source and the detection system comprising a ¼-m monochromator (Oriel 77200), an InGaAs detector and an 8-GHz digital oscilloscope (DSA70804B, Tektronix). The measurements were performed on a crushed sample employing a pinhole to eliminate the radiation trapping effect.

### 3. Crystal growth and structure

#### 3.1. Crystal growth

The as-grown Tm,Ho:CNGG crystal boule is shown in [Figure 1\(a\)](#). It had a cylindrical shape with a uniform cross-section in the central part of the boule (dimensions: diameter ( $\phi$ ): 25 mm  $\times$  length: 35 mm). The cross-section was circular, which is typical for the Cz growth method. The crystal was transparent and neither cracks nor inclusions were observed. An inspection of the boule with a He-Ne laser revealed no scattering centers. The as-grown crystal was yellow-colored, which is attributed to the absorption of color centers in the visible due to the cationic vacancies, a typical behavior for CNGG-type crystals [3]. To eliminate the color centers, the crystal was annealed in air. First, it was heated from RT to 1373 K for 60 h, then it was kept at this temperature for 90 h and finally cooled down to RT for 60 h. As a result, the transparency of the crystal greatly improved, and its color changed to light-yellow, see [Figure 1\(b\)](#).

The actual concentration of the dopant ions was  $N_{\text{Tm}} = 3.24 \times 10^{20} \text{ cm}^{-3}$  (2.64 at.%) and  $N_{\text{Ho}} = 6.76 \times 10^{19} \text{ cm}^{-3}$  (0.55 at.%). Thus, the segregation coefficients for the rare-earth ions,  $K_{\text{RE}} = N_{\text{crystal}}/N_{\text{melt}}$  were  $K_{\text{Tm}} = 0.88$  and  $K_{\text{Ho}} = 1.10$ . For the singly Ho<sup>3+</sup>-doped CNGG crystal grown for comparison, the actual doping level was measured to be 0.55 at.% Ho.

The results of the TG-DSC analysis are shown in [Figure 2](#). The thermogravimetric (TG) analysis shows that Tm,Ho:CNGG has nearly no weight loss in the whole studied temperature range. From the heating curve of the DSC, only one endothermic peak was observed which corresponds to the melting point. The melting temperature of Tm,Ho:CNGG, taken as the onset temperature of this peak, is 1698 K (here, we define the onset temperature as the intersection of the tangent in the flat zone before the peak and the tangent in the maximum slope of the

peak). Tm,Ho:CNGG is completely melted at the peak temperature of 1707 K and the cubic phase is stable below the melting temperature.

### 3.2. Structure refinement

The structure and the phase purity of the as-grown crystal were confirmed by X-ray powder diffraction (XRD), [Figure 3\(a\)](#).

The crystal structure was refined using the Rietveld method, see the details in [Table 1](#). As starting atomic coordinates, we used the data from the undoped CNGG crystal [23]. The starting occupancy factors were taken from [24], considering the determined actual doping concentrations for Tm<sup>3+</sup> and Ho<sup>3+</sup>. Tm,Ho:CNGG is cubic (sp. gr.  $Ia\bar{3}d$  - O<sup>10</sup><sub>h</sub>, No. 230) and isostructural to the undoped CNGG crystal. The lattice constant  $a = 12.4952(1)$  Å and the unit-cell volume  $V = 1950.87$  Å<sup>3</sup> (the number of formula units in the unit-cell  $Z = 8$ ). The calculated crystal density is  $\rho_{\text{calc}} = 5.064$  g/cm<sup>3</sup>. The  $R$ -factors for the refinement were as follows:  $R_{\text{wp}} = 7.76\%$  and  $R_{\text{exp}} = 6.13\%$  (the reduced  $\chi$ -squared value  $\chi^2 = (R_{\text{wp}}/R_{\text{exp}})^2 = 1.60$ ). The determined fractional atomic coordinates ( $x, y, z$ ) are listed in [Table 2](#). No other phases except of the cubic one are found in the pattern.

According to the determined atomic coordinates, we drew the structure of Tm,Ho:CNGG. [Figure 3\(b\)](#) shows its projection in the  $b$ - $c$  plane. CNGG-type crystals belong to the family of cubic multicomponent garnets with a general formula  $\{A\}_3[B]_2(C)_3O_{12}$ , where  $\{A\}$ ,  $[B]$ , and  $(C)$  are dodecahedral (Wyckoff symbol:  $24c$ ), octahedral ( $16a$ ), and tetrahedral ( $24d$ ) sites, respectively [3]. Stoichiometric CNGG has a chemical formula of  $\text{Ca}_3\text{Nb}_{1.5}\text{Ga}_{3.5}\text{O}_{12}$ , or, equivalently,  $\{\text{Ca}_3\}[\text{Nb}_{1.5}\text{Ga}_{0.5}](\text{Ga}_3)\text{O}_{12}$ . Such a garnet can be synthesized only by precipitation at a temperature  $<1633$  K in the form of polycrystalline material [1]. For a real crystal, its composition deviates from the stoichiometric one, e.g., let us consider the studied crystal of Tm,Ho:CNGG. The Ca<sup>2+</sup> ions are located in dodecahedral sites (with a coordination number, C.N. = VIII). The rare-earth ions (Tm<sup>3+</sup> and Ho<sup>3+</sup>, in our case) replace for the Ca<sup>2+</sup> ones in these sites. The corresponding ionic radii for VIII-fold oxygen coordination are  $R_{\text{Ca}} = 1.12$  Å,  $R_{\text{Tm}} = 0.994$  Å and  $R_{\text{Ho}} = 1.105$  Å [25] which explains the observed decrease of the lattice constant for the Tm,Ho:CNGG crystal as compared to undoped CNGG ( $a = 12.5969(1)$  Å [26]). There are four shorter (2.366 Å) and four longer (2.499 Å) metal - oxygen (M - O) interatomic distances in the  $[\text{Ca}|\text{Tm}|\text{HoO}_8]$  dodecahedra, [Figure 4](#). The Ga<sup>3+</sup> and Nb<sup>5+</sup> cations are randomly distributed over both the octahedral (C.N. = VI) and tetrahedral (C.N. = IV) sites, see the occupancy factors (O.F.) in [Table 2](#). The corresponding

M – O bond lengths are 2.024 Å (×6) and 1.642 Å (×4), [Figure 4](#). The bond lengths and geometric parameters of the [MO<sub>n</sub>] polyhedrons in Tm,Ho:CNGG are summarized in [Table 3](#).

The random distribution of the Ga<sup>3+</sup> and Nb<sup>5+</sup> cations over the octahedral and tetrahedral sites determines the disordered crystal structure. The inhomogeneous broadening of the spectral bands of the rare-earth ions in Ca<sup>2+</sup> sites occurs due to the various multi-ligands in the second coordination sphere. The shortest intermetallic distances are: Ca|Tm|Ho – Ca|Tm|Ho = 3.692 Å (×6), Ca|Tm|Ho – Nb1|Ga1 = 3.370 Å (×4) and Ca|Tm|Ho – Nb2|Ga2 = 3.692 Å (×4) and 3.014 Å (×2).

According to the structure refinement data, the actual crystal composition is the following: {Ca<sub>2.9043</sub>Tm<sub>0.0792</sub>Ho<sub>0.0165</sub>}[Nb<sub>1.42</sub>Ga<sub>0.58</sub>](Ga<sub>2.754</sub>Nb<sub>0.246</sub>)O<sub>12</sub>.

### 3.3. Raman spectroscopy

Raman spectroscopy is known to be a sensitive tool to study the structural features of CNGG crystals [1,3]. The unpolarized Raman spectrum of Tm,Ho:CNGG is shown in [Figure 5](#).

The two groups of vibrations which are most sensitive to structure alteration are found in the long-frequency part of the Raman spectra of CNGG-type crystals (at ~700-900 cm<sup>-1</sup>) [1]. They are assigned to the symmetric stretching modes ( $\nu_s$ ) of isolated metal oxygen tetrahedra [M<sub>2</sub>O<sub>4</sub>]. For Tm,Ho:CNGG, the tetrahedral sites are occupied by both Ga<sup>3+</sup> and Nb<sup>5+</sup> cations. The lines at lower frequencies correspond to the [Ga<sub>2</sub>O<sub>4</sub>] groups and those at higher frequencies – to the [Nb<sub>2</sub>O<sub>4</sub>] ones. The appearance of satellite peaks indicates the structural distortion of the [M<sub>2</sub>O<sub>4</sub>] tetrahedra due to a nearby cationic vacant position [1]. At lower frequencies, there are two lines at 746 cm<sup>-1</sup> (C<sub>1</sub>, vacancies) and at 777 cm<sup>-1</sup> (C<sub>2</sub>, undistorted [Ga<sub>2</sub>O<sub>4</sub>]). At higher frequencies, there are such at 827 cm<sup>-1</sup> (C<sub>4</sub>, undistorted [Nb<sub>2</sub>O<sub>4</sub>]) and 869 cm<sup>-1</sup> (C<sub>5</sub>, vacancies). The relative intensity of the satellite C<sub>1</sub> and C<sub>5</sub> lines is weak indicating a moderate distortion of the [M<sub>2</sub>O<sub>4</sub>] tetrahedra. Note that the introduction of RE<sup>3+</sup> ions (Tm<sup>3+</sup> and Ho<sup>3+</sup> in our case) in dodecahedral sites reduced the concentration of vacancies in these sites and in the octahedral ones while it almost did not affect those in the tetrahedral sites [3]. The RE<sup>3+</sup> ion effect is revealed by a frequency shift of the C<sub>2</sub> and C<sub>4</sub> lines. Indeed, for stoichiometric undoped CNGG, they are found at 763 cm<sup>-1</sup> and 832 cm<sup>-1</sup>, respectively [1]. The broad pedestal seen in the long-frequency part is due to the luminescence of the Ho<sup>3+</sup> ion.

## 4. Optical spectroscopy

### 4.1. Judd-Ofelt modeling: Ho<sup>3+</sup> ions



The transition probabilities for the  $\text{Tm}^{3+}$  ion in CNGG were analyzed previously using the standard Judd-Ofelt (J-O) theory [12]. Consequently, we focused on the J-O analysis only for the  $\text{Ho}^{3+}$  ion. To this aim, the absorption spectra were measured for both  $\text{Tm,Ho:CNGG}$  (codoped) and  $\text{Ho:CNGG}$  (singly doped) crystals, [Figure 6](#). Here, the assignment of the  $\text{Tm}^{3+}$  and  $\text{Ho}^{3+}$  transitions is after [27]. The analysis of the  $\text{Ho}^{3+}$  transitions in absorption for both crystals yielded similar results. Below, for clarity, we will describe only the results achieved for  $\text{Ho:CNGG}$ .

The calculations were performed using the standard J-O theory [28,29] and its modification accounting for an intermediate configuration interaction (ICI) [30,31]. The reduced squared matrix elements  $U^{(k)}$ ,  $k = 2, 4, 6$  for  $\text{Ho}^{3+}$  transitions in absorption and emission were taken from [32]. The refractive indices were calculated from the Sellmeier equation [3]. The J-O formalism was applied to electric-dipole (ED) contributions to intensities of the  $4f-4f$   $\text{Ho}^{3+}$  transitions. The contribution of magnetic-dipole (MD) transitions with  $\Delta J = J - J' = 0, \pm 1$  was calculated separately within the Russell–Saunders approximation on wave functions of  $\text{Ho}^{3+}$  under the assumption of a free-ion.

For the standard J-O theory, the ED line strengths of the  $J \rightarrow J'$  transitions  $S^{\text{ED}}(JJ')$  are calculated as [28,29]:

$$S_{\text{calc}}^{\text{ED}}(JJ') = \sum_{k=2,4,6} U^{(k)} \Omega_k, \quad (1a)$$

$$U^{(k)} = \langle (4f^n)SLJ || U^{(k)} || (4f^n)S'L'J' \rangle^2. \quad (1b)$$

Here,  $U^{(k)}$  are the reduced squared matrix elements and  $\Omega_k$  are the intensity (J–O) parameters (for both,  $k = 2, 4, 6$ ).

Within the ICI approximation, the ED line strengths are given by [31]:

$$S_{\text{calc}}^{\text{ED}}(JJ') = \sum_{k=2,4,6} U^{(k)} \mathcal{Q}_k^{\text{ED}}, \quad (2a)$$

$$\mathcal{Q}_k^{\text{ED}} = \Omega_k [1 + 2R_k (E_J + E_{J'} - 2E_f^0)]. \quad (2b)$$

The J-O (intensity) parameters  $\mathcal{Q}_k^{\text{ED}}$  are the linear functions of energies of the two multiplets ( $E_J$  and  $E_{J'}$ ) involved in the transition, where  $E_f^0$  is the mean energy of the  $4f^n$  configuration and  $R_k$  ( $k = 2, 4, 6$ ) are the parameters representing the configuration interaction. There are 6 free parameters, namely  $\Omega_k$  and  $R_k$  ( $k = 2, 4, 6$ ).

The measured and calculated absorption oscillator strengths ( $f_{\text{exp}}^{\Sigma}$  and  $f_{\text{calc}}^{\Sigma}$ , respectively) are listed in [Table 4](#). The root-mean-square (rms) deviation between these values is lower when using the ICI approximation (1.260) as compared to the standard J-O theory (2.784). Moreover,

the former model predicts better the oscillator strength for the  ${}^5I_8 \rightarrow {}^5I_7$  transition. Thus, we used the ICI model for further calculations. The determined intensity parameters ( $\Omega_k$  and  $R_k$  for the ICI approximation and  $\Omega_k$  for the standard J-O theory) are listed in [Table 5](#).

The probabilities of spontaneous radiative transitions  $A_{\text{calc}}^{\Sigma}(JJ')$ , the luminescence branching ratios  $B(JJ')$  and the radiative lifetimes of the excited-states  $\tau_{\text{rad}}$  (up to  ${}^5S_2+{}^5F_4$ ) are listed in [Table 6](#) (as calculated within the ICI approximation).

The radiative lifetime of the upper laser level of  $\text{Ho}^{3+}$  ( ${}^5I_7$ ) in Tm,Ho:CNGG is 8.41 ms. Xue *et al.* determined the  ${}^5I_7$  luminescence lifetime for 0.5 at.% Ho:CNGG as 7.07 ms [21] which agrees well with our calculation. Ryabochkina *et al.* evaluated  $\tau_{\text{rad}} \sim 13$  ms from the reciprocity method which seems to be notably overestimated [20].

#### 4.2. $\text{Tm}^{3+} \leftrightarrow \text{Ho}^{3+}$ energy transfer

$\text{Tm}^{3+}, \text{Ho}^{3+}$  codoped materials can be easily excited at  $\sim 0.8 \mu\text{m}$  (to the  ${}^3H_4 \text{Tm}^{3+}$  state). Due to the efficient cross-relaxation for neighbor  $\text{Tm}^{3+}$  ions,  ${}^3H_4 + {}^3H_6 \rightarrow {}^3F_4 + {}^3F_4$ , the lower-lying  $\text{Tm}^{3+}$  excited-state is populated. As the barycenter of the  ${}^3F_4 \text{Tm}^{3+}$  state is located slightly above that of the  ${}^5I_7 \text{Ho}^{3+}$  state, an efficient  $\text{Tm}^{3+} \rightarrow \text{Ho}^{3+}$  energy-transfer (ET) may take place leading to the  ${}^5I_7 \rightarrow {}^5I_8 \text{Ho}^{3+}$  emission at  $>2 \mu\text{m}$ . [Figure 7\(a\)](#) shows the luminescence spectrum of the Tm,Ho:CNGG crystal.

The Tm,Ho:CNGG emission is broadband and it spans from  $\sim 1.6$  to  $2.12 \mu\text{m}$  originating from both the  ${}^3F_4 \rightarrow {}^3H_6 \text{Tm}^{3+}$  and  ${}^5I_7 \rightarrow {}^5I_8 \text{Ho}^{3+}$  transitions. As compared to a singly  $\text{Tm}^{3+}$ -doped crystal, the spectrum extends to longer wavelengths.

To determine the parameters of the bidirectional  ${}^3F_4 (\text{Tm}^{3+}) \leftrightarrow {}^5I_7 (\text{Ho}^{3+})$  ET and to assess the suitability of Tm,Ho:CNGG for laser operation, we performed luminescence decay studies. The luminescence was excited at 1615 nm (directly to the  ${}^3F_4 \text{Tm}^{3+}$  state) and monitored at 1800 nm (purely  $\text{Tm}^{3+}$  emission) and at 2080 nm (purely  $\text{Ho}^{3+}$  emission).

The measured luminescence decay curves plotted in a semi-log scale are shown in [Figure 7\(b\)](#). For the  $\text{Tm}^{3+}$  ion, the luminescence intensity first decreases fast (within few hundreds of  $\mu\text{s}$ ) and then the decay becomes single-exponential. For  $\text{Ho}^{3+}$ , a fast rise of luminescence intensity is observed which is synchronized with the  $\text{Tm}^{3+}$  decay, indicating the ET. The decay of  $\text{Ho}^{3+}$ , 1 ms after the excitation pulse is also single-exponential with the same time constant as for  $\text{Tm}^{3+}$ , indicating a thermal equilibrium in the  $\text{Tm}^{3+}, \text{Ho}^{3+}$  system.

The measured luminescence decay curves were fitted by the model of Walsh *et al.* [33]:

$$\frac{n_2(t)}{n_2(0)} = \frac{\beta}{\alpha + \beta} \exp(-t/\tau) + \frac{\alpha}{\alpha + \beta} \exp(-(\alpha + \beta)t), \quad (3a)$$

$$\frac{n_7(t)}{n_7(0)} = \frac{\alpha}{\alpha + \beta} \exp(-t/\tau) - \frac{\alpha}{\alpha + \beta} \exp(-(\alpha + \beta)t). \quad (3b)$$

Here,  $t$  is the time after the excitation pulse,  $n_2$  and  $n_7$  are the fractional populations of the  ${}^3F_4$   $Tm^{3+}$  and  ${}^5I_7$   $Ho^{3+}$  manifolds,  $\alpha = P_{28}N_{Ho}$  and  $\beta = P_{71}N_{Tm}$  are the transfer rates,  $P_{28}$  is the parameter of the direct nonradiative transfer of energy  $Tm^{3+} \rightarrow Ho^{3+}$ ,  $P_{71}$  is the parameter of the back ET,  $Ho^{3+} \rightarrow Tm^{3+}$  and  $\tau$  is the thermal equilibrium decay time. The result of the fitting is shown in [Figure 7\(b\)](#). The best-fit parameters are  $P_{28} = 5.917 \times 10^{-23} \text{ cm}^3 \mu\text{s}^{-1}$ ,  $P_{71} = 0.679 \times 10^{-23} \text{ cm}^3 \mu\text{s}^{-1}$  and  $\tau = 6.80 \text{ ms}$ . The determined  $\tau$  value is in between the radiative lifetimes for the  ${}^3F_4$   $Tm^{3+}$  state (4.11 ms [12]) and the  ${}^5I_7$   $Ho^{3+}$  state (8.41 ms, this work).

The ratio of the ET parameters,  $\Theta = P_{71}/P_{28}$ , referred to as the equilibrium constant, is 0.115. It shows how the  $Tm^{3+}$  and  $Ho^{3+}$  ions share the excitation energy and its value highlights the predominantly direct  $Tm^{3+} \rightarrow Ho^{3+}$  ET. The value of  $\Theta$  for  $Tm, Ho:CNGG$  is close to that for  $Tm, Ho:YAG$  ( $\Theta = 0.12$ ) [33]. The fractional population of  $Ho^{3+}$  ions in the  ${}^5I_7$  manifold in the steady-state regime  $f_{Ho} = \alpha/(\alpha + \beta) > 64\%$  and thus the fractional population of  $Tm^{3+}$  ions in the  ${}^3F_4$  manifold is  $f_{Tm} = 1 - f_{Ho} < 36\%$ . These values further emphasize the feasibility of  $Tm, Ho:CNGG$  crystals for efficient Ho laser operation.

### 4.3. Transition cross-sections

The absorption cross-sections,  $\sigma_{abs}$ , for the  ${}^3H_6 \rightarrow {}^3H_4$   $Tm^{3+}$  transition in  $Tm, Ho:CNGG$  are shown in [Figure 8\(a\)](#). The maximum  $\sigma_{abs} = 0.54 \times 10^{-20} \text{ cm}^2$  at 786.3 nm and the full width at half maximum (FWHM) of the absorption band is 28.4 nm. Thus,  $Tm, Ho:CNGG$  is very attractive for pumping with AlGaAs laser diodes emitting at  $\sim 0.8 \mu\text{m}$  due to the low sensitivity of the pump absorption to the temperature drift of the diode wavelength.

The absorption,  $\sigma_{abs}$ , and stimulated-emission (SE),  $\sigma_{SE}$ , cross-sections for the  ${}^3F_4 \leftrightarrow {}^3H_6$   $Tm^{3+}$  and  ${}^5I_7 \leftrightarrow {}^5I_8$   $Ho^{3+}$  transitions at  $\sim 2 \mu\text{m}$  are summarized in [Figure 8\(b\)](#). The SE cross-sections were calculated using the Füchtbauer–Ladenburg (F-L) formula [34]:

$$\sigma_{SE}(\lambda) = \frac{\lambda^5}{8\pi \langle n \rangle^2 \tau_{rad} c} \frac{W'(\lambda)}{\int \lambda W'(\lambda) d\lambda}, \quad (4)$$

where  $\lambda$  is the wavelength,  $\langle n \rangle$  is the refractive index at the mean emission wavelength  $\langle \lambda_{em} \rangle$  calculated using the Sellmeier formulas [3],  $c$  is the speed of light,  $\tau_{rad}$  is the radiative lifetime of the emitting level ( ${}^3F_4$  for  $Tm^{3+}$  and  ${}^5I_7$  for  $Ho^{3+}$ ) and  $W'(\lambda)$  is the luminescence spectrum calibrated for the spectral response of the set-up. For the  $Tm^{3+}$  ion,  $\tau_{rad}({}^3F_4) = 4.11 \text{ ms}$  and for  $Ho^{3+}$ ,  $\tau_{rad}({}^5I_7) = 8.41 \text{ ms}$ . The results for  $Tm^{3+}$  were previously described [12], so that we focus primarily on the  $Ho^{3+}$  ion.

For the  ${}^5I_8 \rightarrow {}^5I_7$   $\text{Ho}^{3+}$  transition in absorption, the maximum is  $\sigma_{\text{abs}} = 0.46 \times 10^{-20} \text{ cm}^2$  at 1920.2 nm. The SE cross-section for this transition in emission is  $\sigma_{\text{SE}} = 0.47 \times 10^{-20} \text{ cm}^2$  at 2080.7 nm and the emission bandwidth (FWHM)  $\Delta\lambda_{\text{em}}$  is 70.2 nm.

The transitions of both  $\text{Tm}^{3+}$  and  $\text{Ho}^{3+}$  ions at  $\sim 2 \mu\text{m}$  represent a quasi-three level laser scheme with reabsorption. Thus, to estimate the laser wavelength, the tuning range and the gain bandwidth, the gain spectra are calculated:

$$\sigma_{g,\text{Tm(Ho)}} = \beta_{\text{Tm(Ho)}} \sigma_{\text{SE,Tm(Ho)}} - (1 - \beta_{\text{Tm(Ho)}}) \sigma_{\text{abs,Tm(Ho)}}, \quad (5)$$

where,  $\beta_{\text{Tm}} = N_2({}^3F_4)/N_{\text{Tm}}$  and  $\beta_{\text{Ho}} = N_7({}^5I_7)/N_{\text{Ho}}$  are the inversion levels for  $\text{Tm}^{3+}$  and  $\text{Ho}^{3+}$  ions, respectively, and  $N_2$  and  $N_7$  are the populations of the  ${}^3F_4$   $\text{Tm}^{3+}$  and  ${}^5I_7$   $\text{Ho}^{3+}$  upper laser levels, respectively. In the case of a codoped material, the  $\sigma_{g,\text{Tm}}$  and  $\sigma_{g,\text{Ho}}$  spectra cannot be treated independently and cannot be used directly to explain the spectral behavior of wavelength tunable or ML lasers because the populations of the excited-states of  $\text{Tm}^{3+}$  and  $\text{Ho}^{3+}$  ions are linked to each other in the thermal equilibrium regime by the bidirectional ET.

The effective gain cross-section for a codoped material is:

$$\sigma_{g,\text{eff}} = \left( \beta_{\text{Ho}} \sigma_{\text{SE,Ho}} - (1 - \beta_{\text{Ho}}) \sigma_{\text{abs,Ho}} \right) \frac{N_{\text{Ho}}}{N_{\text{tot}}} + \left( \beta_{\text{Tm}} \sigma_{\text{SE,Tm}} - (1 - \beta_{\text{Tm}}) \sigma_{\text{abs,Tm}} \right) \frac{N_{\text{Tm}}}{N_{\text{tot}}}, \quad (6)$$

Here, the gain cross-section is defined with respect to the total ( $\text{Tm} + \text{Ho}$ ) doping concentration of  $N_{\text{tot}} = N_{\text{Tm}} + N_{\text{Ho}}$ . Assuming population of only the  $({}^3H_6, {}^3F_4)$   $\text{Tm}^{3+}$  and  $({}^5I_8, {}^5I_7)$   $\text{Ho}^{3+}$  multiplets, the condition of the thermal equilibrium state established by the bidirectional  $\text{Tm}^{3+} \leftrightarrow \text{Ho}^{3+}$  ET reads:

$$P_{28}(\text{Tm} \rightarrow \text{Ho}) \cdot N_2({}^3F_4) \cdot (N_{\text{Ho}} - N_7({}^5I_7)) = P_{71}(\text{Ho} \rightarrow \text{Tm}) \cdot N_7({}^5I_7) \cdot (N_{\text{Tm}} - N_2({}^3F_4)). \quad (7)$$

From Eq. (7), we arrive at:

$$\beta_{\text{Ho}} = \frac{\Omega \beta_{\text{Tm}}}{1 + (\Omega - 1) \beta_{\text{Tm}}}. \quad (8)$$

Here,  $\Omega = 1/\Theta$  where  $\Theta$  describes the rates of the direct and back ET, as described above.

The calculated  $\sigma_{g,\text{eff}}$  spectra are shown in [Figure 9](#) for different inversion rates for  $\text{Tm}^{3+}$  ions  $\beta_{\text{Tm}}$ . The gain spectra are smooth and broad. For small  $\beta_{\text{Tm}} < 0.04$ , the spectra are almost flat extending beyond 2.1  $\mu\text{m}$ . With increasing inversion rate, the local maximum in the gain spectra experiences a blue-shift from  $\sim 2100$  nm to 2081 nm. The local peak at 2081 nm due to the  ${}^5I_7 \rightarrow {}^5I_8$   $\text{Ho}^{3+}$  emission dominates the spectra for  $\beta_{\text{Tm}} > 0.05$ . At shorter wavelengths, the gain is first due to both  $\text{Tm}^{3+}$  and  $\text{Ho}^{3+}$  ions and then, below approximately 2  $\mu\text{m}$  – mostly to the  $\text{Tm}^{3+}$  ions. For  $\beta_{\text{Tm}} = 0.15$ , the gain bandwidth (FWHM)  $\Delta\lambda_{\text{g}}$  exceeds 150 nm.

## 5. Laser operation

### 5.2. Laser set-up: Diode-pumped laser

The diode-pumped laser performance of Tm,Ho:CNGG was studied in a simple plano-plano (microchip-type) laser cavity, [Figure 10](#). The rectangular crystal with a thickness of 3.3 mm and an aperture of  $3 \times 3 \text{ mm}^2$  was wrapped in In-foil from all 4 lateral sides for better heat removal. It was fixed in a Cu-holder cooled by circulating water (12 °C). Both end-facets of the crystal were polished to laser quality and remained uncoated. The laser cavity was composed by a flat pump mirror (PM) coated for high transmission (HT) at  $\sim 0.80 \text{ }\mu\text{m}$  (the pump wavelength) and for high reflection (HR) at 1.8-2.1  $\mu\text{m}$ , and a set of flat output couplers (OCs) with measured transmission  $T_{\text{OC}} = 0.2\%-9\%$  at the laser wavelength. Both cavity mirrors were placed close to the crystal end-faces resulting in a geometrical cavity length of  $\sim 3.5 \text{ mm}$ .

As a pump source, we employed a fiber-coupled (fiber core diameter: 200  $\mu\text{m}$ , N.A. = 0.22) AlGaAs laser diode emitting unpolarized output at  $\sim 802 \text{ nm}$  ( $M^2 > 80$ , i.e., almost “top-hat” pump beam). The diode emission wavelength was stabilized by circulating water. The pump beam was collimated and focused into the crystal through the PM using an antireflection (AR) coated lens assembly (reimaging ratio: 1:1, focal length:  $f = 30 \text{ mm}$ ). The pump spot diameter in the focus was  $2w_{\text{P}} = 200 \text{ }\mu\text{m}$  and the confocal parameter was  $2z_{\text{R}} = 1.8 \text{ mm}$ . Pumping was realized in a double-pass configuration, as all the OCs provided partial reflection at the pump wavelength ( $R \approx 40\%$ ). The double-pass absorption was calculated from pump-transmission measurements under non-lasing conditions at the threshold,  $\eta_{\text{abs}}(2\text{-passes}) = 43.8 \pm 0.5\%$ .

The spectra of the laser emission were measured with a spectrum analyzer (WaveScan, 1000-2600 nm, APE GmbH) with a resolution of 0.2 nm.

### 5.2. Diode-pumped laser performance

The input-output characteristics of the CW diode-pumped Tm,Ho:CNGG laser are shown in [Figure 11\(a\)](#). The laser generated a maximum output power of 413 mW at 2088.4 nm with a slope efficiency  $\eta$  of 15.9% (vs. the absorbed pump power  $P_{\text{abs}}$ , fitting the linear part of the output dependence). The laser threshold was at  $P_{\text{abs}} = 0.37 \text{ W}$  and the optical-to-optical laser efficiency  $\eta_{\text{opt}}$  was 5.3% (all the characteristics are specified for  $T_{\text{OC}} = 1.5\%$ ). The laser threshold gradually increased with output coupling, from 0.18 W ( $T_{\text{OC}} = 0.2\%$ ) up to 0.60 W ( $T_{\text{OC}} = 9\%$ ). For  $T_{\text{OC}} > 3\%$ , the slope efficiency deteriorated which is common for Tm,Ho lasers and is related to enhanced upconversion in the gain medium causing more serious thermal

issues. A thermal roll-over in the output dependences was observed for  $P_{\text{abs}} > 2.5$  W. Thus, the power scaling was limited by the risk of thermal fracture.

The laser emission was unpolarized. With increasing the output coupling, the laser spectra exhibited a blue-shift, from 2102.2 nm ( $T_{\text{OC}} = 0.2\%$ ) to 2069.8 nm ( $T_{\text{OC}} = 9\%$ ), [Figure 11\(b\)](#). This shift is related to the weaker reabsorption losses at high inversion levels  $\beta$  and is typical for quasi-three-level lasers. This spectral behavior is also in line with the gain spectra, [Figure 9](#). The laser emission is related solely to  $\text{Ho}^{3+}$  ions (the  $^5\text{I}_7 \rightarrow ^5\text{I}_8$  transition).

### 5.3. Laser set-up: Mode-locked laser

A standard X-shaped cavity as shown in [Figure 12](#) was employed to investigate the mode-locked laser performance of Tm,Ho:CNGG. The sample used had a size of  $3 \times 3 \times 6$  mm<sup>3</sup> with both end-faces ( $3 \times 3$  mm<sup>2</sup>) antireflection (AR) coated for the wavelength ranges of 780-810 and 1800-2300 nm. To remove the generated heat during laser operation, it was tightly mounted in a Cu-holder which was water cooled to 14 °C. The laser element was placed at normal incidence between two folding concave mirrors  $M_1$  and  $M_2$  (radius of curvature,  $\text{RoC} = 100$  mm) with a separation of  $\sim 108$  mm. The pump source applied was a narrow-band CW Ti:Sapphire laser with a maximum output power of 3.6 W at 785.6 nm. The pump beam was focused into the laser crystal through the folding mirror  $M_1$  using a focusing lens (focal length:  $f = 70$  mm) resulting in a spot diameter  $2w_p$  of  $\sim 60$   $\mu\text{m}$  (in the crystal). The calculated size of the laser mode in the crystal was 31  $\mu\text{m}$  for both the tangential and sagittal planes. The crystal was pumped in a single-pass. The pump absorption under lasing conditions was weakly dependent on the pump level and output coupling and amounted to  $61 \pm 1\%$ .

For CW laser experiments, we used a flat highly-reflective mirror  $M_3$  and a set of flat OCs with  $T_{\text{OC}} = 0.2\%$ - $3.0\%$  at the laser wavelength. For the wavelength tuning experiment, a Lyot filter was inserted near the OC. The Lyot filter was a 3.2 mm-thick quartz plate with the optical axis at  $60^\circ$  to the surface. Finally, for ML operation, three chirped mirrors (Layertec GmbH),  $\text{CM}_1$ – $\text{CM}_3$ , were employed for intracavity dispersion compensation, each of them providing a group delay dispersion,  $\text{GDD} = -125$  fs<sup>2</sup> per bounce. The GaSb-based SESAM used as a saturable absorber exhibits an anti-resonant design at 2- $\mu\text{m}$  wavelength and contained two quantum wells separated by 10 nm. The QWs are placed 50 nm below the surface, realized by a 50 nm thick GaSb cap layer. The measured relaxation time (slow component) for the uncoated SESAM amounted to 21 ps [35]. The flat highly-reflective mirror  $M_3$  was replaced by a concave chirped mirror ( $\text{CM}_1$ ),  $\text{RoC} = 100$  mm) forming a waist on the SESAM. The

calculated mode radius at the SESAM was 107 and 105  $\mu\text{m}$  in the tangential and sagittal planes, respectively.

The laser spectrum was measured using a spectrum analyzer (WaveScan, extended IR, APE GmbH), the radio frequency (RF) spectrum – by a fast InGaAs PIN photodiode (ET-50000, EOT Inc) and a RF spectrum analyzer (R&S FSP7, Rohde&Schwarz), and the autocorrelation (AC) trace - by a commercial autocorrelator (pulseCheck, APE GmbH).

#### 5.4. CW wavelength tuning

With the highly-reflective mirror  $M_3$  and without the Lyot filter, the laser operated in CW (free-running) regime, [Figure 13\(a,b\)](#). It generated 425 mW at 2083.7 nm with  $\eta = 22.9\%$  (for  $T_{OC} = 3.0\%$ ). The laser threshold slightly increased with the output coupling, from  $P_{abs} = 80$  mW ( $T_{OC} = 0.2\%$ ) to 136 mW ( $T_{OC} = 3.0\%$ ). The output dependences were linear and the power scaling was limited by the available pump. Lower threshold as compared to diode pumping is explained by the better quality of the pump beam. The somewhat better slope efficiency is probably due to the better mode-matching. The laser wavelength experienced a blue-shift with increasing  $T_{OC}$ , from 2097.4 and 2099.4 nm ( $T_{OC} = 0.2\%$ ) to 2083.7 nm ( $T_{OC} = 3.0\%$ ), the same behaviour as in the case of diode-pumping and agrees with the gain spectra.

Wavelength tuning of the Tm,Ho:CNGG laser was studied by inserting the Lyot filter into the cavity, close to the OC. The tuning performance was characterized with the 0.2% and 0.5% OCs. A continuous tuning of the emission wavelength from 1932.5 to 2142.1 nm ( $T_{OC} = 0.5\%$ ) and from 1940.3 to 2144.6 nm ( $T_{OC} = 0.2\%$ ) was achieved, equivalent to  $>200$  nm tuning range in both cases (determined at the zero power level). This highlights the suitability of the Tm,Ho:CNGG disordered crystal for the generation of ultrashort laser pulses. The maximum in the tuning curves was observed at  $\sim 2088$  nm.

The effective gain cross-section,  $\sigma_{g,eff}$ , spectra for Tm,Ho:CNGG were calculated to explain the observed wavelength tuning performance, [Figure 13\(c\)](#). For the calculation, we used  $\beta_{Tm} = 0.11$ , so that, according to Eq. (8),  $\beta_{Ho} = 0.52$ . The gain spectrum reasonably agrees with the measured wavelength tunability curve. In the same graph, we show the gain spectrum for  $\text{Ho}^{3+}$  ions only,  $\sigma_{g,Ho}$ . It is narrower and cannot explain the wavelength tuning below 2  $\mu\text{m}$ . Thus, both  $\text{Ho}^{3+}$  and  $\text{Tm}^{3+}$  ions contribute to the observed tuning range.

In a previous work on  $\text{Tm}^{3+},\text{Na}^+:\text{CNGG}$  [12], a tuning range of 168 nm (1885-2053 nm) was achieved. Thus,  $\text{Tm}^{3+},\text{Ho}^{3+}$  codoping extends this range to well beyond 2  $\mu\text{m}$  when using CNGG as host.

### 5.5. Femtosecond mode-locked laser operation

The ML performance was studied with the CMs and the GaSb based SESAM inserted into the cavity, leading to a total physical cavity length of about 1.68 m. Considering the group velocity dispersion of the laser crystal, approximately  $-31 \text{ fs}^2/\text{mm}$  at  $\sim 2.06 \text{ }\mu\text{m}$ , the total round-trip GDD amounted to  $-1622 \text{ fs}^2$ . ML operation was first achieved by employing the 1.5% OC. The absorbed pump power was set at  $P_{\text{abs}} = 2.0 \text{ W}$ . After carefully optimizing the separations of  $M_1$ - $M_2$  and  $CM_3$ -SESAM, and adjusting the position of the SESAM and the  $\text{Tm,Ho:CNGG}$  crystal, stable and self-starting ML operation was obtained. Pulses as short as 100 fs were generated without external compression. The corresponding autocorrelation trace and optical spectrum were measured, see [Figure 14\(a,b\)](#). The emission spectrum was centered at 2067 nm and had a FWHM of 45.3 nm. This leads to a time-bandwidth product (TBP) of 0.318, corresponding to nearly Fourier-limited pulses. The measured average output power of the mode-locked laser was 105 mW.

With the aim to achieve shorter pulses, an OC with lower transmission,  $T_{\text{OC}} = 0.5\%$ , was employed. As expected, shorter pulses were obtained and its duration amounted to 73 fs without external compression. The corresponding autocorrelation trace and optical spectrum are shown in [Figure 14\(c,d\)](#). The spectrum was centered at 2061 nm and had a FWHM of 61.5 nm. This leads to a TBP of 0.317, again corresponding to nearly Fourier-limited pulses. The measured average output power of the mode-locked laser was lower compared to the performance with the 1.5% OC, 36 mW, under the same  $P_{\text{abs}}$ .

The stability of the ML regime of operation was assessed by measurements with a RF spectrum analyzer. The RF spectrum shows an extinction ratio above the noise level of  $\sim 81 \text{ dB}$  for the fundamental beat note, measured with a resolution bandwidth (RBW) of 100 Hz, [Figure 15\(a\)](#). The uniform harmonic beat notes recorded in [Figure 15\(b\)](#) indicate stable mode-locking without any Q-switching or other instabilities. The pulse repetition rate of  $\sim 89.3 \text{ MHz}$  corresponds to the cavity length of  $\sim 1.68 \text{ m}$ . The results were similar for the two studied OCs.

The obtained pulses are shorter than in the previous work with the  $\text{Tm,Ho:CNGG}$  crystal [22] where a SWCNT saturable absorber was employed: Pan *et al.* reported on 83 fs pulses at 2081 nm (emission bandwidth: 57.5 nm). This shortening is probably due to the better matching of the pump and laser modes for the designed cavity employing an AR-coated laser crystal placed at normal incidence (in [22], the crystal was placed at the Brewster angle).

## 6. Conclusions



To conclude,  $\text{Tm}^{3+}, \text{Ho}^{3+}$ -codoped disordered CNGG garnet is an attractive material for broadly tunable and sub-100 fs mode-locked lasers emitting slightly above 2  $\mu\text{m}$  owing to its attractive spectroscopic properties, i.e., (i) inhomogeneously broadened and smooth emission spectra at  $\sim 2 \mu\text{m}$ , (ii) efficient and predominantly unidirectional (direct)  $\text{Tm}^{3+} \rightarrow \text{Ho}^{3+}$  energy transfer, as expressed by the equilibrium constant  $\Theta = P_{71}/P_{28} = 0.115$ , (iii) a relatively long lifetime of the  $\text{Ho}^{3+}$  upper laser level (8.41 ms, radiative) and the thermal equilibrium lifetime for the  $\text{Tm}^{3+}, \text{Ho}^{3+}$  system (6.80 ms, measured) leading to a relatively low laser threshold, and (iv) the contribution of both  $\text{Tm}^{3+}$  and  $\text{Ho}^{3+}$  ions to the gain spectra featuring a bandwidth of more than 150 nm. We report on the first diode-pumped laser operation of  $\text{Tm}, \text{Ho}:\text{CNGG}$  yielding 413 mW at 2088.4 nm with a slope efficiency of 15.9%. A tuning range of  $\sim 200$  nm (with the laser wavelength up to 2144.6 nm) is demonstrated for this crystal. Finally, when employing a GaSb SESAM, sub-100 fs nearly Fourier-limited pulses (73 fs, without external compression) are generated.

Further work with  $\text{Tm}, \text{Ho}:\text{CNGG}$  crystals should focus on their codoping by univalent alkali metal cations ( $\text{Na}^+$ ,  $\text{Li}^+$  or their combination) serving two aims: (i) elimination of the unwanted cationic vacancies and (ii) possible further extension of the gain bandwidth.

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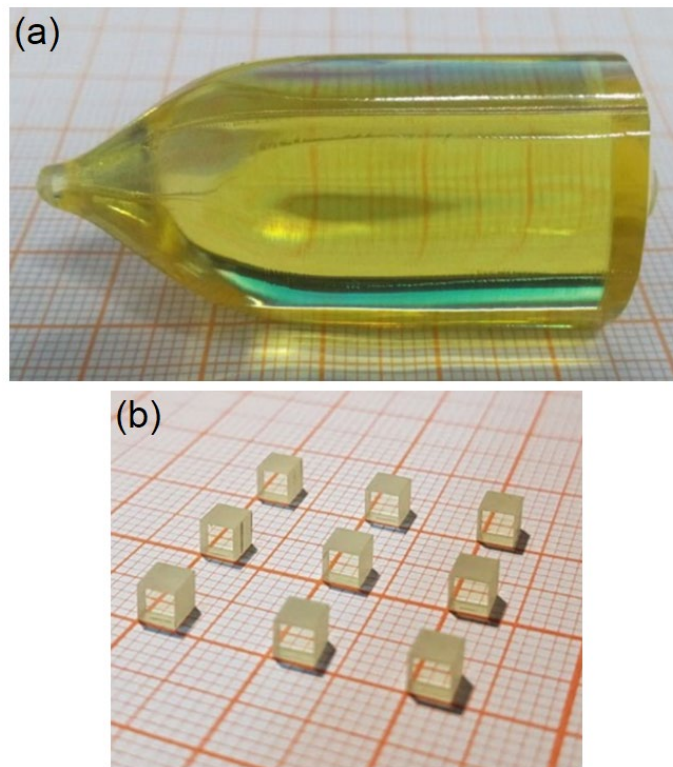
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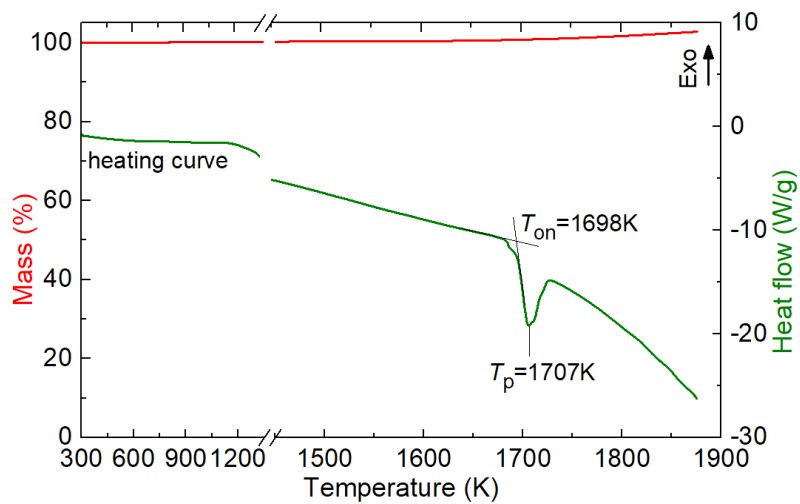
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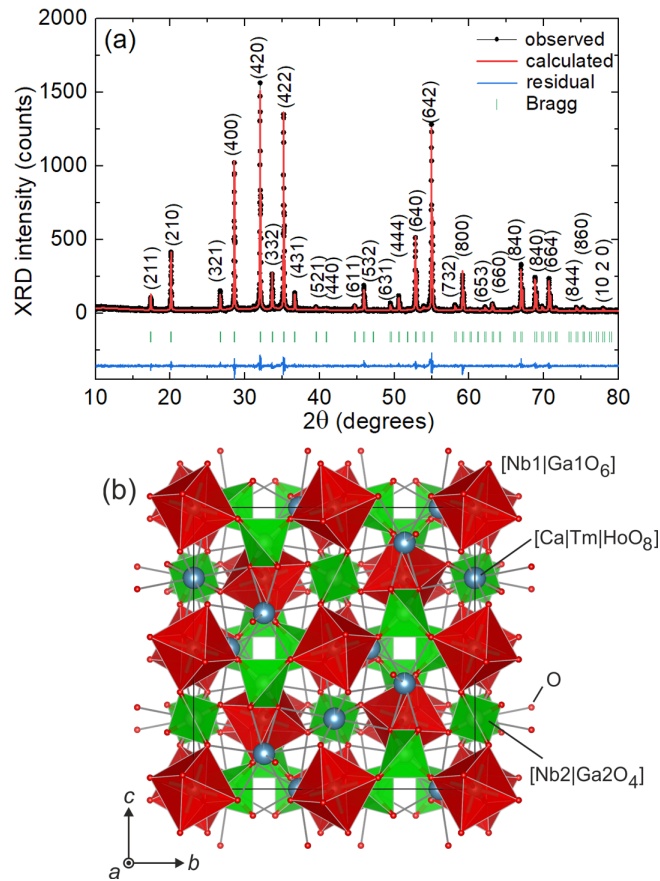
## List of figure captions



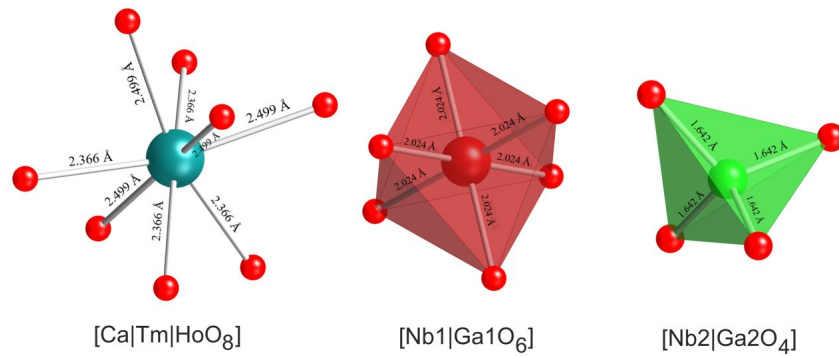
**Figure 1.** (a) Photograph of the as-grown 2.64 at.% Tm, 0.55 at.% Ho:CNGG crystal, the growth direction is along the [111] axis; (b) polished laser elements fabricated from the crystal boule which are annealed in air at 1373 K.



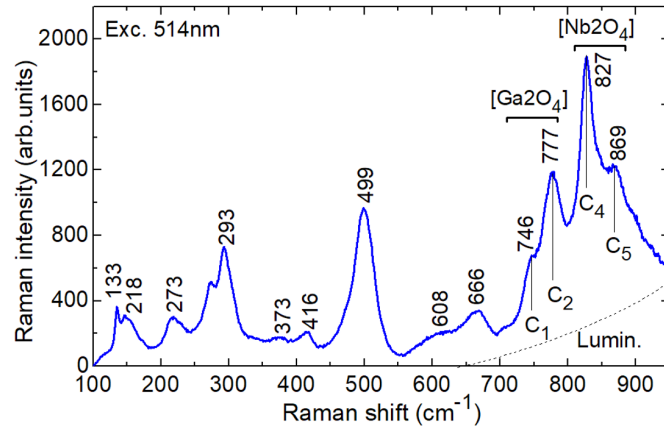
**Figure 2.** TG-DSC curve of the Tm,Ho:CNGG crystal.



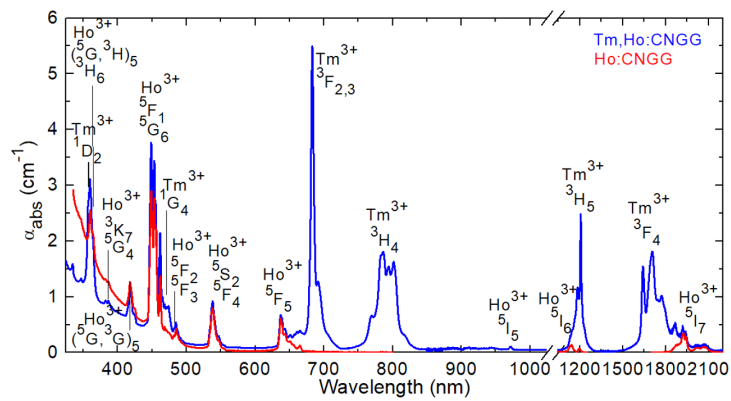
**Figure 3.** (a) X-ray powder diffraction (XRD) pattern of the 2.64 at.% Tm, 0.55 at.% Ho:CNGG crystal showing the results of the Rietveld refinement, numbers denote the Miller's indices ( $hkl$ ); (b) fragment of crystal structure in projection on the  $b$ - $c$  plane.



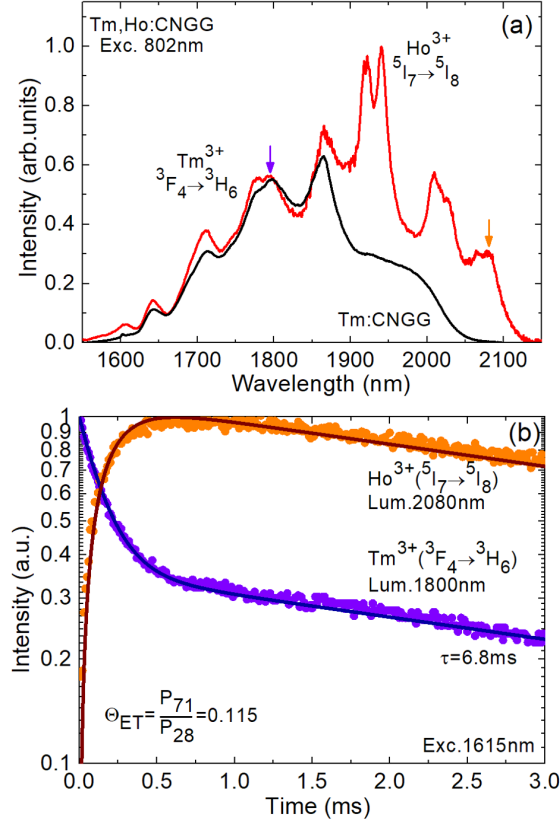
**Figure 4.** Oxygen coordination and metal – oxygen interatomic distances for the dodecahedral sites (Ca|Tm|Ho), octahedral sites (Nb1|Ga1) and tetrahedral sites (Nb2|Ga2) in the Tm,Ho:CNGG garnet.



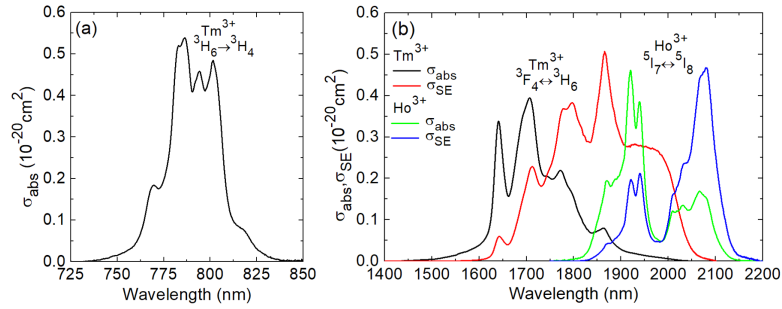
**Figure 5.** Unpolarized Raman spectrum of the Tm,Ho:CNGG crystal,  $\lambda_{\text{exc}} = 514$  nm. Numbers indicate the Raman energy in  $\text{cm}^{-1}$ .  $C_i$  are the bands due to symmetric stretching vibrations in  $[\text{M}_2\text{O}_4]$  ( $\text{M}_2 = \text{Ga}_2[\text{Nb}_2]$ ) tetrahedra; Dashed line – luminescence background.



**Figure 6.** Absorption spectra of 2.64 at.% Tm, 0.55 at.% Ho:CNGG and 0.55 at.% Ho:CNGG.

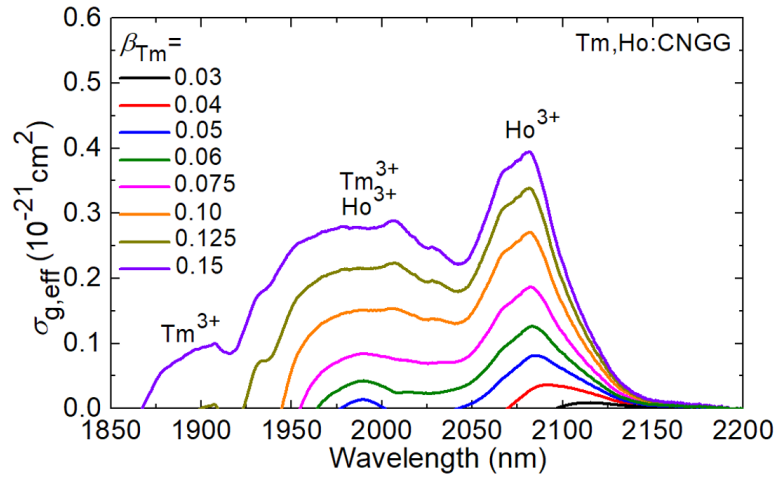


**Figure 7.** (a) Luminescence spectrum of the Tm, Ho: CNGG crystal at  $\sim 2 \mu\text{m}$ ,  $\lambda_{\text{exc}} = 802 \text{ nm}$ . The spectrum of singly-doped Tm: CNGG crystal is given for comparison. The *arrows* indicate the wavelength selected for the decay studies; (b) luminescence decay curves for the Tm, Ho: CNGG crystal measured at 2080 nm ( $\text{Ho}^{3+}$  emission) and at 1800 nm ( $\text{Tm}^{3+}$  emission),  $\lambda_{\text{exc}} = 1615 \text{ nm}$ : *symbols* – experimental data, *curves* – their modeling with Eq. (3).

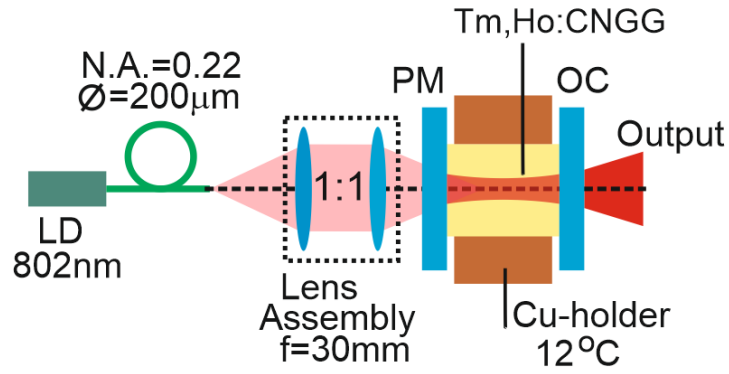


**Figure 8.** Spectroscopy of  $\text{Tm}^{3+}$  and  $\text{Ho}^{3+}$  ions in CNGG: (a) absorption cross-section,  $\sigma_{\text{abs}}$ , for the  ${}^3\text{H}_6 \rightarrow {}^3\text{H}_4$   $\text{Tm}^{3+}$  transition; (b) absorption,  $\sigma_{\text{abs}}$ , and stimulated-emission (SE),  $\sigma_{\text{SE}}$ , cross-sections for the  ${}^3\text{F}_4 \leftrightarrow {}^3\text{H}_6$   $\text{Tm}^{3+}$  and  ${}^5\text{I}_7 \leftrightarrow {}^5\text{I}_8$   $\text{Ho}^{3+}$  transitions.

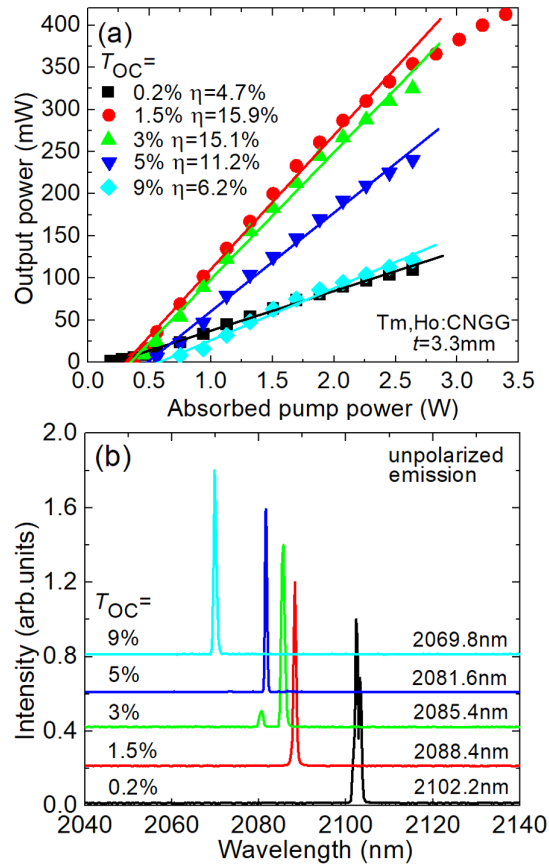




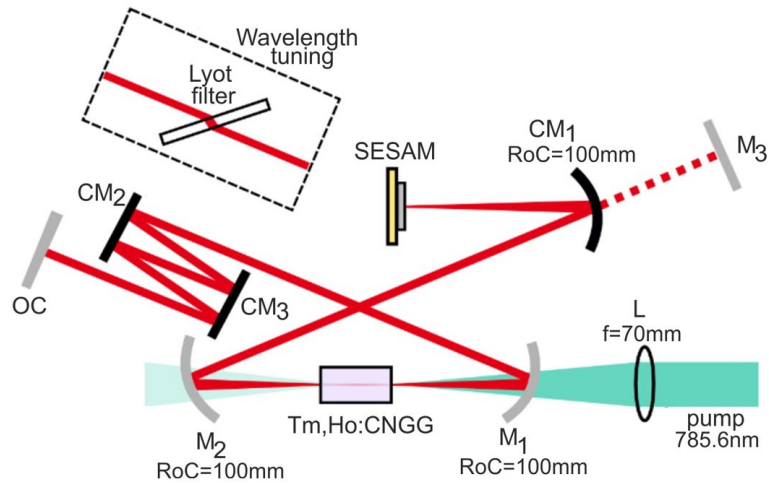
**Figure 9.** Effective gain cross-sections,  $\sigma_{g,\text{eff}}$ , for the 2.64 at.% Tm, 0.55 at.% Ho:CNGG at  $\sim 2 \mu\text{m}$ :  $\beta_{\text{Tm}} = N_2(^3\text{F}_4)/N_{\text{Tm}}$  is the inversion level for  $\text{Tm}^{3+}$  ions; calculated using Eqs. (6)-(8).



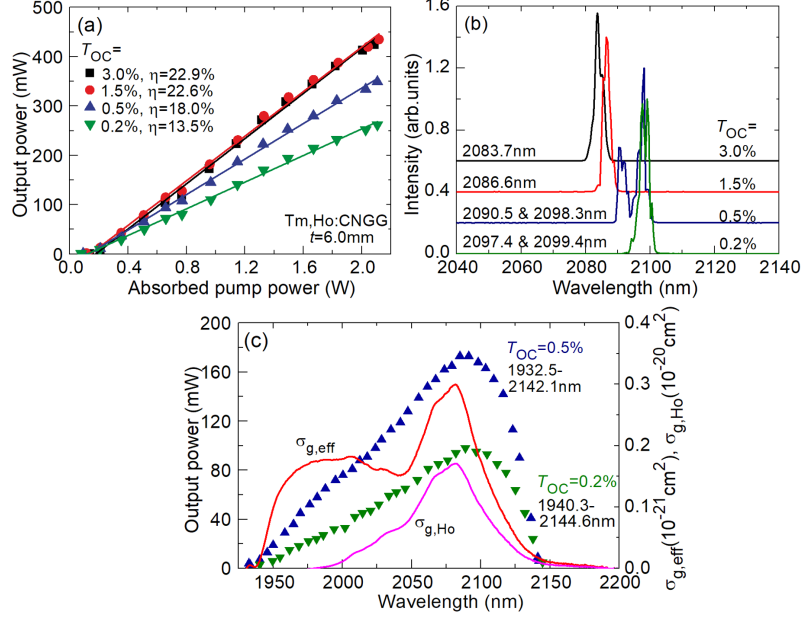
**Figure 10.** Scheme of the CW diode-pumped Tm, Ho:CNGG laser: LD – laser diode, PM – pump mirror, OC – output coupler.



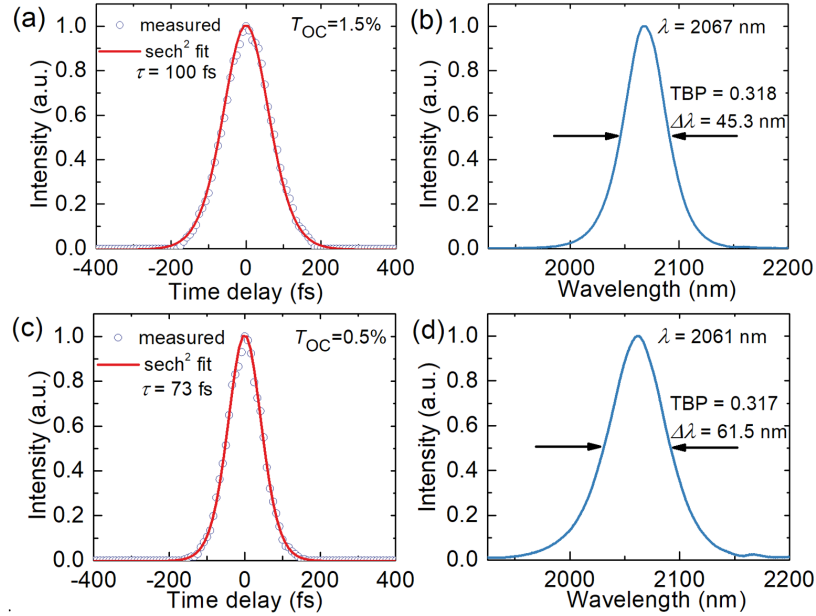
**Figure 11.** Continuous-wave diode-pumped Tm,Ho:CNGG laser: (a) input-output dependences,  $\eta$  – slope efficiency; (b) typical laser emission spectra measured at maximum  $P_{\text{abs}}$ .



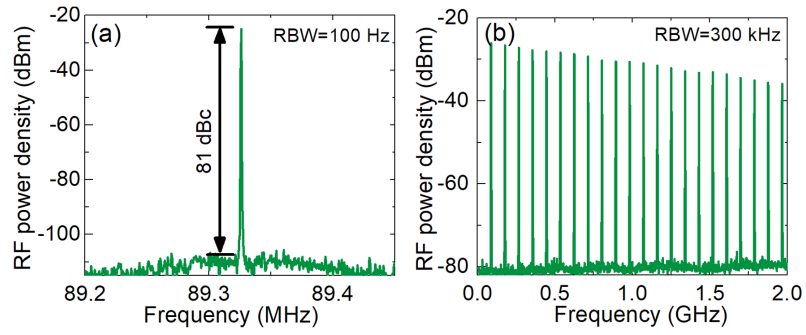
**Figure 12.** Layout of the mode-locked Tm,Ho:CNGG laser (L: lens; M<sub>1</sub> and M<sub>2</sub>: dichroic folding mirrors; M<sub>3</sub>: plane highly-reflective mirror; CM<sub>1</sub>, CM<sub>2</sub> and CM<sub>3</sub>: chirped mirrors; OC: output coupler).



**Figure 13.** CW laser performance of the Tm,Ho:CNGG laser pumped by a Ti:Sapphire laser at 785.6 nm: (a,b) free-running operation, (a) input-output dependences,  $\eta$  – slope efficiency; (b) typical laser emission spectra measured at  $P_{\text{abs}} = 2.0$  W; (c) *symbols*: wavelength tuning for  $T_{\text{OC}} = 0.2\%$  and  $0.5\%$ , *curves*: calculated effective gain,  $\sigma_{g,\text{eff}}$ , Eq. (6), and  $\text{Ho}^{3+}$  gain,  $\sigma_{g,\text{Ho}}$ , Eq. (5), spectra for  $\beta_{\text{Tm}} = 0.11$  and  $\beta_{\text{Ho}} = 0.52$ . The laser emission is linearly polarized.



**Figure 14.** SESAM mode-locked Tm,Ho:CNGG laser: (a,c) autocorrelation traces (*symbols* - measured data, *red curves* - their  $\text{sech}^2$  fits) and (b,d) corresponding spectra of the laser emission. (a,b)  $T_{\text{OC}} = 1.5\%$ , (c,d)  $T_{\text{OC}} = 0.5\%$ .  $P_{\text{abs}} = 2.0$  W.



**Figure 15.** Radio frequency (RF) spectra of the SESAM mode-locked Tm,Ho:CNGG laser: (a) fundamental beat note and (b) 2.0 GHz wide span (RBW: resolution bandwidth).  $T_{OC} = 0.5\%$ .  $P_{abs} = 2.0$  W.

## List of Tables

**Table 1.** Crystallographic data and structure refinement parameters for Tm,Ho:CNGG.

Parameters	Values
Chemical formula	$\text{Ca}_{2.895}\text{Tm}_{0.09}\text{Ho}_{0.015}\text{Nb}_{1.628}\text{Ga}_{3.25}\text{O}_{12}$
System	cubic
Space group	$Ia\bar{3}d$
Space group IT number	230
Lattice constants, $a = b = c$ (Å)	12.4952(1)
$\alpha = \beta = \gamma$ (deg.)	90
Unit-cell volume $V$ (Å <sup>3</sup> )	1950.87
$2\theta$ range (deg.)	10 - 80
Radiation	Cu Ka1 ( $\lambda = 1.5418$ Å)
Calculated density (g/cm <sup>3</sup> )	5.063
Refinement	Rietveld refinement with FULLPROF software
Reliability factors	$R_p = 5.80$ , $R_{wp} = 7.76$ , $R_{exp} = 6.13$ and $\chi^2 = 1.60$

**Table 2.** Fractional atomic coordinates ( $x$ ,  $y$ ,  $z$ ), occupancy factors (O.F.) and isotropic displacement parameters ( $B_{\text{iso}}$ ) for the Tm,Ho:CNGG crystal.

Atom	Wyckoff symbol	$x$	$y$	$z$	O.F.	$B_{\text{iso}}$ , $\text{\AA}^2$
Ca	24c	0.1250	0	0.2500	0.9681	1.22(3)
Tm	24c	0.1250	0	0.2500	0.0264	1.22(3)
Ho	24c	0.1250	0	0.2500	0.0055	1.22(3)
Ga1	16a	0	0	0	0.2900	0.98(1)
Nb1	16a	0	0	0	0.7100	0.98(1)
Nb2	24d	0.3750	0	0.2500	0.0820	0.75(1)
Ga2	24d	0.3750	0	0.2500	0.9180	0.75(1)
O	96h	0.0398	0.0488	0.6555	1	1.08(3)

**Table 3.** Bond lengths and geometric parameters of the  $[\text{MO}_n]$  polyhedrons in Tm,Ho:CNGG.

Parameter	Polyhedron		
	[Ca Tm HoO <sub>8</sub> ]	[Nb1 Ga1O <sub>6</sub> ]	[Nb2 Ga2O <sub>4</sub> ]
Bonds length M-O ( $\text{\AA}$ )	2.4991(0) $\times 4$ 2.3662(3) $\times 4$	2.0242(1) $\times 6$ -	1.6423(7) $\times 4$ -
Average bond length ( $\text{\AA}$ )	2.43270	2.0238	1.6425
Polyhedral volume ( $\text{\AA}^3$ )	24.5481	11.046	2.2504
Distortion index (bond length)	0.02762	0.0	0.0
Quadratic elongation	-	1.0004	1.0071
Bond angle variance ( $\text{deg.}^2$ )	-	1.3591	28.636
Effective coordination number	7.76	6.00	4.00

**Table 4.** Measured and calculated absorption oscillator strengths of the  $\text{Ho}^{3+}$  ion in CNGG<sup>a</sup>.

Transition	$\langle E_J \rangle$ ,	$\Gamma$ ,	$f_{\text{exp}}^{\Sigma} \times 10^6$	$f_{\text{calc}}^{\Sigma} \times 10^6$	$f_{\text{calc}}^{\Sigma} \times 10^6$
${}^5\text{I}_8 \rightarrow {}^{2S'+1}\text{L}'_J$	$\text{cm}^{-1}$	$\text{cm}^{-1}\text{nm}$		J-O	ICI
${}^5\text{I}_7$	5092	35.618	1.563	1.715 <sup>ED+</sup> 0.580 <sup>MD</sup>	1.037 <sup>ED+</sup> 0.580 <sup>MD</sup>
${}^5\text{I}_6$	8658	6.597	0.834	1.196 <sup>ED</sup>	0.955 <sup>ED</sup>
${}^5\text{F}_5$	15414	8.977	3.608	4.790 <sup>ED</sup>	3.671 <sup>ED</sup>
${}^5\text{S}_2 + {}^5\text{F}_4$	18531	9.738	5.584	5.360 <sup>ED</sup>	5.363 <sup>ED</sup>
${}^5\text{F}_3$	20497	3.484	2.467	1.273 <sup>ED</sup>	1.649 <sup>ED</sup>
${}^5\text{F}_{1,2} + {}^3\text{K}_8$ $+ {}^5\text{G}_6$	21980	32.779	26.799	28.043 <sup>ED+</sup> 0.132 <sup>MD</sup>	26.653 <sup>ED+</sup> 0.132 <sup>MD</sup>
${}^3\text{G}_5$	24060	5.582	5.360	6.624 <sup>ED</sup>	5.448 <sup>ED</sup>
${}^5\text{G}_{2,5} + {}^3\text{H}_6$	27607	11.790	15.138	9.648 <sup>ED</sup>	15.096 <sup>ED</sup>
<i>rms dev.</i>				2.784	1.260

<sup>a</sup> $\langle E_J \rangle$  - energy of the “center of gravity” of the absorption band,  $\Gamma$  – integrated absorption coefficient,  $f_{\text{exp}}^{\Sigma}$  and  $f_{\text{calc}}^{\Sigma}$  – experimental and calculated absorption oscillator strengths, respectively, ED and MD stand for the electric-dipole and magnetic-dipole contributions, respectively.

**Table 5.** Intensity parameters of the  $\text{Ho}^{3+}$  ion in CNGG.

Parameter	J-O theory	ICI theory
$\Omega_2 \times 10^{20}, \text{cm}^2$	4.107	11.132
$\Omega_4 \times 10^{20}, \text{cm}^2$	3.977	3.901
$\Omega_6 \times 10^{20}, \text{cm}^2$	1.383	2.179
$R_2 \times 10^4, \text{cm}$	-	0.398
$R_4 \times 10^4, \text{cm}$	-	0.136
$R_6 \times 10^4, \text{cm}$	-	0.093

**Table 6.** Calculated probabilities of spontaneous radiative transitions of the Ho<sup>3+</sup> ion in CNGG<sup>a</sup>.

Emitting state	Terminal state	$\langle\lambda\rangle$ , nm	$A_{\text{calc}}^{\Sigma}(JJ')$ , s <sup>-1</sup>	$B(JJ')$ , %	$A_{\text{tot}}$ , s <sup>-1</sup>	$\tau_{\text{rad}}$ , ms
<sup>5</sup> I <sub>7</sub>	<sup>5</sup> I <sub>8</sub>	1964	76.3 <sup>ED</sup> +42.6 <sup>MD</sup>	1	118.9	8.41
<sup>5</sup> I <sub>6</sub>	<sup>5</sup> I <sub>7</sub>	2804	29.0 <sup>ED</sup> +24.3 <sup>MD</sup>	0.183	291.6	3.43
	<sup>5</sup> I <sub>8</sub>	1155	238.3 <sup>ED</sup>	0.817		
<sup>5</sup> I <sub>5</sub>	<sup>5</sup> I <sub>6</sub>	3820	10.7 <sup>ED</sup> +8.8 <sup>MD</sup>	0.072	271.3	3.69
	<sup>5</sup> I <sub>7</sub>	1617	149.5 <sup>ED</sup>	0.551		
	<sup>5</sup> I <sub>8</sub>	886.8	102.4 <sup>ED</sup>	0.377		
<sup>5</sup> I <sub>4</sub>	<sup>5</sup> I <sub>5</sub>	4833	11.7 <sup>ED</sup> +4.4 <sup>MD</sup>	0.088	184.3	5.42
	<sup>5</sup> I <sub>6</sub>	2134	67.7 <sup>ED</sup>	0.367		
	<sup>5</sup> I <sub>7</sub>	1211	84.0 <sup>ED</sup>	0.456		
	<sup>5</sup> I <sub>8</sub>	749.3	16.5 <sup>ED</sup>	0.089		
<sup>5</sup> F <sub>5</sub>	<sup>5</sup> I <sub>4</sub>	4824	0.15 <sup>ED</sup> +0.03 <sup>MD</sup>	<0.001	4735.5	0.21
	<sup>5</sup> I <sub>5</sub>	2414	18.4 <sup>ED</sup> +0.9 <sup>MD</sup>	0.004		
	<sup>5</sup> I <sub>6</sub>	1479	208.4 <sup>ED</sup> +2.8 <sup>MD</sup>	0.044		
	<sup>5</sup> I <sub>7</sub>	968.4	964.7 <sup>ED</sup>	0.204		
	<sup>5</sup> I <sub>8</sub>	648.6	3540.2 <sup>ED</sup>	0.748		
<sup>5</sup> S <sub>2</sub> + <sup>5</sup> F <sub>4</sub>	<sup>5</sup> F <sub>5</sub>	3212	66.9 <sup>ED</sup> +8.2 <sup>MD</sup>	0.005	15698	0.064
	<sup>5</sup> I <sub>4</sub>	1928	179.0 <sup>ED</sup> +0.08 <sup>MD</sup>	0.011		
	<sup>5</sup> I <sub>5</sub>	1378	488.4 <sup>ED</sup> +0.3 <sup>MD</sup>	0.031		
	<sup>5</sup> I <sub>6</sub>	1013	1162.6 <sup>ED</sup>	0.074		
	<sup>5</sup> I <sub>7</sub>	744.1	3138.0 <sup>ED</sup>	0.200		
	<sup>5</sup> I <sub>8</sub>	539.6	10654 <sup>ED</sup>	0.679		

<sup>a</sup> $\langle\lambda\rangle$  - mean wavelength of the emission band,  $A_{\text{calc}}^{\Sigma}(JJ')$  – probability of radiative spontaneous transition,  $B(JJ')$  – luminescence branching ratio,  $A_{\text{tot}}$  – total probability of radiative spontaneous transitions,  $\tau_{\text{rad}}$  – radiative lifetime of the excited state, ED and MD – electric-dipole and magnetic-dipole contributions, respectively.