

IGNASI MEREDIZ SOLÀ

**TEMPORAL ANALYSIS OF ASYMMETRIC VOLATILITY IN
CRYPTOCURRENCIES**

BACHELOR'S THESIS

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Title, Abstract and Keywords (in Catalan, Spanish and English)

English

Title: Temporal analysis of asymmetric volatility in cryptocurrencies.

Abstract: In this paper a temporal analysis has been carried out to observe the relationship between the inefficiency caused by the uninformed investors and the asymmetric volatility in the cryptocurrency market. This analysis is done applying the TGARCH model with a rolling windows approach. This work concludes that asymmetric volatility and inefficiency appear in the cryptocurrencies studied. This asymmetry in the volatility makes positive shocks to generate more volatility than negative ones, which is the opposite effect found in traditional financial markets, except for gold. Nevertheless, these factors are not constant throughout the period. In the two major cryptocurrencies neither of the two effects is found, confirming that the higher market capitalization and liquidity improve efficiency and no asymmetry in the volatility is generated. In the other cryptocurrencies, the relationship between inefficiency and asymmetric volatility is found. This relationship could be caused by the Fear of Missing Out (FOMO) effect of uninformed investors and it appears in a different period depending on the cryptocurrency.

Keywords: Cryptocurrencies, TGARCH, Asymmetric volatility.

Catalan

Títol: Anàlisi temporal de la volatilitat asimètrica en les criptomonedes.

Resum: En aquest treball s'ha fet un anàlisi temporal per observar la relació entre la ineficiència causada pels inversors no informats i la volatilitat asimètrica en el mercat de les criptomonedes. Aquest estudi s'ha fet aplicant el model TGARCH amb un anàlisi de finestres consecutives. Aquest treball conclou que la ineficiència i la volatilitat asimètrica apareixen en les criptomonedes estudiades. Aquesta asimetria en la volatilitat fa que els *shocks* positius generin més volatilitat que els *shocks* negatius, que és l'efecte contrari que es troba als mercats financers tradicionals, excepte per l'or. Però, aquests factors no són constants durant tot el període. En les dues criptomonedes més grans no s'ha trobat cap dels dos efectes, confirmant que l'elevada capitalització de mercat i la liquiditat milloren l'eficiència i no es genera asimetria en la volatilitat. A les altres criptomonedes, s'ha trobat relació entre la ineficiència i volatilitat asimètrica. Aquesta relació és causada per l'efecte de la por a perdre l'oportunitat dels inversors no informats, i apareix en un període diferent en funció de la criptomoneda.

Paraules clau: Criptomonedes, TGARCH, Volatilitat asimètrica

Spanish

Título: Análisis temporal de la volatilidad asimétrica en las criptomonedas.

Resumen: En este trabajo se ha llevado a cabo un análisis temporal para observar la relación en la ineficiencia causada por los inversores no informados y la volatilidad asimétrica en el mercado de las criptomonedas. Este estudio se ha hecho aplicando el modelo TGARCH con un análisis de ventanas consecutivas. Este trabajo concluye que la ineficiencia y la volatilidad asimétrica aparecen en las criptomonedas estudiadas. Esta asimetría en la volatilidad hace que los *shocks* positivos generen más volatilidad que los *shocks* negativos, que es el efecto contrario que se encuentra en los mercados financieros tradicionales, excepto para el oro. Pero, estos factores no son constantes durante todo el período. En las dos criptomonedas más grandes no se ha encontrado ninguno de los dos efectos, confirmando que la elevada capitalización de mercado y la liquidez mejoran la eficiencia y no se genera asimetría en la volatilidad. En las otras criptomonedas se ha encontrado relación entre la ineficiencia y la volatilidad asimétrica. Esta relación parece estar causada por el efecto del miedo a perder la oportunidad de los inversores no informados, y aparece en diferentes períodos dependiendo de la criptomoneda.

Palabras clave: Criptomonedas, TGARCH, Volatilidad asimétrica.

Presentation

The objective of this thesis is to apply an empirical model to study cryptocurrencies in a way that it has not been done before.

The main motivation to do this thesis is that I have always been really interested in developing research in the economics field. For example, in my high school thesis I analyzed unemployment in Spain and my hometown, Reus, compared to Germany, United Kingdom and Brazil. This enabled me to find the difference in the structure of the labor market of these countries. Furthermore, I have recently become more interested in developing empirical economic projects. Currently, I am collaborating with Dr. Aurelio Fernández Bariviera with a scholarship granted by the Spanish Ministry of Education, Culture and Sport for the academic year 2018-2019. The topic we are mainly focused in is: "Statistical analysis of time series in cryptocurrencies".

This has brought me to continue my studies and try to enter a PhD program in Economics in the future. Consequently, this is an amazing opportunity to be more involved doing research in this field and to develop my own thesis. This can help in different ways since it will show me the enormous difficulty of creating your own project from zero. This involves thinking an economic problem or analysis which has not been done before, finding out the better way to solve it, look for the data and think about the reasons and consequences of the results found. Moreover, this thesis will give me more information about the activities done in a carrier of an economics researcher and start to think in which field I am more interested in.

The main courses which relates to this thesis are econometrics, statistics and mathematics since the development of an empirical model as I am doing is comprised in these three fields. More particularly, the model used is for time series, which is a specific part of all the econometrics material covered in the degree. Moreover, this topic is also related to the finance courses since it is studied from a financial point of view within the econometrics analysis, discussing most of the concepts used in this course such as asset prices, volatility or returns, for example.

In addition, this thesis has helped me to develop other important competences. I have needed to be creative in the sense to propose a work which it has not been done before in order to contribute to the existing literature. It will be useful to develop my writing and oral skills since I have to write a thesis being convincing in my arguments and then, explain it and defend it to a tribunal. Furthermore, in my case, as I have written and I have to present it in English, it also enables me to develop my third language. Lastly, as I am using R, which is a statistical software highly used in the

economic research field and the data analytics world, it will help me not only to learn to use this software but also to learn some programming basic skills, highly useful for any programming language, with many useful applications.

1. Introduction

The main objective of this thesis is to revisit a financial empirical aspect of the cryptocurrency market. Using a TGARCH model and estimating with a sliding windows approach, it will provide evidence of the time evolution of the asymmetric volatility of the cryptocurrencies and its relationship with the inefficiency caused by uninformed investors. Since Bitcoin's development in 2009, several economists have studied the volatility patterns of the cryptocurrencies for its huge changes in prices and the need to characterize them in an asset class to pass the appropriate regulation. Inspired by Baur & Dimpfl (2018), this paper applies the methodology developed by Glosten, Jagannathan, & Runkle (1993) and the serial correlation approach of Avramov, Chordia, & Goyal (2006) to conduct a rolling windows analysis to detect the main trends of asymmetry in the volatility through the period covered. It also studies its relationship to the inefficiency caused by uninformed traders as one of its potential causes. Moreover, the article by Bouri, Azzi, & Dyhrberg (2017) analyses the asymmetric volatility of Bitcoin before and after the 2013 price crash. This topic has been argued to be important so as to detect which type of investors are predominant in a financial market (informed vs uninformed investors). And in the case that it is dominated by uninformed investors, to find out if they actually generate asymmetry in the volatility. This procedure is developed in the R platform, which is an open source statistical software which contains a wide range of functions and packages.

The main source of the data of the cryptocurrencies is coinmarketcap.com, a webpage that aggregates and reports the trading activity of more than 2000 cryptocurrencies in more than 8900 global exchanges. This webpage is widely used as a source for data by some of the scientific articles published in the main journals since it is a reliable source of information for cryptocurrencies data. Coinmarketcap.com publishes all the necessary information to develop the model proposed here. Moreover, there have been other sources of information such as the papers covered by the literature review, the news related to the cryptocurrency world, the description of the *rugarch* package, and so on.

Bitcoin (BTC), which was developed in 2008 by an anonymous person (or a group of people) whose name is Satoshi Nakamoto, is based on a peer-to-peer network (Nakamoto, 2009). It has been taken into consideration by the media since its price exponentially grew until in December of 2017 it reached its maximum, when the price of one BTC was almost \$20,000 (Morris, 2017). This meant an enormous percentage growth since in June 2009 the price of 1 BTC was 0.0001 USD. However, last year prices collapsed by three-quarters. Currently, as of April 2019, the price of one Bitcoin

is \$5,285.14. During all this period, several other cryptocurrencies have been developed with some characteristics that make them different from Bitcoin. For example, Ethereum incorporates an environment that allows *smart contracts*. At present, this massive growth of the cryptocurrency markets is seen by the fact that there are more than 2100 cryptocurrencies available with a market capitalization of over \$169B (April 29, 2019). However, Bitcoin remains the largest virtual currency in circulation with almost 55% of the market capitalization (“CoinMarketCap”). All cryptocurrencies are backed by the blockchain technology. This underlying technology (Böhme, Christin, Edelman, & Moore, 2015) reduces the need for a central third-party institution to serve as authorities of trust, and it is needed to maintain decentralized systems. This technology has caught the experts’ attention and the media, as well, since its technology seems to have many applications to other fields such as health records or voting systems.

In this context, a lot of scientific papers have been published regarding Bitcoin and cryptocurrencies, most of which are either related to the economics and finance field or to the engineering and computer science one, studying different facts from each one. As an example, published research on Bitcoin began in 2012, and until January 2019 there have been 1162 papers on this topic. Some people focus on the new technology, while others focus on the tremendous returns. Regardless of the focus, cryptocurrencies are and should be of interest to the economics and finance research communities because of their potential to disrupt financial stability.

From an economics perspective, an important discussion referring to cryptocurrencies arose: if the cryptocurrencies should be considered either a currency or an asset. Yermack (2013) asserts that Bitcoin is not a currency as it performs poorly as a unit of account and as a store of value, even though neither cryptocurrencies nor common currencies have any intrinsic value. The high volatility of Bitcoin prices has damaged Bitcoin’s usefulness as a unit of account, which means that Bitcoin behaves more like a speculative investment than a currency, based on the fact that its market capitalization is significantly higher than the economic transactions it facilitates. Another remarkable difference compared to the currencies is that the US Dollar (as an example) is backed by a government, in which people trust, whereas cryptocurrencies are “private money” introduced by the private sector.

Baur, Hong, & Lee (2018) find that Bitcoin is mainly a speculative investment. Corbet, Lucey, Peat, & Vigne (2018) argue that Bitcoin is a speculative asset rather than a currency and this is not altered by the introduction of futures trading. Other studies are focused on categorizing Bitcoin into a certain asset class, which has been proved to be

difficult for its great differences from the rest of the assets. Bitcoin has been found to have a weak correlation with both risky financial assets and safe-haven assets (Bouri, Molnár, Azzi, Roubaud, & Hagfors, 2017; Corbet, Meegan, Larkin, Lucey, & Yarovaya, 2018), which does suggest that Bitcoin belongs to a unique and uncorrelated asset class.

From a legal perspective, Krugman (2018) suggests that policymakers should work to strengthen the supervision of Bitcoin trading since a part of the transactions of cryptocurrencies seem to be mostly for illegal activities such as thefts, drug dealing and tax evasion. Furthermore, even though Bitcoin occupies only a small area of the financial markets compared with more traditional asset classes, Chevapatrakul & Mascia (2018) suggests that it is important to design of an effective regulatory framework. This is necessary to protect investors from very aggressive price movements and containing potential spillovers of risk spreading from the cryptocurrency market to other markets.

Although the considerable agreement between the experts on the idea that cryptocurrencies should not be considered a currency, there are also a lot of factors which can be analyzed from Bitcoin and the rest of the cryptocurrencies from an economics and finance point of view. All this can not only help to better understand the cryptocurrency to better know which its behavior is in order to regulate them in a better way but also to understand more aspects from the financial markets in general.

For example, currently Bitcoin is widely recognized, and various aspects of its price properties have been investigated. Urquhart (2016) concludes (and it was tested by other papers) that Bitcoin was, initially, an inefficient market; but it could be in the process of moving towards a more efficient market. Moreover, Dyhrberg (2016) asserts that Bitcoin has a place in the financial markets and in portfolio management as it can be classified as something in between gold and the American dollar. Cheah & Fry (2015) provide empirical evidence to address the existence of bubbles in Bitcoin markets and find Bitcoin exhibits speculative bubbles.

The research carried out in this paper is of interest of many different groups. For investors in the cryptocurrency world to make better decisions. For policymakers, who have to pass regulation depending on the nature of this asset. Moreover, researchers in any financial market since it intends to shed light on the behavior and consequences of the dominance in a market of the uninformed investors.

Concerning this work, its most direct contribution would be to analyze what the time evolution of the two measures used in Baur & Dimpfl (2018) has been. The model used

is based on two equations. One to detect if there is a serial correlation between returns from one period to the other, which is a proxy for inefficiency. This inefficiency could be caused by uninformed investors (Avramov et al., 2006). And, the other one to find out whether positive or negative lagged shocks have a different effect on volatility. The initial hypothesis of the paper is that uninformed investors, who cause inefficiency in the market, generate asymmetric volatility because of the Fear of Missing Out (FOMO) effect. Regarding the novelty, these two measures, are made for the entire period in the original paper and this paper studies whether the measures remain constant in the period or have been changing. In this second case, if they have varied, possible explanations will be drawn. Moreover, doing it for several cryptocurrencies will show if the evolution is similar or different trends are found. In the latter case, an evaluation will be done on the reasons why they could be different.

The main conclusion of this paper leads us to the fact that there is asymmetric volatility in cryptocurrencies, which means that positive shocks raise more volatility than negative ones. Moreover, it has a relationship with inefficiency in eight of the ten cryptocurrencies studied. However, these two characteristics do not appear constantly throughout the period and they do not behave equally for all the cryptocurrencies, showing that this relationship appears in different periods depending on the cryptocurrency.

The rest of the paper is structured as follows. Section 2 includes the literature review. Section 3 describes the econometric model and the software used. Section 4 introduces the data. Section 5 presents the results. Section 6 provides the conclusion.

2. Literature review

Volatility is a relevant factor for measuring the efficiency of any asset. In financial markets, higher average return is usually a reward for bearing higher risk (higher volatility). For example, the Volatility Index (VIX) is a key market risk indicator that reflects market sentiment and investor expectation. It is widely used by market participants in their management risk strategy. Higher values of the VIX indicate more market uncertainty and vice-versa. Consequently, a lot of papers have studied the main characteristics of the volatility in the financial markets. Engle (1982) introduced a new class of stochastic processes which is called autoregressive conditional heteroscedastic (ARCH) processes. This model is based on the fact that the recent past gives information about the one-period forecast variance. The Generalized ARCH

(GARCH) models (Bollerslev, 1986) assume that variance does not follow a linear trend and it tends to cluster.

GARCH-type models are widely used also in the cryptocurrency market for several purposes for all its potential applications. The presence of long memory and persistent volatility justifies the application of GARCH-type models (Bariviera, 2017). Katsiampa (2017) compares the performance of numerous GARCH-type models. The volatility of other cryptocurrencies have received less attention. Catania, Grassi, & Ravazzolo (2019) focus on a comparison of prediction models for Bitcoin, Ethereum, Litecoin, and Ripple volatility. Therefore, GARCH models are widely used for several studies of cryptocurrencies. These models can capture the not linear pattern of volatility.

From this basic model, some others have been developed to capture specific patterns of volatility, which cannot be studied with the original GARCH model. More specifically, the TGARCH, which is one of the developments from the original GRACH model, was developed by (Glosten et al., 1993). This model is able to separate the effects on the volatility between good and bad news (defined or identified as lagged positive or negative shocks or innovations). Therefore, the main innovation of this model is that it allows to capture asymmetries in the conditional variance equation by introducing a dummy variable depending on the sign of the previous shock sign. This means that, positive and negative innovations have different effects on future conditional variance. So, this model is suitable for the purpose of this article in the sense that it allows to separate the effects of positive and negative shocks.

In this way, asymmetric volatility has been widely studied in the last three decades reaching the main conclusion, with a high agreement between all researchers, that negative shocks generate more volatility than positive shocks for the equity markets. There are two explanations for this asymmetry, the leverage effects and volatility feedback. On the one hand, the leverage effect theory states that a drop in the value of the stock (negative innovation) increases financial leverage, which makes the stock riskier and increases its volatility (Black, 1976; Christie, 1982). On the other hand, volatility feedback is based on the fact that if volatility is priced, an anticipated increase in volatility raises the required return on equity, leading immediate stock price decline (Pindyck, 1984; French, Schwert, & Stambaugh, 1987; Campbell & Hentschel, 1992).

However, gold behaves in a different way in terms of asymmetric volatility. Baur, (2012) found that in the gold market, positive shocks increase the volatility by more than negative shocks, which is the opposite result to what is found in the other financial markets. The authors attribute this difference to the safe-haven effects of gold, which

means that gold is widely used among investors and governments to protect the wealth since gold is uncorrelated with the other markets. Therefore, if investors tend to predict negative future returns in the rest of the markets, we can expect the volatility raises and the price of gold goes up.

The safe-haven characteristic has been also studied for cryptocurrencies. Although, cryptocurrencies (mainly Bitcoin for its maturity) have some characteristics similar to gold for its low correlation with the traditional markets, it cannot still be compared to gold for its remarkable differences. For example, Bouri, Molnár, et al. (2017) find that Bitcoin acts as a poor hedge, but it is suitable for diversification purposes. In addition, Dyhrberg (2016) states that Bitcoin has a place on the financial markets and in portfolio management as it can be classified as something in between gold and the American dollar. Baur, Dimpfl, & Kuck (2018), replicating the same study but extending the data and applying the TGARCH model, add that Bitcoin exhibits distinctively different return, volatility and correlation characteristics compared to other assets including gold and the US dollar. Smales (2018) states that it should not currently be considered as a safe haven, even if it were to meet the existing criteria related to return correlation. Klein, Pham Thu, & Walther (2018) find that Bitcoin and gold feature fundamentally different properties as assets and linkages to equity markets.

Furthermore, in a similar way to gold, the same effect was found in the cryptocurrency market in terms of asymmetric volatility for almost all of 20 major cryptocurrencies (Baur & Dimpfl, 2018). However, it is important to note that the asymmetric volatility parameter was significant for three out of the twenty cryptocurrencies studied in this paper. This shows that, even though some signs of asymmetric volatility are found for cryptocurrencies, they could not be constant for all the period and equal for all cryptocurrencies. As explained above, the safe-haven effect does not seem to be the reason for this asymmetry as in the gold market. Therefore, the authors suggest that the predominance of uninformed investors in this market makes the volatility higher after positive shocks for the Fear of Missing Out (FOMO) effect. FOMO is based on the idea that uninformed investors tend to react more after positive news (a positive shock in the previous period) than negative news. The main reason is the fact that they make their decisions based on the idea not to lose this tempting opportunity (in periods where the prices are increasing) rather than a fundamental analysis of the asset, which is what informed investors do. Then, the market has a lower reaction to negative shocks because market is dominated by informed investors. Thus, rational traders move prices closer to its fundamental value while noise (uninformed) traders move prices away from fundamentals. FOMO is one of the reasons that explain financial bubbles since

decisions are made based on previous day returns instead of the fundamental value of the asset.

It has been widely studied that uninformed investors (non-informational liquidity-driven trading activity) leads to enhanced volatility while informed trading leads to a decline in volatility (Hellwig, 1980; Wang, 1993). Following this research, Avramov et al. (2006) find that informed (uninformed) trades result in zero (non-zero) serial correlation in returns. Thus, uninformed trading seems to cause autocorrelation between different periods returns. Consequently, if the appearance of uninformed investors is after a positive shock, the reason why they appear could be the FOMO effect, not to lose this opportunity. Hence, if uninformed investors have an impact on the relationship between present and lagged returns (as suggested by Avramov et al. (2006)), using an autoregressive of the return can signal the appearance of uninformed investors in a market. So, the next step is to investigate if the periods where there are uninformed investors coincide with the period of asymmetric volatility.

Moreover, one of the characteristics that facilitates the appearance of the FOMO effect is the exposure of the cryptocurrency market on social media or forums. This enormous exposure generates that uninformed investors enter the market guided by the news or comments written on these webpages rather than a fundamental analysis value of the asset. For example, Shen, Urquhart, & Wang (2019) show that the number of tweets is a significant driver of Bitcoin's next day trading volume and realized volatility. Because of the dominance of uninformed investors in the cryptocurrency market during some periods, financial analysts rarely recommend or rate cryptocurrencies; therefore, cryptocurrency markets seem to be dependent on socially constructed opinions.

Moreover, Bouri et al. (2017) test whether asymmetric volatility could change before and after a price crash. They do so by dividing their subsample in two periods (before and after the price crash of 2013). They find that before the crisis of 2013 the volatility was asymmetric in the opposite way of the traditional assets, whereas this asymmetry is not found after the price crash. This study proves the importance of adding a temporal analysis to this estimator in order to find some differences and to determine the causes of this asymmetry.

This paper intends to shed light on the temporal evolution of asymmetric volatility for cryptocurrencies. It uses a rolling sample method to study the trends during the sample period and evaluate whether there is a relationship with the inefficiency caused by uninformed traders. This can give a better image of the reasons that cause the asymmetry in the volatility (being higher after a positive shock, according to previous

papers). And, in case it is found, to determine and study the factors that cause this asymmetry to be different from the other markets. In addition, studying several cryptocurrencies can show different trends between them, allowing to find out the reasons of these differences.

3. Methodology

In this section, we explain in detail the methodology used in this paper. This comprises a description of the model used, their properties, the program used (R software) and an example of a script to estimate the model.

3.1. Econometric approach

As introduced in the last section, the model applied to study the asymmetric volatility in the cryptocurrency market is the TGARCH model. This model is based on the assumption that positive and negative shocks have a different effect on volatility. This model is the same used in the two papers dealing with the same topic for cryptocurrencies (Bouri et al., 2017; Baur & Dimpfl, 2018).

In order to detect serial correlation, equation (1) considers an autoregressive model of order 1 (AR (1)) for the conditional mean of the returns. This is suggested by Avramov et al. (2006) to signal the presence of noise (uninformed) traders. Furthermore, the conditional volatility of Bitcoin returns is modeled in equation (2) using the TGARCH model of Glosten et al., (1993). The gamma parameter in equation (2), determines if there is asymmetry in the volatility in cryptocurrencies and in which sign.

The model then reads as follows:

$$r_t = \theta_0 + \theta_1 r_{t-1} + \varepsilon_t \quad (1)$$

$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \gamma \varepsilon_{t-1}^2 I(\varepsilon_{t-1} < 0) + \beta h_{t-1} \quad (2)$$

$$\varepsilon_t \sim \text{i.i.d. } N(0, h_t)$$

$I(\cdot)$ is an indicator function which is 1 if $\varepsilon_{t-1} < 0$ and 0 otherwise. Both equations must be stationary. In equation (1), θ_1 must be between -1 and 1. Moreover, in equation (2) the restriction $\alpha + \beta + \frac{1}{2} \gamma < 1$ needs to hold for the variance equation to describe a stationary variance process. $\frac{1}{2}$ is the associated probability of ε_{t-1} being negative, which means that $I(\cdot)$ is equal to 1. The variance model parameters α and β are restricted to be positive and γ can take any value between -1 and 1.

In eq. (1), r_{t-1} is the lagged daily returns that considers the presence of serial correlation. Fama (1970) proposed the Efficient Market Hypothesis (EMH). This theory states that efficient asset prices must follow a random walk and thus, they cannot be predicted from previous period news. So, θ_1 is a measure to study the efficiency. If θ_1 is different from 0, it means that current returns could have a correlation with previous day returns, and thus, it is a sign of inefficiency. Consequently, if θ_1 is positive (negative), returns are positively (negatively) correlated with the previous period returns. As higher is this coefficient, higher will be the inefficiency. A potential explanation if θ_1 is different from 0 (making the market inefficient) is that when uninformed investors dominate the market, making decisions based on previous day shocks rather than the fundamental value of the asset, they generate a correlation of consecutive periods returns (Avramov et al., 2006).

In Eq. (2), ω is the constant, α represents the ARCH term which measures the impact of past innovations on current variance, β is related with the persistence of the conditional variance, ε is the shock or innovation, and γ captures any potential asymmetric effect of lagged shocks on the volatility.

γ , which is the parameter of interest from equation (2), can be zero, positive or negative. The rationale behind the conclusion from its results are the following:

- If $\gamma = 0$, the impact of the shock does not change depending on its sign.

If γ is different from 0, there are several options:

- If $\varepsilon_{t-1} > 0$, $I = 0$, and the effect of the innovation on the variance (h_t) is given by α .
- If $\varepsilon_{t-1} < 0$, $I = 1$, and the effect of the shock on the variance (h_t) is $\alpha + \gamma$.

Then,

- If $\gamma < 0$, it means that the variance is lower when the previous shock is negative compared to when the previous shock is positive ($\alpha + \gamma < \alpha$). Consequently, positive shocks raise more volatility than negative ones.
- If $\gamma > 0$, it means that the variance is higher when the previous innovation is negative compared to when the previous shock is positive ($\alpha + \gamma > \alpha$). Consequently, negative shocks raise more volatility than positive ones.

Consequently, if γ is positive, then a negative shock generates more volatility than a positive shock of the same magnitude. This is the situation found for the traditional financial markets. In contrast, if γ is negative, then a positive shock generates more volatility than a negative shock of the same magnitude. This situation is associated with the gold and the cryptocurrency markets.

Furthermore, in order to understand other potential causes of the inefficiency and/or asymmetric volatility, a proxy for the liquidity is calculated. The Amihud illiquidity measure (Amihud, 2005) is chosen based on its robustness and simplicity. It requires only daily trade data. The Amihud illiquidity ratio is defined as,

$$ILLIQ_T^i = \frac{1}{D_T} \sum_{t=1}^{D_T} \frac{|R_t|}{P_t^i V_t^i}$$

Where:

- D_T is the number of traded days in year T. In this model, $D_T = 730$ since each rolling window is of two years, or 730 days.
- $r_{t,T}^i$ is the daily return of asset i on day t in USD, which it has been calculated for the TGARCH model.
- V_t^i is the daily volume traded of asset i on day t, and P_t^i is the daily price of asset i on day t in USD.

This measure provides an understanding on the relationship between volume and price change, providing us with a proxy on the price impact of daily aggregate trades. Consequently, as lower is the result, higher is the liquidity of the asset since it means that changes in prices (returns) have a low effect on the value of the volume traded.

Defined by Wei (2018), there are five groups regarding liquidity in the cryptocurrency market. So, they sort our cryptocurrencies based on the Amihud illiquidity ratio into 5 groups, with group 1 being most liquid and group 5 being least liquid and report their return characteristics. The numbers that separate each group are (from the more liquid group to the less liquid group): (<0,00001; 0,00011; 0,00191; 0,00960; 0,03581). This measure can be a useful reference for the cryptocurrencies studied in this paper.

Finally, a rolling windows analysis has been used to calculate and to study the evolution of the variables of interest explained above. The idea behind rolling analysis is to construct 'new' observations using samples of consecutive observations to conduct a temporal analysis of a time series. This rolling sample approach works as follows: we compute any estimation for the first 730 observation (which comprises a period of two years), then we discard the first one and add the following of the time series and continue this way until the end of data. Thus, each estimate is calculated from data samples of the same size. In this way, it is possible to see the evolution of the parameters studied.

Furthermore, the estimators are found maximizing the likelihood derived from the model proposed and the sample. This is a measure of adjustment of the model to the data. So, the natural logarithm of the maximum likelihood of the model. Since the number parameters and observations are the same (the model and the number of observations (730) are equivalent between cryptocurrencies), this estimator is comparable between cryptocurrencies and it will show in which cryptocurrencies and periods the model is more adjusted.

3.2. R software

To do all these estimations, the R software is used. R is suitable for this paper because it is an integrated suite of software facilities for data manipulation, calculation and graphical display, widely used for economists. Among other things it has:

- An effective data handling and storage facility.
- A suite of operators for calculations on arrays, in particular matrices.
- A large, coherent, integrated collection of intermediate tools graphical facilities for data analysis.
- A well-developed, simple and effective programming language which includes conditionals, loops, user defined recursive functions and input and output facilities.

More specifically, as it is an open source software, any person can upload a package for any specific purpose. Regarding to this article, there is a package called *rugarch* which is highly suitable for the purpose of this paper. This package aims to provide for a comprehensive set of methods for modelling univariate GARCH processes, including fitting, filtering, forecasting, simulation as well as diagnostic tools including plots and various tests. (Ghalanos, 2017).

The main reason for choosing this software is that it allows to calculate the model with a rolling windows approach. For this purpose, the function *for loops* has been used. Furthermore, this software is useful to create several kinds of graphs to better understand the estimation and have an explicit image of the results obtained. For example, the *xts* package facilitates the process to creates graphs, shading the significant values, which is a useful visual way to see the relationship between the two estimators studied in this paper. Furthermore, it is easy to write commands to make all the other calculations done in this article such as the daily returns, daily volatility and the Amihud's illiquidity ratio. *For a detailed script to estimate the model, see the Appendix.*

4. Data

To have a higher variety of reliable results, the methodology used to collect the data consists of taking the 40th cryptocurrencies with higher market capitalization (as of April 29, 2019) to have a representative sample and then take the cryptocurrencies with at least three years of data. As making a rolling sample of two years (730 observations), it is thought that it is important to have at least 365 rolling estimators of each cryptocurrency.

This leaves with eleven cryptocurrencies (Bitcoin, Ethereum, Ripple, Litecoin, Tether, Stellar, Monero, Dash, NEM, Dogecoin and Decred). However, from this sample we needed to take out Tether since, as a stable coin with a really low level of volatility, it was not suitable for this TGARCH model with a rolling windows approach. Consequently, we download daily transaction data of these ten cryptocurrencies from coinmarketcap.com for the period April 28, 2013 (or the earliest data available for each cryptocurrency) to April 28, 2019, and all denominated in US dollar. Historical daily data on coinmarketcap.com dates back to April 28, 2013, when active Bitcoin trading started.

This database includes daily open, close, maximum and minimum price, the value of the volume of transactions and the market capitalization. These data are volume weighted averages from a large number of different exchanges. Prices are calculated by averaging prices quoted at major exchanges weighted by volume. Statistics are updated every five minutes.

In addition to providing timely information coins listed on coinmarketcap.com, it must also satisfy rigorous assessment criteria. Firstly, based on information such as the total number in circulation coins must be deemed to constitute a genuine cryptocurrency. Secondly, coins must be traded on a public exchange that is more than thirty days old and with an active Application Programming Interface (API) available. Essentially this means that all listed cryptocurrencies must be genuinely tradable. Thirdly, listed coins must satisfy a transparency requirement and have a public URL that displays the total supply (total coins used so far).

The rigorous methodological aspects have made coinmarketcap.com as one of the most used databases for scientific research about cryptocurrencies.

Table 1. Summary of the cryptocurrencies studied (April 29, 2019).

N is the number of return observations per time series, *RW* is the number of rolling estimators, *MC* is the market capitalization in USD, % *MC* is the percentage of market capitalization from the total of the market, *V* (24h) is the value of the volume for the last 24h and % *V* is percentage of the total volume traded the last 24h for all cryptocurrency market.

	N	RW	Market cap	% MC	V (24h)	% V
Bitcoin	2191	1462	92,893,095,762	54.93%	13,634,259,231	31.90%
Ethereum	1360	631	16,493,724,849	9.75%	5,967,266,846	13.96%
Ripple	2093	1364	12,379,483,842	7.32%	891,708,918	2.09%
Litecoin	2191	1462	4,191,772,639	2.48%	4,191,772,639	9.81%
Stellar	1727	998	1,860,329,710	1.10%	200,833,716	0.47%
Monero	1803	1074	1,031,329,467	0.61%	61,711,356	0.14%
Dash	1899	1170	954,908,266	0.56%	257,607,575	0.60%
NEM	1488	759	502,692,486	0.30%	24,384,110	0.06%
Dodgecoin	1960	1231	295,174,806	0.17%	31,361,580	0.07%
Decred	1173	444	224,958,578	0.13%	1,248,745	0.003%
Total			169,099,930,477	77.37%	42,734,045,368	59.11%

As shown in table 1, these cryptocurrencies accumulate almost 80% of the market capitalization and almost 60% of the daily volume traded. So, these ten cryptocurrencies comprise a representative selection of the cryptocurrency market. From now on, in all the tables to present the results, the cryptocurrencies will be presented by order of market capitalization (as of April 2019).

We calculate daily cryptocurrency returns to be the logarithmic difference of the closing price between two consecutive days:

$$R_t = (\ln(P_t) - \ln(P_{t-1})) * 100$$

where $\ln(P_t)$ is the natural logarithm of the close price at time t and $\ln(P_{t-1})$ is the natural logarithm of the close price the day before.

Furthermore, it is also calculated the daily price volatility, defined as the logarithmic difference between intraday highest and lowest prices, expressed with the following equation,

$$\text{ReturnVolatility}_t = (\ln(P_t^{\text{high}}) - \ln(P_t^{\text{low}})) * 100$$

This measure is used by Bariviera (2017). This calculation can give a better understanding of the different characteristic for cryptocurrencies. Both the mean and the standard deviation are presented in Table 2.

Table 2. Descriptive statistics of the daily returns and the daily volatility.

N is the number of daily return/daily volatility observations per time serie, *Sd* the standard deviation, *Min* is the minimum, *Med* is the median of the series, *Max* is the maximum, *Skew* and *Kurt* are skewness and kurtosis. The right columns are the Mean and the Standard deviation of the daily volatility.

	N	Daily returns							Daily volatility	
		Mean	Sd	Min	Med	Max	Kurt	Skew	Mean	Sd
Bitcoin	2191	0.17	4.31	-26.62	0.19	35.75	8.04	-0.17	4.96	4.87
Ethereum	1360	0.30	7.56	-130.2	-0.09	41.23	67.19	-3.41	8.37	7.80
Ripple	2093	0.19	7.57	-61.63	-0.29	102.74	28.19	2.04	7.03	8.41
Litecoin	2191	0.13	6.63	-51.39	-0.02	82.90	25.20	1.74	6.95	7.46
Stellar	1727	0.21	7.88	-36.64	-0.35	72.31	15.57	1.96	10.05	9.29
Monero	1803	0.20	7.29	-32.54	-0.10	58.46	5.96	0.71	10.24	10.20
Dash	1899	0.30	7.95	-46.05	-0.20	126.32	42.27	2.99	9.44	11.38
NEM	1488	0.37	8.51	-36.15	-0.01	99.56	17.48	1.92	11.02	9.16
Dogecoin	1960	0.11	7.94	-58.04	-0.33	116.63	32.75	2.19	8.73	9.98
Decred	1173	0.27	7.97	-34.20	-0.13	44.11	4.36	1.05	11.66	8.84
Average		0.22	7.36	-51.35	-0.13	78.00	24.70	1.10	8.85	8.74

Table 2 exhibits the descriptive statistics of the daily returns of the ten cryptocurrencies studied in this paper. The mean of daily returns of the ten cryptocurrencies is positive and ranging from 0.13% (Litecoin) to 0.37% (NEM), with an average mean return of 0.22%. Bitcoin has the lowest standard deviation which it seems to be due to its maturity and the fact that it is the first cryptocurrency.

All the kurtosis values are higher than 3. That is, all the cryptocurrencies follow a leptokurtic distribution, which means that there are more outliers than the normal distribution. Regarding the skewness, Bitcoin and Ethereum (the two biggest in market capitalization) are negative skewed which means that the mass of the distribution is concentrated on the right of the figure. The rest of the cryptocurrencies are positive skewed, which means that the mass of the distribution is concentrated on the left of the figure.

Furthermore, the mean of the daily volatility ranges from 4.96% (Bitcoin) to 11.02% (Decred), which means a considerable intraday price variation. The standard deviation of the daily returns (which measures the volatility of the returns from one day to another) is bigger than standard deviation of the daily volatility.

Results from Table 2 suggest that there is some relationship between risk (standard deviation and daily volatility) and reward (returns), but this relationship it is not clear for all the cryptocurrencies.

Moreover, as studied in several articles, there is a clear and positive relationship between liquidity and efficiency in the sense that the number of investors is positively correlated with the efficiency. This happens because the more trading there is, the more information about the market is available, and thus, the investors can make more informed decisions. For this reason, the Amihud's illiquidity ratio (Amihud, 2005) is calculated. This ratio was used for cryptocurrencies by Wei (2018), confirming that efficiency is positively correlated with liquidity. So, the Amihud's illiquidity ratio is calculated for the ten cryptocurrencies. Since coinmarketcap.com publishes the daily value (24 hours) of the volume traded, it is not necessary to multiply price for volume, since this variable is already published for every day. To better read the results and easily compare them between cryptocurrencies, all the ratios have been multiplied by 100,000,000 (10^8).

Table 3. Amihud's illiquidity ratio descriptive statistics.

Sd the standard deviation, Min is the minimum, 1Q is the first quartile, Med is the median of the series, 3Q is the third quartile and Max is the maximum.

	Mean	Sd	Min	1Q	Med	3Q	Max
Bitcoin	0.03	0.03	0.001	0.01	0.02	0.05	0.08
Ethereum	0.30	0.52	0.003	0.03	0.07	0.19	2.27
Ripple	3.40	3.42	0.02	0.56	2.80	4.44	14.56
Litecoin	0.47	0.34	0.01	0.14	0.45	0.84	0.93
Stellar	119.51	113.81	0.27	12.71	38.49	247.66	281.69
Monero	36.12	39.34	0.14	0.70	19.05	64.45	110.09
Dash	16.28	16.01	0.05	0.96	13.54	30.92	59.35
NEM	525.88	774.84	0.40	8.73	19.72	1251.67	2043.10
Dogecoin	10.95	6.33	0.47	4.78	11.90	17.49	18.32
Decred	55.07	36.06	3.16	13.54	62.54	83.33	115.33
Average	76.80	99.07	0.45	4.22	16.86	170.10	264.57

Table 3 shows the Amihud's illiquidity ratio for the ten cryptocurrencies studied. The lower the ratio is, the more liquid the market is. Following the criterion of Wei (2018), all the cryptocurrencies are from the high liquidity group (all of them are lower than 1,000 ($0.0001 \cdot 10^8$), with the exception of NEM whose initial period belongs to the second group. However, there are some remarkable differences between them. For example, the mean of Bitcoin, which is the most liquid cryptocurrency, is ten thousand time smaller than the mean of the least liquid, which is NEM. The evolution of liquidity is similar for all cryptocurrencies because, as predicted, the more mature (more observations) the market becomes, the more liquid it is. These graphs have not been displayed since they did not add information to the study of this paper. Nevertheless, this does not happen comparing different cryptocurrencies. For example, Ethereum,

which is a young cryptocurrency compared to the others, is the second in terms of liquidity. The main reason is that Ethereum developed quickly since it is not only a cryptocurrency because it also incorporates an environment that allows *smart contracts*.

5. Results

The initial hypothesis presented in this paper is as follows: asymmetric volatility with higher volatility after positive shocks than negative shocks is caused by uninformed investors. Uninformed investors are those who rely on the previous news to invest, not on the fundamental value as the informed investors do. So, these uninformed investors enter the market after positive shocks because of FOMO effect, so raising volatility more after positive shocks than negative ones.

From now on, the estimator that measures the asymmetry of volatility will be defined by “Gamma” and the estimator which measures the serial correlation of the conditional returns by “AR”, to make the explanations clearer. The use of the sliding window approach, it allows the observation of a time-varying pattern of these estimators.

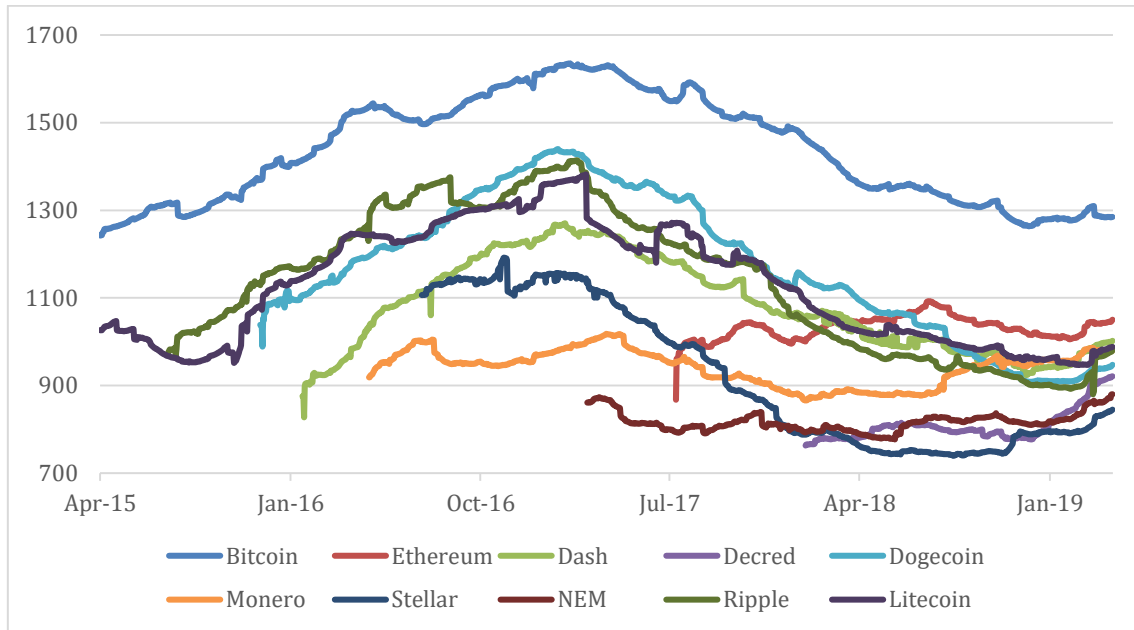
Therefore, this means that there is an expected relationship between AR (uninformed investors) and Gamma (asymmetric volatility). A negative Gamma suggests that positive asymmetric volatility exists, and a relationship with the AR estimator is expected because the uninformed investors cause it because of the FOMO, that appears after a positive shock. Moreover, the Gamma estimator may be positive in some periods. This would mean that negative shocks raise more volatility than positive shocks do, as it is the case for common assets, and other explanations should be investigated.

In the following figure, the natural logarithm of the maximum likelihood estimation (MLE) for all of the rolling estimations is shown in order to find out in which periods and cryptocurrencies the model is better adjusted.

As shown in Figure 1 (in the next page), Bitcoin is the cryptocurrency which the model is more adjusted to the data. It is important to note that the date that appears in the x-axis is the final date of the observation. In addition, Figure 1 (in the next page) also shows that the natural logarithm of the MLE follows a similar pattern for all the cryptocurrencies since they all have its maximum at about the first half of 2017 and most of them tend to decrease. Therefore, this generalized decrease could be due to

the price bubble of the cryptocurrency market of the second half of 2017, who has made the model less adjusted for the data.

Figure 1. Evolution of the natural logarithm of the MLE.



It is also remarkable to observe that all the cryptocurrencies, except for Bitcoin, tend to converge to a MLE of 950 being Ethereum the second one, which it is shown in Figure 1. So, Bitcoin and Ethereum present some characteristics (such as market capitalization) that make this model more adjusted for them.

In addition, the process to exhibit and analyze the results is the following. Firstly, the results are displayed in tables, showing the main statistics of the parameter estimators in all the series unconditional and conditional to the significance of the estimator (i.e. considering only the estimator values significantly different from 0 at a 10% significance level). This process can give the first conclusion of the result, which is to figure out if these estimators are constant throughout the period or if they are only found in some specific periods of the section covered. Moreover, it can give some hints if the relationship between inefficiency and asymmetric volatility actually exists. Secondly, the graphs of the two parameter estimators are displayed for all the cryptocurrencies, shading, in grey, the significant observations. This visual analysis will clearly show if this relationship holds for all the cryptocurrencies and it will allow to separate them (in case there are differences).

Table 4. Descriptive statistics of the Gamma coefficients.

Mean is the average of the results found, Sd the standard deviation, Min is the minimum, Max is the maximum, N is the number of significant estimators and % is the percentage of significant estimations from the total of rolling estimations.

	Gamma				Gamma significant (10%)					
	Mean	Sd	Max	Min	N	%	Mean	Sd	Max	Min
Bitcoin	-0.02	0.05	0.08	-0.15	67	5%	-0.12	0.01	-0.10	-0.15
Ethereum	-0.04	0.02	0.003	-0.08	1	0.2%	-0.02		-0.02	-0.02
Ripple	-0.46	0.19	-0.07	-0.93	329	24%	-0.45	0.04	-0.35	-0.50
Litecoin	-0.07	0.04	0.02	-0.27	793	54%	-0.09	0.03	-0.04	-0.15
Stellar	-0.27	0.22	-0.02	-0.87	226	23%	-0.32	0.30	-0.02	-0.87
Monero	-0.15	0.06	-0.02	-0.30	398	37%	-0.16	0.06	-0.02	-0.30
Dash	-0.09	0.10	0.22	-0.31	128	11%	-0.06	0.04	-0.02	-0.17
NEM	-0.01	0.18	0.33	-0.45	55	7%	-0.34	0.05	-0.27	-0.45
Dogecoin	-0.15	0.11	0.09	-0.41	322	26%	-0.13	0.08	0.08	-0.39
Decred	-0.03	0.01	0.003	-0.04	172	39%	-0.03	0.004	-0.02	-0.04
Average	-0.13	0.10	0.06	-0.38	210	23%	-0.17	0.07	-0.08	-0.30

Table 4 displays the Gamma coefficients for ten cryptocurrencies studied. The first conclusion from this table is that almost all the significant Gamma coefficients are negative, except for some of them in Dogecoin. The number of significant coefficients range from 1 (0.2%) for Ethereum to 793 (54%) for Litecoin.

The two cryptocurrencies with a higher variance of values are Stellar, whose Gamma coefficients range from -0.02 to -0.87, followed by Dogecoin, whose Gamma coefficients range from 0.08 to -0.39. The two cryptocurrencies with the lower variance are Decred, whose values range from -0.02 to -0.04, followed by Bitcoin, whose coefficients range from -0.1 to -0.15.

Nevertheless, it is important to note that in seven of the cryptocurrencies there are some positive Gamma coefficient, even though there are only seven significant observation in Dogecoin.

The first conclusion, then, is that cryptocurrencies have indeed some periods of asymmetric volatility that positive shocks raise more volatility than negative ones. However, this fact is not constant over time, since the average of significant Gamma estimators is 23%. Moreover, it also confirms the results found in Baur & Dimpfl (2018). From the common cryptocurrencies studied in both papers, the two significant Gamma coefficients in their paper (Litecoin and Monero) are the two cryptocurrencies with more significant Gamma coefficients, as shown in Table 4.

Table 5. Descriptive statistics of the AR coefficients.

Mean is the average of the results found, Sd the standard deviation, Min is the minimum, Max is the maximum, N is the number of significant estimators and % is the percentage of significant estimations from the total of rolling estimations.

	AR				AR significant (10%)					
	Mean	Sd	Max	Min	N	%	Mean	Sd	Max	Min
Bitcoin	0.03	0.02	0.08	-0.06	21	1%	0.08	0.003	0.08	0.07
Ethereum	0.05	0.01	0.09	-0.02	54	9%	0.07	0.01	0.09	0.07
Ripple	0.05	0.03	0.17	-0.02	249	18%	0.08	0.01	0.12	0.08
Litecoin	-0.03	0.04	0.03	-0.12	219	15%	-0.06	0.03	0.01	-0.12
Stellar	-0.001	0.09	0.14	-0.19	276	28%	-0.13	0.05	0.14	-0.19
Monero	-0.04	0.05	0.04	-0.14	323	30%	-0.10	0.02	-0.06	-0.14
Dash	-0.06	0.04	0.01	-0.14	362	31%	-0.11	0.02	-0.01	-0.14
NEM	-0.07	0.02	0.0002	-0.13	48	6%	-0.12	0.005	-0.11	-0.13
Dogecoin	-0.06	0.04	0.05	-0.14	39	3%	-0.10	0.01	-0.08	-0.14
Decred	-0.11	0.01	-0.07	-0.15	432	97%	-0.11	0.01	-0.07	-0.15
Average	-0.02	0.04	0.05	-0.11	202	24%	-0.05	0.02	0.004	-0.08

Table 5 exhibits the AR coefficients for the ten cryptocurrencies examined. The mean of the coefficient of the three cryptocurrencies with a higher market capitalization are positive (Bitcoin, Ethereum and Ripple). In contrast, the mean of the coefficient for the rest of the cryptocurrencies are negative, with the exception of Stellar, which have some significant positive AR estimators. The cryptocurrency with the lower number of significant AR coefficients is Bitcoin with 21 (1%) and the one with the higher number of significant observations is Decred with 432 (97%).

The conclusion from these results is that cryptocurrencies have some periods of inefficiency, which it could be associated to the presence of uninformed traders. Taking all the series, in all the cryptocurrencies, except for Decred that all of them are negative, there are both positive and negative coefficients.

Table 6. Descriptive statistics of AR and Gamma significant coefficients.

% is the percentage of significant estimations from the total of rolling estimations, Mean is the average of the results found, Sd the standard deviation, Min is the minimum, Max is the maximum.

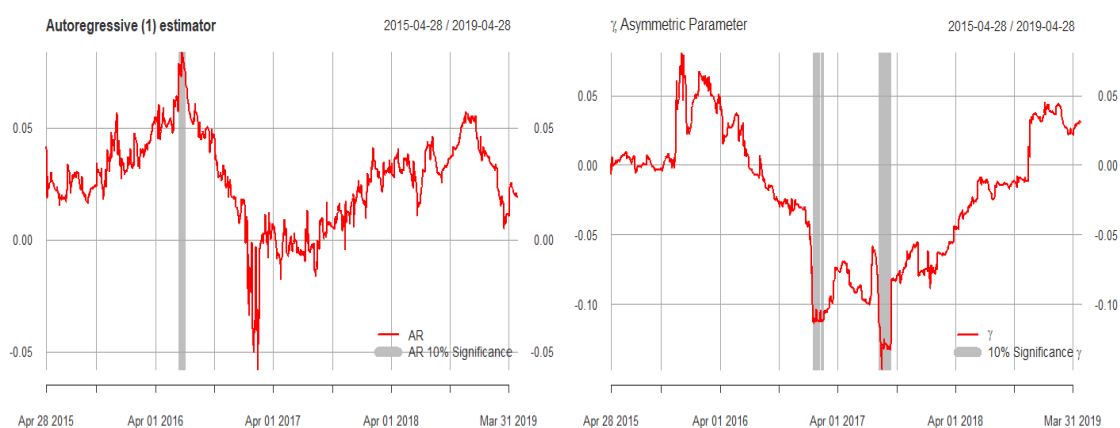
	AR significant (10%)					Gamma significant (10%)				
	%	Mean	Sd	Max	Min	%	Mean	Sd	Max	Min
Bitcoin	1%	0.08	0.003	0.08	0.07	5%	-0.12	0.01	-0.10	-0.15
Ethereum	9%	0.07	0.01	0.09	0.07	0.2%	-0.02	-	-0.02	-0.02
Ripple	18%	0.08	0.01	0.12	0.08	24%	-0.45	0.04	-0.35	-0.50
Litecoin	15%	-0.06	0.03	0.01	-0.12	54%	-0.09	0.03	-0.04	-0.15
Stellar	28%	-0.13	0.05	0.14	-0.19	23%	-0.32	0.30	-0.02	-0.87
Monero	30%	-0.10	0.02	-0.06	-0.14	37%	-0.16	0.06	-0.02	-0.30
Dash	31%	-0.11	0.02	-0.01	-0.14	11%	-0.06	0.04	-0.02	-0.17
NEM	6%	-0.12	0.00	-0.11	-0.13	7%	-0.34	0.05	-0.27	-0.45
Dogecoin	3%	-0.10	0.01	-0.08	-0.14	26%	-0.13	0.08	0.08	-0.39
Decred	97%	-0.11	0.01	-0.07	-0.15	39%	-0.03	0.004	-0.02	-0.04
Average	24%	-0.05	0.02	0.004	-0.08	23%	-0.17	0.07	-0.08	-0.30

Table 6 presents the significant Gamma coefficients together with the AR significant coefficients for the cryptocurrencies studied. This table gives a sign that some part of the asymmetry in the volatility of cryptocurrencies seems to be related to the inefficiency since the percentage of significant coefficients of both is similar in most of the cryptocurrencies.

The third conclusion is as follows: it seems to be a relationship between AR and Gamma in the sense that some of the percentage of significant values are similar. The average of significant AR estimators is 24%, really close to the 23% found for Gamma estimators. However, it is important to see which the trends of these estimators are to better analyze this relationship.

Consequently, displaying the results of the temporal analysis for both estimators (with the rolling windows approach) for all the cryptocurrencies, it will be possible to find the trends of this asymmetric volatility and to study if the inefficiency caused by uninformed investors is the cause. The date that appears in the x-axis is the end date for each rolling estimator.

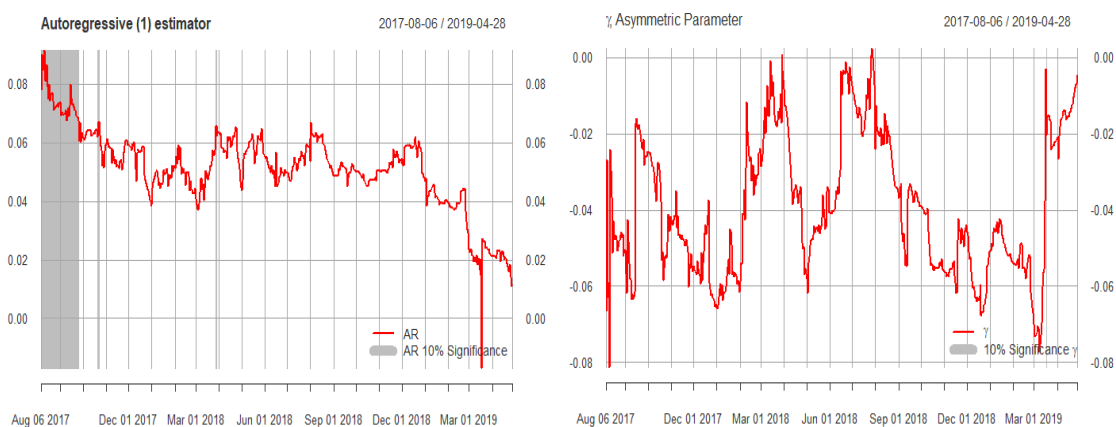
Figure 2. Evolution of the AR and Gamma estimations for Bitcoin.



Almost all the AR coefficients for Bitcoin are positive, except for a period in 2017 when it is negative. Bitcoin has twenty-one (1.44%) consecutive significant AR values between June and July of 2016. All of them positive between 0.07 and 0.08.

As asymmetry is concerned, the series for Gamma is positive at the beginning, it becomes negative between 2016 and 2017 and it remains negative almost until the end of the series (the significant coefficients are in this period), but it finishes being positive again, making an u-shape time series. There are sixty-seven (4,58%) significant Gamma coefficients divided into two periods but all of them in 2017. All of them are negative and between -0.09 and -0.15. So, Bitcoin does have a period of asymmetric volatility, though it is not remarkable since they do not coincide within the periods of inefficiency.

Figure 3. Evolution of the AR and Gamma estimations for Ethereum.

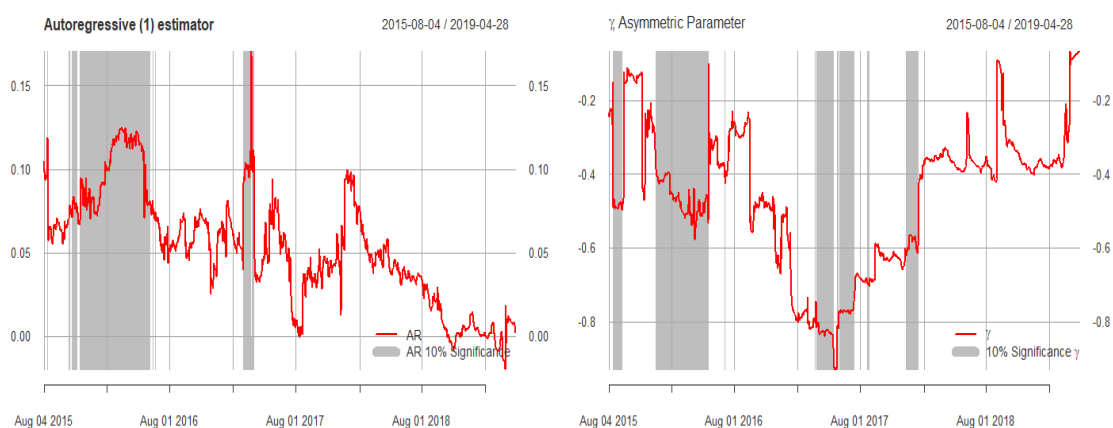


The AR estimator for Ethereum follows a clear process. It is almost always positive, starting close to 0.09 and slowly getting closer to zero. Ethereum has fifty-four (9%)

significant AR coefficients. All of them are positive and between 0.07 and 0.09. Most of the significant observations are at the beginning of the series. So, most of them are during the initial period (2015 - 2017) of the cryptocurrency. This seems to be a sign of initial inefficiency of the cryptocurrency, even though this period of inefficiency rapidly changed. However, as shown in Figure 5, this period of inefficiency does not coincide with a period of asymmetric volatility.

The pattern of the Gamma estimator is irregular even though the mean seems to be constant around -0.04. Regarding asymmetry, there is only one significant observation, showing that Ethereum does not reject the null hypothesis for Gamma.

Figure 4. Evolution of the AR and Gamma estimations for Ripple.

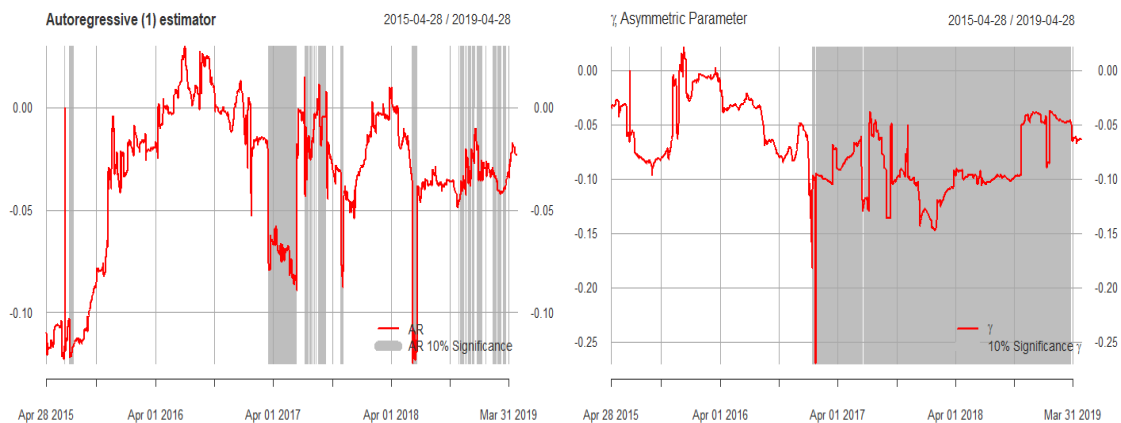


The AR estimator for Ripple starts at 0.10 and it linearly decrease until approaching zero at the end of the period covered. Ripple has 249 (18%) of significant AR values. Most of them are concentrated on the first quarter of the series.

The Gamma coefficient has a u-shaped trend (starting and ending close to zero and having its lowest value at around -0.09). Moreover, there are 329 (24%) significant gamma coefficients, half of them in the initial period of the cryptocurrency and the other half in the middle of the time series. All of them are negative and ranging between -0,35 and -0,50.

Therefore, Ripple does show a relationship between both estimators at the beginning of the series.

Figure 5. Evolution of the AR and Gamma estimations for Litecoin.

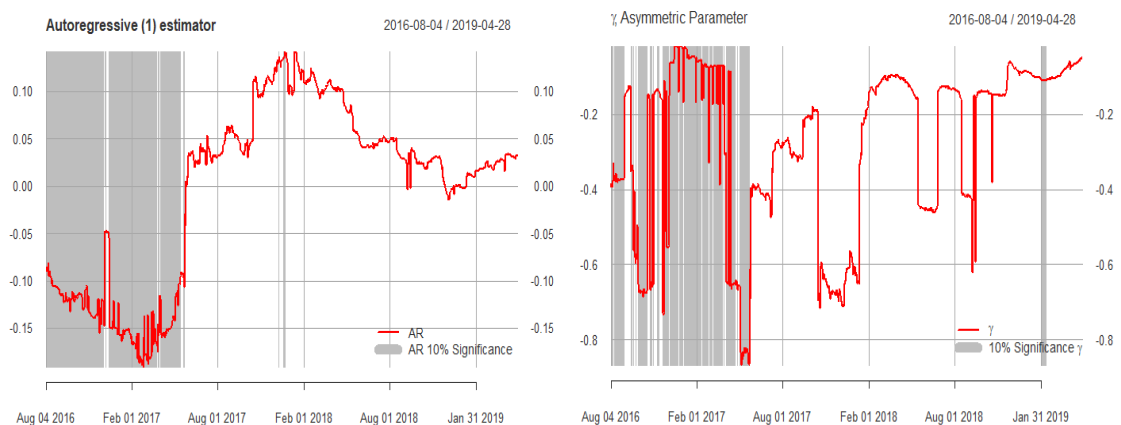


Litecoin AR estimator follows an irregular pattern with two periods (in 2016 and 2018) of positive coefficients. It has 219 (15%) significant AR coefficients. All of them are concentrated in the middle and in the end of the series. Most of the coefficients are negative, ranging from 0.01 to -0.12. The positive significant coefficients are at the end of 2017.

The Gamma estimator almost follows a horizontal line, but the average of the coefficients of the first half of the series is higher (with some positive coefficients) than in the second half (when almost all the coefficients are significant.) From the beginning of 2017 (second half of the series) almost all the Gamma coefficients are significant and negative, which means that there are 793 (54%) significant observations. All the coefficients are negative, ranging from -0.04 to -0.15.

Consequently, Litecoin does show some signs of a relationship between the two estimators at the end of the series.

Figure 6. Evolution of the AR and Gamma estimations for Stellar.

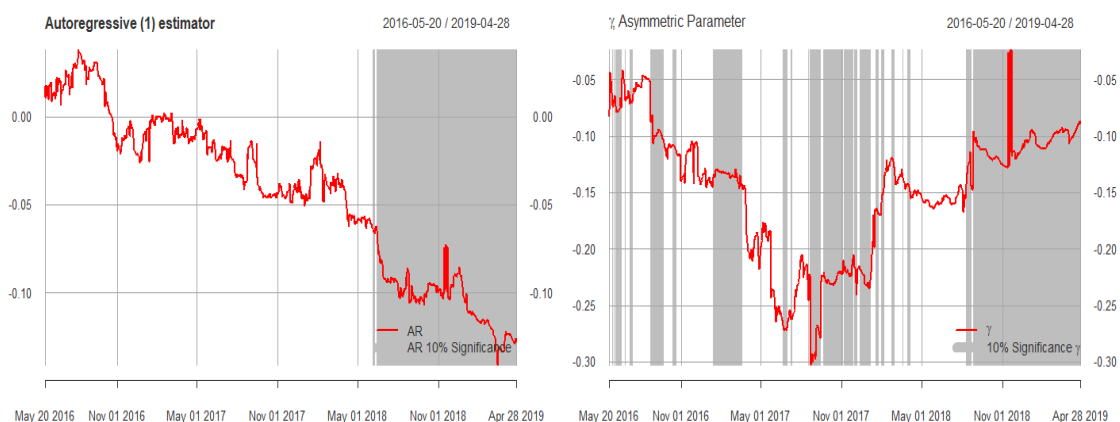


The AR estimator for Stellar starts with significant and negative coefficient at about -0.15, then when it is not significant anymore (with some exceptions), it quickly becomes positive. At the end of the series it goes to 0, but it finishes with an upward trend (approaching 0.05). Stellar has 276 (28%) of significant AR values. Most of them are concentrated on the first quarter of the observations.

The Gamma estimator series is irregular, but the asymmetry of the variance seems to be reducing in an upward trend of the series getting closer to 0. Moreover, there are 226 (23%) of significant gamma coefficients, most of them are in the first quarter of the series. All of them are negative and ranging between -0,02 and -0,89, which represents an enormous variation.

In this case, there is a clear relationship between inefficiency and asymmetry of volatility since the coefficients are significant in the initial period.

Figure 7. Evolution of the AR and Gamma estimations for Monero.

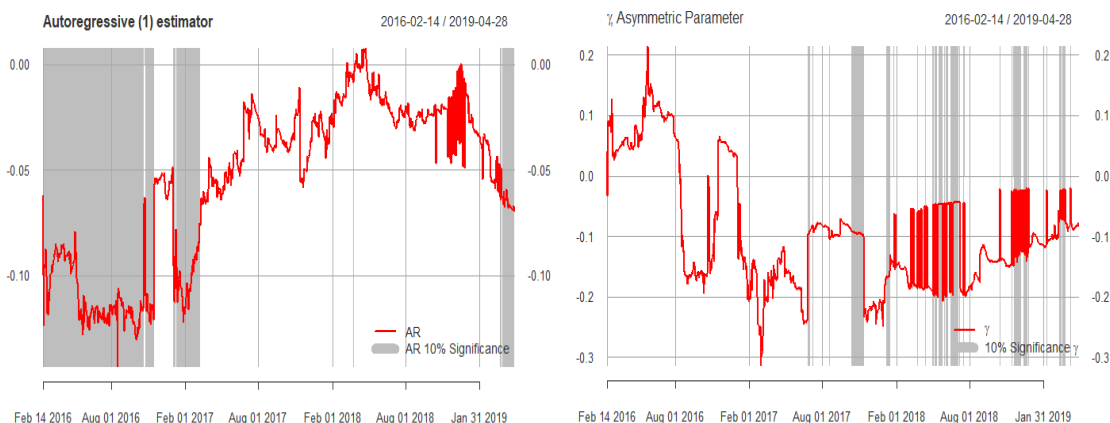


The trend of the AR estimator for Monero is pretty clear since it starts being positive and close to 0 and it has a downward trend finishing around -0.15. The lowest values are the significant ones, starting at about -0.06 (in the middle of 2018). Monero has 323 (30%) of significant AR values, all of them negative, at the end of the series and ranging from -0.06 to -0.14.

The Gamma estimator follows a u-shape trend having the lowest values between -0.05 and -0.1, with some estimators at the end going close to 0. Furthermore, there are 398 (37%) significant gamma coefficients, most of them are in the last quarter of the series. However, there are some significant estimators dispersed throughout the series. All of them are negative and ranging between -0.02 to -0.30.

Therefore, in this case there is a clear relationship between the inefficiency caused by uninformed traders and the asymmetric volatility. Nevertheless, there are some periods of asymmetric volatility which cannot be explained by inefficiency.

Figure 8. Evolution of the AR and Gamma estimations for Dash.

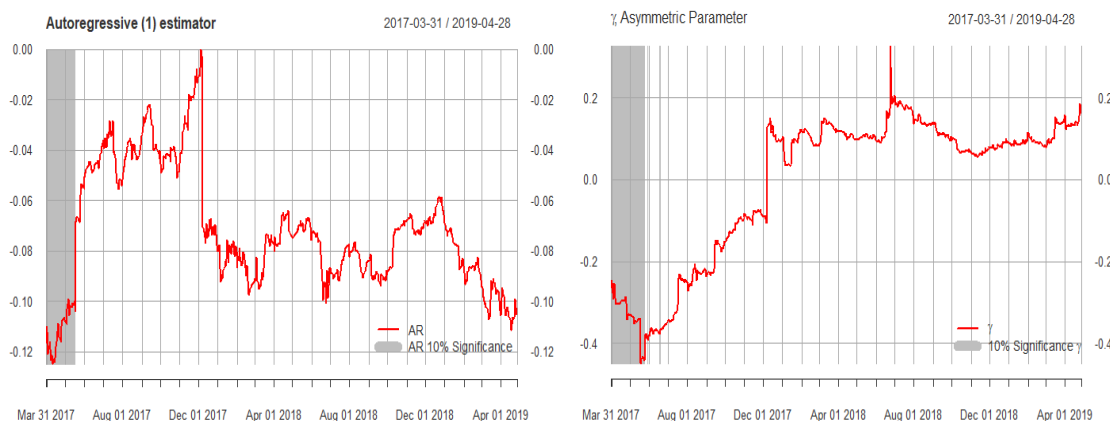


The AR estimator for Dash follows an upward trend until the middle of 2018 (the maximum values of these periods are positive), when it starts to decrease again. Dash has 362 (31%) of significant AR estimators. All of them negative and ranging from -0.01 to -0.14. They are mostly concentrated in the first quarter of the series.

Moreover, the Gamma estimator follows an irregular trend, decreasing in the first year and then going slowly upward. There are 128 (11%) significant Gamma coefficients, all of them negative and ranging from -0.02 to -0.17, mostly concentrated in the second half of the period.

In this case, the inefficiency appears in the initial period of the cryptocurrency, but the asymmetry is found in the second half of the period covered. Dash does not follow the initial hypothesis presented in this paper.

Figure 9. Evolution of the AR and Gamma estimations for NEM.

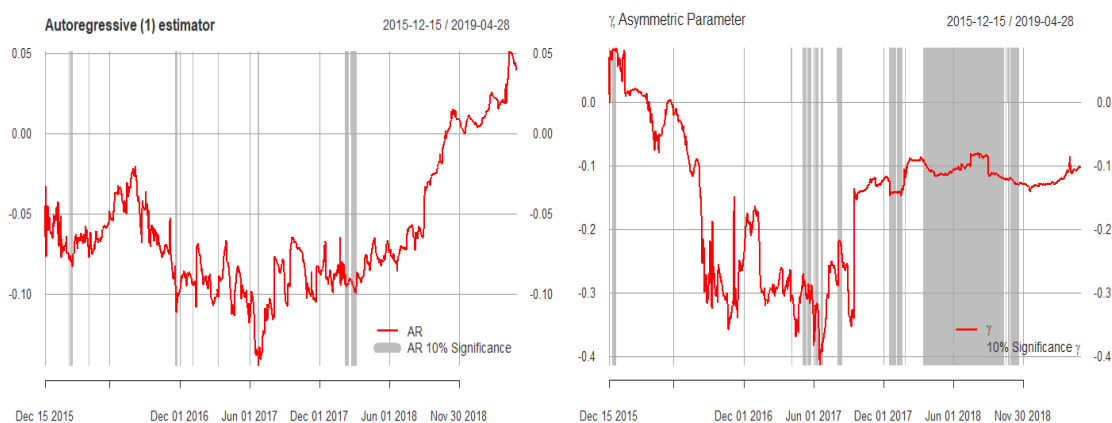


The NEM's AR coefficient quickly increases from -0.13, getting closer to 0 and then declining at the end of 2017 and stabilizing at around between -0.08 and -0.1. Almost all the lowest values of the series are significant. NEM has 48 (6%) of significant AR values, all of them negative and ranging between -0.11 and -0.13. They are all in the first months of the series.

The Gamma estimator follows an upward trend starting at around -0,03 and stabilizing between 0.01 and 0.02. Again, the lowest values are the significant ones. Moreover, there are 55 (7%) significant Gamma coefficients, all of them negative and ranging from -0.27 to -0.45.

Both estimators follow the same trend since all the significative values are at the beginning of the series and they both become not significant at about the same date (May 2017). Moreover, regarding all the series (not only the significant values), in December 2017 they both experience a change since AR estimator goes from 0 to -0.08 and the Gamma estimator change the sign from negative to positive.

Figure 10. Evolution of the AR and Gamma estimations for Dogecoin.

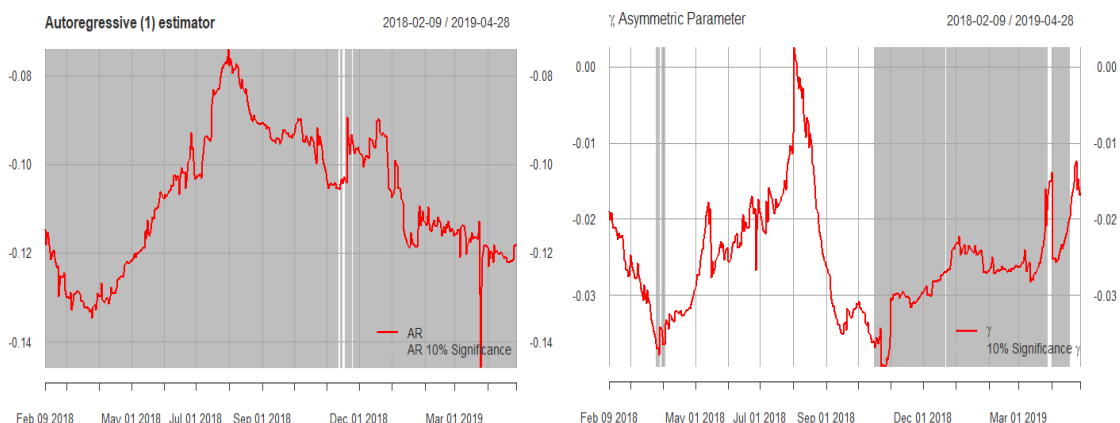


The Dogecoin AR estimator follows an almost linear trend at around -0.05 until the end of 2018, when it starts to increase, and it gets to 0.05. Dogecoin has 39 (3%) significant AR values, all of them negative and ranging between -0.08 and -0.14.

The Gamma estimator starts being positive (the significant positive values are found there) and it decreases until the middle of 2017 with a mean of -0.3, approximately, then, it stabilizes at around -0.01 until the end (most of the significant values are found in this period). Moreover, there are 322 (26,16%) significant Gamma coefficients, ranging from 0.08 to -0.39, dispersed around the time series but mostly concentrated in the second half of the observations. There are 9 significant gamma coefficients in December 2015, which does not seem to be remarkable.

In this case, there is not inefficiency but there is a period of asymmetric volatility at the end. So, Dogecoin does not follow the initial hypothesis of this paper.

Figure 11. Evolution of the AR and Gamma estimations for Decred.



The AR estimator for Decred follows an inverse u-shape series, growing from -0.13 to -0.08 until August 2018 when it starts to decrease to -0.12. Almost all the AR coefficients (97%) are significant. All of them are negative and ranging between -0.07 and -0.15

The Gamma estimator series decreases for the first three months, going from -0.02 to -0.04. Then, it increases until August 2018 becoming positive and close to 0 estimators, when it rapidly decreases in one month at about -0.03 again. Finally, it slowly grows to arrive to estimators close to -0.01 (all the significant values are in this last period). There are 172 (39%) significant Gamma coefficients, all of them negative and ranging between -0.02 and -0.04. The vast majority of the significant gamma coefficients are in the second half of the time series.

The first half of both series follow the same trend, both having in August 2018 its maximum. At this point, both series change its trend. At the end of October 2018, the Gamma estimators starts to have a regular positive trend (and it is the period when it is also significant). Therefore, asymmetric volatility is related to inefficiency at the second half of the series for Decred, but there is a period inefficiency in the first half that does not coincide with a period of asymmetric volatility.

To conclude, all the results seen in this section confirm that almost all cryptocurrencies have some periods of asymmetric volatility. Moreover, since Gamma it is negative in almost all the cases, positive shocks raise more volatility than negative ones. In addition, the temporal analysis also shows that this pattern is not constant throughout the period. These results confirm those found by previous authors analyzing specifically this topic. This means that it confirms the results that Baur & Dimpfl (2018)

find in the sense that all cryptocurrencies present signs of asymmetric volatility but it not significant in all periods of the cryptocurrencies. Moreover, it also confirms the results that Bouri, Azzi, et al. (2017) find because after the price crash of 2013, there are not significant signs of asymmetric volatility for Bitcoin.

Furthermore, in this paper a temporal analysis (with a sliding windows approach) for each estimator is done. This approach has shown that eight out of the ten cryptocurrencies covered in this paper present some signs of a relationship between the inefficiency and the asymmetric volatility. This fact confirms the idea that inefficiency caused by uninformed investors generate asymmetry in the volatility by the FOMO effect, which appears after positive shocks.

In addition, even though this relationship is found in eight out of the cryptocurrencies studied, it is not found in the same period of the cryptocurrency or it is not found in the same intensity in terms of number of significant estimators. Consequently, we have thought that it would be important to check the evolution of prices for all the cryptocurrencies in order to find the origin of this behavior. Following the initial hypothesis, we expect that the cryptocurrencies in which the relationship between inefficiency and asymmetric volatility appears later is because a sign of the price attracted the uninformed investors.

Consequently, depending on the relationship between inefficiency caused by uninformed investors and asymmetric volatility, four groups of cryptocurrencies are found:

- I. Low levels of inefficiency, symmetric volatility: Bitcoin and Ethereum have low levels of both inefficiency and asymmetric volatility. Bitcoin follows an irregular pattern, whereas Ethereum has an inefficiency period in the beginning, but it does not coincide significant asymmetry in the volatility. Consequently, it seems that the cryptocurrencies with highest market capitalization and liquidity are less inefficient and do not present consistent signs of asymmetric volatility during this period, which is consistent with the initial hypothesis presented. Moreover, it is also consistent with the results found by Baur & Dimpfl (2018) because they observe that Bitcoin and Ethereum have a different behavior.
- II. Inefficiency correlated with asymmetric volatility in the beginning of the series: Ripple, Stellar and NEM. They all present significant periods of inefficiency and asymmetry in the volatility in the beginning of the cryptocurrencies. This is consistent with the initial hypothesis since there is a relationship between

inefficiency (which could be caused by uninformed traders) and the asymmetric volatility.

If this relationship is in the beginning, a possible explanation is that the lack of liquidity of the beginning of the cryptocurrency and the shortage of information make all the investors less informed, and thus, they are guided by previous shocks (the FOMO effect after positive shocks). It could be also the case that uninformed investors enter these markets in the beginning since they see an investment opportunity to take advantage of the initial period of the cryptocurrency to make money, also because of the FOMO effect.

A specific case of this group is NEM, which has less than 10% significant observations for both estimators. What seem to happen for NEM is that even though it is the third cryptocurrency with the lowest market capitalization, it grew faster in the initial period of the cryptocurrency compared with the rest of the cryptocurrencies, whose development was slower.

- III. Inefficiency correlated with asymmetric volatility in the end of the series: Monero, Litecoin and Decred. In these cryptocurrencies there is a relationship between inefficiency and asymmetric volatility in the end of both series.

Monero and Litecoin present some significant periods of asymmetric volatility which do not coincide with periods of inefficiency, suggesting that there must be another potential explanation for this asymmetry.

The case for Decred is a bit different since it has this same relationship but almost all the period is inefficient. Consequently, in the second half of the series, inefficiency caused by uniformed investor could be causing asymmetry in the volatility in Decred. However, the first half of the series is inefficient but does not coincide with any period of asymmetry in the volatility.

This group mainly follows the initial hypothesis since the inefficiency is related to the asymmetric volatility. The explanation which it is found for these cryptocurrencies is that its prices grew to higher levels during the first half of 2017 (which is the period of the price bubble) than the cryptocurrencies from the second group. This fact can be checked in Figure 12. Moreover, Table 7 shows that the mean of the prices from December 2016 on for the cryptocurrencies. On the one hand, the mean of the prices for the cryptocurrencies from this group are 74.43 for Litecoin, 112.43 for Monero and 39.59 for Decred. On the other hand, the average of the prices for the cryptocurrencies from group (II) are 0.42 for Ripple, 0.15 for Stellar and 0.21 for NEM. Therefore, the main reason why the uninformed

investors chose mostly these cryptocurrencies from 2017 on could be that its price was more appealing than in the others because it was higher, and it was seen as an opportunity to make more money.

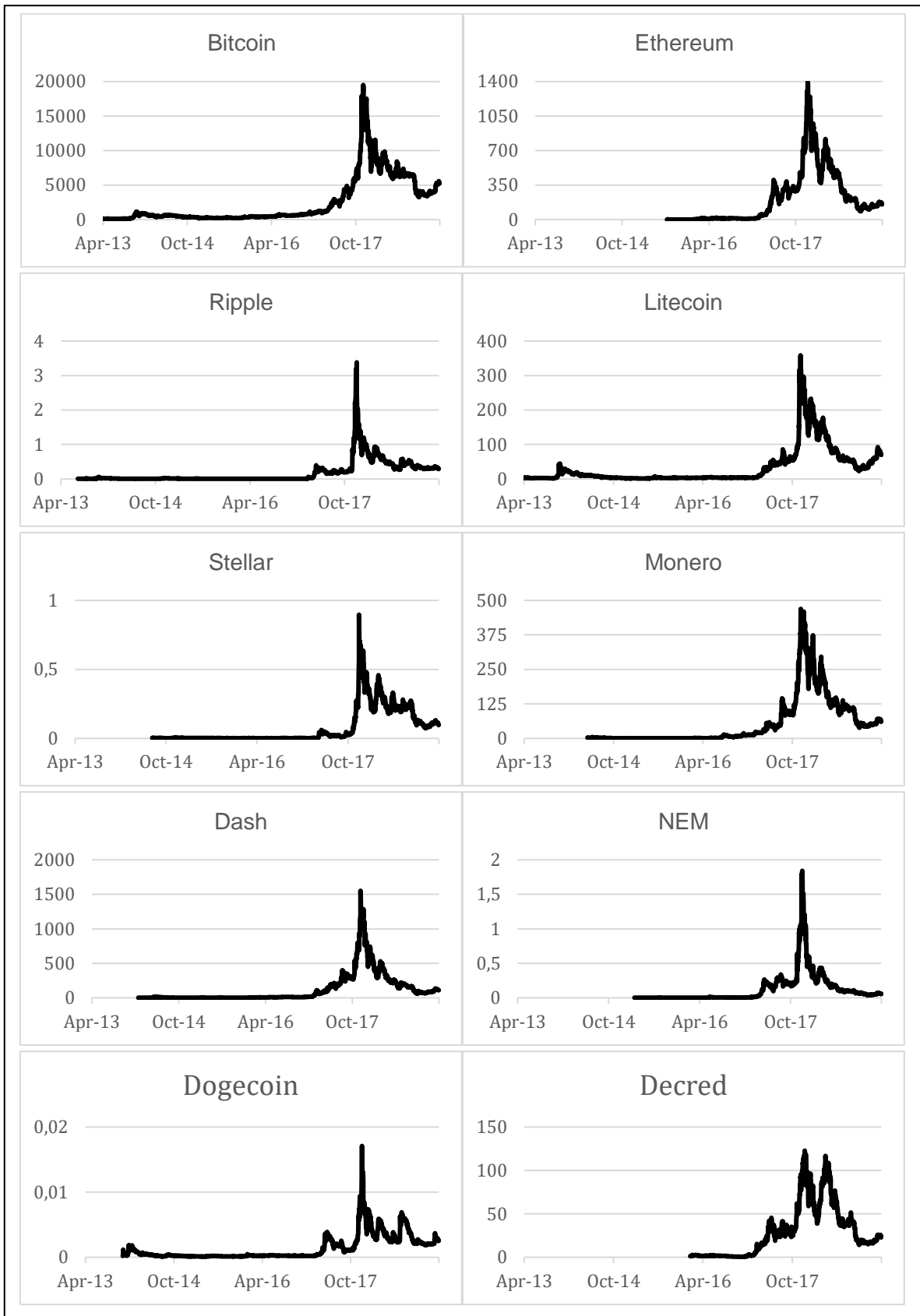
- IV. No relationship between inefficiency and asymmetric volatility: Dash and Dogecoin. On the one hand, Dash has an inefficiency period in the beginning and asymmetric volatility at the end, which does not follow a regular trend. On the other hand, Dogecoin has not a remarkable period of inefficiency, whereas a period of asymmetric volatility is found at the end. Therefore, this group is the only one which does not follow the initial hypothesis of the paper and it should be further studied why it is different.

Table 7. Summary of the descriptive statistics of the cryptocurrencies prices, divided in two subsamples.

N is the number of observations in the corresponding period and Sd the standard deviation.

	All period			Beginning to 31-Dec.-16			01-Jan.-17 to 28-Apr.-19		
	N	Mean	Sd	N	Mean	Sd	N	Mean	Sd
Bitcoin	2191	2408.2	3351.9	1343	419.71	214.36	848	5557.5	3574.6
Ethereum	1360	204.68	263.63	512	7.26	5.06	848	323.88	271.50
Ripple	2093	0.18	0.33	1245	0.01	0.01	848	0.42	0.42
Litecoin	2191	32.28	52.74	1343	5.67	5.98	848	74.43	65.06
Stellar	1727	0.08	0.13	879	0.002	0.001	848	0.15	0.15
Monero	1803	53.92	86.22	955	1.97	2.69	848	112.43	96.63
Dash	1899	120.97	214.36	1051	4.88	3.38	848	264.85	255.94
NEM	1488	0.12	0.23	640	0.002	0.002	848	0.21	0.26
Dogecoin	1960	0.001	0.002	1112	0.0003	0.0003	848	0.003	0.002
Decred	1173	29.01	29.74	325	1.40	0.52	848	39.59	28.62
Average		284.95	399.91		44.09	23.2		637.35	429.32

Figure 12. Price evolution of all the cryptocurrencies studied.



6. Conclusion

Cryptocurrencies have received a lot of attention from the media in the last years due to both its novelty and how its fast-growth prices. In this context of high volatility of prices, we analyze its asymmetric pattern. In previous papers (Bouri, Azzi, et al., 2017; Baur & Dimpfl, 2018) some evidence of asymmetric volatility in cryptocurrencies is found. This asymmetry is the opposite to the one found for the common assets since positive shocks raise more volatility than negative ones. So, the initial hypothesis of this paper is that there is a relationship between inefficiency, caused by uninformed investors moved by the Fear of Missing Out (FOMO) effect, and the asymmetric volatility. FOMO states that uninformed investors tend to react more after a positive shock than a negative one because they rely more on news of previous periods returns than a fundamental analysis of the asset. And this generates asymmetric volatility.

In this paper, a temporal analysis is carried out with a sliding windows approach. The main conclusion is that asymmetric volatility exists for cryptocurrencies, positive shocks raising more volatility than negative ones. This confirms the results found in previous papers. Furthermore, there is a relationship between inefficiency (caused by uninformed investors) and asymmetric volatility for eight of the ten cryptocurrencies studied. Nevertheless, neither it is constant throughout time nor does appear in the same period in all the cryptocurrencies. Leaving aside the two cryptocurrencies in which this relationship is not found, this leaves with three groups: if neither of both facts is found, if the relationship is found in the beginning of the series or if it is found at the end.

These various trends suggest some potential different explanations. In the biggest cryptocurrencies by market capitalization and liquidity (Bitcoin and Ethereum), there is not significant evidence of any of the two characteristics, which means that in the more developed financial markets, uninformed investors do not dominate. Furthermore, on the one hand, if the relationship is found in the beginning of the cryptocurrency it could mean that uninformed investors enter these markets to take advantage of the initial period (for the low levels of liquidity and the lack of information) of the cryptocurrency as an investment opportunity, guided by the FOMO effect. On the other hand, if the relationship appears at the end it means that uninformed investors entered the market for some reason. And it is found that in the three cryptocurrencies where the relationship appears in the end, the mean of its prices (above all from 2017 on) is substantially higher than the three cryptocurrencies in which the relationship is found in the beginning. This could imply that uninformed investors entered these markets because the price served as a claim of the opportunity, guided by the FOMO effect.

These results suggest that inefficiency caused by uninformed investors could be a remarkable part of the asymmetric volatility generated in the cryptocurrency market.

Regarding future research, it would be interesting to investigate whether the fact that the uninformed agents move through the FOMO would imply a certain sign of AR (theta) parameter. Furthermore, it could be useful to study more precisely the reasons that make both Dash and Dogecoin to follow a different trend than the other eight cryptocurrencies. In addition, it may be beneficial to analyze for more cryptocurrencies to better understand of all the concepts which are found in this paper. Lastly, it would be profitable to try different methods to deeply study the relationship between inefficiency and asymmetric volatility.

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Appendix

An example of a R script used to calculate the model for Bitcoin is the following:

- 1) Export the document in csv format and require the *rugarch* package.

```
setwd("C:/Users/MEDIA/Desktop/TFG/TFGDocumentos")
require(rugarch)
Bitcoin = read.csv("Bitcoin.csv", header = TRUE, sep = ";", dec = ",")
dataset = Bitcoin
```

- 2) Calculate the returns and the variables to compute the Amihud's illiquidity ratio, as defined in the paper.

```
bitcoin_price = dataset$Close
```

```

lbitcoin_price = log(bitcoin_price)
BTC_returns = diff(lbitcoin_price, lag=1)
volume = dataset$Volume
liquidityvolume = volume[2:2192]
illqBTC = (abs(BTC_returns))/(liquidityvolume)

```

3) Specificity the model.

```

spec1=ugarchspec(variance.model = list(model = "gjrGARCH", garchOrder = c(1, 1),
submodel = NULL, external.regressors = NULL, variance.targeting = FALSE),
mean.model = list(armaOrder = c(1,0), include.mean = TRUE, archm = FALSE,
archpow = 1, arfima = FALSE, external.regressors = NULL, archex = FALSE),
distribution.model = "norm")

```

4) Calculate some variables for the loop and create the matrices to keep the results.

```

T=length(BTC_returns)
M=730
B=T-M+1
result_coef=matrix(0,nrow=B,ncol=6)
result_stdev=matrix(0,nrow=B,ncol=6)
result_pvalue=matrix(0,nrow=B,ncol=6)
result_mcov=matrix(0,nrow=B,ncol=15)
result_loglik=matrix(0,nrow=B,ncol=1)
result_liquidity=matrix(0,nrow=B,ncol=1)

```

5) Applicate the *for loop* function to calculate the results of any of the variables of interest and print it in the correct matrix.

```

for (i in 1:B){
  BTCtest = BTC_returns[i:(M+i-1)]
  fit2 = ugarchfit(data=BTCtest,spec=spec1)
  result_coef[i,] = fit2 @fit$coef
  result_stdev[i,] = fit2 @fit$robust.se.coef
  result_pvalue[i,] = 2*(1-pnorm(abs(fit2 @fit$robust.tval)))
  result_mcov[i,] = fit2 @fit$robust.cvar[upper.tri(diag(1,6))]
  result_loglik[i,] = fit2 @fit$LLH
  result_liquidity[i,] = (sum(illqBTC[i:(M+i-1)]))/730
  print(c("i=",i))
}

```

6) Create of data frames of the results, naming the variables of the results and export them as csv files.

```

BTCcoef = as.data.frame (result_coef)
BTCsd = as.data.frame (result_stdev)

```

```

BTCpvalue = as.data.frame (result_pvalue)
BTCmcov = as.data.frame (result_mcov)
BTCloglik = as.data.frame (result_loglik)
BTCliquidity = as.data.frame (result_liquidity)

names(BTCcoef) = c("mu", "ar", "omega", "alpha", "beta", "gamma")
names(BTCsd) = c("mu", "ar", "omega", "alpha", "beta", "gamma")
names(BTCpvalue) = c("mu", "ar", "omega", "alpha", "beta", "gamma")

write.csv2(BTCcoef, "BTCcoef.csv")
write.csv2(BTCliquidity, "BTCliquidity.csv")
write.csv2(BTCsd, "BTCsd.csv")
write.csv2(BTCpvalue, "BTCpvalue.csv")
write.csv2(BTCloglik, "BTCloglik.csv")
write.csv2(BTCmcov, "BTCmcov.csv")

```

- 7) Use the function `xts` to facilitate the creation of the graph, adding the final date of the first and last rolling estimation.

```

require(xts)
seriests=xts(coefGamma,seq(from=as.Date("2015-08-04"), to=as.Date("2019-04-28"),
by="day"), frequency=52)

```

- 8) To select the non-significant observations at a 10%.

```

#Selecciono los casos no significativos al 10%
sel_sig=(result_pvalue[,6]>0.1)

```

- 9) The column is between -1 and 1.

```

shade <- cbind(upper = rep(1, dim(seriests)[1]), lower = rep(-1, dim(seriests)[1]))

```

- 10) In the cases in which the estimator is non-significant, which are in `sel_sig`, assign a 0.

```

shade[sel_sig,1]=0
shade[sel_sig,2]=0

```

- 11) Transform to an `xts` object to create the graph with the significant values shaded in grey.

```

shade <- xts(shade, index(seriests))

```

```
plot(seriests,type="l",col="red",main=c(expression(gamma)," , Asymmetric Parameter"))
addPolygon(shade, col = "gray", on = -1)
legendname1=expression(gamma)
legendname2=expression('10% Significant' ~ gamma)
addLegend("bottomright", on=1,
          legend.names = c(legendname1,legendname2),
          lty=c(1, 1), lwd=c(2, 10),
          col=c("red","grey"))
```

* From 7) it has been done again (with the necessary changes) to create also a graph for the autoregressive (1) estimator.