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Delegation and Information Sharing in Oligopoly

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Abstract

Information sharing in oligopoly has been analyzed by assuming that firms behave as a sole economic agent. In this paper I assume that ownership and management are separated. Managers are allowed to falsely report their costs to owners and rivals. Under such circumstances, if owners want to achieve information sharing they must use managerial contracts that implement truthful cost reporting by managers as a dominant strategy. I show that, contrary to the classical result, without the inclusion of message-dependent payments in managerial contracts there will be no information sharing. On the other hand, with the inclusion of such publicly observable payments and credible ex-ante commitment by owners not to modify these payments, there will be perfect information sharing without the need for third parties.

Keywords: Information sharing, Delegation, Managerial Contracts.
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1 Introduction

The paper investigates the motivation of oligopolists for sharing their private information when ownership and management are separated. Some of the classic results in the literature of information sharing are Gal-Or (1985, 1986), Sakai (1985), Shapiro (1986) and Vives (1984). These papers show that whether information sharing is advantageous depends on several factors: the nature of competition (Cournot or Bertrand), the type of information ("Common value" or "private value"), the type of uncertainty (about cost or demand) and the relationship between the products (substitutes or complements). For example, Gal-Or (1986) shows for the duopoly case with unknown private costs and firms which compete in quantities, that information sharing is a dominant strategy. More recently, Raith (1996) and Jin (2000) showed that the incentives for information sharing are determined by only two factors: (1) the profitability of informing rivals about their own profit function, which is determined by the slope of reaction curves, and (2) the profitability of informing rivals about one's own profit function, which is always positive. The sum of these two effects determines whether information is shared or not. On the other hand, Ziv (1993) showed that the results also depend crucially on the assumption that the private information revealed by a firm is verifiable. Otherwise, ex-post there may be an incentive problem for the firm to reveal the true information.

The models in the above literature assume that firms behave as a sole economic agent. However, if ownership and management are separated, Fershtman and Judd (1987), Sklivas (1987) and Vickers (1985) have shown that the behavior of firms may deviate from the predictions of the classical Cournot model. In particular, they show that firms behave more aggressively than under the Cournot model. In this paper I investigate whether the separation of ownership and management also affects the classic results on the incentive for sharing information in oligopoly.

I analyze subgame perfect Nash equilibria (SPNE further on) in a three-stage game that includes two firms with an owner and a manager in each. At stage one of the game, owners determine their manager's compensation scheme, at stage two each manager observes his production costs privately and sends a publicly observable message, and at stage three managers compete in quantities. The game is solved by backward induction. As in Fershtman and Judd (1987), the managerial compensation scheme is assumed to be a linear combination of profits and sales. Following Ziv (1993), I analyze as benchmarks the cases of no information sharing and perfect information sharing. I prove in Proposition 2 that the classic result from Gal-Or (1986) that firms which have private information about their production costs in a Cournot game prefer information sharing to no information sharing generalizes to the case in which ownership and management are separated. Then, again following Ziv (1993), I analyze the incentives for information sharing when cost messages are unverifiable. Here I distinguish two cases: message independent payment contracts and payment contracts that are allowed to depend on a manager's cost message. In the first case (Proposition 3), the implementation of a truth-telling mechanism implies that managers are paid according to sales only and there will be no information transfer. In the second case (Proposition 4) owners can implement truth-telling without any costs and there will be perfect information sharing.

The paper shows that with the separation of ownership and management the truth-telling problem originally stated by Ziv (1993) can be solved in a simple way. In Ziv (1993), the truth-telling problem is solved by the introduction of a message cost. This cost has the undesirable property that the implementation of truth-telling may require "burning money", which reduces the advantage of information sharing, or may require illegal transfers between rival firms. In my model, information sharing is profitable if owners can credibly commit themselves to remunerate their managers with message-dependent payments. However, if they cannot commit to such contracts there will be no information sharing. Thus, the main conclusion from the paper is that the possibility of information sharing in an industry with firms in which ownership and management is separated depends on whether owners can credibly implement message-dependent payment schemes.

The paper is organized as follows. Section 2 sets out the model and discusses its assumptions. Section 3 analyzes as benchmarks the cases of no information sharing and truthful information sharing with verifiable messages. Section 4 analyzes truth-telling equilibria when messages are unverifiable and managers may make false cost reports. Finally, Section 5 draws conclusions.

2 The model

2.1 The framework

Consider a related version of the Fershtman and Judd (1987) and Ziv (1993) model. A duopoly faces the following inverse demand function

$$p = a - q_1 - q_2,$$

where p is the price and q_i, $i = 1, 2$, is the output produced by firm i. Each firm includes an owner and a manager and has constant cost per unit of production $c_i \in [\underline{c}, \overline{c}]$. I assume that c_i, $i = 1, 2$, are i.i.d, and that $f(c_i) > 0$, $\forall c_i \in [\underline{c}, \overline{c}]$, where $f(c_i)$ is the density function. Furthermore, to simplify the exposition let $E(c_i) = 1$ and denote $Var(c_i) = \sigma^2$.[1] All this is public information. I will analyze the following delegation game:

[1] Concerning a and $E(c_i)$, my results will depend only on their quotient. Therefore assuming $E(c_i) = 1$ is equivalent to a rescaling of a.

Stage 1. The owner of each firm decides (simultaneously) the incentive structure of managerial wage contracts. These wage contracts are of the form

$$w_i = A_i + B_i O_i \qquad \text{with} \qquad O_i = \alpha_i \pi_i + (1-\alpha_i) S_i = (a - q_i - q_j - \alpha c_i)\, q_i,$$

where $\pi_i = (p - c_i)\, q_i$ and $S_i = pq_i$ are firm i's profit and sales function, respectively. We assume that managers are risk neutral, that α_i, A_i and B_i are constants, where $\alpha_i \geq 0$ and $B_i \geq 0$. In other words, at stage one owners 1 and 2 select simultaneously α_1, A_1 and B_1, and α_2, A_2 and B_2, respectively, to maximize $\Pi_i = \pi_i - w_i$ under the constraint that manager i expects to earn at least his reservation wage $\underline{w}$.

Stage 2. The managers observe (simultaneously) their firm's production costs. After observing the signal, managers can send a publicly observable message $m_i \in [\underline{c}, \overline{c}]$ about the signal they received (which can be different from the true one) or send no message.

Stage 3. Every manager decides his output (simultaneously).

I make the following assumptions:

Assumption 1. i) $a > 2\overline{c} - \underline{c}$, ii) $a > 3$, iii) $a > 1 + 2\sigma^2$.

The first two assumptions are standard and guarantee that $q_i > 0\ \forall c_1, c_2$. The third assumption is a restriction on the distribution of the unit cost of production and guarantees that in equilibrium $\alpha_i < 1$. The assumption is satisfied, for example, if we assume that costs are symmetrically distributed about their mean and that realizations close to the mean are more likely than realizations at the border.[2]

2.2 Comments

As Fershtman and Judd (1987) pointed out, the assumption concerning the managerial incentive scheme indirectly implies that owners can observe only profit and sales figures but not the number of units sold. The linearity assumption is somehow restrictive but should not influence the results qualitatively. Also, the restriction that a manager's compensation scheme does not directly depend on the competitors' sales or profits may be justified by assuming that the owner's information about their own profits and sales is much better than their information about their competitors' or that legal restrictions impede its inclusion. The assumption that α_i and B_i are non-negative can be justified by the legality problem of managerial penalties of profit

[2] If realizations close to the mean are more likely, an upper limit for σ^2 is the variance of the uniform distribution: $\sigma^2 \leq \frac{(\overline{c}-\underline{c})^2}{12}$. If the distribution is symmetric and if 1i) holds, $\frac{(\overline{c}-\underline{c})^2}{12} < \frac{(a-1)}{6} < \frac{(a-1)}{2}$. Thus, $\sigma^2 < \frac{(a-1)}{2}$.

or sales increases. Furthermore, it should be noted that the parameters of the managerial wage contract, A_i, B_i and α_i can be interpreted in two ways. On the one hand, it can be assumed that these values are determined in the model *before* information about messages and quantities becomes available. Thus, a low value of α_i for example, can be interpreted as an ex-ante firm-specific investment in sales and advertising activities, which implies that managers must be compensated this way to solve generalized principal-agent problems. On the other hand, A_i, B_i and α_i may be functions of observable information revealed at later stages of the game, like the messages sent by firm i's manager at stage 2. In that case we interpret the managerial compensation scheme in a narrow sense like a contract.

In any case, the choice of A_i, B_i and α_i is a device for the owner to precommit to a behavior that is otherwise not credible for its rival. This allows to solve the truthful revelation problem in a simple way. As already noted by Ziv (1993), former models "ignore the problem of revealing truthful information to competitors by assuming the existence of some exogenous player who transforms the correct information (Novshek and Sonnenschein (1982)) or by assuming that data is verifiable (Gal-Or (1986))." Ziv (1993) himself endogenizes the incentives for truthful information sharing. However, the mechanisms he proposes have their own problems. Signalling by "burning money" may imply higher costs than benefits from information sharing. For example, under the assumption of my model without delegation (or $\alpha_i = 1$) he shows that information sharing is never beneficial for firms with such a mechanism. The alternative of using direct payments from each firm to its competitor may avoid this problem but, as Ziv himself mentions, the "legality of such transactions is questionable" because they may be seen as a device to force collusion. Therefore, choosing a certain managerial compensation scheme (observable to rivals) is a device for solving the truthful revelation problem in a simple way without including third parties.

3 Two benchmarks

3.1 No information sharing

As a first benchmark consider the case of no information sharing ($m_i = \{\emptyset\}$). At the *third stage* managers compete à la Cournot taking into account their rivals' expected production costs. Because managers are risk neutral, they act to maximize O_i and the values of A_i and B_i are irrelevant for their choice of q_i. Manager i's maximization problem is

$$\max_{q_i} E_{c_j} O_i(q_i, q_j) = (a - q_i - E_{c_j} q_j - \alpha_i c_i) q_i, \tag{1}$$

where $E_{c_j} q_j = \int_{\underline{c}}^{\overline{c}} q_j(\alpha_i, \alpha_j, c_i, c_j) f_j(c_j) dc_j$. The SPNE values at stage 3 are given by:[3]

$$q_{i(n)}(\alpha_i, \alpha_j) = \frac{1}{6}\left(2a - \alpha_i(1 + 3c_i) + 2\alpha_j\right). \tag{2}$$

At *stage 2* manager i observes his production costs but sends no message. Finally, *at stage 1*, owner i's maximization problem is

$$\begin{aligned} \max_{\alpha_i, A_i, B_i} E_{c_i,c_j}\Pi_{i(n)}(\alpha_i, \alpha_j) \quad &= \quad \int_{\underline{c}}^{\overline{c}} \int_{\underline{c}}^{\overline{c}} \Big[\Big(a - q_{i(n)}(\alpha_i, \alpha_j) - q_{j(n)}(\alpha_i, \alpha_j) - c_i\Big) q_{i(n)}(\alpha_i, \alpha_j) \\ &\qquad -w_{i(n)}(\alpha_i, \alpha_j)\Big] f_i(c_i) f_j(c_j) dc_i dc_j, \qquad (3) \\ s.t. \qquad &\int_{\underline{c}}^{\overline{c}} \int_{\underline{c}}^{\overline{c}} w_{i(n)}(\alpha_i, \alpha_j) f_i(c_i) f_j(c_j) dc_i dc_j \geq \underline{w}. \qquad (4) \end{aligned}$$

Notice that A_i is chosen such that (4) is binding. Consequently, owner i's objective function is $E_{c_i,c_j}\Pi_{i(n)}(\alpha_i, \alpha_j) = E_{c_i,c_j}\pi_{i(n)}(\alpha_i, \alpha_j) - \underline{w}$. Furthermore, including the third stage equilibrium values the owners' maximization problem is equivalent to solve:

$$\max_{\alpha_i} E_{c_i,c_j}\pi_{i(n)}(\alpha_i, \alpha_j) = \frac{1}{9}\left(a + \alpha_j - 2\alpha_i\right)\left(a + \alpha_j + \alpha_i - 3\right) - \frac{1}{4}\alpha_i\sigma^2\left(\alpha_i - 2\right). \tag{5}$$

The first-order conditions of the Nash equilibrium at stage 1 are:

$$\alpha_i = 1 - 2\frac{a + \alpha_j - 2}{8 + 9\sigma^2}. \tag{6}$$

This allows us to establish the following result.[4]

Lemma 1. *The SPNE values in the case of no information sharing are given by*

$$\alpha_{i(n)} = \begin{cases} 1 - 2\frac{a-1}{10+9\sigma^2} & \textit{for} \quad a < 6 + \frac{9}{2}\sigma^2 \\ 0 & \textit{for} \quad a \geq 6 + \frac{9}{2}\sigma^2 \end{cases}, \textit{and} \qquad q_{i(n)} = \frac{1}{3}a - \frac{1}{6}\left(3c_i - 1\right)\alpha_{i(n)}.$$

The owners' expected profits are

$$E_{c_i,c_j}\pi_{i(n)} = \frac{1}{9}a\left(a - 3\right) + \frac{1}{18}\alpha_{i(n)}\left(6 + 2a + 9\sigma^2\right) - \frac{1}{36}\alpha_{i(n)}^2\left(8 + 9\sigma^2\right).$$

[3]Hereafter I shall use the subscript n to denote the case of *no* information sharing.

[4]To simplify the notation I denote $E_{c_i,c_j}\pi_{i(n)}(\alpha_i, \alpha_j) = E_{c_i,c_j}\pi_i\left(q_{i(n)}(\alpha_i, \alpha_j), q_{j(n)}(\alpha_j, \alpha_i)\right)$, and $E_{c_i,c_j}\pi_{i(n)} = E_{c_i,c_j}\pi_i\left(q_{i(n)}(\alpha_{i(n)}, \alpha_{j(n)}), q_{j(n)}(\alpha_{j(n)}, \alpha_{i(n)})\right)$.

Owners will increase the incentives for profits if a decreases and if σ rises. Thus, in large markets with low uncertainty about costs, firms will be more aggressive sellers. Notice that we can analyze the case without delegation (or when delegation is not essential because the objectives of owners and managers coincide) by imposing the restriction $\alpha_i = 1$ at the first stage. This implies that quantities at the third stage of the game are chosen to maximize profits. So, if there is no delegation the owner's expected profit is $E_{c_i,c_j}\pi_{i(n)}(1,1) = \frac{1}{9}(a-1)^2 + \frac{1}{4}\sigma^2$. Also, $E_{c_i,c_j}\pi_{i(n)}(1,1) - E_{c_i,c_j}\pi_{i(n)} > 0$.[5] The owners of both firms prefer managers to maximize profits. However, if owner 1 sets $\alpha_1 = 1$, owner 2 increases his profits by choosing $\alpha_2 < 1$. Therefore, as shown by Fershtman and Judd (1987), in equilibrium each owner sets $\alpha_i < 1$.

3.2 Truthful information sharing

Now consider the case of truthful information sharing with verifiable messages ($m_i = c_i$). At the *third stage* managers compete à la Cournot and know their rivals' production costs. Manager i's maximization problem is

$$\max_{q_i} O_i(q_i, q_j) = (a - q_i - q_j - \alpha_i c_i)q_i. \tag{7}$$

The SPNE values at stage 3 are given by:[6]

$$q_{i(v)}(\alpha_i, \alpha_j) = \frac{1}{3}a + \frac{1}{3}\alpha_j c_j - \frac{2}{3}\alpha_i c_i. \tag{8}$$

At *stage 2* the manager observes his production costs and sends a publicly observable and verifiable message. At *stage 1*, the owner of firm i solves

$$\max_{\alpha_i} E_{c_i,c_j}\pi_{i(v)}(\alpha_i, \alpha_j) = \frac{1}{9}(a - 3 + \alpha_j + \alpha_i)(a + \alpha_j - 2\alpha_i) - \frac{\sigma^2}{9}\left(-6\alpha_i - \alpha_j^2 + 2\alpha_i^2\right). \tag{9}$$

The first-order conditions of the Nash equilibrium at stage 1 are:

$$\alpha_i = -\frac{1}{4}\frac{a + \alpha_j - 6 - 6\sigma^2}{1 + \sigma^2}. \tag{10}$$

Lemma 2. *The SPNE values in the case of truthful information sharing with verifiable messages are given by*

$$\alpha_{i(v)} = \begin{cases} 1 - \frac{a-1-2\sigma^2}{4\sigma^2+5} & \textit{for} \quad a < 6(\sigma^2+1) \\ 0 & \textit{for} \quad a \geq 6(\sigma^2+1) \end{cases} \textit{, and} \qquad q_{i(v)} = \frac{1}{3}a + \left(\frac{1}{3}c_j - \frac{2}{3}c_i\right)\alpha_{i(v)}.$$

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[5]It can be checked easily that $E_{c_i,c_j}\pi_{i(n)}(1,1) - E_{c_i,c_j}\pi_{i(n)} = \begin{cases} \frac{1}{9}\left(28 + 27\sigma^2\right)\frac{(a-1)^2}{(10+9\sigma^2)^2} & \text{for} \quad a < 6 + \frac{9}{2}\sigma^2 \\ \frac{1}{9}(a+1) + \frac{\sigma^2}{4} & \text{for} \quad a \geq 6 + \frac{9}{2}\sigma^2 \end{cases}.$

[6]Hereafter the case of information sharing with *v*erifiable information is denoted by v.

The owners' expected profits are

$$E_{c_i,c_j}\pi_{i(v)} = \frac{1}{9}a\,(a-3) + \frac{1}{9}\alpha_{i(v)}\left(3+a+6\sigma^2\right) - \frac{1}{9}\alpha_{i(v)}^2\left(2+\sigma^2\right).$$

Notice that by Assumption 1iii) $\alpha_{i(v)} < 1$. Lemma 2 is the result of Theorem 3 in Fershtman and Judd (1987). Again, owners will increase the incentives for profits if a decreases and if σ rises. That is, in large markets with low uncertainty about costs, firms will be more aggressive sellers. In the case of information sharing without delegation, the owner's profit is $E_{c_i,c_j}\pi_{i(v)}(1,1) = \frac{1}{9}(a-1)^2 + \frac{5}{9}\sigma^2$. As with the case of no information sharing, we get $E_{c_i,c_j}\pi_{i(v)}(1,1) - E_{c_i,c_j}\pi_{i(v)} > 0$ under Assumption 1iii).

3.3 Benefits from information sharing

Suppose owners do not delegate the quantity choice to managers. A comparison of the owners' profits under information sharing and no information sharing then yields the result of Gal-Or (1986):

Proposition 1. *For any nondegenerate distribution of c_1 and c_2, without quantity delegation, owners strictly prefer truthful information transfer to no information sharing.*

Proof. $E_{c_i,c_j}\pi_{i(v)}(1,1) - E_{c_i,c_j}\pi_{i(n)}(1,1) = \frac{11}{36}\sigma^2 > 0.$ *q.e.d.*

As pointed out by Raith (1996) who analyzed the incentives for information sharing in oligopoly in a general model,[7] the incentives for revealing information are determined by two separate effects: "(1) Letting the rivals acquire a better knowledge of their respective profit functions leads to a higher correlation of strategies, the profitability of which is determined by the slope of the reaction curves. (2) Letting the rivals acquire a better knowledge of one's own profit function is *always* profitable." In our model both effects are positive.[8] Therefore, information sharing unambiguously is profitable for both firms. We show that the result remains valid when the quantity choice decision in each firm is delegated to a manager.

Proposition 2. *For any nondegenerate distribution of c_1 and c_2 owners prefer truthful transfer to no information sharing. Furthermore, $\alpha_{i(v)} \geq \alpha_{i(n)}$.*

[7] The model is general in the sense that it allows different information structures (common value, independent value and perfect signals), different forms of competition (Cournot and Bertrand), different kinds of uncertainty (about demand or costs), the degree of information revelation (complete, partial, none at all), and that it imposes no (fewer) restrictions on the number of firms (the demand function).

[8] In fact Ziv (1993), who did not impose the symmetry assumption on the distribution of costs, showed that the first effect is $\frac{4}{36}Var(c_j)$ and that the second effect is $\frac{7}{36}Var(c_i)$.

Proof. First, we prove the last statement. For $a < 6+\frac{9}{2}\sigma^2$ we have $\alpha_{i(v)}-\alpha_{i(n)} = \sigma^2\frac{-a+21+18\sigma^2}{(10+9\sigma^2)(4\sigma^2+5)} > 0$. For $6+\frac{9}{2}\sigma^2 \leq a < 6+6\sigma^2$ we have $\alpha_{i(v)} - \alpha_{i(n)} = \left(1-\frac{a-1-2\sigma^2}{4\sigma^2+5}\right) > 0$. Finally, for $a \geq 6+6\sigma^2$ we get $\alpha_{i(v)} - \alpha_{i(n)} = 0$. Now consider the following decomposition of the profits from information sharing:

$$\begin{aligned} E_{c_i,c_j}\pi_{i(v)} - E_{c_i,c_j}\pi_{i(n)} &= \left[E_{c_i,c_j}\pi_{i(v)} - E_{c_i,c_j}\pi_{i(n)}(\alpha_{i(v)},\alpha_{j(v)})\right] \\ &\quad + \left[E_{c_i,c_j}\pi_{i(n)}(\alpha_{i(v)},\alpha_{j(v)}) - E_{c_i,c_j}\pi_{i(n)}\right] \\ &= \left[\frac{1}{36}\alpha_{i(v)}\sigma^2\left(6+5\alpha_{i(v)}\right)\right] \\ &\quad + \left[\frac{1}{36}\left(\alpha_{i(v)}-\alpha_{i(n)}\right)\left(2\left(2a+6+9\sigma^2\right)-\left(8+9\sigma^2\right)\left(\alpha_{i(n)}+\alpha_{i(v)}\right)\right)\right] \\ &\geq 0, \end{aligned} \tag{11}$$

because $\alpha_{i(v)} \geq \alpha_{i(n)}$ and $0 \leq \alpha_i \leq 1$ imply that both terms are non-negative. *q.e.d.*

In expression (11) we can distinguish two terms (in square brackets). The first term measures the benefits from information sharing for any given value of α_i. This term will always be positive and reaches its maximum $(\frac{11}{36}\sigma^2)$ in the case where managers are compensated in function of profits only. The second term measures the gains from the increase in the profit share in the manager's compensation scheme due to information sharing. This term will also be non-negative. Thus within our model of delegation we can identify a third effect of information sharing: (3) Letting one's rival owners acquire better knowledge of their respective profit functions increases the profit share in one's own manager's compensation scheme, which is always profitable. Furthermore, with delegation the size of effects (1) and (2) increases in the profit share of the manager's incentive scheme.

4 Truth-telling equilibria

We now consider the case of information-sharing with unrestricted messages. Let $g_i(m_i) = \int_{\underline{c}}^{\overline{c}} c_i f_i(c_i|m_i)dc_i$ be the conditional expected cost of firm i given the message it sends, calculated by firm j. At *stage 3,* manager i's maximization problem is given by

$$\max_{q_i} E_{c_j}\left[O_i(q_i,q_j)|m_i,m_j\right] = \left(a - q_i - E_{c_j}\left(q_j|m_i,m_j\right) - \alpha_i c_i\right) q_i. \tag{12}$$

Reaction functions are

$$q_i = \frac{1}{2}\left(a - E_{c_j}\left(q_j|m_i,m_j\right) - \alpha_i c_i\right). \tag{13}$$

Taking expectations we get[9]

$$E_{c_i}\left(q_i|m_i\right)=\frac{1}{2}\Big(a-E_{c_j}\left(q_j|m_j\right)-\alpha_i g_i\left(m_i\right)\Big). \tag{14}$$

Solving for equilibrium expected quantities in (14) and substituting in (13) yields the SPNE values at stage 3:[10]

$$q_{i(u)}\left(\alpha_i,\alpha_j,m_i,m_j\right)=\frac{1}{6}\left(2a-\alpha_i\left(g_i\left(m_i\right)+3c_i\right)+2\alpha_j g_j(m_j)\right). \tag{15}$$

At *stage 2* managers observe their production costs and send a message $m_i\in[\underline{c},\overline{c}]$ to the manager of their respective rival firm (observable also to the owners of both firms). Manager i chooses m_i to maximize his expected wage function

$$E_{c_j}w_{i(u)}\left(m_i,m_j\right)=A_i+\frac{1}{36}B_i\left(2a-\alpha_i\left(g_i(m_i)+3c_i\right)+2\alpha_j g_j(m_j)\right)^2. \tag{16}$$

Since messages are not restricted on truthful messages, firm j's manager will believe the message of firm i's manager only if it maximizes the wage function. Furthermore, because at stage 2 messages are sent simultaneously, firm i's manager has not observed the message of firm j's manager. So $g_j=E(c_j)=1$. To find the incentive mechanism that implements truth-telling as a dominant strategy, suppose that the manager of firm j believes the message of firm i to be true, i.e. $g_i(m_i)=m_i$. Suppose, A_i, B_i and α_i are constants. Then the solution to manager i's maximization problem is

$$m_{i(u)}=\arg\max_{m_i\in\{\underline{c},\overline{c}\}}\left\{A_i+\frac{1}{36}B_i\left(2a-\alpha_i\left(m_i+3c_i\right)+2\alpha_j\right)^2\right\}=\underline{c}\quad for\quad \alpha_i>0. \tag{17}$$

In general, $c_i\neq\underline{c}$, and $m_{i(u)}\neq c_i$. Therefore truth-telling cannot be implemented as a dominant strategy for the manager of firm i if $\alpha_i>0$. For $\alpha_i=0$ the manager is indifferent between sending a true message or any other message. However, if lying implies some additional cost, such as a loss of reputation for example, we can assume that the manager will send the true message.

Lemma 3. *If A_i, B_i and α_i are constants, for any $\alpha_i>0$ truth-telling cannot be implemented as a dominant strategy. The SPNE values in the case where truth-telling is a weakly dominant strategy are given by*

$$\alpha_{i(u)}=0\quad and\quad q_{i(u)}=\frac{a}{3}.$$

[9]Notice that $E_{c_j}(q_j\mid m_i,m_j)=E_{c_j}(q_j\mid m_j)$ and $E_{c_i}(c_i\mid m_i,m_j)=g_i$ because firms' messages and costs are independent. Thus, $E_{c_i}\left[E_{c_j}(q_j\mid m_i,m_j)\mid m_i,m_j\right]=E_{c_i}\left[E_{c_j}(q_j\mid m_j)\mid m_i\right]=E_{c_j}(q_j\mid m_j)$.

[10]Hereafter the case of information sharing with *u*nverifiable information and implementation of truthful information sharing is denoted by u.

The owners' expected profits are

$$E_{c_i,c_j}\pi_{i(u)} = \frac{1}{9}a\,(a-3)\,.$$

The managers of a firm will believe the message of the rival firm's manager only if the owners of the rival firm choose $\alpha_{i(u)} = 0$. Otherwise, the manager just ignores the message and behaves as if the rival firm had sent no message at all. A comparison of the owners' expected profits under truthful information sharing with non-verifiable messages and no information sharing yields the following result.

Proposition 3. *For any nondegenerate distribution of c_1 and c_2 owners prefer no information sharing to truthful information sharing.*

Proof.

$$E_{c_i,c_j}\pi_{i(u)} - E_{c_i,c_j}\pi_{i(n)} = \begin{cases} \frac{1}{36}\left(-12-9\sigma^2+2a\right)\frac{24+108\sigma^2+56a+54a\sigma^2+81\sigma^4}{\left(10+9\sigma^2\right)^2} < 0 & \text{for} \quad a < 6+\frac{9}{2}\sigma^2 \\ 0 & \text{for} \quad a \geq 6+\frac{9}{2}\sigma^2. \end{cases}$$

q.e.d.

If the managerial wage contract cannot depend on messages, the only way to implement truth-telling is to use pure sales contracts. However, with pure sales contracts there is no gain for owners from information sharing, but just a loss due to lower profit share in the managerial wage contract. Therefore, owners are worse off under information sharing in this case.

Now suppose we allow for wages to depend directly on messages. First, notice that owners cannot be better off when messages are unverifiable than when messages are verifiable because the former implies a further restriction in the owners' maximization problem. Suppose first that only parameter A_i depends on the manager's cost message, while B_i and α_i are independent of messages. Now, firm j's manager believes the message of firm i's manager to be true iff

$$m_{i(u)} = \arg\max_{m_i \in \{\underline{c},\overline{c}\}} \left\{ A_i(m_i) + \frac{1}{36}B_i\left(2a - \alpha_i\left(m_i + 3c_i\right) + 2\alpha_j\right)^2 \right\}. \tag{18}$$

The first-order constraint is

$$A_i'(m_{i(u)}) + \frac{2}{36}B_i\left(2a - \alpha_i\left(m_{i(u)} + 3c_i\right) + 2\alpha_j\right)(-\alpha_i) = 0. \tag{19}$$

Owner i can implement truth-telling (i.e. $m_{i(u)} = c_i$) at *stage 2* by choosing

$$A_{i(u)}(m_i) = \frac{1}{9}\alpha_i B_i\left(\left(a+\alpha_j\right)m_i - \alpha_i m_i^2\right) + k_i, \tag{20}$$

where k_i is chosen such that the manager's participation constraint is fulfilled.[11] Including first and second stage equilibrium values, owner i's maximization problem at *stage 1* can be written as

$$\max_{\alpha_i} E_{c_i,c_j}\pi_{i(u)}(\alpha_i,\alpha_j)=\frac{1}{9}\left(a-3+\alpha_j+\alpha_i\right)\left(a+\alpha_j-2\alpha_i\right)-\frac{\sigma^2}{9}\left(-6\alpha_i-\alpha_j^2+2\alpha_i^2\right). \quad (21)$$

This problem coincides with that when messages are verifiable. Therefore, we immediately get the following proposition.

Proposition 4. *If managers' wage functions can directly depend on cost messages, owners can implement a wage scheme that induces truthful cost revelation and yields higher benefits under information sharing than under no information sharing.*

Notice that this result implies that it is not necessary to analyze the optimality of other message schemes that are based on message-dependent B_i or α_i, or combinations of the two. The best owners can achieve is what they achieve in the verifiable information case. Modifying the managerial payment scheme by including message-dependent payments is a simple way to solve the truth-telling problem pointed out by Ziv (1993). On the one hand, it is not necessary to "burn money", which may imply that information sharing is too costly. On the other hand, it is not necessary to use direct payments between rivals, which may be illegal. However, in our model the result that owners can implement message-dependent payment schemes without any cost relies on our assumption that managers are risk neutral. With risk-averse managers, the possibility of modifying A_i may be limited and it may be necessary to modify other components in the managerial wage scheme, e.g. to reduce α_i, which in turn reduces owners' profits. There will then be a trade-off between gains from information sharing and losses due to an increase in the sales component in managerial wage schemes.

Taken together, Propositions 3 and 4 seem to indicate that whether the managerial payment scheme can depend on cost messages determines the possibility of information sharing. However, what is really important is not whether message-dependent payments between owners and managers can be included but that *all* such payments are observable to rivals and that owners can credibly commit *ex-ante* not to modify these payments ex-post. Proposition 3 shows that if this is not the case, there will be no information sharing, and Proposition 4 shows, that if this is the case, there will be perfect information sharing.

[11]This follows immediatly by substitution of $m_{i(u)}=c_i$ in (19), integration of (19) respective c_i, and resubstitution of $c_i=m_i$.

5 Conclusions

Communication of private information has been analyzed by assuming that firms behave as a sole economic agent. A classic result is that ex-ante firms in a Cournot game are interested in sharing private cost information. However, ex-post the incentives to share information disappear. Therefore, information sharing takes place only if cost messages are verifiable and ex-ante commitment, for example by delegation to a third party, is possible. When ownership and management are separated, this third party becomes unnecessary. Owners can commit ex-ante to information sharing by choosing their manager's compensation scheme suitably. Furthermore, even if cost messages are unverifiable, but owners can credibly implement message-dependent payment schemes, there will be perfect information sharing (at least when managers are risk neutral). However, if this is not the case, there will be no information sharing at all. Information sharing therefore depends on the implementability of publicly observable message-dependent payment schemes.

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