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Free Mobility and Taste-Homogeneity of Jurisdiction Structures*

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Abstract

We consider a population of agents distributed on the unit interval. Agents form jurisdictions in order to provide a public facility and share its costs equally. This creates an incentive to form large entities. Individuals also incur a transportation cost depending on their location and that of the facility which makes small jurisdictions advantageous. We consider a fairly general class of distributions of agents and generalize previous versions of this model by allowing for non-linear transportation costs. We show that, in general, jurisdictions are not necessarily homogeneous. However, they are if facilities are always intraterritory and transportation costs are superadditive. Superadditivity can be weakened to strictly increasing and strictly concave when agents are uniformly distributed.

Keywords: Consecutiveness, stratification, local public goods, coalition formation, country formation.

JEL Classification: C71 (Cooperative Games), D71 (Social Choice; Clubs; Committees; Associations), H73 (Interjurisdictional Differentials and Their Effects).

1 Introduction

In many situations agents partition into jurisdictions, groups or coalitions in order to provide a public service or facility. Examples are hospitals, schools, museums, police protection, parks, post offices or libraries; but also the formation of a country can be thought of in similar terms. Given such a situation, a very elementary question with a long tradition in public finance is when we can expect jurisdictions to be homogeneous in terms of the population contained. In

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this paper we consider a generalized version of a popular model of jurisdiction formation and investigate when free mobility of individuals implies that jurisdictions are taste-homogeneous.

Since the seminal work of Tiebout (1956), an important literature has studied local public good economies where agents can move freely across jurisdictions.[1] Free mobility is considered an appealing concept as it mimics social institutions, at least in Western democracies where freedom to migrate is constitutionally guaranteed (Scotchmer (2002)). Our notion of a free mobility equilibrium is standard. It simply requires that each individual prefers his own jurisdiction to any other existing jurisdiction. It is an elementary notion in the sense that several researchers have build on it and complemented it through additional requirements.[2]

Tiebout also suggested that individuals would sort into taste-homogeneous jurisdictions (Wooders (1999)). In the present paper we analyze a generalized version of a popular spatial model of jurisdiction formation in which agents are distributed on the unit interval. In the context of this model, taste-homogeneous jurisdictions are connected entities, that is, they are intervals (Bogomolnaia et al. (2008a)).[3]

We analyze a popular model in which the size of jurisdictions is determined by the following trade-off. As the population of jurisdictions increases there arise economics of scale because citizens' costs for public goods can be spread among more citizens. But as jurisdictions become larger they also become more heterogeneous and thus more difficult to manage. In our model the former aspect is formalized through an equal sharing rule for the costs of the facility of a jurisdiction. The latter aspect is captured through majority voting on the location of the facility within each jurisdiction and the fact that citizens incurs transportation costs from their location to that of the facility. Alesina et al. (2004) provide empirical support for this trade-off.

Transportation costs in this model are often called heterogeneity costs because the spatial structure of this model has two interpretations. On one hand, the geographical distance to the local public good and, on the other, the ideological distance indicating how close policies are to the individual's preferences. Alesina and Spolaore (1997, 2003 and 2006) and Alesina et al. (2004) argue that both dimension are correlated and that the model captures both.

Although we consider a fairly general class of distributions of agents, our main innovation on the modelling side is to introduce non-linear transportation costs. This generalization is common in models of spatial competition of firms and captures the realistic feature that these

[1] We will not attempt to survey this large body of work here. Scotchmer (2002) offers a recent discussion of the literature. The Tiebout Hypothesis (1956) asserts that the movement of consumers to jurisdictions where their wants are best satisfied (consumers "vote with their feet") and competition between jurisdictions in the provision of local public goods for residents will lead to "market-like" outcomes in which jurisdictions offer differentiated packages of public goods.

[2] For instance, Jéhiel and Scotchmer (1997) and Alesina and Spolaore (1997, 2003 and 2006)) complement it through a stability condition. Greenberg and Weber (1986 and 1993), Demange (1994), Alesina and Spolaore (1997, 2003 and 2006)), Conley and Konishi (2002) and in some sense Jéhiel and Scotchmer (2001) require in addition immunity to coalitional deviations.

[3] Connected structures are also called consecutive or stratified. This feature of jurisdiction structures is stressed for instance in Westhoff (1977), Greenberg and Weber (1986), Brueckner (1994) and Conley and Wooders (2001). It is also important in political science, see Greenberg and Weber (1985) and Brams, Jones and Kilgour (2002).

Table 1: Are jurisdictions necessarily homogenous?

		Transportation costs	
		superadditive	str. concave
Distribution	any	yes	no
	uniform	yes	yes

costs are increasing but–depending on the application of the model–at a decreasing or increasing rate.

We show that, in general, jurisdictions are not necessarily homogeneous. However, we provide conditions under which they are. Connected jurisdictions can be guaranteed if facilities are always intraterritory and transportation costs are superadditive. The condition that facilities have to be intraterritory is important in situations in which an unconnected jurisdiction has a multiplicity of median voters that do not belong to this jurisdiction. We show that this condition cannot be relaxed to extraterritory locations. Superadditivity can be weakened to strictly increasing and strictly concave when agents are uniformly distributed (see Table 1). In the context of country formation facilities are often interpreted as governments which in reality are always intraterritory. The model provides, hence, for a wide class of transportation costs an explanation for the fact that in reality almost all countries are geographically connected (Alesina and Spolaore (2003)).

There is a growing literature that uses variations of the present model in order to analyze properties of jurisdiction structures.[4] To the best of our knowledge, within this framework the possibility of unconnected entities when individual agents are allowed to move freely has not been pointed out.[5] Bogomolnaia et al. (2007) consider a special case of the present set-up in which agents are uniformly distributed and transportation costs are linear. They show that jurisdictions are connected under the stronger notion when migrant sets agents have to be of positive measure. Similar results have also been obtained in the models of jurisdiction formation of Jéhiel and Scotchmer (1997 and 2001) which also assume a continuum of agents. Haeringer (2000) and Bogomolnaia et al. (2008b) postulate models of finite societies and prove that entities are connected. None of these papers finds a similar interplay to ours between assumptions on the distribution of agents and transportation costs (see Table 1). Moreover, the importance of intraterritory facility choices has not been emphasized.

The paper is organized as follows. The next section presents the model. Sections 3 and 4 focus on extraterritory and intraterritory facility choices, respectively. The last section contains some concluding remarks.

[4] This literature includes Alesina and Spolaore (1997, 2003 and 2006), Cechlárová et al. (2001), Le Breton and Weber (2003), Goyal and Staal (2004), Haimanko et al. (2005 and 2007) and Drèze et al. (2007 and 2008).

[5] This result, however, parallels those in closely related models when groups of agents can jointly create new jurisdictions (Bogomolnaia et al. (2007 and 2008b)), in a different type of local public good economy (Conley and Wooder (2001)) and in political science (Greenberg and Weber (1985) and Brams, Jones and Kilgour (2002)).

2 The Model

Consider a population of agents located on the unit interval $[0, 1]$. The agents' distribution is given by a cumulative distribution function F, defined over $[0, 1]$. We suppose that F is continuous and strictly increasing.[6] We denote by λ the measure on $[0, 1]$ induced by the distribution function F with the total mass $\lambda([0, 1])$ equal to 1. To save on notation we identify an individual with her location t.

The population is partitioned in several jurisdictions each providing its members with access to its own public facility. A jurisdiction is denoted by S and might not be an interval. We consider jurisdictions to be the union of a finite number of closed (and proper) intervals.[7] The size (or measure of population) of jurisdiction S is denoted by s. Notice that in general s is different from the share of the interval $[0, 1]$ that jurisdiction S occupies. A jurisdiction structure $\mathcal{P} = \{S_1, ..., S_N\}$ is any partition of $[0, 1]$ in a finite number of jurisdictions such that $\lambda(S_n \cap S_m) = 0$, for all $n \neq m$ and $\cup_{n=1}^{N} S_n = [0, 1]$.[8]

The total cost $c(t, S)$ of an agent t in jurisdiction S is given by

$$c(t, S) = h(|t - l|) + \frac{g}{s},$$

where g represents the cost of the facility in S and is assumed to be independent of its location l and the number of users.[9] Throughout we assume that g is shared equally among the members of the jurisdiction. In addition, each individual incurs a transportation or heterogeneity cost $h(|t - l|)$ from her location t to that of the facility l. We assume that $h(\cdot)$ is continuous with $h(0) = 0$ and $h(x) > 0$ for all $x > 0$.

In the context of the present model the previous literature has dealt with the case in which transportation costs are proportional to distance. Inspired by the literature on spatial competition of firms we generalize this requirement here. We will look at two properties of $h(\cdot)$, the first of which is the following.

Definition 2.1 $h(\cdot)$ *is superadditive, if for all* $x, y \in \mathbb{R}_+$

$$h(x + y) \geq h(x) + h(y).$$

Note that any convex function is superadditive but that the reverse implication does not hold (Bruckner and Ostrow (1962)). Notice also that the case of linear heterogeneity costs considered

[6] An important implication of this is to rule out the existence of mass points. The non-existence of mass points is important (see footnote 12 immediately after Lemma 4.1).

[7] This precludes the possibility of isolated agents forming an enclave. As will become clear from the analysis such agents would belong to multiple jurisdictions. As such a formulation does not yield new insights we exclude this case.

[8] We assign jurisdictions their subindex by the following procedure. Jurisdiction S_1 starts at point zero. Starting at zero and going to the right, the first point belonging to another jurisdiction than S_1 belongs to S_2. The next point belonging neither to S_1 nor S_2 forms part of S_3, and so on.

[9] As pointed out in Alesina and Spolaore (1997 and 2003) we could postulate that costs are linear in size s and add the fixed cost g. As long as $g > 0$ this would not affect the results.

in the previous literature is a special case in which the inequality holds with equality. A second property we consider is that transportation costs are increasing in distance but at a decreasing rate, that is, $h(\cdot)$ is strictly concave.

Jurisdictions provide a public facility which can be located anywhere on $[0,1]$. The decision over the location of the facility is taken by majority voting. This implies, since individual utilities (costs) are single-peaked with respect to l, that the median voter determines l. In unconnected coalitions exists the possibility that the median voter is not unique. In these cases there exists an interval of median voter locations and only the endpoints of this interval belong to the jurisdiction. There exists therefore a continuum of possible location choices. We distinguish two classes of choices: intra- and extraterritorital locations.

Definition 2.2 *Denote by M_S the set of median voters in a jurisdiction S. Rule α chooses $l = \alpha \inf\{M_S\} + (1-\alpha)\sup\{M_S\}$. When $\alpha \in (0,1)$ the rule is extraterritory and intraterritory otherwise.*

Notice that, while rule $\alpha = 1/2$ is a natural choice made e.g. in Bogomolnaia et al. (2007 and 2008a), rule $\alpha = 0$ and rule $\alpha = 1$ have the advantage to insure that the location of the facility belongs to the territory of the jurisdiction. The latter is an appealing property–not only under Alesina and Spolaore's (1997 and 2003) geographical interpretation of this model.

We are interested in investigating the implications of free mobility of agents on the taste-homogeneity of jurisdiction structures. As in the context of this simple model the location of an agent captures the individual's preferences, taste-homogenous structures can be considered to be connected (Bogomolnaia et al. (2008a)).

Definition 2.3 *A jurisdiction structure $\mathcal{P} = \{S_1, ..., S_N\}$ is connected if every $S_n \in \mathcal{P}$ is an interval.*

Free mobility formalizes the appealing property of jurisdiction structures not to contain agents who are dissatisfied with their jurisdiction because they could migrate (or switch) to another jurisdiction and obtain lower total costs.[10]

Definition 2.4 *A jurisdiction structure $\mathcal{P} = \{S_1, ..., S_N\}$ is a Free Mobility Equilibrium [FME] if for all $t \in [0,1]$,*

$$c(t,S) \leq c(t,S'), \text{ for all } S' \in \mathcal{P},$$

where S is a jurisdiction in $\mathcal{P}$ to which t belongs.

Notice that, since we consider jurisdictions to be (collections of) closed intervals, an agent on the border between two jurisdictions belongs to both. The definition of a FME implies then that these individuals are indifferent between both jurisdictions. This property has been called border indifference (Bogomolnaia et al. (2007)).

[10]While this notion is standard there exist different denominations for it. We use the terminology of Jéhiel and Scotchmer (1997), Scotchmer (2002) and Conley and Konishi (2002). Bogomolnaia and Jackson (2002) and Bogomolnaia et al. (2008a) call this stability notion Nash stability. Demange (1994) uses the term Tiebout equilibrium.

3 Extraterritory Facility Choices

As we have seen, in unconnected coalitions exists the possibility that the median voter is not unique. We start our analysis by showing by means of an example that under any rule $\alpha \in (0,1)$ a FME is not necessarily connected. Notice that e.g. Bogomolnaia et al. (2007 and 2008a) consider $\alpha = 1/2$.

Example 1 *Let agents be uniformly distributed. Consider $\mathcal{P} = \{S_1, S_2\}$ with*

$$S_1 = [0, 1/4 - \beta] \cup [1/4, 1/4 + \beta] \cup [3/4, 1] \text{ and } S_2 = cl([0,1] \setminus S_1), \text{ where } \beta \in [0, 1/4].$$

Consider a rule $\alpha \in [1/2, 1)$. (Because of the underlying symmetry the extension to $\alpha \in (0, 1/2)$ is straightforward.) Choose $\beta = (2 - 1/\alpha)/4$. It follows that $l_1 = l_2 = 1/2$ and $s_1 = s_2 = 1/2$. Thus, agents are indifferent between S_1 and S_2 implying that this structure is a FME.

Notice that for the above reasoning the shape of the transportation costs $h(\cdot)$ and the costs g of the facility play no role. It also extends immediately to non-uniform distributions which are symmetric around 1/2. What is crucial is the possibility to locate the facilities of two different jurisdictions at the same position. This is no longer possible with intraterritory facility choices. To see this consider $\alpha = 1$. We obtain $\beta = 1/4$ and $S_1 = [1/4, 1/2] \cup [3/4, 1]$. It is still true that $l_1 = 1/2$ but now l_2 switches to 1/4. This opens the door to guaranteeing the existence of connected structures under intraterritory facility choices which is the focus of the next section.[11]

4 Intraterritory Facility Choices

In the remainder of the sequel we suppose that all jurisdictions employ rule $\alpha = 1$ (again because of the underlying symmetry the extension to $\alpha = 0$ is straightforward). We start by formally showing that with intraterritory facility choices the force underlying Example 1 is no longer active.

Lemma 4.1 *Under rule α with $\alpha \in \{0, 1\}$, $l \neq l'$ for all S, $S' \in \mathcal{P}$.*

Proof. See Appendix A. ■

This fact allows guaranteeing the existence of homogeneous jurisdiction structures in fairly general situations.[12]

[11] The attentive reader will realize that Example 1 contradicts Proposition 1 in Bogomolnaia et al. (2008a), as they assume that $\alpha = 1/2$. This is due to the fact that their proof omits the case in which the facilities of two different jurisdictions are located at the same position. To highlight the role of this possibility we state Lemma 4.1 below. Notice that (as we will show shortly) the results in Bogomolnaia et al. (2008a) are true assuming an intraterritory facility choice.

[12] Notice that Lemma 4.1 is no longer true if jurisdictions can choose different intraterritory facility choices or if there are mass points in the distribution of agents. As a consequence, a FME is not necessarily connected. To see this consider $\mathcal{P} = \{S_1, S_2\}$ with $S_1 = [0, 1/4] \cup [1/2, 3/4]$ and $S_2 = [1/4, 1/2] \cup [3/4, 1]$. Suppose S_1 chooses $\alpha = 0$, while S_2 has $\alpha = 1$. Under the uniform distribution, $s_1 = s_2 = l_1 = l_2 = 1/2$. As all agents are indifferent between S_1 and S_2, $\mathcal{P}$ is a FME. The same is true if a mass of 1/2 of the agents is located at 1/2 and the remaining mass of 1/2 is uniformly distributed, as in our model individuals at 1/2 belong to both S_1 and S_2.

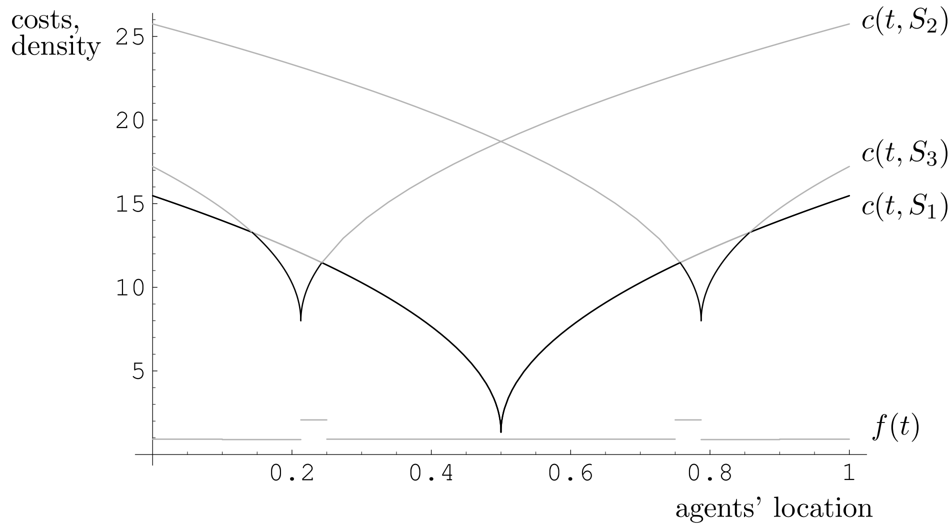

Figure 1: The jurisdictions in Example 2.

Proposition 4.1 *Let $h(\cdot)$ be superadditive. If $\mathcal{P} = \{S_1, ..., S_N\}$ is a FME, then it is connected.*

Proof. See Appendix B. ■

We show now that the fact that transportation costs are increasing at an increasing rate, which is implied by superadditivity, is important.

Example 2 *Let $g = 1$ and let agents be distributed according to the following density function*

$$f(t) = \begin{cases} 0.89 & \textit{if} \quad t \in [0.1, 0.21] \cup [0.79, 0.9] \\ 2.07 & \textit{if} \quad t \in (0.21, 0.25] \cup [0.75, 0.79) \\ 0.92 & \textit{otherwise} \end{cases}.$$

Assume heterogeneity costs are $h(x) = 20\sqrt{x}$. We show that the following partition $\mathcal{P}$ is a FME:

$$\mathcal{P} = \{S_1 = [0, 0.14] \cup [0.24, 0.76] \cup [0.86, 1], S_2 = [0.14, 0.24], S_3 = [0.76, 0.86]\}.$$

The jurisdiction sizes are $(s_1, s_2, s_3) = (3/4, 1/8, 1/8)$ and the facilities are located at $(l_1, l_2, l_3) = (0.5, 0.21, 0.79)$. This example is represented in Figure 1. In the upper part it shows total costs for each agent in each jurisdiction. The continuous black function is the lower envelope of the costs $c(t, S_i)$ for $i = 1, 2, 3$. The gray parts continue the costs associated to a given jurisdiction. Because of border indifference, jurisdictions are limited by the intersections of the lowest $c(t, S_i)$ for $i = 1, 2, 3$. The gray horizontal lines in the lower part of Figure 1 indicate $f(t)$. For the parameter values specified above, two small jurisdictions embedded in a larger one constitute a FME.

Notice that because the median locations are unique the example does not depend on a particular choice rule among median locations. The intuition for Example 2 is that relaxing superadditivity creates the possibility that, say, $c(t, S_1)$ and $c(t, S_3)$ intersect twice. In such a case it is advantageous for individuals to belong to jurisdictions whose facility is far away but offer a large population. In Figure 1 this creates an incentive for agents close to 0 and 1 to belong to S_1. However, notice that, as $h(\cdot)$ is strictly increasing, agents on the borders of the embedded jurisdictions are not equally well off. Therefore, l_2 and l_3 are only median locations because the distribution of agents counterbalances the asymmetry arising from increasing transportation costs. If the distribution of agents is sufficiently uniform, this is not possible. The following result refers to the uniform distribution which is frequently used in combination with linear transportation costs (e.g. Alesina and Spolaore (1997, 2003 and 2006) or Bogomolnaia et al. (2007 and 2008a)).

Proposition 4.2 *Assume that individuals are uniformly distributed. Let $h(\cdot)$ be twice differentiable with $h'(\cdot) > 0$ and $h''(\cdot) < 0$. If $\mathcal{P} = \{S_1, ..., S_N\}$ is a FME, then it is connected.*

Proof. See Appendix B. ■

It is an important question whether—assuming the uniform distribution—the requirements on $h(\cdot)$ can be weakened further. As can be seen from the proof provided in Appendix B, the role of the strict concavity of $h(\cdot)$ is to limit the number of potential intersections of the cost functions associated to different jurisdictions.[13] However, we conject (but have been unable to prove) that the assumption of the uniform distribution of agents is strong enough to guarantee that jurisdiction structures are connected even when cost functions have a less regular shape and one imposes only that they are strictly increasing. On the other hand, it is easy to see that if $h(\cdot)$ is not strictly increasing, a FME is not necessarily connected. The reason is that if $h(\cdot)$ is horizontal for some values an entire jurisdiction can be located in this region that has the facility at its midpoint.[14]

5 Concluding Remarks

This paper generalized a popular model of jurisdiction formation by allowing for non-linear transportation costs. We analyzed when a fundamental notion of free mobility of agents implies that jurisdictions are taste-homogeneous, which in our model simply means connected.

Our results have implications for further study of the spatial model of jurisdiction formation. On the one hand, they yield a tractable model for further study. For example, Alesina and Spolaore (1997, 2003 and 2006) start their analysis assuming that countries are connected and say that this is realistic and can be motivated by introducing "additional costs on governments of countries that are not geographically connected (1997, p. 1030)". Given this fundamental building block of the model, they apply then successively different notions of stability starting

[13] Differentiability is only assumed to simplify the proof.

[14] The details of a simple example are available on request.

with a free mobility requirement.[15] Our results imply that it is not necessary to impose such additional costs. Free mobility alone is sufficient to generate connected countries. Moreover, this is true in the basic model, in the extension to general distribution functions that Alesina and Spolaore propose and it is true in alternative extensions to non-linear transportation costs.

On the other hand, our results hold true under stronger stability notions. For example, Bogomolnaia et al. (2008a) analyze the present model with linear transportation costs and examine free mobility equilibria and local Nash stability. The latter is a stronger notion requiring that no group of agents of positive but arbitrary small size finds it advantageous to migrate. They show that every locally Nash stable structure is a free mobility equilibrium. As their proof only requires the continuity of the individual cost functions, it still holds in the setting of the present paper and implies that any local Nash stable structure is taste-homogeneous.

Our model is a step toward generalizing a popular model of jurisdiction formation by allowing for non-linear transportation costs, but more work is needed. On one hand, it is important to know whether our conjecture that the assumption of the uniform distribution of agents in combination with strictly increasing cost functions is enough to guarantee that jurisdiction structures are connected. On the other hand, it is an important future research question to analyze further notions of stability. Such an analysis could refer to a combination of group and individual stability notions and might become tractable when using our result that jurisdictions are connected.

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[15] As their stability notion allows only for agents at a border to switch country, it is weaker than ours. However, under equal sharing of government provision costs, total costs are increasing in transportation costs. Thus, in connected countries it is true that if no agent on a border prefers to switch country no other agent wishes to do so either.

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A Appendix A: Auxiliary Lemmatas

In this Appendix we derive some elementary results which are used in the proofs of Propositions 4.1 and 4.2. Given two jurisdictions S and S' we denote the location of the facilities by l and l', and the sizes by s and s', respectively. We start by offering a proof for Lemma 4.1 which is restated below.

Lemma A.1 *Under rule α with $\alpha \in \{0, 1\}$, $l \neq l'$ for all S, $S' \in \mathcal{P}$.*

Proof. Notice that under rule α with $\alpha \in \{0, 1\}$, $l \in S$ and $l' \in S'$. Suppose $l = l'$. Given that jurisdictions are the union of a finite number of closed (and proper) intervals, both jurisdictions can have a finite number of points in common. However, each of which is the infimum of an interval belonging to one jurisdiction, say S, and supremum of an interval belonging to other jurisdiction, say S'. If such a point belongs to both sets of medians M_S and $M_{S'}$, then it is $\sup\{M_S\}$ and $\inf\{M_{S'}\}$. If $\alpha = 0$, rule α chooses the former but not the latter, and vice versa for $\alpha = 1$. ■

Lemma A.2 *Let $h(\cdot)$ be strictly increasing. Consider a FME $\mathcal{P}$ containing S and S', where $l < l'$.*

(i) *If $s \geq s'$, then $c(i, S) < c(i, S')$, $\forall i \leq l$ and if $s \leq s'$, then $c(i, S) > c(i, S')$, $\forall i \geq l'$.*

(ii) *There exists a unique $k \in (l, l')$ such that $c(k, S) = c(k, S')$. Moreover, $c(i, S) < c(i, S')$ for all $i \in [l, k)$ and $c(i, S) > c(i, S')$ for all $i \in (k, l']$.*

Proof. Suppose $s \geq s'$ and consider $i \leq l$. As $g/s \leq g/s'$ and $h(l - i) < h(l' - i)$, we have $c(i, S) < c(i, S')$. As the case $s \leq s'$ is similar, this proves (i). From $S, S' \in \mathcal{P}$ and the properties of the distribution function follows that $s, s' > 0$. W.l.o.g. assume that $s \geq s'$. Notice that part (i) implies that if $t' \in S'$, then $t' > l$, as $\mathcal{P}$ is a FME. Consider the interval $[l, l']$ and define $\Delta(i, S, S') := c(i, S) - c(i, S')$. Since $h(\cdot)$ is strictly increasing, $\Delta(i, S, S')$ is strictly increasing. Suppose $\Delta(l', S, S') \leq 0$. Then it is impossible that $\mathcal{P}$ is a FME and l' is a median location. Therefore we have $\Delta(l', S, S') > 0$. By part (i) $\Delta(l, S, S') < 0$ holds. By Bolzano's Theorem there exists $k \in (l, l')$ such that $\Delta(k, S, S') = 0$. The remaining affirmations of the statement of (ii) follow from $\Delta(i, S, S')$ being strictly increasing. ■

Lemma A.3 *Let $h(\cdot)$ be strictly increasing. Consider S, $S' \in \mathcal{P}$ with $l < l'$. $\mathcal{P}$ is not a FME if there exist $t \in S$ and $t' \in S'$ such that either*

(i) *$t' \leq l < l' \leq t$ or*

(ii) *$l \leq t' < t \leq l'$.*

Proof. By Lemma A.2 part (i), $t' \leq l < l'$ implies $s < s'$, while $l < l' \leq t$ requires $s < s'$. This contradiction proves (i). For (ii) consider k as specified in Lemma A.2 part (ii). If $k \in (l, t)$, then $t \notin S$ and if $k \in (t', l')$, then $t' \notin S'$. ■

Lemma A.4 *Let $h(\cdot)$ be superadditive. Consider S, $S' \in \mathcal{P}$ with $l < l'$. $\mathcal{P}$ is not a FME if either*

(i) *there exist $t \in S$ such that $l' \leq t$ or*

(ii) *there exist $t' \in S'$ such that $t' \leq l$.*

Proof. We prove only the first part as the second can be proven similarly. By Lemma A.2 part (ii) it holds that $c(l', S') < c(l', S)$. This is the same as

$$h(l' - l) > g\frac{s - s'}{ss'}.$$

In a FME $t \in S$ implies

$$h(t-l) - h(t-l') \leq g\frac{s-s'}{ss'}.$$

Together we have

$$h(t-l) < h(l'-l) + h(t-l'),$$

contradicting superadditivity. ■

Lemma A.5 *Let $h(\cdot)$ be twice differentiable with $h'(\cdot) > 0$ and $h''(\cdot) < 0$. Consider a FME $\mathcal{P}$ containing S and S', where $l < l'$.*

(i) *If there exist $t \in S$ such that $l' \leq t$, then $l' < t$ and there exists a unique $\hat{k} \in (l, l')$ and a unique $\check{k} \in (l', t]$ such that $i \in S'$ implies that $i \in [\hat{k}, \check{k}]$. Moreover, if $i \in (\hat{k}, \check{k})$, then $i \notin S$.*

(ii) *If there exist $t' \in S'$ such that $t' \leq l$, then $t' < l$ and there exists a unique $\hat{k} \in [t', l)$ and a unique $\check{k} \in (l, l')$ such that $i \in S$ implies that $i \in [\hat{k}, \check{k}]$. Moreover, if $i \in (\hat{k}, \check{k})$, then $i \notin S'$.*

Proof. We prove only the first part as the second can be proven similarly. Since $t \in S$ and $\mathcal{P}$ is a FME, $c(t, S') \geq c(t, S)$. By Lemma A.2 $c(l', S) > c(l', S')$, so $t > l'$ holds. We have that $h(t-l') < h(t-l)$ and $t \in S$. This implies $s > s'$. By Lemma A.2 there exists a unique $\hat{k} \in (l, l')$ such that $c(\hat{k}, S) = c(\hat{k}, S')$ and $c(i, S') > c(i, S)$ for all $i < \hat{k}$. Moreover, $c(i, S) > c(i, S')$ for all $i \in (\hat{k}, l']$. Consider the interval $[l', 1]$ and define $\Delta(i, S, S') := c(i, S) - c(i, S')$. Notice that by Lemma A.2 $\Delta(l', S, S') > 0$. Moreover, for all $i \in [l', 1]$, $h(i-l) > h(i-l')$. Together with $h'(\cdot) > 0$ and $h''(\cdot) < 0$ this implies that $\Delta(i, S, S')$ is strictly decreasing. Remember that, since $t \in S$ and $\mathcal{P}$ is a FME, $c(t, S') \geq c(t, S)$. If $\Delta(t, S, S') = 0$, then set $\check{k} = t$. If $\Delta(t, S, S') < 0$, then by Bolzano's Theorem, there exist $\check{k} \in (l', t)$ such that $\Delta(\check{k}, S, S') = 0$. Since $\Delta(i, S, S')$ is strictly decreasing, $\check{k}$ is unique. Moreover, $c(i, S) > c(i, S')$ for all $i \in (l', \check{k})$ and $c(i, S) < c(i, S')$ for all $i > \check{k}$. ■

Lemma A.6 *Let $h(\cdot)$ be twice differentiable with $h'(\cdot) > 0$ and $h''(\cdot) < 0$. Consider S, $S' \in \mathcal{P}$ with $l < l'$. $\mathcal{P}$ is not a FME if there exist $t \in S$ and $t' \in S'$ such that either*

(i) *$l < l' < t < t'$ or*

(ii) *$t < t' < l < l'$.*

Proof. We prove only the first part as the second can be proven similarly. Suppose $l < l' < t < t'$ with $t \in S$ and $t' \in S'$. Lemma A.5 states that there exist unique $\hat{k}$ and $\check{k}$ such that $t' \in S'$ implies that $t' \in [\hat{k}, \check{k}]$, where $\check{k} \leq t$. This contradicts $t < t'$. ■

B Appendix B: Proofs for Propositions 4.1 and 4.2

B.1 Proof of Proposition 4.1

We start by introducing some additional notation. Given an unconnected jurisdiction $S \in \mathcal{P}$ denote the parts (intervals) of S by $S^1, S^2, ..., S^M$ such that $\cup_{m=1}^{M} S^m = S$. Indicate the part containing l by S^l. Let $S \in \mathcal{P}$ be the unconnected jurisdiction with the lowest subindex. Define $b_1 := \sup\{S^1\}$ and

$$b_2 := \begin{cases} \inf\{S^2\} & \text{if} \quad S^1 = S^l \\ \inf\{S^l\} & \text{if} \quad S^1 \neq S^l \end{cases} .$$

Let $S' \in \mathcal{P}$ be such that $b_1 = \inf\{S'\}$.

Let $\mathcal{P}$ be an unconnected FME. By definition $b_1 < b_2$. Given rule $\alpha = 1$, we have $l \neq b_2$ and $b_1 < l'$. Also, by Lemma A.1 $l \neq l'$. Thus, it is sufficient to consider the following four cases:

1. $l \leq b_1 < l' \leq b_2$. As $l < l' \leq b_2 \in S$ holds, application of Lemma A.4 part (i) yields a contradiction.

2. $l \leq b_1 < b_2 < l'$. Since $b_1 \in S'$ and $b_2 \in S$ holds, application of Lemma A.3 part (ii) yields a contradiction.

3. $b_1 < l' \leq b_2 < l$. As $b_1 \in S$ and $b_1 < l' < l$ holds, application of Lemma A.4 part (ii) yields a contradiction.

4. $b_1 < b_2 < l < l'$. Since $b_1 \in S'$ and $b_1 < l < l'$ holds, application of Lemma A.4 part (ii) yields a contradiction. ■

B.2 Proof of Proposition 4.2

Consider again the four cases of the proof of Proposition 4.1. Notice that we still can apply Lemma A.3. However, Lemma A.4 cannot be applied. The following Lemma replaces Lemma A.4 by combining Lemmatas A.5 and A.6 with the assumption of uniformly distributed agents, which proves the desired result.

Lemma B.1 *Let individuals be uniformly distributed and $h(\cdot)$ be twice differentiable with $h'(\cdot) > 0$ and $h''(\cdot) < 0$. Consider S, $S' \in \mathcal{P}$ with $l < l'$. $\mathcal{P}$ is not a FME if either*

1. *there exist $t \in S$ such that $l' \leq t$ or*

2. *there exist $t' \in S'$ such that $t' \leq l$.*

Proof. We prove only the first part as the second can be proven similarly. Consider a FME $\mathcal{P}$ containing S and S' with $l, t \in S$, $l' \in S'$ and $l < l' \leq t$. Given $S \in \mathcal{P}$, without loss of generality, let S' be such that l' is the closest such facility to l. We start again by introducing some additional notation. Since S is unconnected, denote by S^1 the part containing l and by S^2 the part containing t. Similarly, denote by $S^{1\prime}$ the part of S' containing l'. Define $b_1 := \sup\{S^1\}$, $b_2 := \inf\{S^{1\prime}\}$, $b_3 := \sup\{S^{1\prime}\}$ and $b_4 := \inf\{S^2\}$. Let $\mathcal{L}$ be the set of facility locations in $[b_3, b_4]$.

We proof the Lemma through a sequence of claims.

Claim B.1 *There exists an (open) interval $A \subset [b_3, b_4]$ such that for all $i \in A$ we have $i \notin S \cup S'$.*

Proof. Lemma A.5 implies that there exists a unique $\hat{k} \in [b_1, b_2]$ and a unique $\check{k} \in [b_3, b_4]$ such that $t' \in S'$ implies that $t' \in [\hat{k}, \check{k}]$. From the properties of $h(\cdot)$ we have $l' - \hat{k} < \check{k} - l'$. Lemma A.5 also establishes that there exists no $i \in S$ with $i \in [l', \check{k})$. With the uniform distribution l' can only be a median location if there exists an interval of agents $i \notin S \cup S'$ with $i \in [b_3, \check{k})$. ■

Claim B.2 *For all $i \in [b_3, b_4]$ with $i \in S'' \in \mathcal{P} \setminus \{S, S'\}$, we have $l'' \in [b_3, b_4]$.*

Proof. Suppose $i \in [b_3, b_4]$ with $i \in S'' \in \mathcal{P} \setminus \{S, S'\}$. Then i must belong to an interval of citizens of S'' and we can assume that i belongs to the interior of this interval. Hence $i \in (b_3, b_4)$. Notice that $l'' \notin (l, l')$ by assumption. Assume $l'' < l$. Since $l'' < l < i < t$ with $i \in S''$ and $t \in S$, Lemma A.6 part (i) yields a contradiction. Assume $l'' > t$. We have $l < i < t \leq l''$ and the statement follows from Lemma A.3 part (ii). ■

Claim B.3 *Let $S'' \in \mathcal{P} \setminus \{S, S'\}$ be such that $l'' = \sup\{\mathcal{L}\}$. We have that $\sup\{S''\} \leq b_4$ and that if $i \in (l'', \sup\{S''\})$, then $i \in S''$.*

Proof. Consider $S'' \in \mathcal{P} \setminus \{S, S'\}$ such that $l'' = \sup \mathcal{L}$. Suppose $\sup\{S''\} > b_4$. We have $l < l'' < t < \sup\{S''\}$ with $t \in S$ and $\sup\{S''\} \in S''$, contradicting Lemma A.6 part (i). Assume there exists $i \in (l'', \sup\{S''\})$ with $i \in \hat{S} \neq S''$. From Claim B.2 and $l < l' < l'' = \sup \mathcal{L}$ follows that $\hat{l} < l''$. We have $\hat{l} < l'' < i < \sup\{S''\}$ with $i \in \hat{S}$ and $\sup\{S''\} \in S''$, contradicting again Lemma A.6 part (i). ■

Claim B.4 *$\mathcal{P}$ cannot be a FME.*

Proof. Consider $S'' \in \mathcal{P} \setminus \{S, S'\}$ such that $l'' = \sup \mathcal{L}$. Suppose $\inf\{S''\} \leq l$. We have $\inf\{S''\} \leq l < l'' < t$ with $\inf\{S''\} \in S''$ and $t \in S$, contradicting Lemma A.3 part (i). Suppose $\inf\{S''\} > l$. Denote by $\hat{S} \in \mathcal{P}$ the jurisdiction $\hat{S} \neq S''$ such that $\sup\{S''\} \in \hat{S}$. By Claim B.2 and $l < l' < l'' = \sup \mathcal{L}$, $\hat{l} < l''$. By border indifference we have $c(\sup\{S''\}, S'') = c(\sup\{S''\}, \hat{S})$. Assume $\inf\{S''\} < \hat{l}$. We have $\inf\{S''\} < \hat{l} < l'' < \hat{t}$ with $\inf\{S''\} \in S''$ and $\hat{t} \in \hat{S}$, contradicting Lemma A.3 part (i). Assume $\inf\{S''\} \geq \hat{l}$. Notice that S'' may be unconnected to the left of l'', but is connected on the right by Claim B.3. As agents are uniformly distributed and $h(\cdot)$ is symmetric in distance, $c(\inf\{S''\}, S'') \geq c(\sup\{S''\}, S'')$. However, as $h(\cdot)$ is strictly increasing, we have $c(\inf\{S''\}, \hat{S}) < c(\sup\{S''\}, \hat{S})$. Together we have

$$c(\inf\{S''\}, S'') \geq c(\sup\{S''\}, S'') = c(\sup\{S''\}, \hat{S}) > c(\inf\{S''\}, \hat{S}),$$

contradicting that $\inf\{S''\} \in S''$. ■

This concludes the proof of Lemma B.1. ■