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Multi-stage oligopoly models with nested logit demand structures: A simplifying approach

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Abstract

Solving multi-stage oligopoly models by backward induction can easily become a complex task when firms are multi-product and demands are derived from a nested logit framework. This paper shows that under the assumption that within-segment firm shares are equal across segments, the analytical expression for equilibrium profits can be substantially simplified. The size of the error arising when this condition does not hold perfectly is also computed. Through numerical examples, it is shown that the error is rather small in general. Therefore, using this assumption allows to gain analytical tractability in a class of models that has been used to approach relevant policy questions, such as for example firm entry in an industry or the relation between competition and location. The simplifying approach proposed in this paper is aimed at helping improving these type of models for reaching more accurate recommendations.

JEL classification numbers: L11, L13

Keywords: Nested logit, multinomial logit, multi-stage oligopoly models, analytical solution of multi-stage games.

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1 Introduction

Multi-stage games of oligopolistic competition constitute a popular modeling approach in Industrial Organization. In particular, models where in the last stage firms compete in prices and in previous stages they make a series of decisions based on the profitability realized in the last stage have been widely used to model firm entry, firm location or product choice. The key ingredient of the class of models considered in this paper is that the price competition stage is modeled following the random utility framework (see for example Ben-Akiva & Lerman, 1985). This means that each consumer buys just one unit of a differentiated good to maximize utility. So, the focus is on models where the last stage of the game is solved following a multinomial or a nested logit approach.

The nested logit is a common approach to the modelization of demand in the context of product differentiation. It is particularly appropriate whenever substitution patterns are stronger within certain subsets of products, that can then be grouped into different nests. This also allows to soften the incidence of the *Independence of Irrelevant Alternatives* problem of the standard multinomial logit, where all products are assumed to be equal substitutes of each other.

However, the analytical complexity of demand expressions for the nested logit can render the equilibrium profits expressions of the last stage of the game rather complicated. Then, solving the game by backward induction may become a very difficult, if not impossible, task. Following a nested logit approach, this paper shows how the expression for equilibrium profits in the last subgame can become as tractable as the expression that would be derived from a simple multinomial logit. This is true when the conditional market shares of the firm are equal across its market segments. In this case, each segment is like a small replica of the whole market in terms of firm performance, such that a firm's share in any segment is identical to the firm's market share.

In the real world this assumption may not be satisfied. However, whether the error made in assessing firm's profits under such an assumption is important is an empirical question whose answer will depend on the particular industry of interest. To address this point, the approximation error and the conditions under which it will be low enough to

make that assumption admissible are given. Moreover, numerical simulations to show that the size of the error for many regular market structures is very small, or even negligible, are provided.

Multi-stage oligopoly games of product differentiation have been used in a wide variety of contexts.¹ For example, Chisholm & Norman (2004) study location decisions by multiproduct firms where first a leader chooses its locations, then the followers choose theirs and finally they all compete in prices within a multinomial logit demand framework. Although the authors recognize that more flexible product substitution patterns could be considered, they explicitly exclude the nested logit setup, perhaps due to its added complexity that could compromise the analytical tractability of their model. Anderson & de Palma (1992) propose a nested logit demand structure where consumers choose first the firm and then a product of that firm. They show the existence of a symmetric equilibrium with multiproduct firms in models where firms make sequential decisions on entry, number of products and prices. In this paper, symmetry is not required and the demand structure is more complex because consumers are assumed to first choose among product types (or segments) and then they choose a particular product in the given segment. Each firm is allowed to sell various products within each segment. However, it is assumed that an equilibrium exists in the price competition stage of the game. Grossmann (2007) studies a model where firms choose first the number of products and then they compete *à la Cournot*. Grossmann also considers the case of a nested logit demand as in Anderson & de Palma (1992), but restricting the analysis to a duopoly, for the sake of analytical tractability. The results in this paper are intended to help softening the computational burden in these kind of approaches. Other approaches to demand in multi-stage oligopoly models stick to the frameworks commonly known as the “linear” or the “circular city”. In these models the functional form of demand allows for quite tractable expressions of equilibrium profit functions. Having simpler expressions eases the resolution of these games by using backward induction. However, that approach to demand is not the most

¹This does not pretend to be an exhaustive list but an illustration of the variety of applications of these type of models.

adequate in the case of durable goods, where consumers are assumed to be occasional rather than frequent buyers of the product. Examples of this literature include Zhou & Vertinsky (2001), who study firm entry and location decisions in growing markets; Christou & Vettas (2005), who consider firm location choices when rivals' quality is not known; Pal & Sarkar (2002) or Janssen *et al.* (2005) for the study of multi-store competition and location decisions; Economides (1993) for models of entry, location and quality choice. The results proposed in this paper may help in extending these type of analyses to setups where demand is derived from a random utility framework.

The paper is organized as follows. Section 2 presents the model and derives the equivalence result between multinomial and nested logit expressions for equilibrium profits. Section 3 computes the approximation error in case the condition for equivalence does not hold exactly. Section 4 provides numerical simulations to compute the size of errors under alternative industry configurations and illustrates the validity of the approximation using real data for the Spanish car market. Section 5 concludes.

2 Comparison of equilibrium profits under multinomial and nested logit models

Consider a multi-stage oligopoly model where firms in the last stage simultaneously choose prices and in the previous stages makes an entry, R&D investment or some other strategic decision. Assume that the market is segmented as a result of vertical or horizontal product differentiation. Firms commercialize products in several segments (but not necessarily in all of them) and are allowed to sell more than one product in the same segment. In the last stage, each firm maximizes profits choosing prices for all its products:

$$\max_{\{p_{gli}\}_{i,g}} \Pi_l = \sum_{g=1}^{\mathfrak{R}} \sum_{i=1}^{N_{gl}} (p_{gli} - c_{gli}) MS_{gli} \quad (1)$$

where g, l, i are segment, firm and product indexes, respectively. G is the total number of segments. N_{gl} is the number of goods of firm l in segment g . M is the market size. S_{gli} is the market share of product i of segment g for firm l . p and c are, respectively, price and

marginal cost. It is assumed that marginal costs are constant. However, they can differ across products.²

The first-order conditions for this problem can be written as:

$$\begin{aligned}
MS_{gli} + \alpha (p_{gli} - c_{gli}) M \frac{\partial S_{gli}}{\partial p_{gli}} + \sum_{j \neq i} \alpha (p_{glj} - c_{glj}) M \frac{\partial S_{glj}}{\partial p_{gli}} \\
+ \sum_{r \neq g} \sum_{j=1}^{N_{rl}} \alpha (p_{rlj} - c_{rlj}) M \frac{\partial S_{rlj}}{\partial p_{gli}} = 0 \quad , \quad g = 1, \dots, G \quad , \quad i = 1, \dots, N_{gl}
\end{aligned} \tag{2}$$

From these derivatives, depending on the underlying demand structure we will obtain different equilibria. To see this, consider two popular demand specifications in discrete choice problems: the nested and the multinomial logit.

2.1 Nested logit

Consider the nested logit model by Anderson *et al.* (1992). Assume a nesting structure where consumers decide first the segment and then choose a product within that segment. Each consumer buys only one product to maximize utility. The indirect utility from product i of firm l in segment g is given by:

$$U_{gli} = k_{gli} - \alpha p_{gli} + \varepsilon_{gli} = u_{gli} + \varepsilon_{gli} \tag{3}$$

where k is some index of product quality summarizing observed product characteristics. The price coefficient, α , represents the marginal utility of income. Finally, ε_{gli} is an idiosyncratic shock following an extreme value distribution. Then, the market share of product i of firm l , conditional on choosing segment g , is:

²It is assumed for simplicity in the exposition that there are no fixed costs. Their absence is innocuous given that the analysis of the paper focuses on the price competition stage of the game. They are of course relevant when solving the previous stages of the game. However, their functional form may differ across models (depending on whether we consider an entry decision, or an investment in capacity, for example). A general formulation to cover that variety of cases would just complicate the notation without changing the results.

$$S_{li/g} = \frac{\exp\left(\frac{u_{gli}}{\mu_2}\right)}{\sum_{j=1}^{N_g} \exp\left(\frac{u_{gmj}}{\mu_2}\right)} \quad (4)$$

where μ_2 is a parameter representing the degree of heterogeneity of products within segment g . The attractiveness of each segment is given by its inclusive value:

$$A_g = \mu_2 \ln \sum_{m=1}^N \sum_{j=1/g}^{N_{gl}} \exp\left(\frac{u_{gmj}}{\mu_2}\right) \quad (5)$$

such that the probability of choosing nest g is given by:

$$S_g = \frac{\exp\left(\frac{A_g}{\mu_1}\right)}{\sum_{r=1}^G \exp\left(\frac{A_r}{\mu_1}\right)} \quad (6)$$

where μ_1 measures heterogeneity across segments, with $\mu_1 \geq \mu_2$. Therefore, the total market share, S_{gli} , is:

$$S_{gli} = S_g * S_{li/g} \quad (7)$$

The price derivatives are then:

$$\frac{\partial S_{gli}}{\partial p_{gli}} = -S_{gli} \left[\frac{1}{\mu_1} (1 - S_g) S_{li/g} + \frac{1}{\mu_2} (1 - S_{li/g}) \right] \alpha \quad (8)$$

$$\frac{\partial S_{glj}}{\partial p_{gli}} = -S_{glj} \left[\frac{1}{\mu_1} (1 - S_g) S_{li/g} - \frac{1}{\mu_2} S_{li/g} \right] \alpha \quad (9)$$

$$\frac{\partial S_{rlj}}{\partial p_{gli}} = \frac{\alpha}{\mu_1} S_{rlj} S_{gli} \quad (10)$$

Substituting the derivatives in (2), the first-order conditions become after some algebra:

$$\begin{aligned} (p_{gli} - c_{gli}) S_{gli} &= \frac{\mu_2}{\alpha} S_{gli} + \left(1 - \frac{\mu_2}{\mu_1}\right) S_{li/g} \sum_{i=1}^{N_{gl}} (p_{gli} - c_{gli}) S_{gli} \\ &+ \frac{\mu_2}{\mu_1} S_{gli} \sum_{g=1}^G \sum_{j=1}^{N_{gl}} (p_{gli} - c_{gli}) S_{gli} \quad , \quad i = 1, \dots, N_{gl} \quad , \quad g = 1, \dots, G \end{aligned} \quad (11)$$

Summing over g and i and defining $S_l = \sum_{g=1}^G \sum_{i=1}^{N_{gl}} S_{gli}$ as the total market share of firm

l and $S_{gl} = \sum_{i=1}^{N_{gl}} S_{gli}$ as the market share of firm l in segment g , expression (11) becomes:

$$\Pi_l = \frac{\mu_2}{\alpha} M S_l + \left(1 - \frac{\mu_2}{\mu_1}\right) M \sum_{g=1}^G \left[\sum_{i=1}^{N_{gl}} (p_{gli} - c_{gli}) S_{gli} \right] \sum_{i=1}^{N_{gl}} S_{li/g} + \frac{\mu_2}{\mu_1} S_l \Pi_l \quad (12)$$

Finally, after solving for Π_l we get the expression for the equilibrium profits:

$$\Pi_l = \frac{\mu_1 \mu_2}{\alpha (\mu_1 - \mu_2 S_l)} M S_l + \frac{\mu_1 - \mu_2}{\mu_1 - \mu_2 S_l} M \sum_{g=1}^G \left[\sum_{i=1}^{N_{gl}} (p_{gli} - c_{gli}) S_{gli} \right] \sum_{i=1}^{N_{gl}} S_{li/g} \quad (13)$$

2.2 Multinomial logit

The multinomial logit is a simpler and more tractable approach. However, compared with the nested logit, it gives rise to less realistic substitution patterns between products. The multinomial logit can be obtained as a particular case of the nested logit when $\mu_1 = \mu_2$.

In that case:

$$S_{gli} = \frac{\exp\left(\frac{u_{gli}}{\mu_2}\right)}{\sum_{j=1}^N \exp\left(\frac{u_{gmj}}{\mu_2}\right)} \quad (14)$$

with price derivatives:

$$\frac{\partial S_{gli}}{\partial p_{gli}} = -\frac{\alpha}{\mu_2} S_{gli} (1 - S_{gli}) \quad (15)$$

$$\frac{\partial S_{glj}}{\partial p_{gli}} = \frac{\alpha}{\mu_2} S_{glj} S_{gli} \quad (16)$$

$$\frac{\partial S_{rlj}}{\partial p_{gli}} = \frac{\alpha}{\mu_2} S_{rlj} S_{gli} \quad (17)$$

and equilibrium profits:

$$\Pi_l = \frac{\mu_2}{\alpha} \frac{S_l}{1 - S_l} M \quad (18)$$

2.3 The equivalence between nested and multinomial logit

The main difference between the expressions for equilibrium profits comes from the second summand of (13). The fact that $\sum_{i=1}^{N_{gl}} S_{li/g}$ can vary across nests prevents any further attempt to simplify (13). However, the following assumption will allow such a simplification.

Assumption 1 *The market share of a firm conditional to belonging to a given segment is the same across segments, i.e., $\sum_{i=1}^{N_{gl}} S_{li/g} = \sum_{i=1}^{N_{rl}} S_{li/r} \forall g, r \in G$.*

This means that every firm has the same degree of performance in all the segments where it is present. However, the shares of each product within nests are allowed to be different, i.e. $S_{li/g} \neq S_{lj/g}$ in general, and this implies that we can have $S_{gli} \neq S_{glj}$, but also $S_{gli} \neq S_{rli}$ because although $S_{li/g} = S_{li/r}$ still $S_g \neq S_r$.

Proposition 1 *Under assumption 1, the expressions of equilibrium profits in the last stage of the game for the nested and the multinomial logit are identical.*

Proof. Using assumption 1 we can take $\sum_{i=1}^{N_{gl}} S_{li/g}$ out when summing over g in (13):

$$\Pi_l = \frac{1}{\alpha} \frac{\mu_1 \mu_2}{\mu_1 - \mu_2 S_l} M S_l + \frac{\mu_1 - \mu_2}{\mu_1 - \mu_2 S_l} M \sum_{i=1}^{N_{sl}} S_{li/g} \left[\sum_{g=1}^G \sum_{i=1}^{N_{gl}} (p_{gli} - c_{gli}) S_{gli} \right] \quad (19)$$

$$\Pi_l \left(1 - \frac{\mu_1 - \mu_2}{\mu_1 - \mu_2 S_l} \sum_{i=1}^{N_{gl}} S_{li/g} \right) = \frac{1}{\alpha} \frac{\mu_1 \mu_2}{\mu_1 - \mu_2 S_l} M S_l \quad (20)$$

Now recall that: $S_l = \sum_{g=1}^G \sum_{i=1}^{N_{gl}} S_{gli} = \sum_{g=1}^G \sum_{i=1}^{N_{gl}} S_g S_{li/g} = \sum_{g=1}^G S_g \sum_{i=1}^{N_{gl}} S_{li/g}$, but given assumption 1: $\sum_{g=1}^G S_g \sum_{i=1}^{N_{gl}} S_{li/g} = \sum_{i=1}^{N_{gl}} S_{li/g} \sum_{g=1}^G S_g = \sum_{i=1}^{N_{gl}} S_{li/g} * 1$. Then $S_l = \sum_{i=1}^{N_{gl}} S_{li/g}$ and hence:

$$\Pi_l \left(1 - \frac{\mu_1 - \mu_2}{\mu_1 - \mu_2 S_l} S_l \right) = \frac{1}{\alpha} \frac{\mu_1 \mu_2}{\mu_1 - \mu_2 S_l} M S_l \quad (21)$$

$$\Pi_l = \frac{\mu_2}{\alpha} \frac{S_l}{1 - S_l} M \quad (22)$$

which is identical to (18). ■

Nevertheless, it is important to clarify that the equilibria in the price competition stage will be different for both specifications. Or alternatively, from an empirical point of view, the estimates of parameters μ and α consistent with the observed data on p and S_l will be different in a nested or in a multinomial setup.

When firm's conditional market shares are equal across segments the total market share of the firm is identical to the share within each segment. Within each group, all products compete in a multinomial logit fashion. Therefore, from the point of view of the firm, the whole market is just the addition of several markets that differ only in their size (which is the share of the segment on the total market). The firm is able to attain exactly the same share in each of these smaller multinomial logit markets. So, in practical terms, it is as if the whole market was a multinomial logit one. In other words, under assumption 1, each segment is like a smaller replica of the whole market. As a consequence, the nested logit setup can be understood as a repetition of identical (in terms of profitability) multinomial logit setups and so it is natural to obtain a similar symmetry between nested and multinomial equilibrium profits.

The usefulness of this result can be better illustrated with an example. Consider a standard two-stage empirical model of entry where each firm first decide whether to introduce a product and then in the second stage all firms set prices simultaneously. We can solve the price competition stage for the equilibrium vector of prices, P^* . Once these are known, under assumption 1 we can solve for the first stage of the game, assessing the profitability of the entry decision, by plugging them in (22) instead of using (13), which is much more cumbersome. In this manner, we can combine the more flexible substitution patterns of a demand derived from a nested logit specification with the simplicity of a multinomial logit when solving for the previous stages of the game.

3 Approximation error

It could be the case that in the context of a particular industry or problem of interest the conditional market shares of the firm cannot be regarded as being exactly identical across

segments. There may be significant or large differences. Therefore, it is important to know the bias that assumption 1 would be introducing in the computation of equilibrium profits in the price competition subgame, should it not hold perfectly.

Definition 1 Denote the firm's conditional market share by $S_{gl}^c = \sum_{i=1}^{N_{gl}} S_{li/g}$ and its total share by $S_l = \sum_{g=1}^G S_g S_{gl}^c$. The difference $d_{gl} = S_{gl}^c - S_l$, is a measure of the departure from assumption 1 for each segment g where firm l is producing.

Proposition 2 When assumption 1 does not hold, the percentage approximation error in equilibrium profits due to the use of (22) instead of (13) in the price competition stage of the game is: $\Pi_{error} = -(1 - \mu_2) \frac{S_l}{1 - S_l} \sum_{g=1}^G \frac{\pi_{gl}}{\Pi_l^{true}} \frac{d_{gl}}{S_l}$, where $\Pi_l^{true} = \Pi_l$, as given by expression (13).

Proof. Substituting d_{gl} in (13) and rearranging terms gives:

$$\Pi_l \left(\frac{\mu_1 - \mu_2 S_l - \mu_1 S_l + \mu_2 S_l}{\mu_1 - \mu_2 S_l} \right) = \frac{\mu_1 \mu_2}{\alpha (\mu_1 - \mu_2 S_l)} M S_l + \frac{\mu_1 - \mu_2}{\mu_1 - \mu_2 S_l} \sum_{g=1}^G \pi_{gl} d_{gl} \quad (23)$$

where $\pi_{gl} = M \sum_{i=1}^{N_{gl}} (p_{gli} - c_{gli}) S_{gli}$ such that $\sum_{g=1}^G \pi_{gl} = \Pi_l$. Then, renaming Π_l as Π_l^{true} :

$$\Pi_l^{true} = \frac{\mu_1 \mu_2}{\alpha (\mu_1 - \mu_1 S_l)} M S_l + \frac{\mu_1 - \mu_2}{\mu_1 - \mu_1 S_l} \sum_{g=1}^G \pi_{gl} d_{gl} \quad (24)$$

Now recall that if we obviate the differences in firm's (within-segment) conditional market shares and we assume they are equal (and thus equal to S_l) the corresponding expression for profits would be:

$$\Pi_l^{appr} = \frac{\mu_2}{\alpha (1 - S_l)} M S_l \quad (25)$$

Therefore, we can define the percentage error of approximation as:

$$\Pi_{error} = \frac{\Pi_l^{appr} - \Pi_l^{true}}{\Pi_l^{true}} = \frac{-1}{\Pi_l^{true}} \frac{\mu_1 - \mu_2}{\mu_1 - \mu_1 S_l} \sum_{g=1}^G \pi_{gl} d_{gl} \quad (26)$$

Normalizing $\mu_1 = 1$ and rearranging terms:

$$\Pi_{error} = -(1 - \mu_2) \frac{S_l}{1 - S_l} \sum_{g=1}^G \frac{\pi_{gl}}{\Pi_l^{true}} \frac{d_{gl}}{S_l} \quad (27)$$

■

Therefore, the percentage error of approximation is a function of the similarity parameter, the ratio of firm's market share to rivals' share and a weighted sum of the percentage difference between firm's conditional shares and firm's total share using as weights the proportion of each segment profits over firm's total profit. As μ_2 decreases, the absolute value of the error increases. If $\mu_2 = 1$ the error vanishes because in that case we are back to the standard multinomial setting and thus there is no error by definition. Expression (27) shows that the non-fulfillment of assumption 1 gives rise to some error through the term d_{gl} . However, the total error is also a function of other elements that could offset the deviations induced by d_g . Therefore, the approximation error may be very small even for large violations of assumption 1 (see section 4 below), meaning that the multinomial logit can be, in general, a good approximation of the nested logit in terms of profitability. Moreover, notice that:

Remark 1 *The approximation error for firm l depends only on firm's l shares and profits.*

Remark 2 *A higher firm share will increase the error through higher $\frac{S_l}{1-S_l}$.*

Remark 3 *The term $\sum_{g=1}^G \frac{\pi_{gl}}{\Pi_l^{true}} \frac{d_{gl}}{S_l}$ can be positive or negative depending on the sign of the $\frac{d_{gl}}{S_l}$ terms and their respective weights.*

In particular, $\frac{d_{gl}}{S_l}$ can be positive or negative and greater or lower than one in absolute value. It will trivially be zero when the conditional market shares of the firm are equal across segments. The weight $\frac{\pi_{gl}}{\Pi_l^{true}}$ is a priori in the interval $[0, 1]$. However, if we are willing to consider the possibility of having negative profits in some (or all) segments that needs not to be the case. Nevertheless, if we have in mind a medium or long run situation, where unprofitable products (firms) exit the market we can restrict ourselves to the case of non-negative profits. In this way the sum is indeed a weighted average.

Remark 4 *If the relative weight of each segment in firm's total profit is similar (equal) to each segment share, then the approximation error will likely be close to (equal) zero, because $\sum_{g=1}^G S_g d_{gl} = 0$.³*

This in turn would imply that the ranking of profitability or market performance is very similar (equal) across segments and that each segment is a replica of size S_g of the whole market. This remark is important because it shows that the approximation error can be very small even in cases where the assumption of equal conditional shares is not met by a large amount (d_{gl} large).

Remark 5 *In general, the larger the difference between $\frac{\pi_{gl}}{\Pi_{true}}$ and the respective segment share, S_g , the larger will be the approximation error.*

4 Applications

4.1 Numerical simulations for approximation errors

In the previous section we have derived an analytical expression for the approximation error and provided some insights on the factors that can make it higher or lower. Here, I will compute the error size for alternative market structures. Equation (27) is a function of four elements that are bounded between 0 and 1: The dissimilarity parameter (μ_2) ranges between 0 (perfect nesting) and 1 (multinomial case); market shares, $S_l \in [0, 1]$ by definition; the ratio of firm's variable profits in a segment to firm's total profits can be assumed to be in the $[0, 1]$ interval in the long run, as argued in the previous section; and the firm's share conditional to segment g , S_{gl}^c , which determines d_{gl} , is also between 0 and 1 by definition.

In a theoretical model S_l , S_{gl}^c and d_{gl} are function of equilibrium prices (and characteristics) as given by equations 4 and 7. One possible approach to the simulation exercise

$$^3 \sum_{g=1}^G S_g d_{gl} = \sum_{g=1}^G S_g (S_{gl}^c - S_l) = \sum_{g=1}^G S_g S_{gl}^c - \sum_{g=1}^G S_g S_l = S_l - S_l \sum_{g=1}^G S_g = 0$$

would be to parametrize some industry by specifying the number of firms, their cost functions, the number of segments, the number of products and their characteristics, etc. and computing the equilibrium prices, shares and profits to obtain the approximation errors for each possible combination.

However, the number of possibilities quickly grows beyond the limits of feasibility as soon as we start to combine alternative values for each of the primitives involved. Therefore, the approach I will follow consists on relying on the boundness of market shares that compose expression (27) to compute the approximation error for each possible combination of (equilibrium) values of each of its four ingredients. In doing so, I obviate the fact that some combinations may not take place in some oligopoly models, but still I can compute how large the error would be if it was possible to have such combinations.

More precisely, I assume a market where products can be segmented in four groups with $\mu_2 = 0.4$, meaning a value of 0.6 for the similarity parameter within nests. The ratio $\frac{\pi_{sl}}{\Pi_l^{true}}$, the conditional market share, S_{gl}^c and the segment share, S_g can take six values, from 0 to 1 in steps of 0.2, in each of the four segments.⁴ This implies 1296 alternative combinations for S_{gl}^c (6^4), and 56 different possibilities for S_g and $\frac{\pi_{gl}}{\Pi_l^{true}}$ (notice that for a combination to be valid both segment shares and ratio of profits must sum up to 1 across segments, this restricts the total number of valid possibilities). Next, I compute all possible values of S_l that can be obtained from the combination of S_{gl}^c and S_g ($1296 \times 56 = 72576$). Then, S_l and S_{gl}^c are used to compute the 72576 possible values of $\frac{d_{gl}}{S_l}$ in each of the 4 segments. This is combined with the 56 possible alternatives for $\frac{\pi_{gl}}{\Pi_l^{true}}$ to compute the associated error to each of the 4 064 256 possible combinations (72576×56). In this manner we can have the approximation error for quite different industry structures, for example cases where the ratio of profits is similar to segment shares while the relative difference is very high, or very small. The only parameters that are restricted to not vary are μ_2 and the number of segments. μ_2 enter the expression of the approximation error linearly, so it is easy to infer what would be the impact of lowering or increasing it. The

⁴The Matlab script used to perform the numerical simulations is available from the author upon request.

number of segments is chosen somewhat arbitrarily to fit standard segmentation patterns, but also to prevent the number of possible combinations to become prohibitively large.

Figure 1 shows the absolute value of approximation errors (z-axis) for each of the possible combinations of $\frac{d_{gl}}{S_l}$ (x-axis) and $\frac{\pi_{gl}}{\Pi_l^{true}}$ (y-axis) described above. Each point of the x-axis represents one of the 72576 possible combinations of $\frac{d_{gl}}{S_l}$, each of these combinations has 4 elements, one for each segment. The 56 combinations of $\frac{\pi_{gl}}{\Pi_l^{true}}$ are represented in the y-axis. Each of them is also composed of 4 elements. The z-axis represents the errors associated with all possible combinations of 4-tuples of $\frac{\pi_{gl}}{\Pi_l^{true}}$ and $\frac{d_{gl}}{S_l}$ as given by expression (27). The 4-tuples in x- and y-axis are sorted according to the sum across segments of the absolute values of $\frac{d_{gl}}{S_l}$ and $\frac{\pi_{gl}}{\Pi_l^{true}}$, respectively. Therefore, elements located closer to the origin are meant to represent smaller aggregate deviations. The z-axis is scaled in per unit terms (for example, a value of 0.1 in the z-axis represents an error of 10% as computed from expression (27)). Figure 1 provides several insights: firstly, errors can be very large and very small, depending on how the elements combine. Secondly, $\frac{d_{gl}}{S_l}$ is the leading factor in increasing the size of the error. Thirdly, approximation errors become of unreasonable magnitude only when the relative differences in shares, $\frac{d_{gl}}{S_l}$, are also extremely high. As it was said before, we can compute errors for all possible combinations, even for those combinations that are not too likely to occur.

Having these facts in mind, it is more interesting to focus on industry structures that may be of particular interest in theoretical or applied work. For instance, consider again an industry with 4 segments of equal size ($S_1 = S_2 = S_3 = S_4 = 0.25$) and $\mu_2 = 0.4$. Now, the conditional share of a firm in any segment is restricted to take values between 0 and 0.2 in intervals of 0.05 ($S_{s_l}^c = 0, 0.05, 0.10, 0.15, 0.2$). This means that in each segment there is at least 5 operating firms. The set of values that $\frac{\pi_{gl}^v}{\Pi_l^{v,true}}$ can take is trimmed to exclude the extreme cases where the firm obtains more than 90% or less than 10% of its total profits from only one segment. The idea here is that we want to concentrate on industry configurations where firms are really making business in more than one segment. Under this new parametrization we can compute again approximation errors. The result is plotted in Figure 2 and we can observe that now errors are always below 20% and

in fact they are in almost all cases even below 10%. Interestingly, we obtain such low approximation errors even for large differences between conditional shares and total shares. Recall that d_{gl} measures how well our assumption is satisfied. Figure 3 plots the sum of the absolute values of $\frac{d_{gl}}{S_l}$ across segments, i.e., for each of the 625 possible 4-tuples of $\frac{d_{gl}}{S_l}$ that correspond to the x-axis of Figure 2, the term $\sum_{g=1}^4 \left| \frac{d_{gl}}{S_l} \right|$ is computed. The values are sorted from smaller to larger as in Figure 2 and the y-axis of Figure 3 is also expressed in per unit terms. By comparing Figure 3 with the shape of Figure 2 along the x-axis we can see that the largest approximation errors correspond to the largest deviations from the assumption of equal conditional shares. But the approximation errors remain low (below 10%) even for large deviations of 300% to 400%. It is not until deviations reach values over 400% that approximation errors get larger, and even in that case they remain below 20%. This occurs because, as noted in section 3, the approximation error depends on several factors that can keep the error small even if assumption 1 is not close to be fulfilled (see remark 4). This suggests that this approximation can therefore be applied even if the conditional market shares of firms are not really that close one another. The suitability of the approximation will be determined also by the particular structure of the industry of interest. Just by knowing the number of segments and firms' shares we can be able to recover the approximation errors for alternative combinations of $\frac{\pi_{gl}}{\Pi_l^{true}}$ (assuming we cannot know firms' profits) using expression (27). If the errors so computed are small enough then we can employ the approximation for our analysis, regardless of how large the differences between firms' conditional shares might be.

So far, we have looked only to one particular industry configuration. Let's see now what happens when we move to alternative settings. If we reduce the least number of firms per segment to 4 (i.e., S_{gl}^c can be 0.25) we observe that the maximum of the errors is a bit above 20% but still they are mostly below 10% (Figure 4). When we allow for only at most two firms per segment we start having some larger approximation errors, although still in most cases they are very low (Figure 5). Therefore, as the number of firms in each segment increases the approximation error quickly decreases. This happens because the approximation error as computed in expression (27) is increasing in firm's

market share, S_l ,⁵ and the smaller the number of firms the larger will tend to be, in general, their market shares.

The previous results were obtained assuming equally sized segments. If we consider a more polarized structure where for example two of the segments are large and two are small ($S_g = 0.1, 0.4, 0.4, 0.1$) we will have higher errors (compare Figures 2 and 6), but still similar to those of Figure 5. Assuming a less polarized segmentation, $S_g = 0.2, 0.3, 0.3, 0.2$, will significantly reduce the size of errors (Figure 7) to the levels that we have in the base case of Figure 2. Therefore, when the size of the segments is not too different the approximation errors are small. As was shown in section 3 (remark 4), the approximation error is smaller the more similar are segment shares, S_g , and relative profits, $\frac{\pi_{gl}}{\Pi_l^{true}}$. It turns out that when segment shares are equal or similar, the differences between S_g and $\frac{\pi_{gl}}{\Pi_l^{true}}$ tend to be smaller on average as compared with cases where the segment shares are more polarized. As a consequence, the approximation errors are smaller in those cases.

In summary, the comparison of results for alternative specific industry configurations suggests that, with four segments, when we have five or more firms in each segment and/or when segments have a relatively similar size the approximation errors from using a simplified expression for equilibrium profits will be very small or even negligible. Only when the number of firms is very small and the size of segments is very polarized we may have some particular industry configuration for which the approximation error gets larger. Arbitrarily large errors are only likely to occur when we consider situations that simultaneously combine several extreme (and perhaps unlikely) cases. Therefore, irrespective of what the equilibrium strategies may be, the numerical simulations show that whenever the equilibrium market shares associated to them lie within a certain interval, the second summand of equation (13) will be small enough to ensure that the approximation error from using equation (22) is also small.

⁵Recall remark 2 in section 3.

4.2 A real data example

The numerical examples give some insights on the type of situations where the approximation in Proposition 2 may be valid. This subsection illustrates its application to data for the Spanish car market during the 1990's. The data set contains monthly information on new car registrations for all firms in the market from January 1990 to December 2000. This is a segmented market where up to eight different groups of products are defined, based on both vertical and horizontal differentiation. These segments are: Small-Mini, Small, Compact, Intermediate, High-Intermediate, Luxury, Sport and Minivan. Therefore, the data set provides information on real market shares at the segment and market levels.⁶ Unfortunately, the information on profitability is not available. During these eleven years a total of 33 brands are present in the market, which gives a total of 363 possible firm-year observations. Nevertheless, due to entry and exit, not all the firms are operating all the time. The observed number of firm-year combinations is 323. During the sample period, most firms operate in 3 to 5 segments. Only a few firms in some given years sell products in 7 or 8 segments.

In this market, the difference between conditional shares and market shares varies a lot across firm-year observations. Figure 8 displays the sum, for each firm-year, of d_{gl} relative to S_l in absolute value in all segments (measured in per unit terms). Figure 10 presents the sum of absolute values of deviations in absolute terms, i.e., $\sum_g d_{gl}$. In both cases it is clear that some firms are very far from satisfying assumption 1, while in other cases it may be plausible. Figures 9 and 11 show that the frequency of extreme violations of assumption 1 is small, but that still large differences exist. Consequently, we can anticipate that by making use of the approximation proposed in this paper, we will incur in some approximation error. However, as discussed in the previous section, the fact that d_{gl} is large does not necessarily imply that the approximation error is also large.

Therefore, for each firm-year combination the approximation error as given by expression (27) is computed.⁷ The value of μ_2 is assumed to be 0.4, as in the numerical

⁶Some additional information on product characteristics is also available. However, this is not relevant for the application here. See Moral & Jaumandreu (2007) for more details on the data set.

⁷The period of reference for the approximation errors is the year because this avoids the influence of

examples. The ratio $\frac{\pi_{gt}}{\Pi_t^{true}}$ is also unknown and therefore the same approach of the previous subsection is followed: For each firm-year all possible combinations of segment relative profitability are computed, assuming that it must be between 0 and 1 and increasing it in steps of 0.05 (i.e. 5%). In order to avoid extremely unrealistic cases, the combinations where at least one segment represents less than 10% or more than 90% of total profits, are excluded. For example, in 1990 Ford was operating in five segments, this implied 771 possible combinations of relative profitability like (0.1, 0.1, 0.1, 0.1, 0.6) or (0.4, 0.1, 0.25, 0.15, 0.1), but not (0.05, 0, 0, 0, 0.95) or (0.05, 0.05, 0.2, 0.2, 0.5) because they would imply that some segments contribute less than 10% or more than 90% to total profits. The number of possible combinations varies as the number of segments where the firm is present varies. So, for instance, in 1994 Ford was selling cars in six segments and then the total number of possible combinations was 923, like (0.1, 0.1, 0.1, 0.1, 0.1, 0.5) and so on.

Under these conditions, Figure 12 plots the size of approximation errors. The x-axis contains each observed firm-year combination, the y-axis represents the maximum number of possible combinations of relative profitability. We can see that errors are in general very small for all firms in all years, with just very few outliers. The largest error in absolute value is of 36%.

Figure 13 displays the histogram of approximation errors and shows that they are almost always below 10%. That error of 36% actually corresponds to the brand Renault in 1990. In that year, Renault was operating in five segments selling a total of 164798 units (57673 in Small, 70033 in Compact, 34983 in High-Intermediate, 1178 in Luxury and 931 in Minivan). Regarding the relative profitability per segment, a total of 771 combinations, as described above, were possible for Renault in 1990. It turns out that the largest approximation error of 36% corresponds to a particular combination of relative profitability that assigns a very low weight to the Small and Compact segments while assigning a large one to Minivan and Luxury. More precisely, the error of 36% is the result

seasonal patterns and the potential perturbations that more volatile, higher frequency data could induce. Moreover, the year is usually one important period of reference for firms in terms of sales and profits objectives.

of assuming that the Small and Compact segments represent only a 20% of Renault's total profits in 1990, while they account for more than 77% of sales in that period, and at the same time the Minivan segment is assumed to generate the 60% of total profits being just a 0.5% of Renault's sales. It seems very unlikely that this combination of profitability per segment may be close to the true one. However, even in this extreme situation the approximation error is just a 36%. The remaining outliers in Figure 13 can be explained in the same way.

Therefore, this example shows that even for unreasonable combinations of relative profitability, the maximum error we can have for any firm in the Spanish market is not larger than 36%. Whenever the relative profitability is not completely dissociated of market shares, the approximation error is very small, as Figure 13 shows. This means that the equilibrium profits from using a nested logit in the price competition stage can be fairly well approximated by a multinomial logit expression.

5 Concluding remarks

This paper proposes an approach to facilitate the backward induction solution of multi-stage oligopoly games where in the last stage multi-product firms compete in prices in the context of a nested logit framework with two nests. I show that when firms have similar conditional market shares across segments the analytical expression for equilibrium profits of a nested logit is identical to the multinomial logit counterpart. However, the equilibrium prices and quantities will obviously be different in both specifications. What becomes identical is the functional form of equilibrium profits with respect to the strategic variables and parameters of the problem in the last subgame. However, this is enough to simplify the resolution of the previous subgames of the problem because the new expression, while still non-linear, is much less involved.

I also compute the approximation error that occurs if conditional shares are not exactly equal across segments. When dealing with real data, even if the assumption is plausible it is likely that conditional shares are not numerically identical. Hence, it is important to know by how much the approximated equilibrium profits in the last stage of the game

would differ as compared to the true ones. I show, using numerical simulations, that for many regular industry configurations the approximation error is actually small or negligible even when the assumption is not fulfilled by a large amount. Therefore, in these cases the equilibrium profits in a simpler multinomial framework can be regarded as a good approximation to those that would obtain in a nested logit context.

Finally, the simplifying approach proposed in the paper is also applied to real data from the Spanish car market in 1990-2000. The approximation errors turn out to be very small, even if for some firms the assumption of equal conditional shares across segments is far from being satisfied. This suggests that the scope for the application of this simplifying approach can be broad.

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Figures

Figure 1: Approximation errors for all the combinations of $\frac{\pi_{gl}}{\Pi_l^{true}}$, S_{gl}^c and S_g taking values (0, 0.2, 0.4, 0.6, 0.8, 1)

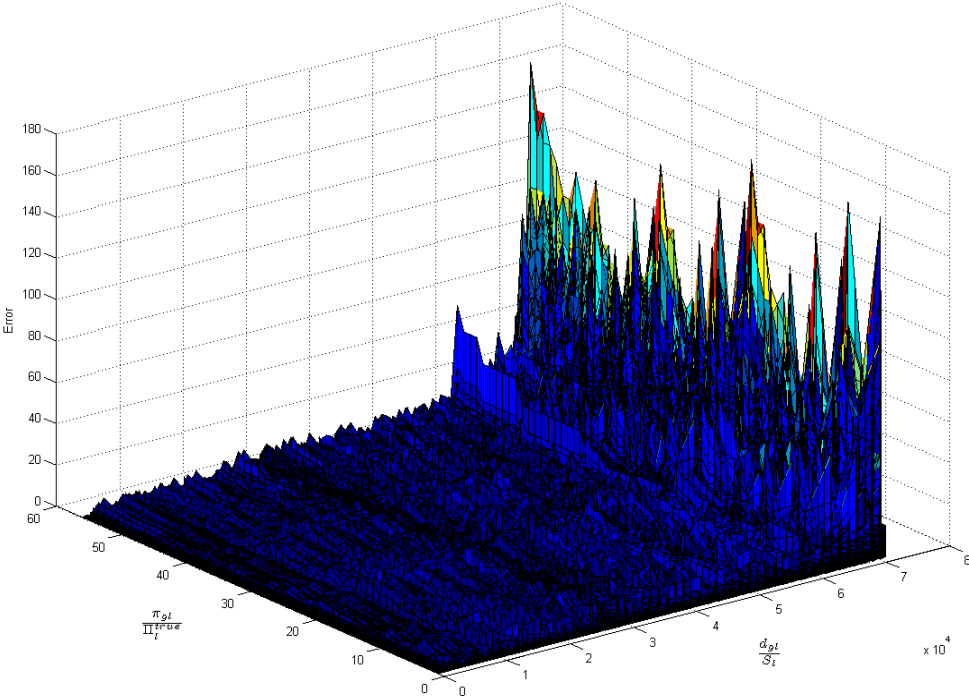


Figure 2: Approximation errors for all the combinations of $S_g = (0.25, 0.25, 0.25, 0.25)$, $\frac{\pi_{gl}}{\Pi_l^{true}} = (0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9)$ and $S_{gl}^c = (0, 0.05, 0.1, 0.15, 0.2)$

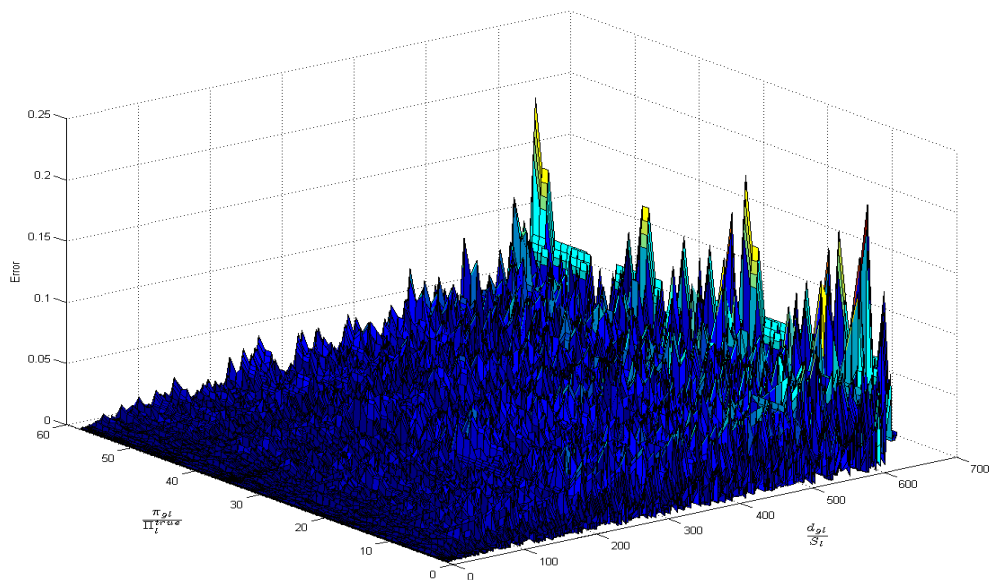


Figure 3: Sum of absolute values of relative differences across segments for the configurations of the x-axis of Figure 2

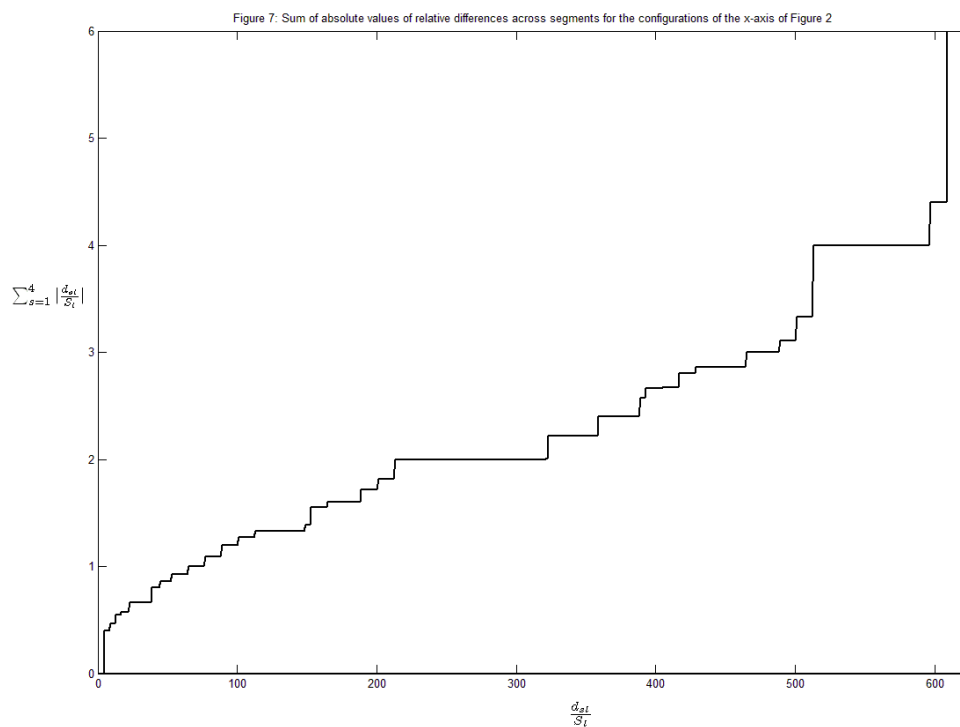


Figure 4: Approximation errors for all the combinations of $S_g = (0.25, 0.25, 0.25, 0.25)$, $\frac{\pi_{gl}}{\Pi_l^{true}} = (0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9)$ and $S_{gl}^c = (0, 0.05, 0.1, 0.15, 0.2, 0.25)$

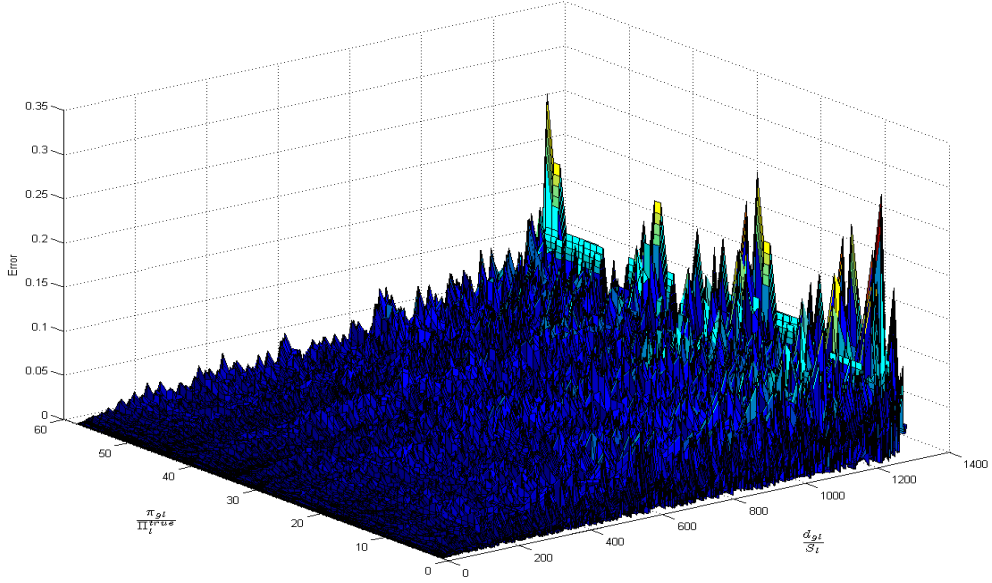


Figure 5: Approximation errors for all the combinations of $S_g = (0.25, 0.25, 0.25, 0.25)$, $\frac{\pi_{gl}}{\Pi_l^{true}} = (0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9)$ and $S_{gl}^c = (0, 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5)$

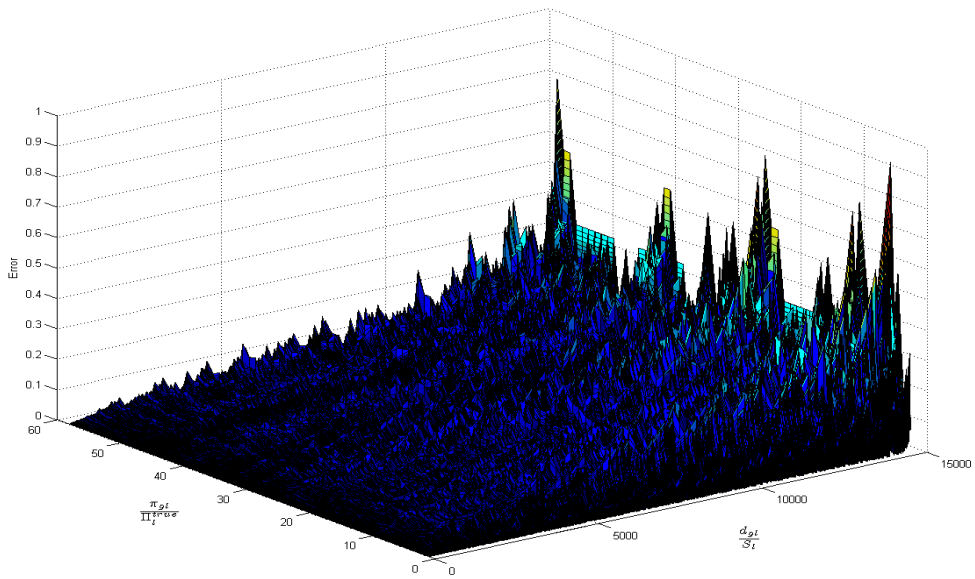


Figure 6: Approximation errors for all the combinations of $S_g = (0.1, 0.4, 0.4, 0.1)$, $\frac{\pi_{gl}}{\Pi_i^{true}} = (0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9)$ and $S_{gl}^c = (0, 0.05, 0.1, 0.15, 0.2)$

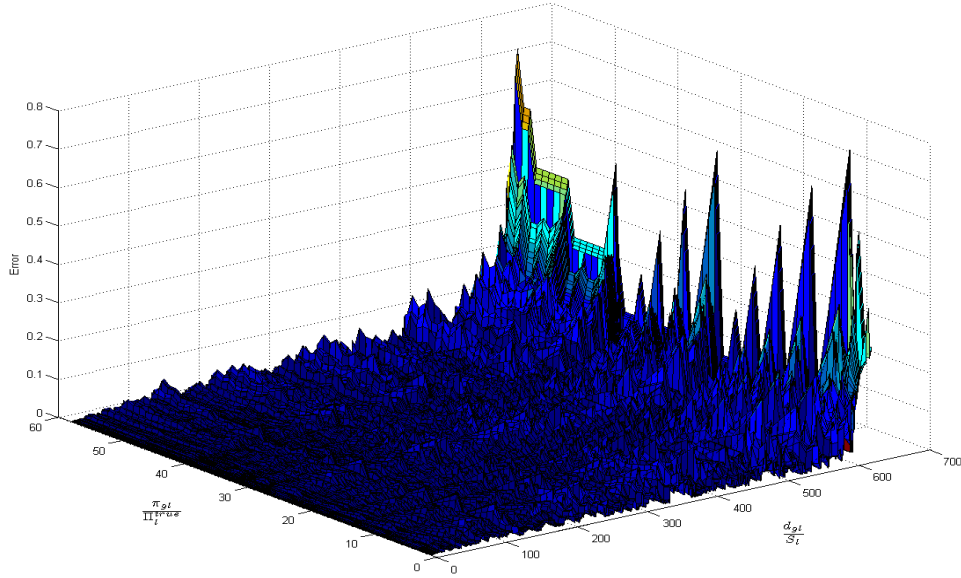


Figure 7: Approximation errors for all the combinations of $S_g = (0.2, 0.3, 0.3, 0.2)$, $\frac{\pi_{gl}}{\Pi_i^{true}} = (0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9)$ and $S_{gl}^c = (0, 0.05, 0.1, 0.15, 0.2)$

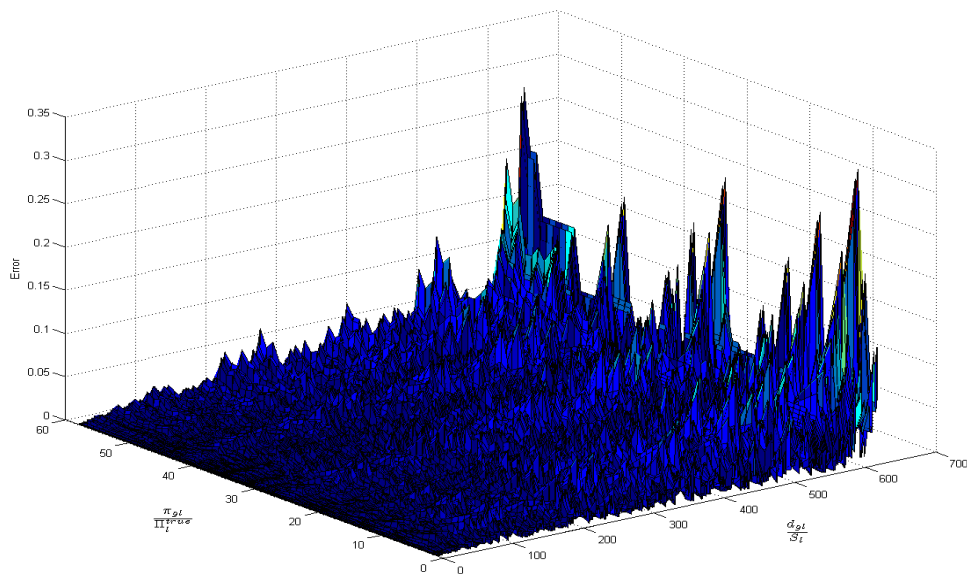


Figure 8: Assessment of the degree of fulfillment of Assumption 1 in the data for the Spanish car market (in relative terms)

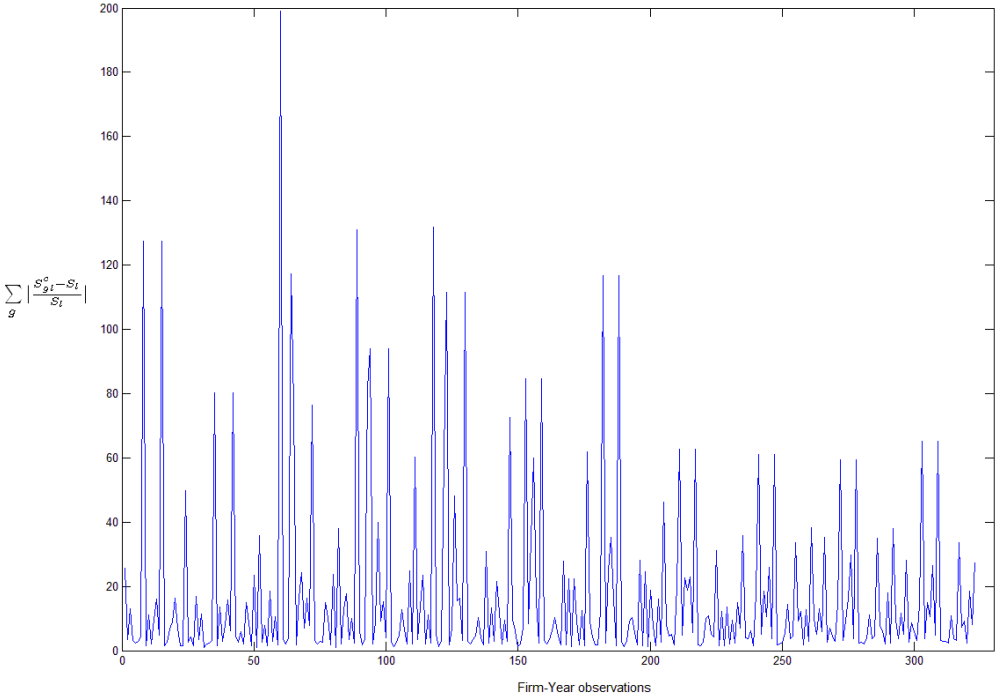


Figure 9: Histogram of the departures from Assumption 1 (in relative terms) for all firms in the Spanish car market

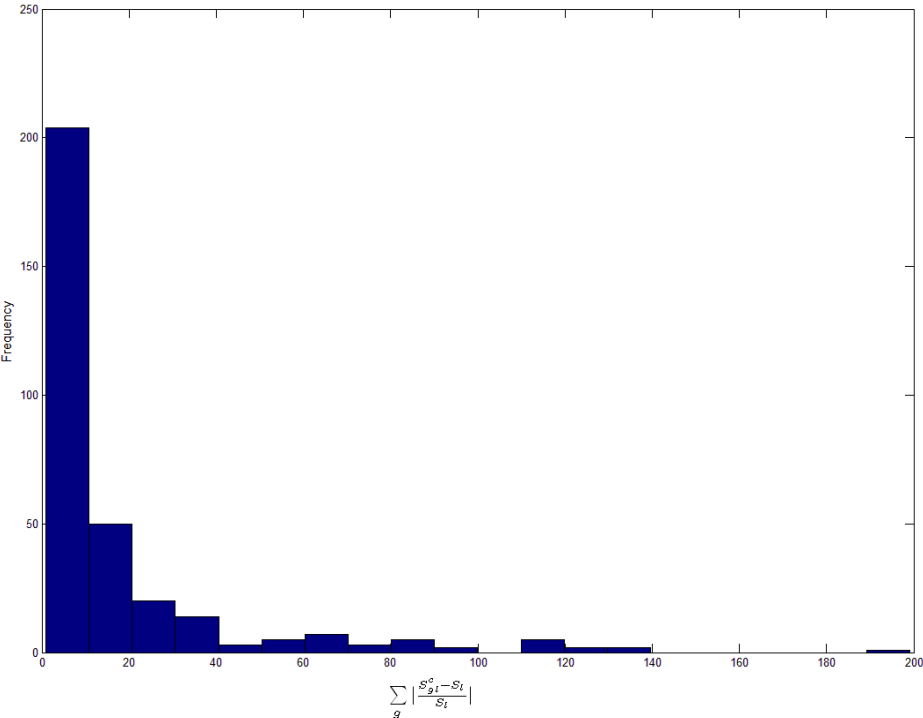


Figure 10: Assessment of the degree of fulfillment of Assumption 1 in the data for the Spanish car market (in absolute terms)

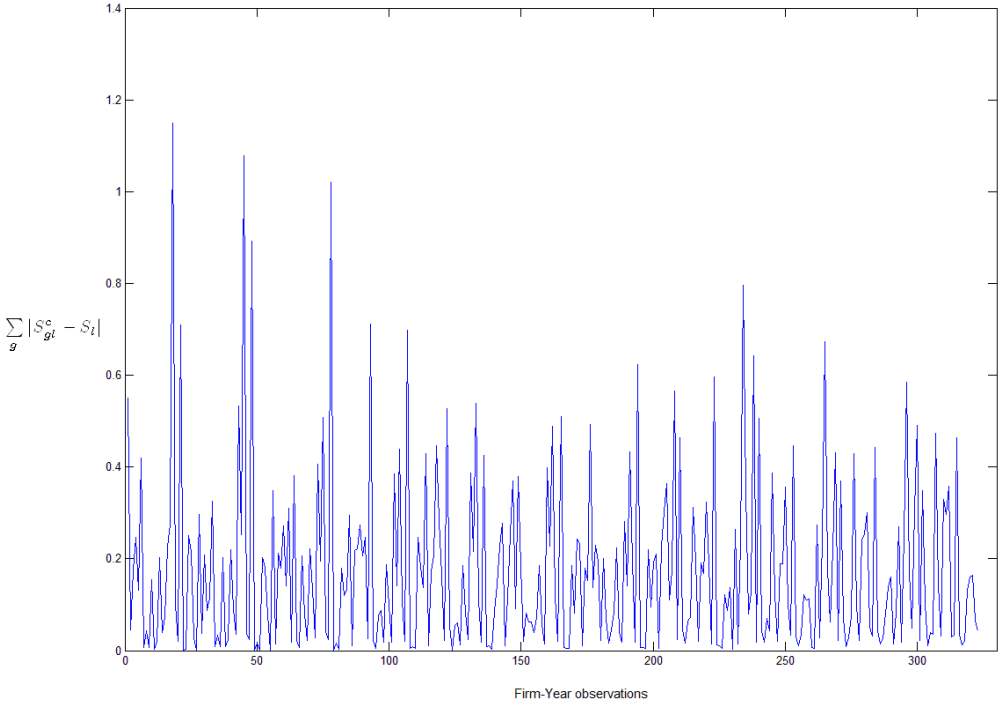


Figure 11: Histogram of the departures from Assumption 1 (in absolute terms) for all firms in the Spanish car market

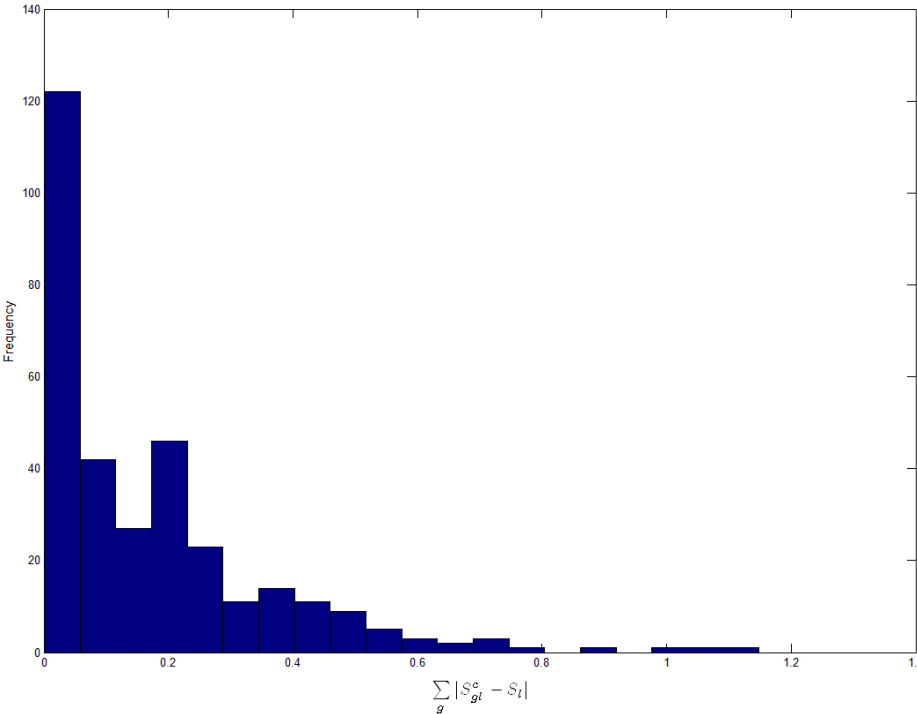


Figure 12: Approximation errors using data on the Spanish car market and considering $\frac{\pi_{gl}}{\Pi_i^{true}} = (0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9)$

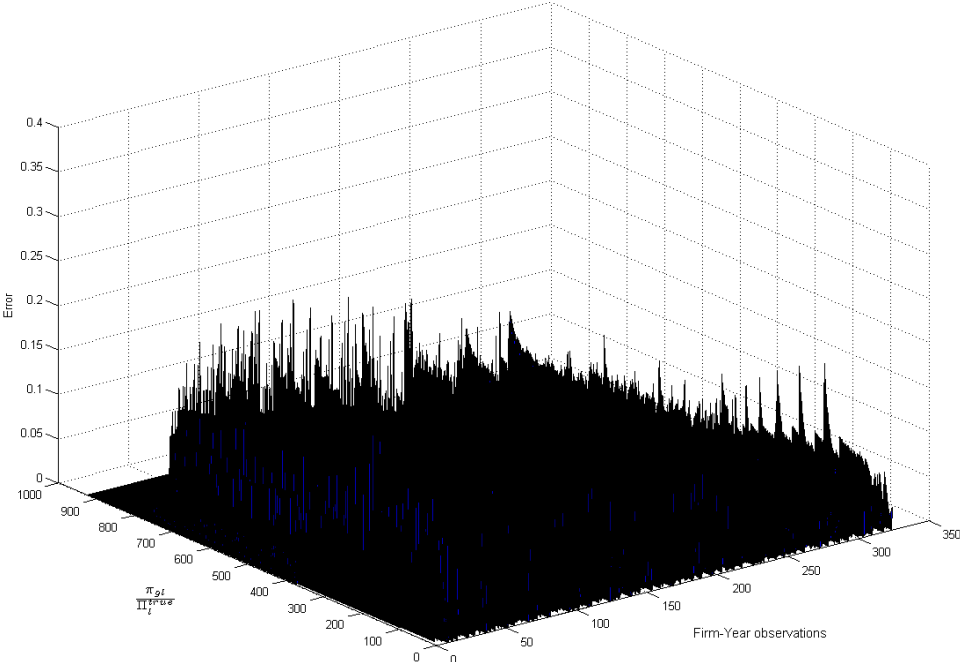


Figure 13: Histogram of approximation errors using data on the Spanish car market and considering $\frac{\pi_{gl}}{\Pi_i^{true}} = (0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9)$

