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Fair bounds based solidarity.

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Abstract

How should scholarships be distributed among the (public) higher education students? We raise this situation as a redistribution problem. Following the approach developed in Fleurbaey (1994) and Bossert (1995), redistribution should be based on the notion of *solidarity* and it re-allocates resources taking into account only agents' *relevant* characteristics. We also follow Luttens (2010a), who considers that compensation of relevant characteristics must be based on a lower bound on what every individual deserves. In doing so, we use the so-called *fair bound* (Moulin (2002)) to define an *egalitarian* redistribution mechanism and characterize it in terms of *non-negativity, priority in lower bound* and *solidarity*. Finally, we apply our approach to the scholarships redistribution problem.

Keywords: Redistribution mechanism, Lower bounds, Scholarship, Solidarity
JEL classification: C71, D63, D71.

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1. Introduction

Public higher education is often partially borne by governments. However, as mentioned in Johnstone (2004), *"higher education is also costly [...], especially when governments are also besieged with other pressing public needs, many of which seem more politically compelling than the claims of higher education"*. Then, this higher education cost must be shared by: (1) the government; and (2) parents/students.

Governments also typically provide some types of financial aid (*scholarships*) for higher education, consisting of grants, work-study programs and tuition waivers. The most common scholarships consider both the student's academic achievement, or high scores on standardized tests, and the student family's financial record. The first characteristic (that we will call *effort* or *relevant characteristic*) does not deserves compensation. The second one (that we will call *skill* or *irrelevant characteristic*) is the one that should be compensated in order to *equalize the opportunities among students coming from different financial situations.* That is, the purpose is to compensate the inequality due to irrelevant characteristics (skill), while preserving inequality due to relevant characteristics (effort).

In this paper, we consider this situation as an example of a redistribution problem, that we name *scholarship redistribution problem*, where each student should be assigned an amount, x_i, which represents the scholarship she receives (if positive) or additional tuition fees she must pay (if negative).[1]

If we suppose that inequalities among agents (students) are determined by unequal exerted effort levels and different innate skills (or family income), the aim of fair income redistribution is to guarantee an equal income for individuals exerting the same effort (*the principle of compensation*) and to perform equal income transfers to individuals with equal skills (*the principle of natural reward*). It is well known that, in many contexts, there does not exist a redistribution mechanism that satisfies both the principle of compensation and the principle of natural reward simultaneously. As a result, the literature has concentrated on dealing with such trade-off between both principles. A specific route along those lines has been to strengthen the principle of compensation to the solidarity one, a principle with a long tradition in the

[1] Redistribution means that the sum of all x_i equals 0.

theory of justice.[2]

Specifically, we follow the model developed in Bossert (1995). He proposes a quasi-linear approach to this problem and establishes that the re-allocation of resources must only consider a set of *relevant* characteristics. These features elicit compensations that are assigned on a *additive solidarity* basis.

Recently, in Luttens (2010a) a strengthening of the solidarity principle has been proposed. An income gain (loss), generated by a change in the skill profile, is shared on the basis of the information contained in a lower bound in what every individuals must receive. Our model follows the analysis in Luttens (2010a) by considering a lower bound on what each individual deserves. By requiring some *solidarity conditions relative to the lower bound*, and the *respect of this lower bound* we obtain compatibility with a version of the principle of natural reward.[3]

The article is organized as follows. In the next section, we present the model and introduce the basic definitions and axioms. Section 3 proposes and characterizes our egalitarian mechanism. Finally, Section 4 is devoted to apply our approach to the scholarship redistribution problem. Some final remarks are contained in Section 5 and the proof of our main result is given in an Appendix.

2. The model

2.1. ***Fair monetary compensation model***

As we have already mentioned, we adopt the approach developed by Bossert (1995). Let us denote by $N = \{1, ..., n\}$ the finite population of size $n \geq 2$. Individuals are distinguished by two characteristics: *skill* and *effort*:

- The characteristic which elicits compensation, skill, is given by a real number $y \in \mathbb{Y}$, where $\mathbb{Y}$ is an interval of $\mathbb{R}_+$. The skill profile is the vector $y_N = (y_1, \ldots, y_n)$. Individuals' skills are compensated by an

[2] See Fleurbaey and Maniquet (2007) for a survey in this literature. We follow this paper for notation and definitions.

[3] We will use the Equal Resource for Uniform Talent (ERUT) condition (Fleurbaey (1994),Bossert (1995)). This condition is incompatible with additive solidarity (see Fleurbaey and Maniquet (2007)).

amount x_i of a transferable resource (*money*). Note that x_i is a real number that can be positive (*subsidies*) or negative (*taxes*).

- The characteristic which does not elicit compensation, effort, is also a real number $z \in \mathbb{Z}$, where $\mathbb{Z}$ is an interval of $\mathbb{R}_+$. The effort profile is $z_N = (z_1, \ldots, z_n)$. Without loss of generality we assume that individuals are ranked: $z_1 \leq z_2 \leq \ldots \leq z_n$.

An *economy* consists of the pair that contains skill and effort profiles, $e = (y_N, z_N)$. Let $\mathcal{E}$ be the set of economies, $\mathcal{E} \subseteq \mathbb{Y}^n \times \mathbb{Z}^n$. Given an economy $e = (y_N, z_N) \in \mathcal{E}$, it is assumed that (quasi-linear) utility functions $u : \mathbb{R} \times \mathbb{Y} \times \mathbb{Z} \rightarrow \mathbb{R}$ are as follows:

$$u(x_i, y_i, z_i) = x_i + v(y_i, z_i).$$

Utility measures a monetary outcome (final outcome after redistribution). The pre-tax income function, $v : \mathbb{Y} \times \mathbb{Z} \rightarrow \mathbb{R}_{++}$, is supposed to be strictly increasing in y, and that it is not additively separable in y and z, $v(y_i, z_i) \neq v_1(y_i) + v_2(z_i)$. The total sum of pre-tax incomes is denoted by $R = \sum_{i \in N} v(y_i, z_i)$.[4]

An allocation $x_N = (x_1, ..., x_n) \in \mathbb{R}^n$ is the vector defined by transferable resources x_i. We assume, for simplicity, that the total amount to be distributed is $\Omega = 0$, so that we are looking at a *redistribution problem* (subsidies coincide with taxes). An allocation for an economy $e \in \mathcal{E}$ is *feasible* whenever $\sum_{i \in N} x_i = 0$. We denote by $F(e)$ the set of feasible allocations for economy e. Note that all feasible allocations are Pareto efficient since we rule out free disposal in the definition of feasibility. An allocation (redistribution) mechanism is a function $S : \mathcal{E} \rightarrow \mathbb{R}^n : \forall e \in \mathcal{E}, S(e) \subseteq F(e)$.

We assume, as in Luttens (2010a), that individuals, because of the effort they exert, have some *claim* on the total pre-tax income R. Let $g : \mathbb{Z} \rightarrow \mathbb{R}_{++}$ be the *claims function* that assigns to each individual, i, with an effort level, z_i, a claim, $g(z_i)$ that depends on the individual's effort only. We assume

[4] When v is additively separable in y and z, a natural way to redistribute income (that satisfies both the principle of compensation and the principle of natural reward) is to make each individual's income after redistribution equal to the average contribution of y_N plus the individual contribution of z_i in the income generating process (Bossert (1995)).

that function $g(z)$ is continuous and strictly increasing in z. We denote the total sum of claims by $C = \sum_{i \in N} g(z_i)$. It will be a conflicting claims problem whenever $C > R$. One particular example to model claims within the context of fair income redistribution is to use median incomes as claims: $g_m(z_i) = v(\bar{y}, zi)$, for all $i \in N$, where $\bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i$.

2.2. ***Solidarity***

Before presenting the solidarity axiom, some notation will be helpful. Given two economies which only differ on skill profiles, $e = (y_N, z_N)$ and $e' = (y'_N, z_N)$, changes in any function, $h(e)$, are denoted by: $\Delta h = h(e') - h(e)$. This notation will be used to represent changes in functions u, v and g, as well as changes in variable x.

In this context, what does *solidarity* mean? Solidarity is a well known principle in the literature on redistribution (see Thomson (1988), Roemer (1986) and Fleurbaey and Maniquet (2007)) where it appears as a way of compensating irrelevant characteristics. The main idea is that a change in the resources affects all agents in the same direction. We use here the following strengthening of Solidarity, due to Bossert (1995), which is based on the argument that there is no reason to make some agents benefit unequally from variations in the profile. In particular, it would be undesirable to let an agent whose characteristics are improved to benefit more than other agents. This notion of solidarity (*additive solidarity*) implies that if the effort they exert is the same, then each agent will finish at the same utility level after redistribution. Formally,

Axiom 1. ADDITIVE SOLIDARITY *(AS, Bossert (1995))*
For each $e = (y_N, z_N), e' = (y'_N, z_N) \in \mathcal{E}$,

$$\Delta u_i = \Delta u_j, \quad \forall i, j \in N.$$

Remark 1. *Note that the increment on the amount to be shared* ΔR *(positive, or negative) only comes from changes in the skill, and not in the effort.*

2.3. ***Principle of Natural Reward***

We now analyze the principle of natural reward, namely, the goal of compensating only irrelevant characteristics (skill) and not other characteristics (effort). The main idea is that an agent with a better skill than another one should not receive more resources in the redistribution. The axiom we will use here is due to Fleurbaey (1994) and Bossert (1995).[5]

Axiom 2. EQUAL RESOURCE FOR UNIFORM TALENT *(ERUT, Fleurbaey (1994) and Bossert (1995))*
For each $e = (y_N, z_N) \in \mathcal{E}$,

$$\textit{if } \forall i, j \in N \; y_i = y_j, \; \textit{then } x_i = 0 \quad \forall i \in N.$$

2.4. ***Lower bounds***

The notion of a lower bound (on what each individual deserves) is a classical tool in the literature on conflicting claims (see, for instance, Giménez-Gómez and Marco-Gil (2008) and Dominguez (forthcoming)). In the context of compensation, Moulin (1994) suggests to define a bound based on what an agent would obtain if others shared her efforts.

Axiom 3. EGALITARIAN BOUND *(EB, Moulin (1994))*
For each $e = (y_N, z_N) \in \mathcal{E}$,

$$\forall i \in N, \; u_i \leq \frac{R}{n} \quad \textit{or} \quad \forall i \in N, \, : u_i \geq \frac{R}{n}.$$

Although this axiom clearly has a favor of compensation, one can argue that it also contains a pint of natural reward, because it forbids excessive compensation (see Fleurbaey and Maniquet (2007) for additional details).

Recently, Luttens (2010a) introduces the idea of *solidarity from a lower bound*. This notion implies equalizing the increment of utility only for agents that have identical increment in their lower bound.[6]

Formally,

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[5] Note that this is a mild axiom, and the allocation mechanism that assigns $x_i = 0$ for all i fulfills it.

[6] Luttens uses in his definition of solidarity the *minimal rights* lower bound (O'Neill (1982)): Given an economy $e = (y_N, z_N)$ and a claims function g, the minimal rights lower bound is defined for each $i \in N$ by: $m_i(e, g) = min \left\{ g(z_i), \max \left\{ 0, R - \sum_{j \in N \setminus \{i\}} g(z_j) \right\} \right\}$.

Axiom 4. Additive Solidarity for equal changes in minimal rights *(AS*, Luttens (2010a))*

For each $e = (y_N, z_N), e' = (y'_N, z_N) \in \mathcal{E}$, *if* $\Delta m_i = \Delta m_j$ *then*

$$\Delta u_i = \Delta u_j.$$

This axiom proposes a different strengthening of the ethical principle of Solidarity by sharing the increment in the total pre-tax income, generated by a change in the skill profile, on the basis of the information contained in individuals' minimal rights, that depends on her exerted effort level, but does not depend on her skill. In Luttens (2010b) this axiom is used to characterize bankruptcy rules.

In general, given an economy $e = (y_N, z_N)$ and a claims function $g(z)$, a *generic* lower bound b is a function that assigns to any agent $i \in N$, $b_i(e, g)$ a minimum amount that should be guaranteed to individual i. An elemental fairness principle implies that if there is not enough resources to satisfy all claims, then no agent should have guaranteed an amount higher than her claim; that is,

$$R \leq C \Rightarrow b_i(e, g) \leq g(z_i) \quad \forall i \in N.$$

We are interested in lower bounds such that an increasing increment in the total pre-tax income, generated by a change in the skill profile, provides an increment in the lower bound, wherever possible (*monotone* lower bound).

Definition 1. *For each* $e = (y_N, z_N), e' = (y'_N, z_N) \in \mathcal{E}$, *and each claims function* $g(z)$, *a bound* $b(e, g)$ *is said to be monotone if*

$$\Delta R > 0 \quad \Rightarrow \quad \begin{cases} \Delta b_i > 0, \textit{ for some } i \in N \textit{ ; or} \\ \\ b_i(e, g) = g(z_i), \textit{ for all } i \in N \end{cases}$$

This is a very restrictive requirement and most of the lower bounds defined in the bankruptcy literature fails it. However, we think it is a desirable property, since it ensures that the extra resources generated by changes in the skill profile are distributed.

The lower bound we use, that we call *fair bound* (Moulin (2002)), follows the idea of solidarity proposed in Bossert (1995) since it guarantees a strictly positive amount of the resources to each agent, independently of the other agents' claims. Moreover, this lower bound is monotone. Specifically, the so-called *fair bound* provides an egalitarian distribution of the resources, truncated by the claim.

Definition 2. *(Moulin (2002)) Given an economy* $e = (y_N, z_N)$ *and a claims function* $g(z)$*, the fair bound* $f(e, z)$ *is defined, for each* $i \in N$*, by:*

$$f_i(e, g) = \min \left\{ g(z_i), \frac{R}{n} \right\}.$$

Remark 2. *Obviously the fair bound is related to the EB property (Axiom 3, Moulin (1994)). The difference is that the egalitarian sharing of* R *is truncated by the claims function. On the other hand, it can be interpreted in terms of sustainability (see Herrero and Villar (2002)):*

$$f_i(e, g) = \begin{cases} g(z_i) & \textit{if this claim is sustainable} \\ \frac{R}{n} & \textit{in other case} \end{cases}$$

2.5. ***Additional Axioms***

Now we introduce some additional requirements that an allocation mechanism S should satisfy. The first property establishes that when the total pre-tax income (*resources*) equals the aggregate claim, then each agent's utility equals to her claim. The second one estates that if the resources are not enough to satisfy the aggregate claim, then individual monetary outcome can not exceed the corresponding claim. Both are usual properties in the literature about conflicting claims problems.

Axiom 5. Claims Feasibility *(CF)*

An allocation mechanism, S, satisfies claims feasibility, if for each $e \in \mathcal{E}$*, each* $x_N \in S(e)$ *and each* $i \in N$*, if* $R = C$*, then* $u(x_i, y_i, z_i) = g(z_i)$*.*

Axiom 6. Claims Boundedness *(CB)*

An allocation mechanism, S, satisfies claim-boundedness, if for each $e \in \mathcal{E}$*, each* $x_N \in S(e)$ *and each* $i \in N$*, then* $R \leq C$ *implies* $u(x_i, y_i, z_i) \leq g(z_i)$*.*

Next properties were introduced by Luttens (2010a), where the lower bound being used is the *minimal rights*. We define them for a generic lower bound $b_i(e, g)$. The first one is a direct adaptation of Axiom 4.

Axiom 7. ADDITIVE SOLIDARITY FOR EQUAL CHANGES IN LOWER BOUNDS *(ASB)*

An allocation mechanism, S, satisfies additive solidarity for equal changes in lower bound, if for each pair of economies $e = (y_N, z_N)$ and $e = (y'_N, z_N)$ in $\mathcal{E}$, each $x_N \in S(e)$ and $x'_N \in S(e')$ and each $i, j \in N$, if $\Delta b_i = \Delta b_j$, then $\Delta u_i = \Delta u_j$.

Axiom 8. PRIORITY IN LOWER BOUND *(PB)*

An allocation mechanism, S, satisfies priority in lower bound, if for each pair of economies $e = (y_N, z_N)$ and $e = (y'_N, z_N)$ in $\mathcal{E}$, each $x_N \in S(e)$ and $x'_N \in S(e')$ and each $i \in N$, if $N_1 = \{i \in N : \Delta b_i = 0\} \neq \varnothing$, then $\sum_{i \in N \setminus N_1} \Delta u_i = \Delta R$.

Finally, we require that each individual receives a minimum amount of the resources guaranteed by the lower bounds.

Axiom 9. RESPECT OF LOWER BOUND *(RB)*

An allocation mechanism, S, satisfies respect of lower bound $b_i(e, g)$, if for each $e \in \mathcal{E}$, each $x_N \in S(e)$ and each $i \in N$, then $u(x_i, y_i, z_i) \geq b_i(e, g)$.

3. Lower bounds based egalitarian mechanisms

This section provides the definition of our redistribution mechanism, which is based on the idea that changes in the income should be based on changes in the *lower bound*. Moreover, we analyze the properties it fulfills and provide a characterization when the lower bound being used is the *fair bound*.

Definition 3. *Given a claims function $g(z)$, and a lower bound function $b(e, g)$, a lower bounds based egalitarian mechanism, S_b, allocates resources for each economy $e \in \mathcal{E}$ and each $i \in N$, as follows:*

$$(x_i)_{S_b} = -v(y_i, z_i) + d_i,$$

where d_i is defined by:

1. *If $R \leq C$, and $j \in \{0, 1, \ldots, n-1\}$ is chosen such that* $\sum_{i<j \in N} g(z_i) + (n-j+1)g(z_j) < R \leq \sum_{i \leq j \in N} g(z_i) + (n-j)g(z_{j+1}),$

$$d_i \equiv \begin{cases} g(z_i) & \forall i \leq j \in N, \\ \dfrac{R - \sum\limits_{i \leq j \in N} g(z_i)}{n-j} & \textit{otherwise.} \end{cases}$$

2. *If $C < R \leq ng(z_j)$, and $j \in \{1, \ldots, n\}$ is chosen such that $ng(z_{j-1}) < R$*

$$d_i \equiv \begin{cases} g(z_i) + \alpha_i & \forall i < j \in N, \\ g(z_i) + \dfrac{R - C - \sum\limits_{i < j \in N} \alpha_i}{n-j} & \textit{otherwise.} \end{cases}$$

3. *If $R > ng(z_n)$,*

$$d_i \equiv g(z_i) + \alpha_i + \frac{R - ng(z_n)}{n},$$

$$\textit{where } \alpha_i = \max \left\{ 0, \frac{ng(z_i) - C - \sum\limits_{h<i \in N: ng(z_h) \geq C} \alpha_h}{n-i+1} \right\}.$$

Next propositions show general properties of the *lower bounds based egalitarian mechanism* which are independent on the lower bound we use. The elemental proof is omitted.

Proposition 1. *For any claims function $g(z)$ and any lower bound $b(e, g)$, S_b fulfills claims feasibility (CF) and claims boundedness (CB).*

Proposition 2. *For any lower bound $b(e, g)$, if the claims function being used is $g_m(z_i) = v(\bar{y}, zi)$, then S_b fulfills the Equal Resource for Uniform Talent axiom (ERUT).*

3.1. ***Fair bounds based egalitarian mechanism.***

In this section, RB^f, ASB^f and PB^f denote the use of the *fair bound* in RB, ASB and PB properties, respectively. When we use this lower bound, we will refer to our mechanism as *fair bounds based egalitarian mechanism* and denote it as S_f. Our main result characterizes this mechanism in terms of the above mentioned axioms.

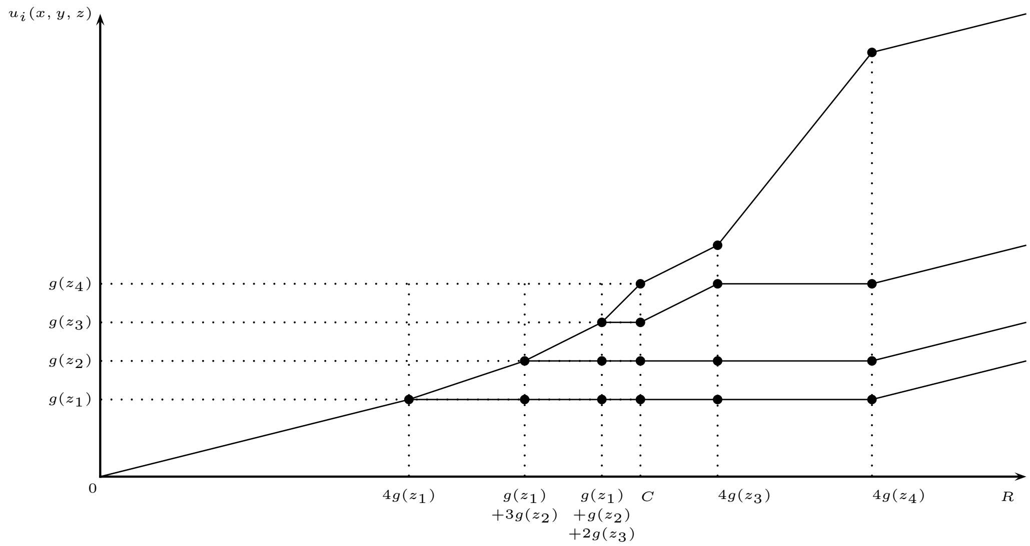

Figure 1: **Fair bounds based egalitarian mechanism.** The horizontal and vertical axis represent different levels of the resources, R, and the total income received by each agent, in a four-agent problem, respectively.

Theorem 1. *$S = S_f \Leftrightarrow S$ satisfies CF, ASB^f, RB^f and PB^f.*

Proof. *See Appendix.*

Figure 1 shows that from zero to the aggregate claim, the *fair bounds based egalitarian mechanism* retrieves the *Constrained Equal Awards* rule.[7] That is, if we focus in the conflicting claims problems, $R < C$, then this mechanism share the income equally among agents until each agent receives her claims. The successive increments are distributed among those agents who experiment changes in their *fair bounds.* Note that by the definition of this bound, only when the resources are bigger than n times the largest claim, no bound changes. So, from this point, the resources are distributed equally among agents again. It must be noticed that this mechanism also satisfies *non-negativity* (which is implied by *respect of fair bound*) and *claim-boundedness.*

[7] See Thomson (2003) for a formal definition of this rule.

4. An actual issue: Scholarship

Let us return to the *scholarship redistribution problem.* For each problem and each student, $i \in N$, we associate an economy $e = (y, z)$, where y_i (the skill) represents the family income (the characteristic which elicits compensation), and z_i (the effort) the academic achievement. Let us consider that the economic effort, after redistribution, is defined by $u(x_i, y_i, z_i) = x_i + v(y_i, z_i)$.

The pre-social economic effort is given by $v(y_i, z_i) = -\alpha(y_i)c(z_i)$. This function contains the higher education cost for a student with an effort level z_i. The component, $c(z_i)$, corresponds to the cost paid by students (their families), which includes tuition and fees, room and board, books and supplies, etc. This function will depend on the effort of students, since their effort determines the number of years to finish college, the total number of subjects, etc. On the other hand, the scalar function α represents the *relative* importance of this cost with respect to the family income. Thus, it is strictly decreasing, convex and we consider it normalized such that $\lim_{y \to 0} \alpha(y) = 1$ and $\lim_{y \to +\infty} \alpha(y) = 0$.

Finally, solidarity is obtained by means of some monetary compensations x_i, representing either admission fees (if negative) or scholarships (if positive). Each student has a claim function, $g_m(z_i) = v(\bar{y}, z_i)$, which represents the economic effort made by a family with an average income $\bar{y} = \frac{1}{n} \sum y_i$. Let $R = \sum v(y_i, z_i)$. and $C = \sum g(z_i)$. It is clear that by convexity of $\alpha(y_i)$, $R \leq C$.

Note that the final cost paid by each student is $c(z_i) - x_i$. Our mechanism equalizes utilities. This fact implies that students have different costs that depend on their income.

4.1. ***Properties.***

Given a *scholarship redistribution problem*, a social planner will propose a redistribution based on the following requirements: (1) Whenever possible, students payment only depends on the effort and no in the family income (*claims feasibility*). (2) If resources increase, then they will be equally distributed among students with equal changes in her fair bounds (*additive solidarity for equal changes in fair bound*). (3) Extra resources are distributed among students that have incremented their effort (*priority in fair bound*). And (4) No one receives more than it deserves due her effort (*respect to fair bound*).

In this case, by Theorem 1 the *fair bounds based egalitarian mechanism* is the only one satisfying all of them. This mechanism implies that,

- If the effort is the same, then the redistribution of the resources is entirely based on income differences (the characteristic that elicits compensation). So that, transfers from the richest students to the poorest ones are made with respect to the income gap.
- If all the students have the same income, no compensation is made.

Finally, with the aim of clarifying the behavior of this redistribution proposal, we present a numerical example.

Example 1. *Let us consider three different levels of effort (L, M, and H):*

Cost / Effort Level	*L(low)*	*M(medium)*	*H(high)*
$c(z)$	70	60	40

Now, consider three different levels of income (l, m, and h):

Income	*l(low)*	*m(medium)*	*h(high)*
α	0.95	0.60	.30

Note that each student is identified by a pair of characteristics (A,a), where $A \in \{L, M, H\}$ *and* $a \in \{l, m, h\}$*. Finally, consider that* $\bar{\alpha} = \alpha(\bar{y}) = 0,50$.

- ***Case 1***: *Three groups of individuals that only differ in their family income* (l, m, h)*. That is, each student has the same level of effort,* $z_i = M$.

Group	$v = -\alpha c$	$g(z)$	x_f	u	*Final cost* $= c - x_f$
(M, l)	−48	−30	14	−34	46
(M, m)	−36	−30	2	−34	58
(M, h)	−18	−30	−16	−34	76

- ***Case 2***: *The level of effort of the second group (l) increases from M to H.*

Group	$v = -\alpha c$	$g(z)$	x_f	u	*Final cost* $= c - x_f$
(M, l)	-48	-30	15	-33	45
(H, m)	-33	-27.5	0	-33	55
(M, h)	-18	-30	-15	-33	75

- ***Case 3***: *The level of effort is not uniform within each group. In the l income group, half of the students exert high effort (H), and the others exert medium effort (M). Students in the m group are divided between H and L. And in the h group, students exert effort levels M and L.*

Group	$v = -\alpha c$	$g(z)$	x_f	u	*Final cost* $= c - x_f$
(H, l)	-38	-20	7.75	-30.25	32.25
(M, l)	-48	-30	17.75	-30.25	42.25
(H, m)	-24	-20	-6.25	-30.25	46.25
(L, m)	-42	-35	7	-35	63
(M, h)	-18	-30	-12.25	-30.25	72.25
(L, h)	-21	-35	-14	-35	84

5. Final comments

In this paper we have analyzed *redistribution problems* by considering the existence of a lower bound on what individuals deserve. Particularly, we have proposed and characterized a new egalitarian mechanism which reallocates the resources based on changes in the *relevant* characteristics, represented by the *fair bound*. This mechanism fulfills both solidarity and natural reward axioms: *non-negativity* and *claim-boundedness*. A fact that Luttens (2010b) only achieves in a very restricted domain.

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Appendix: Proof of Theorem 1

Given an economy $e = (y_N, x_N)$, we define $\overline{e} = (\overline{y}_N, z_N)$ where $\overline{y}_N$ is chosen such that $\overline{R} < ng(z_1)$. So PB^f implies that the "initial income" for $\overline{e}$ is:[8]

$$\overline{x}_i = -v(\overline{y}_i, z_i) + \frac{\overline{R}}{n}, \qquad \forall i \in N.$$

Now, consider $\Delta R = R - \overline{R}$ due to the skill change from $\overline{y_i}$ to y_i of individual $i \in N$. By denoting $\Delta u_i = x_i + v(y_i, z_i) - (\overline{x}_i + v(\overline{y}_i, z_i))$, we know that $\sum_{i \in N} \Delta u_i = \Delta R > 0$.

One of the following three situations occurs for each $i \in N$:

Case 1: $R < C$

(a) If $R \leq ng(z_1)$, from the definition of the *fair bound*, $b_i = \frac{R}{n}$.

Then, by PB^f and ASB^f,

$$\Delta u_i = \frac{\Delta R}{n}.$$

Therefore,

$$x_i = -v(y_i, z_i) + \frac{R}{n}.$$

which coincides with case (1) in Definition 3.

(b) If $ng(z_1) \leq R \leq g(z_1) + (n-1)g(z_2)$, by definition of the *fair bound*,

$$f_1(e, g) = g(z_1) \leq f_i(e, g) = \frac{R}{n}, \forall i \geq 2.$$

[8] Remember that we assume, without loss of generality, that $z_1 \leq z_2 \leq \ldots \leq z_n$ and, being g strictly increasing, we have $g(z_1) \leq g(z_2) \leq ... \leq g(z_n)$.

By PB^f, $\sum_{i \in N} \Delta u_i = \Delta R$. By ASB^f, $\Delta u_i = \Delta u_j$, $\forall i, j \geq 2$. By RB^f, $x_1 \geq -v(y_1, z_1) + g(z_1)$. By PB^f and CF, $x_1 \leq -v(y_1, z_1) + g(z_1)$. Then,

$$\Delta u_1 = g(z_1) - \frac{\overline{R}}{n}, \text{ and } \Delta u_i = \frac{R - g(z_1) - \frac{\overline{R}}{n}}{n-1}, \forall i \geq 2.$$

Therefore,

$$x_1 = -v(y_1, z_1) + g(z_1),$$
$$x_i = -v(y_i, z_i) + \frac{R - g(z_1)}{n-1}, \quad \forall i \geq 2,$$

which coincides with case (1) of Definition 3.

(c) If $g(z_1) + (n-1)g(z_2) \leq R \leq ng(z_2)$, by definition of the *fair bound*,

$$f_1(e, g) = g(z_1) \leq f_i(e, g) = \frac{R}{n}, \forall i \geq 2.$$

By PB^f, ASB^f, RB^f, and CF,

$$\Delta u_1 = g(z_1) - \frac{\overline{R}}{n}, \Delta u_2 = g(z_2) - \frac{\overline{R}}{n}, \text{ and}$$
$$\Delta u_i = \frac{R - g(z_1) - \frac{\overline{R}}{n}}{n-1}, \forall i \geq 3.$$

Therefore,

$$x_1 = -v(y_1, z_1) + g(z_1), \; x_2 = -v(y_2, z_2) + g(z_2),$$
$$x_i = -v(y_i, z_i) + \frac{R - g(z_1)}{n-1}, \quad \forall i \geq 3,$$

which coincides with case (1) of Definition 3.

(d) If $ng(z_2) \leq R \leq g(z_1) + g(z_2) + (n-2)g(z_3)$, by definition of the *fair bound*,

$$f_1(e, g) = g(z_1), f_2(e, g) = g(z_2) \leq f_i(e, g) = \frac{R}{n}, \forall i \geq 3.$$

By PB^f, ASB^f, RB^f, and CF,

$$\Delta u_1 = g(z_1) - \frac{\overline{R}}{n}, \Delta u_2 = g(z_2) - \frac{\overline{R}}{n} \text{ and}$$
$$\Delta u_i = \frac{R - g(z_1) - \frac{\overline{R}}{n}}{n-1}, \forall i \geq 3.$$

Therefore,

$$x_1 = -v(y_1, z_1) + g(z_1),\ x_2 = -v(y_2, z_2) + g(z_2),$$
$$x_i = -v(y_i, z_i) + \frac{R - g(z_1)}{n-1},\ \forall i \geq 3,$$
which coincides with case (1) of Definition 3.

(e) If $g(z_1) + g(z_2) + (n-2)g(z_3) \leq R \leq ng(z_3)$, by definition of the *fair bound*,
$$f_1(e,g) = g(z_1), f_2(e,g) = g(z_2) \leq f_i(e,g) = \frac{R}{n},\ \forall i \geq 3.$$
By PB^f, ASB^f, RB^f, and CF,
$$\Delta u_1 = g(z_1) - \frac{\overline{R}}{n},\ \Delta u_2 = g(z_2) - \frac{\overline{R}}{n},\ \Delta u_3 = g(z_3) - \frac{\overline{R}}{n} \text{ and}$$
$$\Delta u_i = \frac{R - g(z_1) - \frac{\overline{R}}{n}}{n-1},\ \forall i \geq 4.$$
Therefore,
$$x_1 = -v(y_1, z_1) + g(z_1),$$
$$x_2 = -v(y_2, z_2) + g(z_2),$$
$$x_3 = -v(y_3, z_3) + g(z_3),$$
$$x_i = -v(y_i, z_i) + \frac{R - g(z_1)}{n-1},\ \forall i \geq 4,$$
which coincides with case (1) of Definition 3.

Note that it is straightforwardly to replicate this reasoning until $R = C$.

Case 2: $R = C$

By CF, $x_i = -v(y_i, z_i) + g(z_i)$ for each $i \in N$.

Case 3: $R > C$

By CF and PB^f, $x_i \geq -v(y_i, z_i) + g(z_i) + \Delta u_i$ for each $i \in N$. Consider $i \in N$ is the first agent such that $ng(z_i) \geq C$,

(a) If $C \leq R \leq ng(z_i)$, by definition of the *fair bound*,
$$f_h(e,g) = f_h(\overline{e}, g),\ \forall h < i \in N;$$
$$f_j(e,g) = f_{j+1}(e,g) > f_h(e,g),\ \forall j \geq i \in N.$$
Then, by PB^f and ASB^f,

$$\Delta u_h = 0,\ \forall h < i \in N;\ \Delta u_j = \frac{R - C}{n - i + 1},\ \forall j \geq i \in N.$$

Note that $\forall h < i \in N, \alpha_h = 0$, where α_h is the constant that appears in Definition 3. Therefore,

$$x_h = -v(y_h, z_h) + g(z_h),\ \forall h < i \in N;$$
$$x_j = -v(y_j, z_j) + g(z_j) + \frac{R - C}{n - i + 1},\ \forall j \geq i \in N,$$

which coincides with case (2) of Definition 3.

(b) If $ng(z_i) \leq R \leq ng(z_{i+1})$, by definition of the *fair bound*,

$$f_h(e, g) = f_h(\overline{e}, g),\ \forall h < i \in N;$$
$$f_i(e, g) = g(z_i) > f_i(\overline{e}, g)$$
$$f_j(e, g) = f_{j+1}(e, g) > f_i(\overline{e}, g),\ \forall j > i \in N.$$

By PB^f, $\sum\limits_{j \geq i \in N} \Delta u_j = \Delta R$. By ASB^f, $\Delta u_j = \Delta u_{j+1}$, for each $j > i \in N$. By RB^f, $x_i \geq -v(y_i, z_i) + g(z_i)$. Finally, we can easily check that if we consider an economy e' such that $R' = ng(z_i)$, then $x'_i = -v(y'_i, z_i) + g(z_i) + \dfrac{ng(z_i) - C}{n - i + 1}$. Thus, by PB^f, $x_i \geq -v(y'_i, z_i) + g(z_i) + \dfrac{ng(z_i) - C}{n - i + 1}$. Moreover, $f_i(e, g) = g(z_i) = f_i(e', g)$, and $f_j(e, g) = f_{j+1}(e, g) > f_j(e', g)_j$ for each $j > i \in N$. So, by PB^f and ASB^f, $x_i^{'} \leq -v(y_i^{'}, z_i) + g(z_i) + \frac{ng(z_i) - C}{n - i + 1}$. Then,

$$\Delta u_h = 0,\ \forall h < i \in N;$$
$$\Delta u_i = \frac{ng(z_i) - C}{n - i + 1} \equiv \alpha_i,$$
$$\Delta u_j = \frac{R - C - \alpha_i}{n - j + 1},\ \forall j > i \in N.$$

Note that $\forall h < i \in N, \alpha_h = 0$. Therefore,

$$x_h = -v(y_h, z_h) + g(z_h) + \alpha_h,\ \forall h \leq i \in N,$$
$$x_j = -v(y_j, z_j) + g(z_j) + \frac{R - C - \alpha_i}{n - i},\ \forall j > i \in N,$$

which coincides with case (2) of Definition 3.

Again, it is straightforwardly to continue with this reasoning until $R = ng(z_n)$.

(c) If $R = ng(z_n)$, by the definition of the *fair bound*,

$$f_h(e, g) = f_h(\overline{e}, g),\ \forall h < n,$$

$$f_n(e, g) > f_n(\overline{e}, g).$$

Then, by PB^f and ASB^f,

$$\Delta u_h = 0,\ \forall h < n;$$

$$\Delta u_n = R - C - \sum_{h<n} \alpha_h.$$

Therefore, as in case (b), by RB^f, PB^f and ASB^f,

$$x_h = -v(y_h, z_h) + g(z_h) + \alpha_h,\ \forall h < n;$$

$$x_n = -v(y_n, z_n) + g(z_n) + R - C - \sum_{h<n} \alpha_h,$$

which coincides with case (2) of Definition 3.

(d) If $R \geq ng(z_n)$, by definition of the *fair bound*,

$$f_i(e, g)_i = f_i(\overline{e}, g),\ \forall i \in N.$$

Then, by ASB^f,

$$\Delta u_i = \frac{R - C - \sum_{i \in N} \alpha_i}{n},\ \forall i \in N.$$

Therefore,

$$x_i = -v(y_i, z_i) + g(z_i) + \alpha_i + \frac{R - C - \sum_{i \in N} \alpha_i}{n},\ \forall i \in N,$$

which coincides with case (3) of Definition 3.

q.e.d.