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A note on Luttens' *Minimal rights based solidarity.*

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Abstract

Following the approach developed by Luttens (2010), we consider a model where individuals with different levels of skills exert different levels of effor. Specifically, we propose a redistribution mechanism based on a lower bound on what every individual deserves: the so-called *minimal rights* (O'Neill (1982)). Our refinement of Luttens' mechanism ensures at the same time *minimal rights based solidarity*, *participation* (non-negativity) and *claims feasibility.*

Keywords: Redistribution mechanism, Minimal rights, Solidarity, Participation, Claims feasibility
JEL classification: C71, D63, D71.

1. Introduction

The redistribution mechanism proposed in Luttens (2010), although is based on the minimal rights lower bound fails to respect it. Moreover, as the author suggests, "a debatable property of the minimal rights based egalitarian mechanism is that the poorest individuals might end up with a negative income after redistribution when R is sufficiently low." We propose a refinement of Luttens' mechanism by requiring respect of minimal rights, which solves the initial Luttens' shortcomings. Note that minimal rights supposes a very weak notion of guarantee: it requires that each agent receives at least what is left of the resources after the other claims have been fully compensated, or zero if this amount is negative. So, if the claims are high enough no agent receives anything.

Specifically, we follow the model developed in Bossert (1995). He proposes a quasi-linear approach to this problem and establishes that the re-allocation of resources must only consider a set of relevant characteristics. These features elicit compensations that are assigned on a additive solidarity basis.

The article is organized as follows. In the next section, we present the model and introduce the basic definitions. Section 3 proposes and characterizes our respect of minimal rights based egalitarian mechanism. Some final remarks compare our mechanism with the ones presented in Luttens (2010). The Appendix gathers the proof of our characterization result.

2. The model

2.1. ***Fair monetary compensation model***

We adopt the approach developed by Bossert (1995). Let us denote by $N = \{1, ..., n\}$ the finite population of size $n \geq 2$. Individuals are distinguished by two characteristics: skill and effort. The skill elicits compensation and it is given by a real number $y \in \mathbb{Y}$, where $\mathbb{Y}$ is an interval of $\mathbb{R}_+$. The skill profile is the vector $y_N = (y_1, \ldots, y_n)$. Individuals' skills are compensated by an amount x_i of a transferable resource (money). The effort does not elicit compensation and it is also a real number $z \in \mathbb{Z}$, where $\mathbb{Z}$ is an interval of $\mathbb{R}_+$. The effort profile is $z_N = (z_1, \ldots, z_n)$. Without loss of generality we assume that individuals are ranked: $z_1 \geq z_2 \geq \ldots \geq z_n$.

An economy consists of the pair that contains skill and effort profiles, $e = (y_N, z_N)$. Let $\mathcal{E}$ be the set of economies, $\mathcal{E} \subseteq \mathbb{Y}^n \times \mathbb{Z}^n$. Given an economy $e = (y_N, z_N) \in \mathcal{E}$, it is assumed that (quasi-linear) utility functions $u : \mathbb{R} \times \mathbb{Y} \times \mathbb{Z} \to \mathbb{R}$ are as follows: $u(x_i, y_i, z_i) = x_i + v(y_i, z_i)$.

Utility measures a monetary outcome (final outcome after redistribution). The pre-tax income function, $v : \mathbb{Y} \times \mathbb{Z} \to \mathbb{R}_{++}$, is supposed to be strictly increasing in y, and that it is not additively separable in y and z, $v(y_i, z_i) \neq v_1(y_i) + v_2(z_i)$.[1] The total sum of pre-tax incomes is denoted by $R = \sum_{i \in N} v(y_i, z_i)$.

[1] When v is additively separable in y and z, a natural way to redistribute income (that satisfies both the principle of compensation and the principle of natural reward) is to make each individual's income after redistribution equal to the average contribution of y_N plus the individual contribution of z_i in the income generating process (Bossert (1995)).

An allocation $x_N = (x_1, ..., x_n) \in \mathbb{R}^n$ is the vector defined by transferable resources x_i. We assume that the total amount to be distributed is *zero*, so that we are looking at a redistribution problem (subsidies coincide with taxes). Then, an allocation for economy $e \in \mathcal{E}$ is feasible whenever $\sum_{i \in N} x_i = 0$. We denote by $F(e)$ the set of feasible allocations for economy e. Note that all feasible allocations are Pareto efficient since we rule out free disposal in the definition of feasibility. An allocation (redistribution) mechanism is a function $S : \mathcal{E} \to \mathbb{R}^n : \forall e \in \mathcal{E}, S(e) \subseteq F(e)$.

We assume, as in Luttens (2010), that individuals, because of the effort they exert, have some claim on the total pre-tax income R. Let $g : \mathbb{Z} \to \mathbb{R}_{++}$ be the claims function that assigns to each individual, i, with an effort level, z_i, a claim, $g(z_i)$, that depends on the individual's effort only. We assume that function $g(z)$ is continuous and strictly increasing in z. We denote the total sum of claims by $C = \sum_{i \in N} g(z_i)$, and $C_{-i} = \sum_{j \neq i \in N} g(z_j)$. It will be a conflicting claims problem whenever $C > R$.

2.2. ***Axioms***

Before introducing the axioms, we provide the definition of the minimal rights lower bound (O'Neill (1982)). This bound guarantees to each agent the amount that is left when the rest of them have received their claim, or zero if this amount is negative. Associated with it, respect of minimal rights states that each individual should receive at least her minimal right.

Definition 1. Minimal rights *(O'Neill (1982))*
For each $e = (y_N, z_N) \in \mathcal{E}$, *the minimal rights,* m, *for each* $i \in N$, *is*

$$m_i(e, g) = min \left\{ g(z_i), \left[R - \sum_{j \in N \setminus \{i\}} g(z_j) \right]_+ \right\},$$

where $[a]_+ = max \{0, a\}$.

Axiom 1. Respect of minimal rights *(RMR)*
For each $e = (y_N, z_N) \in \mathcal{E}$, *and each* $i \in N$, $u_i(e, g) \geq m_i(e, g)$.

The following axiom states that no individual can incur in losses. It is an immediate consequence of the respect of minimal rights, since $m_i(e, g)$ is always non-negative.

Axiom 2. PARTICIPATION *(Maniquet (1998))*
For each $e = (y_N, z_N) \in \mathcal{E}$, and each $i \in N$, $u_i(e, g) \geq 0$.

Next axiom, *claims feasibility*, is a usual assumption which requires that when the total pre-tax income (resources) equals the aggregate claim, then each individual's utility equals to her claim.

Axiom 3. CLAIMS FEASIBILITY *(CF)*
For each $e = (y_N, z_N) \in \mathcal{E}$, and each $i \in N$, if $R = C$, then $u_i(e, g) = g(z_i)$.

Before presenting the solidarity axiom, some notation will be helpful. Given two economies which only differ on skill profiles, $e = (y_N, z_N)$ and $e' = (y'_N, z_N)$, changes in any function or variable are denoted by $\Delta h = h(e', g) - h(e, g)$. This notation will be used to represent changes in the utility function, u, the minimal rights vector, m, as well as changes in the total pre-tax income, R.

The following axiom requires an equal treatment of two individuals who exert the same effort in the allocation of the extra resources, only when their minimal rights change equally.

Axiom 4. ADDITIVE SOLIDARITY FOR EQUAL CHANGES IN MINIMAL RIGHTS *(AS^*, Luttens (2010))*
For each $e = (y_N, z_N), e' = (y'_N, z_N) \in \mathcal{E}$, and each $i, j \in N$, if $\Delta m_i = \Delta m_j$, then $\Delta u_i = \Delta u_j$.

Note that the application of the previous axiom can lead to some agent be below her minimal right, $m_i(e, g)$, which seems to go against the idea behind the axiom itself. We modify this axiom so that it could not happen.

Axiom 5. ADDITIVE SOLIDARITY FOR EQUAL SIGNIFICANT CHANGES IN MINIMAL RIGHTS *(AS^{**})*
For each $e = (y_N, z_N), e' = (y'_N, z_N) \in \mathcal{E}$, and each $i, j \in N : i < j$, if $\Delta m_i = \Delta m_j$, then $u_j(e', g) = [u_j(e, g) + \Delta u_i]_+$.

Remark 1. *Note that, whenever $u_j(e', g) > 0$, then AS^{**} coincides with AS^*.*

The next axiom establishes that the changes in the resources should be shared among those individuals with changes in their bounds.

Axiom 6. PRIORITY *(PRI, Luttens (2010))*
For each $e = (y_N, z_N), e' = (y'_N, z_N) \in \mathcal{E}$, *if* $N_2 = \{i \in N : \Delta m_i \neq 0\} \neq \varnothing$, *then* $\sum_{i \in N_2} \Delta u_i = \Delta R$.

If the minimal rights vector equals to zero, $m = 0$, then an increasing in the resources does not affect each agent's minimal rights in the same way, since it depends on how far the differences $R - \sum_{j \in N \setminus \{i\}} g(z_j)$ are from zero. We introduce a weaker condition that says priority should be applied only if at least one agent has a strictly positive minimal right.

Axiom 7. WEAK PRIORITY *(WPRI)*
For each $e = (y_N, z_N), e' = (y'_N, z_N) \in \mathcal{E}$, *if* $N_2 = \{i \in N : \Delta m_i \neq 0\} \neq \varnothing$, *and there is some* i *such that* $m_i \neq 0$, *then* $\sum_{i \in N_2} \Delta u_i = \Delta R$.

3. Respect minimal rights based egalitarian mechanism.

This section provides a refinement of Luttens' mechanism, which is based on the fulfillment of the respect of minimal rights axiom. Our main result characterizes this mechanism in terms of *claims feasibility*, and *minimal rights based solidarity*. Hereinafter, and for notational convenience, we will denote the utility function by $u_i(g, R)$ and the minimal rights lower bound by $m_i(g, R)$, for a given R.

Definition 2. *A Respect minimal rights based Egalitarian mechanism,* $S_{gMRE/RMR}$ *allocates resources for each* $e \in \mathcal{E}$ *and each* $i \in N$*, as follows:*

(1) $R \geq C_{-n}$,

$$(u_i(g,R))_{S_{gMRE/RMR}} = g(z_i) + \frac{R - C}{n} \qquad \forall i \in N$$

(2) $C_{-k} \leq R \leq C_{-(k+1)}$,

$$(u_i(g,R))_{S_{gMRE/RMR}} = \begin{cases} (u_i(g, C_{-(k+1)}))_{S_{gMRE/RMR}} & \forall i \geq k+1 \\ (u_i(g, C_{-(k+1)}))_{S_{gMRE/RMR}} + \dfrac{R - C_{-(k+1)}}{k} & \forall i < k+1 \end{cases}$$

(3) $G_{n-1} \leq R \leq C_{-1}$,

$$(u_i(g,R))_{S_{gMRE/RMR}} = (u_i(g,C_{-1}))_{S_{gMRE/RMR}} + \frac{R - C_{-1}}{n} \qquad \forall i \in N$$

(4) $G_{n+1-k} \leq R \leq G_{n+2-k}$ *for* $k = 2, 3, \ldots, n-2$,

$$(u_i(g,R))_{S_{gMRE/RMR}} = \begin{cases} 0 & \forall i \geq n+2-k \\ (u_i(g,G_{n+2-k})_{S_{gMRE/RMR}} + \dfrac{R - G_{n+2-k}}{k} & \forall i < n+2-k \end{cases}$$

where $G_s = \sum_{i=2}^{s-1} g(z_i) - (s-2)g(z_s)$.

Theorem 1. $S = S_{gMRE/RMR} \Leftrightarrow S$ *satisfies* CF, AS^{**} *and* $WPRI$.

Proof. See Appendix 1.

The following proposition, which can be straightforwardly obtained from the proof of Theorem 1, highlights the fact that our solution meets the boundedness in which it is based.

Proposition 1. $S_{gMRE/RMR}$ *satisfies respect of minimal rights and participation.*

4. Final comments.

In this paper we have analyzed redistribution problems by means of a lower bound on what individuals deserve. We have modified the Luttens' mechanism so that our proposal not only makes *claims feasibility* and *participation* compatible but also it fullfils the bound on which is based (*RMR*). Our proposal behaves as Luttens' *CF*-mechanism for large R. But we obtain that, for small R, (i) no-one can incur in a negative income, and (ii) no-one can receive more than her claim when the resources are not enough to satisfy the aggregate claim (*claim-boundedness*), two usual requirements in conflicting claims problems. All these differences can be observed in the following figures.

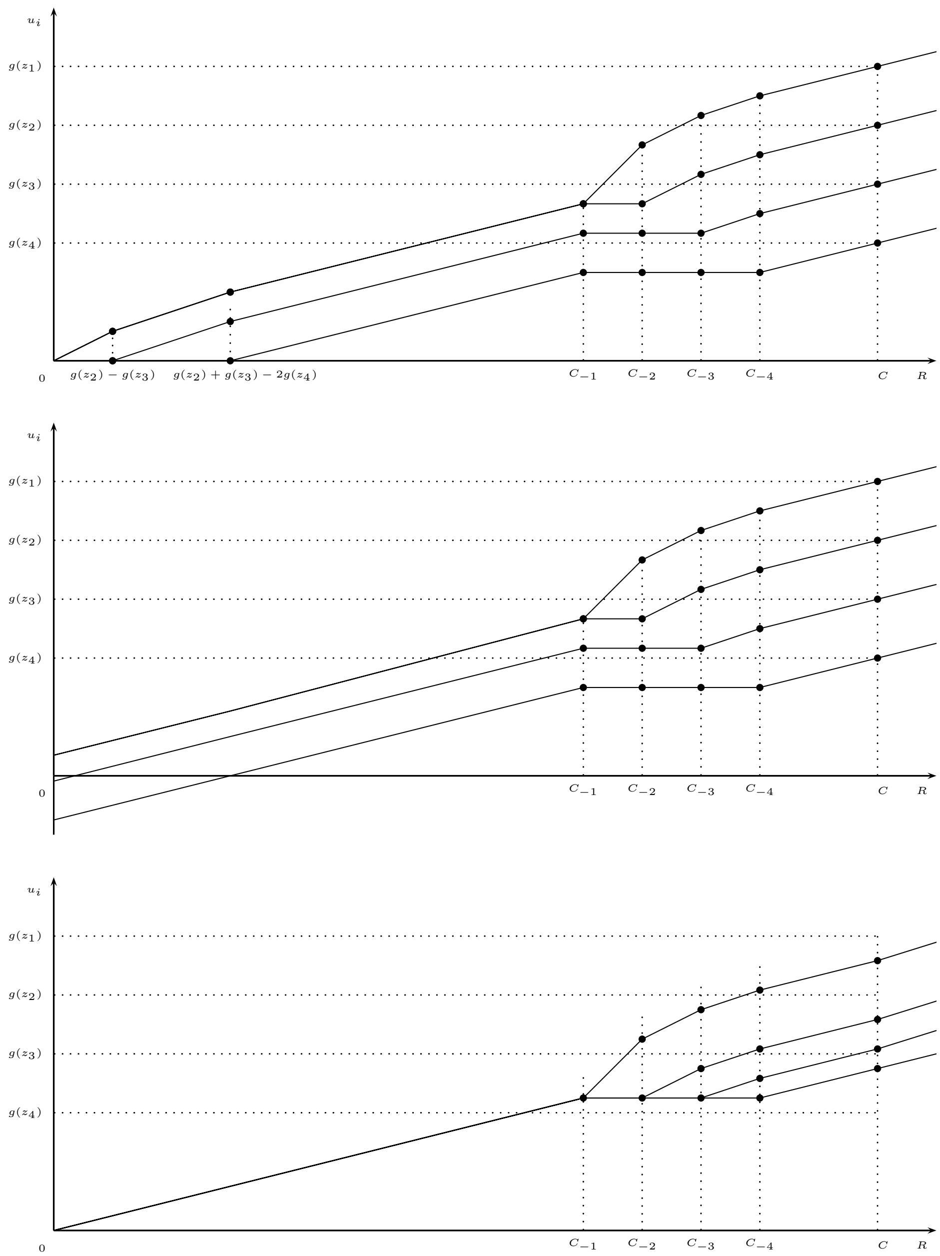

Figure 1: **Respect minimal rights based egalitarian mechanisms.** The horizontal and vertical axis represent different levels of the resources, R, and the total income received by each individual, in a four-individual problem, respectively. The top figure corresponds to our mechanism, while the central and bottom correspond to the mechanisms defined by Luttens (CF and P, respectively).

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Appendix: Proof of Theorem 1.

Given an economy $e = (y_N, z_N)$, we define $\overline{e} = (\overline{y}_N, z_N)$ where $\overline{y}_N$ is chosen such that $\overline{R} = C$. Note that, for each $i \in N$, $m_i(g, \overline{R}) = g(z_i)$. Hence the "initial income" is $u_i(g, \overline{R}) = g(z_i)$, for each $i \in N$. One of the following situations occurs:

1. $R \geq C$
   For each $i \in N$, $R - C_{-i} \geq C - C_{-i} = g(z_i)$. Thus, $m_i(g, R) = g(z_i)$, and $m_i(g, R) - m_i(g, \overline{R}) = 0$. By AS^{**}, $u_i(g, R) = g(z_i) + \frac{R-C}{n}$, which coincides with case **(1)** of Definition 2.
2. $C_{-n} \leq R \leq C$
   For each $i \in N$, $R - C_{-i} = (R - C_{-n}) + (C_{-n} - C_{-i}) = (R - C_{-n}) + (g(z_i) - g(z_n)) = (R - C) + g(z_i) < g(z_i)$, and $R - C_{-i} \geq C_{-n} - C_{-i} = g(z_i) - g(z_n) \geq 0$. Thus, $m_i(g, R) = (R - C_{-n}) + (g(z_i) - g(z_n)) = (R - C) + g(z_i)$. Hence, $m_i(g, R) - m_i(g, \overline{R}) = R - C$. By AS^{**}, $u_i(g, R) = g(z_i) + \frac{R-C}{n}$, which coincides with case **(1)** of Definition 2.

3. $C_{-(n-1)} \leq R \leq C_{-n}$
For each $i \in N$, $R - C_{-i} = (R - C_{-n}) + (C_{-n} - C_{-i}) = (R - C_{-n}) + (g(z_i) - g(z_n)) = (R - C) + g(z_i) < g(z_i)$. For each $i \leq n-1$, $R - C_{-i} \geq 0$, and $R - C_{-n} < 0$. Thus, $m_i(g, R) = (R - C) + g(z_i)$ for each $i \leq n-1$ and $m_n(g, R) = 0$. Hence, $m_i(g, R) - m_i(g, C_{-n}) = (R - C) + g(z_i) - ((C_{-n} - C) + g(z_i)) = R - C_{-n}$, for each $i \leq n-1$ and $m_n(g, R) - m_n(g, C_{-n}) = 0$. By $WPRI$ and AS^{**}, $u_i(g, R) = u_i(g, C_{-n}) + \frac{R - C_{-n}}{n-1}$, for each $i \leq n-1$ and $u_n(g, R) = u_n(g, C_{-n})$ which coincides with case **(2)** of Definition 2.
4. $C_{-(k-1)} \leq R \leq C_{-k}$
For each $i \in N$, $R - C_{-i} = (R - C_{-k}) + (C_{-k} - C_{-i}) = (R - C_{-k}) + (g(z_i) - g(z_k)) = (R - C) + g(z_i) < g(z_i)$. For each $i \leq k-1$, $R - C_{-i} \geq 0$, and $R - C_{-i} < 0$, otherwise. Thus, $m_i(g, R) = (R - C) + g(z_i)$, for each $i \leq k-1$, and $m_i(g, R) = 0$, otherwise. Hence, $m_i(g, R) - m_i(g, C_{-k}) = (R - C) + g(z_i) - ((C_{-k} - C) + g(z_i)) = R - C_{-k}$, for each $i \leq k-1$ and $m_i(g, R) - m_n(g, C_{-k}) = 0$. By $WPRI$ and AS^{**}, $u_i(g, R) = u_i(g, C_{-k}) + \frac{R - C_{-k}}{k-1}$, for each $i \leq k-1$ and $u_i(g, R) = u_i(g, C_{-k})$, otherwise, which coincides with case **(2)** of Definition 2.
5. $G_{n-1} \leq R \leq C_{-1}$
For each $i \in N$, $R - C_{-i} = (R - C_{-1}) + (C_{-1} - C_{-i}) = (R - C_{-1}) + (g(z_i) - g(z_1)) < 0$. Thus, $m_i(g, R) = 0$. Hence, $m_i(g, R) - m_i(g, C_{-1}) = 0$. By AS^{**}, $u_i(g, R) = u_i(g, C_{-1}) + \frac{R - C_{-1}}{n}$, which coincides with case **(3)** of Definition 2.
6. $G_{n+1-k} \leq R \leq G_{n+2-k}$, for $k = 2, 3, \ldots, n-2$
For each $i \in N$, $R - C_{-i} < 0$. Thus, $m_i(g, R) = 0$. Hence, $m_i(g, R) - m_i(g, C_{-1}) = 0$. By AS^{**}, $u_i(g, R) = u_i(g, G_{n+2-k}) + \frac{R - G_{n+2-k}}{k}$, for each $i \leq n + 2 - (k+1)$ and $u_i(g, R) = 0$, otherwise, which coincides with case **(4)** of Definition 2.

q.e.d.