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Corruption in representative democracy

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DEPARTAMENT D'ECONOMIA – CREIP
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Corruption in representative democracy[†]

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Abstract

A parliament with n members, distributed among two parties, decides whether to accept or reject a certain proposal. Each member of the parliament votes in favour or against. If there are at least t members in favour, the proposal is accepted; otherwise it is rejected. A non-member of the parliament, the briber, is interested in having the proposal accepted. To this end, he is willing to bribe members to induce them to vote in favour. It is compared a parliament with party discipline, where members vote according to the party line, and a parliament without party discipline, where members vote according to their own opinion. The paper determines, for given values of n and t, the average number of members that the briber has to bribe in each case (with the average taken with respect to all the possible allocations of members between parties and their votes, and also with respect to those allocations inducing the briber to bribe). The results show that a parliament with parties with party discipline is more costly for the briber to be bribed.

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1. Introduction

Voting is the typical procedure to make collective decisions. In democratic societies citizens express their opinions by voting for the President (or the Prime Minister), the members of the Parliament, and the local authorities. One desirable feature of voting procedures is that voters reveal their opinions sincerely. If it is easy to strategically influence voting outcomes, one cannot be confident of the reliability and legitimacy of political decisions. The internal manipulability of voting procedures (manipulability by voters) is also related to the external manipulability or pressure exercized on voters by non-voters interested in certain outcomes of the voting procedure. This pressure can be legal (lobbying in the United States) or illegal (bribery and corruption). We would like to focus on the illegal procedures.

A parallel line of research has paid attention to the question of how electoral systems are related to corruption. One of the first theoretical works on this topic was Myerson (1993), who investigated the connection between some electoral rules and corruption. His findings on the effectiveness of the electoral rules in eliminating corruption from a parliament were contested by some empirical works (Persson et al. (2003), Rose-Ackerman (2005), and Birch (2007), for instance) and by later work by Myerson (1999) himself.

The studies on electoral systems and corruption have generally tended to focus on electoral rules to elect parliaments, and little attention seems to have been paid to decision-making in the parliament once elected. For example, Charron (2011) studies empirically the connection between party systems and corruption. Taking the electoral formula as a proxy for the number of parties, he finds that multipartism in countries with dominance of single-member districts is associated with higher levels of corruption, while the party system's relationship with corruption plays no role in countries with proportional representation.

Of course, we cannot deny the connection between the electoral rules and the structure of the elected parliament. But different electoral rules may generate the same structure, or different structures can be the outcome of the same rule. Mainly, the electoral rules influence both the distribution of seats between parties and the number of parties in the parliament. For example, proportional representation with party lists always leads to a parliament with parties, giving no way for the no-party members and no tool for the voter to elect a particular member of the parliament. With the closed party list voters are forced to elect the whole party but not a particular person. The closed list system is used in many countries, such as Russia, South Africa, Spain, Portugal, and Argentina. On the contrary, there are countries which use the open list system: Sweden, Denmark, Norway, Austria, Belgium, and the European Union in the election of the European parliament. Under an open list a voter can influence the structure of the parliament, not only the distribution of seats between parties, but also the presence of particular candidates in a party. A closed list system is more likely to give rise to a parliament with strong party discipline, whereas the open list gives more freedom to the members, as the next election of the members mostly depends on the opinion of voters rather than the party's. In a highly democratic world with open list system the members of the parliament should be elected because of their own merits, not because of the fact that the party he belongs entered the parliament. Such a parliament of individually elected members can be seen as a parliament without parties. Questions decided by such a parliament do not depend (or at least intend not to depend) on the political background of parties. Accordingly, such a parliament could be seen as a desirable one for a highly democratic society. Though we do not consider the parliament from the strategic point of view of re-electing, it is interesting to notice~~,~~ that political systems with fair executive reelections seem~~s~~ to have less myopic and more electoral conscious politicians and, therefore, less corruption (Linz (1990), Linz and Stepan (1996), Bailey and Valenzuela (1997), and Rose-Ackerman (1999)).

Much less attention seems to have been devoted to the connection between corruption and the structural characteristics of the parliaments, such as size of the parliament, the number of parties with representation, and the decision rules adopted by the parliament. These characteristics are likely to have a crucial influence on the level of political corruption inside the parliament. There are some empirical works dealing with the issue: Lederman et al. (2005) show that democracies, parliamentary systems, political stability, and freedom of the press are all associated with lower corruption; and Pelizzo (2006) shows that the potential for corruption is inversely related to parties' levels of institutionalization – so that the more a party is institutionalized, the less likely it is to become involved in corrupt practices.

A model of voting inside the parliament is proposed to compare bribery costs in different parliaments. The bribery cost is defined as the number of members needed to be bribed to achieve the desirable result averaged over all possible states of the parliament, which is assumed to have only two parties. A possible justification for this restrictive assumption is that, for specific motions, opinions may be easily become polarized or dichotomic, so that it is as if only two parties existed. In fact, voting for a particular question could split the parliament in two parts, one representing the government's party and the other the opposition parties, or one representing left-wing politics and the other representing right-wing politics. On the one hand, in a parliament with party discipline members vote according to the party line and the briber needs to bribe the entire party, and the number of members of the party serves as a proxy for the cost of bribing the whole party. On the other hand, in a parliament without party discipline members vote according to their own opinion. The persistence of illegal procedures (such as bribing votes) could be explained by a low cost of bribing. This raises the question of whether party discipline makes corruption more or less costly than the absence of party discipline, and thereby, stimulate or prevent corruption.

The comparison of parliaments with and without party discipline suggests the following intuition: in case of party discipline under the assumption that members of the party are voting strongly according to the party line on average a briber needs to bribe more votes than it is exactly needed to achieve the desired outcome; in case of no party discipline, there is no state-where it is needed to bribe seats in excess. This intuition has nonetheless to be qualified because the number of states with the respect to which the average bribery cost is calculated differs in each case, so it is not obvious which of the two is higher. It is offered precise formulae to determine these costs in the two cases and for every voting rule based on an acceptance threshold. The main result (Proposition 3) states that, for parliaments with at least four members, bribery costs are higher with party than without party discipline.

2. Model and definitions

There is a **parliament** with n members. Each member belongs to one of two parties. The parliament has to make a political **decision** concerning a certain proposal using some voting rule. Each member of the parliament chooses either Y (the vote representing acceptance of the proposal) or N (the vote representing rejection).

To make a decision of whether to accept or reject the proposal, the parliament resorts to a voting rule. Any such rule is assumed to be characterized by a natural number $t \in \{1,2,\ldots,n\}$. This number is a **threshold** representing the minimum number of Y votes guaranteeing that the proposal is accepted. Standard majority rule constitutes the typical voting rule adopted by parliaments to make ordinary decisions. In this case, the threshold is $t^{maj} = \frac{n}{2} + 1$ when n is even and $t^{maj} = \frac{n+1}{2}$ when n is odd. To make crucial decisions such as constitutional amendments, a qualified majority is generally used. For example, constitutional changes in Spain demand 3/5 of parliament support to be approved (the so-called "ordinary procedure"), whereas for global changes of the constitution the necessary support is at least 2/3 ("aggravated procedure"). In terms of our model, the procedures correspond to $t = \frac{3n}{5}$ and $t = \frac{2n}{3}$, respectively. The case $t = n$ would correspond to the demand for unanimous support for a change.

There is an exogenous agent, not a member of the parliament, who is interested in having the proposal accepted. Such an agent will act as a **briber**, bribing members, if necessary, to vote Y (if the briber were interested in rejection, the model could be redefined by just changing t to $t' = n - t$). A huge and powerful corporation, to the extent that it has enough resources to bribe parties, could act as a briber. For example, the domestic automobile producing company which is interested in a law that increases the state duty for imported cars; or huge internet-providers cooperating with mobile operators lobbing for separate tarification of TCP/IP telephony.

Party discipline can be present or absent. The existence of party discipline means that, for each of the two parties, all the members of the party must cast the same vote: either all choose Y or all choose N. In this case, rather than parliament members, it will be said that parties vote either Y or N. When there is no party discipline, all members of the parliament are free to choose Y or N. Definition 1 next defines the concept of possible state of the parliament under party discipline. Definition 2 does the same when party discipline is absent.

Definition 1. *Party-discipline case*. Given n, a possible state ω_n of the parliament in the presence of party discipline is a pair $\omega_n = (d, s)$ such that $d \in \{0, 1, 2\}$ is the number of parties voting Y and s is the size of the smallest party in case that both parties vote the same (that is, if $d \neq 1$) and is the size of the party voting Y when parties vote differently (if $d = 1$).

Remark 1. If $d \neq 1$ (that is, both parties cast the same vote), then $s \in \{1, \dots, n/2\}$ if n is even and $s \in \{1, \dots, (n-1)/2\}$ if n is odd. If $d = 1$ (parties vote differently), then $s \in \{1, \dots, n-1\}$.

Remark 2. By Remark 1, for a given n, the total number of possible states under party discipline is $n/2 + n/2 + n - 1 = 2n - 1$ if n is even and $(n-1)/2 + (n-1)/2 + n - 1 = 2n - 2$ if n is odd.

In the absence of party discipline the concept of state of the world is simpler. The term s does not matter and the term d now means the number of members voting Y.

Definition 2. *No-party-discipline case*. Given n, a possible state ω_n of the parliament in the absence of party discipline is a number $\omega_n = d \in \{0, 1, \dots, n\}$ representing the number of members voting Y.

Given n, with or without party discipline, define Ω_n to be set of states. With the presumption that a certain n is given, ω and Ω will be written instead of, respectively, ω_n and Ω_n. For any finite set S, let $|S|$ denote the number of members of S.

Definition 3. Given threshold t and state ω, $m(\omega)$ is defined to be the minimum number of members of the parliament that have to be bribed to ensure that the parliament accepts the proposal, that is, at least t parliament members vote Y. Specifically, in the party-discipline case, letting $\omega = (d, s)$,

(i) if $d = 2$, then $m(\omega) = 0$;
(ii) if $d = 1$, then $m(\omega) = 0$ if $s \geq t$ and $m(\omega) = n - s$ if $s < t$;
(iii) if $d = 0$, then $m(\omega) = s$ if $s \geq t$; $m(\omega) = n$ if $s < t$ and $n - s < t$; and $m(\omega) = n - s$ if $s < t$ and $n - s \geq t$,

and in the no-party-discipline case, letting $\omega = d$,
(i) if $d < t$, then $m(\omega) = t - d$;
(ii) if $d \geq t$, then $m(\omega) = 0$.

From the briber's perspective, some member of the parliament has to be bribed in state ω if $m(\omega) \neq 0$. The event that some member of the parliament has to be bribed could be then defined as $\{\omega \in \Omega : m(\omega) \neq 0\}$.

Definition 4. Given n and t, the average number of members of the parliament that are needed to be bribed to ensure that the proposal is accepted is $A = \frac{\sum_{\omega \in \Omega} m(\omega)}{|\Omega|}$.

Definition 5. Given n and t, the average number of members of the parliament that are needed to be bribed to ensure that the proposal is accepted conditional on the fact that some member has to be bribed is $C = \frac{\sum_{\omega\in\Omega} m(\omega)}{|\{\omega\in\Omega:\, m(\omega)\neq 0\}|}$.

The number A will be interpreted as the average bribery cost for the briber (or, for short, average bribery cost). The number C will be interpreted as the conditional average bribery cost for the briber (or, for short, conditional bribery cost). A_d and C_d will define the values of A and C with party discipline and A_{nd} and C_{nd} without party discipline.

The conditional bribery cost C is a proxy for the real price the briber has to pay on average if he has to bribe. The average bribery cost A could be used to compare different parliaments in the sense that it takes into account all possible states of the world, and includes the likelihood to take a positive decision. Is it preferable to have a high or a low A? If it is high enough, it can deter the briber because of the high cost; but if at the same time C is low enough, the briber may have a strong incentive to act. And if A is low, it appears to encourage corruption, even though C can be high.

3. Results

First consider the party-discipline case – a parliament with two parties, the members of which are obliged to cast the vote of the party.

Proposition 1.
With party discipline the average and conditional bribery cost are

$$A_d = \frac{(t-1)(n-t/2)}{n-1/2} + \frac{n}{4(n-1/2)} \qquad C_d = \frac{(t-1)(n-t/2)}{\frac{t-1}{2}+\frac{n}{4}} + \frac{n}{2(t-1)+n} \qquad if\ t > \frac{n}{2}\,, even\ n.$$

$$A_d = \frac{(t-1)(n-t/2)}{n-1} \qquad C_d = \frac{(t-1)(n-t/2)}{\frac{t-1}{2}+\frac{n-1}{4}} \qquad if\ t > \frac{n}{2}, odd\ n.$$

$$A_d = \frac{(t-1)\left(n-\frac{3t}{4}\right)}{n-1/2} + \frac{n(n+2)}{8(2n-1)} \qquad C_d = \frac{(t-1)\left(n-\frac{3t}{4}\right)}{\frac{t-1}{2}+\frac{n}{4}} + \frac{n(n+2)}{8(t-1)+4n} \qquad if\ t \le \frac{n}{2}, even\ n.$$

$$A_d = \frac{(t-1)\left(n-\frac{3t}{4}\right)}{n-1} + \frac{(n-1)(n+1)}{8(2n-2)} \qquad C_d = \frac{(t-1)\left(n-\frac{3t}{4}\right)}{\frac{t-1}{2}+\frac{n-1}{4}} + \frac{(n-1)(n+1)}{8(t-1)+4(n-1)} \quad if\ t \le \frac{n}{2}\,, odd\ n.$$

Remark 3. With relatively big values of n, the difference between odd and even n is small. Since $\frac{n}{4(n-1/2)} < 1$ and, if $t > 1$, $\frac{n}{2(t-1)+n} < 1$

$$\lfloor A_d \rfloor \approx \frac{(t-1)(n-t/2)}{n-1/2} \qquad \lfloor C_d \rfloor \approx \frac{(t-1)(n-t/2)}{\frac{t-1}{2}+\frac{n}{4}} \qquad if\ t > \frac{n}{2}\,, even\ n.$$

For $t > n/2$ $A_d \approx t - \frac{t}{2}\frac{t}{n}$. Note that t/n expresses the threshold as a fraction of the parliament. So, the average bribery cost is the threshold t minus the proportion *t/n* of $t/2$. For instance, if $n = 99$ and $t = 66$, then $A_d \approx 66 - 33(2/3) = 44$: under party discipline, in parliament with 99 members and threshold 66, some 44 members will have to be bribed of average.

For $t \leq n/2$ $A_d \approx t - \frac{3}{4}t\frac{t}{n} + \frac{n}{16}$. In other words, the average cost is the threshold plus one sixteen of n minus the proportion *t/n* of $3t/4$.

Second, it is considered the no-party discipline case, when parliament members vote on their own.

Proposition 2.
Without party discipline the average and conditional bribery cost are

$$A_{nd} = \frac{t(t+1)}{2(n+1)} \qquad C_{nd} = \frac{t+1}{2}.$$

Remark 4. With relatively big t the bribery cost C_{nd} tends to $t/2$. It means that on average if the briber has to pay, he has to pay approximately half of the members needed to achieve the threshold.

Remark 5. Approximately $A_{nd} \approx \frac{t}{2}\frac{t}{n}$. Thus, A_{nd} tends to the proportion *t/n* of $t/2$.

Proposition 3 next provides a general result on the comparability of bribery costs between the discipline and non-discipline cases.

Proposition 3.
For $3 < t < n$, $A_d > A_{nd}$ and $C_d > C_{nd}$: the average and conditional bribery costs with party discipline are greater than the average and conditional bribery costs without party discipline.

Remark 6. Under the unanimity rule or when $n = t$ the average bribery costs are equal for the party-discipline and no-party-discipline cases; the conditional bribery costs with party discipline are greater than the conditional bribery costs without party discipline.

4. Concluding remarks and discussion

As any commodity, demand for the corruption is likely to be regulated by its price: more corruption will be presumably observed the smaller the price paid by the briber. So to reduce corruption it seems that the parliament structure has to be defined to make corruption costly. For general voting rules (except the unanimity rule) the model predicts that a parliament with party discipline is more costly to be bribed. This result agrees with the conclusions of Lederman et al. (2005) and Pelizzo (2006), who showed that parliament systems with parties with strong discipline, organization, and party line are all associated with lower corruption. Under the unanimity rule the average bribery costs are equal for the two cases, while the conditional bribery cost is greater with party discipline.

The no-party discipline case can be considered as an example of voting among the citizenship for a certain proposal. In real life there are certain questions that cannot be expected to be answered sincerely by the members of the parliament. For instance, voting for reducing the number of members of the parliament. Besides, there are some crucial decisions which maybe should be voted for by all the inhabitants of a country. For example, should the form of government change from monarchy to republic? Is part of the nation (Scotland, Québec, Catolonia) to be allowed to secede from the whole nation (United Kingdom, Canada, Spain)? Why such a crucial question should be under the voting of only a few hundred of representatives?

Proposition 3 and Remark 6 suggest the following conjecture: the same average and conditional bribery costs in a parliament with party discipline can be reproduced in a parliament without party discipline by changing structural characteristics such as size or voting rule. To achieve the same value of the average cost as in the party-discipline case in the case of no-party-discipline the threshold should be increased, and the number of members should be decreased.

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Appendix

Proof of Proposition 1.

Part 1: even n and $t > n/2$. By Remark 2, $|\Omega| = 2n - 1$. To compute $\sum_{\omega\in\Omega} m(\omega)$ it is only necessary to consider the states $\omega = (d, s)$ such that $d \in \{0,1\}$, since $m(\omega) = 0$ when $d = 2$.

Case 1: $d = 0$. If $s \in \{1, \dots, n - t\}$, then, since $t > n/2$, $n - s > t$. Therefore, $m(\omega) = n - s$. If $s \in \{n - t + 1, \dots, n/2\}$, then both parties are smaller than t. This means that $s < t$ and $n - s < t$ and, accordingly, there is a need to bribe both parties: $m(\omega) = n$. As a result, $\sum_{\omega\in\Omega \text{ with } d=0} m(\omega) = \sum_{s=1}^{n-t}(n - s) + n\big(n/2 - (n - t)\big) = ((t - 1)(n - t) + nt)/2$.

Case 2: $d = 1$. If $s \in \{1, \dots, t - 1\}$ then $m(\omega) = n - s$. Since the party voting positively is smaller than t there is a need to bribe the party voting negatively. Therefore, $\sum_{\omega\in\Omega \text{ with } d=1} m(\omega) = \sum_{i=1}^{t-1} n - i = (t - 1)(n - t/2)$.

Consequently, collecting the two cases, $\sum_{\omega\in\Omega} m(\omega) = \frac{(t-1)(n-t)+nt}{2} + \frac{(t-1)(2n-t)}{2} = (t - 1)(2n - t) + \frac{n}{2}$. Since $|\Omega| = 2n - 1$, the average bribery cost is

$$A_d = \frac{(t-1)(2n-t) + n/2}{2n-1} = \frac{(t-1)(n-t/2)}{n-1/2} + \frac{n}{4(n-1/2)}.$$

With the respect to the conditional bribery cost, the number of states when bribing occurs is $|\{\omega \in \Omega: m(\omega) \neq 0\}| = |\{\omega \in \Omega: m(\omega) \neq 0 \; and \; d = 0\}| + |\{\omega \in \Omega: m(\omega) \neq 0 \; and \; d = 1\}| = |\{\omega \in \Omega: d = 0\}| + |\{\omega \in \Omega: m(\omega) \neq 0 \; and \; d = 1\}| = n/2 + (t - 1)$.

To sum up, the conditional bribery cost is

$$C_d = \frac{(t-1)(2n-t) + n/2}{(t-1) + n/2} = \frac{(t-1)(n-t/2)}{\frac{t-1}{2} + \frac{n}{4}} + \frac{n}{2(t-1) + n}.$$

Part 2: even n and $t \leq n/2$. This part coincides with part 1 except for the case 1, So, let $d = 0$.

If $s \in \{1, \dots, t - 1\}$, then, since $n - s > t$. Therefore, $m(\omega) = n - s$. If $s \in \{t, \dots, n/2\}$, then $s \geq t$ and $n - s \geq t$. Thus, $m(\omega) = s$. As a result, $\sum_{\omega\in\Omega \text{ with } d=0} m(\omega) = \sum_{s=1}^{t-1}(n - s) +$

$\sum_{s=t}^{n/2} s = (t-1)(n-t) + n(n+2)/8$.

So, putting the two cases together, $\sum_{\omega\in\Omega} m(\omega) = (t-1)(n-t/2) + (t-1)(n-t) + n(n+2)/8 = (t-1)(2n-3t/2) + n(n+2)/8$. The total number of states and the number of states when bribing occurs is the same as in part 1. So, the average and conditional bribery costs are

$$A_d = \frac{(t-1)(2n-3t/2) + n(n+2)/8}{2n-1} = \frac{(t-1)\left(n-\frac{3t}{4}\right)}{n-1/2} + \frac{n(n+2)}{8(2n-1)};$$

$$C_d = \frac{(t-1)(2n-3t/2) + n(n+2)/8}{(t-1)+n/2} = \frac{(t-1)\left(n-\frac{3t}{4}\right)}{\frac{t-1}{2}+\frac{n}{4}} + \frac{n(n+2)}{8(t-1)+4n}.$$

Part 3: odd n.

Changing n from even to odd eliminate the states when the two parties have the same number of members. The states with equal number of members matter only when parties vote equally. Therefore, by Remark 1, having n odd changes the total number of states to $2n-2$ and the total number of states when bribing occurs to $(n-1)/2 + t - 1$. When both parties vote negatively, the range of s should be changed to $\{1, (n-1)/2\}$ since n is odd. Therefore, for odd n and $> n/2$ $\sum_{\omega\in\Omega \text{ with } d=0} m(\omega) = \sum_{s=1}^{n-t}(n-s) + n\big((n-1)/2 - (n-t)\big) = (t-1)(n-t/2)$; for $t < n/2$ $\sum_{\omega\in\Omega \text{ with } d=0} m(\omega) = \sum_{s=1}^{t-1}(n-s) + \sum_{s=t}^{(n-1)/2} s = (t-1)(n-t) + (n+1)(n-1)/8$;

All these changes lead us to the final formulas of bribery costs for odd n and $t > n/2$

$$A_d = \frac{(t-1)(2n-t)}{2n-2} = \frac{(t-1)(n-t/2)}{n-1};$$

$$C_d = \frac{(t-1)(2n-t)}{t-1+(n-1)/2} = \frac{(t-1)(n-t/2)}{\frac{t-1}{2}+\frac{n-1}{4}};$$

and, for $t < n/2$

$$A_d = \frac{(t-1)(2n-3t/2) + (n-1)(n+1)/8}{2n-2} = \frac{(t-1)\left(n-\frac{3t}{4}\right)}{n-1} + \frac{(n-1)(n+1)}{8(2n-2)};$$

$$C_d = \frac{(t-1)(2n-3t/2) + (n-1)(n+1)/8}{t-1+(n-1)/2} = \frac{(t-1)\left(n-\frac{3t}{4}\right)}{\frac{t-1}{2}+\frac{n-1}{4}} + \frac{(n-1)(n+1)}{8(t-1)+4(n-1)}.$$

Proof of Proposition 2.

For a state $\omega = d$ the number d can take value from 0 to n: $d \in \{0,1,\dots,n\}$. so, there are $n+1$ possible states of the parliament. There is a need to bribe when $d \in \{0,1,\dots,t-1\}$, so, there are t states when there is a need to bribe. For these states the minimum number of members of the parliament that have to be bribed is $\sum_{\omega\in\Omega} m(\omega)$ for $d \in \{0,1,\dots,t-1\}$. This number is then $\sum_{d=0}^{t-1}(t-d) = (1+2+\cdots+t) = t(t+1)/2$. Applying Definitions 4 and 5 we obtain Proposition 2.

Proof of Proposition 3.

The proof consists of proving 3 Lemmas.

Lemma 1. For $2 < t < n$, $A_d > A_{nd}$, for $t = n$ $A_d = A_{nd}$.

Lemma 2. For $2 < \frac{n}{2} < t < n$, $C_d > \frac{(t-1)(2n-t)}{2(n-2)} > C_{nd}$.

Lemma 3. For $3 < t \leq \frac{n}{2}$, $C_d > t-1 > C_{nd}$.

Proof of Lemma 1. For $2 < t < n$, $A_d > \frac{t}{2} > A_{nd}$.

Step 1. For $t > \frac{n}{2}$ and even n, $A_d > \frac{t}{2} > A_{nd}$ for $1 < t < n$, since

$$A_d = \frac{2nt - t^2 - 3n/2 + t}{2n-1} > \frac{t}{2} \leftrightarrow (n-t)(t-3/2) > 0 \text{ for } 1 < t < n;$$

$$A_{nd} = \frac{t(t+1)}{2(n+1)} < \frac{t}{2} \text{ for } t < n.$$

On the other hand, for $t = n$:

$A_{nd} = \frac{t(t+1)}{2(n+1)} = \frac{t}{2} = \frac{2nt-t^2-3n/2+t}{2n-1} = A_d$. As a result, $t = n$ implies $A_{nd} = A_d$:

Step 2. For $t > \frac{n}{2}$, odd n it is true that $A_d > \frac{t}{2} > A_{nd}$ for $2 < t < n$, to see this, observe that

$A_d = \frac{2nt-t^2-2n+t}{2n-2} > \frac{t}{2} \leftrightarrow (n-t)(t-2) > 0$ for $2 < t < n$; and

$A_{nd} < \frac{t}{2}$, as shown in Step 1.

Note that, for $t = n$ or $t = 2$,

$$A_d = \frac{2nt - t^2 - 2n + t}{2n - 2} = \frac{t}{2} = A_{nd}.$$

Step 3. For $t \leq \frac{n}{2}$, even or odd n, such that $n > 3$ it is true that $A_d > A_{nd}$.

Assume that n and t can take non-integer values. Letting derivative be taken with the respect to t, A_d is increasing $(A_d{}' > 0)$ and concave $(A_d{}'' < 0)$ for both even and odd n. A_{nd} is increasing $(A_{nd}{}' > 0)$ and convex $(A_{nd}{}'' > 0)$ for both even and odd n. The smallest value of t is 1, the greatest is $n/2$. As both functions are increasing, one is convex and the other is concave it is sufficient to show that in the extreme values $A_d > A_{nd}$.

When $t = 1$ and $n > 2$, $A_d > A_{nd}$, since

$$A_{nd} = \frac{t(t+1)}{2(n+1)} = \frac{1}{n+1};$$

for even n $\quad A_d = \frac{(t-1)\left(n - \frac{3t}{4}\right)}{n - 1/2} + \frac{n(n+2)}{8(2n-1)} = \frac{n(n+2)}{8(2n-1)} > \frac{1}{n+1}$ for $n > 2$;

for odd n $\quad A_d = \frac{(t-1)\left(n - \frac{3t}{4}\right)}{n - 1} + \frac{(n-1)(n+1)}{8(2n-2)} = \frac{(n-1)(n+1)}{8(2n-2)} > \frac{1}{n+1}$ for $n > 2$.

When $t = n/2$ and $n > 2$, $A_d > A_{nd}$, since

$$A_{nd} = \frac{t(t+1)}{2(n+1)} = \frac{n(n+2)}{8(n+1)};$$

for even n $\quad A_d = \frac{(t-1)(n-3t/4)}{n-1/2} + \frac{n(n+2)}{8(2n-1)} = \frac{5n(n-2) + n(n+2)}{8(2n-1)} > \frac{n(n+2)}{8(n+1)}$ for $n > 2$;

for odd n $A_d = \frac{(t-1)(n-3t/4)}{n-1} + \frac{(n-1)(n+1)}{8(2n-2)} = \frac{5n(n-2) + (n-1)(n+1)}{8(2n-2)} > \frac{n(n+2)}{8(n+1)}$ for $n > 2$.

Proof of Lemma 2.

For $2 < \frac{n}{2} < t < n$, $C_d > \frac{(t-1)(2n-t)}{2(n-2)} > C_{nd}$, since

$$C_{nd} = \frac{t+1}{2} < \frac{(t-1)(2n-t)}{2(n-2)} \leftrightarrow (t-n)(t-3) - 2 < 0;$$

for even n, $$C_d = \frac{(t-1)(n-t/2)}{\frac{t-1}{2}+\frac{n}{4}} + \frac{n}{2(t-1)+n} = \frac{(t-1)(n-t)+n(t-1/2)}{n/2+t-1}$$
$$> \frac{(t-1)(n-t)+n(t-1)}{n/2+t-1} = \frac{(t-1)(2n-t)}{n/2+t-1} > \frac{(t-1)(2n-t)}{2(n-2)} \text{ for } n > 4;$$

for odd n, $$C_d = \frac{(t-1)(n-t/2)}{\frac{t-1}{2}+\frac{n-1}{4}} = \frac{(t-1)(2n-t)}{n/2+t-3/2} > \frac{(t-1)(2n-t)}{2(n-2)} \text{ for } n > 3.$$

Proof of Lemma 3.

For $3 < t \leq \frac{n}{2}$, $C_d > t-1 > C_{nd}$, since

for even n $$C_d = \frac{(t-1)\left(n-\frac{3t}{4}\right)}{\frac{t-1}{2}+\frac{n}{4}} + \frac{n(n+2)}{8(t-1)+4n} = \frac{4(t-1)(4n-3t)+n(n+2)}{4(n+2t-2)}$$
$$> \frac{4(t-1)(4n-3t)}{4(n+2t-2)} > \frac{(t-1)(4n-3t)}{4n-3t} = t-1;$$

for odd n $$C_d = \frac{(t-1)\left(n-\frac{3t}{4}\right)}{\frac{t-1}{2}+\frac{n-1}{4}} + \frac{(n-1)(n+1)}{8(t-1)+4(n-1)} = \frac{4(t-1)(4n-3t)+(n+1)(n-1)}{4(n+2t-3)}$$
$$> \frac{4(t-1)(4n-3t)}{4(n+2t-3)} > \frac{(t-1)(4n-3t)}{4n-3t} = t-1;$$

$$C_{nd} = \frac{t+1}{2} < t-1 \text{ for } t > 3.$$