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Independent Central Banks: Low inflation at no cost? A model with fiscal policy

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Abstract

In this article we extend the rational partisan model of Alesina and Gatti (1995) to include a second policy, fiscal policy, besides monetary policy. It is shown that, with this extension, the politically induced variance of output is not always eliminated nor reduced by delegating monetary policy to an independent and conservative central bank. Further, inflation and output stabilisation will be affected by the degree of conservativeness of the central bank and by the probability of the less inflation averse party gaining power.

Keywords: rational partisan theory; fiscal policy; independent central bank

JEL Classification: E58, E63.

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1 Introduction

By appointing an independent and conservative central bank to take control of monetary policy, Rogoff (1985) showed that average inflation would be reduced. However, this benefit came with a (theoretical) cost of higher output variability. Given that Alesina and Summers (1993) did not find empirical evidence of higher output variability, Alesina and Gatti (1995) developed a theoretical model to illustrate why an independent and conservative central bank might not bring higher volatility of output. They decomposed the variability of output into two sources: the economic volatility, induced by exogenous shocks, and the political volatility, introduced by the uncertainty about the future course of policy. The rational partisan model used by Alesina and Gatti (1995) included two political parties running for office, with different views of the economy. There is only one policy in the model, monetary policy, that is either decided by the party that wins the elections or can be delegated to an independent central bank. The latter option eliminates the political volatility of output, allowing for the possibility that overall volatility of output does not necessarily increase with an independent central bank. Maloney et al. (2003) test a rational partisan model with only monetary policy and find some support for OECD countries that central bank independence reduces the politically induced business cycles volatility.

Monetary policy has been considered an ideal candidate for delegation (see, for instance, Drazen, 2002 and Alesina and Tabellini, 2007, 2008). This is due to its technical nature and the difficulty in judging the ability or talent of the person responsible for taking the decisions. Fiscal policy, on the other hand, is not a clear candidate for delegation, mainly because of its redistributive impact. As fiscal policy can secure a minimum number of voters, politicians will not willingly delegate such policy if they want to be reelected. Therefore, fiscal and monetary policy are implemented in many countries by different authorities that are generally independent. For this reason, an interesting extension of Alesina and Gatti (1995) would be the inclusion of fiscal policy in the model, in order to see whether the politically induced uncertainty is still eliminated or at least reduced by an independent central bank responsible for monetary policy and isolated from electoral cycles.

The next section will develop the rational partisan model with two policies, monetary and fiscal policy. Section 3 and Section 4 will study the effects on inflation and output of the introduction of an independent central bank responsible for monetary policy and, finally, Section 5 will present the conclusions.

2 The Model

In this section, we will extend the analysis of Alesina and Gatti (1995) to consider two instruments (and, thus, two policies), as in Alesina and Tabellini (1987). We will assume that there are two parties competing for office, L (a left-wing party) and R (a right-wing party), and there is an exogenous probability P that party L wins the elections and takes office. Agents (wage setters) in this economy will not know what party will be in office when they form their inflation expectations, π^e. For this reason, their expectations embody electoral uncertainty: $\pi^e = PE(\pi_L) + (1-P)E(\pi_R)$, where $E(\pi_j)$ represents expected inflation if party j is in office ($j = L, R$). Once elections take place, the party in office will attempt to stabilise the economy after the shocks occur, and the optimal values of inflation and taxes will be revealed. This sequential structure of the game is static by nature, as the game ends once the policy instruments are chosen.

If party j is in office, the output is given by

$$x_j = \pi_j - \pi^e - \tau_j - w^* + \varepsilon, \tag{1}$$

where π_j is the actual inflation rate. Moreover, τ_j represents taxes levied on output, w^* denotes the target real wage that workers seek to achieve, and ε is a productivity shock such that $E(\varepsilon) = 0$ and $var(\varepsilon) = \sigma_\varepsilon^2$.

The government j budget constraint is

$$g_j = \tau_j + \pi_j, \tag{2}$$

where g_j denotes the ratio of public expenditures over output when party j is in office. Note that public spending will be financed by a distortionary tax (controlled by the fiscal authority) and/or by money creation (controlled by the authority responsible for monetary policy). Given the static nature of the model, debt is not included.

We assume that the loss function for party j is given by

$$V_{Gj} = \frac{1}{2}\left(\pi^2 + \delta_j(x - x^*)^2 + \gamma_j(g - g^*)^2\right), \tag{3}$$

where δ_j and γ_j represent the relative weights assigned to output and public spending stabilisation with respect to inflation, respectively, and δ_j, $\gamma_j > 0$. Thus, the party in office wishes to minimize the deviations of inflation, output and public spending from some targets.[1]

Alesina and Gatti (1995) assume that both parties share the same goals, but differ in the relative weights attributed to output with respect to inflation. As there is an extra goal in this model, we will also allow for parties to differ in their relative weight of public expenditures with respect to inflation. The actual size and relative importance of these weights is an empirical question. Further, we expect them to change through time and by countries, given that political and economic structures of societies are continuously evolving, and each country is characterised by some idiosyncratic treats that make it unique. For this reason, we will classify the parties according to two criteria:

[1] We set the inflation target to be zero. The results would not change qualitatively with a positive target.

1. Their relative inflation aversion, which will be determined by $m_j = \frac{\frac{1}{\delta_j}+\frac{1}{\gamma_j}}{2}$. We will assume that $m_R > m_L$, i.e., the goal of stabilising inflation is more important for party R than for party L.[2]

2. Their relative weights assigned to output and spending stabilisation in their loss function: $\frac{\delta_j}{\gamma_j}$. For instance, if party L is relatively more interested than party R in achieving the output target, then $\frac{\delta_L}{\gamma_L} > \frac{\delta_R}{\gamma_R}$.[3]

The model developed by Alesina and Gatti concludes that delegating monetary policy to an independent central bank eliminates the politically induced variability of output. In order to study whether the introduction of fiscal policy in the model alters this conclusion, we will consider two cases: first, when monetary and fiscal policy are controlled by the government, and second, when monetary policy is delegated to an independent authority (central bank). The first case will represent an economy with no (or very little) central bank independence, whereas the second case will refer to an economy that has granted independence to its central bank for the conduct of monetary policy. In both cases, the timing of events is as follows: expectations and thus, wages, are set first. Afterwards, elections take place; party L wins with probability P, and party R with probability $1-P$. After the election, the shock ε occurs. In the first case, the government chooses both policies. In the second case, the government and the central bank will simultaneously choose their policy.

2.1 No independent monetary policy

When monetary and fiscal policy are both under the control of the government,[4] the party in government will attempt to minimise its loss function (3) by using two instruments, π and τ. The inflation rates chosen by the two parties if in office and the corresponding outputs in the period immediately after the elections are (where the superscript N indicates no delegation of monetary policy):

$$\pi_L^N = \frac{m_R+2}{(m_L+2)(m_R+1)+P(m_L-m_R)}A - \frac{\varepsilon}{m_L+2}, \quad (4)$$

$$\pi_R^N = \frac{m_L+2}{(m_L+2)(m_R+1)+P(m_L-m_R)}A - \frac{\varepsilon}{m_R+2}, \quad (5)$$

[2]Mathematically, we assume that the arithmetic mean of the weight of inflation relative to output and public spending is higher for party R. Notice that 1 is the weight attributed to inflation in the loss function of parties. In models with only one policy, it is assumed that $\delta_L > \delta_R$ -see, for instance, Alesina (1987) and Alesina and Gatti (1995), and thus in this case $m_L = 1/\delta_L$ and $m_R = 1/\delta_R$, which would correspond to $m_R > m_L$. See also Ferré and Manzano (2013).

[3]As stated before, in models with only one policy, it is assumed that $\delta_L > \delta_R$. However, even though $\delta_L > \delta_R$, it is possible to have either $\frac{\delta_L}{\gamma_L} > \frac{\delta_R}{\gamma_R}$ or $\frac{\delta_L}{\gamma_L} < \frac{\delta_R}{\gamma_R}$.

[4]Notice that this would also include the case where there is a fully dependent central bank, as the central bank would be choosing π in order to minimise (3).

$$x_L^N = x^* - \frac{1}{2\delta_L}\pi_L^N \text{ and} \tag{6}$$

$$x_R^N = x^* - \frac{1}{2\delta_R}\pi_R^N, \tag{7}$$

where $A = x^* + g^* + w^*$.[5]

2.2 Introducing an independent monetary authority

We will now study the case where monetary policy is undertaken by an independent monetary authority. Independence refers to the extent to which the central bank determines monetary policy without political interference.

We will follow Dixit and Lambertini's (2003) claim that, with discretionary policies, fiscal and monetary authorities should be assigned identical goals. Thus, when monetary policy is decided by an independent central bank, its loss function will be

$$V_{CB} = \frac{1}{2}\left(\pi^2 + \delta_{CB}\left(x - x^*\right)^2 + \gamma_{CB}(g - g^*)^2\right), \tag{8}$$

where $\delta_{CB} > 0$ and $\gamma_{CB} \geq 0$.

In this case, the timing of the events is the same, with the only difference that after the shock ε occurs, the central bank will use its instrument (π) to minimise its loss function (8), and the party in government will attempt to minimise its loss function (3) by using the instrument τ. With this institutional specialisation we obtain the following inflation rates and outputs (where superscript D indicates delegation of monetary policy):

$$\pi_L^D = \frac{c_R m_R + 2}{\left(c_L m_L + 2\right)\left(c_R m_R + 1\right) + P\left(c_L m_L - c_R m_R\right)} A - \frac{\varepsilon}{c_L m_L + 2}, \tag{9}$$

$$\pi_R^D = \frac{c_L m_L + 2}{\left(c_L m_L + 2\right)\left(c_R m_R + 1\right) + P\left(c_L m_L - c_R m_R\right)} A - \frac{\varepsilon}{c_R m_R + 2}, \tag{10}$$

$$x_L^D = x^* - \frac{c_L}{2\delta_L}\pi_L^D \text{ and} \tag{11}$$

$$x_R^D = x^* - \frac{c_R}{2\delta_R}\pi_R^D, \tag{12}$$

where we have introduced a new variable, c_j, which is a measure of the degree of the relative conservativeness of the central bank with respect to party j:

$$c_j = \frac{1}{\frac{\frac{\delta_{CB}}{\delta_j} + \frac{\gamma_{CB}}{\gamma_j}}{2}},$$

[5] A detailed derivation of the optimal policies under non-delegation and delegation of monetary policy to an independent central bank can be found in the Appendix (see Propositions A.1 and A.2).

with $j = L, R$.

The notion of conservativeness generally refers to the degree of the central bank's inflation aversion. Our measure of conservativeness compares the relative degree of inflation aversion of the central bank and of the political party. Notice that when $c_j = 1$, the central bank and party j have the same degree of conservativeness, and when $c_j > 1$, the central bank is more conservative than party j.

Remark 1 *If $c_L = 1$ and $c_R = 1$, that is, the central bank is as conservative as both parties, then $\pi_j^D = \pi_j^N$ and $x_j^D = x_j^N$.*

Remark 2 *If $\frac{\delta_L}{\gamma_L} = \frac{\delta_R}{\gamma_R}$, that is, the two parties are identical in their relative interest in stabilising output over spending, then $\pi_L^D = \pi_R^D$ and $x_L^D = x_R^D$. To understand this result note that when $\frac{\delta_L}{\gamma_L} = \frac{\delta_R}{\gamma_R}$, the two parties solve the same optimisation problem and, consequently, as there will be no difference in their behaviour, uncertainty plays no role.*

Rogoff (1985) showed, in a model with only monetary policy, that society's welfare could improve by appointing a more conservative central bank. Ferré and Manzano (2012) demonstrate that, in a model with monetary and fiscal policy, the optimal degree of conservativeness when society's preferences are represented by the government is $c > 1$. For this reason, we will now proceed to focus on cases where the central bank is more conservative than at least one of the political parties. Alesina and Gatti (1995) point out that if political parties are polarised, it might not be easy to reach an agreement to delegate the conduct of monetary policy to an independent institution. They argue, however, that such an agreement will be easier to reach when the independent institution has an inflation aversion that is intermediate. Following these authors, we analyse a central bank more conservative than the left-wing party and less conservative than the right wing party. In our framework this assumption is represented by: $c_L > 1 > c_R$ and we label it "moderately conservative". We also follow the Rogoff tradition and assume that an agreement can be reached to appoint a central bank more conservative than both political parties: $c_L > c_R \geq 1$, which we will refer to as "ultraconservative".

According to Alesina and Gatti (1995), "the institution of an independent and inflation-averse central bank has two benefits: first, it reduces average inflation; second, it eliminates politically induced output variability".[6] We will analyse if these results hold when the model is extended to consider two policies.

[6] Demertzis (2004) carries out numerical simulations on Alesina and Gatti's model and finds that, for an intermediate central bank, inflation might not always be lower.

3 The effects of an independent central bank on inflation

The following expressions show the expected inflation and inflation stabilisation when monetary policy is under the control of the government (N) and when it is decided by an independent central bank (D):[7]

$$E\left(\pi^N\right) = \frac{P\left(m_R+2\right)+\left(1-P\right)\left(m_L+2\right)}{\left(m_L+2\right)\left(m_R+1\right)+P\left(m_L-m_R\right)}A,$$

$$\begin{aligned} E\left((\pi^N)^2\right) &= \frac{P\left(m_R+2\right)^2+\left(1-P\right)\left(m_L+2\right)^2}{\left(\left(m_L+2\right)\left(m_R+1\right)+P\left(m_L-m_R\right)\right)^2}A^2 \\ &+ \left(P\left(\frac{1}{m_L+2}\right)^2+\left(1-P\right)\left(\frac{1}{m_R+2}\right)^2\right)\sigma_\varepsilon^2, \qquad (13) \end{aligned}$$

$$E\left(\pi^D\right) = \frac{P\left(c_R m_R+2\right)+\left(1-P\right)\left(c_L m_L+2\right)}{\left(c_L m_L+2\right)\left(c_R m_R+1\right)+P\left(c_L m_L-c_R m_R\right)}A \text{ and}$$

$$\begin{aligned} E\left((\pi^D)^2\right) &= \frac{P\left(c_R m_R+2\right)^2+\left(1-P\right)\left(c_L m_L+2\right)^2}{\left(\left(c_R m_R+1\right)\left(c_L m_L+2\right)+P\left(c_L m_L-c_R m_R\right)\right)^2}A^2 + \\ &\left(P\left(\frac{1}{c_L m_L+2}\right)^2+\left(1-P\right)\left(\frac{1}{c_R m_R+2}\right)^2\right)\sigma_\varepsilon^2. \qquad (14) \end{aligned}$$

What is the effect of the introduction of an independent central bank, responsible for monetary policy, on expected inflation and inflation stabilisation? The following proposition shows that it will depend on the degree of conservativeness of the central bank.

Proposition 1: *a) By appointing an ultraconsevative ($c_L > c_R \geq 1$) independent central bank responsible for monetary policy, the expected value of inflation is reduced and a higher degree of inflation stabilisation is achieved,*

b) By appointing a moderately conservative ($c_L > 1 > c_R$) independent central bank, the expected value of inflation is reduced and a higher degree of inflation stabilisation is achieved if and only if P is high enough.

In the presence of political uncertainty, inflation is generally lower (in expected terms) and more stable when monetary policy has been delegated to an independent and conservative central bank. In other words, this proposition indicates that when monetary policy is carried out by a conservative and independent central bank, agents expect inflation to be lower and more stable

[7] We look at inflation stabilisation, $E(\pi^2)$, given that the objective of the authorities is to minimise π^2. Recall that $E(\pi^2) = [E(\pi)]^2 + var(\pi)$. The expressions that follow are derived in Lemma A.3.

than if monetary policy was set by the parties. An exception arises in case (b), where the central bank is less conservative than party R. When the probability of party R coming to power is high enough (that is, P is low enough), we expect a lower and more stable inflation without delegating monetary policy to an independent central bank, as this party is already very inflation averse.

4 The effects of an independent central bank on output

The theoretical research that followed Rogoff's article (1985) suggested that central bank independence came at a cost of higher output variability. As Alesina and Summers (1993) did not seem to find evidence of a higher variance of output for OECD countries, Alesina and Gatti (1995) developed a model to explain why such variance might not necessarily increase. The variance of a random variable can be decomposed in two parts: the *politically induced variance* (Var_P), which reflects the fluctuations in the variable induced by electoral uncertainty, and the *economically induced variance* (Var_E), which is due to the exogenous shocks. In Alesina and Gatti (1995), removing the conduct of monetary policy away from the government eliminates the politically induced variance of output. We will study how this result is altered with the introduction of fiscal policy in the analysis.

4.1 The politically induced variance of output

The politically induced variances of output when monetary policy is under the control of the government (N) and when it is decided by an independent central bank (D) are given by (see Lemma A.3):

$$Var_P(x^N) = P\,(1-P)\left(E(x_L^N) - E(x_R^N)\right)^2 \tag{15}$$

and

$$Var_P(x^D) = P\,(1-P)\left(E(x_L^D) - E(x_R^D)\right)^2. \tag{16}$$

The last expression implies that, in general, the politically induced variance of output does not vanish when monetary policy is delegated to an independent central bank. There will only be two scenarios in which this variance vanishes: when there is no political uncertainty ($P = 0, 1$) and when $E(x_L^D) = E(x_R^D)$. Direct computations yield that $E(x_L^D) = E(x_R^D)$ occurs when $\frac{\delta_L}{\gamma_L} = \frac{\delta_R}{\gamma_R}$, and according to Remark 2, in this case the two parties solve the same optimisation problem and, consequently, the political uncertainty introduced by elections does not play any role. These results are summarized in the following proposition:

Proposition 2: *The appointment of an independent central bank when there is more than one policy instrument does not eliminate the variance of output induced by political uncertainty, except when* $\frac{\delta_L}{\gamma_L} = \frac{\delta_R}{\gamma_R}$.

Given that the politically induced variance of output is not eliminated by introducing an independent central bank, in the next lines we will compare this variance with and without delegation of monetary policy. The comparison of $Var_P(x^N)$ and $Var_P(x^D)$ in (15) and (16) is equivalent to comparing the distance between the expected values of output, $\left|E(x_L^i) - E(x_R^i)\right|$, with $i = N, D$. Consequently, the comparison of this type of variances is reduced to the study of expected outputs under both frameworks N and D.

Notice that non-delegation and delegation would coincide when $c_L = c_R = 1$, i.e., monetary policy is undertaken by a central bank that is as conservative as the two parties. We can study the effect of moving towards a moderately conservative central bank ($c_L > 1 > c_R$) by analysing the impact of an increase in c_L and a decrease in c_R, and we can study the effect of introducing an utraconservative central bank ($c_L > c_R \geq 1$) by analysing the consequences of increasing both c_L and c_R. The following result will prove useful in explaining how expected outputs are affected in moving from N (no independent central bank) to D (independent central bank):

Lemma 3: $\frac{\partial}{\partial c_j} E\left(x_j^D\right) < 0$ *and* $\frac{\partial}{\partial c_j} E\left(x_i^D\right) > 0$, $i, j = L, R$, $i \neq j$.

Without any loss of generality, let's focus on an increase in c_L, keeping c_R constant ($c_L > c_R$). This corresponds to a new situation identical to the initial one, except that now monetary policy is undertaken by a more conservative central bank if party L is in office. As Lemma 3 points out, this change in c_L will have two effects on expected outputs: a *direct effect* $\left(\frac{\partial}{\partial c_L} E\left(x_L^D\right) < 0\right)$ and an *indirect effect* $\left(\frac{\partial}{\partial c_L} E\left(x_R^D\right) > 0\right)$. The direct effect arises from the fact that, if monetary policy is now undertaken by a more conservative central bank, expected inflation $E\left(\pi_L^D\right)$ will be lower and, given that $E\left(\pi_L^D\right) - \pi^e$ will decrease, by (1), $E\left(x_L^D\right)$ will be lower. The indirect effect on $E\left(x_R^D\right)$ arises due to the *possibility* of the other party's victory (L), which will lower the overall expected inflation, π^e, as inflation would be decided by a more conservative central bank if that party (L) was in office. Therefore, with a lower π^e, the central bank will have less incentives to inflate if party R is in office (i.e., $E\left(\pi_R^D\right)$ decreases). By virtue of (12), this would have a positive effect on $E\left(x_R^D\right)$.[8]

4.1.1 A moderate central bank

When monetary policy is delegated to a moderately conservative central bank, there will be an increase in c_L and a decrease in c_R with respect to the initial non-delegation situation ($(c_L, c_R) = (1, 1)$). The direct effect of increasing c_L and the indirect effect of decreasing c_R will both generate a decrease in expected output if party L is in office and, consequently, $E\left(x_L^N\right) > E\left(x_L^D\right)$. On the other hand, the indirect effect of increasing c_L and the direct effect of decreasing

[8] Similarly, an increase in c_R keeping c_L constant, would have a direct effect $\left(\frac{\partial}{\partial c_R} E\left(x_R^D\right) < 0\right)$ and an indirect effect $\left(\frac{\partial}{\partial c_R} E\left(x_L^D\right) > 0\right)$.

c_R will both bring an increase in expected output if party R is in office and, consequently, $E\left(x_R^N\right) < E\left(x_R^D\right)$.

The comparison between $Var_P(x^N)$ and $Var_P(x^D)$ will depend on the initial relative size of expected outputs under non-delegation. When party L is relatively less concerned about output than party R $\left(\frac{\delta_L}{\delta_R} \text{ is small}\right)$, then expected output would be lower if party L was in office, i.e. $E\left(x_L^N\right) < E\left(x_R^N\right)$.[9] Given that delegating monetary policy to a moderate central bank decreases expected output for party L and increases expected output for party R, we obtain $E\left(x_L^D\right) < E\left(x_L^N\right) < E\left(x_R^N\right) < E\left(x_R^D\right)$. By (15) and (16), this implies that the politically induced variance of output with an independent central bank is higher than with no independent monetary policy, i.e., $Var_P(x^N) < Var_P(x^D)$. We show this initial situation in the following graph:

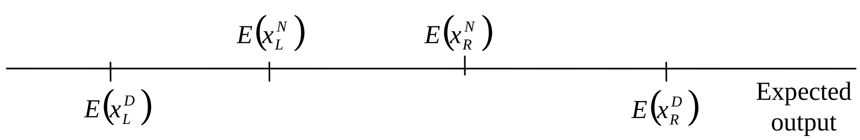

Figure 1. $\frac{\delta_L}{\delta_R}$ is low enough and the central bank is moderately conservative.

When party L gives relatively more weight to output stabilisation $\left(\frac{\delta_L}{\delta_R} \text{ is large}\right)$, then $E\left(x_L^N\right) > E\left(x_R^N\right)$. The following graph shows that in this case the politically induced variance of output is reduced by an independent and moderate central bank:

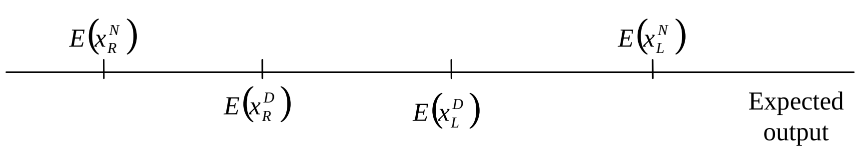

Figure 2. $\frac{\delta_L}{\delta_R}$ is high enough and the central bank is moderately conservative.

[9] Direct computations yield $E\left(x_L^N\right) < E\left(x_R^N\right)$ whenever $\frac{\delta_L}{\delta_R} < \frac{m_R+2}{m_L+2}$.

The following proposition formalizes these results:

Proposition 4: *The appointment of a moderately conservative independent central bank increases the variability of output induced by political uncertainty if and only if* $\frac{\delta_L}{\delta_R}$ *is low enough (*$\frac{\delta_L}{\delta_R} < M$*), where* $\frac{m_R+2}{m_L+2} < M$.[10] *The expression for* M *is provided in the Appendix.*

Proposition 4 and the case presented in Figure 1 are already indicating that the results found by Alesina and Gatti (1995) in the case of one policy cannot be extended when fiscal policy is included. An illustrative case in which the results of Alesina and Gatti would not hold in our model occur when party L cares less about output stabilisation than party R. Politically this might seem puzzling but we must remember that there are more objectives to be considered by the parties and party L could, for instance, be more focused on spending stabilisation. In this case, whenever party L assigns less weight to output stabilisation than party R, $\delta_L < \delta_R$, $\frac{\delta_L}{\delta_R}$ will always be smaller than M, and therefore the political variance with an independent central bank will be larger.

Another illustrative case, perhaps more intuitive, would occur when party L is relatively more focused on output stabilisation than party R, $\delta_L > \delta_R$, being the latter party much more conservative (m_R substantially larger than m_L). In this case, we would only need $\frac{\delta_L}{\delta_R} < \frac{m_R+2}{m_L+2}$. Using the expression of m_L and m_R we have that the previous inequality is equivalent to $4(\delta_L - \delta_R) < \frac{\delta_R}{\gamma_R} - \frac{\delta_L}{\gamma_L}$. Therefore, if party L is relatively less interested in achieving the output target than achieving the public spending target ($\frac{\delta_R}{\gamma_R}$ substantially larger than $\frac{\delta_L}{\gamma_L}$) this condition would be fulfilled.

4.1.2 An ultraconservative central bank

When monetary policy is delegated to an ultraconservative central bank ($c_L > c_R \geq 1$), there will be an increase in both c_L and c_R with respect to the initial non-delegation situation ($(c_L, c_R) = (1,1)$). Now, the direct and indirect effects on expected outputs work in opposite directions. As Lemma 3 indicates notice that, when party j is in office, the increase in c_j decreases $E\left(x_j^D\right)$ (the direct effect), but the increase in c_i increases $E\left(x_j^D\right)$ (the indirect effect), $i, j = L, R$, $i \neq j$. Direct computations yield that, when party L is in office, the direct effect always dominates for expected output and, thus, $E\left(x_L^N\right) > E\left(x_L^D\right)$. By contrast, when party R is in office, the direct effect does not always dominate for expected output.[11]

[10] The critical value for $\frac{\delta_L}{\delta_R}$ is M rather than $\frac{m_R+2}{m_L+2}$. Whenever $\frac{\delta_L}{\delta_R} < M$, expected outputs would follow the relationship shown in Figure 1. Note that $E\left(x_L^N\right) = E\left(x_R^N\right)$ whenever $\frac{\delta_L}{\delta_R} = \frac{m_R+2}{m_L+2}$, but the politically induced variance under an independent central bank is still larger. Finally, when $\frac{\delta_L}{\delta_R}$ is slightly larger than $\frac{m_R+2}{m_L+2}$, $E\left(x_L^N\right) > E\left(x_R^N\right)$ but the politically induced variance of output with the independent central bank would still be higher, due to the movements of expected outputs when moving from N to D.

[11] Notice that the increase in c_L is larger than the increase in c_R; the direct effect is more relevant for $E(x_L)$, whereas it might not always dominate for $E(x_R)$.

The comparison between $Var_P(x^N)$ and $Var_P(x^D)$ will now depend on two factors: the initial relative size of expected outputs under non-delegation and whether the indirect effect dominates when party R is in office. When this indirect effect dominates, $E\left(x_L^N\right) > E\left(x_L^D\right)$ and $E\left(x_R^N\right) < E\left(x_R^D\right)$, and the analysis in this case would be exactly like in the moderate central bank case.

Suppose now that the direct effect dominates, $E\left(x_R^N\right) > E\left(x_R^D\right)$. When $\frac{\delta_L}{\delta_R}$ is small, the introduction of an ultraconservative central bank will reduce expected output for party L more significantly than for party R. The logic for this result is as follows: suppose that δ_R is very large (and, thus $\frac{\delta_L}{\delta_R}$ is small). In this case, $E\left(x_R^D\right)$ will be closer to the output target x^* than $E\left(x_L^D\right)$, and thus $E\left(x_L^D\right) < E\left(x_R^D\right)$. Moreover, as party R prioritises the objective for output in both frameworks, under delegation and non-delegation, expected output is very close to x^*. Therefore, the reduction in expected output when party R is in office is of little significance. Figure 3 illustrates this case, showing that $Var_P(x^N) < Var_P(x^D)$:

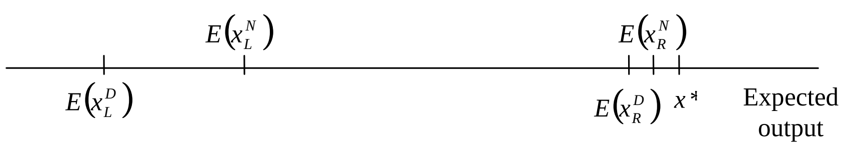

Figure 3. $\dfrac{\delta_L}{\delta_R}$ is very low and the central bank is ultraconservative.

When the direct effect dominates, $E\left(x_R^N\right) > E\left(x_R^D\right)$ and $\frac{\delta_L}{\delta_R}$ is large, the introduction of an ultraconservative central bank will reduce expected output for party R more significantly than for party L. The logic of this result is analogous to the previous case. As can be seen in the following graph, again $Var_P(x^N) < Var_P(x^D)$:

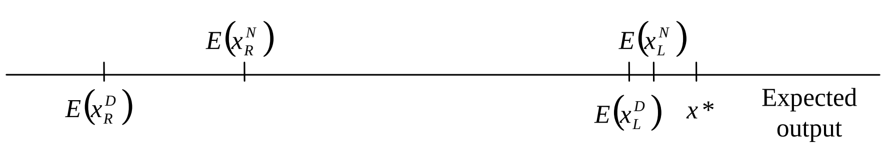

Figure 4. $\dfrac{\delta_L}{\delta_R}$ is very high and the central bank is ultraconservative.

Thus, we find that when the direct effect dominates ($E\left(x_R^N\right) > E\left(x_R^D\right)$) and $\frac{\delta_L}{\delta_R}$ takes an extreme value then $Var_P(x^N) < Var_P(x^D)$. By contrast, when $\frac{\delta_L}{\delta_R}$ is intermediate in value, the reduction in expected output for party L (which is the highest expected output), is now larger than for party R and, therefore, $Var_P(x^N) > Var_P(x^D)$, as shown in Figure 5:

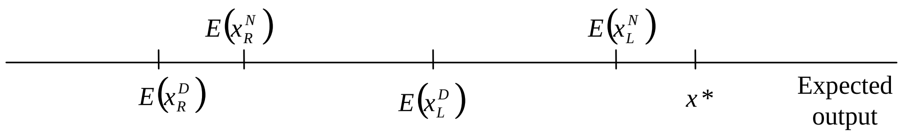

Figure 5. $\frac{\delta_L}{\delta_R}$ takes an intermediate value and the central bank is ultraconservative.

The previous results are summarized in the following proposition:

Proposition 5: *The appointment of an ultraconservative independent central bank increases the politically induced variability of output when:*
a) the indirect effect dominates for $E(x_R)$

$$\text{(i.e., } 1 - c_R + \left(\frac{c_L - c_R}{c_L m_L + 2} + \frac{c_R\left(m_R - m_L\right)\left(c_L - 1\right)}{\left(m_L + 2\right)\left(c_L m_L + 2\right)}\right) m_L P \geq 0\text{)}$$

and $\frac{\delta_L}{\delta_R} < M$,
b) the direct effect dominates for $E(x_R)$

$$\text{(i.e., } 1 - c_R + \left(\frac{c_L - c_R}{c_L m_L + 2} + \frac{c_R\left(m_R - m_L\right)\left(c_L - 1\right)}{\left(m_L + 2\right)\left(c_L m_L + 2\right)}\right) m_L P < 0\text{)}$$

and the value of $\frac{\delta_L}{\delta_R}$ *is not intermediate (i.e., either* $\frac{\delta_L}{\delta_R} < M$ *or* $\frac{\delta_L}{\delta_R} > N$*), where the expressions of* M *and* N *are provided in the Appendix and* $\frac{m_R+2}{m_L+2} < M < \frac{c_L(c_R m_R+2)}{c_R(c_L m_L+2)} < N$.

The condition in part a) of Proposition 5 holds when either party R is much more inflation averse than party L (i.e., whenever $m_R \geqslant m_L + (m_L+2)\frac{Pm_L + c_L m_L + 2}{Pm_L(c_L-1)}$), or when the central bank is not much more conservative than party R (i.e., whenever

$$m_R < m_L + (m_L+2)\frac{Pm_L + c_L m_L + 2}{Pm_L(c_L-1)} \text{ if}$$
$$c_R \leq 1 + \frac{Pm_L(c_L-1)\frac{m_R+2}{(m_L+2)(c_L m_L+2)}}{Pm_L\left(\frac{1}{c_L m_L+2} + \frac{1}{c_L m_L+2}(m_L - m_R)\frac{c_L-1}{m_L+2}\right)+1}).$$

In both cases, the preferences between the central bank and party R are much more similar than those between the central bank and party L. Therefore, the change in expected output when party R is in office due to the possibility that monetary policy will be conducted by the central bank instead of party L is more significative than the change in expected output when party R is in office due to monetary policy being undertaken by the central bank instead of party R. In other words, the indirect effect dominates the direct effect in expected output when party R is in office. When this condition does not hold, the two parties are similarly inflation averse and the central bank is much more conservative than both parties.

When the central bank is ultraconservative, the critical values for $\frac{\delta_L}{\delta_R}$ are M and N. The first critical point, M, is the same as in the moderately conservative central bank (see Proposition 4). Therefore, when $\frac{\delta_L}{\delta_R}$ is smaller than M, $Var_P(x^N) < Var_P(x^D)$, independently of the conservativeness of the central bank. Now, when $\frac{\delta_L}{\delta_R}$ is larger than M, the opposite $\left(Var_P(x^N) > Var_P(x^D)\right)$ will be true as long as $\frac{\delta_L}{\delta_R} < N$. However, when $\frac{\delta_L}{\delta_R} > N$ and the direct effect dominates the indirect effect when party R is in office, $Var_P(x^N) < Var_P(x^D)$.

4.2 The economically induced variance of output

The variances of output due to the economic shocks are given by:

$$Var_E\left(x^N\right) = \left(P\left(\frac{1}{2\delta_L\left(m_L+2\right)}\right)^2 + (1-P)\left(\frac{1}{2\delta_R\left(m_R+2\right)}\right)^2\right)\sigma_\varepsilon^2$$

and

$$Var_E\left(x^D\right) = \left(P\left(\frac{c_L}{2\delta_L\left(c_L m_L+2\right)}\right)^2 + (1-P)\left(\frac{c_R}{2\delta_R\left(c_R m_R+2\right)}\right)^2\right)\sigma_\varepsilon^2.$$

Proposition 6: *The appointment of a moderately conservative independent central bank increases the economically induced variance of output whenever P is large enough. By contrast, the appointment of an ultraconservative central bank always increases the economically induced variance of output.*

This result is in line with the previous literature in that, appointing an independent central bank that is more conservative (than both parties), increases the economically induced variance. However, if the central bank is more conservative than party L but less than party R, then the economically induced variance is higher under delegation whenever P is large enough, that is, when the probability of party L -the less inflation averse party- winning elections is high.

4.3 Output stabilisation

Alesina and Gatti (1995) find that if σ_ε^2 is low enough, delegation of the conduct of monetary policy reduces the variance of output. This is so because, in this case, the relevant component of the volatility of output is the politically induced variance of output. However, the analysis developed in the previous section allows us to conclude that this result is not robust in our framework. We would like to point out that there is another difference between the two models. In Alesina and Gatti (1995), the study of output stabilisation coincides with the study of the variance. To see this, note that applying the standard statistics theory:

$$E\left(\left(x^i - x^*\right)^2\right) = \left(E\left(x^i - x^*\right)\right)^2 + var(x^i - x^*) = \left(E\left(x^i - x^*\right)\right)^2 + var(x^i),\ i = N, D.$$

In Alesina and Gatti (1995), $E\left(x^i - x^*\right) = 0$, $i = N, D$, and hence,

$$E\left(\left(x^i - x^*\right)^2\right) = var(x^i),\ i = N, D,$$

which indicates that in their model to study the output stabilisation term it suffices to analyse the variance of output. However, when $E\left(x^i - x^*\right) \neq 0$, as in our model, this will not be the case. In Lemma A.3 in the Appendix, we rewrite the output stabilisation term as follows:

$$E\left(\left(x^i - x^*\right)^2\right) = P(E(x_L^i - x^*))^2 + (1-P)(E(x_R^i - x^*))^2 + Var_E\left(x^i\right),\ i = N, D.$$

When σ_ε^2 is large enough, the comparison of output stabilisation is reduced to the comparison of the economically induced variance of output. By contrast, when σ_ε^2 is low enough, the comparison of the output stabilisation under delegation and non-delegation is reduced to the comparison of the sum of the first two terms in the previous expression. Notice that we know that the appointment of a moderately conservative independent central bank gives rise to $E\left(x_L^N\right) > E\left(x_L^D\right)$ and $E\left(x_R^N\right) < E\left(x_R^D\right)$. Moreover, since all these expected outputs are smaller than the output target, x^*,we have that

$$0 < E\left(x^* - x_L^N\right) < E\left(x^* - x_L^D\right) \text{ and } 0 < E\left(x^* - x_R^D\right) < E\left(x^* - x_R^N\right).$$

Hence, $\left(E(x_L^D - x^*)\right)^2 > \left(E(x_L^N - x^*)\right)^2$ and $\left(E(x_R^D - x^*)\right)^2 < \left(E(x_R^N - x^*)\right)^2$. Therefore, we can conclude that when σ_ε^2 is low enough and P is high enough, output is more stable under nondelegation, whereas the opposite result is obtained when P is low enough. In other words, whenever the supply shocks are not significant, output stabilisation will be more effective without an independent central bank the more likely is the less inflation party to win the elections.

When an ultraconservative central bank is appointed and the indirect effect dominates, we obtain the same result. If the direct effect dominates, then $E\left(x_L^N\right) > E\left(x_L^D\right)$ and $E\left(x_R^N\right) > E\left(x_R^D\right)$. Hence, $0 < E\left(x^* - x_L^N\right) < E\left(x^* - x_L^D\right)$ and $0 < E\left(x^* - x_R^N\right) < E\left(x^* - x_R^D\right)$. Accordingly, $\left(E(x_L^D - x^*)\right)^2 > \left(E(x_L^N - x^*)\right)^2$ and $\left(E(x_R^D - x^*)\right)^2 > \left(E(x_R^N - x^*)\right)^2$. Therefore, we can conclude that, in this case, when σ_ε^2 is low enough output is more stable under nondelegation.

5 Conclusions

The analysis presented in this article has shown that extending a rational partisan theory model to include two policies, monetary and fiscal policy, can significantly change the results obtained with models with only monetary policy.

First of all, it has been shown that the benefits in terms of inflation (low and stable inflation) of the appointment of an independent central bank depend on the degree of conservativeness of the central bank. An ultraconservative independent central bank always achieves lower and more stable inflation. However, a moderate independent central bank only achieves lower and more stable inflation if the probability of the less inflation averse party winning the elections is high enough.

Other results obtained in this article are related to the politically and economically induced variances of output. When there is more than one policy instrument, the appointment of an independent central bank does not eliminate the politically induced variance of output. Even more, we cannot conclude that this type of variance is reduced when the monetary policy is delegated to the central bank.

The appointment of an ultraconservative central bank unequivocally increases the economically induced variance of output. By contrast, the appointment of a moderately conservative independent central bank increases the economically induced variance of output whenever the probability of the less inflation averse party winning the elections is high enough.

Finally, the analysis illustrates that the stability of output depends on the variance of the supply shocks. When supply shocks are very relevant, the comparison of the stability of output is reduced to the comparison of the economically induced variance of output. When the supply shocks are not significant, we obtain that output is more stable under delegation to a moderate independent central bank provided that the probability of wining the elections by the less inflation averse party is large enough. When an ultraconservative central bank is appointed there are circumstances where we obtain the previous result and there are other circumstances where output is more stable under nondelegation. In this last case a trade-off between inflation and output stabilisation arises.

The model has been presented as a static one-shot game, attempting to extend the seminal model of Alesina and Gatti (1995) to include a second policy and study the main effects of such extension. Once it has been asserted what these effects are, there are some other possible avenues of research. For instance, a natural extension would be to endogenise the probability of a party being elected. In this potentially dynamic setting, public debt could also be introduced in order to affect the probability of being elected by the incumbent party.

Appendix:

We begin the Appendix with the derivation of the optimal policies under non-delegation and delegation, respectively.

Proposition A.1: *The policies chosen by the two parties, if in office, under non-delegation are given by*

$$\begin{aligned}
\pi_L^N &= \frac{m_R+2}{(m_L+2)(m_R+1)+P(m_L-m_R)}A-\frac{\varepsilon}{m_L+2}, \\
\pi_R^N &= \frac{m_L+2}{(m_L+2)(m_R+1)+P(m_L-m_R)}A-\frac{\varepsilon}{m_R+2}, \\
\tau_L^N &= g^*-\left(1+\frac{1}{2\gamma_L}\right)\pi_L^N \textit{ and} \\
\tau_R^N &= g^*-\left(1+\frac{1}{2\gamma_R}\right)\pi_R^N.
\end{aligned}$$

Proof of Proposition A.1: Under non-delegation,[12] the party in office, denoted by j, chooses π and τ in order to solve the following optimisation problem:

$$\min_{\pi,\tau} V_{Gj}=\frac{1}{2}\left(\pi^2+\delta_j(x-x^*)^2+\gamma_j^2(g-g^*)\right).$$

The first order conditions (f.o.c.) of this optimisation problem are given by[13]

$$\begin{aligned}
\frac{\partial}{\partial\pi}V_{Gj} &= \pi+\delta_j(x-x^*)+\gamma_j(g-g^*)=0 \text{ and} \\
\frac{\partial}{\partial\tau}V_{Gj} &= -\delta_j(x-x^*)+\gamma_j(g-g^*)=0.
\end{aligned}$$

Using the Expressions (1) and (2) in the previous two equalities, it follows that

$$\pi_j = \frac{1}{m_j+2}(\pi^e+A-\varepsilon) \text{ and} \tag{17}$$

$$\tau_j = g^*-\frac{\delta_j\left(2\gamma_j+1\right)}{\gamma_j+\delta_j+4\gamma_j\delta_j}(\pi^e+A-\varepsilon), \tag{18}$$

where

$$\begin{aligned}
m_j &= \frac{\frac{1}{\delta_j}+\frac{1}{\gamma_j}}{2} \text{ and} \\
A &= g^*+w^*+x^*.
\end{aligned}$$

[12] To ease the analysis, we drop the superscript N in this proof.

[13] Direct computations yield that the objective function is strictly convex. Therefore, the first order conditions are necessary and sufficient to obtain a minimum. The same comment applies for the remainder optimisation problems.

Rewriting (17) for the two parties, we have

$$\begin{aligned}\pi_L &= \frac{1}{m_L+2}\left(\pi^e + A - \varepsilon\right) \text{ and} \\ \pi_R &= \frac{1}{m_R+2}\left(\pi^e + A - \varepsilon\right).\end{aligned}$$

Recall that $\pi^e = PE(\pi_L) + (1-P)E(\pi_R)$. Taking expectations in the previous expressions and solving for π^e, we get

$$\pi^e = \frac{P\frac{1}{m_L+2} + (1-P)\frac{1}{m_R+2}}{1 - \left(P\frac{1}{m_L+2} + (1-P)\frac{1}{m_R+2}\right)}A. \tag{19}$$

Substituting this expression into (17) and (18) for $j = L, R$, and after some algebra, we obtain the expressions for π_L, π_R, τ_L and τ_R included in the statement of this proposition. ■

Proposition A.2: *Under delegation, the policies chosen by the central bank and the party, if in office, are given by*

$$\begin{aligned}\pi_L^D &= \frac{c_R m_R + 2}{(c_L m_L + 2)(c_R m_R + 1) + P(c_L m_L - c_R m_R)}A - \frac{\varepsilon}{c_L m_L + 2}, \\ \pi_R^D &= \frac{c_L m_L + 2}{(c_L m_L + 2)(c_R m_R + 1) + P(c_L m_L - c_R m_R)}A - \frac{\varepsilon}{c_R m_R + 2}, \\ \tau_L^D &= g^* - \left(1 + \frac{c_L}{2\gamma_L}\right)\pi_L^D \text{ and} \\ \tau_R^D &= g^* - \left(1 + \frac{c_R}{2\gamma_R}\right)\pi_R^D.\end{aligned}$$

Proof of Proposition A.2: Under delegation,[14] the central bank chooses π in order to solve the following optimisation problem:

$$\min_{\pi} V_{CB} = \frac{1}{2}\left(\pi^2 + \delta_{CB}(x - x^*)^2 + \gamma_{CB}(g - g^*)^2\right).$$

The first order condition (f.o.c.) of this optimisation problem is given by

$$\frac{\partial}{\partial \pi} V_{CB} = \pi + \delta_{CB}(x - x^*) + \gamma_{CB}(g - g^*) = 0.$$

In this setup the party in office, denoted by j, chooses τ in order to solve the following optimisation problem:

$$\min_{\tau} V_{G_j} = \frac{1}{2}\left(\pi^2 + \delta_j (x - x^*)^2 + \gamma_j (g - g^*)^2\right).$$

[14] Again to simplify the notation, we drop the superscript D in this proof.

The first order condition (f.o.c.) of this optimisation problem is given by

$$\frac{\partial}{\partial \tau} V_{G_j} = -\delta_j \left(x - x^*\right) + \gamma_j (g - g^*) = 0.$$

Using the expressions (1) and (2) in the f.o.c. of the authorities' problems, and after some algebra, it follows that

$$\pi_j = \frac{\gamma_j \delta_{CB} + \delta_j \gamma_{CB}}{\gamma_j + \delta_j + 2\gamma_j \delta_{CB} + 2\delta_j \gamma_{CB}} \left(\pi^e + A - \varepsilon\right) \text{ and} \tag{20}$$

$$\tau_j = g^* - \frac{\delta_j + \gamma_j \delta_{CB} + \delta_j \gamma_{CB}}{\gamma_j + \delta_j + 2\gamma_j \delta_{CB} + 2\delta_j \gamma_{CB}} \left(\pi^e + A - \varepsilon\right). \tag{21}$$

Rewriting (20) for the two parties, we have

$$\begin{aligned}
\pi_L &= \frac{\gamma_L \delta_{CB} + \delta_L \gamma_{CB}}{\gamma_L + \delta_L + 2\gamma_L \delta_{CB} + 2\delta_L \gamma_{CB}} \left(\pi^e + A - \varepsilon\right) \text{ and} \\
\pi_R &= \frac{\gamma_R \delta_{CB} + \delta_R \gamma_{CB}}{\gamma_R + \delta_R + 2\gamma_R \delta_{CB} + 2\delta_R \gamma_{CB}} \left(\pi^e + A - \varepsilon\right).
\end{aligned}$$

Using the expressions for c_L and c_R, we get

$$\delta_{CB}\gamma_L + \gamma_{CB}\delta_L = \frac{2\delta_L \gamma_L}{c_L} \text{ and } \delta_{CB}\gamma_R + \gamma_{CB}\delta_R = \frac{2\delta_R \gamma_R}{c_R}.$$

Hence,

$$\begin{aligned}
\pi_L &= \frac{1}{c_L m_L + 2} \left(\pi^e + A - \varepsilon\right) \text{ and} \\
\pi_R &= \frac{1}{c_R m_R + 2} \left(\pi^e + A - \varepsilon\right).
\end{aligned}$$

Again recall that $\pi^e = PE\left(\pi_L\right) + (1-P)E\left(\pi_R\right)$. Taking expectations in the previous expressions and solving for π^e, we get

$$\pi^e = \frac{P\left(\frac{1}{c_L m_L + 2}\right) + (1-P)\left(\frac{1}{c_R m_R + 2}\right)}{1 - \left(P \frac{1}{c_L m_L + 2} + (1-P)\left(\frac{1}{c_R m_R + 2}\right)\right)} A.$$

Substituting this expression into (20) and (21) for $j = L, R$, and after some algebra, we obtain the expressions for π_L, π_R, τ_L and τ_R included in the statement of this proposition. ■

Next, we derive a lemma which will be useful to prove some of the coming results:

Lemma A.3: *Consider a random variable z that, conditional on the realization of the shock takes two possible values given by $z_L = E(z_L) + F_L \varepsilon$ and $z_R = E(z_R) + F_R \varepsilon$. Then, the politically induced variance of the variable z is given by*

$$Var_P\left(z\right) = P\left(1-P\right)\left(E(z_L) - E(z_R)\right)^2,$$

and the economically induced variance of the output

$$Var_E(z) = \left(P(F_L)^2 + (1-P)(F_R)^2\right)\sigma_\varepsilon^2.$$

Moreover,

$$E\left(z^2\right) = P(E(z_L))^2 + (1-P)(E(z_R))^2 + \left(P(F_L)^2 + (1-P)(F_R)^2\right)\sigma_\varepsilon^2.$$

Proof of Lemma A.3: Recall that

$$z_L = E(z_L) + F_L\varepsilon \text{ and } z_R = E(z_R) + F_R\varepsilon.$$

Moreover, recall that $E(z) = PE(z_L) + (1-P)E(z_R)$.Using these expressions, direct computations yield

$$E\left((z_L - E(z))^2\right) = (1-P)^2 (E(z_L) - E(z_R))^2 + (F_L)^2 \sigma_\varepsilon^2. \tag{22}$$

Analogously, we have that

$$E\left((z_R - E(z))^2\right) = P^2 (E(z_L) - E(z_R))^2 + (F_R)^2 \sigma_\varepsilon^2. \tag{23}$$

In addition,

$Var(z) = E\left((z - E(z))^2\right) = PE\left((z_L - E(z))^2\right) + (1-P)E\left((z_R - E(z))^2\right).$
Substituting (22) and (23), we get

$$Var(z) = P(1-P)(E(z_L) - E(z_R))^2 + \left(P(F_L)^2 + (1-P)(F_R)^2\right)\sigma_\varepsilon^2.$$

Note that the first term of the right hand side of the previous equality corresponds to the politically induced variance of the random variable z, whereas the second term corresponds to the economically induced variance of z. Finally, recall that $E\left(z^2\right) = (E(z))^2 + var(z)$. Operating, we get

$$E\left(z^2\right) = P(E(z_L))^2 + (1-P)(E(z_R))^2 + \left(P(F_L)^2 + (1-P)(F_R)^2\right)\sigma_\varepsilon^2. \blacksquare$$

Derivation of Expressions (13) and (14): Applying Lemma A.3 for $z = \pi^N$ and $z = \pi^D$, direct computations yield

$$E\left((\pi^N)^2\right) = \frac{P(m_R+2)^2 + (1-P)(m_L+2)^2}{((m_L+2)(m_R+1) + P(m_L - m_R))^2}A^2 + \\ + \left(P\left(\frac{1}{m_L+2}\right)^2 + (1-P)\left(\frac{1}{m_R+2}\right)^2\right)\sigma_\varepsilon^2 \text{ and}$$

$$E\left((\pi^D)^2\right) = \frac{P(c_R m_R+2)^2 + (1-P)(c_L m_L+2)^2}{((c_R m_R+1)(c_L m_L+2) + P(c_L m_L - c_R m_R))^2}A^2 + \\ + \left(P\left(\frac{1}{c_L m_L+2}\right)^2 + (1-P)\left(\frac{1}{c_R m_R+2}\right)^2\right)\sigma_\varepsilon^2. \blacksquare$$

Proof of Proposition 1: a) Let

$$f(c_L, c_R) = E\left(\pi^D\right) = \frac{P\left(c_R m_R + 2\right) + (1-P)\left(c_L m_L + 2\right)}{\left(c_L m_L + 2\right)\left(c_R m_R + 1\right) + P\left(c_L m_L - c_R m_R\right)} A$$

and

$$g(c_L, c_R) = E\left((\pi^D)^2\right) = \frac{P(c_R m_R + 2)^2 + (1-P)(c_L m_L + 2)^2}{((c_R m_R + 1)(c_L m_L + 2) + P(c_L m_L - c_R m_R))^2} A^2 + \\ + \left(P\left(\frac{1}{c_L m_L + 2}\right)^2 + (1-P)\left(\frac{1}{c_R m_R + 2}\right)^2\right)\sigma_\varepsilon^2.$$

Direct computations yield that $f(c_L, c_R)$ and $g(c_L, c_R)$ are decreasing functions in c_L and c_R. Moreover, $f(1,1) = E\left(\pi^N\right)$ and $g(1,1) = E\left((\pi^N)^2\right)$. The combination of these results allows us to conclude that $E\left(\pi^N\right) > E\left(\pi^D\right)$ and $E\left((\pi^N)^2\right) > E\left((\pi^D)^2\right)$ whenever $c_L > c_R \geq 1$.

b) Suppose now that $c_L > 1 > c_R$. First, we focus on the comparison of the expected inflation. Let $h(P) = E\left(\pi^N\right) - E\left(\pi^D\right)$. Differentiating,

$$\begin{aligned} \frac{\partial}{\partial P} h(P) &= \frac{\left(m_R - m_L\right)\left(m_L + 2\right)\left(m_R + 2\right)}{\left(\left(m_L + 2\right)\left(m_R + 1\right) + P\left(m_L - m_R\right)\right)^2} A \\ &\quad - \frac{\left(c_L m_L + 2\right)\left(c_R m_R + 2\right)\left(c_R m_R - c_L m_L\right)}{\left(\left(c_R m_R + 1\right)\left(c_L m_L + 2\right) + P\left(c_L m_L - c_R m_R\right)\right)^2} A. \end{aligned}$$

Direct computations yield $\frac{\partial}{\partial c_L}\left(\frac{\partial}{\partial P} h(P)\right) > 0$ and $\frac{\partial}{\partial c_R}\left(\frac{\partial}{\partial P} h(P)\right) < 0$. Hence, $\frac{\partial}{\partial P} h(P) > \frac{\partial}{\partial P} h(P)|_{\substack{c_L = 1 \\ c_R = 1}} = 0$ since $c_L > 1 > c_R$. Therefore, $h(P)$ is an increasing function in P. Moreover, $h(1) > 0$ and $h(0) < 0$ whenever $c_L > 1 > c_R$. This implies that there exists a unique value $\overline{P}$ such that $h(P) > 0$ (or equivalently, $E\left(\pi^N\right) > E\left(\pi^D\right)$) if and only if $P > \overline{P}$.

In relation to the comparison of the term related to inflation stabilisation, we distinguish three cases:

Case 1: $c_R m_R < c_L m_L$. As $c_L > 1 > c_R$, $E\left((\pi^N)^2\right)|_{P=1} > E\left((\pi^D)^2\right)|_{P=1}$ and $E\left((\pi^N)^2\right)|_{P=0} < E\left((\pi^D)^2\right)|_{P=0}$. Moreover, from Lemma 1, we know that in this case $E\left((\pi^N)^2\right)$ increases in P, whereas $E\left((\pi^D)^2\right)$ decreases in P. Therefore, we can conclude that there exists a value $\overline{P}$ such that $E\left((\pi^D)^2\right) < E\left((\pi^N)^2\right)$ if and only if $P > \overline{P}$.

Case 2: $c_R m_R = c_L m_L$. In this case, from Lemma 1, we know that in this case $E\left((\pi^N)^2\right)$ increases in P, whereas $E\left((\pi^D)^2\right)$ is independent of P. Again, the fact that $E\left((\pi^N)^2\right)|_{P=1} > E\left((\pi^D)^2\right)|_{P=1}$ and $E\left((\pi^N)^2\right)|_{P=0} < E\left((\pi^D)^2\right)|_{P=0}$,allows us to conclude that there exists a value $\overline{P}$ such that $E\left((\pi^D)^2\right) < E\left((\pi^N)^2\right)$ if and only if $P > \overline{P}$.

Case 3: $c_R m_R > c_L m_L$. Let $k(P) = E\left((\pi^N)^2\right) - E\left((\pi^D)^2\right)$. Differentiating,

$$\frac{\partial}{\partial P} k(P) = A^2\left(m_R - m_L\right) \frac{(m_L + m_R + 4)(m_R - m_L)P + (m_L + 2)\left(m_R^2 + (m_L + 5)m_R + 3m_L + 8\right)}{((m_L + 2)(m_R + 1) + P(m_L - m_R))^3}$$

$$-A^2\left(c_Rm_R-c_Lm_L\right)\times$$
$$\times\frac{(c_Lm_L+c_Rm_R+4)(c_Rm_R-c_Lm_L)P+(c_Lm_L+2)\left(c_R^2m_R^2+(c_Lm_L+5)c_Rm_R+3c_Lm_L+8\right)}{((c_Rm_R+1)(c_Lm_L+2)+P(c_Lm_L-c_Rm_R))^3}+$$
$$+\sigma_\varepsilon^2\left((m_R-m_L)\frac{m_L+m_R+4}{(m_R+2)^2(m_L+2)^2}-(c_Rm_R-c_Lm_L)\frac{c_Lm_L+c_Rm_R+4}{(c_Rm_R+2)^2(c_Lm_L+2)^2}\right).$$

After some algebra, we have $\frac{\partial}{\partial c_L}\left(\frac{\partial}{\partial P}k(P)\right)>0$. Now, we distinguish two subcases:

Subcase 3.1: $\frac{\partial}{\partial c_R}\left(\frac{\partial}{\partial P}k(P)\right)\leq 0$. In this case taking into account that $\frac{\partial}{\partial c_L}\left(\frac{\partial}{\partial P}k(P)\right)>0$, $c_L>1$ and $c_R<1$, we get $\frac{\partial}{\partial P}k(P)>\frac{\partial}{\partial P}k(P)|_{\substack{c_L=1\\c_R=1}}>0$.

Subcase 3.2: $\frac{\partial}{\partial c_R}\left(\frac{\partial}{\partial P}k(P)\right)>0$. As $c_R>\frac{c_Lm_L}{m_R}$, in this case $\frac{\partial}{\partial P}k(P)>\frac{\partial}{\partial P}k(P)|_{c_R=\frac{c_Lm_L}{c_R}}>0$.

Therefore, in both subcases we have that $k(P)$ is strictly increasing in P. Taking into account that $k(0)<0$ and $k(1)>0$, we can conclude that there exists a unique value $\overline{P}$ such that $k(P)>0$ (or equivalently, $E\left((\pi^N)^2\right)>E\left((\pi^D)^2\right)$) if and only if $P>\overline{P}$. ■

Proof of Proposition 2: Combining (9), (10), (11) and (12), we have

$$\begin{aligned}E\left(x_L^D\right)&=x^*-\frac{c_L}{2\delta_L}\frac{c_Rm_R+2}{(c_Lm_L+2)(c_Rm_R+1)+P(c_Lm_L-c_Rm_R)}A\text{ and}\\E\left(x_R^D\right)&=x^*-\frac{c_R}{2\delta_R}\frac{c_Lm_L+2}{(c_Lm_L+2)(c_Rm_R+1)+P(c_Lm_L-c_Rm_R)}A.\end{aligned}$$

Hence,

$$E\left(x_L^D\right)-E\left(x_R^D\right)=\frac{1}{2}\frac{\delta_Lc_R(c_Lm_L+2)-\delta_Rc_L(c_Rm_R+2)}{\delta_L\delta_R((c_Lm_L+2)(c_Rm_R+1)+P(c_Lm_L-c_Rm_R))}A,$$

or using the expressions of m_L, m_R, c_L and c_R, it follows that

$$\delta_Lc_R(c_Lm_L+2)-\delta_Rc_L(c_Rm_R+2)=\frac{2\delta_L\delta_R\gamma_L(\delta_L\gamma_R-\gamma_L\delta_R)(2\gamma_{CB}+1)}{\gamma_L(\gamma_L\delta_{CB}+\delta_L\gamma_{CB})(\gamma_R\delta_{CB}+\delta_R\gamma_{CB})}.$$

Combining the previous two equalities, we conclude that $E(x_L^D)=E(x_R^D)$ if and only if $\frac{\delta_L}{\gamma_L}=\frac{\delta_R}{\gamma_R}$. ■

Proof of Lemma 3: Combining (9), (10), (11) and (12), we have

$$E\left(x_L^D\right)=x^*-\frac{c_L}{2\delta_L}\frac{c_Rm_R+2}{(c_Lm_L+2)(c_Rm_R+1)+P(c_Lm_L-c_Rm_R)}A\tag{24}$$

and

$$E\left(x_R^D\right)=x^*-\frac{c_R}{2\delta_R}\frac{c_Lm_L+2}{(c_Lm_L+2)(c_Rm_R+1)+P(c_Lm_L-c_Rm_R)}A.\tag{25}$$

Differentiating, we have the results stated in the statement of this lemma. ■

Proof of Proposition 4: Recall that when monetary policy is delegated to a moderately conservative central bank

$$\begin{aligned} E\left(x_L^N\right) &> E\left(x_L^D\right) \text{ and} \quad (26)\\ E\left(x_R^N\right) &< E\left(x_R^D\right). \quad (27)\end{aligned}$$

Now we distinguish three cases: 1) $\frac{\delta_L}{\delta_R} \le \frac{m_R+2}{m_L+2}$, 2) $\frac{m_R+2}{m_L+2} < \frac{\delta_L}{\delta_R} \le \frac{c_L(c_R m_R+2)}{c_R(c_L m_L+2)}$ and 3) $\frac{\delta_L}{\delta_R} > \frac{c_L(c_R m_R+2)}{c_R(c_L m_L+2)}$.

Case 1: $\frac{\delta_L}{\delta_R} \le \frac{m_R+2}{m_L+2}$. In this case $E\left(x_L^N\right) \le E\left(x_R^N\right)$. Combining this inequality, (26) and (27), it follows that

$$E\left(x_L^D\right) < E\left(x_L^N\right) \le E\left(x_R^N\right) < E\left(x_R^D\right).$$

Hence, $E\left(x_R^D\right) - E\left(x_L^D\right) > E\left(x_R^N\right) - E\left(x_L^N\right)$, i.e., the difference (in absolute value) of the expected terms under delegation is higher than under non-delegation. This allows us to conclude that in this case $Var_P(x^D) > Var_P(x^N)$.

Case 2: $\frac{m_R+2}{m_L+2} < \frac{\delta_L}{\delta_R} \le \frac{c_L(c_R m_R+2)}{c_R(c_L m_L+2)}$. In this case we know that $E\left(x_L^N\right) > E\left(x_R^N\right)$ and $E\left(x_L^D\right) < E\left(x_R^D\right)$. Therefore, to show $Var_P(x^N) < Var_P(x^D)$, it suffices to prove

$$E\left(x_L^N\right) - E\left(x_R^N\right) < E\left(x_R^D\right) - E\left(x_L^D\right). \quad (28)$$

Combining (4), (5), (6) and (7), we have

$$\begin{aligned} E\left(x_L^N\right) &= x^* - \frac{1}{2\delta_L}\frac{m_R+2}{(m_L+2)(m_R+1)+P(m_L-m_R)}A \text{ and}\\ E\left(x_R^N\right) &= x^* - \frac{1}{2\delta_R}\frac{m_L+2}{(m_L+2)(m_R+1)+P(m_L-m_R)}A.\end{aligned}$$

Using the previous two expressions, (24) and (25), (28) can be rewritten as

$$\frac{\delta_L(m_L+2)-\delta_R(m_R+2)}{2\delta_L\delta_R((m_L+2)(m_R+1)+P(m_L-m_R))}A < \frac{-\delta_L c_R(c_L m_L+2)+\delta_R c_L(c_R m_R+2)}{2\delta_L\delta_R((c_L m_L+2)(c_R m_R+1)+P(c_L m_L-c_R m_R))}A,$$

or equivalently,

$$\frac{\delta_L}{\delta_R} < M,$$

with

$$M = \frac{c_L\frac{c_R m_R+2}{(c_L m_L+2)(c_R m_R+1)+P(c_L m_L-c_R m_R)} + \frac{m_R+2}{(m_L+2)(m_R+1)+P(m_L-m_R)}}{c_R\frac{c_L m_L+2}{(c_L m_L+2)(c_R m_R+1)+P(c_L m_L-c_R m_R)} + \frac{m_L+2}{(m_L+2)(m_R+1)+P(m_L-m_R)}}.$$

Moreover, after some algebra we have that $\frac{m_R+2}{m_L+2} < M < \frac{c_L(c_R m_R+2)}{c_R(c_L m_L+2)}$ whenever $c_L > c_R$, $c_L > 1$ and $m_R > m_L$. Therefore, in Case 2 we conclude that there exists a value M such that $Var_P(x^N) < Var_P(x^D)$ if and only if $\frac{\delta_L}{\delta_R} < M$.

Case 3: $\frac{\delta_L}{\delta_R} > \frac{c_L(c_R m_R+2)}{c_R(c_L m_L+2)}$. In this case we know that $E\left(x_L^N\right) > E\left(x_R^N\right)$ and $E\left(x_L^D\right) > E\left(x_R^D\right)$. Therefore, to show $Var_P(x^N) > Var_P(x^D)$, it suffices to prove

$$E\left(x_L^N\right) - E\left(x_R^N\right) > E\left(x_L^D\right) - E\left(x_R^D\right),$$

or equivalently,

$$E\left(x_L^N\right) - E\left(x_L^D\right) > E\left(x_R^N\right) - E\left(x_R^D\right).$$

Using (26) and (27), we know that the left-hand side of the previous inequality is positive and the right-hand side is negative. Consequently, this inequality holds and, this allows us to conclude that in Case 3 $Var_P(x^N) > Var_P(x^D)$. ■

Proof of Proposition 5: We distinguish three cases: 1) $\frac{\delta_L}{\delta_R} \leq \frac{m_R+2}{m_L+2}$, 2) $\frac{m_R+2}{m_L+2} < \frac{\delta_L}{\delta_R} \leq \frac{c_L(c_R m_R+2)}{c_R(c_L m_L+2)}$ and, 3) $\frac{\delta_L}{\delta_R} > \frac{c_L(c_R m_R+2)}{c_R(c_L m_L+2)}$.

Case 1: $\frac{\delta_L}{\delta_R} \leq \frac{m_R+2}{m_L+2}$. In this case $E\left(x_L^N\right) \leq E\left(x_R^N\right)$ and $E\left(x_L^D\right) < E\left(x_R^D\right)$. Therefore, to show $Var_P(x^N) < Var_P(x^D)$, it suffices to prove

$$E\left(x_R^N\right) - E\left(x_L^N\right) < E\left(x_R^D\right) - E\left(x_L^D\right),$$

or equivalently,

$$E\left(x_R^N\right) - E\left(x_R^D\right) < E\left(x_L^N\right) - E\left(x_L^D\right). \tag{29}$$

Notice that the right hand side of the previous inequality is positive since $E\left(x_L^N\right) > E\left(x_L^D\right)$. Next, we distinguish two subcases: 1.1) $E\left(x_R^N\right) \leq E\left(x_R^D\right)$, and 1.2) $E\left(x_R^N\right) > E\left(x_R^D\right)$.

Subcase 1.1 ($E\left(x_R^N\right) \leq E\left(x_R^D\right)$): In this case the left hand side of the previous inequality is negative and hence, (29) holds.

Subcase 1.2 ($E\left(x_R^N\right) > E\left(x_R^D\right)$, i.e., $c_R \frac{c_L m_L+2}{(c_L m_L+2)(c_R m_R+1)+P(c_L m_L-c_R m_R)} > \frac{m_L+2}{(m_L+2)(m_R+1)+P(m_L-m_R)}$**):** In this case, substituting the expressions of $E\left(x_L^N\right)$, $E\left(x_R^N\right)$, $E\left(x_R^D\right)$ and $E\left(x_L^D\right)$ and after some algebra, we have that (29) is equivalent to

$$\frac{\delta_L}{\delta_R} < \frac{m_R+2}{m_L+2} + \frac{2\frac{(m_L+2)(c_L-c_R)+c_R(m_R-m_L)(c_L-1)}{(c_L m_L+2)(c_R m_R+1)+P(c_L m_L-c_R m_R)}}{\left(c_R\frac{c_L m_L+2}{(c_L m_L+2)(c_R m_R+1)+P(c_L m_L-c_R m_R)} - \frac{m_L+2}{(m_L+2)(m_R+1)+P(m_L-m_R)}\right)(m_L+2)}.$$

In this case the second term of the right hand side of the previous inequality is positive. Then, we can conclude that this inequality is satisfied whenever $\frac{\delta_L}{\delta_R} \leq \frac{m_R+2}{m_L+2}$.

Case 2: $\frac{m_R+2}{m_L+2} < \frac{\delta_L}{\delta_R} \leq \frac{c_L(c_R m_R+2)}{c_R(c_L m_L+2)}$. In this case we know that $E\left(x_L^N\right) > E\left(x_R^N\right)$ and $E\left(x_L^D\right) < E\left(x_R^D\right)$. Therefore, to show $Var_P(x^N) < Var_P(x^D)$, it suffices to prove

$$E\left(x_L^N\right) - E\left(x_R^N\right) < E\left(x_R^D\right) - E\left(x_L^D\right).$$

Substituting the expressions of $E\left(x_L^N\right)$, $E\left(x_R^N\right)$, $E\left(x_R^D\right)$ and $E\left(x_L^D\right)$ and after some algebra, we have that the previous inequality is equivalent to

$$\frac{\delta_L}{\delta_R} < M,$$

with

$$M = \frac{c_L \frac{c_R m_R+2}{(c_L m_L+2)(c_R m_R+1)+P(c_L m_L - c_R m_R)} + \frac{m_R+2}{(m_L+2)(m_R+1)+P(m_L-m_R)}}{c_R \frac{c_L m_L+2}{(c_L m_L+2)(c_R m_R+1)+P(c_L m_L - c_R m_R)} + \frac{m_L+2}{(m_L+2)(m_R+1)+P(m_L-m_R)}}.$$

Moreover, after some algebra we have that $\frac{m_R+2}{m_L+2} < M < \frac{c_L(c_R m_R+2)}{c_R(c_L m_L+2)}$ whenever $c_L > c_R$, $c_L > 1$ and $m_R > m_L$. Therefore, in Case 2 we conclude that there exists a value M such that $Var_P(x^N) < Var_P(x^D)$ if and only if $\frac{\delta_L}{\delta_R} < M$.

Case 3: $\frac{\delta_L}{\delta_R} > \frac{c_L(c_R m_R+2)}{c_R(c_L m_L+2)}$. In this case we know that $E\left(x_L^N\right) > E\left(x_R^N\right)$ and $E\left(x_L^D\right) > E\left(x_R^D\right)$. Therefore, to show $Var_P(x^N) > Var_P(x^D)$, it suffices to prove

$$E\left(x_L^N\right) - E\left(x_R^N\right) > E\left(x_L^D\right) - E\left(x_R^D\right),$$

or equivalently,

$$E\left(x_L^N\right) - E\left(x_L^D\right) > E\left(x_R^N\right) - E\left(x_R^D\right). \tag{30}$$

Notice that the left-hand side side of the previous inequality is positive since $E\left(x_L^N\right) > E\left(x_L^D\right)$. Then, we distinguish two subcases:

Subcase 3.1 ($E\left(x_R^D\right) \geq E\left(x_R^N\right)$, i.e.,$1-c_R+\left(\frac{c_L-c_R}{c_L m_L+2} + \frac{c_R(m_R-m_L)(c_L-1)}{(m_L+2)(c_L m_L+2)}\right) m_L P \geq 0$). In this case (30) holds since the left hand side of (30) is positive, whereas the right hand side of (30) is negative. Therefore, we conclude that $Var_P(x^N) > Var_P(x^D)$.

Subcase 3.2: ($E\left(x_R^D\right) < E\left(x_R^N\right)$, i.e.,$1-c_R+\left(\frac{c_L-c_R}{c_L m_L+2} + \frac{c_R(m_R-m_L)(c_L-1)}{(m_L+2)(c_L m_L+2)}\right) m_L P < 0$). Substituting the expressions of $E\left(x_L^N\right)$, $E\left(x_R^N\right)$, $E\left(x_R^D\right)$ and $E\left(x_L^D\right)$ and after some algebra, we have that (30) is equivalent to

$$\frac{\delta_L}{\delta_R} < N,$$

where

$$N = \frac{\frac{c_L}{2} \frac{c_R m_R+2}{(c_L m_L+2)(c_R m_R+1)+P(c_L m_L - c_R m_R)} - \frac{1}{2} \frac{m_R+2}{(m_L+2)(m_R+1)+P(m_L-m_R)}}{\frac{c_R}{2} \frac{c_L m_L+2}{(c_L m_L+2)(c_R m_R+1)+P(c_L m_L - c_R m_R)} - \frac{1}{2} \frac{m_L+2}{(m_L+2)(m_R+1)+P(m_L-m_R)}},$$

with $N > \frac{c_L(c_R m_R+2)}{c_R(c_L m_L+2)}$. ■

Proof of Proposition 6: Combining (4), (5), (6) and (7), we have

$$x_L^N = x^* - \frac{1}{2\delta_L} \frac{m_R+2}{(m_L+2)(m_R+1)+P(m_L-m_R)} A + \frac{1}{2\delta_L} \frac{\varepsilon}{m_L+2}$$

and

$$x_R^N = x^* - \frac{1}{2\delta_R} \frac{m_L+2}{(m_L+2)(m_R+1)+P(m_L-m_R)} A + \frac{1}{2\delta_R} \frac{\varepsilon}{m_R+2}.$$

Hence, applying Lemma A.3,

$$Var_E\left(x^N\right) = \left(P\left(\frac{1}{2\delta_L\left(m_L+2\right)}\right)^2 + (1-P)\left(\frac{1}{2\delta_R\left(m_R+2\right)}\right)^2\right)\sigma_\varepsilon^2.$$

Analogously, combining (9), (10), (11) and (12), we get

$$x_L^D = x^* - \tfrac{c_L}{2\delta_L} \tfrac{c_R m_R + 2}{(c_L m_L + 2)(c_R m_R + 1) + P(c_L m_L - c_R m_R)} A + \tfrac{c_L}{2\delta_L} \tfrac{\varepsilon}{c_L m_L + 2}$$

and

$$x_R^D = x^* - \tfrac{c_R}{2\delta_R} \tfrac{c_L m_L + 2}{(c_L m_L + 2)(c_R m_R + 1) + P(c_L m_L - c_R m_R)} A + \tfrac{c_R}{2\delta_R} \tfrac{\varepsilon}{c_R m_R + 2},$$

and from Lemma A.3,

$$Var_E\left(x^D\right) = \left(P\left(\frac{c_L}{2\delta_L\left(c_L m_L + 2\right)}\right)^2 + (1-P)\left(\frac{c_R}{2\delta_R\left(c_R m_R + 2\right)}\right)^2\right)\sigma_\varepsilon^2.$$

Hence, $Var_E\left(x^N\right) - Var_E\left(x^D\right)$ is a linear function in P. Next we distinguish two cases:

Case 1: $c_R < 1 < c_L$. It is easy to see that $Var_E\left(x^N\right)|_{P=1} - Var_E\left(x^D\right)|_{P=1} < 0$ and that $Var_E\left(x^N\right)|_{P=0} - Var_E\left(x^D\right)|_{P=0} > 0$ whenever $c_R < 1 < c_L$. Hence, we can conclude that there exists a $Var_E\left(x^D\right) > Var_E\left(x^N\right)$ if and only if $P > \overline{\overline{\overline{P}}}$.

Case 2: $c_L > c_R \geq 1$. Direct computations yield that $Var_E\left(x^D\right) > Var_E\left(x^N\right)$ whenever $c_L > c_R \geq 1$. ■

Bibliography

Alesina, A. (1987), Macroeconomics and Politics in a Two-Party System as a Repeated Game, Quarterly Journal of Economics 102, 651-677.

Alesina, A. and R. Gatti (1995), Independent Central Banks: Low Inflation at no Cost?, American Economic Review 85, 196-200.

Alesina, A. and L. Summers (1993), Central Bank Independence and Macroeconomic Performance: Some Comparative Evidence, Journal of Money, Credit, and Banking 25 (2), 151–161.

Alesina, A. and G. Tabellini (2008), Bureaucrats or Politicians? Part II: Multiple Policy Tasks, Journal of Public Economics, 92(3-4), pages 426-447.

Alesina, A. and G. Tabellini (2007), Bureaucrats or Politicians? Part I: A Single Policy Task, American Economic Review, 97(1), 169-179.

Alesina, A. and G. Tabellini (1987), Rules and Discretion with Noncoordinated Monetary and Fiscal Policies, Economic Inquiry 25, 619-630.

Demertzis, M. (2004), Central Bank independence: Low inflation at no cost? A Numerical Simulations Exercise, Journal of Macroeconomics 26, 661–677.

Dixit, A. and L. Lambertini (2003), Interactions of Commitment and Discretion in Monetary and Fiscal Policies, American Economic Review 93, 1522-1542.

Drazen, A. (2002), Central bank Independence, Democracy, and Dollarization, Journal of Applied Economics 1, 1-17.

Ferré, M. and C. Manzano (2013), Rational Partisan Theory with Fiscal Policy and an Independent Central Bank, Working Paper 8, Universitat Rovira i Virgili.

Ferré, M. and C. Manzano (2012), Designing the Optimal Conservativeness of the Central Bank, Economics Bulletin 32(2), 1461-1473.

Maloney, J., A. Pickering and K. Hadri (2003), Political Business Cycles and Central Bank Independence, The Economic Journal 113, 167-181.

Rogoff, K. (1985), The Optimal Degree of Commitment to an Intermediate Monetary Target, Quarterly Journal of Economics 100, 1169-1189.