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Moral Hazard in Repeated Procurement of Services*

Patricia Esteve-González†

Abstract

This paper analyzes repeated procurement of services as a four-stage game divided into two periods. In each period there is (1) a contest stage à la Tullock in which the principal selects an agent and (2) a service stage in which the selected agent provides a service. Since this service effort is non-verifiable, the principal faces a moral hazard problem at the service stages. This work considers how the principal should design the period-two contest to mitigate the moral hazard problem in the period-one service stage and to maximize total service and contest efforts. It is shown that the principal must take account of the agent's past service effort in the period-two contest success function. The results indicate that the optimal way to introduce this 'bias' is to choose a certain degree of complementarity between past service and current contest efforts. This result shows that contests with 'additive bias' ('multiplicative bias') are optimal in incentive problems when effort cost is low (high). Furthermore, it is shown that the severity of the moral hazard problem increases with the cost of service effort (compared to the cost of contest effort) and the number of agents. Finally, the results are extended to more general contest success functions.

JEL classification: C72; D82

Key words: Biased contests; Moral Hazard; Repeated Game; Incentives.

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1 Introduction

The literature on contests analyzes competitive situations in which agents compete by exerting sunk effort in order to maximize their probability of winning a prize. The models in this literature enable explanations of the results observed in many situations such as, for example, rent-seeking, promotional competition, labor market tournaments, sports competitions and social conflicts.[1] These models focus on agents' incentives to choose effort in the contest stage(s) but do not analyze what happens once the winner is determined. In many situations, the contest winner and the contest designer enter into a principal-agent relationship in which the former provides goods or services to the latter (in a service stage) the quality of which is not perfectly observable or verifiable. In fact, many authors illustrate their analysis with examples that involve a principal-agent relationship after the contest stage without explicitly modeling such a conflict of interest in the service stage (Che and Gale 2003; Corchón and Dahm 2011; Fullerton and McAfee 1999; Taylor 1995).[2] However, this has been studied by Siegel (2010) who considers that the winner of a contest must exert effort not only during the contest stage (unconditional effort) but also after the contest stage (conditional effort to winning). He labels these kinds of contest *simple contests* and assumes that the winner commits to exerting a certain effort after the contest which is determined *ex ante*. This paper extends such simple contests in two directions. Firstly, the commitment assumption is relaxed to allow an analysis of situations in which effort is non-verifiable and, therefore, an *ex ante* commitment is not possible. Secondly, repeated simple contests are considered to model repeated procurement of services as a repeated game with two periods and a contest stage and a service stage in each period. This allows an analysis of how past performance should be used in future contest design to solve moral hazard problems at the service stages in the procurement of services.

This paper focuses on a repeated contest with two symmetric agents and a designer to model a situation with repeated procurement of services. There are two periods, 1 and 2, with a contest stage and a service stage in each period (contest 1, service 1, contest 2 and service 2). As a consequence, the relationship between the contest 1 winner and the designer is potentially repeated in the second period. Effort at both contest and service stages is non-verifiable. This paper analyzes how contest 2 should be designed to mitigate the moral hazard

[1] See Konrad (2009) for an overview of this literature.

[2] For example, Corchon and Dahm (2011) illustrate their model with the Olympic Games. Although they only consider the sunk efforts undertaken at the contest stage, they recognize that, once the Games are assigned, the quality of such Games is related to the effort that the host city exerts at the service stage. Then, there is a positive relationship between effort and service quality, and there may be a moral hazard problem when such effort is not contractible.

problem in service 1. This is achieved by introducing a 'bias' that takes account of service 1 effort in the second period *Contest Success Function* (CSF). With such a biased contest, the designer can give an advantage (the bias) to the contest 1 winner if he did not shirk at service 1.[3] The relationship between the bias (the service 1 effort) and the contest 2 effort is modeled by a *Constant Elasticity of Substitution* (CES) function. This allows the designer to establish several types of bias, including additive and multiplicative biases which are the most used in the literature. The designer has lexicographic preferences and values service efforts more than contest efforts. The main objective of the analysis is to determine how the designer should choose the *weight* of the service 1 effort (compared to the contest 2 effort) and the *degree of complementarity* between efforts to maximize, firstly, the total effort exerted by all agents in the service stages and, secondly, total contest effort.

When efforts are perfect substitutes and effort cost is low, it is shown that the designer's optimal strategy is to weight the service 1 and contest 2 efforts equally. This is because an increase in the weight of service 1 effort in the second period CSF has two effects. Firstly, it increases the contest 1 winner's returns of exerting high effort at service 1 and contest 2. Secondly, it decreases the contest 1 loser's returns of exerting high effort at contest 2. When service 1 and contest 2 efforts have similar weights (in the second period CSF), the designer achieves the highest service and contest efforts from both agents because the advantage of the service 1 provider is neither too high (to discourage the contest 1 loser to compete at contest 2) nor too low (such that shirking in service 1 is advantageous for the contest 1 winner). For higher effort cost, the designer should give a higher weight to the service 1 effort. Though this will not allow her to achieve maximum contest effort, it allows her to resolve the moral hazard problem at the service 1 stage. Moreover, the results show that it will be more difficult for the designer to solve the moral hazard problem when agents are impatient and value the present (period 1) more than the future (period 2).

If a more general relationship between service 1 and contest 2 efforts (the CES function) is considered in the second period CSF when efforts have the same weight, it is shown that the designer should choose the degree of complementarity between efforts according to the cost of such efforts. When effort cost is low, the degree of substitutability should be high because both agents are more willing to exert high effort in all periods. For medium effort cost, the degree of substitutability should be neither too low nor too high. On the one hand, if the degree of substitutability is too high, this reduces the contest 1 winner's return from service 1 effort in

[3]Notice that this paper refers to any agent as *he* and to the designer as *she*.

contest 2 and, therefore, his incentive to shirk increases. On the other hand, if the degree of substitutability is too low, both agents have lower incentives to choose high effort at contest 2. Therefore, a medium degree of substitutability allows the designer to achieve the maximum contest and service efforts. Finally, for higher effort cost, the designer should choose a low degree of substitutability as a result of preferring service effort over contest effort. However, it will never be optimal for the designer to consider service 1 effort and contest 2 effort as perfect complements. Again, it is shown that the moral hazard problem will be more severe when agents are impatient.

Analyzing service procurement procedures is relevant for many public and private situations because of the economic importance of the services sector. In 2010, the services sector represented 70% of the world's GDP (World Bank 2013). Repeated service provision after contests, the situation studied in this paper, is commonly used in public procurement which represents around 20% of the GDP in OECD countries and around 14% in that of non-OECD countries (Audet 2002). Thus, in 2012, the US government allocated 516.6 billion dollars to public procurement contracts (US Government Spending 2013), and, in 2009, European countries allocated more than 420 billion euros to public procurement contracts (European Commission 2011). The European Union's contracting rules establish three kinds of procedures (open, restricted and negotiated procedures) in which the contracting authority selects an economic operator who is then contracted to provide a service. Past performance is one of the criteria that the contracting authority can take into account in addition to the cost bidder, the number of services included, the candidate's curriculum vitae, the corporate social responsibility, etc. However, past performance usually has a low weighting in future contests and, especially when Treasury is pressured, it is not taken into account and contracts are assigned to the lowest cost bidder.[4] The results from this paper show that this is not always the optimal policy because past performance should be considered together with current contest bids to mitigate possible moral hazard problems at the service stages. As an example, consider a fireworks contest organized every July by the town council of Tarragona, Spain. The winner of this contest is hired to provide the fireworks in the town festival in September but is not allowed to participate in the next year's fireworks contest. However, as a consequence of this, it has been commonly recognized that contest winners perform higher quality fireworks during the contests than during the festival. Modifying the selection mechanism by introducing a biased contest that takes account of past performance

[4]Operators, however, can be excluded from future contests because they have not paid their social security contributions, have been found guilty of grave professional misconduct, etc.

could resolve this problem.[5]

This paper is closely related to two strands in the literature on contests. As noted above, while most contests are implicitly based on a principal-agent relationship after the contest stage, this has not been explicitly modeled until the contribution of Siegel (2010). In his model, agents compete in an all-pay auction and there is *ex-ante* commitment to exert a certain *ex-post* service effort. He finds that increasing the importance of the service effort (relative to the contest effort) in the all-pay auction increases agents' total equilibrium effort (contest and service). While this increases the win probability of the agent who values the prize highest and reduces the win probability of the agent who values the prize lowest, he finds that, in equilibrium, all agents will increase their efforts to win the prize. This paper extends Siegel's analysis to a repeated contest-service situation which allows examination of contexts in which an *ex ante* commitment is not possible and an incentive mechanism is required to avoid moral hazard problems. Increasing the importance of service effort in this context has a completely different objective. Firstly, it is *past* service effort that is used in the CSF and not future service effort. Secondly, the optimal level of service effort is determined by the trade-off between solving the incentive problem at the service stage and providing a high degree of competition at the contest stage. This paper shows that this trade-off is not trivial and depends on the degree of substitutability of contest 2 and service 1 efforts in the second period CSF, the expense of the effort and the agents' time preferences. Melkonyan (2013) considers a variation of Siegel's model in which the CSF is a Tullock lottery, and contest and service efforts are related through a CES function. While he also analyses contest and service stages and assumes that agents commit to future service effort (thus, as in Siegel 2010, there is no incentive problem in the service provision), his results highlight the role of the degree of substitutability between service and contest effort in providing competition at the contest stage, an effect that is also present in this paper.

The literature on repeated biased contests is the second strand of the literature the paper is related to.[6] Meyer (1992) analyzes promotions as incentive mechanisms through a model with two Lazear and Rosen (1981) contests and 'additive' biases. The first contest is an interim evaluation and the second contest is a promotion. She finds that, with homogeneous agents, the principal's optimal choice to maximize total effort is to add an advantage at the promotion in favor of the winner of the interim evaluation. This additive bias enhances competition in the

[5]There are many other applications of the results in this paper which include research tournaments, electoral campaigns or promotion tournaments in firms.

[6]Regarding repeated service provision with moral hazard, a related paper is Cesi and Albano (2008). However, in their paper the authors focus on service effort and do not consider repeated contests with sunk efforts.

interim evaluation because agents compete for having an advantage in the promotion. Then, at the promotion, the winner of the interim evaluation can use his advantage to reduce his effort (and costs). However, when the advantage is large enough, the loser of the interim evaluation is discouraged in the promotion and competition for the promotion decreases. Therefore, a biased contest creates a trade-off between interim evaluation and promotion efforts. This trade-off is also found in repeated contests with 'multiplicative' bias.[7] For example, Beviá and Corchón (2013) consider conflict situations with two contests where the probability of winning the first contest is introduced as a bias that multiplies agents' efforts in the second contest. As a result, agents compete in the first contest for winning an advantage in the second contest, and agents compete in the second contest for keeping their strengths. These authors show the existence of a discouragement effect in the second contest which increases with the size of the bias. Therefore, to maximize total contest efforts, the size of the bias is limited from below and above.[8] Another example of a multiplicative bias in repeated contests is Ridlon and Shin (2013). These authors show that higher competition in contest 1 (which allows the winner to obtain an advantage in contest 2) compensates lower competition in contest 2 when the size of the bias is low enough.[9] In this paper, there is also a trade-off between service 1 and contest 2 efforts. However, the bias introduced in contest 2 is used to solve the moral hazard problem at the previous service stage and not to increase competition at previous contest stage.

The literature has studied different forms of introducing biases into contests. Dahm and Porteiro (2008) analyze a Tullock CSF with a general function that relates the bias with the contest effort. Special cases of this general function are the two most commonly used biases: the additive bias and the multiplicative bias. They find that the additive bias decreases the stronger agent's incentives to exert high effort while the multiplicative bias discourages the weaker agent. This paper considers a CES function to relate the bias (service 1 effort) with the contest 2 effort. Although this function is less general than the one used by Dahm and Porteiro (2008), it highlights the importance of the degree of substitution between efforts for the results that are obtained. A multiplicative bias implies that efforts are more complementary

[7] The multiplicative bias in the Tullock CSF was introduced by Clark and Riis (1998).

[8] Möller (2012) and Clark and Nilssen (2013) also find a similar trade-off considering learning effects in repeated contests that allow agents to increase their efficiency in future contests. Jofre-Bonet and Pesendorfer (2000) support empirically the existence of such learning effects for the case of highway paving contracts in California. Similar to learning effects, other authors consider pre-contest investment that allows agents to increase their efficiency in contests (see Fu and Lu 2009 and Munster 2007).

[9] Ridlon and Shin (2013) also show that total contest effort is maximized with a multiplicative bias favouring (handicapping) the contest 1 winner when agents are homogeneous (heterogeneous). Alternatively, Moldovanu et al. (2012) introduce awards and punishments in the prize structure instead of biasing contests to maximize total contest effort.

than under an additive bias. This paper analyzes how agents' incentives change when the degree of substitutability is reduced and to what extent introducing a bias in the CSF under repeated procurement can mitigate the moral hazard problem at the service stage.[10]

The rest of this paper is organized as follows. The next section describes the model and the main results are provided in Section 3. In Section 4, the model is extended and additional results are given. Finally, concluding remarks are found in Section 5.

2 The model

Consider three players, a designer and two identical agents, and two periods, $t = 1$ and $t = 2$. In each period, the designer selects one of the agents through a contest (contest stage) and this winning agent is then hired to provide a service (service stage). In each period, once the service is executed, all players observe the winner's effort at the service stage and the designer pays the winner 1 monetary unit. Previously, in $t = 0$, the designer decides the general setting of the second contest. Figure 1 summarizes the timing of the model.

The strategies of agents are their efforts, which are binary: high or low effort.[11] At the t-period contest stage (contest t), agents simultaneously choose their efforts; ; an agent i's effort is $e_{i,t} \in \{0, 1\}$.The winner and the loser of contest t are denoted by w and l respectively. Then, at service t, the winner chooses his effort $s_{w,t} \in \{0, 1\}$. The same cost function is assumed for all stages and all periods with $c_{i,t}(e_{i,t} = 1) = c_{w,t}(s_{w,t} = 1) = c > 0$ and $c_{i,t}(e_{i,t} = 0) = c_{w,t}(s_{w,t} = 0) = 0$.[12] Given that effort is not contractible, as it is non-verifiable by a third party in court, the designer commits to pay the winner of the contest stage even when he shirks at the service stage (chooses low effort).

Contestants are risk neutral and maximize their expected utility,

$$\begin{aligned} E(U_{i,1}) = P_{i,1} \left(1 - c_{w,1}(s_{w,1}) + \delta \left[P_{w,2}[1 - c_{w,2}(s_{w,2})] - c_{w,2}(e_{w,2})\right]\right) + \\ + (1 - P_{i,1})\delta \left[P_{l,2}[1 - c_{w,2}(s_{w,2})] - c_{l,2}(e_{l,2})\right] - c_{i,1}(e_{i,1}). \end{aligned} \tag{1}$$

[10]Hölmstrom (1979) is the first work suggesting that reputation can be used to mitigate moral hazard. Moral hazard problems have been studied extensively in the contract theory literature (see Bolton and Dewatripont 2005 and Laffont and Tirole 1993). Contests and biased contests appear in this literature as a way to provide incentives to agents when their outputs are unobservable or non-verifiable. See also Gibbons and Waldman (1999) for a review on incentive provision by contracts and tournaments.

[11]Agents' effort can be interpreted as the quality of their activity. For example, Corchon and Dahm (2011) observe that the quality of Olympic Games increases with the investment of the hosting city. This example clarifies the positive relationship between quality and effort.

[12]The cost function is a positive constant c that multiplies effort at any stage. This assumption and other aspects of the model will be generalized later.

At contest 1, the representative agent i exerts effort $e_{i,1}$ and wins the contest with probability $P_{i,1}$. If he wins the contest, he is hired at service 1 and exerts the effort $s_{w,1}$. Given this effort, he wins contest 2 with probability $P_{w,2}$. On the other hand, if agent i loses contest 1, he is not hired at service 1 and wins contest 2 with probability $P_{l,2}$. The winner of contest 2 will be hired at service 2 and exert effort $s_{w,2}$. The profits of the second period are discounted by $\delta \leq 1$.

The probability of winning any contest is the simple logistic CSF first introduced by Tullock (1980),[13]

$$P_{i,t}(\hat{e}_{1,t}, \hat{e}_{2,t}) = \begin{cases} \frac{\hat{e}_{i,t}}{\hat{e}_{i,t}+\hat{e}_{j,t}} & \text{if } \hat{e}_{i,t} + \hat{e}_{j,t} > 0 \text{ and } i \neq j \\ \frac{1}{2} & \text{if } \hat{e}_{i,t} + \hat{e}_{j,t} = 0 \text{ and } i \neq j \end{cases}, \tag{2}$$

where $\hat{e}_{i,t}$ is the effective effort of agent i at contest t.[14] At contest 1, this effective effort is equal to the current effort for any agent, $\hat{e}_{i,1} = e_{i,1}$, because agents do not have a past. At contest 2, the effective effort of the loser of contest 1 is his current effort because he is not involved in the provision of service 1. However, at contest 2, the designer can take into account the service 1 effort of the service 1 provider. Equation (3) defines agent w's effective effort at contest 2 which is a CES function,

$$\hat{e}_{w,2} = \left(\gamma s_{w,1}^{\rho} + e_{w,2}^{\rho}\right)^{1/\rho}, \qquad \text{where } \gamma \geq 0 \text{ and } 0 < \rho \leq 1. \tag{3}$$

On the one hand, γ represents the weight that the designer gives to the past effort ($s_{w,1}$). Past effort can be more important ($\gamma \geq 1$) or less important ($\gamma \leq 1$) than current effort ($e_{w,2}$). On the other hand, ρ determines the elasticity of substitution between efforts, which is equal to $1/(1-\rho)$. When $\rho \to 1$, efforts $s_{w,1}$ and $e_{w,2}$ are perfect substitutes and the elasticity of substitution is infinite. At the other extreme, when $\rho \to -\infty$, efforts are perfect complements and the elasticity of substitution is zero. In general, when ρ decreases, the complementarity between efforts increases. We focus on $\rho \in (0, 1]$ because equation (3) is not well defined when an effort is zero (low) and $\rho \leq 0$. Notice also that efforts in equation (3) are additive when $\rho \to 1$ and multiplicative when $\rho \to 0$.[15] As figure 1 shows, the designer determines the contest setting in $t = 0$ by choosing γ and ρ.

[13] The Tullock CSF is axiomatized by Skaperdas (1996), and Corchon and Dahm (2010) give a microfoundation for this CSF. Contestants are uncertain about a characteristic of the decider (the designer) that is relevant for her decision in addition to effort. This fits our model because evaluating services of the same quality (effort) is rather subjective. Then, agents might win the contest probabilistically, and the Tullock CSF relates agents' efforts with their win probabilities.

[14] Under the alternative assumption that the win probability is zero when no agent exerts effort, analogous results to Lemma 1 (below) hold.

[15] Notice that $\hat{e}_{w,2} \to \gamma s_{w,1} e_{w,2}$ when $\rho \to 0$. Clark and Riis (1998) axiomatize this class of CSF.

The designer's objective is to choose the pair (ρ, γ) that maximizes total effort in all periods.[16] Her preferences on agents' effort are lexicographic: she prefers high effort in services (first priority) over high effort in contests (secondary priority). The model is solved by backwards induction to find the Subgame Perfect Equilibria (SPE) in behavior strategies.

The case where $\delta = 0$ can be interpreted as a non-repeated game with just one period ($t = 1$). Then, the winner of the contest stage always shirks at the service stage because his relationship with the designer ends. However, at the contest stage both agents would like to compete as long as effort is cheap enough. This result is formalized in Lemma 1 (a proof can be found in Appendix 6.1).[17]

Lemma 1 *Consider $\delta = 0$. There is always moral hazard because the winner of the contest always shirks at the service stage. Behaviors at the contest stage depend on the cost of effort, more precisely:*

- *When effort cost is low, $c < 1/2$, the unique SPE has both agents exerting high effort at the contest stage (maximum contest effort).*
- *When effort cost is high, $c > 1/2$, the unique SPE has both agents exerting low effort at the contest stage (minimum contest effort).*

It should be noted that, apart from the functional form specified in equation (2), the studied situation is a special case of a Siegel (2010) simple contest in which commitment is not assumed. First, agents exert effort at the contest stage (the unconditional or sunk cost) and, then, the winner has to exert effort at the service stage (the conditional cost to winning).[18]

The next section focuses on the potential repeated relationship between the designer and the provider of service 1 in period 2 and the setting with repeated procurement of services is analyzed. In such a context, the designer can avoid the moral hazard problem at service 1

[16] Note that there are situations where efforts at contest stages, in addition to efforts at service stages, are useful for the designer. For example, in public procurement, contests give a sign of transparency and information from the public agencies to citizens. In the fireworks example, contests are performances that increase social welfare. In promotion tournaments, agents' activity can be profitable for the firm. There are further comments on other designer's preferences in the concluding section.

[17] When $c = 1/2$, agents are completely indifferent about their effort choice at the contest stage. There is, hence, a multiplicity of equilibria in behavior strategies. Since the situation is symmetric, focusing on symmetric behavior strategies opens the door for agents choosing any probability of entering the contest "in between" the two pure strategies described in Lemma 1.

[18] In Siegel (2010), the parameter $0 \leq \alpha \leq 1$ represents the part of the cost which is sunk and C is the total cost of the game for the winner. Then, $\alpha C = c_i(e_i)$ is the sunk cost at the contest stage and $(1 - \alpha)C = c_i(s_i)$ is the conditional cost exerted by the winner at the service stage. Given that Siegel does not allow for shirking at the service stage, our setting reduces to $c_i(s_i = 1) = c_i(e_i = 1) = c$. Equation (1), thus, is a special case of Siegel's setting in which $C = 2c$ and $\alpha = 1/2$.

by considering the winner's effort at service 1 when resolving contest 2. The conditions under which potential repetition of a contract with the same agent is most effective are analyzed. From Lemma 1 we know that when the effort cost is high ($c > 1/2$), there is no incentive to compete at the contest stage and moral hazard at the service stage cannot be avoided. Therefore, in what follows, we focus on the case $c < 1/2$ to analyze optimal procurement design under repeated contracting.

3 Results

3.1 The weight of the past when efforts are perfect substitutes

Consider the designer's problem of the optimal choice of γ, the bias that allows her to weight past (service) effort differently from current (contest) effort in the second period CSF. Assuming that $\rho = 1$ and $\delta = 1$, equation (3) is

$$\hat{e}_{w,2} = \gamma s_{w,1} + e_{w,2}. \tag{4}$$

The game is solved by backward induction. At service 2, the winner of contest 2 shirks ($s_{w,2} = 0$) as high effort is costly but does not produce future returns (see Lemma 1). At contest 2, an agent's probability of winning depends on his effective effort. If the designer does not take into account the agent's past behavior, i.e. sets $\gamma = 0$, the effective effort is equal to the current effort for both agents. Therefore, the winner of the previous contest has no advantage over the loser of that contest. This situation corresponds to the case in Lemma 1. On the contrary, if the designer considers also the agent's past behavior, i.e. chooses $\gamma > 0$, the previous winner has an advantage over the loser in winning contest 2 when he chose high effort at service 1 as shown in equation (4). For example, consider that the designer weights present and past effort in contest 2 equally ($\gamma = 1$). Then, the winner of contest 1 has twice the probability of also winning contest 2 if he does not shirk in service 1 and if both agents choose high effort in contest 2. Thus, by considering past effort in contest 2, the designer creates incentives for the winner of contest 1 not to shirk at service 1. Whether the moral hazard problem at service 1 can be avoided depends on effort cost c. It turns out that when the past is less important than the present ($\gamma \leq 1$), the moral hazard problem is avoided only when effort is cheap enough. However, when the past is more important than the present ($\gamma > 1$), the moral hazard problem in service 1 is avoided for any $c < 1/2$. Proposition 1 states these results formally (all calculations are in Appendix 6.2).

Proposition 1 *When $\gamma > 0$, potential repetition can avoid moral hazard at service 1. At service 2, the winner of contest 2 always shirks. Four different types of SPE exist:*

SPE I *When $c < \min\{\gamma/(2\gamma+4), 1/((\gamma+2)(\gamma+1))\}$, moral hazard is mitigated and contests effort is maximum. The unique SPE has both agents exerting high effort in both contests, and the winner of contest 1 also chooses high effort at service 1.*

SPE II *When $\gamma \leq 1$ and $\gamma/(2\gamma+4) < c < 1/2$, moral hazard cannot be avoided but contests effort is maximum. The unique SPE has both agents exerting high effort in both contests, but the winner of contest 1 shirks in service 1.*

SPE III *When $\gamma > 1$ and $1/((\gamma+2)(\gamma+1)) < c < 1/(\gamma+1)$, moral hazard is mitigated but contests effort is not maximum. The unique SPE has both agents exerting high effort in contest 1. The winner of contest 1 exerts high effort at service 1 and no effort at contest 2. The loser of contest 1 exerts high effort at contest 2.*

SPE IV *When $\gamma > 1$ and $c > 1/(\gamma+1)$, moral hazard is mitigated but contest 2 effort is minimum. The unique SPE has both agents exerting high effort at contest 1 and no effort at contest 2. The winner of contest 1 exerts high effort at service 1.*

Figure 2 displays the SPE of Proposition 1.[19] In order to develop an intuition for the results, consider first the left area of Figure 2 in which the weight of the past is lower than 1. The lower the parameter γ, the more severe the moral hazard problem. The concave part of the solid thick line separates the area in which the winner of contest 1 shirks (Region II) from the area in which he exerts high effort (Region I).

Now, consider the right area of Figure 2 in which the weight of the past is larger than 1. When γ and effort cost increase, the returns of high effort at contest 2 decrease while the returns of exerting high effort at service 1 increase. Consequently, moral hazard is always avoided in period 1 ($s_{w,1} = 1$). Taking advantage of his high effort in service 1, the winner of contest 1 exerts no effort in contest 2 when effort is expensive enough (Region III). The convex part of the solid thick line separates the area in which the winner of contest 1 exerts high effort in contest 2 (Region I) from the area in which he shirks in contest 2 (Region III). In Region IV, the loser of contest 1 is completely discouraged from competing in contest 2 because effort is

[19]The name of regions in Figure 2 corresponds to the name of SPE in Proposition 1.

rather expensive and his rival, the winner in contest 1, has an advantage because of his past service effort. In this case, at contest 2, the loser of contest 1 is pre-empted by the winner of contest 1. The dashed thick line separates the area where the loser of contest 1 exerts high effort in contest 2 (Region III) from the area in which he shirks in contest 2 (Region IV).

Given that the designer's utility is lexicographic (high service effort has priority over high contest effort), she has the following preference relationship over the SPE: $SPEI \succ SPEIII \succ SPEIV \succ SPEII$. Figure 2 shows that the designer can achieve her most preferred outcome most easily by choosing $\gamma = 1$. In other words, when $\gamma = 1$, SPE I is possible for higher effort cost ($c < 1/6$) than for other values of γ different from 1. However, SPE I is not achievable for larger values of effort cost. The designer can then always avoid her least preferred outcome SPE II with low effort at service 1 by choosing $\gamma > 1$. In this case, she should choose γ not too high as she prefers SPE III over SPE IV. Summarizing these results, it is found that the designer's optimal choice of γ is weighting past service and present contest effort almost equally ($\gamma = 1$) in the second period CSF.

To assess the importance of the discount factor for the results in Proposition 1, Figure 3 considers two alternative scenarios with discount factors lower than 1 ($\delta = 0.8$ and $\delta = 0.2$). In these cases, it is shown that $\gamma = 1$ is no longer the optimal choice for the designer. This is because more impatient agents must be compensated by a higher weight of past service effort in contest 2. As δ decreases, the optimal value of γ must increase to maintain the incentive of the winner of contest 1 to exert high effort in service 1. When agents give no value to the future ($\delta = 0$), agents play a one period game and the results of Lemma 1 apply: SPE II is the only equilibrium outcome. Focusing on the case with $\delta = 1$, in the next subsection it is assumed that $\gamma = 1$ and the effect of changing the degree of substitution between efforts in equation (3) on agents' incentives to exert effort is analyzed.

3.2 The role of complementarities between equally weighted efforts

Consider the designer's optimal choice of ρ which determines the elasticity of substitution between past effort ($s_{w,1}$) and current effort ($e_{w,2}$) in the second period CSF. Assuming $\gamma = 1$ and $\delta = 1$,[20] the provider of service 1 has the following effective effort at contest 2,

$$\hat{e}_{w,2} = (s_{w,1}^{\rho} + e_{w,2}^{\rho})^{\frac{1}{\rho}}. \tag{5}$$

[20]Note that $\gamma = 1$ is the optimal choice for the designer in the previous subsection only when $\delta = 1$. Note also that for $\gamma = 0$, stating any ρ has no effect on agents' incentives.

The game is solved by backward induction. Again, the winner of contest 2 shirks at service 2. At contest 2, the agent's win probability depends on the effective efforts. If the designer states $\rho = 1$, both efforts are perfect substitutes as in equation (4). However, when $\rho < 1$, some complementarity between efforts is introduced, and the winner of contest 1 has more incentives to exert high effort at both service 1 and contest 2. For example, consider that the loser of contest 1 exerts high effort at contest 2 and the winner of contest 1 exerts high effort at both service 1 and contest 2. Then, the winner of contest 1 has at least twice the probability of also winning contest 2 as his opponent since the gap between both probabilities increases when ρ decreases.[21] In equilibrium, whether the moral hazard problem at service 1 is avoided depends again on effort cost. It turns out that when the complementarity is high enough, the moral hazard is avoided for any $c < 1/2$. However, this implies a reduction of competition in contest 2. Proposition 2 states these results formally (all calculations are in Appendix 6.3).[22]

Proposition 2 *For any $0 < \rho \leq 1$, potential repetition can avoid moral hazard at service 1. At service 2, the winner of contest 2 always shirks. Three different types of SPE exist:*

SPE I *When $c < \min\left\{1/(2^{1/\rho}+1), (2^{1/\rho}-1)/(2(2^{1/\rho}+1))\right\}$, moral hazard is mitigated and contests effort is maximum. The unique SPE has both agents exerting high effort in contests 1 and 2, and the winner of contest 1 also chooses high effort at service 1.*

SPE II *When $(2^{1/\rho}-1)/(2(2^{1/\rho}+1)) < c < 1/2$, moral hazard cannot be avoided but contests effort is maximum. The unique SPE has both agents exerting high effort in contests 1 and 2, but the winner of contest 1 shirks in service 1.*

SPE V *When $0 < \rho < ln(2)/ln(3)$ and $1/(2^{1/\rho}+1) < c < (2^{1/\rho}-1)/(2(2^{1/\rho}+1))$, moral hazard is mitigated but contests effort is not maximum. The unique SPE has both agents exerting high effort at contest 1 and the winner of contest 1 also exerts high effort at service 1. However, at contest 2, the winner of contest 1 exerts high effort with probability $r_{w,2} = (2^{1/\rho}+1)/(2^{1/\rho}-1) - 2c(2^{1/\rho}+1)/(2^{1/\rho}-1)$, and the loser of contest 1 exerts high effort with probability $q_{l,2} = 2c(2^{1/\rho}+1)/(2^{1/\rho}-1)$.*

Figure 4 illustrates the Regions for each SPE in Proposition 2. Consider first the right area of Figure 4 in which efforts $s_{w,1}$ and $e_{w,2}$ are better substitutes ($\rho \to 1$). The lower the

[21]The cause of this implication is that $s^{\rho}_{w,1} + e^{\rho}_{w,2}$ is larger than one when both efforts are high. Given that the effective effort $(s^{\rho}_{w,1} + e^{\rho}_{w,2})^{\frac{1}{\rho}}$ is convex for $\rho \in (0, 1]$, it increases when ρ decreases.

[22]To be precise, when $\delta = 1$ and the cost of effort is $(2^{1/\rho}-1)/(2(2^{1/\rho}+1)) < c < 1/2$, the winner of contest 1 is indifferent whether to shirk or not at service 1. Therefore, there is SPE III in addition to SPE II (see Proposition 1). Since the contest 1 winner's indifference disappears when $\delta < 1$, we focus here on SPE II.

parameter ρ, the less severe the moral hazard problem because the returns of exerting high effort twice (at service 1 and contest 2) increase and the winner of contest 1 is more willing to exert high effort at higher cost. The thick solid line separates the area in which the winner of contest 1 shirks (Region II) from the area in which he exerts high effort (Region I). Note that when the designer decreases ρ, the winner of contest 1's probability of winning contest 2 increases exponentially if he exerted high effort at service 1 while the opposite effect is found for the loser of contest 1. The thick solid line represents the first effect while the dashed thick line represents the latter one.

Now consider the left of Figure 4 in which efforts are more complementary ($0 < \rho < ln2/ln3$). Between the solid thick line and the dashed thick line there is Region V which corresponds to SPE V. Here, moral hazard in period 1 is avoided and agents play behavior strategies at contest 2. When $c = 1/4$, both agents have the same probability of exerting high effort. If effort cost increases, the winner of contest 1 has fewer incentives to exert high effort because of his advantage from not shirking, and, consequently, the loser of contest 1 has more incentives to exert high effort. If, however, effort cost is lower than $1/4$, the winner of contest 1 has more incentives to exert high effort and this discourages the loser of contest 1.[23]

The designer has the following preference relationship over the SPE: $SPEI \succ SPEV \succ SPEII$. Figure 4 shows that the designer can achieve her preferred outcome most easily by choosing $\rho = ln2/ln3$ (SPE I is possible for $c < 1/4$). However, when effort is too expensive and SPE I is not achievable, the designer can avoid her least preferred outcome with moral hazard at service 1, SPE II, by choosing $\rho < ln2/ln3$. By increasing complementarity, she has a high effort in service 1 at the expense of decreasing competition in contest 2. In this situation, it seems that the designer should increase complementarity as much as possible in order to obtain SPE V, which is the unique alternative to SPE II.

Consider the limit case $\rho \to 0$ with the effective effort

$$\hat{e}_{w,2} = s_{w,1} e_{w,2}. \tag{6}$$

Note that equation (6) gives the minimum effort between $s_{w,1}$ and $e_{w,2}$ because of the assumed values for the binary effort. Therefore, this effective effort considers past and current efforts as perfect complements. On the one hand, when any of both efforts is equal to zero, such effective effort is equal to zero. On the other hand, when the winner of contest 1 exerts high service 1

[23]These behaviors are consistent with the other SPE. When $c = 1/(2^{1/\rho} + 1)$, $(r_{w,2}, q_{l,2}) = (1, 2/(2^{1/\rho} - 1))$ and when $c = (2^{1/\rho} - 1)/2(2^{1/\rho} + 1)$, $(r_{w,2}, q_{l,2}) = (2/(2^{1/\rho} - 1), 1)$.

and contest 2 efforts, his effective effort at contest 2 is never higher than 1 and he then has no advantage in exerting high effort at service 1. The results show that SPE I is achievable for cheaper effort but moral hazard cannot be avoided for higher values of effort cost. Proposition 3 states these results formally (all calculations are in Appendix 6.4).

Proposition 3 *When $\rho \to 0$, potential repetition can avoid moral hazard at the period 1 service stage. At service 2, the winner of contest 2 always shirks. Two different types of SPE exist:*

SPE I *When $c < 1/4$, moral hazard is mitigated and contests effort is maximum. The unique SPE has both agents exerting high effort in both contests, and the winner of contest 1 also chooses high effort at service 1.*

SPE VI *When $1/4 < c < 1/2$, there is moral hazard and contests effort is not maximum. The unique SPE has both agents exerting high effort at contest 1, the winner shirks at both service 1 and contest 2, and only the loser of contest 1 exerts high effort at contest 2.*

In equilibrium, the winner of contest 1 always chooses $s_{w,1} = e_{w,2}$. Then, when effort is cheap enough, he still has incentives to choose high effort in both service 1 and contest 2 ($s_{w,1} = e_{w,2} = 1$) although he has no advantage in contest 2. However, when effort cost is higher, his incentives to exert high effort twice decrease because it is too expensive and he has no advantage in contest 2. That is, the win probability of the service 1 provider at contest 2 is much the same as his opponent's win probability. Therefore, when complementarity is too high, the bias of the second period CSF stops being an advantage and the winner of contest 1 has incentives to shirk not only at service 1, but also at contest 2. Then, only the loser of contest 1 has incentives to exert high effort at contest 2.

The designer has the following preference relationship over the SPE: $SPEI \succ SPEVI$. Note that the designer's most preferred equilibrium is achievable for $c < 1/4$, as in Proposition 2. However, for higher effort costs in Proposition 3, moral hazard at service 1 cannot be avoided and contests effort cannot be maximized. In other words, $SPEII \succ SPEVI$. The discontinuity between the results of Propositions 2 and 3 is due to the assumption of binary effort. This model cannot analyze the negative interval $\rho \in (-\infty, 0]$ but the results of both propositions are explanatory enough to conclude that increasing complementarities between efforts avoids moral hazard as long as such level of complementarity is not too high.

The CES function from equation (5) can be analyzed as the level of linearity of such a function in addition to the degree of complementarity between efforts. The level of linearity between the past and the present, determined by ρ, changes the relationship between efforts $s_{w,1}$ and $e_{w,2}$ going from perfect complements to perfect substitutes. In these two extreme points, equation (5) is linear. In the first case, when $\rho = 1$, the bias is additive and an increase from a low to a high effort $e_{w,2}$ increases the effective effort in one unit. In the second case, when $\rho \to 0$, the bias is multiplicative and an increase in $e_{w,2}$ increases the effective effort in $s_{w,1}$. However, when $\rho \in (0, 1)$, an increase in $e_{w,2}$ increases the effective effort in $(s_{w,1}^{\rho} + 1)^{\frac{1}{\rho}-1}$, which is higher than one if the contest 1 winner exerted high effort at service 1.[24] Therefore, reducing the linearity of the relationship between efforts from both extremes increases the contest 1 winner's returns of exerting high effort twice, mitigating the studied moral hazard problem. The designer's optimal choice of the degree of complementarity between efforts depends on how expensive the effort is: the higher the cost, the higher the complementarity between efforts, so long as they are not perfect complements.

In relation to the importance of the discount factor in the results of Proposition 2, Figure 5 considers two alternative scenarios with the discount factors $\delta = 0.8$ and $\delta = 0.2$. When the discount factor decreases, the optimal ρ is lower than $ln2/ln3$. Then, similar to Subsection 3.1, when agents are more impatient, the designer must increase the degree of complementarity between efforts in the second period CSF to increase the dependence between efforts from different periods, i.e. $s_{w,1}$ and $e_{w,2}$. As δ decreases, the optimal value of ρ also decreases in order to give enough incentives to the winner of contest 1 to exert high effort at service 1. However, when agents give no value to the future ($\delta \to 0$), results of Lemma 1 apply and SPE II is the only equilibrium.

Summing up, the studied moral hazard problem is mitigated when the second period CSF takes account of past and current efforts as more substitute efforts, so long as effort cost is not too high and agents are not too impatient. In such cases, the designer's optimal choice is considering a bit of complementarity between efforts $s_{w,1}$ and $e_{w,2}$, around $\rho = ln2/ln3$, depending on c and δ. This increases the service 1 provider's returns of exerting high effort at both service 1 and contest 2. However, when efforts are too complementary, the bias in the second period CSF (service 1 effort) is more a condition to participate in contest 2 than an advantage for the service 1 provider. Therefore, the moral hazard problem is less (more) severe when the degree of substitution (complementarity) of both efforts is high enough.

Regarding only the two most common biases used by the literature (the multiplicative and

[24] Consider the effective effort of the contest 1 winner from equation (5) when $\rho \in (0, 1)$. If the winner of contest 1 did shirk at service 1, exerting high effort at contest 1 increases his effective effort in one unit. However, if he exerted high effort at service 1, exerting high effort at contest 1 increases his effective effort in $2^{\frac{1}{\rho}-1}$.

additive ones), the designer should consider an additive bias in the second period CSF instead of a multiplicative bias. In this model, the multiplicative bias corresponds to considering efforts as perfect complements and does not provide the correct incentives to avoid moral hazard in service stages and to enhance competition in contest stages. However, the additive bias provides an advantage to the previous service provider that increases the probability of avoiding moral hazard at service 1, although it may dampen the competition at contest 2.

In this model, introducing an additive bias is more optimal than introducing a multiplicative bias. Then, the next section focuses on the additive bias to generalize some assumptions of the model in Section 2. Consider a base model where the service 1 provider's effective effort in contest 2 has perfect substitutes efforts ($\rho = 1$) with the same weight ($\gamma = 1$),

$$\hat{e}_{w,2} = s_{w,1} + e_{w,2}. \tag{7}$$

The results with this effective effort (base model) are a special case of Propositions 1 and 2 for $\rho = \gamma = 1$ which are summarized in Corollary 1.[25]

Corollary 1 *When $\gamma = \rho = 1$, potential repetition can avoid moral hazard at service 1. At service 2, agent w always shirks. Two different types of SPE exist:*

SPE I *When $c < \delta/6$, moral hazard is mitigated and contests effort is maximum. The unique SPE has both players exert high effort in both contests, and the winner of contest 1 also chooses high effort at service 1.*

SPE II *When $\delta/6 < c < 1/2$, moral hazard cannot be avoided but contests effort is maximum. The unique SPE has both agents exert high effort in both contests, but the winner of contest 1 shirks in service 1.*

Subsection 4.1 considers a larger pool of potential providers, and Subsection 4.2 relax the assumption that the cost of effort in both contest stage and service stage are equal. Then, Subsection 4.3 turn to the all-pay auction CSF and, finally, the last subsection considers continuous efforts with a generalization of the Tullock's CSF.

[25] Notice that there is continuity when effort cost is $c = \delta/6$. There are multiple equilibria, in all of which both agents compete in contest stages and the winner of contest 2 shirks at service 2. However, the winner of contest 1 is indifferent between exerting effort or not at service 1. Notice also that agent w is indifferent between shirking or not when $\delta/6 < c < 1/2$ and $\delta = 1$, see footnote 22.

4 Extensions

4.1 Large pool of providers

This subsection extends the base model to several agents (two or more) as in many procurement of services situations.[26] Consider the base model but now there are n agents where n is any positive natural number equal or higher than 2. Then, the CSF is

$$P_{i,t}(\hat{e}_{1,t}, \hat{e}_{2,t}, \ldots, \hat{e}_{i,t}, \ldots, \hat{e}_{n,t}) = \begin{cases} \frac{\hat{e}_{i,t}}{\sum_{j=1}^{N} \hat{e}_{j,t}} & \text{if } \sum_{j=1}^{N} \hat{e}_{j,t} > 0 \\ \frac{1}{N} & \text{otherwise} \end{cases}, \tag{8}$$

where the effective effort is given by equation (7).

When $c > 1/N$, agents do not exert effort either at the contest stages or at the service stages. Then, the results focus on $c < 1/N$. It is found that the moral hazard problem can be avoided when effort is cheap enough. Nevertheless, this problem is more severe when either agents are more impatient (the discount factor δ decreases) or the number of agents increases. Proposition 4 states these results more formally (all calculations are in Appendix 6.5).[27]

Proposition 4 *Potential repetition can avoid moral hazard at service 1. At service 2, the winner of contest 2 always shirks. Two different types of SPE exist:*

SPE I *When $c < \delta(N-1)/(N^2+N)$, moral hazard is mitigated and contests effort is maximum. The SPE has all agents exerting high effort in both contests, and the winner of contest 1 also chooses high effort at service 1.*

SPE II *When $\delta(N-1)/(N^2+N) < c < 1/N$, moral hazard cannot be avoided but contests effort is maximum. The SPE has all agents exerting high effort in both contests, but the winner of contest 1 shirks in service 1.*

Notice that the severity of the moral hazard problem increases with the number of agents because, then, the contest 1 winner competes against more competitors and there is more uncertainty about the repetition of his contractual relationship with the designer. In other words, the larger the pool of providers, the lower the contest 1 winner's probability of winning contest 2. Consequently, the designer should consider a few number of competitors in order

[26]For instance, the public procurement negotiated procedures in the European Union states that public agencies must invite to three candidates at least.

[27]Note that the winner of contest 1 is indifferent between shirking or not at service 1 when $c = \delta(N-1)/(N^2+N)$.

to reduce the uncertainty about the future repetition of the contract while keeping the level of competition.[28]

4.2 The cost of the effort depends on the stage

Now it is analyzed the base model with two agents but considering that the effort cost at contest stages can differ from the effort cost at service stages. Then, the service effort cost function is $c_s(s_{w,t}) = c_s s_{w,t}$ and the contest effort cost function is $c_c(e_{i,t}) = c_c e_{i,t}$. In this case, the results depend on which stage has a more expensive effort.

Firstly, consider that service effort is more expensive than contest effort ($c_s \geq c_c$). The results, which are analogous to Corollary 1, are explained formally in Proposition 5 (all calculations are in Appendix 6.6).

Proposition 5 *When $c_s \geq c_c$ or both $c_s \leq c_c$ and $0 < \delta c_c < \delta/6$, potential repetition can avoid moral hazard at service 1. At service 2, agent w always shirks. Two different types of SPE exist:*

SPE I *When $c_s < \delta/6$, moral hazard is mitigated and contests effort is maximum. The unique SPE has both agents exerting high effort in both contests, and the winner of contest 1 also chooses high effort at service 1.*

SPE II *When $\delta/6 < c_s < 1/2$, moral hazard cannot be avoided but contests effort is maximum. The unique SPE has both agents exerting high effort in both contests, but the winner of contest 1 shirks at service 1.*

Consider now that contest effort is more expensive than service effort ($c_c \geq c_s$). It turns out that a trade-off between high service 1 effort and high contest 2 efforts may appear. Proposition 6 explains the results formally (all calculations are in Appendix 6.6).

Proposition 6 *When both $c_c \geq c_s$ and $\delta/6 < \delta c_c < \delta 1/2$, potential repetition can avoid moral hazard at service 2. At service 2, the winner of contest 2 always shirks. Three different types of SPE exist:*

SPE I *When $c_s < \delta/6$, moral hazard is mitigated and contests effort is maximum. The unique SPE has both agents exerting high effort in both contests, and the winner of contest 1 also chooses high effort at service 1.*

[28] This conclusion is consistent with Che and Gale (2003) and Fullerton and McAfee (1999).

SPE II *When $\delta/6 < \delta c_c < c_s < 1/2$, moral hazard cannot be avoided but contests effort is maximum. The SPE has both agents exerting high effort in both contests, but the winner of contest 1 shirks in service 1.*

SPE III *When $\delta/6 < c_s < \delta c_c < 1/2$, moral hazard is mitigated but contest effort is not maximum. The SPE has both atents exerting high effort in contest 1. The winner of contest 1 exerts high effort at service 1 and no effort at contest 2. The loser of contest 1 exerts high effort at contest 2.*

In Propositions 5 and 6, SPE I is achieved when both kinds of effort cost are low enough. However, the moral hazard problem is less severe when the service effort cost is lower than the contest effort cost. In this case, when the service effort cost is still lower than the discounted contest effort cost but effort costs are too high to achieve SPE I, the designer can achieve SPE III as an alternative to SPE II.[29] Then, the moral hazard problem is solved to the detriment of competition at contest 2.

4.3 All Pay Auction CSF

In order to check the robustness of the base case result, this subsection analyzes the base model with all-pay auctions. This CSF is a deterministic function which is used commonly in the contest literature (see, for example, Corchón and Dahm 2011; Epstein et al. 2011; Konrad 2009; Siegel 2010),

$$P_{i,t}(\hat{e}_{i,t}, \hat{e}_{j,t}) = \begin{cases} 1 & \text{if } \hat{e}_{i,t} > \hat{e}_{j,t} \text{ and } i \neq j \\ 1/2 & \text{if } \hat{e}_{i,t} = \hat{e}_{j,t} \text{ and } i \neq j \\ 0 & \text{if } \hat{e}_{j,t} > \hat{e}_{i,t} \text{ and } i \neq j \end{cases}. \tag{9}$$

The effective effort is given by equation (7). When effort cost is low enough, moral hazard is avoided but the designer's most preferred equilibrium (SPE I) does not exist for any effort cost. Proposition 7 states the results more formally (all calculations are in Appendix 6.7),

Proposition 7 *With all-pay auctions at contest stages, potential repetition can avoid moral hazard at service 1. At service 2, the winner of contest 2 always shirks. Two different types of SPE exist:*

[29] As in Section 3.1, the designer's preferences on the SPE are $SPEI \succ SPEIII \succ SPEII$.

SPE VII *When $c < \delta/2$, moral hazard is mitigated but contests effort is not maximum. The SPE has both agents exerting high effort at contest 1, and the winner of contest 1 also chooses high effort at service 1. However, in contest 2, the winner of contest 1 exerts high effort with probability $r_{w,2} = 1 - 2c$, and the loser of contest 1 exerts high effort with probability $q_{l,2} = 2c$.*

SPE II *When $\delta/2 < c < 1/2$, moral hazard cannot be avoided but contests effort is maximum. The SPE has both agents exerting high effort in both contests, but the winner of contest 1 shirks in service 1.*

Notice that the alternative SPE VII to SPE I avoids moral hazard at service 1 for a large interval of effort cost ($c < \delta/2$), but competition at contest 2 is lower. In SPE VII both agents have the same probability of competing at contest 2 when $c = 1/4$. When the effort cost increases, the winner of contest 1 has lower incentives to exert high effort because of his advantage and this increases the incentives of the loser of contest 1 to exert high effort at contest 2. When the effort cost decreases from $c = 1/4$, the winner of contest 1 has higher incentives to exert high effort and this desincentives the loser of contest 1. Notice that SPE VII is very similar to SPE V (Section 3.2) but with lower probabilities of exerting high effort at contest 2 for both agents.

Notice also that the interval for SPE II with all-pay auctions is smaller than with the Tullock CSF. Comparing Proposition 7 with Corollary 1, we can see that the designer should choose Tullock contests when effort cost is quite low because she can obtain SPE I which is preferred to SPE VII. On the other hand, when effort cost is higher and SPE I is not achievable in the Tullock contests ($\delta/6 < c < \delta/2$), the designer should state all-pay auctions because she can obtain SPE VII as an alternative to SPE II. Finally, for high effort cost ($\delta/2 < c < 1/2$), the designer is indifferent between choosing any of both CSF because any of them cannot avoid moral hazard in service 1. However, if agents are not impatient ($\delta \to 1$), the designer should organize all-pay auctions in contest stages for higher effort costs.

4.4 Continuous efforts and a generalized CSF

In order to check the robustness of the base case result, this subsection analyzes the base model with continuous efforts $e_{i,t}, s_{w,t} \in \Re_+$, as in many literature on contests (Beviá and Corchón 2012; Che and Gale 2003; Epstein et al. 2011).[30] This allow us to generalize the Tullock CSF

[30] Assuming continuous efforts means assuming not only that agents can choose between infinite number of strategies, but also that the designer can observe these strategies.

in Section 2,

$$P_{i,t}(\hat{e}_{i,t}, \hat{e}_{j,t}) = \begin{cases} \frac{\hat{e}^r_{i,t}}{\hat{e}^r_{i,t}+\hat{e}^r_{j,t}} & \text{if } \hat{e}^r_{i,t} + \hat{e}^r_{j,t} > 0 \\ \frac{1}{2} & \text{otherwise} \end{cases}, \quad (10)$$

where the exponent can be interpreted as the degree of noise in the selection process and satisfies $r \in (0, 2]$.[31] Here, the effective effort is equation (4) instead of equation (7) because moral hazard is not avoided with the last.

The results show that both agents exert the same effective effort $e_{l,2} = \hat{e}_{w,2} = r/4c$ at contest 2.[32] This interior solution implies that there is a trade-off between the contest 2 effort of the provider of service 1 and the size of the weight of service 1 effort at the second period CSF. Proposition 8 states this result more formally (all calculations are in Appendix 6.8),

Proposition 8 *When efforts are continuous, potential repetition can avoid moral hazard at service 1. At service 2, the winner of contest 2 always shirks. Two different types of SPE exist:*

- *When $\delta\gamma \leq 1$, moral hazard cannot be avoided. The SPE has both agents exerting effort $e_{i,t} = r/4c$ at contest stages, and the winner of contest 1 shirks $s_{w,t} = 0$ at service 1.*[33]
- *When $\delta\gamma \geq 1$, moral hazard is mitigated but efforts decrease in contest 2. The SPE has both players exerting the same effort at contest 1 $e_{i,1} = (r/4c)[1+(\delta r/4)-(r/4\gamma)]$, and the winner of contest 1 exerts the effort $s_{w,1} = r/(4c\gamma)$ at service 1. However, this agent does not compete at the period 2 contest stage $e_{w,2} = 0$ while his opponent does $e_{l,2} = r/4c$.*

Notice that efforts at contest stages decrease when the effort cost increases and when r decreases (noise increases).[34] On the one hand, when the past is not important ($\delta\gamma \leq 1$), moral hazard is not avoided at service 1. This result is analogous to SPE II in the discrete game. On the other hand, when the past is important enough ($\delta\gamma \geq 1$), the moral hazard problem is mitigated but competition at contest 2 decreases.[35] Given these results, the designer's optimal choice is $\gamma = 1/\delta$ because while moral hazard in service 1 is avoided with a sufficiently high

[31] At contest 2, when the second order condition of agent l's expected utility is considered, the usual condition of concavity is found, $r < 2$. However, agent w's condition of concavity is $r < 2 + 4c\gamma s_{w,1}$. Since $2 \leq 2 + 4c\gamma s_{w,1}$, the level of noise must be $r < 2$ to maximize both agents' expected utilities. See Pérez-Castrillo and Verdier (1992).

[32] The fact that $e_{l,2} = \hat{e}_{w,2}$ comes from the addition of effective effort. See Dahm and Porteiro (2008) for a related result in a different context and other function forms.

[33] Note that the equilibrium efforts when the past is not important ($\delta\gamma \leq 1$) coincide with the well known result of the Tullock contest with homogeneous contestants, see Konrad (2009).

[34] A very high noise is analogous to ignoring the past: the probability of winning contest 2 is one half regardless the effort at service 1.

[35] The winner of contest 1 prefers not to shirk at service 1 and not compete at contest 2 because costs are lower $((r/4)[1 + (\delta r/4) - (r/4\gamma)] + (r/4\gamma) + 0)$ than when he shirks at service 1 but competes at contest 2 $((r/4) + 0 + (\delta r/4))$.

weight of the past ($\delta\gamma \geq 1$), such effort at service 1 decreases with γ.[36] This result supports the results of the base model.

5 Conclusions

This paper studies a moral hazard problem in repeated procurement of services by a repeated model. There are two periods and two homogeneous and risk-neutral agents who compete each period in a contest stage for being hired and provide a service in a service stage. The designer wants to maximize, in the first place, the total service effort and, in the second place, the total contest effort. In order to mitigate the moral hazard problem and maximize service efforts, the designer chooses how to bias the second contest to reward (punish) the first provider who exerted a high (low) effort in the first service. In the second contest, the past service effort is considered as a bias and a CES function to relate past service effort with current contest effort in the second period CSF. The results show that the designer's optimal choice is to consider these efforts more (less) substitutes when effort cost is low (high). This model differs from other repeated contest models in considering a principal-agent relationship between the winner and the designer after the contest in a service stage and, moreover, analyzing the incentive effects of repeating the model on the studied moral hazard problem.

The results of this paper provide some valuable insights into how to improve procedures commonly applied in the repeated procurement of services. Past performance should be taken into account in the design of future contests to mitigate moral hazard problems in the provision of services. However, it can disincentivize competition in future contests. Therefore, it becomes particularly important to determine the optimal degree to which contests should be biased towards past performance. It is shown that when effort cost for the agents is high, the designer should penalize a former service provider for low performance in future contests (i.e., allow for a high degree of complementarity between service and contest effort in the CSF, as for example with a multiplicative bias). On the other hand, when effort cost is low, the designer should allow a former service provider to compensate low performance in past services by higher competition effort in future contests (i.e., allow for a low degree of complementarity between service and contest effort in the CSF, as for example with an additive bias). Furthermore, the optimal degree of complementarity between past service effort and future contests effort increases when agents become more impatient or the future repetition of the procurement procedure is more

[36] When $\delta\gamma = 1$, the winner of contest 1 is indifferent between shirking or not at service 1. Then, he mus state a $\delta\gamma$ slightly higher than one. However, the sum of both efforts $s_{w,1}$ and $e_{w,2}$ will be never larger than $r/4c$.

uncertain.

Another feature of the results is the assessment between additive and multiplicative biases in contests. While the models in the literature have commonly used additive biases and multiplicative biases without evaluating whether these biases are optimal from the designer's point of view (if there is a designer), this paper provides an analysis of the conditions under which either additive biases or multiplicative biases are indeed the designer's best choice. The theoretical results of this paper might also be valuable for a designer with different preferences from those assumed here.[37] Some interesting extensions of the model are left for future research, such as the role of risk aversion, heterogeneous agents and different CSF in the determination of the optimal bias.

6 Appendix

6.1 Proof of Lemma 1

In the service stage, the winner of the contest will shirk because the relationship with the designer ends here and his payoffs are higher $1 - c < 1$.[38] Anticipating this, both agents maximize the following expected utility for the reperesentative agent i,

$$E(U_i(r,q)) = r\left[q\left(\frac{1}{2} - c\right) + (1-q)(1-c)\right] + (1-r)\left[(1-q)\frac{1}{2}\right], \tag{11}$$

where r (q) is the probability that agent i (j) exerts high effort. Taking the derivative, we obtain

$$\frac{\partial E(U_i(r,q))}{\partial r} = \frac{1}{2} - c. \tag{12}$$

The agents' behaviour strategy at the contest stage depends on how expensive the effort is. When it is higher than 1/2, the first derivative is negative and no agent exerts high effort in the competition. If the cost of effort is lower than 1/2, the first derivative is positive and, then, both agents exert high effort at the contest stage. Finally, when the cost of effort is equal to 1/2, both agents are indifferent between exerting high effort or not. Given the symmetric situation, we expect symmetric behaviour strategies with $q = r$.

[37]For example, consider a designer who wants to mitigate moral hazard at service stages but consider contest efforts wasteful. In this case, her optimal choice is considering efforts as more complements even when effort cost is low. See Epstein et al. (2011) for a general model in which the designer can have different preferences on contests efforts.

[38]This result is repeated in all service 2 of the repeated model.

6.2 Proof of Proposition 1

The winner of contest 2 shirks in providing service 2. At contest 2, the effective effort of contest 1 winner (agent w) depends on his effort at service 1. If he shirked, his effective effort is his current effort and Lemma 1 describes the period 2 outcome. If, however, agent w exerted high effort at service 1, his expected utility is

$$E(U_{w,2}(r_{w,2}, q_{l,2}|s_{w,1} = 1)) = r_{w,2}\left[q_{l,2}\left(\frac{1+\gamma}{2+\gamma} - c\right) + (1 - q_{l,2})(1 - c)\right] + \\ + (1 - r_{w,2})\left[q_{l,2}\left(\frac{\gamma}{1+\gamma}\right) + (1 - q_{l,2})(1)\right]. \quad (13)$$

r (q) is the probability that agent w (l) exerts high effort. The derivative is

$$\frac{\partial E(U_{w,2}(r_{w,2}, q_{l,2}|x_{w,1} = 1))}{\partial r_{w,2}} = \frac{q_{l,2}}{(1+\gamma)(2+\gamma)} - c. \quad (14)$$

Agent l maximizes

$$E(U_{l,2}(r_{w,2}, q_{l,2}|s_{w,1} = 1)) = q_{l,2}\left[r_{w,2}\left(\frac{1}{(\gamma+2)} - c\right) + (1 - r_{w,2})\left(\frac{1}{(\gamma+1)} - c\right)\right]. \quad (15)$$

The derivative is

$$\frac{\partial E(U_{l,2}(r_{w,2}, q_{l,2}|s_{w,1} = 1))}{\partial q_{l,2}} = (\gamma + 2) - c(1+\gamma)(2+\gamma) - r_{w,2}. \quad (16)$$

When $\gamma \leq 1$, both agents' reaction functions imply

$$\{r_{w,2}, q_{l,2}\} = \begin{cases} \{0, 1\} & \text{if } 1/((1+\gamma)(2+\gamma)) < c < 1/2 \\ \{r_{w,2}, 1\} & \text{if } c = 1/((1+\gamma)(2+\gamma)), \text{ where } r_{w,2} \in [0, 1] \\ \{1, 1\} & \text{if } 0 < c < 1/((1+\gamma)(2+\gamma)) \end{cases}. \quad (17)$$

When $\gamma > 1$, both agents' reaction functions imply

$$\{r_{w,2}, q_{l,2}\} = \begin{cases} \{0, 0\} & \text{if } 1/(1+\gamma) < c < 1/2 \\ \{0, q_{l,2}\} & \text{if } c = 1/(1+\gamma), \text{ where } q_{l,2} \in [0, 1] \\ \{0, 1\} & \text{if } 1/((1+\gamma)(2+\gamma)) < c < 1/(1+\gamma) \\ \{r_{w,2}, 1\} & \text{if } c = 1/((1+\gamma)(2+\gamma)), \text{ where } r_{w,2} \in [0, 1] \\ \{1, 1\} & \text{if } 0 < c < 1/((1+\gamma)(2+\gamma)) \end{cases}. \quad (18)$$

At service 1, agent w maximizes his expected utility,

$$E(U_{w,1}(x_{w,1})) = x_{w,1}[1 - c + \delta E(U_{w,2}(r_{w,2}, q_{l,2})|s_{w,1} = 1)] + \\ + (1 - x_{w,1})[1 + \delta E(U_{w,2}(r_{w,2}, q_{l,2})|s_{w,1} = 0)], \tag{19}$$

were $x_{w,1}$ is the probability that agent w exerts high effort. Taking the derivative, we obtain

$$\frac{\partial E(U_{w,1}(x_{w,1}))}{\partial x_{w,1}} = \delta \left[E(U_{w,2}(r_{w,2}, q_{l,2})|s_{w,1} = 1) - E(U_{w,2}(r_{w,2}, q_{l,2})|s_{w,1} = 0)\right] - c. \tag{20}$$

His behaviour strategy when $\gamma > 1$ is not to shirk. When $\gamma \leq 1$, his behaviour strategy is summarized in the following equation,

$$x_{w,1}(c) = \begin{cases} 0 & \text{if } \gamma/(2\gamma + 4) < c < 1/2 \\ [0, 1] & \text{if } c = \gamma/(2\gamma + 4) \\ 1 & \text{if } 0 < c < \gamma/(2\gamma + 4) \end{cases} . \tag{21}$$

At contest 1, both agents have no past and their effective efforts are their current efforts by definition. Given that results of each agent are symmetric, we solve this stage for the representative agent i who maximizes his expected utility

$$\begin{aligned} E(U_{i,1}(r_{i,1}, q_{j,1})) = r_{i,1}q_{j,1} &\left(\frac{1}{2}E(U_{w,1}(x_{w,1})) + \frac{\delta}{2}E(U_{l,2}(r_{w,2}, q_{l,2})|x_{w,1}) - c\right) + \\ &+ r_{i,1}(1 - q_{j,1})\left(E(U_{w,1}(x_{w,1})) - c\right) + \\ &+ (1 - r_{w,2})q_{j,1}\left(\delta E(U_{l,2}(r_{w,2}, q_{l,2})|x_{w,1})\right) + \\ &+ (1 - r_{w,2})(1 - q_{j,1})\left(\frac{1}{2}E(U_{w,1}(x_{w,1})) + \frac{\delta}{2}E(U_{l,2}(r_{w,2}, q_{l,2})|x_{w,1})\right). \end{aligned} \tag{22}$$

$r_{i,1}$ ($q_{j,1}$) denotes agent i's (j's) probability of exerting high effort at contest 1. The derivative is

$$\frac{\partial E(U_{i,1}(r_{i,1}, q_{j,1}))}{\partial r_{i,1}} = \frac{1}{2}E(U_{w,1}(x_{w,1})) - \frac{\delta}{2}E(U_{l,2}(r_{w,2}, q_{l,2})|x_{w,1}) - c. \tag{23}$$

Focussing on $\delta = 1$, we obtain that the agent i's response function for any $\gamma > 0$ is exerting high effort.

The SPE sequence of efforts on the path $\{(r_{i,1}, q_{j,1}), (x_{w,1}), (r_{w,2}, q_{l,2}), (x_{w,2})\}$ when $\gamma \leq 1$ is

$$\begin{cases} \{(1,1),(0),(1,1),(0)\} & \text{if } \frac{\gamma}{(2\gamma+4)} < c < \frac{1}{2} \\ \{(1,1),(x_{w,1}),(1,1),(0)\} & \text{if } c = \frac{\gamma}{(2\gamma+4)}\text{, where } x_{w,1} \in [0,1] \\ \{(1,1),(1),(1,1),(0)\} & \text{if } 0 < c < \frac{\gamma}{(2\gamma+4)} \end{cases}. \tag{24}$$

And the obtained SPE sequence of efforts on the path when $\gamma > 1$ can be described as

$$\begin{cases} \{(1,1),(1),(0,0),(0)\} & \text{if } \frac{1}{(1+\gamma)} < c < \frac{1}{2} \\ \{(1,1),(1),(0,q_{l,2}),(0)\} & \text{if } c = \frac{1}{(1+\gamma)}\text{, where } q_{l,2} \in [0,1] \\ \{(1,1),(1),(0,1),(0)\} & \text{if } \frac{1}{(1+\gamma)(2+\gamma)} < c < \frac{1}{(1+\gamma)} \\ \{(1,1),(1),(r_{w,2},1),(0)\} & \text{if } c = \frac{1}{(1+\gamma)(2+\gamma)}\text{, where } r_{w,2} \in [0,1] \\ \{(1,1),(1),(1,1),(0)\} & \text{if } 0 < c < \frac{1}{(1+\gamma)(2+\gamma)} \end{cases}. \tag{25}$$

6.3 Proof of Proposition 2

The winner of contest 2 shirks in providing service 2. At contest 2, Lemma 1 describes the period 2 outcome if agent w shirked at service 1. Otherwise, his expected utility is

$$\begin{aligned} E(U_{w,2}(r_{w,2}, q_{l,2}|s_{w,1} = 1)) = r_{w,2}\left[q_{l,2}\left(\frac{2^{1/\rho}}{2^{1/\rho}+1} - c\right) + (1-q_{l,2})(1-c)\right] + \\ + (1-r_{w,2})\left[q_{l,2}\left(\frac{1}{2}\right) + (1-q_{l,2})(1)\right]. \end{aligned} \tag{26}$$

The derivative is

$$\frac{\partial E(U_{w,2}(r_{w,2}, q_{l,2}|x_{w,1} = 1))}{\partial r_{w,2}} = q_{l,2}(2^{1/\rho} - 1) - 2c(2^{1/\rho} + 1). \tag{27}$$

Agent l maximizes

$$E(U_{l,2}(r_{w,2}, q_{l,2}|s_{w,1} = 1)) = q_{l,2}\left[r_{w,2}\left(\frac{1}{2^{1/\rho}+1} - c\right) + (1-r_{w,2})\left(\frac{1}{2} - c\right)\right]. \tag{28}$$

The derivative is

$$\frac{\partial E(U_{l,2}(r_{w,2}, q_{l,2}|s_{w,1} = 1))}{\partial q_{l,2}} = \frac{2^{1/\rho}+1}{2^{1/\rho}-1}(1-2c) - r_{w,2}. \tag{29}$$

When $ln(2)/ln(3) \leq \rho \leq 1$, both agents' reaction functions imply

$$\{r_{w,2}, q_{l,2}\} = \begin{cases} \{0,1\} & \text{if } (2^{1/\rho}-1)/(2(2^{1/\rho}+1)) < c < 1/2 \\ \{r_{w,2}, 1\} & \text{if } c = (2^{1/\rho}-1)/(2(2^{1/\rho}+1)) \text{, where } r_{w,2} \in [0,1] \\ \{1,1\} & \text{if } 0 < c < (2^{1/\rho}-1)/(2(2^{1/\rho}+1)) \end{cases} . \quad (30)$$

When $0 \leq \rho \leq ln(2)/ln(3)$, both agents' reaction functions imply

$$\{r_{w,2}, q_{l,2}\} = \begin{cases} \{0,1\} & \text{if } (2^{1/\rho}-1)/(2(2^{1/\rho}+1)) < c < 1/2 \\ \{r_{w,2}, 1\} & \text{if } c = (2^{1/\rho}-1)/(2(2^{1/\rho}+1)) \\ \left\{\frac{(2^{1/\rho}+1)(1-2c)}{(2^{1/\rho}-1)}, \frac{2c(2^{1/\rho}+1)}{(2^{1/\rho}-1)}\right\} & \text{if } 1/(2^{1/\rho}+1) < c < (2^{1/\rho}-1)/(2(2^{1/\rho}+1)) \\ \{1, q_{l,2}\} & \text{if } c = 1/(2^{1/\rho}+1) \\ \{1,1\} & \text{if } 0 < c < 1/(2^{1/\rho}+1) \end{cases} . \quad (31)$$

At service 1, agent w maximizes his expected utility given equation (19). His behaviour strategy for any parameter $0 \leq \rho \leq 1$ is summarized in the following equation,

$$x_{w,1}(c) = \begin{cases} [0,1] & \text{if } (2^{1/\rho}-1)/(2(2^{1/\rho}+1)) \leq c < 1/2 \\ 1 & \text{if } 0 < c < (2^{1/\rho}-1)/(2(2^{1/\rho}+1)) \end{cases} . \quad (32)$$

At contest 1, both agents maximize their expected utility which is equation (22) when $\delta = 1$. We obtain again that both agents always exert high effort at contest 1. To sum up, the obtained SPE can be described by the sequence of efforts on the path $\{(r_{i,1}, q_{j,1}), (x_{w,1}), (r_{w,2}, q_{l,2}), (x_{w,2})\}$ when $ln(2)/ln(3) \leq \rho \leq 1$,

$$\begin{cases} \{(1,1),(0),(1,1),(0)\} & \text{if } \frac{(2^{1/\rho}-1)}{2(2^{1/\rho}+1)} < c < \frac{1}{2} \text{ and } \delta < 1 \\ \{(1,1),(x_{w,1}),(1,1),(0)\} & \text{if } c = \frac{2^{1/\rho}-1}{2(2^{1/\rho}+1)} \text{, where } x_{w,1} = [0,1] \\ \{(1,1),(1),(1,1),(0)\} & \text{if } 0 < c < \frac{2^{1/\rho}-1}{2(2^{1/\rho}+1)} \end{cases} . \quad (33)$$

The obtained SPE through the sequence of efforts on the path when $0 < \rho < ln(2)/ln(3)$ can be described as

$$\begin{cases} \{(1,1),(0),(1,1),(0)\} & \text{if } \frac{2^{1/\rho}-1}{2(2^{1/\rho}+1)} < c < \frac{1}{2} \text{ and } \delta < 1 \\ \{(1,1),(x_{w,1}),(r_{w,2},1),(0)\} & \text{if } c = \frac{2^{1/\rho}-1}{2(2^{1/\rho}+1)} \text{, where } x_{w,1} = [0,1], r_{w,2} = [\frac{1-2c}{2c}, 1] \\ \left\{(1,1),(1),(\frac{(2^{1/\rho}+1)(1-2c)}{2^{1/\rho}-1}, \frac{2c(2^{1/\rho}+1)}{2^{1/\rho}-1}),(0)\right\} & \text{if } \frac{1}{2^{1/\rho}+1} < c < \frac{2^{1/\rho}-1}{2(2^{1/\rho}+1)} \\ \{(1,1),(1),(1,q_{l,2}),(0)\} & \text{if } c = \frac{1}{2^{1/\rho}+1} \text{, where } q_{l,2} \in [\frac{2}{2^{1/\rho}-1}, 1] \\ \{(1,1),(1),(1,1),(0)\} & \text{if } 0 < c < \frac{1}{2^{1/\rho}+1} \end{cases} \quad (34)$$

.

6.4 Proof of Proposition 3

The winner of contest 2 shirks in providing service 2. At contest 2, the agent w's effective effort depends on his effort at service 1. If he shirked, his effective effort is zero, his expected utility is equation (35) and agent l's expected utility is equation (36).

$$E(U_{w,2}(r_{w,2}, q_{l,2}|s_{w,1} = 0)) = r_{w,2}\left[q_{l,2}\,(-c) + (1 - q_{l,2})(\frac{1}{2} - c)\right] + \frac{(1 - r_{w,2})(1 - q_{l,2})}{2} \tag{35}$$

$$E(U_{l,2}(r_{w,2}, q_{l,2}|s_{w,1} = 0)) = q_{l,2}\,[1 - c] + (1 - q_{l,2})\left[\frac{1}{2}\right] \tag{36}$$

Taking derivatives, it turns out that only agent l competes with high effort. If, however, agent w exerted high effort at service 1, their expected utilities are symmetric,

$$E(U_{w,2}(r_{w,2}, q_{l,2}|s_{w,1} = 1)) = r_{w,2}\left[q_{l,2}\left(\frac{1}{2} - c\right) + (1 - q_{l,2})(1 - c)\right] + \frac{(1 - r_{w,2})(1 - q_{l,2})}{2} \tag{37}$$

$$\begin{aligned} E(U_{l,2}(r_{w,2}, q_{l,2}|s_{w,1} = 1)) = q_{l,2}&\left[r_{w,2}\left(\frac{1}{2} - c\right) + (1 - r_{w,2})(1 - c)\right] + \\ &+ \frac{(1 - q_{l,2})(1 - r_{w,2})}{2}. \end{aligned} \tag{38}$$

Taking derivatives, it is obtained that both agents compete.

At service 1, agent w maximizes his expected utility given equation (19). His behaviour strategy is summarized in the following equation,

$$x_{w,1}(c) = \begin{cases} 1 \text{if } 0 < c < 1/4 \\ [0, 1] \text{ if } c = 1/4 \\ 0 \text{if } 1/4 < c < 1/2 \end{cases}. \tag{39}$$

At contest 1, both agents maximize equation equation (22) when $\delta = 1$. We obtain that both agents exert high effort at contest 1 only when $c < 1/4$. Otherwise, no agent exerts high effort. To sum up, the obtained SPE can be described by the sequence of efforts on the path

$\{(r_{i,1}, q_{j,1}), (x_{w,1}), (r_{w,2}, q_{l,2}), (x_{w,2})\}$,

$$\begin{cases} \{(0,0),(0),(0,1),(0)\} & \text{if } \frac{1}{4} < c < \frac{1}{2} \\ \{(r_{i,1}, q_{j,1}),(x_{w,1}),(1,1),(0)\} & \text{if } c = \frac{1}{4}, \text{ where } x_{w,1} = [0,1] \\ \{(1,1),(1),(1,1),(0)\} & \text{if } 0 < c < \frac{1}{4} \end{cases} . \tag{40}$$

6.5 Proof of Proposition 4

The winner of contest 2 shirks in providing service 2. At contest 2, results depend on the agent w's effort at service 1. If he shirked, all agents are equal and they maximize their expected utility by assuming that the other agents have the same behaviour $q_{j-i,2}$

$$E(U_{i,2}(r_{i,2}, q_{j-i,2}|s_{w,1} = 0)) = r_{i,2}\left[q_{j-i,2}\left(\frac{1}{N} - c\right) + (1 - q_{j-i,2})(1-c)\right] + \\ + (1 - r_{w,2})\left[q_{j-i,2}\,(0) + (1 - q_{j-i,2})\left(\frac{1}{N}\right)\right]. \tag{41}$$

Taking the derivative, we have

$$\frac{\partial E(U_{i,2}(r_{i,2}, q_{j-i,2}|x_{w,1} = 0))}{\partial r_{i,2}} = q_{j-i,2}(2 - N) + N(1-c) - 1, \tag{42}$$

where $N > 2$. Assuming that all agents are equal, agents' reaction functions imply that they compete as long as $0 < c < 1/N$.[39] If, however, agent w exerted high effort at service 1, his expected utility is

$$E(U_{w,2}(r_{w,2}, q_{l,2}|s_{w,1} = 1)) = r_{w,2}\left[q_{l,2}\left(\frac{2}{N+1} - c\right) + (1 - q_{l,2})(1-c)\right] + \\ + (1 - r_{w,2})\left[q_{l,2}\left(\frac{1}{N}\right) + (1 - q_{l,2})(1)\right], \tag{43}$$

where $q_{l,2}$ brings together all the losers of contest 1 and it is assumed that these agents have the same behaviour. The derivative is

$$\frac{\partial E(U_{w,2}(r_{w,2}, q_{l,2}|x_{w,1} = 1))}{\partial r_{w,2}} = q_{l,2}(N-1) - cN(N+1). \tag{44}$$

Agents l maximize

$$E(U_{l,2}(r_{w,2}, q_{l,2}|s_{w,1} = 1)) = q_{l,2}\left[r_{w,2}\left(\frac{1}{N+1} - c\right) + (1 - r_{w,2})\left(\frac{1}{N} - c\right)\right]. \tag{45}$$

[39] This assumption is equivalent to the assumption $c < 1/2$ in the case with two agents.

The derivative is

$$\frac{\partial E(U_{l,2}(r_{w,2}, q_{l,2}|s_{w,1}=1))}{\partial q_{l,2}} = (N+1)(1-cN) - r_{w,2}. \tag{46}$$

The reaction functions of both types of agents imply

$$\{r_{w,2}, q_{l,2}\} = \begin{cases} \{0,1\} & \text{if } (N-1)/(N(N+1)) < c < 1/N \\ \{r_{w,2}, 1\} & \text{if } c = (N-1)/(N(N+1)), \text{ where } r_{w,2} \in [0,1] \\ \{1,1\} & \text{if } 0 < c < (N-1)/(N(N+1)) \end{cases}. \tag{47}$$

At service 1, agent w maximizes his expected utility given equation (19). His behaviour strategy is summarized in the following equation,

$$x_{w,1}(c) = \begin{cases} 0 & \text{if } \delta(N-1)/(N(N+1)) < c < 1/N \\ [0,1] & \text{if } c = \delta(N-1)/(N(N+1)) \\ 1 & \text{if } 0 < c < \delta(N-1)/(N(N+1)) \end{cases}. \tag{48}$$

At contest 1, all agents maximize their expected utility, which is equation (22), and, again, it turns out that all agents always exert high effort at contest 1. To sum up, the obtained SPE can be described by the sequence of efforts on the path $\{(r_{i,1}, q_{j,1}), (x_{w,1}), (r_{w,2}, q_{l,2}), (x_{w,2})\}$

$$\begin{cases} \{(1,1),(0),(1,1),(0)\} & \text{if } \delta(N-1)/(N(N+1)) < c < 1/N \\ \{(1,1),(x_{w,1}),(1,1),(0)\} & \text{if } c = \delta(N-1)/(N(N+1)), \text{ where } x_{w,1} = [0,1] \\ \{(1,1),(1),(1,1),(0)\} & \text{if } 0 < c < \delta(N-1)/(N(N+1)) \end{cases}. \tag{49}$$

6.6 Proof of Propositions 5 and 6

The winner of contest 2 shirks in providing service 2. At contest 2, the agent w's effective effort depends on his effort at the period 1 service stage. If he shirked, both agents exert high effort at contest 2 when $c_c < 1/2$. If, however, agent w exerted high effort at service 1, his expected utility is

$$E(U_{w,2}(r_{w,2}, q_{l,2}|s_{w,1}=1)) = r_{w,2}\left[q_{l,2}\left(\frac{2}{3} - c_c\right) + (1-q_{l,2})(1-c_c)\right] +$$
$$(1-r_{w,2})\left[q_{l,2}\left(\frac{1}{2}\right) + (1-q_{l,2})(1)\right]. \tag{50}$$

The derivative is

$$\frac{\partial E(U_{w,2}(r_{w,2}, q_{l,2}|x_{w,1}=1))}{\partial r_{w,2}} = q_{l,2} - 6c_c. \tag{51}$$

Agent l maximizes

$$E(U_{l,2}(r_{w,2}, q_{l,2}|s_{w,1}=1)) = q_{l,2}\left[r_{w,2}\left(\frac{1}{3} - c_c\right) + (1-r_{w,2})\left(\frac{1}{(2)} - c_c\right)\right]. \tag{52}$$

The derivative is

$$\frac{\partial E(U_{l,2}(r_{w,2}, q_{l,2}|s_{w,1}=1))}{\partial q_{l,2}} = 3 - 6c_c - r_{w,2}. \tag{53}$$

Both agents' reaction functions imply

$$(r_{w,2}, q_{l,2}) = \begin{cases} (0,1) & \text{if } 1/6 < c_c < 1/2 \\ (r_{w,2}, 1) & \text{if } c_c = 1/6, \text{ where } r_{w,2} \in [0,1] \\ (1,1) & \text{if } 0 < c_c < 1/6 \end{cases}. \tag{54}$$

At service 1, agent w maximizes his expected utility,

$$\begin{aligned} E(U_{w,1}(x_{w,1})) = x_{w,1}[1 - c_s + \delta E(U_{w,2}(r_{w,2}, q_{l,2})|s_{w,1}=1)]+ \\ + (1-x_{w,1})[1 + \delta E(U_{w,2}(r_{w,2}, q_{l,2})|s_{w,1}=0)]. \end{aligned} \tag{55}$$

The derivative is

$$\frac{\partial E(U_{w,1}(x_{w,1}))}{\partial x_{w,1}} = \delta\left[E(U_{w,2}(r_{w,2}, q_{l,2})|s_{w,1}=1) - E(U_{w,2}(r_{w,2}, q_{l,2})|s_{w,1}=0)\right] - c_s. \tag{56}$$

Results depend on which cost is the highest. When $c_s \geq c_c$, agent w's behaviour strategy is

$$x_{w,1}(c_s) = \begin{cases} 0 & \text{if } \delta/6 < c_s < 1/2 \\ [0,1] & \text{if } c_s = \delta/6 \\ 1 & \text{if } 0 < c_s < \delta/6 \end{cases}. \tag{57}$$

When $c_c \geq c_s$, agent w's behaviour strategy is

$$x_{w,1}(c_s, c_c) = \begin{cases} 0 & \text{if both } \delta/6 < c_s < 1/2 \text{ and } 0 < \delta c_c < c_s < 1/2 \\ [0,1] & \text{if either } c_s = \delta/6 < \delta c_c < 1/2 \text{ or } \delta/6 < c_s = \delta c_c < 1/2 \\ 1 & \text{if either } 0 < c_s < \delta/6 \text{ or } \delta/6 \leq c_s < \delta c_c < 1/2 \end{cases}. \tag{58}$$

At contest 1, both agents maximize their expected utility,

$$\begin{aligned}E(U_{i,1}(r_{i,1}, q_{j,1})) = r_{i,1}q_{j,1}&\left(\frac{1}{2}E(U_{w,1}(x_{w,1})) + \frac{\delta}{2}E(U_{l,2}(r_{w,2}, q_{l,2})|x_{w,1}) - c_c\right) + \\ &+ r_{i,1}(1-q_{j,1})\left(E(U_{w,1}(x_{w,1})) - c_c\right) + \\ &+ (1-r_{w,2})q_{j,1}\left(\delta E(U_{l,2}(r_{w,2}, q_{l,2})|x_{w,1})\right) + \\ &+ (1-r_{w,2})(1-q_{j,1})\left(\frac{1}{2}E(U_{w,1}(x_{w,1})) + \frac{\delta}{2}E(U_{l,2}(r_{w,2}, q_{l,2})|x_{w,1})\right).\end{aligned} \tag{59}$$

Taking the derivative, we obtain

$$\frac{\partial E(U_{i,1}(r_{i,1}, q_{j,1}))}{\partial r_{i,1}} = \frac{1}{2}E(U_{w,1}(x_{w,1})) - \frac{\delta}{2}E(U_{l,2}(r_{w,2}, q_{l,2})|x_{w,1}) - c_c. \tag{60}$$

The agents' response function in both cases ($c_c \geq c_s$ and $c_c \leq c_s$) is to compete with high effort. Therefore, both agents always exert effort at contest 1. To sum up, the obtained SPE can be described by the sequence of efforts on the path $\{(r_{i,1}, q_{j,1}), (x_{w,1}), (r_{w,2}, q_{l,2}), (x_{w,2})\}$ when $c_c \leq c_s$ or when both $c_c \geq c_s$ and $\delta c_c < \delta/6$,

$$\begin{cases} \{(1,1),(0),(1,1),(0)\} & \text{if } \frac{\delta}{6} < c_s < \frac{1}{2} \\ \{(1,1),(x_{w,1}),(1,1),(0)\} & \text{if } c_s = \frac{\delta}{6}, \text{ where } x_{w,1} \in [0,1] \\ \{(1,1),(1),(1,1),(0)\} & \text{if } 0 < c_s < \frac{\delta}{6} \end{cases}. \tag{61}$$

When both $c_c \geq c_s$ and $\delta/6 < \delta c_c < 1/2$, the obtained SPE can be described as the sequence of efforts on the path

$$\begin{cases} \{(1,1),(0),(1,1),(0)\} & \text{if } \frac{\delta}{6} < \delta c_c < c_s < \frac{1}{2} \\ \{(1,1),(x_{w,1}),(r_{w,2},1),(0)\} & \text{if } \frac{\delta}{6} < c_s = \delta c_c < \frac{1}{2}, \text{ where } r_{w,2}, x_{w,1} \in [0,1] \\ \{(1,1),(1),(0,1),(0)\} & \text{if } \frac{\delta}{6} < c_s < \delta c_c < \frac{1}{2} \\ \{(1,1),(x_{w,1}),(r_{w,2},1),(0)\} & \text{if } \frac{\delta}{6} < c_s = \delta c_c < \frac{1}{2}, \text{ where } r_{w,2}, x_{w,1} \in [0,1] \\ \{(1,1),(1),(1,1),(0)\} & \text{if } 0 < c_s < \frac{\delta}{6} \end{cases}. \tag{62}$$

6.7 Proof of Proposition 7

The winner of contest 2 shirks in providing service 2. At contest 2, the agent w's effective effort depends on his effort at service 1. If he shirked, his effective effort is his current effort and Lemma 1 describes the period 2 outcome. If, however, agent w exerted high effort at service 1, his expected utility is

$$E(U_{w,2}(r_{w,2}, q_{l,2}|s_{w,1} = 1)) = r_{w,2}(1 - c) + (1 - r_{w,2}) \left[q_{l,2} \left(\frac{1}{2} \right) + (1 - q_{l,2})(1) \right]. \tag{63}$$

Taking the derivative, we have

$$\frac{\partial E(U_{w,2}(r_{w,2}, q_{l,2}|x_{w,1} = 1))}{\partial r_{w,2}} = q_{l,2} - 2c. \tag{64}$$

Agent l maximizes

$$E(U_{l,2}(r_{w,2}, q_{l,2}|s_{w,1} = 1)) = q_{l,2} \left[r_{w,2}(0 - c) + (1 - r_{w,2}) \left(\frac{1}{2} - c \right) \right]. \tag{65}$$

The derivative is

$$\frac{\partial E(U_{l,2}(r_{w,2}, q_{l,2}|s_{w,1} = 1))}{\partial q_{l,2}} = 1 - 2c - r_{w,2}. \tag{66}$$

When $c < 1/2$, both agents' reaction functions imply

$$(r_{w,2}, q_{l,2}) = (1 - 2c, 2c). \tag{67}$$

At service 1, agent w maximizes his expected utility given equation (19). His behaviour strategy is summarized in the following equation,

$$x_{w,1}(c) = \begin{cases} 0 & \text{if } \delta/2 < c < 1/2 \\ [0, 1] & \text{if } c = \delta/2 \\ 1 & \text{if } 0 < c < \delta/2 \end{cases}. \tag{68}$$

At contest 1, both agents maximize equation (22) and it turns out that both agents always exert effort at contest 1. To sum up, the obtained SPE can be described by the sequence of efforts on the path $\{(r_{i,1}, q_{j,1}), (x_{w,1}), (r_{w,2}, q_{l,2}), (x_{w,2})\}$:

$$\begin{cases} \{(1,1), (0), (1,1), (0)\} & \text{if } \frac{\delta}{2} < c < \frac{1}{2} \\ \{(1,1), (x_{w,1}), (r_{w,2}, q_{l,2}), (0)\} & \text{if } c = \frac{\delta}{2}, \\ & \text{where } x_{w,1} \in [0,1], r_{w,2} \in [1-2c, 1], q_{l,2} \in [2c, 1] \\ \{(1,1), (1), (1-2c, 2c), (0)\} & \text{if } 0 < c < \frac{\delta}{2} \end{cases} \tag{69}$$

6.8 Proof of Proposition 8

The winner of contest 2 shirks in providing service 2. At contest 2, first we find the interior solution of the problem. Agent w maximizes his expected utility

$$E(U_{w,2}(\hat{e}_{w,2}, e_{l,2})) = \frac{\hat{e}^r_{w,2}}{\hat{e}^r_{w,2} + e^r_{l,2}} - ce_{w,2}. \tag{70}$$

The derivative is

$$\frac{\partial E(U_{w,2}(\hat{e}_{w,2}, e_{l,2}))}{\partial e_{w,2}} = \frac{r(\hat{e}_{w,2})^{r-1}\hat{e'}_{w,2}e^r_{l,2}}{(\hat{e}^r_{w,2} + e^r_{l,2})^2} - c, \tag{71}$$

where $\hat{e'}_{w,2} = \partial\hat{e}_{w,2}/\partial e_{w,2}$. Agent l maximizes

$$E(U_{l,2}(\hat{e}_{w,2}, e_{l,2})) = \frac{e^r_{l,2}}{\hat{e}^r_{w,2} + e^r_{l,2}} - ce_{l,2}. \tag{72}$$

The derivative,

$$\frac{\partial E(U_{l,2}(\hat{e}_{w,2}, e_{l,2}))}{\partial e_{l,2}} = \frac{re^{r-1}_{l,2}\hat{e}^r_{w,2}}{(\hat{e}^r_{w,2} + e^r_{l,2})^2} - c, \tag{73}$$

is equalized to zero. Then, the cost in both reaction function is isolated and, by equalizing both functions, agent l's optimal effort must fulfil the condition

$$e_{l,2} = \frac{\hat{e}_{w,2}}{\hat{e'}_{w,2}}. \tag{74}$$

Taking the first order conditions from equation (72) and replacing equation (74), the following condition arises

$$\frac{r}{4c} = \hat{e}_{w,2}. \tag{75}$$

Taking equations (74) and (75), the optimal effort agent l is

$$\frac{r}{4c} = e^*_{l,2}. \tag{76}$$

At service 1, agent w maximizes his expected utility

$$E(U_{w,1}(s_{w,1})) = 1 - cs_{w,1} + \delta[\frac{\hat{e}^r_{w,2}}{\hat{e}^r_{w,2} + (\frac{r}{4c})^r} - ce_{w,2}], \tag{77}$$

where $\hat{e}_{w,2} = \gamma s_{w,1} + e_{w,2}$. Given the result of equation (75), we have

$$e_{w,2} = \frac{r}{4c} - \gamma s_{w,1}. \tag{78}$$

Taking into account the last equation, the derivative of the expected utility is

$$\frac{\partial E(U_{w,1}(s_{w,1}))}{\partial s_{w,1}} = \delta\gamma - 1. \tag{79}$$

Agent w's behaviour strategy is summarized by the following equation

$$s_{w,1}(\gamma, \delta) = \begin{cases} 0 & \text{if } \delta\gamma \leq 1 \\ \frac{r}{\gamma 4c} & \text{if } \delta\gamma \geq 1 \end{cases}. \tag{80}$$

At contest 1, both agents are equal and maximize their expected utility. When $\delta\gamma \leq 1$, they maximize

$$E(U_{i,1}(e_{i,1}, e_{j,1})) = \frac{e_{i,1}^r}{e_{i,1}^r + e_{j,1}^r}\left[1 + \delta(\frac{1}{2} - \frac{r}{4})\right] + \left(1 - \frac{e_{i,1}^r}{e_{i,1}^r + e_{j,1}^r}\right)\delta\left[\frac{1}{2} - \frac{r}{4}\right] - ce_{i,1} \tag{81}$$

Taking the derivative, we obtain

$$\frac{\partial E(U_{i,1}(e_{i,1}, e_{j,1}))}{\partial e_{i,1}} = \frac{re_{i,1}^{r-1}e_{j,1}^r}{(e_{i,1}^r + e_{j,1}^r)^2} - c. \tag{82}$$

Assuming that both agents have symmetric behaviour, the optimal effort at this stage when $\delta\gamma \leq 1$

$$e_{i,1}^* = \frac{r}{4c}. \tag{83}$$

To sum up, the obtained SPE can be described by the sequence of efforts on the path when $\delta\gamma \leq 1$:

$$\{(e_{i,1}, e_{j,1}), (s_{w,1}), (e_{w,2}, e_{l,2}), (s_{w,2})\} = \left\{\left(\frac{r}{4c}, \frac{r}{4c}\right), (0), \left(\frac{r}{4c}, \frac{r}{4c}\right), (0)\right\} \tag{84}$$

When $\delta\gamma \geq 1$, agents maximize their expected utility at contest 1,

$$E(U_{i,1}(e_{i,1}, e_{j,1})) = \frac{e_{i,1}^r}{e_{i,1}^r + e_{j,1}^r}\left[1 - \frac{r}{4\gamma} + \frac{\delta}{2}\right] + \left(1 - \frac{e_{i,1}^r}{e_{i,1}^r + e_{j,1}^r}\right)\delta\left[\frac{1}{2} - \frac{r}{4}\right] - ce_{i,1}. \tag{85}$$

The derivative is

$$\frac{\partial E(U_{i,1}(e_{i,1}, e_{j,1}))}{\partial e_{i,1}} = \frac{re_{i,1}^{r-1}e_{j,1}^r}{(e_{i,1}^r + e_{j,1}^r)^2}\left[1 + \frac{r}{4}\left(\delta - \frac{1}{\gamma}\right)\right] - c. \tag{86}$$

Assuming that both agents have symmetric behaviour, the optimal effort at this stage when

$\delta\gamma \leq 1$ is

$$e_{i,1}^{*} = \frac{r}{4c}\left[1 + \frac{r}{4}\left(\delta - \frac{1}{\gamma}\right)\right]. \tag{87}$$

To sum up, the obtained SPE can be described by the sequence of efforts on the path when $\delta\gamma \leq 1$:

$$\left\{\left(\frac{r}{4c}\left[1 + \frac{r}{4}\left(\delta - \frac{1}{\gamma}\right)\right], \frac{r}{4c}\left[1 + \frac{r}{4}\left(\delta - \frac{1}{\gamma}\right)\right]\right), \left(\frac{r}{4\gamma c}\right), \left(0, \frac{r}{4c}\right), (0)\right\} \tag{88}$$

References

Audet, D. (2002). *Government procurement: A synthesis report.* OECD Journal on Budgeting, 2(3), 149 – 194.

Beviá, C., & Corchón, L. (2013). Endogenous strength in conflicts. *International Journal of Industrial Organization*, 31(3), 297 – 306.

Bolton, P., & Dewatripont, M. (2005). *Contract Theory.* Massachussets Institute of Technology, Cambridge.

Cesi, B., & Albano, G.L. (2008). Past Performance Evaluation in Repeated Procurement: A Simple Model of Handicapping. *3rd International Public Procurement Conference Proceedings*: 875 – 898.

Che, Y.K., & Gale, I. (2003). Optimal Design of Research Contests. *American Economic Review*, 93(3), 646 – 671.

Clark, D.J., & Nilssen, T. (2013). Learning by doing in contests. *Public Choice*, 156: 329 – 343.

Clark, D.J., & Riis, C. (1998). Contest Success Functions: An Extension. *Economic Theory*, 11(1), 201 – 204.

Corchón, L., & Dahm, M. (2010). Foundations for contest success functions. *Economic Theory*, 43(1), 81 – 98.

Corchón, L., & Dahm, M. (2011). Welfare maximizing contest success functions when the planner cannot commit. *Journal of Mathematical Economics*, 47(3), 309 – 317.

Dahm, M., & Porteiro N. (2008). Biased Contests. *Public Choice*, 136(1-2), 55 – 67.

Epstein, G.S., Mealem, Y., & Nitzan, S. (2011). Political Culture and Discrimination in Contests. *Journal of Public Economics*, (95), 88 – 93.

European Commission (2011). *Public Procurement in Europe: costs and effectiveness.* Retrieved in March 2013 from http://ec.europa.eu/internal_market/publicprocurement/modernising_rules/evaluation

Fullerton, R.L., & McAfee, R.P. (1999). Auctioning Entry into Tournaments. *Journal of Political Economy*, 107(3), 573 – 605.

Fu, Q., & Lu, J. (2009). Contest with Pre-contest Investment. *Economics Letters*, 103, 142 – 145.

Gibbons, R., & Waldman, M. (1999). 'Careers in organizations: theory and evidence', in Ashenfelter, O., and Card, D. (editors). *Handbook of Labor Economics*, Volume 3. Elsevier Science B.V.

Hölmstrom, B. (1979). Moral Hazard and Observability. *Bell Journal of Economics*, 10(1), 74 – 91.

Jofre-Bonet, M., & Pesendorfer, M. (2000). Bidding behaviour in a repeated procurement auction: A summary. *European Economic Review*, 44(4-6), 1006 – 1020.

Konrad, K. (2009). *Strategy and Dynamics in Contests.* Oxford University Press Inc. (New York).

Laffont, JJ., & Tirole, J. (1999). *A Theory of Incentives in Procurement and Regulation.* Massachussets Institute of Technology, Cambridge.

Lazear, E.P., & Rosen, S. (1981). Rank-Order Tournaments as Optimum Labor Contracts. *Journal of Political Economy*, 89(5), 841 – 864.

Levin, J. (2002). Multilateral Contracting and the Employment Relationship. *The Quarterly Journal of Economics*, 117(3), 1075 – 1103.

Melkonyan, T. (2013). Hybrid Contests. *Journal of Public Economic Theory*, 15(6), 968 – 992.

Meyer, M. (1992). Biased Contests and Moral Hazard: Implications for Career Profiles. *Annals of Economics and Statistics*, (25/26), 165 – 187.

Moldovanu, B., Sela, A., & Shi, X. (2012). Carrots and Sticks: Prizes and Punishments in Contests. *Economic Inquiry*, 50(2), 1453 – 462.

Möller, M. (2012). Incentives versus competitive balance. *Economics Letters*, 117(2), 505 – 508.

Münster, J. (2007). Contests with Investment. *Managerial and Decision Economics*, 28, 849 – 862.

Pérez-Castrillo, D., and Verdier, T. (1992). A General Analysis of Rent-Seeking Games. *Public Choice*, 73(3), 335 – 350.

Ridlon, R. & Shin, J. (2013). Favoring the Winner or Loser in Repeated Contests. *Marketing Science*, 32(5), 768 – 785.

Siegel, R. (2010). Asymmetric Contests with Conditional Investments. *American Economic Review*, 100(5), 2230 – 2260.

Skaperdas, S. (1996). Contest Success Functions. *Economic Theory*, 7(2), 283 – 290.

Taylor, C.R. (1995). Digging for Golden Carrots: An Analysis of Research Tournaments. *American Economic Review*, 85(4), 872 – 890.

Tullock, G. (1980). 'Efficient Rent Seeking', in Buchanan J., Tollison R., and Tullock G. (editors). *Toward a theory of the rent-seeking society.* College Station: Texas A & M University Press.

US Government Spending (2013). *Economic Policy & External Debt.* Retrieved in April 2013 from http://usaspending.gov/explore

World Bank (2013). *Economic Policy and External Debt: Services, etc., value added (% of GDP).* Retrieved in March 2013 from http://data.worldbank.org/

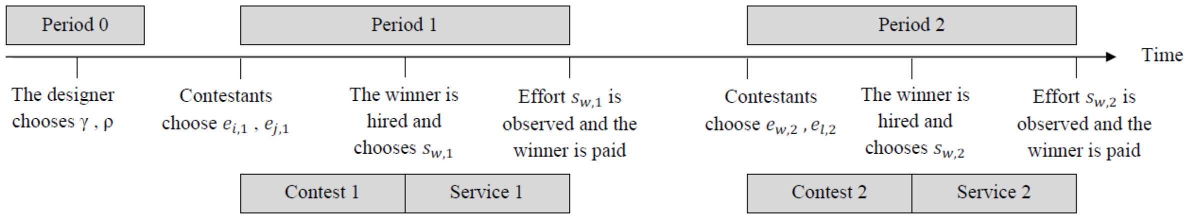

Figure 1: Timing of the model. First, at $t = 0$, the designer chooses the setting of contest 2 (the biased contest) by choosing γ and ρ. At period 1, there are two stages: a contest stage in which the winner is selected, and a service stage in which the winner of contest 1 provides a service. At period 2, there is also a contest stage and a service stage. Contest 2 is biased through γ and ρ in order to consider the effort exerted at service 1 by the winner of contest 1 at the second CSF.

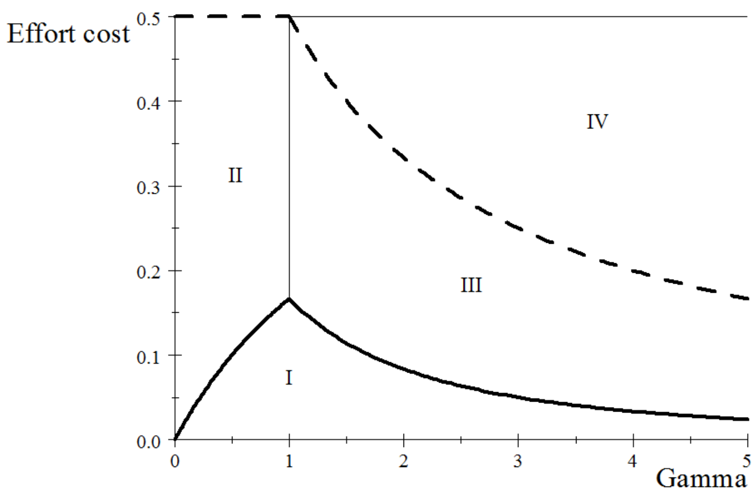

Figure 2: Relationship between SPE, γ (gamma) and effort cost for $\rho = 1$ and $\delta = 1$. In SPE I, moral hazard is mitigated and contests effort is maximum. In SPE II, moral hazard cannot be avoided but contests effort is maximum. In SPE III, moral hazard is mitigated but contests effort is not maximum. In SPE IV, moral hazard is mitigated but contest 2 effort is minimum. Then, the designer's preferences over the SPE are $I \succ III \succ IV \succ II$.

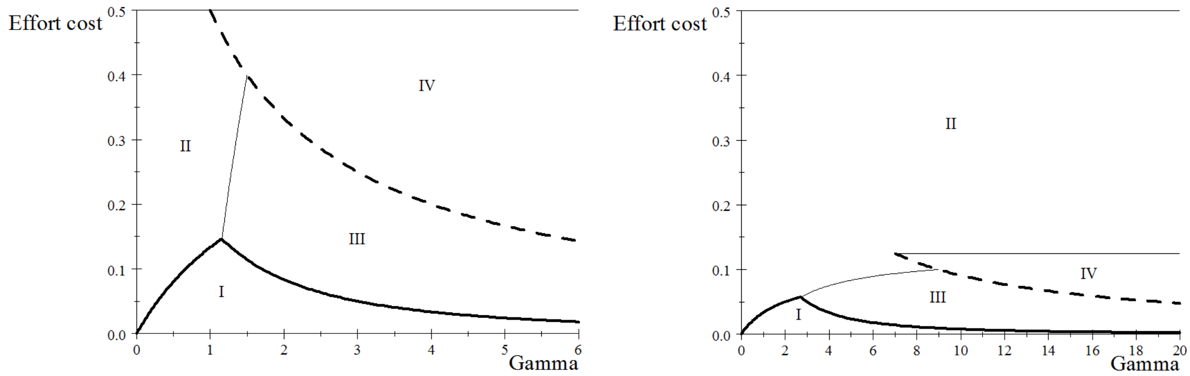

Figure 3: Relationship between SPE, γ and effort cost for $\rho = 1$ when $\delta = 0.8$ (left) and $\delta = 0.2$ (right).

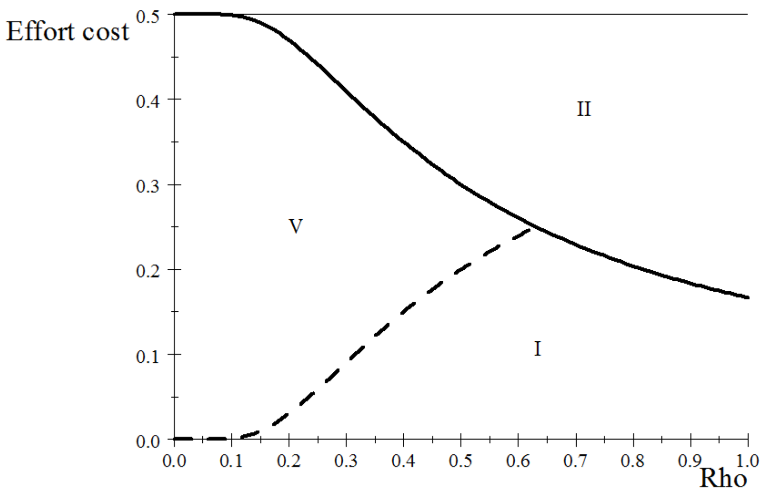

Figure 4: Relationship between SPE, ρ (rho) and effort cost for $\gamma = 1$ and $\delta = 1$. In SPE I, moral hazard is mitigated and contests effort is maximum. In SPE II, moral hazard cannot be avoided but contests effort is maximum. In SPE V, moral hazard is mitigated but contests effort is not maximum. At the limit, when $\rho \to 0$, there is SPE I for cost effort lower than 1/4 and SPE VI for higher cost efforts. In SPE VI, there is moral hazard and contests effort is not maximum. Then, the designer's preferences over the SPE are $I \succ V \succ II \succ VI$.

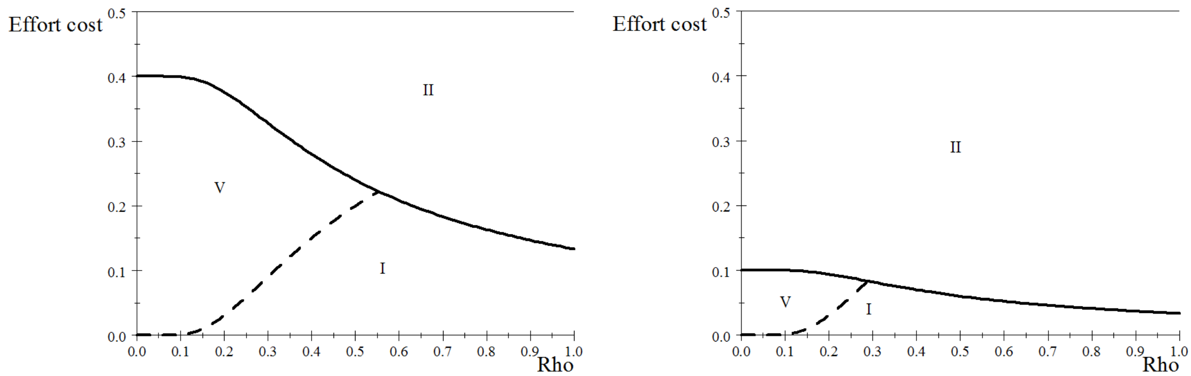

Figure 5: Relationship between SPE, ρ and effort cost for $\gamma = 1$ when $\delta = 0.8$ (left) and $\delta = 0.2$ (right).