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**DEPARTAMENT D'ECONOMIA – CREIP**  
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# Gathering support from rivals: the two rivals case

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**Abstract.** Two voters must choose between two alternatives. Voters vote in a fixed linear order. If there is not unanimity for any alternative, the procedure is repeated. At every stage, each voter prefers the same alternative to the other, has utilities decreasing with stages, and has an impatience degree representing when it is worth voting for the non-preferred alternative now rather than waiting for the next stage and voting for the preferred alternative. Intuition suggests that the more patient voter will get his preferred alternative. I found that in the unique solution of the sequential voting procedure obtained by backward induction, the first voter get his preferred alternative at the first stage independently from his impatience rate.

**Keywords:** sequential voting, impatience rate, multi-stage voting, unanimity

## 1 Introduction

There is no doubt that voting is a highly important decision-making procedure used every day and almost everywhere. Political decisions are taken through voting. Decisions in a big company are also taken by voting. It is common that voting takes place only in one stage, making easy the process of decision taking. In this case the voters do not have the possibility of changing their vote.

There are some voting procedures where there are several stages until the decision is taken. Jury trials constitute one such example. The voting does not stop until the majority or all the voters in case of unanimity agree on the decision. The election of the Catholic Pope can also provide an example: till the super majority of cardinals agree on a decision the voting must continue. So, the main question is why and when voters agree to change their votes and when the decision is taken. To answer this question a simple model for two voters is proposed.

The model represents the situation in which two individuals must agree on choosing one alternative of the two by voting. The voting procedure is sequential. Voters are arranged in a fixed linear order. At each stage voters cast their votes in that order. If two votes are different, then the procedure is repeated in a new stage.

At every stage, each voter prefers the same alternative to the other and has utilities decreasing with stages. The decrease of the utilities represents the cost of the delay. Each voter has an impatience degree indicating when it is worth voting for the non-preferred alternative now rather than waiting for the next stage and voting for the preferred alternative.

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Since both voters know the impatience degree of themselves and the other voter, intuition suggests that the more patient voter will manage to get his preferred alternative. It is shown that in the unique solution of the sequential voting procedure obtained by backward induction, the first voter gets his preferred alternative at the first stage.

The paper is organized as follows. First it is provided in Section 2 a short review of the related literature, studying similar models or problem of decision-making. Next the description of the model and the assumptions are proposed in Section 3. Section 4 presents the result of paper and shows the necessity of each of the assumed condition. Section 5 concludes the paper with some remarks.

## 2 Related literature

The closest model to the one presented here is a model by Kwiek [1]. He considers a decision-making conclave choosing between two alternatives under a super-majority rule (including unanimity). If a decision is not reached in the first round of voting, then the procedure repeats in the next round, and so on, until the required supermajority is reached. The delay in time is increasingly costly to each player. The question that is asked is: which rule offers higher utilitarian welfare? In answering this question he finds that there is a subgame perfect Nash equilibrium that leads to a unique voting outcome in the first round. This outcome coincides with the alternative preferred by the pivotal voter with the greater indifference time (or, in other words, impatience degree).

Compte and Jehiel [2] study collective search processes. They construct a model in which at each stage the committee is proposed to accept or to decline a certain proposal, in case of rejection the procedure passes to the next stage and a new proposal is considered. They study which members have more impact on the decision under different majority rules. One of the interesting results is that under unanimity when proposals vary along a single dimension the extremists determine the final decision. In their framework the extremists are the voters with more intense preferences and therefore with the highest degree of patience. Applying the results of Compte and Jehiel to the model considered in this paper it is expected that the voters with higher degree of patience are those who define the result.

Ponsatí and Sákovics [3] show the uniqueness of equilibrium in a model with many players, two alternatives and delay costs. Baron and Ferejohn [4] study a dynamic model of bargaining in legislatures, when at each round a randomly selected voter makes a proposal to vote by a committee.

The idea of the uniqueness of the subgame perfect equilibrium is similar to the result of Rubinstein [5], where the proposal of the first individual is accepted by the other individual. Besides, there is a huge literature on voting by conformity. For instance, Bernheim [6] states that the voters are willing to conform because they recognize that even small departures from the social norm will seriously impair their status. Despite this penalty, agents with sufficiently extreme preferences refuse to conform. Applying this idea to the model considered here suggests that the voters with higher degree of patience (extreme voters) are not likely to conform with the first voter.

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Some researchers suggest that observing the actions of the other agents would induce individuals to believe that these agents are better informed and therefore, these individuals are likely to imitate their behavior (see, for instance, Banerjee [7]). Herrera and Martinelli [8] develop a model based on the idea that voters follow a leader and attract other voters to follow him too. Rodríguez-Álvarez and Rivas [9] also study the effect of the presence of leaders between the voters on the information transmission among themselves. In the model studied here leadership can be presented as taking the initiative and voting first.

### 3 Related literature

There are two individuals, named 1 and 2. There are two alternatives,  $a$  and  $b$ . A moment in time is denoted by  $t$ . Time is discrete:  $t \in \{1, 2, 3, \dots\}$ . Each individual  $i \in \{1, 2\}$  is endowed with a utility function  $u_i$  defined over the set of pairs  $(c, t)$ , where  $c \in \{a, b\}$  and  $t$  is a moment in time.

The two individuals are supposed to be engaged in a sequential voting procedure in which each voter votes for one of the two alternatives following a fixed ordering of the two individuals, where 1 designates the first individual in the ordering and 2 designates the second individual. The voting procedure stops when both voters vote for the same alternative. In this case, the outcome of the voting procedure is represented by a pair  $(c, t)$ , where  $c \in \{a, b\}$  is the alternative for which both voters have voted and  $t$  is the moment in time at which the voting took place. If the voting procedure does not end, then the outcome is represented by the symbol  $\emptyset$ . The voting procedure is defined as follows.

#### Stage 1

Individual 1 votes for  $a$  or votes for  $b$ . Knowing this choice, individual 2 next votes for  $a$  or votes for  $b$ . If both individuals vote for the same alternative  $c$ , then the procedure ends and, for  $i \in \{1, 2\}$ , individual  $i$  gets utility  $u_i(c, 1)$ . If the individuals do not vote for the same alternative, then the procedure moves to stage 2.

#### Stage $t$

If stage  $t$  is reached, then again individual 1 votes for  $a$  or votes for  $b$ . Knowing this choice, individual 2 next votes for  $a$  or votes for  $b$ . If both individuals vote for the same alternative  $c$ , then the procedure ends and, for  $i \in \{1, 2\}$ , individual  $i$  gets utility  $u_i(c, t)$ . If the individuals do not vote for the same alternative, then the procedure moves to stage  $t + 1$ .

The procedure represents the collective decision mechanism by means of which voters carry on voting until they reach a unanimous decision.

The individuals' utility functions are assumed to satisfy the following four conditions. Let  $\alpha_i$  designate the most preferred alternative by individual  $i$  and  $\beta_i$  designate

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nate the other least preferred alternative for of individual  $i$ .

**Persistence.** For each individual  $i \in \{1, 2\}$  and for all  $t$ ,  $u_i(\alpha_i, t) > u_i(\beta_i, t)$ .

Condition **Persistence** says that each individual has an alternative that is always more preferred than the other: the individual either prefers  $a$  over  $b$  at each time  $t$  or prefers  $b$  over  $a$  at each time  $t$ . This assumption is natural: a supporter of a left party today would rather prefer the left party to the right one tomorrow, too.

**Impatience.** For each individual  $i \in \{1, 2\}$ , alternative  $c \in \{a, b\}$ , and times  $t > t'$ ,  $u_i(c, t) < u_i(c, t')$

Condition **Impatience** asserts that the more the time passes to make a decision, the smaller is the corresponding utility. Any voting in stages induces the time delay of the decision, and time is usually associated with money costs. It is easy to find examples when the faster one decides the smaller are the expenditures: for instance, plane or train tickets become more expensive with time. As another example, consider a board of directors who needs to make a decision, in which company to invest: while no decision is made, the money is in the bank but is affected by inflation.

**Reversion.** For each individual  $i \in \{1, 2\}$ , there is the smallest  $t_i \geq 2$  (called reversal time) such that  $u_i(\beta_i, t_i) > u_i(\alpha_i, t_i + 1)$ .

Condition **Reversion** holds that, for each individual  $i$ , there is at least one stage  $t_i$  making the individual prefer to obtain the least preferred alternative now rather than to get a moment immediately later the most preferred alternative. Intuitively,  $t_i$  represents the moment at which  $i$  loses his patience: it no longer pays to wait for the possibility of obtaining in the future the most preferred alternative by disregarding the possibility of obtaining now the least preferred alternative. When there is a cost of the delay, or in other words, the utility is decreasing with each stage, the voters would rather agree to obtain something now than to wait and continue losing.

**Termination.** For each individual  $i \in \{1, 2\}$ , alternative  $c \in \{a, b\}$ , and all  $t$   
 $u_i(c, t) > u_i(\emptyset)$ .

The outcome  $\emptyset$  different from each pair  $(c, t)$ , corresponds to the situation in which the procedure never stops. Condition **Termination** states that, for each individual  $i$ , the utility of outcome  $\emptyset$  is smaller than the utility of any other outcome. Any voting procedure induces some costs, so it is clear that the voters would prefer to stop the procedure rather than to experience these costs at every stage.

## 4 Result

**Proposition.** Assuming Persistence, Impatience, Reversion, and Termination the outcome of the only subgame perfect equilibrium of the sequential voting procedure is  $(\alpha_1, 1)$ .

The proposition states that, in the only solution of the sequential voting procedure obtained by backward induction, the individual voting first obtains his most preferred option immediately, at the first stage of the procedure.

**Proof.** If both individuals prefer most the same alternative  $c$ , then it is easily verified that in the only subgame perfect equilibrium both choose  $c$  at stage 1. If  $\alpha_1 \neq \alpha_2$ , then, without loss of generality, suppose that  $\alpha_1 = a$  (so  $\alpha_2 = b$ ). It must be shown that, at stage 1, both individuals vote for  $a$ .

By Reversion, consider  $t_2$ . If, on the one hand, 1 chooses  $a$ , then: (i) by choosing  $a$  individual 2 stops the procedure and gets  $u_2(a, t_2)$ ; and (ii) by choosing next  $b$ , 2 makes the procedure enter stage  $t_2 + 1$ . By entering stage  $t_2 + 1$ , three types of outcomes may result.

- (i) First, the outcome  $\emptyset$  corresponding to an unending procedure. By Termination, the utility for 2 of this outcome is smaller than  $u_2(a, t_2)$ .
- (ii) Second, the outcome corresponding to choosing  $a$  at stage  $t' \geq t_2$ . In that case, 2 gets  $u_2(a, t')$ , which, by Impatience, is smaller than  $u_2(a, t_2)$ .
- (iii) And third, the outcome corresponding to choosing  $b$  at stage  $t' \geq t_2$ . In that case, 2 gets  $u_2(b, t')$ , which, by the definition of  $t_2$ , is smaller than  $u_2(a, t_2)$ .

As a consequence, the best choice for 2 at stage  $t_2$  when 1 has chosen  $a$  is to choose  $a$  and stop the procedure.

If, on the other hand, 1 chooses  $b$  at stage  $t_2$ , then: (i) by choosing  $b$  next individual 2 stops the procedure and gets  $u_2(b, t_2)$ ; and (ii) by choosing next  $a$ , 2 makes the procedure enter stage  $t_2 + 1$ . By entering stage  $t_2 + 1$ , three types of outcomes may result.

- (i) First, the outcome  $\emptyset$  corresponding to an unending procedure. By Termination, the utility for 2 of this outcome is smaller than  $u_2(b, t_2)$ .
- (ii) Second, the outcome corresponding to choosing  $b$  at stage  $t' \geq t_2$ . In that case, 2 gets  $u_2(b, t')$ , which, by Impatience, is smaller than  $u_2(b, t_2)$ .
- (iii) And third, the outcome corresponding to choosing  $a$  at stage  $t' \geq t_2$ . In that case, 2 gets  $u_2(a, t')$ . Since  $\alpha_2 = b$ , by Persistence and Impatience,  $u_2(a, t') < u_2(b, t') < u_2(b, t_2)$ .

In sum, the best choice for 2 at stage  $t_2$  when 1 has chosen  $b$  is to choose  $b$ .

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Now consider the decision of individual 1 at stage  $t_2$ . If 1 chooses  $a$ , then, since 2 will choose also  $a$  as show above, 1 gets  $u_1(a, t_2)$ . If 1 chooses  $b$ , then, since, as just shown, 2 will choose  $b$  as well, 1 gets  $u_1(b, t_2)$ . As  $\alpha_1 = a$ ,  $u_1(a, t_2) > u_1(b, t_2)$ . Consequently, the best choice for 1 at stage  $t_2$  is  $a$ .

To recap, it has been shown that, at stage  $t_2$ , by backward induction, both individuals vote for  $a$ . Taking this result as the base case of an induction argument, choose  $t'' < t_2$  and suppose that, for each  $t \in \{t'' + 1, t'' + 2, \dots, t_2\}$ , backward induction leads both individuals to choose  $a$ . It has to be proved that, by backward induction, both individuals also pick  $a$  at stage  $t''$ . This result would conclude the proof.

To this end, choose  $t'' < t_2$  and suppose that, for each  $t \in \{t'' + 1, t'' + 2, \dots, t_2\}$ , backward induction leads both individuals to choose  $a$ . In particular, this implies that, at stage  $t'' + 1$ , 1 gets  $u_1(a, t'' + 1)$ , whereas 2 gets  $u_2(a, t'' + 1)$ .

If 1 chooses  $a$  at stage  $t''$ , then: (i) by choosing  $a$ , 2 stops the procedure and gets  $u_2(a, t'')$ ; and (ii) by choosing  $b$ , 2 forces the procedure to enter stage  $t'' + 1$ , where, by the induction hypothesis, 2 would get  $u_2(a, t'' + 1)$ . By Impatience,  $u_2(a, t'') > u_2(a, t'' + 1)$  and, therefore, 2 would choose  $a$  if 1 chose  $a$  at stage  $t''$ .

If 1 chooses  $b$  at stage  $t''$ , then: (i) by choosing  $b$  next individual 2 stops the procedure and gets  $u_2(b, t'')$ ; and (ii) by choosing next  $a$ , 2 makes the procedure enter stage  $t'' + 1$  and gets  $u_2(a, t'' + 1)$ . Since  $\alpha_2 = b$ , by Impatience and Reversion,  $u_2(b, t'') > u_2(b, t'' + 1) > u_2(a, t'' + 1)$ .

To show that 1 will choose  $a$  and thereby finish the proof, it remains to be verified that, by choosing instead  $b$ , 1 will get less than  $u_1(a, t'')$ . To see this, suppose 1 chooses  $b$ . If, on the one hand, 2 chooses  $b$ , the procedure stops and 1 gets  $u_1(b, t'')$ , which, since  $\alpha_1 = a$ , by Persistence is smaller than  $u_1(a, t'')$ . If, on the other hand, 2 selects  $a$ , then the procedure reaches stage  $t'' + 1$ , where, by the induction hypothesis, 1 would get  $u_1(a, t'' + 1)$ . By Impatience,  $u_1(a, t'' + 1) < u_1(a, t'')$ . ■

The proposition holds under four assumptions: Persistence, Consistence, Reversion, and Termination. The following examples show the necessity of each of these assumptions.



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4.1 Example 1. Dropping Persistence.

Consider the utility functions from Table 1. Persistence does not hold for individual 1 at stage 2:  $u_1(\beta_1, 2) > u_1(\alpha_1, 2)$ . It is easy to see that Impatience, Reversion and Termination hold for both individuals.

Table 1.

Stage $t$	Individual 1		Individual 2	
	$u_1(a, t)$	$u_1(b, t)$	$u_2(a, t)$	$u_2(b, t)$
1	15	11	6	8
2	8	10	5	7
3	6	4	4	6
4	3	2	1	3
$\infty$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

Stage 3. By Reversion, since  $t_2 = 3$ , individual 2 prefers any outcome at stage 3 rather than to pass to the next stage. Knowing this, 1 chooses  $a$  since it is the best option. Consequently, the outcome is  $(a, 3)$ : individual 1 gets 6 and individual 2 gets 4.

Stage 2. If 1 chooses  $a$  at stage 2, then: (i) by choosing  $a$ , 2 stops the procedure and gets 5; and (ii) by choosing  $b$ , 2 forces the procedure to enter stage 3 and gets 4. If 1 chooses  $b$  at stage 2, then: (i) by choosing  $b$  next individual 2 stops the procedure and gets 7; and (ii) by choosing next  $a$ , 2 makes the procedure enter stage 3 and gets 4. In both cases individual 2 agrees with the decision of 1.

If at the stage 2 the individual 1 chooses  $a$ , then, since 2 will choose also  $a$  as show above, 1 gets 8. If 1 chooses  $b$ , then, since, as just shown, 2 will choose  $b$  as well, 1 gets 10. Consequently, the best choice for 1 at stage 2 is  $b$  and the outcome at the stage 2 is  $(b, 2)$ .

Stage 1. If 1 chooses  $a$  at stage 2, then: (i) by choosing  $a$ , 2 stops the procedure and gets 6; and (ii) by choosing  $b$ , 2 forces the procedure to enter stage 2 and gets 7. If 1 chooses  $b$  at stage 1, then: (i) by choosing  $b$  next individual 2 stops the procedure and gets 8; and (ii) by choosing next  $a$ , 2 makes the procedure enter stage 2 and gets 7. In both cases individual 2 votes for  $b$ .

If 1 chooses  $a$ , then he makes the procedure to enter next stage and gets 10; if 1 chooses  $b$  then he gets 11. Consequently, the best choice for 1 at stage  $t = 1$  is  $b$ .

The example shows that if the Persistence is dropped at least for one stage for the individual 1, then assuming Impatience, Reversion, and Termination the outcome of the only subgame perfect equilibrium of the sequential voting procedure is  $(\beta_1, 1)$ .

**4.2 Example 2. Dropping Impatience.**

Consider the utility functions from Table 2. Impatience does not hold for individual 2 at stages 1-2:  $u_2(a, 1) < u_2(b, 1)$ . At the same time it is clear that Persistence, Reversion, and Termination hold.

**Table 2**

Stage $t$	Individual 1		Individual 2	
	$u_1(a, t)$	$u_1(b, t)$	$u_2(a, t)$	$u_2(b, t)$
1	8	6	4	9
2	7	5	6	8
3	4	3	3	7
4	1	2	1	2
$\omega$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

Stage 3. By analogy with the previous example: the outcome is  $(a, 3)$ : individual 1 gets 4 and individual 2 gets 3.

Stage 2. If 1 chooses  $a$  at stage 2, then: (i) by choosing  $a$ , 2 stops the procedure and gets 6; and (ii) by choosing  $b$ , 2 forces the procedure to enter stage 3 and gets 3. If 1 chooses  $b$  at stage 2, then: (i) by choosing  $b$  next individual 2 stops the procedure and gets 8; and (ii) by choosing next  $a$ , 2 makes the procedure enter next stage and gets 3. In both cases the best choice for individual 2 is to agree with 1.

If 1 chooses  $a$ , then he gets 7; if 1 chooses  $b$ , then he gets 5. Consequently, the best choice for 1 at stage 2 is  $a$  and the outcome at the stage 2 is  $(a, 2)$ .

Stage 1. If 1 chooses  $a$  at stage 2, then: (i) by choosing  $a$ , 2 stops the procedure and gets 4; and (ii) by choosing  $b$ , 2 forces the procedure to enter stage 2 and gets 6. If 1 chooses  $b$  at stage 1, then: (i) by choosing  $b$  next individual 2 stops the procedure and gets 9; and (ii) by choosing next  $a$ , 2 makes the procedure enter stage 2 and gets 6. In both cases the best choice for individual 2 is to vote for  $b$ .

Since 2 anyway chooses  $b$ , if 1 chooses  $a$ , then he makes the procedure enter next stage and gets 7; if 1 chooses  $b$  he gets 6. Consequently, the best choice for 1 at stage 1 is to vote for  $a$  and therefore to pass to the next stage.

Assuming Persistence, Reversion, and Termination the outcome of the only subgame perfect equilibrium of the sequential voting procedure is  $a_1$  reached at stage 2.

Consider the following utility functions presented in Table 3.. For individual 2 the Impatience is not hold for the stages 2-3. Simultaneously, Persistence, Reversion, and Termination hold for both voters and all observed stages.

**Table 3**

Stage	Individual 1	Individual 2
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$t$	$u_1(a, t)$	$u_1(b, t)$	$u_2(a, t)$	$u_2(b, t)$
1	8	7	4	9
2	6	5	6	8
3	3	2	2	3
$\infty$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

Stage 2. By analogy with the previous examples it is obtained that at stage 2 the outcome is  $(a, 3)$ : individual 1 gets 6 and individual 2 gets 6.

Stage 1. If 1 chooses  $a$  at stage 2, then: (i) by choosing  $a$ , 2 stops the procedure and gets 4; and (ii) by choosing  $b$ , 2 forces the procedure to enter stage 3 and gets 6. If 1 chooses  $b$  at stage 2, then: (i) by choosing  $b$  next individual 2 stops the procedure and gets 8; and (ii) by choosing next  $a$ , 2 makes the procedure enter next stage and gets 6. The best choice for individual 2 is to choose  $b$  regardless of the decision of individual 1. If 1 chooses  $a$ , then he makes the procedure to enter the next stage and he gets 6. If 1 chooses  $b$ , then he gets 7 at current stage. Consequently, the best choice for 1 at stage 2 is  $b$ .

This example shows that assuming Persistence, Reversion, and Termination the outcome of the only subgame perfect equilibrium of the sequential voting procedure is  $(b, 1)$ .

### 4.3 Example 3. Dropping Reversion.

Assume that Reversion does not hold for individual 2. It means that there is not even one stage when the utility of his non preferred alternative is greater than the utility of his preferred alternative at the next stage. Consider the following utility functions, see Table 3. It is clear that Persistence, Impatience, and Termination hold for both voters. Assume that the Reversion does not hold for any stage for individual 2.

The dropping of Reversion makes individual 2 always vote for his preferred alternative  $b$ . If 1 does not vote for the same alternative, then the procedure reaches  $\emptyset$  outcome. Knowing this, individual 1 has incentives to vote  $b$ , stop the procedure, and get something yet greater than  $\emptyset$ . Since Persistence, Impatience, and Termination do hold for both voters, by backward induction it is obtained that the outcome of the only subgame perfect equilibrium of the sequential voting procedure is  $(a, 1)$ .

### 4.4 Example 4. Dropping Termination.

Assume that the Termination is dropped for individual 2. He always has a possibility to disagree with the individual 1 and to force the procedure to pass to the next stage, or in the other words, to obtain the outcome  $(\emptyset)$ . Consider the following utility functions:

**Table 4.**

Stage $t$	Individual 1		Individual 2	
	$w_1(a, t)$	$w_1(b, t)$	$w_2(a, t)$	$w_2(b, t)$
1	6	5	8	13
2	4	2	5	7
	$\emptyset$	$\emptyset$	10	

At the same time Persistence, Impatience, and Reversion hold for both voters.

Stage 1. If 1 chooses  $a$ , then: (i) by choosing  $a$ , 2 stops the procedure and gets 8; and (ii) by choosing  $b$ , 2 forces the procedure to enter stage  $2$  and as it is shown above, he can obtain the 10 from the  $(\emptyset)$  outcome. Consequently, the best choice is  $b$ .

If 1 chooses  $b$  at stage  $2$ , then: (i) by choosing  $b$  next individual 2 stops the procedure and gets 13; and (ii) by choosing next  $a$ , 2 makes the procedure enter next stage and so on until to get 10. The best choice for individual 2 is to choose  $b$  if 1 chooses  $b$ .

Since 2 anyway chooses  $b$ , if 1 chooses  $a$ , then he makes the procedure to enter the next stage and so on until he gets  $\emptyset$ . If 1 chooses  $b$ , then he gets 5. Consequently, the best choice for 1 at stage  $2$  is  $b$ .

Assuming Persistence, Impatience, and Reversion, the outcome of the only subgame perfect equilibrium of the sequential voting procedure is  $(\beta_{11})$ .

To recap all the example above, it has been shown that if one of the four assumptions is dropped at least for one voter, while the other assumption hold for both voters, the proposition does not hold and the outcome of the only subgame perfect equilibrium of the sequential voting procedure can be  $(\alpha_{11})$ ,  $(\alpha_1, \delta)$ , or  $(\alpha_1, \delta)$ .

## 5 Concluding remarks

The result in this paper disproves the previous natural intuition that the most patient voter wins the voting procedure. In other words, being more patient does not guarantee the victory of your preferred alternative. In the present framework, what really matters is the order of the voters.

It is easy to provide real-life examples of the fact that the first one who takes the initiative wins. Consider a situation of two players: a firm and a labor union in case of strike. If the strike happens already, it is to tell that the union sets up some claims to the enterprise. Though it seems that the enterprise is more patient, disposing more resources, in case of strike it is likely that the enterprise agrees with the claims of the union. In case when the strike is about to happen the enterprise has the power of the first move: it can prevent a strike by making an offer to the workers that will not be

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rejected. This offer is less generous than the offer in case when strike already happened.

Since the result states that the first who proposes wins, the rivals may be more interested in voting first than in being more patient. For example, consider two parties making a coalition, so they have to choose which of their leaders will be the new head. The key is who will propose first his candidature.

If the order of the voters is not exogenous and defined by the voters themselves, then it is likely that the most extreme voter would take the initiative and votes first. Therefore, in this case the result in some degree coincides with Compte and Jehiel [2] and Rodríguez-Álvarez and Rivas [9].

The model can be extended along at least two directions. First, to modify existing elements of the model: to have any number of voters, more than two alternatives, to introduce explicitly a final stage, etc. For instance, the case when the number of alternatives is equal to the number of voters can be seen as a committee forced to choose one of them as a chairman. Second, to modify the procedure. The most natural extension is not to fix the order of the voters and make it random at each stage. The presence of bribing seems to be challenging and promising. The bribing can be presented in different ways: utility transfer between the voters, direct payments, increasing the probability to vote first, etc. The extension to  $n$  voters allows to see what happens if coalitions are allowed; or what happens if the unanimity is not required. Maybe in this case the veto power of the individuals is reduced and the impatience degree matters.

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