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Unilateral Effects Screens for Partial Horizontal Acquisitions: The Generalized HHI and GUPPI*

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Abstract

Recent years have witnessed an increased interest, by competition agencies, in assessing the competitive effects of partial acquisitions. We propose a generalization to a partial horizontal acquisition setting of the two most traditional indicators used to screen unilateral anti-competitive effects: the Helfindahl-Hirschman Index and the Gross Upward Price Pressure Index. The proposed generalized indicators can deal with all types of acquisitions that may lessen competition in the industry: acquisitions by owners that are internal to the industry (rival firms) and engage in cross-ownership, as well as acquisitions by owners that are external to the industry and engage in common-ownership. Furthermore, these indicators can deal with direct and indirect acquisitions, which may or may not correspond to control, and nest full mergers as a special case. We provide an empirical application to several acquisitions in the wet shaving industry. The results seem to suggest that (i) a full merger induces higher unilateral anti-competitive effects than a partial controlling acquisition involving the same firms, (ii) a partial controlling acquisition induces higher unilateral anti-competitive effects than a partial non-controlling acquisition involving the same firms and the same financial stakes, and (iii) an acquisition by owners that are internal to the industry induces higher unilateral anti-competitive effects than an acquisition (involving the same firms and the same stakes) by external owners that participate in more than one competitor firm.

JEL Classification: L13, L41, L66

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1 Introduction

Full acquisitions complete and permanently eliminate competition among the firms involved in the transaction. This constitutes the basic element of a merger analysis. In contrast, *partial acquisitions* do not completely and permanently eliminate competition among firms. Nevertheless, they may present significant competitive concerns and, as a consequence, competition agencies have taken an increased interest in assessing their competitive effects.

Following the long theoretical literature in industrial organization, agencies have typically focused on partial acquisitions that give rise to a *cross-ownership* structure, i.e., acquisitions by owners that are *internal* to the industry, rival firms. Some recent examples include the UK Competition Commission assessment of the BskyB's proposed acquisition of a 17.9% stake in ITV and the European Commission assessment of the News Corporation's proposed acquisition of an approximately 25% stake in Premiere.

However, the phenomenal growth of private equity investment in recent years has led agencies to focus also on partial acquisitions that give rise to a *common-ownership* structure, i.e., acquisitions by owners that are *external* to the industry, but participate in more than one competitor firm. A recent example includes the FTC assessment of the Kinder Morgan buyout by (among others) private equity funds managed and controlled by the Carlyle Group and Riverstone Holdings LLC, which already hold a significant partial ownership position in Magellan Midstream, a major competitor of Kinder Morgan.

The competitive effects of partial acquisitions giving rise to cross-ownership or common-ownership structures depend heavily on whether the ownership right involved in the acquisition is a *financial* or a *corporate control* interest. The former refers to the right of the (partial) owner to receive the stream of profits generated by the operations and investments of the acquired firm, while the latter refers to the right of the (partial) owner to influence the decisions of the target firm. We need to identify and distinguish the two rights because partial horizontal acquisitions that do not result in effective control present competitive concerns distinct from partial acquisitions involving effective control. When a party (internal or external to the industry) acquires a partial financial interest in a firm, it acquires a share of its profits. Such acquisition can lessen competition by reducing the incentive of the acquiring party to compete aggressively because it shares in the losses thereby inflicted on that rival. On the other hand, when a party (internal or external to the industry) acquires corporate control in a firm, it acquires the ability to influence the competitive conduct of the target firm. Such influence can lessen competition because it may be used to induce the rival to compete less aggressively against the acquiring party.

Brito *et al.* (2014a) propose an empirical structural methodology to quantitatively assess the unilateral competitive effects of partial horizontal acquisitions. However, competition agencies are typically given a very short period to analyze a potential acquisition upon receiving its notification, with little data available before deciding whether to issue a second request. In this paper, we propose a generalization to a partial horizontal acquisition setting of the two most traditional indicators used to screen unilateral anti-competitive effects: the *Helfindahl-Hirschman Index* (HHI), typically suitable for Cournot homogeneous-product industries, and the *Gross Upward Price Pressure Index* (GUPPI), typically suitable for Bertrand differentiated-product industries.

The proposed generalized indicators bypass the demand estimation required by a structural methodology and can be computed with the data submitted in a typical notification to the competition agency. Furthermore, they can deal with acquisitions that give rise to either a cross-ownership structure, or a common-ownership structure, or both. Moreover, they can also deal with *direct* and *indirect* acquisitions of either financial interests, or corporate control, or both.[1] This issue is particularly important for antitrust purposes because indirect partial ownership interests may constitute a way of evading antitrust rules that limit direct ownership in rivals. Finally, the proposed generalized indicators nest full mergers as a special case, retrieving the standard HHI and GUPPI typically used in merger simulation.

We also provide an empirical application of the two proposed generalized indicators to several acquisitions in the wet shaving industry. On December 20, 1989, the Gillette Company, which had been the market leader for years and accounted for 50% of all razor blade units sales, contracted to acquire the wet shaving businesses of Wilkinson Sword in the United States (among other operations) to Eemland Management Services BV (Wilkinson Sword's parent company) for $72 million. It also acquired a 22.9 percent of the nonvoting equity shares of Eemland for about $14 million. On January 10, 1990, the Department of Justice (DoJ) instituted a civil proceeding against Gillette. The complaint alleged that the effect of the acquisition by

[1] An owner has an indirect partial ownership interest in firm B if it holds a partial ownership interest in firm A and, in turn, firm A holds a partial ownership interest in firm B.

Gillette may have been substantially to lessen competition in the sale of wet shaving razor blades in the United States. Shortly after the case was filed, Gillette voluntarily rescinded the acquisition of Eemland's wet shaving razor blade business in the United States, but went through with the acquisition of 22.9% *nonvoting* equity interest in Eemland. The DoJ approved the acquisition after being assured that this stake would be passive. These two acquisitions (one involving a partial interest and another a full merger), and two additional hypothetical ones, are screened below. The results seem to suggest that (i) a full merger induces higher unilateral anti-competitive effects than a partial controlling acquisition involving the same firms, (ii) a partial controlling acquisition induces higher unilateral anti-competitive effects than a partial non-controlling acquisition involving the same firms and the same financial stakes, and (iii) an acquisition by owners that are internal to the industry induces higher unilateral anti-competitive effects than an acquisition (involving the same firms and the same stakes) by external owners that participate in more than one competitor firm.

This paper is organized as follows: Section 2 reviews the literature, Section 3 presents the theoretical framework, Section 4 develops the two proposed generalized indicators, Section 5 provides the above mentioned empirical application and Section 6 concludes.

2 Literature Review

This section reviews the literature relative to screening indicators for partial horizontal acquisitions within Cournot homogeneous- and Bertrand differentiated-product industries. Table 1 summarizes the schematic of the literature relative to both settings, according to the types of owners and the nature of the partial acquisition.

2.1 Cournot Homogeneous-Product Industries

The literature on unilateral effects screening indicators for partial horizontal acquisitions within Cournot homogeneous-product industries began with Reynolds and Snapp (1986). They examine the impact of acquisitions that give rise to a *cross-ownership* structure of *direct financial interests*. They show that, in markets where entry is difficult, cross-shareholding by rival firms involving partial financial interests (even if relatively small) could result in lower equilibrium market output and higher equilibrium market prices. They propose to screen such effects using a modified HHI. However, their proposal can not screen acquisitions of corporate control neither of indirect stakes nor of common-ownership by owners that are external to the industry. Bresnahan and Salop (1986) build on Reynolds and Snapp (1986) by introducing the distinction between financial interest and corporate control. They consider different direct financial and *corporate control* cross-shareholding arrangements and propose a set of modified HHIs to screen the unilateral effects of each of those alternative arrangements. However, their proposal can not address acquisitions of all types of corporate control arrangements, neither of indirect stakes nor of common-ownership by owners that are external to the industry. Flath (1992) builds on Bresnahan and Salop (1986) and extends the analysis by treating the more general case in which *indirect* partial cross-ownership interests are also present. However, to do so, he focus on acquisitions that give rise to a cross-ownership structure of financial interests. Further, he does not propose indicators to screen whether the analyzed acquisitions lead to unilateral anti-competitive effects. Dietzenbacher *et al.* (2000) build on Flath (1992) and propose a modified HHI to screen acquisitions that give rise to a cross-ownership structure of *direct* and *indirect* financial interests. However, their proposal can not address acquisitions of corporate control arrangements, neither of common-ownership by owners that are external to the industry. O'Brien and Salop (2000) extend Bresnahan and Salop (1986)'s modified HHI to a richer set of financial interest and corporate control scenarios. However, to do so, they focus only on direct acquisitions that involve owners that are external to the industry, i.e., they screen the unilateral effects of acquisitions that give rise to a *common-ownership* structure of direct interests. Their proposal can not screen acquisitions of indirect stakes neither of cross-ownership by rival firms. We propose to extend the literature by deriving generalizations of both the standard and the modified HHI to screen whether partial horizontal acquisitions lead to unilateral anti-competitive effects for settings involving all types of owners (internal and external to the industry), and acquisitions (direct and indirect, involving control or not, partial or full).

2.2 Bertrand Differentiated-Product Industries

The literature on unilateral effects screening indicators for partial horizontal acquisitions within Bertrand differentiated-product industries began with O'Brien and Salop (2000). They examine the impact of acquisitions that give rise to a *common-ownership* structure of *direct financial* and *corporate control* interests. They do so building on Shapiro (1996)'s diversion ratio approach.

TABLE 1

*Literature Schematic**

	Cross Ownership	Common Ownership	Both Types
Panel A: Cournot Homogeneous-Product Industries			
Direct FI	Reynolds and Snapp (1986)	–	–
Direct FI and CC	Bresnahan and Salop (1986)†	O'Brien and Salop (2000)	–
Direct and Indirect FI	Flath (1992)‡ DSV (2000)	–	–
Direct and Indirect FI and CC	–	–	Current Paper
Panel B: Bertrand Differentiated-Product Industries			
Direct FI and CC	–	O'Brien and Salop (2000)◇	–
Direct and Indirect FI	DSV (2000)▷	–	–
Direct and Indirect FI and CC	–	–	Current Paper

* FI and CC denote Financial Interest and Corporate Control, respectively. DSV (2000) denotes Dietzenbacher et al. (2000). † Bresnahan and Salop (1986) model direct FI and CC, but can not address all types of CC arrangements. ‡ Flath (1992) model direct and indirect FI, but does not propose an indicator. ◇ O'Brien and Salop (2000) model direct FI and CC and propose a PPI, a measure which lead afterwards to the GPPUI, but was never generalized to partial horizontal acquisition settings. ▷ DSV (2000) model direct and indirect FI, but do not propose an indicator.

They screen the unilateral effects of such partial ownership interests using a summary measure of the economic pressure to change prices in response to a change in the direct financial and corporate control cross-shareholding arrangements. They refer to this measure as a Price Pressure Index (PPI). However, their proposal can not address acquisitions of indirect stakes neither of cross-ownership by rival firms. Dietzenbacher *et al.* (2000), on the other hand, build on Flath (1992) and examine the impact of acquisitions that give rise to a *cross-ownership* structure of *financial* and *corporate control* interests, both *direct* and *indirect.* However, they do not propose indicators to screen whether the analyzed acquisitions lead to unilateral anti-competitive effects. Farrel and Shapiro (2010) build on O'Brien and Salop (2000)'s PPI to develop an Upward Pricing Pressure (UPP) test to screen the unilateral anti-competitive effects of mergers. A test that gave rise, afterwards, to the GUPPI, proposed by Salop and Moresi (2009). However, to the best of our knowledge, it was never generalized to partial horizontal acquisition settings.[2] We propose to extend the literature by deriving generalizations of the standard GPPUI to screen whether partial horizontal acquisitions lead to unilateral anti-competitive effects for settings involving all types of owners (internal and external to the industry), and acquisitions (direct and indirect, involving control or not, partial or full).

3 The Theoretical Framework

This section introduces the theoretical framework under which the partial horizontal acquisitions' screening indicators (in the context of both Cournot homogeneous-product and Bertrand differentiated-product industries) are derived. The general setting is adapted from Brito *et al.* (2014a) to cope with acquisitions that give rise to a common-ownership structure (and are not explicitly addressed in Brito *et al.*, 2014a's application).

3.1 The Setup

There are N single-product firms, indexed by $j \in \Im \equiv \{1, ..., N\}$. There are also K owners, indexed by $k \in \Theta \equiv \{1, \ldots, K\}$, who may include not just owners $\Theta \backslash \Im$ that are external to the

[2]The only exception, although following a different nature of indicator, is Brito *et al.* (2014b). They build on both O'Brien and Salop (2000) and Dietzenbacher *et al.* (2000) to extend the analysis to settings involving aquisitions that give rise to both a *cross-* and a *common-ownership* structure of *financial and corporate control interests*, either *direct or indirect.* They propose sufficient statistics for the effects of partial ownership (and divestiture of partial ownership) on consumer welfare. However, they do so only for a duopoly setting.

industry (and can engage in common-ownership), but also owners from the subset $\Im$ of firms that are internal to the industry (and can engage in cross-ownership).[3]

As discussed above, the competitive effects of partial acquisitions giving rise to cross-ownership or common-ownership structures depend heavily on whether the ownership right involved in the acquisition is a financial or a corporate control interest. In order to capture the distinction between these two rights, we consider that the total stock of each firm j is composed of voting stock and non-voting (preferred) stock, with the latter giving the holder a share of the profits but no right to vote for the Board or to participate in other decisions.

The degree of financial interest of owner k in firm j is represented by $0 \leq \phi_{kj} \leq 1$, with $\sum_{k\in\Theta} \phi_{kj} = 1$, which denotes the shareholder's holdings of *total stock* in the firm, regardless of whether it be voting or non-voting stock. The degree of corporate control of owner k over the decision making of firm j is denoted by $0 \leq \gamma_{kj} \leq 1$, with $\sum_{k\in\Theta} \gamma_{kj} = 1$, a measure of corporate control that will be a function of the owner's holdings of *voting stock* in the firm. The larger the holdings of voting stock in a firm, the greater the degree of control over the decision making will typically be. However the relationship may not necessarily be linear. For instance, an owner holding 49% of voting stock in a firm may have no control over the decision making of the firm if one other owner holds 51%. In contrast, an owner holding 10% of voting stock in a firm may have effective control over the decision making of the firm if each of the remaining owners hold a tiny amount of voting stock.

3.2 Firm's Aggregate Profit

We model financial cross-ownership among rival firms in the lines of Reynolds and Snapp (1986), Bresnahan and Salop (1986), Flath (1992), and Dietzenbacher *et al.* (2000). To do so, we distinguish between a firm's *operating* and *aggregated* profit. The reason being that, in an industry characterized by cross-ownership among rival firms, the aggregate profit of a firm includes not just the stream of profits generated by the firm's own operations, but also a share in its rivals' aggregate profits due to the ownership stake in these firms.

Let π_j and Π_j denote the operating and the aggregated profit of firm $j \in \Im$, respectively.

[3] The set $\Theta \backslash \Im$ denotes the set Θ excluding the firms in $\Im$.

The operating profit, π_j, is generated by the firm's own operations and can be written as follows:

$$\pi_j = (p_j - mc_j)\, q_j - c_j, \tag{1}$$

where p_j, mc_j, q_j, and c_j denote the price, the (assumed constant) marginal cost, the quantity, and the fixed cost, respectively, of firm j. However, the aggregated profit of firm j includes not just π_j, but also a share in the aggregate profits of all the rivals in which firm j has an ownership stake. We make the following assumption regarding the distribution of those profits among rivals:

Assumption 1 *Each firm's aggregate profit is distributed among owners proportionally to the total stock owned, regardless of whether it be voting or non-voting stock.*

Under Assumption 1, firm j receives a profit stream from its ownership stake in firm g that corresponds to the percentage ϕ_{jg} of firm g's total stock owned. Therefore, the aggregate profit of *each* firm $j \in \Im$ can be written as follows:

$$\Pi_j = \pi_j + \sum\nolimits_{g \in \Im \backslash j} \phi_{jg} \Pi_g, \tag{2}$$

where the second term denotes the returns on the cross-holdings of firm j in all other rival firms.[4]

The set of the above N aggregate profit equations implicitly determines the aggregate profit of each firm as a function of the operating profits of *all* the firms in the industry over which the firm has (direct or indirectly) a financial stake on. In order to see why this is the case, let $\mathbf{F}^*$ denote the $N \times N$ *financial* cross-ownership matrix with zero diagonal elements, $\phi_{jj} = 0$, and off-diagonal elements $0 \leq \phi_{jg} \leq 1$ (if $j \neq g \in \Im$) representing the percentage held by firm j on firm g's total stock. Note that the specification allows for the special case of full acquisitions, which just corresponds to setting $\phi_{jg} = 1$ and $\gamma_{jg} = 1$ for $j \neq g$. In vector notation, the aggregate profit equations become:

$$\mathbf{\Pi} = \boldsymbol{\pi} + \mathbf{F}^* \mathbf{\Pi}, \tag{3}$$

where $\mathbf{\Pi}$ and $\boldsymbol{\pi}$ are $N \times 1$ vectors of aggregate and operating profits, respectively. In order to

[4] The set $\Im \backslash j$ denotes the set $\Im$ not including firm j.

solve for those profits explicitly, we make the following assumption regarding the cross-ownership financial structure of the firms in the market:

Assumption 2 *The rank of* $(\mathbf{I} - \mathbf{F}^*)$ *equals the number of firms in the market.*

Under Assumption 2, matrix $(\mathbf{I} - \mathbf{F}^*)$ is invertible, which implies it is possible to solve for the aggregate profit equation in terms of the vector of operating profits:

$$\mathbf{\Pi} = (\mathbf{I} - \mathbf{F}^*)^{-\mathbf{1}} \boldsymbol{\pi}, \tag{4}$$

where $\mathbf{I}$ denotes a $N \times N$ identity matrix.

3.3 Manager's Objective Function

We model corporate control cross-ownership, as well as financial and corporate control common-ownership in the lines of O'Brien and Salop (2000). To do so, we make the following assumption regarding the objective of the manager of the firm:

Assumption 3 *The manager of the firm maximizes a (control) weighted sum of the owners returns.*

In a standard oligopoly model with no partial ownership interests, barring any market imperfections that preclude efficient contracting between the owners and the manager, the former will typically agree (and give the appropriate incentives) that the latter should maximize profits. However, in a common-ownership setting, owners may have conflicting interests and, therefore, may not agree on the best course of action for the firm. As O'Brien and Salop (2000) argue, an owner of firm j who also has a large financial interest in rival firm g typically wants firm j to pursue a less aggressive strategy than the strategy desired by an owner with no financial interest in firm j. In this situation, the manager must weight the conflicting interests of the different owners according to the corporate-control structure of the firm, which determines each owner's influence over decision-making within the firm.

Assumption 3 considers that the owners' interests are captured by their corresponding returns and, as such, the objective function of the manager of firm j can be written as follows:

$$\varpi_j = \sum\nolimits_{k \in \Theta/j} \gamma_{kj}^* R_k, \tag{5}$$

where γ_{kj}^* measures the *normalized* degree of control of owner k over the manager of firm j (to be discussed below), and R_k is the return of owner k.[5] Furthermore, it allows for a wide variety of plausible corporate-control structures. Under this formulation, a higher weight on the return of a particular owner is associated with a greater degree of influence by that owner over the manager. Different control scenarios then correspond to different sets of control weights for the different owners.

The return of owner $k \in \Theta$ varies depending on whether that owner is internal or external to the industry. If the owner is external to the industry, $k \notin \Im$, we can assume it only cares about the returns of equity holding in the different rival firms.[6] If, on the other hand, the owner is internal to the industry, $k \in \Im$, we assume it cares about a control weighted sum of the owners equity holdings' returns. In short, the return of owner k can be written as follows:[7]

$$R_k = \begin{cases} \sum_{g \in \Im} \phi_{kg} \Pi_g & \text{if } k \notin \Im \\ \varpi_k & \text{if } k \in \Im \end{cases}. \tag{6}$$

The *normalization* of the degree of control of each owner over the manager of the firm is required so that the returns of the internal owners can be compared to those of the external owners.[8] It is given by:

[5] Without loss of generality, we assume the firm does not constitute itself as a owner, which translates into the set Θ/j (that denotes the set Θ not including firm j). Some firms do possess own shares. However, because a firm's interests are ultimately their owners interests, in these cases, the control weight of those shares is ultimately distributed among the owners according to their corresponding control weight.

[6] Of course, external owners care about the returns of their entire portfolio of holdings, not only those relative to the industry. However, for the purposes of evaluating the unilateral anti-competitive effects of acquisitions, only the latter are relevant.

[7] Because an internal owner's interests represent ultimately external owner's interests, it is important to note that the sum of all external onwers returns is indeed equal to the sum of the firms operational profits. For all $k \in \Theta$, we have that: $\sum_{k \notin \Im} R_k = \sum_{k \notin \Im} \sum_{g \in \Im} \phi_{kg} \Pi_g = \sum_{g \in \Im} \sum_{k \notin \Im} \phi_{kg} \Pi_g = \sum_{g \in \Im} \left(1 - \sum_{k \in \Im} \phi_{kg}\right) \Pi_g = \sum_{g \in \Im} \Pi_g - \sum_{g \in \Im} \sum_{k \in \Im} \phi_{kg} \Pi_g = \mathbf{1}'\mathbf{\Pi} - \mathbf{1}'\mathbf{F}^*\mathbf{\Pi} = (\mathbf{1}' - \mathbf{1}'\mathbf{F}^*)\,\mathbf{\Pi} = \mathbf{1}'\left(\mathbf{I} - \mathbf{F}^*\right)\left(\mathbf{I} - \mathbf{F}^*\right)^{-1} \boldsymbol{\pi} = \mathbf{1}'\boldsymbol{\pi} = \sum_{g \notin \Im} \pi_g$, where $\mathbf{1}$ denotes a $(N \times 1)$ vector of ones.

[8] As an illustration consider a firm with two owners (both external): one with a 40% stake and another with a 60% stake (with both stakes involving financial and corporate control). Assuming that no normalization is imposed, the manager of the firm weights the returns of the two owners (according to their control stake): $\varpi = 0.4R_1 + 0.6R_2$. If we assume further that the two external owners do not engage in common-ownership, the objective function of the manager is given by: $\varpi = 0.4\,(0.4\Pi) + 0.6\,(0.6\Pi) = 0.52\Pi$. This would suggest that the manager only cares about 52% of the firm's aggregated profit, which is not correct. Absent common-ownership, both owners agree that the manager should maximize the full aggregated profits. For maximization purposes, the 0.52 weight is not relevant, but if this objective function enters the objective function of a rival manager, the

$$\gamma_{kj}^{*} = \begin{cases} \left(\frac{1-\sum_{m\in\Im/j}\gamma_{mj}}{\sum_{\substack{m\in\Theta \\ m\notin\Im}}\gamma_{mj}\phi_{mj}} \right)\gamma_{kj} & \text{if } k \notin \Im \wedge \sum_{\substack{m\in\Theta \\ m\notin\Im}}\gamma_{mj}\phi_{mj} > 0 \\ \gamma_{kj} & \text{otherwise} \end{cases} . \tag{7}$$

Substituting equation (5) into equation (6) yields that the objective function of the manager of *each* firm $j \in \Im$ can, in fact, be written as follows:

$$\varpi_j = \sum\nolimits_{k\in\Im/j} \gamma_{kj}\varpi_k + \left(\frac{1-\sum_{m\in\Im/j}\gamma_{mj}}{\sum_{\substack{m\in\Theta \\ m\notin\Im}}\gamma_{mj}\phi_{mj}} \right) \sum\nolimits_{\substack{k\in\Theta \\ k\notin\Im}} \gamma_{kj} \sum\nolimits_{g\in\Im} \phi_{kg}\Pi_g, \tag{8}$$

where the first term involves owners that are internal to the industry and the second term involves owners that are external to the industry.

The set of the above N objective function equations implicitly determines the objective function of each firm's manager as a function of the operating profits of *all* the firms over which the owners of the firm have (direct or indirectly) a financial and/or a corporate stake on. In order to see why this is the case, let $\mathbf{C}^*$ denote the $N \times N$ *control* cross-ownership matrix with zero diagonal elements, $\gamma_{jj} = 0$, and off-diagonal elements $0 \leq \gamma_{jg} \leq 1$ (if $j \neq g$) representing the measure of firm j's degree of control over the manager of firm g. Let also $\mathbf{F}$ and $\mathbf{C}$ denote the $(K-N) \times N$ *financial* and *control* common-ownership matrices with typical element ϕ_{kj} and γ_{kj}, respectively.[9] Finally, let $\mathbf{A1} = diag\left(\mathbf{C}^{\intercal}\mathbf{F}\right)$ denote the $N \times N$ diagonal matrix (with diagonal elements a_{jj}^1) formed by substituting zeros for all off-diagonal elements of $\mathbf{C}'\mathbf{F}$, and let $\mathbf{A2}$ denote the $N \times N$ diagonal matrix (with diagonal elements a_{jj}^2) formed with the elements of $(\mathbf{1} - \mathbf{C}^{*\intercal}\mathbf{1})$, where $\mathbf{1}$ denote a $N \times 1$ vector of ones.

In vector notation, the objective function equations become:

$$\boldsymbol{\varpi} = \mathbf{C}^{*\intercal}\boldsymbol{\varpi} + \mathbf{B}\mathbf{C}^{\intercal}\mathbf{F}\boldsymbol{\Pi}, \tag{9}$$

where $\boldsymbol{\varpi}$ denotes the $N \times 1$ vector of the managers objective functions, and $\mathbf{B}$ denotes the

weight must be normalized to its true value. That is the purpose of the normalization.

[9] Note that both $\mathbf{F}$ and $\mathbf{C}$ matrices are defined only in terms of the set of owners external to the industry. The interests of the set of owners that are internal to the industry are taken into account in matrices $\mathbf{F}^*$ and $\mathbf{C}^*$.

$N \times N$ normalization diagonal matrix with diagonal elements, b_{jj}, given by:

$$b_{jj} = \begin{cases} a_{jj}^2/a_{jj}^1 & \text{if } a_{jj}^1 > 0 \\ 1 & \text{if } a_{jj}^1 = 0 \end{cases} . \tag{10}$$

In order to solve for the N objective function explicitly, we make the following assumption regarding the cross-ownership control structure of the firms in the market:

Assumption 4 *The rank of* $(\mathbf{I} - \mathbf{C}^{*\intercal})$ *equals the number of firms in the market.*

Under Assumption 4, matrix $(\mathbf{I} - \mathbf{C}^{*\intercal})$ is invertible, which implies it is possible to solve for the objective function equation in terms of the vector of operating profits:

$$\boldsymbol{\varpi} = (\mathbf{I} - \mathbf{C}^{*\intercal})^{-\mathbf{1}} \mathbf{B}\mathbf{C}^{\intercal}\mathbf{F}\boldsymbol{\Pi} = (\mathbf{I} - \mathbf{C}^{*\intercal})^{-\mathbf{1}} \mathbf{B}\mathbf{C}^{\intercal}\mathbf{F} (\mathbf{I} - \mathbf{F}^{*})^{-1} \boldsymbol{\pi} (\mathbf{p}) , \tag{11}$$

where $\mathbf{I}$ denotes the identity matrix and the second equality is obtained by simple substitution of the aggregate profit equation (4). This result implies, as discussed above, that the objective function of the manager of *each* firm j is entirely equivalent to a weighted sum of the operating profits of *all* the firms in the industry over which the owners of firm j have (direct or indirectly) a financial and/or a corporate stake on:

$$\varpi_j = \sum\nolimits_{k \in \Theta} \gamma_{kj} R_k = \sum\nolimits_{g \in \Im} l_{jg} \pi_g, \tag{12}$$

where l_{jg} denotes a weight that depends on the financial interest and the corporate control that firm j holds over firm g. In particular, l_{jg}, denotes the typical element of the $N \times N$ matrix **L:**

$$\mathbf{L} = (\mathbf{I} - \mathbf{C}^{*\intercal}\mathbf{C}^{*\intercal})^{-\mathbf{1}} (\mathbf{C}^{*\intercal}\mathbf{B} + \mathbf{C}^{\intercal}\mathbf{F}) (\mathbf{I} - \mathbf{F}^{*})^{-1} \tag{13}$$

The weights l_{jg}, for any $j, g \in \Im$, capture the two dimensions of partial ownership: financial interest (represented in matrices $\mathbf{F}^*$ and $\mathbf{F}$) and corporate control (represented in matrices $\mathbf{C}^*$ and $\mathbf{C}$), as well as the two acquisition settings: cross-ownership (represented in matrices $\mathbf{F}^*$ and $\mathbf{C}^*$) and common-ownership (represented in matrices $\mathbf{F}$ and $\mathbf{C}$). Without loss of generality, we normalize the weight on the own-operating profit to be one by dividing the objective function of the manager of each firm $j \in \Im$ by l_{jj}. This implies that the manager of firm $j \in \Im$ maximizes the following, entirely equivalent, objective function:

$$\varpi_j' = \sum\nolimits_{g \in \Im} \frac{l_{jg}}{l_{jj}} \pi_g = \sum\nolimits_{g \in \Im} w_{jg} \pi_g, \tag{14}$$

where $w_{jg} = l_{jg}/l_{jj}$ for any $j, g \in \Im$ denotes the typical element of the $N \times N$ *normalized* weight matrix $\mathbf{W}$:

$$\mathbf{W} = diag\,(\mathbf{L})^{-1}\,\mathbf{L}, \tag{15}$$

and $diag\,(\mathbf{L})$ is the $N \times N$ matrix formed by substituting zeros for all off-diagonal elements of $\mathbf{L}$.

Our derived objective function generalizes a variety of ownership settings:

1. In the absence of cross- and common-ownership, $\mathbf{F}^*$ and $\mathbf{C}^*$ constitute null matrices and $\mathbf{C}^\intercal\mathbf{F}$ constitutes a diagonal matrix.[10] This implies that $\mathbf{W} = diag(\mathbf{C}^\intercal\mathbf{F})^{-\mathbf{1}}\,\mathbf{C}^\intercal\mathbf{F}$, which, since $\mathbf{C}^\intercal\mathbf{F}$ is diagonal, yields $\mathbf{W} = \mathbf{I}$ and reduces the objective function of firm j's manager to $\varpi_j' = \pi_j$. In other words, with no partial ownership interests of any kind, owners typically agree that the manager should maximize operating profits.

2. In cases of cross-ownership structures of financial interests, $\mathbf{C}^*$ and $\mathbf{C}^\intercal\mathbf{F}$ constitute a null and a diagonal matrix, respectively. This yields the objective function in Flath (1992) and Dietzenbacher *et al.* (2000) since $\mathbf{L} = \mathbf{C}^\intercal\mathbf{F}\,(\mathbf{I} - \mathbf{F}^*)^{-\mathbf{1}}$ and $\mathbf{C}^\intercal\mathbf{F}$ constitutes a diagonal matrix, implying that $\mathbf{W} = diag\,(\mathbf{L})^{-1}\,\mathbf{L} = diag\left((\mathbf{I} - \mathbf{F}^*)^{-\mathbf{1}}\right)^{-1}(\mathbf{I} - \mathbf{F}^*)^{-\mathbf{1}}$.

3. In cases of common-ownerhip of financial and control interests, $\mathbf{F}^*$ and $\mathbf{C}^*$ constitute null matrices, which yields the objective function of the manager in O'Brien and Salop (2000) since $\mathbf{W} = diag(\mathbf{C}^\intercal\mathbf{F})^{-\mathbf{1}}\,\mathbf{C}^\intercal\mathbf{F}$.

4 The Proposed Generalized Indicators

This section develops the two proposed generalized screening indicators for partial horizontal acquisitions within Cournot homogeneous- and Bertrand differentiated-product industries.

[10] In order to see why this is the case note that, in the absence of cross-ownership, all the off-diagonal elements of $\mathbf{F}^*$ and $\mathbf{C}^*$ are zero, while, in the absence of common-ownership, the external owners of a given firm do not have an interest on rival firms, which implies that all the off-diagonal elements of $\mathbf{C}^\intercal\mathbf{F}$ are zero.

4.1 Cournot Homogeneous-Product Industries: The Generalized HHI

In a Cournot homogeneous-product industry, we have that $p_1 = \ldots = p_j = \ldots = p_N = p$, for all $j \in \Im$, with p being determined by the inverse market demand function, $p(Q)$, where $Q = \sum_{g\in\Im} q_g$ denotes the aggregate industry output level. Under this setting, competition agencies often use market concentration as an useful indicator to screen the likely anti-competitive effects of an acquisition. Our proposed generalized HHI is structurally constructed as follows. The manager of firm j solves:

$$\max_{q_j} \varpi'_j = \sum_{g\in\Im} w_{jg}\pi_g = \sum_{g\in\Im} w_{jg}\left(\left(p\left(Q\right) - mc_g\right) q_g - c_g\right). \tag{16}$$

The Cournot-Nash equilibrium in quantities, $\left(q_1^{ne}, \ldots, q_j^{ne}, \ldots, q_N^{ne}\right)$ and consequently $Q^{ne} = \sum_{g\in\Im} q_g^{ne}$, for an interior solution is characterized by the following system of first-order conditions, for all $j \in \Im$:

$$\left(p\left(Q^{ne}\right) - mc_j\right) + \sum_{g\in\Im} w_{jg}\frac{\partial p\left(Q^{ne}\right)}{\partial Q} q_g^{ne} = 0, \tag{17}$$

which makes use of the fact that $w_{jj} = 1$ and $\partial Q^{ne}/\partial q_j = 1$. This result establishes that an extra unit of output, on the one hand, increases the objective function of the manager of firm j by the difference between price and marginal cost. However, on the other hand, this extra unit impacts the market price by $\partial p\left(Q^{ne}\right)/\partial Q$, which affects the revenues (and thereby the objective function), not only of the units already produced by firm j, but also of the units produced by all the other rival firms g in which the owners of firm j have, direct or indirectly, a financial and/or a corporate stake on (which is weighted by w_{jg}, for $j \neq g$).

Multiplying both sides of each first-order condition by $Q^{ne}/p\left(Q^{ne}\right)Q^{ne}$ yields:

$$\left(p\left(Q^{ne}\right) - mc_j\right)\frac{Q^{ne}}{p\left(Q^{ne}\right)Q^{ne}} + \sum_{g\in\Im} w_{jg}\frac{\partial p\left(Q^{ne}\right)}{\partial Q}\frac{Q^{ne}}{p\left(Q^{ne}\right)Q^{ne}} q_g^{ne} = 0, \tag{18}$$

which after some rearranging becomes:

$$\frac{p\left(Q^{ne}\right) - mc_j}{p\left(Q^{ne}\right)} = -\sum_{g\in\Im} w_{jg}\frac{\left(q_g^{ne}/Q^{ne}\right)}{\left(\partial Q^{ne}/\partial p\left(Q\right)\right)\left(p\left(Q\right)/\left(Q^{ne}\right)\right)} = \frac{1}{\eta}\sum_{g\in\Im} w_{jg} s_g^{ne}, \tag{19}$$

where the second equality makes use of the fact that $\eta = -\left(\partial Q^{ne}/\partial p\left(Q\right)\right)\left(p\left(Q\right)/\left(Q^{ne}\right)\right)$ denotes the (assumed constant) absolute value of the elasticity of demand, and $s_g^{ne} = \left(q_g^{ne}/Q^{ne}\right)$ denotes the output share of firm g, for all $g \in \Im$. This implies that the price-cost margin to price ratio of firm j is proportional to a weighted sum of the own-output share and the output share of all the other rival firms in which the owners of firm j have a stake on.

Multiplying both sides of each first-order condition by s_j^{ne} and summing over all firms, we can express the simultaneous market solution as:

$$\sum\nolimits_{j\in\Im}\left(\frac{p\left(Q^{ne}\right)-mc_j}{p\left(Q^{ne}\right)}\right)s_j^{ne}=\frac{1}{\eta}\sum\nolimits_{j\in\Im}\sum\nolimits_{g\in\Im}w_{jg}s_g^{ne}s_j^{ne} \tag{20}$$

which establishes that the output share-weighted margins to price ratio of all the firms in the industry is proportional (with the scale factor being $1/\eta$) to a measure of concentration. This measure incorporates the interests (direct and indirect, involving control or not, partial or full) of all types of owners (internal and external to the industry) and establishes our proposal.

Definition 1 *The generalized HHI is given by:*

$$GHHI=\sum\nolimits_{j\in\Im}\sum\nolimits_{g\in\Im}w_{jg}s_g^{ne}s_j^{ne}=\mathbf{s}^{\intercal}\mathbf{W}\mathbf{s}, \tag{21}$$

where $\mathbf{s}$ *is the* $N\times 1$ *vector of output market shares and* $\mathbf{W}$ *denotes the normalized weight matrix described above.*

There are four important aspects about our proposed indicator:

1. In the absence of cross- and common-ownership, we have (as discussed in section 4) that all the off-diagonal elements of $\mathbf{F}^*$, $\mathbf{C}^*$ and $\mathbf{C}^{\intercal}\mathbf{F}$ are zero. This implies that $\mathbf{W}=\mathbf{I}$, which reduces the GHHI to the standard HHI: GHHI= $\mathbf{s}^{\intercal}\mathbf{s}$ =HHI.

2. Using the standard HHI to measure the concentration of cross- or common-ownership structures induces a bias: GHHI= $\mathbf{s}^{\intercal}\mathbf{W}\mathbf{s}=\mathbf{s}^{\intercal}\mathbf{s}+\mathbf{s}^{\intercal}\left(\mathbf{W}-\mathbf{I}\right)\mathbf{s}$ =HHI+$\mathbf{s}^{\intercal}\left(\mathbf{W}-\mathbf{I}\right)\mathbf{s}$, which for $\mathbf{W}\neq\mathbf{I}$ implies that GHHI differs from the standard HHI.

3. In cases of cross-ownership structures of financial interests, we have (as discussed in section 4) that $\mathbf{L}=\mathbf{C}^{\intercal}\mathbf{F}\left(\mathbf{I}-\mathbf{F}^*\right)^{-\mathbf{1}}$, with $\mathbf{C}^{\intercal}\mathbf{F}$ constituting a diagonal matrix. This implies that $\mathbf{W}=diag\left(\mathbf{L}\right)^{-1}\mathbf{L}=diag\left(\left(\mathbf{I}-\mathbf{F}^*\right)^{-\mathbf{1}}\right)^{-1}\left(\mathbf{I}-\mathbf{F}^*\right)^{-\mathbf{1}}$, which reduces the GHHI to Dietzenbacher *et al.* (2000)'s modified HHI: GHHI= $\mathbf{s}^{\intercal}diag\left(\left(\mathbf{I}-\mathbf{F}^*\right)^{-\mathbf{1}}\right)^{-1}\left(\mathbf{I}-\mathbf{F}^*\right)^{-\mathbf{1}}\mathbf{s}.$

4. In cases of common-ownership, we have (as discussed in section 4) that $\mathbf{L} = \mathbf{C}^{\intercal}\mathbf{F}$, which implies that $\mathbf{W} = diag\left(\mathbf{C}^{\intercal}\mathbf{F}\right)^{-1}\mathbf{C}^{\intercal}\mathbf{F}$. As a consequence, the GHHI reduces to O'Brien and Salop (2000)'s modified HHI: GHHI= $\mathbf{s}^{\intercal} diag\left(\mathbf{C}'\mathbf{F}\right)^{-1}\mathbf{C}^{\intercal}\mathbf{F}\mathbf{s}$.

Having established our proposed structural measure of concentration, we can relatively straightforward derive an indicator to screen the anti-competitive effects of an acquisition. To do so, consider now an hypothetical acquisition, which can be partial or full, prompted by internal or external owners, and involve corporate control or not. Let $\tilde{\Im}$ denote the subset of firms (direct and indirectly) involved in the acquisition. Independently of the particulars of the acquisition, it will definitely impact the weights in matrix $\mathbf{W}$ for any $j, g \in \tilde{\Im}$. Let $\tilde{\mathbf{W}}$ denote the post-acquisition weight matrix $\mathbf{W}$, with weights given by $\tilde{w}_{jg}$ for any $j, g \in \Im$. Note that for the subset of firms not involved (direct or indirectly) in the acquisition, i.e., for any $j, g \in \Im \backslash \tilde{\Im}$, we have $\tilde{w}_{jg} = w_{jg}$. Finally, let $\left(\tilde{q}_1^{ne}, \ldots, \tilde{q}_j^{ne}, \ldots, \tilde{q}_N^{ne}\right)$ and $\tilde{Q}^{ne} = \sum_{g \in \Im} \tilde{q}_g^{ne}$ denote the interior Cournot-Nash equilibrium in quantities *post-acquisition*, which, assuming a setting of no efficiency gains, is characterized by the following simultaneous market solution:

$$\sum_{j \in \Im} \left(\frac{p\left(\tilde{Q}^{ne}\right) - mc_j}{p\left(\tilde{Q}^{ne}\right)} \right) \tilde{s}_j^{ne} = \frac{1}{\eta} \sum_{j \in \Im} \sum_{g \in \Im} \tilde{w}_{jg} \tilde{s}_g^{ne} \tilde{s}_j^{ne}, \tag{22}$$

where $\tilde{s}_j^{ne}$ denotes the post-acquisition output market share of firm $j \in \Im$.

The above result implies that the difference between the post- and the pre-acquisition output share-weighted margins to price ratio is given by:

$$\frac{1}{\eta} \left(\sum_{j \in \Im} \sum_{g \in \Im} \tilde{w}_{jg} \tilde{s}_g^{ne} \tilde{s}_j^{ne} - \sum_{j \in \Im} \sum_{g \in \Im} w_{jg} s_g^{ne} s_j^{ne} \right) = \frac{1}{\eta} \left(\widetilde{\mathrm{GHHI}} - \mathrm{GHHI} \right), \tag{23}$$

where $\widetilde{\mathrm{GHHI}}$ denotes the post-acquisition GHHI. The higher the post-acquisition GHHI and the increase in the GHHI, the greater the unilateral effects impact of the acquisition on the output share-weighted margins to price ratio and, as a consequence, the greater the likelihood that competition agencies should decide to issue a second request to conduct a more detailed analysis of the acquisition.

4.2 Bertrand Differentiated-Product Industries: The Generalized GUPPI

In a Bertrand differentiated-product industry, competition agencies rely much more on GUPPI than HHI for diagnosing the unilateral effects of an acquisition. Our proposed generalized GUPPI is structurally constructed as follows. The manager of firm j solves:

$$\max_{p_j} \varpi'_j = \sum\nolimits_{g \in \Im} w_{jg} \pi_g = \sum\nolimits_{g \in \Im} w_{jg} \left(\left(p_g - mc_g \right) q_g \left(\mathbf{p} \right) - c_g \right), \tag{24}$$

where $q_g(\mathbf{p})$ is the quantity demanded for the product of firm g, which is a function of the $N \times 1$ vector $\mathbf{p}$ of prices for all the products available in the industry. The Bertrand-Nash equilibrium in prices $\left(p_1^{ne}, \ldots, p_j^{ne}, \ldots, p_N^{ne} \right)$ for an interior solution is characterized by the following system of first-order conditions, for all $j \in \Im$:

$$q_j \left(\mathbf{p}^{ne} \right) + \left(p_j^{ne} - mc_j \right) \frac{\partial q_j \left(\mathbf{p}^{ne} \right)}{\partial p_j} + \sum\nolimits_{g \neq j \in \Im} w_{jg} \left(p_g^{ne} - mc_g \right) \frac{\partial q_g \left(\mathbf{p}^{ne} \right)}{\partial p_j} = 0, \tag{25}$$

which makes use of the fact that $w_{jj} = 1$. This result establishes that an one unit increase in price by firm j, on the one hand, increases the firm's revenues (and thereby the operating profits and the objective function of the manager) by the number of units already produced by the firm. However, on the other hand, it impacts the quantity demanded for the firm's product, by $\partial q_j \left(\mathbf{p}^{ne} \right) / \partial p_j$, which affects the objective function of the manager of firm by the difference between price and marginal cost. Furthermore, it also impacts the quantity demand for the products of all the other rival firms g in which the owners of firm j have, direct or indirectly, a financial and/or a corporate stake on, by $\partial q_g \left(\mathbf{p}^{ne} \right) / \partial p_j$, for $j \neq g$, which affects the objective function of the manager of firm j by the difference between the rivals price and marginal cost (which is weighted by w_{jg}, for $j \neq g$).

After some rearranging, we have that:

$$p_j^{ne} = mc_j - q_j \left(\mathbf{p}^{ne} \right) \left(\partial q_j \left(\mathbf{p}^{ne} \right) / \partial p_j \right)^{-1} + \sum\nolimits_{g \neq j \in \Im} w_{jg} \left(p_g^{ne} - mc_g \right) DR_{gj}, \tag{26}$$

where $DR_{gj} = - \left(\partial q_g \left(\mathbf{p}^{ne} \right) / \partial p_j \right) \left(\partial q_j \left(\mathbf{p}^{ne} \right) / \partial p_j \right)^{-1}$ denotes the diversion ratio from product j to product g, which quantifies, if the price of product j were to rise, how much of the displaced demand for the product switches to product g.

Consider now (as discussed above) an hypothetical acquisition, which can be partial or full, prompted by internal or external owners, and involve corporate control or not. Let $\tilde{\Im}$ denote the subset of firms (direct and indirectly) involved in the acquisition. Independently of the particulars of the acquisition, it will definitely impact the weights in matrix $\mathbf{W}$ for any $j, g \in \tilde{\Im}$. Let $\tilde{\mathbf{W}}$ denote the post-acquisition weight matrix $\mathbf{W}$, with weights given by w_{jg} for any $j, g \in \Im$ and $\tilde{w}_{jg}$ for any $j, g \in \tilde{\Im}$. The idea behind our proposed generalized GUPPI is to use information local to the pre-acquisition Bertrand-Nash equilibrium $\left(p_1^{ne}, \ldots, p_j^{ne}, \ldots, p_N^{ne}\right)$ to predict, under a setting of no efficiency gains, the directional price impacts of acquisitions (in the line of Cheung, 2011; and Jaffe and Weyl, 2013). To do so, we assume that the price of product j is the only variable that re-equilibrates after the acquisition, i.e., we ignore the re-equilibration of the remaining variables (all the other prices, all the quantities and all the price-effects). Finally, let $\left(\tilde{p}_1^{ne}, \ldots, \tilde{p}_j^{ne}, \ldots, \tilde{p}_N^{ne}\right)$ denote the Bertrand-Nash equilibrium in prices *post-acquisition*, which for an interior solution is characterized by the following system of first-order conditions, for all $j \in \tilde{\Im}$:

$$\begin{aligned}\tilde{p}_j^{ne} \quad = \quad & mc_j - q_j\left(\tilde{\mathbf{p}}^{ne}\right)\left(\partial q_j\left(\tilde{\mathbf{p}}^{ne}\right)/\partial p_j\right)^{-1} + \sum\nolimits_{g \neq j \in \tilde{\Im}} \tilde{w}_{jg}\left(\tilde{p}_g^{ne} - mc_g\right) DR_{gj} \\ & + \sum\nolimits_{g \neq j \in \Im \backslash \tilde{\Im}} w_{jg}\left(\tilde{p}_g^{ne} - mc_g\right) DR_{gj},\end{aligned} \tag{27}$$

which, under the no re-equilibration assumption, can be re-written as:

$$\begin{aligned}\tilde{p}_j^{ne} \quad = \quad & mc_j - q_j\left(\mathbf{p}^{ne}\right)\left(\partial q_j\left(\mathbf{p}^{ne}\right)/\partial p_j\right)^{-1} + \sum\nolimits_{g \neq j \in \tilde{\Im}} \tilde{w}_{jg}\left(p_g^{ne} - mc_g\right) DR_{gj} \\ & + \sum\nolimits_{g \neq j \in \Im \backslash \tilde{\Im}} w_{jg}\left(p_g^{ne} - mc_g\right) DR_{gj},\end{aligned} \tag{28}$$

since we have that $\tilde{p}_g^{ne} = p_g^{ne}$ for $g \neq j \in \Im$, $q_j\left(\mathbf{p}^{ne}\right) = q_j\left(\tilde{\mathbf{p}}^{ne}\right)$ for $j \in \Im$, and $\partial q_g\left(\tilde{\mathbf{p}}^{ne}\right)/\partial p_j = \partial q_g\left(\tilde{\mathbf{p}}^{ne}\right)/\partial p_j$ for all $g \in \Im$.

The above result implies that the difference between the post- and pre-acquisition price of product $j \in \tilde{\Im}$ is given by:

$$\left(\tilde{p}_j^{ne} - p_j^{ne}\right) = \sum\nolimits_{g \neq j \in \tilde{\Im}} \left(\tilde{w}_{jg} - w_{jg}\right)\left(p_g^{ne} - mc_g\right) DR_{gj}, \tag{29}$$

which establishes that product j's upward pricing pressure, gross of efficiency gains, is a function of the change in the weights in matrix $\mathbf{W}$, of the pre-acquisition price-cost margins, and of the diversion ratios, all of which referent solely to the products in $\tilde{\Im}$, i.e., to the products of the firms involved in the acquisition. Multiplying both sides of the above result by $1/p_j^{ne}$ establishes our proposal.

Definition 2 *The generalized GUPPI for product $j \in \tilde{\Im}$ is given by:*

$$GGUPPI_j = \sum\nolimits_{g \neq j \in \tilde{\Im}} \left(\tilde{w}_{jg} - w_{jg}\right) \left(p_g^{ne} - mc_g\right) DR_{gj}/p_j^{ne}, \tag{30}$$

where $\tilde{w}_{jg} - w_{jg}$ denotes, the change in the normalized weight matrix elements post- and pre-acquisition for products j and $g \in \tilde{\Im}$, p_j^{ne} and p_g^{ne} denotes the pre-acquisition price of product j and $g \in \tilde{\Im}$, respectively, mc_g denotes the pre-acquisition marginal cost of product $g \in \tilde{\Im}$, and DR_{gj} denotes the pre-acquisition the diversion ratio from product j to product $g \in \tilde{\Im}$.

The higher the level of GUPPI for each product involved in the acquisition, the greater the unilateral impact of the acquisition on their prices and, as a consequence, the greater the likelihood that competition agencies should decide to issue a second request to conduct a more detailed analysis of the acquisition. Finally, note that, in the absence of cross- and common-ownership, we have (as discussed in section 4) that all the off-diagonal elements of $\mathbf{F}^*$, $\mathbf{C}^*$ and $\mathbf{C}^\intercal\mathbf{F}$ are zero, which implies $\mathbf{W} = \mathbf{I}$. In cases of full acquisitions, this leads to $(\tilde{w}_{jg} - w_{jg}) = 1$ for $j, g \in \tilde{\Im}$ and $g \neq j$, which reduces the GGUPPI to the standard GUPPI:

$$\text{GGUPPI}_j = \sum\nolimits_{g \neq j \in \tilde{\Im}} \left(p_g^{ne} - mc_g\right) DR_{gj}/p_j^{ne} = \text{GUPPI}_j. \tag{31}$$

5 Empirical Application

This section presents an empirical application of the GHHI and the GGUPPI to several acquisitions in the wet shaving industry, with the objective of providing a step-by-step illustration of how to compute the two proposed indicators.

On December 20, 1989, the Gillette Company, contracted to acquire the wet shaving businesses of Wilkinson Sword trademark outside of the 12-nation European Community (which included the United States operations) from Eemland Management Services BV (Wilkinson Sword's parent company) for \$72 million. It also acquired a 22.9% of the nonvoting equity shares of Eemland for about \$14 million. At that time, consumers in the United States annually purchased over \$700 million of wet shaving razor blades at the retail level. Five firms supplied all but a nominal amount of these blades: *The Gillette Company*, which had been the market leader for years, *American Safety Razor Company*, *BIC Corporation*, *Warner-Lambert Company*, and *Wilkinson Sword Inc.*.

On January 10, 1990, the DoJ instituted a civil proceeding against Gillette. The complaint alleged that the effect of the acquisition by Gillette may have been substantially to lessen competition in the sale of wet shaving razor blades in the United States. Shortly after the case was filed, Gillette voluntarily rescinded the acquisition of Eemland's wet shaving razor blade business in the United States. Gillette said it decided to settle the case to avoid the time and expense of a lengthy trial.

However, Gillette still went through with the acquisition of 22.9% nonvoting equity interest in Eemland and of all worldwide assets and businesses of Wilkinson Sword trademark from Eemland, apart from the United States and the European Community. Because Eemland kept the Wilkinson Sword's United States wet shaving razor blades business, Gillette had become one of the largest, if not the largest, shareholder in a competitor. The DoJ (1990) allowed the acquisition provided that:

> Gillette and Eemland shall not agree or communicate an effort to persuade the other to agree, directly or indirectly, regarding present or future prices or other terms or conditions of sale, volume of shipments, future production schedules, marketing plans, sales forecasts, or sales or proposed sales to specific customers... (page 7)

In other words, the DoJ approved Gillette's 22.9% stake in Wilkinson Sword after being assured that this stake would be passive. Indeed, Gillette claimed it was merely making an investment. However, even when the acquiring firm cannot influence the conduct of the target firm, the partial acquisition may still raise antitrust concerns. The reason being that the partial acquisition may reduce the incentive of the acquiring firm to compete aggressively because it shares in the losses thereby inflicted on that rival. We examine this question by screening the unilateral effects of this stake. As a comparison, we also examine Gillette's initial proposed 100% acquisition of Wilkinson Sword to screen the counterfactual unilateral effects have Gillette not voluntarily rescinded the acquisition of Eemland's wet shaving razor business in the US. Finally, we also screen two additional hypothetical acquisitions. We examine an hypothetical acquisition of 22.9% voting equity interest in Wilkinson Sword by Gillette, in order to illustrate the differential impact of acquiring a voting and a nonvoting equity interest. Further, we examine an hypothetical acquisition of 22.9% voting equity interest in Wilkinson Sword by Berkshire Hathaway, Inc., Gillette's largest external owner, in order to illustrate the differential impact of an acquisition giving rise to a cross- and a common-ownership structure.

5.1 The Normalized Weight Matrix

In order to apply the two proposed indicators to the above setting, we have to calculate the normalized weight matrix $\mathbf{W}$. To do so, we require information on both the financial and the corporate control structure of the five firms in the industry. We make the following assumption regarding the measure of each owner's degree of control over the manager of a firm:

Assumption 5 *The control weight an owner has over the decision making of a firm is equal to the share of voting rights she owns in the firm.*

Assumption 5 constitutes a natural benchmark, since, as discussed above, the degree of corporate control an owner has (over the decision making of a firm) is a function of the voting rights she holds in the firm. However, it is merely illustrative. As suggested by Goppelsroeder *et al.* (2008), we can, alternatively, measure the owners' degree of control (over the decision making of a firm) by the Shapley–Shubik (1954) power index or the Banzhaf (1965) power index.

We begin by describing the financial and the corporate control structure of the five firms *pre-acquisition*, i.e., prior to December 20, 1989. To do so, we make use of two sources of information: for US-based firms, we analyze the proxy statements (schedule 14A) filled by firms with the Securities and Exchange Commission, while for Europe-based firms, we analyze the Commission of the European Communities' official decision regarding the full acquisition initially proposed. Two comments are in order relative to this information. First, although we are describing the pre-acquisition 1989' structures, we use data from 1990, which was the earliest year available.[11] This implicitly assumes that from 1989 to 1990 the financial and control structure of the firms did not suffer relevant variations other than the ones described above. Second, public data is restricted to identify large external owners, whose interest (directly or together with affiliates)

[11] The only exceptions are the data referent to American Safety Razor Company and BIC Corporation, which earliest year available was 1994 and 1993, respectively.

typically exceeds 5%. As a consequence, we must make an assumption relative to the financial and control weight of the remaining minority external owners. We make the following:

Assumption 6 *Minority external owners do not engage in common-ownership.*

Assumption 6 constitutes a natural benchmark and it is merely illustrative. It implies that each firm's minority external owners agree on the best strategy to pursue. As a consequence, both indicators are invariant to the financial and control weigh of each of those owners. Therefore, we may aggregate without loss of generality the financial and control weigh of each firm's minority external owners into a single fictitious external owner. Naturally, in cases involving common-ownership among the minority external owners, this aggregation is not innocuous, since those owners will have conflicting views on the best strategy to pursue. In those cases, the financial and control weight of each owner matters, which implies that a careful evaluation of all the *individual* weights is essential.

Table 2 presents the financial and the corporate control stake (under Assumptions 5 and 6) of each owner, both internal and external, over the firms in the industry. Let $\Im \equiv \{1, \ldots, 5\}$ denote the set of owners that are internal to the industry, each of which is indexed by j, and $\Theta \backslash \Im \equiv \{6, \ldots, 19\}$ denote the set of owners that are external to the industry, each of which is indexed by k. Table 2, Panel A addresses the ownership stakes of the *five* internal owners. It suggests that, pre-acquisition, the firms in the industry did not engage in cross-ownership. Table 2, Panel B addresses the ownership stakes of the *fourteen* external owners (including the fictitious minority owners). It suggests that, pre-acquisition, external owners also did not engage in common-ownership at all.

Having described the financial and the corporate control structure of the five firms, we can begin to convert that information into the four matrices that are instrumental in computing the weight matrix $\mathbf{W}$: matrices $\mathbf{F}^*$ and $\mathbf{C}^*$, which capture cross-ownership among internal owners, and matrices $\mathbf{F}$ and $\mathbf{C}$, which capture common-ownership from external owners.

We address first the former. Matrices $\mathbf{F}^*$ and $\mathbf{C}^*$ denote the financial and corporate control cross-ownership matrices, respectively. In our application, they are captured by (5×5) matrices. The diagonal elements are, by definition, zero. The off-diagonal elements, ϕ_{jg} and γ_{jg}, represent the financial and corporate control cross-ownership stake of firm j on firm g, respectively, for

TABLE 2

*Financial and Corporate Control**

j/k		ASR		BIC		WL		WS		G	
		F	CC	F	CC	F	CC	F	CC	F	CC
Panel A: Internal Owners											
01	American Safety Razor Company	–	–	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
02	BIC Corporation	0.00	0.00	–	–	0.00	0.00	0.00	0.00	0.00	0.00
03	Warner-Lambert Company	0.00	0.00	0.00	0.00	–	–	0.00	0.00	0.00	0.00
04	Wilkinson Sword, Inc.	0.00	0.00	0.00	0.00	0.00	0.00	–	–	0.00	0.00
05	The Gillette Company	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	–	–
Panel B: External Owners											
06	Equitable (a)	14.40	14.40	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
07	Allsop Venture Partners III, LP	12.40	12.40	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
08	Goldman Sachs Group, LP	7.80	7.80	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
09	Scudder Stevens and Clarck	7.00	7.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	Leucadia-Mezzanine (b)	6.10	6.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
11	Grantham Mayo Van Otter	5.10	5.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
12	ASR Minority Owners	47.20	47.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
13	Bruno Bich	0.00	0.00	77.70	77.70	0.00	0.00	0.00	0.00	0.00	0.00
14	BIC Minority Owners	0.00	0.00	22.30	22.30	0.00	0.00	0.00	0.00	0.00	0.00
15	WL Minority Owners	0.00	0.00	0.00	0.00	100.00	100.00	0.00	0.00	0.00	0.00
16	Eemland Management Services BV	0.00	0.00	0.00	0.00	0.00	0.00	100.00	100.00	0.00	0.00
17	Berkshire Hathaway, Inc.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	10.80	10.80
18	State Street Bank and Trust Co.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	6.00	6.00
19	G Minority Owners	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	83.20	83.20

* Figures are in percentage points. Adapted from Schedule 14A (proxy statement) information and 93/525/EEC decision. F denotes each external owner's holdings of total stock. CC denotes, under Assumption 5, each external owner's holdings of voting stock. ASR, B, WL, WS and G denote American Safety Razor Company, BIC Corporation, Warner-Lambert Company, Wilkinson Sword, Inc., and The Gillette Company, respectively. (a) Equitable denotes the cumulative ownership of Equitable Capital Partners, LP, Equitable Deal Flow Fund, LP, Equitable Capital Partners (Retirement Fund), LP, and The Equitable Life Assurance Society of the United States. (b) Leucadia-Mezzanine denotes the cumulative ownership of Leucadia Investors, Inc. and Mezzanine Capital and Income Trust 2001 PLC.

all $j \neq g \in \Im$. In both cases, the rows and columns are ordered from $j = 1$ to $j = 5$. Given that pre-acquisition firms in the industry do not engage in cross-ownership, we have that $\phi_{jg} = 0$ and $\gamma_{jg} = 0$ for all $j \neq g \in \Im$. This implies that $\mathbf{F}^*$ and $\mathbf{C}^*$ pre-acquisition constitute null matrices:

$$\mathbf{F}^* = \begin{bmatrix} 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \end{bmatrix} \mathbf{C}^* = \begin{bmatrix} 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \end{bmatrix}.$$

We now address the latter. Matrices $\mathbf{F}$ and $\mathbf{C}$ denote the financial and corporate control common-ownership matrices, respectively. In our application, they are captured by (14×5) matrices. The typical element is given by ϕ_{kj} and γ_{kj}, respectively, for all $j \in \Im$ and all $k \in \Theta \backslash \Im$. The rows are ordered from $k = 6$ to $k = 19$, while the columns are ordered from $j = 1$ to $j = 5$. For instance, external owner Berkshire Hathaway, indexed as $k = 17$, has a financial and a corporate control stake on Gillette, indexed as $j = 5$, of 10.8%. As a consequence, we have that $\phi_{17,5} = 0.108$ and $\gamma_{17,5} = 0.108$. Formally, pre-acquisition matrices $\mathbf{F}$ and $\mathbf{C}$ are given by:

$$\mathbf{F} = \begin{bmatrix} 0.144 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.124 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.078 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.070 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.061 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.051 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.472 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.777 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.223 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 1.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.108 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.060 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.832 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 0.144 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.124 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.078 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.070 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.061 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.051 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.472 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.777 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.223 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 1.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.108 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.060 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.832 \end{bmatrix}.$$

Having constructed matrices **C** and **F**, we have all the necessary information to compute pre-acquisition matrices **A** and **B**, as described in section 3.3. In our application, this computation yields:

$$\mathbf{A} = \begin{bmatrix} 0.276 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.653 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 1.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.707 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 3.621 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 1.530 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 1.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 1.413 \end{bmatrix}.$$

Finally, we can compute the pre-acquisition weight matrix **L** and the pre-acquisition normalized weight matrix **W**, as described in section 3.3. To do so, we just make use of matrices

$\mathbf{F}^*$, $\mathbf{C}^*$, $\mathbf{C}$, $\mathbf{F}$ and $\mathbf{B}$. This computation yields:

$$\mathbf{L} = \begin{bmatrix} 0.276 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.653 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 1.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.707 \end{bmatrix} \mathbf{W} = \begin{bmatrix} 1.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 1.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 1.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 1.000 \end{bmatrix},$$

which implies that, absent cross- and common-ownership, and barring any market imperfections that preclude efficient contracting between the owners and the manager, the former agree (and give the appropriate incentives) that the latter should maximize own-operating profits. This constitutes the pre-acquisition benchmark by which all the four acquisitions discussed above are going to be evaluated below.

Gillette Acquires a 100% Voting Equity Interest in Wilkinson Sword

The (hypothetical) acquisition of 100% voting equity interest in Wilkinson Sword by Gillette gives rise to a cross-ownership structure in the industry (since Wilkinson Sword's ownership changes from an external owner, Eemland, to an internal owner). Comparing with the pre-acquisition structure, this implies changes to matrices $\mathbf{F}^*$ and $\mathbf{C}^*$, as well as to matrices $\mathbf{F}$ and $\mathbf{C}$, which induce (Appendix A describes the step-by-step computational details) the following *post-acquisition* normalized weight matrix $\tilde{\mathbf{W}}$:

$$\tilde{\mathbf{W}} = \begin{bmatrix} 1.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 1.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 1.000 & 1.000 \\ 0.000 & 0.000 & 0.000 & 1.000 & 1.000 \end{bmatrix}.$$

This result implies that the managers of Wilkinson Sword and Gillette are, post-acquisition, perfectly aligned. The two firms behave, effectively, as a single entity, in the sense that their owners agree (and give the appropriate incentives) that managers should maximize their joint operating profits.

Gillette Acquires a 22.9% Voting Equity Interest in Wilkinson Sword

In order to illustrate the differential impact of a full merger and a partial acquisition (of a voting equity interest), we consider here the (hypothetical) acquisition of 22.9% *voting* equity interest in Wilkinson Sword by Gillette. This acquisition gives rise to a *partial* cross-ownership structure in the industry, in which Gillette and Eemland, an internal and an external owner, respectively, *share* financial and corporate control interests in Wilkinson Sword. Comparing with the pre-acquisition structure, this implies changes to matrices $\mathbf{F}^*$ and $\mathbf{C}^*$, as well as to matrices $\mathbf{F}$ and $\mathbf{C}$, which induce (Appendix A describes the step-by-step computational details) the following *post-acquisition* normalized weight matrix $\tilde{\mathbf{W}}$:

$$\tilde{\mathbf{W}} = \begin{bmatrix} 1.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 1.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 1.000 & 0.354 \\ 0.000 & 0.000 & 0.000 & 0.229 & 1.000 \end{bmatrix}.$$

This result implies that the managers of Wilkinson Sword and Gillette should maximize a weighted average of the two firms operating profits. However, the weights attributed by the managers to the two firms are not symmetric. The reason is as follows. While the manager of Wilkinson Sword should internalize Gillette's operating profits because Gillette holds a *control* stake in Wilkinson Sword, the manager of Gillette should internalize Wilkinson Sword's operating profits because Gillette holds a *financial* stake in Wilkinson Sword. This suggests that partial aquisitions of *voting* interests align the interests of the firms involved in the acquisition in the same qualitative vein as a full merger. The only difference is solely on the weight given to the rival firm operations.

Gillette Acquires a 22.9% Nonvoting Equity Interest in Wilkinson Sword

In order to illustrate the differential impact of acquiring a voting and a nonvoting equity interest, we consider here the (actual) acquisition of 22.9% *nonvoting* equity interest in Wilkinson Sword by Gillette. This acquisition gives rise to a *partial* cross-ownership structure in the industry, in which Eemland, an external owner, fully controls Wilkinson Sword, but shares the financial interest in the firm with Gillette, an internal owner. Comparing with the pre-acquisition structure, and since the equity interest transacted involves no voting, this implies that there are no changes in control and thus that matrices $\mathbf{C}^*$ and $\mathbf{C}$ remain unchanged. However, it does imply changes to matrices $\mathbf{F}^*$ and $\mathbf{F}$, which capture financial interests. These changes induce (Appendix A describes the step-by-step computational details) the following *post-acquisition* normalized weight matrix $\tilde{\mathbf{W}}$:

$$\tilde{\mathbf{W}} = \begin{bmatrix} 1.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 1.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 1.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.229 & 1.000 \end{bmatrix}.$$

This result implies that the manager of Wilkinson Sword should maximize own-operating profits. The reason being that while Wilkinson Sword has two owners, only one of them, Eemland, has control over the manager. And Eemland only cares about the returns of the equity it holds in Wilkinson Sword. Furthermore, it also implies that the manager of Gillette should maximize a weighted average of Gillette and Wilkinson Sword's operating profits. The reason being that Gillette's aggregate profit, which determines the equity returns of Gillette's owners, includes not just the own-firm's operating profits, but also the 22.9% share in Wilkinson Sword's aggregate profits (which in this case, coincide with Wilkinson Sword's operating profits since the firm does not engage in cross-ownership). This suggests that partial acquisitions of *nonvoting* interests change the incentives of the *acquiring* firm, but not of the *acquired* firm.

Berkshire Hathaway Acquires a 22.9% Nonvoting Equity Interest in Wilkinson Sword

In order to illustrate the differential impact of an acquisition giving rise to a cross- and a common-ownership structure, we consider here the (hypothetical) acquisition of 22.9% *nonvoting* equity interest in Wilkinson Sword by Berkshire Hathaway, Gillette's largest external owner. This acquisition gives rise to a *partial* common-ownership structure in the industry, in which Berkshire Hathaway, an external owner, participates in two competing firms, Gillette and Wilkinson Sword. Comparing with the pre-acquisition structure, and since internal owners are not at all involved in the operation, this implies that matrices $\mathbf{F}^*$ and $\mathbf{C}^*$ remain unchanged. Further, since the equity interest transacted involves no voting, this implies that matrix $\mathbf{C}$ also remains unchanged. However, it does imply changes to matrices $\mathbf{F}$, which capture financial ownership by external owners. These changes induce (Appendix A describes the step-by-step computational details) the following *post-acquisition* normalized weight matrix $\tilde{\mathbf{W}}$:

$$\tilde{\mathbf{W}} = \begin{bmatrix} 1.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 1.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 1.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.035 & 1.000 \end{bmatrix}.$$

This result implies that the manager of Wilkinson Sword should maximize own-operating profits. The reason being that although Wilkinson Sword has two owners, only one of them, Eemland, has control over the manager. And Eemland only cares about the returns of the equity it holds in Wilkinson Sword. Furthermore, it also implies that the manager of Gillette should maximize a weighted average of Gillette and Wilkinson Sword's operating profits, since post-acquisition Gillette's largest external owner, Berkshire Hathaway, holds stakes on both firms. This suggests that partial acquisitions by external owners that participate in more than one competitor firm align the interests of the firms involved in the acquisition in the same qualitative vein as an acquisition by internal owners. The only difference is solely on the weight given to the rival firm operations.

5.2 The Generalized HHI

Competition agencies often use market concentration as an useful indicator to screen the likely competitive effects of an acquisition. In order to apply our proposed generalized market concentration measure, the generalized HHI, to a particular setting, we require information not only about the normalized weight matrix $\mathbf{W}$ (discussed above), but also about the pre-acquisition output shares of the firms in the industry. This latter information is included in the data submitted in a typical notification to a competition agency, and for that reason does not increase the information requirements of unilateral effects analyses. Table 3 presents the pre-acquisition output shares of each firm $j \in \Im \equiv \{1, \ldots, 5\}$ in our illustration. The data is adapted from

TABLE 3

*Firm Pre-Acquisition Output Shares**

j		output share
01	American Safety Razor Company	1%
02	BIC Corporation	20%
03	Warner-Lambert Company	14%
04	Wilkinson Sword, Inc.	3%
05	The Gillette Company	50%

* Figures adapted from DoJ (1990).

the text published by the DoJ (1990) referent to the United States of America v. The Gillette Company, *et al.* case (Civil Action No. 90-005390-0053-TFH). It suggests that Gillette is the dominant firm, accounting for 50% of all razor blade units. BIC is the second biggest-selling firm with 20%, followed by Warner-Lambert with 14% of unit sales. Wilkinson and American Safety Razor have very residual output shares.

We use the normalized weight matrices $\mathbf{W}$ and $\tilde{\mathbf{W}}$ calculated above, and the DoJ (1990) output share data to compute the generalized HHI pre- and post-acquisition for each of the cases discussed. To do so, we make use of equation (21). The results are summarized in Table 4. The pre-acquisition industry has a generalized HHI of $3,106$ $(= (1)^2 + (20)^2 + (14)^2 + (3)^2 + (50)^2)$. This result makes clear that in the absence of cross- and common-ownership, the GHHI reduces to the standard HHI. Further, it suggests that the wet shaving industry was highly concentrated even before December 20, 1989.

The 100% voting equity interest acquisition in Wilkinson Sword initially proposed by Gillette would have induced a post-acquisition industry with a generalized HHI of $3,406$ $(= (1)^2+(20)^2+ (14)^2 + (3)^2 + (3)\,(50) + (50)\,(3) + (50)^2)$. As discussed above, since the acquisition constitutes a full merger, this measure coincides with the standard HHI. The results suggests that the acquisition would have induced an increase in concentration of more than 200 points (in an already highly concentrated industry), an impact sufficiently high for the DoJ to presume that the acquisition would likely enhance market power, justifying the civil proceeding instituted against Gillette.

Gillette voluntarily rescinded the above 100% voting equity interest acquisition. Had Gillette considered a partial acquisition of 22.9% of the voting equity interests of Wilkinson Sword, the post-acquisition industry would have a generalized HHI of $3,193$ $(\approx (1)^2 + (20)^2 + (14)^2 + (3)^2 + 0.354\,(3)\,(50) + 0.229\,(50)\,(3) + (50)^2)$. This result suggests that a full merger induces a higher

TABLE 4

*Generalized HHI**

	WS independent shareholder structure	WS acquired by G 100% voting	G 22.9% voting	G 22.9% nonvoting	BH 22.9% nonvoting
Generalized HHI	3,106	3,406	3,193	3,140	3,111
Δ Generalized HHI	–	300	87	34	5

* WS, G, and BH denote Wilkinson Sword, Gillette, and Berkshire Hathaway, respectively. Δ Generalized HHI denotes the change in GHHI pre- and post-acquisition.

increase in concentration than a partial acquisition of a voting interest. In the particular case at hands, the acquisition would have involved an increase in concentration of less than 100 points, which implies that it was unlikely to have adverse competitive effects, a result which ordinarily requires no further analysis.

However, Gillette did not consider a partial voting equity interest acquisition, but a *non-voting* one. The 22.9% nonvoting equity interest acquisition in Wilkinson Sword induced a post-acquisition industry with a generalized HHI of $3,140$ ($\approx (1)^2 + (20)^2 + (14)^2 + (3)^2 + 0.229\,(50)\,(3) + (50)^2$). This result suggests that the acquisition of a control stake induces a higher increase in concentration than the acquisition of solely a financial stake. In the particular case at hands, the acquisition involved an increase in concentration of less than 100 points, which implies that it was also unlikely to have adverse competitive effects. This seems to validate the decision of DoJ not to challenge the operation.

Finally, had the 22.9% nonvoting equity interest in Wilkinson Sword been acquired by Berkshire Hathaway, Gillette's largest external owner, the post-acquisition industry would have a generalized HHI of $3,111$ ($\approx (1)^2 + (20)^2 + (14)^2 + (3)^2 + 0.035\,(50)\,(3) + (50)^2$). This implies that the increase in concentration induced by Berkshire Hathaway's 22.9% nonvoting equity interest acquisition in Wilkinson Sword is lower than the one induced by Gillette's direct 22.9% nonvoting equity interest acquisition (since Berkshire Hathaway holds solely a share of Gillette). A result that suggests that acquisitions that give rise to common-ownership structures in which external owners *partially* participate in more than one competitor firm may induce a lower increase in concentration than acquisitions that give rise to cross-ownership structures.

TABLE 5

*Prices, Margins and Diversion Ratios**

j		WS	G
Panel A: Prices and Margins ($)			
Price		1.540	4.036
Margin		0.375	0.447
Panel B: Diversion Ratios			
04	Wilkinson Sword, Inc.	-1.000	0.225
05	The Gillette Company	0.003	-1.000

* Price and margin figures are in USD. WS and G denote Wilkinson Sword and Gillette, respectively.

5.3 The Generalized GUPPI

In order to apply our proposed generalized GUPPI to a particular setting, we require information on the normalized weight matrices pre- and post-acquisition *for all firms*, as well as information on the pre-acquisition prices, margins, and diversion ratios *for the firms which weights exhibit changes pre- and post-acquisition.* We already discussed the calculation of the normalized weight matrices pre- and post-acquisition for each of the cases under examination. An analysis of the results makes clear that, in all cases, only the weights associated with Wilkinson Sword and Gillette do change. This implies that, in our application, we require information solely on the pre-acquisition prices, margins, and diversion ratios of those two firms. This information is included in the data submitted in a typical notification to a competition agency, and for that reason does not increase the information requirements of unilateral effects analyses.

Table 5 presents the pre-acquisition prices, margins, and diversion ratios for Wilkinson Sword and Gillette. Since these firms are multi-product firms, the data refers to the median razor package of each firm (across all their products) and is computed using the demand and cost estimates in Brito *et al.* (2014a). It suggests that Wilkinson Sword median prices per package are relatively lower than Gillette's, $1.54 versus $4.04, although the two firms generate slightly the same margin per package. Further, it suggests that roughly one-quarter of the unit sales lost by Gillette if its price were to rise would be captured by Wilkinson Sword, while in the reverse case, the value is considerable smaller: only 0.3% of the unit sales lost by Wilkinson Sword if its price were to rise would be captured by Gillette. In other words, Gillette's customers see Wilkinson Sword products as relatively good substitutes, but the same is not true for Wilkinson Sword's customers. The reason may lay in the fact that Gillette's products are more expensive than Wilkinson Sword's.

We use the normalized weight matrices $\mathbf{W}$ and $\tilde{\mathbf{W}}$ calculated above, and Table 5's data to compute the generalized GUPPI for each of the acquisitions discussed. To do so, we make use of equation (30). The results are summarized in Table 6. According to this indicator, the 100% voting equity interest acquisition in Wilkinson Sword initially proposed by Gillette would have induced a slight upward pricing pressure in industry's products. In order to see why, note that the difference between $\tilde{\mathbf{W}}$ and $\mathbf{W}$ is given by:

$$\tilde{\mathbf{W}} - \mathbf{W} = \begin{bmatrix} 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 1.000 \\ 0.000 & 0.000 & 0.000 & 1.000 & 0.000 \end{bmatrix},$$

which makes clear that (i) only the elements referent simultaneously to Wilkinson Sword and Gillette, as discussed above, change, and (ii) in full acquisitions the GGUPPI coincides with the standard GUPPI. This implies that the acquisition's GGUPPIs are thus given by: $\text{GGUPPI}_j = 0$ for $j = \{1, 2, 3\}$, $\text{GGUPPI}_4 = (1.000)\,(0.447)\,(0.003)\,/\,(1.540) = 0.087\%$, and $\text{GGUPPI}_5 = (1.000)\,(0.375)\,(0.225)\,/\,(4.036) = 2.091\%$. This result suggests that the acquisition would have induced an upward pricing pressure of 0.087% and 2.091% in Wilkinson Sword and Gillette's products, respectively. Further, it confirms the idea, suggested by the generalized HHI, that the acquisition would likely enhance Gillette's market power. However the impact is relatively small, which calls into question DoJ's civil proceeding against Gillette.

Gillette voluntarily rescinded the above 100% voting equity interest acquisition. Had Gillette considered a partial acquisition of 22.9% of the voting equity interests of Wilkinson Sword, the results seem to suggest that the impact would have been even lower. In order to see why, note that, in this case, the difference between $\tilde{\mathbf{W}}$ and $\mathbf{W}$ would have been given by:

$$\tilde{\mathbf{W}} - \mathbf{W} = \begin{bmatrix} 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.354 \\ 0.000 & 0.000 & 0.000 & 0.229 & 0.000 \end{bmatrix},$$

which indicates that, similarly to the full merger case, only the elements referent simultaneously to Wilkinson Sword and Gillette change. This implies that the acquisition's GGUPPIs are thus given by: $\text{GGUPPI}_j = 0$ for $j = \{1, 2, 3\}$, $\text{GGUPPI}_4 = (0.354)\,(0.447)\,(0.003)\,/\,(1.540) = 0.031\%$, and $\text{GGUPPI}_5 = (0.229)\,(0.375)\,(0.225)\,/\,(4.036) = 0.479\%$. This result suggests that the acquisition would have induced an upward pricing pressure of 0.031% and 0.479% in Wilkinson Sword and Gillette's products, respectively. Further, it confirms the idea, suggested by the generalized HHI, that a full merger induces a higher upward pricing pressure than a partial acquisition of a voting interest.

However, Gillette did not consider a partial voting equity interest acquisition, but a *non-voting* one. The results relative to the 22.9% nonvoting equity interest acquisition in Wilkinson Sword seem to validate the decision of DoJ not to challenge the operation, after being assured that the stake would be passive. The upward pricing pressure in the industry's products is screened to be in fact very small. In order to see why, note that the difference between $\tilde{\mathbf{W}}$ and $\mathbf{W}$ is given by:

$$\tilde{\mathbf{W}} - \mathbf{W} = \begin{bmatrix} 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.229 & 0.000 \end{bmatrix},$$

which indicates that only the weight of Gillette's manager on Wilkinson Sword's operating profits does change. This implies that solely Gillette's products will exhibit an upward pricing pressure. The acquisition's GGUPPIs are thus given by: $\text{GGUPPI}_j = 0$ for $j = \{1, 2, 3, 4\}$ and $\text{GGUPPI}_5 = (0.229)\,(0.375)\,(0.225)\,/\,(4.036) = 0.479\%$. This result suggests that the acquisition was unlikely to have adverse competitive effects, since it involved an upward pricing pressure of only 0.479% in Gillette's products. Further, it confirms the idea, suggested by the generalized HHI, that the acquisition of a control stake induces more adverse competitive effects than the acquisition of solely a financial stake, since the upward pricing pressure is lower than in the previous case.

Finally, had the 22.9% nonvoting equity interest in Wilkinson Sword been acquired by Berkshire Hathaway, Gillette's largest external owner, the upward pricing pressure in the industry's products would also have been very small. In order to see why, note that, in this case, the

Table 6
*Generalized GUPPI**

		WS acquired by			
		G	G	G	BH
		100%	22.9%	22.9%	22.9%
j		voting	voting	nonvoting	nonvoting
01	American Safety Razor Company	0.000%	0.000%	0.000%	0.000%
02	BIC Corporation	0.000%	0.000%	0.000%	0.000%
03	Warner-Lambert Company	0.000%	0.000%	0.000%	0.000%
04	Wilkinson Sword, Inc.	0.087%	0.031%	0.000%	0.000%
05	The Gillette Company	2.091%	0.479%	0.479%	0.073%

* WS, G, and BH denote Wilkinson Sword, Gillette, and Berkshire Hathaway, respectively.

difference between $\tilde{\mathbf{W}}$ and $\mathbf{W}$ would have been given by:

$$\tilde{\mathbf{W}} - \mathbf{W} = \begin{bmatrix} 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.035 & 0.000 \end{bmatrix},$$

which indicates that only the weight of Gillette's manager on Wilkinson Sword's operating profits changes. This implies that solely Gillette's products will exhibit an upward pricing pressure. The acquisition's GGUPPIs are thus given by: $\text{GGUPPI}_j = 0$ for $j = \{1, 2, 3, 4\}$ and $\text{GGUPPI}_5 = (0.035)\,(0.375)\,(0.225)\,/\,(4.036) = 0.073\%$. This implies that the upward pricing pressure in Gillette's products induced by Berkshire Hathaway's 22.9% nonvoting equity interest acquisition in Wilkinson Sword is lower than the one induced by Gillette's direct 22.9% nonvoting equity interest acquisition (since Berkshire Hathaway holds solely a share of Gillette). A result that confirms, as suggested by the generalized HHI, that acquisitions that give rise to common-ownership structures in which external owners *partially* participate in more than one competitor firm may induce a lower upward pricing pressure than acquisitions that give rise to cross-ownership structures.

6 Conclusions

This paper puts forward proposals that suggest how to improve the two most traditional indicators – the Helfindahl-Hirschman Index and the Gross Upward Price Pressure Index – used by competition agencies, typically in phase I-type of investigations, to screen potential anti-competitive unilateral effects regarding partial horizontal acquisitions. The proposed generalized indicators can deal with all types of acquisitions that may lessen competition in the industry: acquisitions by owners that are internal to the industry (rival firms) and engage in cross-ownership, as well as acquisitions by owners that are external to the industry and engage in common-ownership. Furthermore, these indicators can deal with direct and indirect acquisitions, which may or may not correspond to control, and nest full mergers as a special case.

We provide an empirical application of the two indicators to several acquisitions in the wet shaving industry. The results seem to suggest that (i) a full merger induces higher unilateral anti-competitive effects than a partial controlling acquisition involving the same firms, (ii) a partial controlling acquisition induces higher unilateral anti-competitive effects than a partial non-controlling acquisition involving the same firms and the same financial stakes, and (iii) an acquisition by owners that are internal to the industry induces higher unilateral anti-competitive effects than an acquisition (involving the same firms and the same stakes) by external owners that participate in more than one competitor firm.

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Appendix A

The Normalized Weight Matrix

In this appendix, we present the step-by-step computation details of the post-acquisition normalized weight matrix for each of the cases considered in our application.

Gillette Acquires a 100% Voting Equity Interest in Wilkinson Sword

The (hypothetical) acquisition of 100% voting equity interest in Wilkinson Sword by Gillette gives rise to a cross-ownership structure in the industry. Comparing with the pre-acquisition structure, this implies changes to matrices $\mathbf{F}^*$ and $\mathbf{C}^*$, as well as to matrices $\mathbf{F}$ and $\mathbf{C}$.

We address first the former. Let $\tilde{\mathbf{F}}^*$ and $\tilde{\mathbf{C}}^*$ denote the cross-ownership matrices *post-acquisition.* The diagonal elements are, by definition, zero. The off-diagonal elements that refer to the financial and corporate control cross-ownership of *or* in American Safety Razor, BIC, and Warner-Lambert remain unchanged: $\tilde{\phi}_{jg} = \phi_{jg}$ and $\tilde{\gamma}_{jg} = \gamma_{jg}$ for $(j \vee g) \in \{1, 2, 3\}$. Further, the financial and corporate control cross-ownership stakes of Wilkinson Sword on Gillette also remain unchanged: $\tilde{\phi}_{4,5} = \phi_{4,5}$ and $\tilde{\gamma}_{4,5} = \gamma_{4,5}$. However, the financial and corporate control cross-ownership stakes of Gillette on Wilkinson Sword increase to $\tilde{\phi}_{5,4} = 1$ (since the operation involves the acquisition of 100% equity interest) and to $\tilde{\gamma}_{5,4} = 1$ (since the equity interest acquisition allows Gillette to fully determine the decisions of Wilkinson Sword), respectively. This implies the following post-acquisition $\tilde{\mathbf{F}}^*$ and $\tilde{\mathbf{C}}^*$ matrices:

$$\tilde{\mathbf{F}}^* = \begin{bmatrix} 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 1.000 & 0.000 \end{bmatrix} \tilde{\mathbf{C}}^* = \begin{bmatrix} 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 1.000 & 0.000 \end{bmatrix}.$$

We now address the latter. Let $\tilde{\mathbf{F}}$ and $\tilde{\mathbf{C}}$ denote the common-ownership matrices *post-acquisition.* All elements relative to the financial and corporate control ownership stakes on American Safety Razor, BIC, Warner-Lambert, and Gillette remain unchanged: $\tilde{\phi}_{kj} = \phi_{kj}$ and $\tilde{\gamma}_{kj} = \gamma_{kj}$ for $j \in \{1, 2, 3, 5\}$ and all $k \in \Theta \backslash \Im$. Further, the financial and corporate control ownership stakes on Wilkinson Sword remain unchanged *for all external owners except Eemland*: $\tilde{\phi}_{k4} = \phi_{k4}$ and $\tilde{\gamma}_{k4} = \gamma_{k4}$ for all $k \in \Theta \backslash \Im$ and $k \neq \{16\}$. However, the financial and corporate control ownership stakes of Eemland on Wilkinson Sword are reduced to $\tilde{\phi}_{16,4} = 0$ (since Eemland sells the full 100% equity interest in the operation) and to $\tilde{\gamma}_{16,4} = 0$ (since the equity interest transacted involved voting), respectively. This implies the following post-acquisition $\tilde{\mathbf{F}}$ and $\tilde{\mathbf{C}}$ matrices:

$$\tilde{\mathbf{F}} = \begin{bmatrix} 0.144 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.124 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.078 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.070 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.061 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.051 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.472 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.777 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.223 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.108 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.060 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.832 \end{bmatrix} \quad \tilde{\mathbf{C}} = \begin{bmatrix} 0.144 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.124 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.078 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.070 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.061 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.051 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.472 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.777 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.223 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.108 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.060 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.832 \end{bmatrix}.$$

Having constructed matrices $\tilde{\mathbf{C}}$ and $\tilde{\mathbf{F}}$, we have all the necessary information to compute the post-acquisition matrices $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{B}}$. In our application, this computation yields:

$$\tilde{\mathbf{A}} = \begin{bmatrix} 0.276 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.653 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.707 \end{bmatrix} \quad \tilde{\mathbf{B}} = \begin{bmatrix} 3.621 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 1.530 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 1.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 1.413 \end{bmatrix}.$$

Finally, we can compute the post-acquisition weight matrix $\tilde{\mathbf{L}}$ and the post-acquisition normalized weight matrix $\tilde{\mathbf{W}}$. Again, to do so, we just make use of matrices $\tilde{\mathbf{F}}^{*}$, $\tilde{\mathbf{C}}^{*}$, $\tilde{\mathbf{C}}$, $\tilde{\mathbf{F}}$ and $\tilde{\mathbf{B}}$.

This computation yields:

$$\tilde{\mathbf{L}} = \begin{bmatrix} 0.276 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.653 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 1.000 & 1.000 \\ 0.000 & 0.000 & 0.000 & 0.707 & 0.707 \end{bmatrix} \quad \tilde{\mathbf{W}} = \begin{bmatrix} 1.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 1.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 1.000 & 1.000 \\ 0.000 & 0.000 & 0.000 & 1.000 & 1.000 \end{bmatrix}.$$

Gillette Acquires a 22.9% Voting Equity Interest in Wilkinson Sword

The (hypothetical) acquisition of 22.9% *voting* equity interest in Wilkinson Sword by Gillette gives rise to a *partial* cross-ownership structure in the industry, in which Gillette and Eemland, an internal and an external owner, respectively, *share* financial and corporate control interests in Wilkinson Sword. Comparing with the pre-acquisition structure, this implies changes to matrices $\mathbf{F}^*$ and $\mathbf{C}^*$, as well as to matrices $\mathbf{F}$ and $\mathbf{C}$.

We address first the former. Let $\tilde{\mathbf{F}}^*$ and $\tilde{\mathbf{C}}^*$ denote the cross-ownership matrices *post-acquisition*. The diagonal elements are, by definition, zero. The off-diagonal elements that refer to the financial and corporate control cross-ownership of *or* in American Safety Razor, BIC, and Warner-Lambert remain unchanged: $\tilde{\phi}_{jg} = \phi_{jg}$ and $\tilde{\gamma}_{jg} = \gamma_{jg}$ for $(j \vee g) \in \{1, 2, 3\}$. Further, the financial and corporate control cross-ownership stakes of Wilkinson Sword on Gillette also remain unchanged: $\tilde{\phi}_{4,5} = \phi_{4,5}$ and $\tilde{\gamma}_{4,5} = \gamma_{4,5}$. However, the financial and corporate control cross-ownership stakes of Gillette on Wilkinson Sword increase to $\tilde{\phi}_{5,4} = 0.229$ (since the operation involves a partial acquisition of 22.9% equity interest) and to $\tilde{\gamma}_{5,4} = 0.229$ (since the equity interest transacted involves voting), respectively. This implies the following post-acquisition $\tilde{\mathbf{F}}^*$ and $\tilde{\mathbf{C}}^*$ matrices:

$$\tilde{\mathbf{F}}^* = \begin{bmatrix} 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.229 & 0.000 \end{bmatrix} \quad \tilde{\mathbf{C}}^* = \begin{bmatrix} 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.229 & 0.000 \end{bmatrix}.$$

We now address the latter. Let $\tilde{\mathbf{F}}$ and $\tilde{\mathbf{C}}$ denote the common-ownership matrices *post-*

acquisition. All elements relative to the financial and corporate control ownership stakes on American Safety Razor, BIC, Warner-Lambert, and Gillette remain unchanged: $\tilde{\phi}_{kj} = \phi_{kj}$ and $\tilde{\gamma}_{kj} = \gamma_{kj}$ for $j \in \{1, 2, 3, 5\}$ and all $k \in \Theta \backslash \Im$. Further, the financial and corporate control ownership stakes on Wilkinson Sword remain unchanged *for all external owners except Eemland*: $\tilde{\phi}_{k4} = \phi_{k4}$ and $\tilde{\gamma}_{k4} = \gamma_{k4}$ for all $k \in \Theta \backslash \Im$ and $k \neq \{16\}$. However, the financial and corporate control ownership stakes of Eemland on Wilkinson Sword are reduced to $\tilde{\phi}_{16,4} = 0.771$ (since Eemland sells only 22.9% equity interest in the operation) and to $\tilde{\gamma}_{16,4} = 0.771$ (since the equity interest transacted involves voting), respectively. This implies the following post-acquisition $\tilde{\mathbf{F}}$ and $\tilde{\mathbf{C}}$ matrices:

$$\tilde{\mathbf{F}} = \begin{bmatrix} 0.144 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.124 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.078 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.070 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.061 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.051 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.472 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.777 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.223 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.771 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.108 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.060 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.832 \end{bmatrix} \quad \tilde{\mathbf{C}} = \begin{bmatrix} 0.144 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.124 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.078 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.070 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.061 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.051 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.472 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.777 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.223 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.771 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.108 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.060 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.832 \end{bmatrix}.$$

Having constructed matrices $\tilde{\mathbf{C}}$ and $\tilde{\mathbf{F}}$, we have all the necessary information to compute the post-acquisition matrices $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{B}}$. In our application, this computation yields:

$$\tilde{\mathbf{A}} = \begin{bmatrix} 0.276 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.653 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.594 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.707 \end{bmatrix} \quad \tilde{\mathbf{B}} = \begin{bmatrix} 3.621 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 1.530 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 1.682 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 1.414 \end{bmatrix}.$$

Finally, we can compute the post-acquisition weight matrix $\tilde{\mathbf{L}}$ and the post-acquisition normalized weight matrix $\tilde{\mathbf{W}}$. To do so, we just make use of matrices $\tilde{\mathbf{F}}^*$, $\tilde{\mathbf{C}}^*$, $\tilde{\mathbf{C}}$, $\tilde{\mathbf{F}}$ and $\tilde{\mathbf{B}}$. This computation yields:

$$\tilde{\mathbf{L}} = \begin{bmatrix} 0.276 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.653 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.647 & 0.229 \\ 0.000 & 0.000 & 0.000 & 0.162 & 0.707 \end{bmatrix} \tilde{\mathbf{W}} = \begin{bmatrix} 1.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 1.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 1.000 & 0.354 \\ 0.000 & 0.000 & 0.000 & 0.229 & 1.000 \end{bmatrix}.$$

Gillette Acquires a 22.9% Nonvoting Equity Interest in Wilkinson Sword

The acquisition of 22.9% nonvoting equity interest in Wilkinson Sword by Gillette gives rise to a *partial* cross-ownership structure in the industry, in which Eemland, an external owner, fully controls Wilkinson Sword, but shares the financial interest in the firm with Gillette, an internal owner. Comparing with the pre-acquisition structure, and since the equity interest transacted involves no voting, this implies that there are no changes in control and thus that matrices $\mathbf{C}^*$ and $\mathbf{C}$ remain unchanged. However, it does imply changes to matrices $\mathbf{F}^*$ and $\mathbf{F}$, which capture financial interests.

We address first the former. Let $\tilde{\mathbf{F}}^*$ denote the financial cross-ownership matrix *post-acquisition*. The diagonal elements are, by definition, zero. The off-diagonal elements that refer to the financial cross-ownership of *or* in American Safety Razor, BIC, and Warner-Lambert remain unchanged: $\tilde{\phi}_{jg} = \phi_{jg}$ for $(j \vee g) \in \{1, 2, 3\}$. Further, the financial cross-ownership stake of Wilkinson Sword on Gillette also remains unchanged: $\tilde{\phi}_{4,5} = \phi_{4,5}$. However, the financial cross-ownership stake of Gillette on Wilkinson Sword increases to $\tilde{\phi}_{5,4} = 0.229$ (since the operation involves a partial acquisition of 22.9% equity interest). This implies the following post-acquisition $\tilde{\mathbf{F}}^*$ matrix:

$$\tilde{\mathbf{F}}^* = \begin{bmatrix} 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.229 & 0.000 \end{bmatrix}.$$

We now address the latter. Let $\tilde{\mathbf{F}}$ denote the financial common-ownership matrix *post-acquisition.* All elements relative to the financial ownership stakes on American Safety Razor, BIC, Warner-Lambert, and Gillette remain unchanged: $\tilde{\phi}_{kj} = \phi_{kj}$ for $j \in \{1, 2, 3, 5\}$ and all $k \in \Theta \backslash \Im$. Further, the financial ownership stakes on Wilkinson Sword remain unchanged *for all external owners except Eemland*: $\tilde{\phi}_{k4} = \phi_{k4}$ for all $k \in \Theta \backslash \Im$ and $k \neq \{16\}$. However, the financial ownership stake of Eemland on Wilkinson Sword is reduced to $\tilde{\phi}_{16,4} = 0.771$ (since Eemland sells only 22.9% equity interest in the operation). This implies the following post-acquisition $\tilde{\mathbf{F}}$ matrix:

$$\tilde{\mathbf{F}} = \begin{bmatrix} 0.144 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.124 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.078 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.070 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.061 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.051 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.472 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.777 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.223 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.771 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.108 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.060 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.832 \end{bmatrix}.$$

Having constructed matrix $\tilde{\mathbf{F}}$, we can use it, jointly with matrix $\mathbf{C}$, which remains unchanged, to compute the post-acquisition matrices $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{B}}$. In our application, this computation yields:

$$\tilde{\mathbf{A}} = \begin{bmatrix} 0.276 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.653 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.771 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.707 \end{bmatrix} \quad \tilde{\mathbf{B}} = \begin{bmatrix} 3.621 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 1.530 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 1.297 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 1.415 \end{bmatrix}.$$

Finally, we can compute the post-acquisition weight matrix $\tilde{\mathbf{L}}$ and the post-acquisition normalized weight matrix $\tilde{\mathbf{W}}$. To do so, we just make use of matrices $\tilde{\mathbf{F}}^{*}$, $\mathbf{C}^{*}$, $\mathbf{C}$, $\tilde{\mathbf{F}}$ and $\tilde{\mathbf{B}}$. This computation yields:

$$\tilde{\mathbf{L}} = \begin{bmatrix} 0.276 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.653 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.771 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.162 & 0.707 \end{bmatrix} \quad \tilde{\mathbf{W}} = \begin{bmatrix} 1.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 1.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 1.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.229 & 1.000 \end{bmatrix}.$$

Berkshire Hathaway Acquires a 22.9% Nonvoting Equity Interest in Wilkinson Sword

The (hypothetical) acquisition of 22.9% *nonvoting* equity interest in Wilkinson Sword by Berkshire Hathaway gives rise to a *partial* common-ownership structure in the industry, in which Berkshire Hathaway, an external owner, participates in two competing firms, Gillette and Wilkinson Sword. Comparing with the pre-acquisition structure, and since internal owners are not at all involved in the operation, this implies that matrices $\mathbf{F}^{*}$ and $\mathbf{C}^{*}$ remain unchanged. Further, since the equity interest transacted involves no voting, this implies that matrix $\mathbf{C}$ also remains unchanged. However, it does imply changes to matrices $\mathbf{F}$, which capture financial ownership by external owners. Let $\tilde{\mathbf{F}}$ denote the financial common-ownership matrix *post-acquisition.* All elements relative to the financial ownership stakes on American Safety Razor, BIC, Warner-Lambert, and Gillette remain unchanged: $\tilde{\phi}_{kj} = \phi_{kj}$ for $j \in \{1, 2, 3, 5\}$ and all $k \in \Theta \backslash \Im$. Further, the financial ownership stakes on Wilkinson Sword remain unchanged *for all external owners except Eemland and Berkshire Hathaway*: $\tilde{\phi}_{k4} = \phi_{k4}$ for all $k \in \Theta \backslash \Im$ and $k \neq \{16, 17\}$. However, the financial ownership stake of Eemland on Wilkinson Sword is reduced to $\tilde{\phi}_{16,4} = 0.771$, while the corresponding financial ownership stake of Berkshire Hathaway increases to $\tilde{\phi}_{17,4} = 0.229$ (since Eemland sells 22.9% equity interest in the operation). As a consequence the post-acquisition $\tilde{\mathbf{F}}$ and $\tilde{\mathbf{C}}$ matrices are given by:

$$\tilde{\mathbf{F}} = \begin{bmatrix} 0.144 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.124 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.078 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.070 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.061 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.051 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.472 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.777 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.223 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.771 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.229 & 0.108 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.060 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.832 \end{bmatrix}.$$

Having constructed matrices $\tilde{\mathbf{F}}$, we can use it, jointly with matrix $\mathbf{C}$, which remains unchanged, to compute the post-acquisition matrices $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{B}}$. In our application, this computation yields:

$$\tilde{\mathbf{A}} = \begin{bmatrix} 0.276 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.653 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.771 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.707 \end{bmatrix} \quad \tilde{\mathbf{B}} = \begin{bmatrix} 3.621 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 1.530 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 1.297 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 1.413 \end{bmatrix}.$$

Finally, we can compute the post-acquisition weight matrix $\tilde{\mathbf{L}}$ and the post-acquisition normalized weight matrix $\tilde{\mathbf{W}}$. To do so, we just make use of matrices $\mathbf{F}^*$, $\mathbf{C}^*$, $\mathbf{C}$, $\tilde{\mathbf{F}}$ and $\tilde{\mathbf{B}}$. This computation yields:

$$\tilde{\mathbf{L}} = \begin{bmatrix} 0.276 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.653 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.771 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.025 & 0.707 \end{bmatrix} \tilde{\mathbf{W}} = \begin{bmatrix} 1.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 1.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 1.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.035 & 1.000 \end{bmatrix}.$$