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**Leontief versus Ghosh: two faces of the
same coin**

Antonio Manresa (CREB, XREAP)
Ferran Sancho

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Antonio Manresa

Departament de Teoria Econòmica
Universitat de Barcelona, Spain
manresa@ub.edu

Ferran Sancho

Department of Economics
Universitat Autònoma de Barcelona, Spain
ferran.sancho@uab.cat

Abstract

In this paper we explore the sectoral and aggregate implications of some endogeneization rules (i.e. on value-added and final demand) which have been common in the Leontief model and have been recently proposed in the Ghosh model. We detect that these rules may give rise in both models to some allegedly pathological behavior in the sense that sectoral or aggregate output, very often, may not follow the logical and economically expected direct relationship with some underlying endogenous variables—namely, output and value-added in the Ghosh model and output and consumption in the Leontief model. Because of the common mathematical structure, whatever is or seems to be pathological in the Ghosh model also has a symmetric counterpart in the Leontief model. These would not be good news for the inner consistency of these linear models. To avoid such possible inconsistencies, we propose new and simple endogeneization rules that have a sound economic interpretation.

Keywords: demand-driven model, supply driven model, extended linear models.

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1. Introduction

Both the classical Leontief and Ghosh models have been subjected to criticism because of the way they handle the role of some key variables. In Ghosh, for instance, output depends on value-added and a change in a given sector's value-added will rise output everywhere with no need for additional value-added in the rest of sectors. There is more output overall but value-added is, in general, insensitive to this fact. In Leontief, in turn, output depends on final demand (say, consumption for simplicity). Any change in final consumption will raise output everywhere but this will not get reflected in any additional demand for consumption, even though overall there is more output and income around. We may conclude that consumption is insensitive to income, a fact that also runs against standard economic logic.

To fix these ideas, and to simply matters, let's consider the following two accounting expressions coming from a typical (or aggregated) 2×2 input-output table:

$$\left. \begin{aligned} x(1) &= z(1,1) + z(1,2) + d(1,h) + d(1,f) \\ x(2) &= z(2,1) + z(2,2) + d(2,h) + d(2,f) \end{aligned} \right\} \quad (\text{L.1})$$

$$\left. \begin{aligned} x(1) &= z(1,1) + z(2,1) + v(1,h) + v(1,f) \\ x(2) &= z(1,2) + z(2,2) + v(2,h) + v(2,f) \end{aligned} \right\} \quad (\text{G.1})$$

Where all magnitudes can be considered in monetary or in (conveniently re-defined) physical units, and they are labeled in the following way:

$x(i)$: total production in sector i ,

$z(i, j)$: intermediate inputs flowing from sector i to sector j ,

$d(i, k)$: final demand for good i with home ($k=h$) or foreign origin ($k=f$),

$v(i, k)$: value-added applied in sector i with home ($k=h$) or foreign origin ($k=f$).

Clearly, accounting expressions (L.1) correspond to the revenues that each sector receives, while accounting expressions (G.1) are the payments of each sector to the suppliers of resources. Revenues and payments have to be equal for each sector.

A matrix representation of the previous accounting balances of resources and/or payments comes in handy:

$$\mathbf{x} = \mathbf{Z} \cdot \mathbf{e} + \mathbf{d}^h + \mathbf{d}^f \quad (\text{L.2})$$

$$\mathbf{x} = \mathbf{Z}' \cdot \mathbf{e} + \mathbf{v}^h + \mathbf{v}^f \quad (\text{G.2})$$

where \mathbf{Z} (and its transpose, \mathbf{Z}') is the standard matrix of intermediate inputs flows. The column vector \mathbf{e} is the unit vector and the rest of magnitudes, also column vectors, have the obvious components as appearing in the previous (L.1) and (G.1) expressions.

The well-known Leontief (L) and Ghosh (G) models can be obtained by defining the following ‘technical’ coefficients:

$$a(i, j) = z(i, j) / x(j)$$

$$b(i, j) = z(i, j) / x(i)$$

Which can be introduced into expressions (L.2) and (G.2), and solving for the vector of total production we obtain the classical expressions:

$$\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1} (\mathbf{d}^h + \mathbf{d}^f) \quad (\text{L.3})$$

$$\mathbf{x} = (\mathbf{I} - \mathbf{B})^{-1} (\mathbf{v}^h + \mathbf{v}^f)$$

where $\mathbf{A} = [a(i, j)]$ and $\mathbf{B} = [b(j, i)]$ are the 2×2 matrices endowed with the above ‘technical’ coefficients¹. And $a(i, j)$ denotes its i row and j column element, its interpretation being the amount of good i used to produce one unit of good j . While, in contrast, $b(j, i)$ denotes its j column and i row element and can be interpreted as the fraction of j used in the production of good i .

The extended versions of these models, to overcome these unwanted properties, establishes a relationship between output \mathbf{x} and final consumption \mathbf{d} (or a part of it) by way of introducing the missing connection $\mathbf{d} = \phi_L(\mathbf{v})$ between consumption \mathbf{d} and value-added \mathbf{v} . In Ghosh the relationship between output and value-added (or a part of it) is made explicit linking the required value-added in each sector to put out a unit of aggregate consumption, i.e. $\mathbf{v} = \phi_G(\mathbf{d})$.

$$\text{(Extended Leontief)} \quad \mathbf{x} : \mathbf{v}_L \rightarrow \mathbf{v}_L(\mathbf{h}) \rightarrow \mathbf{d}_L \rightarrow \mathbf{d}_L(\mathbf{h})$$

$$\text{(Extended Ghosh)} \quad \mathbf{x} : \mathbf{d}_G \rightarrow \mathbf{d}_G(\mathbf{h}) \rightarrow \mathbf{v}_G \rightarrow \mathbf{v}_G(\mathbf{h})$$

¹ Observe that our \mathbf{B} matrix is in fact the transposed of the standard Ghosh matrix. This allows us to have throughout the same normalized vector post-multiplication notation.

Guerra and Sancho (2011) are the first to propose a Ghosh extended model version by incorporating value-added as an endogenous variable. They posit final consumption for each good to be a positive fraction of the corresponding sectorial output making the consumption coefficients fully coherent with the output allocation coefficients while sectorial value-added depends on aggregate consumption. In their version of the Ghosh model, the only exogenous variable is what we may call a public input and therefore production is driven from this supply side variable of the economy.

In a recent article, however, Oosterhaven (2012) points out an allegedly anomalous property of the so called extended Ghosh model by means of a numerical example based in the 3×3 input-output table of Guerra and Sancho (2011). The pinpointed anomaly is the possibility of having an inverse relationship between production and value-added in a sector. This means that we may have that total production in a sector increases while its value-added diminishes. This possibility is, according to Oosterhaven, even worse than leaving that variable unchanged. Furthermore, one can also see in the same example that total production of the economy may go in the opposite direction of total value-added. We share with Osterhaven that this lack of realism in the production process, may indeed be a serious criticism for any model thus built. Furthermore, this author also shows, by some numerical examples, that such an undesirable property no longer applies to the demand-driven Leontief extended model.

The main purpose of this paper is to discuss the logic and structure of these unwanted and anomalous properties - going beyond results based only on numerical examples -. We do that in section 3, by analyzing the general and explicit solutions of the equilibrium values of both, Ghosh and Leontief models, in a 2×2 scenario; sufficient to identify and characterize the main issues. Since both linear models share the same mathematical structure, whatever property –good or bad– of the Ghosh model may have, we find a precise and formal counterpart in the Leontief model.

Unfortunately, what we find is that the aforementioned unwanted properties of the extended Ghosh model, i.e. the pathological behavior of a negative relationship between some sectorial production and its corresponding value-added, is **unavoidable**. This means that the model is biased in the sense that we will always observed this kind of behavior, at least in one sector, no matter what are the parameters of the model. However, at the aggregate, the same negative relationship may happen for some parameter values.

However, this is not the end of the story. With respect to the extended Leontief model, we can easily see from our analysis that the same troublesome relationship between production and value added, at the aggregate level, become true. Furthermore, at the sectorial level, we also show that at least for one sector we will **always** have a negative relationship between production and domestic demand. That systematic type of behavior makes this model biased towards a relationship (i.e. sectorial production and its domestic final demand) that economic reality do not support.

These are bad news, because they may point to a theoretical weakening of both models, and hence of some of the results ensuing from their possible empirical applications.

In Section 2 we comment and debate Oosterhaven's viewpoint. We "transpose" the 3×3 numerical input-output matrix appearing in Guerra and Sancho (2011) to exemplify that the anomalies in the extended Ghosh model are immediately transferred to the extended Leontief model. Thanks to this transposing trick, therefore, our Leontief example fully and numerically coincides with Oosterhaven's example for the extended Ghosh model. No actual calculations are needed. From the discussion it becomes clear that all these apparent negative traits of the extended models are based on a particular way of endogeneizing the driving forces in each model.

An open question thus arises: Is there any other way to incorporate final consumption and value-added endogenously in these models that may avoid such assumedly bad properties? This issue is explored in Section 4 where we show that a way to avoid it in the Ghosh model is by somewhat mimicking Leontief's extended model, and other way around. The aggregate problem still continuous in both models, but can also be trivially avoided by assuming that the share of value-added on output is the same for all sectors. Finally, Section 5 concludes.

2. A "transposed" countering example based on the Guerra-Sancho IO table.

In this section we first want to clarify the different nature of the numerical exercises performed by Guerra and Sancho (2011) and later on by Oosterhaven (2012), taking advantage that both use the same input-output table appearing in Guerra and Sancho (2011). Secondly, we show, also by means of numerical exercises, that the extended Leontief model can be subject too to major criticisms concerning the equilibrium relationship between a change in production and domestic final demand –at the sectorial level– and a change in production and value added –at the aggregate level. The example we use is built in a straightforward manner simply by "transposing" the input-output table of Guerra and Sancho (2011).

Regarding the first issue, it can be seen that Oosterhaven (2012, page 107) produces the results of his Table 2 based on the IO table of Guerra and Sancho (2011) and uses them to lay down the foundations of his criticism of the extended Ghosh model. His argument,

based on a numerical example, examines the impact of reallocating resources among sectors, i.e. neutrally moving five units of final demand in extended L, and similarly for five units of value-added in extended G. For instance, in the last scenario of his Table 2 – reproduced below with two decimal digits as our Table 2b– five units of public services shift from sector 3 to sector 2. As a result, he remarks that output in sectors 1 and 2 go up (+0.63 and +4.82) whereas their value-added go down (-0.17 and -0.09). He also finds that total output goes up (+1.92) with total value-added in the economy going down (-0.60).

As a result of the implementation of these two simultaneous shocks, i.e. the neutral shift, he concludes that the above figures show that the equilibrium magnitudes of the extended Ghosh model present a perverse or anomalous behavior. What Guerra and Sancho (2011) examine, however, is different. They check the response of equilibrium magnitudes in a standard comparative statics exercise, whereby a parameter is shocked and equilibrium is readjusted. Under this scrutiny, they verify that magnitudes in the extended Ghosh model behave in the right and expected way following the *single* exogenous shock in one of the driving variables. In marked contrast, Oosterhaven calculates his outcomes introducing *two* shocks at the same time.

Thanks to the additive separability of the equilibrium magnitudes that follow from linearity, we will run Oosterhaven’s simulation in two separate steps and we will verify that all magnitudes behave correctly in the isolated runs (as Guerra and Sancho demonstrate). When these separate and well-behaved equilibria are aggregated, we reencounter Oosterhaven’s figures and the alleged anomaly as a result.

Let us consider to this effect what would be, according to Oosterhaven, the most anomalous case, namely the last scenario reproduced in Table 2b. We proceed to solve the extended Ghosh model in two runs, first introducing a five unit increase in public services in sector 2, and then reducing in five units the corresponding level in sector 3. The results appear in our Table 1 below.

TABLE 1. Output, value-added and consumption, same scenarios as in Oosterhaven (2012)

Scenario	Extended Ghosh model								
	(+5 in 2)			(-5 in 3)			(+5 in 2 & -5 in 3)		
Sector	Δx	Δv	Δf	Δx	Δv	Δf	Δx	Δv	Δf
1	+5.41	+1.28	+1.89	- 4.78	-1.45	-1.67	+0.63	-0.17	+0.22
2	+9.04	+0.64	+0.45	- 4.22	-0.72	-0.21	+4.82	-0.08	+0.24
3	+7.06	+2.55	+2.12	-10.59	-2.89	-3.18	- 3.53	-0.34	- 1.06
Total	+21.51	+4.47	+4.47	-19.59	-5.06	-5.06	+1.92	-0.60	-0.60

Source: Interindustry table from Table 2a in Guerra and Sancho (2011)

A first observation from the two first columns is that all equilibrium magnitudes behave nicely. The external addition of five new units of the public input in sector 2 gives rise to additional output, value-added and consumption in *all* sectors, with a higher effect in the output of the receiving sector 2 itself (+9.04). Similarly, the reduction of five units in sector 3 reduces *all* relevant magnitudes, with a greater impact in the fall of output of the now less endowed sector 3 (-10.59). Of course, and because of additive separability, the aggregation of the block of columns 1 and 2 in Table 1 here yields Oosterhaven's last block of columns in his Table 2, which is the third block of data in our Table 1. When positives and negatives are added up, the corresponding addition results end up being positive or negative, depending on the relative strengths of the direct, indirect and induced effects implicit in the separated results. This is what we see turns out to occur in the third block of columns in our Table 1. Oosterhaven's anomalies are simply the result of the interplay of two counteracting equilibrium states. We hope this exercise will help to clarify the different nature of the numerical exercises performed by the aforementioned authors.

We now turn our focus to the symmetrical critique of the extended Leontief model that Oosterhaven did not pursue. If we stick for the time being with Oosterhaven's type of exercise, we see that in his scenario 1 in Table 2b, the output for good 1 goes up (+4.75) while its demand goes down (-0.77). And the same type of behavior can be observed in scenario 2 for the second sector. We surmise that this behavior should be considered as equally strange or anomalous as when output is reported to go up while value-added goes down.

As a matter of fact, this anomalous behavior of the extended Leontief model can become even worse, as we will see now. Just take the input-output table in Guerra and Sancho (2011) and transpose it. Using this trick, we do not need to recalculate any results, the same data as in Oosterhaven's Table 2 will do (but the skeptical reader can do the algebra and verify that all checks out correct). We only need to reinterpret the numbers and the variables that appear in our Table 2a below.

Observe now the block of columns for the second scenario in Table 2a. Final consumption for goods 1 and 2 goes down (-0.17 and -0.09) whereas their respective sectorial output goes up (+0.63 and +4.82). Even worse, total output increases (+1.92) with aggregate demand going overall down (-0.60), a very worrying anomaly indeed for any sensible economic model. There is less demand for cars, gasoline and goods in general (-0.60) and less value-added (-0.60) around but the economy produces more cars, gasoline and goods!. Should the extended Ghosh results be really anomalous –following Oosterhaven's viewpoint– then so would be the extended Leontief results and according to him both models would be equally doomed.

Let us now define an 'anomaly' as a situation where equilibrium magnitudes in a given row do not share the same sign. These anomalous results can be easily located in Table 2a-b using the clear yellow color (for Ghosh) and the blue color (for Leontief).

We look first at the total rows, which show the aggregate behavior of both models. The same type of anomaly is observed and shared in both extended models (block of columns 2 in Table 2a for extended L and block of columns 4 in Table 2b for extended G) with aggregate output increasing with value-added and consumption decreasing. Hence both models may suffer the exact same type of anomaly in the aggregate. We will explore further this issue in Section 3 below.

Let us now take a look at the sectorial performance of both extended models. From the figures in Table 2a-b we observe that in *all* simulations there will be at least a sector behaving abnormally. In the Leontief case, changes in production and value-added always behave correctly with the same sign, but final consumption may not do so and have a different sign. In the case of Ghosh, we see that changes in production and final consumption move always in the same direction, but value-added may not behave so. Our analysis below of these alleged anomalies, discussed in a 2×2 general case for both models, can therefore be seen that extend to the 3×3 case, as Oosterhaven (2012) first pointed out but for the Ghosh model only.

TABLE 2a. Output, value-added and final demand for the two extended models, same scenarios as in Oosterhaven (2012)

	Extended Leontief model						Extended Ghosh model					
Scenario	(+5 in 1 & -5 in 3)			(+5 in 2 & -5 in 3)			(+5 in 1 & -5 in 3)			(+5 in 2 & -5 in 3)		
Sector	Δx	Δv	Δf	Δx	Δv	Δf	Δx	Δv	Δf	Δx	Δv	Δf
1	+8.20	+2.87	+0.64	+0.63	+0.22	-0.17	+4.75	-0.77	+0.95	-2.35	-1.41	-0.47
2	+1.66	+0.09	+0.32	+4.82	+0.24	-0.09	-1.65	-0.11	-0.17	+1.95	-0.20	+0.20
3	-2.36	-0.71	+1.29	-3.53	-1.06	-0.34	-5.80	-0.67	-2.32	-6.40	-1.21	-2.55
Total	+7.50	+2.25	+2.25	+1.92	-0.60	-0.60	-2.70	-1.54	-1.54	-6.75	-2.82	-2.82

Source: built from "transposing" the IO data from Table 2a in Guerra and Sancho (2011)

TABLE 2b. Oosterhaven's results with two decimal digits

	Extended Leontief model						Extended Ghosh model					
Scenario	(+5 in 1 & -5 in 3)			(+5 in 2 & -5 in 3)			(+5 in 1 & -5 in 3)			(+5 in 2 & -5 in 3)		
Sector	Δx	Δv	Δf	Δx	Δv	Δf	Δx	Δv	Δf	Δx	Δv	Δf
1	+4.75	+0.95	-0.77	-2.35	-0.47	-1.41	+8.20	+0.64	+2.87	+0.63	-0.17	+0.22
2	-1.65	-0.17	-0.11	+1.95	+0.20	-0.20	+1.66	+0.32	+0.09	+4.82	-0.09	+0.24
3	-5.80	-2.32	-0.67	-6.40	-2.55	-1.21	-2.36	+1.29	-0.71	-3.53	-0.34	-1.06
Total	-2.70	-1.54	-1.54	-6.75	-2.82	-2.82	+7.50	+2.25	+2.25	+1.92	-0.60	-0.60

Source: Oosterhaven (2012), Table 2, pp.107, built from the given IO data in Guerra and Sancho (2012)

All this numerical evidence suggests the need to examine in a more formal way the equilibrium properties of the extended Ghosh and Leontief models as a way to identify and isolate the root of these possible anomalies. We proceed in Section 3, with the help of a simple 2×2 setup, enough for our purposes here, to find explicit expressions for the equilibrium relationships and categorize the different possibilities that may arise under the standard extensions of Ghosh and Leontief, while providing a way in both models to check for possible anomalies in behavior.

3. The 2×2 L-G models

3.1. The classical models

We will start by considering the following two accounting expressions coming from a typical (or aggregated) 2×2 input-output table:

$$\left. \begin{aligned} x(1) &= z(1,1) + z(1,2) + d(1,h) + d(1,f) \\ x(2) &= z(2,1) + z(2,2) + d(2,h) + d(2,f) \end{aligned} \right\} \quad (\text{L.1})$$

$$\left. \begin{aligned} x(1) &= z(1,1) + z(2,1) + v(1,h) + v(1,f) \\ x(2) &= z(1,2) + z(2,2) + v(2,h) + v(2,f) \end{aligned} \right\} \quad (\text{G.1})$$

All magnitudes can be considered in monetary or in (conveniently re-defined) physical units, and they are labeled in the following way:

$x(i)$: total production in sector i ,

$z(i, j)$: intermediate inputs flowing from sector i to sector j ,

$d(i, k)$: final demand for good i with home ($k=h$) or foreign origin ($k=f$),

$v(i, k)$: value-added applied in sector i with home ($k=h$) or foreign origin ($k=f$).

Clearly, accounting expressions (L.1) correspond to the revenues that each sector receives according to the use or employment of its production, while accounting expressions (G.1) are the payments of each sector to the suppliers of resources used in its production process. Since revenues and payments have to be equal for each sector, we see that corresponding sums in both expressions are the same.

A matrix representation of the previous accounting balances of resources and/or payments comes in handy:

$$\mathbf{x} = \mathbf{Z} \cdot \mathbf{e} + \mathbf{d}^h + \mathbf{d}^f \quad (\text{L.2})$$

$$\mathbf{x} = \mathbf{Z}' \cdot \mathbf{e} + \mathbf{v}^h + \mathbf{v}^f \quad (\text{G.2})$$

where \mathbf{Z} (and its transpose, \mathbf{Z}') is the standard matrix of intermediate inputs flows. The column vector \mathbf{e} is the unit vector and the rest of magnitudes, also column vectors, have the obvious components as appearing in the previous (L.1) and (G.1) expressions.

In order to “jump” from the previous accounting expressions to the well-known Leontief (L) and Ghosh (G) models we define the following ‘technical’ coefficients for output production and output allocation, respectively:

$$a(i, j) = z(i, j) / x(j)$$

$$b(i, j) = z(i, j) / x(i)$$

We now introduce these coefficients into expressions (L.2) and (G.2), which will allow us to rewrite them in matrix terms as:

$$\mathbf{x} = \mathbf{A} \cdot \mathbf{x} + \mathbf{d}^h + \mathbf{d}^f \quad (\text{L.3})$$

$$\mathbf{x} = \mathbf{B} \cdot \mathbf{x} + \mathbf{v}^h + \mathbf{v}^f \quad (\text{G.3})$$

where $\mathbf{A} = [a(i, j)]$ and $\mathbf{B} = [b(j, i)]$ are the 2×2 matrices endowed with the above ‘technical’ coefficients². Notice that these coefficients are related, for all i and j , in the following generic way:

$$a(i, j) \cdot x(j) = z(i, j) = b(i, j) \cdot x(i)$$

By construction, in matrix \mathbf{A} , $a(i, j)$ denotes its i row and j column element, its interpretation being the amount of good i used to produce one unit of good j . In matrix \mathbf{B} , in contrast, $b(j, i)$ denotes its j column and i row element and can be interpreted as the fraction of j used in the production of good i . The classical L and G models are those derived from (L.3) and (G.3), by assuming that coefficients in \mathbf{A} and \mathbf{B} are fixed parameters. The values of \mathbf{x} are the unknowns of these models, which can be obtained as a function of the rest of the variables, as defined above. For exogenously given vectors \mathbf{d} and \mathbf{v} we can solve for production \mathbf{x} and obtain the classical expressions for the Leontief and Ghosh reduced forms:

$$\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1} (\mathbf{d}^h + \mathbf{d}^f) \quad (\text{L.4})$$

$$\mathbf{x} = (\mathbf{I} - \mathbf{B})^{-1} (\mathbf{v}^h + \mathbf{v}^f) \quad (\text{G.4})$$

Observe that the two models have the same mathematical structure and so they must share the very same properties. Furthermore, from an empirically based point of view, we have to notice that both models can use either accounting structure to solve for the same initial

² Observe that our \mathbf{B} matrix is in fact the transposed of the standard Ghosh matrix. This allows us to have throughout the same normalized vector post-multiplication notation.

value \mathbf{x} . In other words, we may distinguish both models just because exogenous variables have different names and interpretations. As Dietzenbacher (1997) concluded: “The Ghosh models and the Leontief models can be viewed as each others mirror-image”. In the next sub-section we will see that the extended models can in fact be seen as “two faces of the same coin”.

3.2. The extended 2×2 models

3.2.1. General considerations

In the two classical models, the variables driving the models are exogenous, i.e. in the L model this is final demand whereas in the G model is value-added. A change in total production, due to any other change in the model characteristics, will leave such an exogenous variable unchanged. In expression (L.4) a change in foreign final demand \mathbf{d}^f will affect total output \mathbf{x} . Domestic demand \mathbf{d}^h , however, remains surprisingly insensible to the new equilibrium output despite the economic fact that producing more output would inevitably require more primary factors, or value-added in our terminology. This extra income in the form of value-added does not have any effect on final domestic demand, again a rather surprising fact. Similarly in (G.4), a change in external value-added \mathbf{v}^f (say, through immigration in sector 1) will impact total output \mathbf{x} in all sectors, but this output changes will have no effect whatsoever in the remaining levels of domestic value-added \mathbf{v}^h in both sectors. These insensibility properties of the two models put serious limitations to any model that aims to be considered either for theoretical purposes or for empirical applications and they have been widely discussed in the literature (Guerra and Sancho (2011) for the L model and most notably Oosterhaven (1988, 1989, 2012) and De Mesnard (2009), among others, for the G model).

A way to overcome such unwanted property has been to extend the classical models to make the exogenous variable, or more precisely some part of it, into an endogenous one. This procedure yields the so-called ‘extended’ L and G models in the literature. The idea is simply to add some extra conditions to the (L.4) and (G.4) equations in some sensible economic way. The general way to add more equations to the basic ones can be represented by the features of these two composed mappings:

$$\text{(Extended Leontief)} \quad \mathbf{x} : \mathbf{v}_L \rightarrow \mathbf{v}_L(\mathbf{h}) \rightarrow \mathbf{d}_L \rightarrow \mathbf{d}_L(\mathbf{h})$$

$$\text{(Extended Ghosh)} \quad \mathbf{x} : \mathbf{d}_G \rightarrow \mathbf{d}_G(\mathbf{h}) \rightarrow \mathbf{v}_G \rightarrow \mathbf{v}_G(\mathbf{h})$$

In the case of the extended Ghosh model (with variables identified by the submarker G), we have that value-added is a function of output \mathbf{x} through the composed mapping $\mathbf{v}_G(\mathbf{h}) = (\mathbf{v}_G \circ \mathbf{d}_G)(\mathbf{x})$. This means that given an amount of production \mathbf{x} we obtain a certain amount of internal demand $\mathbf{d}_G(\mathbf{h})$ which, in turn, determines the home value-added of the

economy $\mathbf{v}_G(\mathbf{h})$. In particular, the way that this mapping is specified in Guerra and Sancho (2011) in their extended Ghosh model, for $j=1, 2$, can be seen to be the following:

$$d_G(j, h) = d_G(x_G(j)) = \alpha(j) \cdot x_G(j) \quad (1a)$$

$$v_G(j, h) = v_G(d_G(j, h)) = \beta(j) \cdot (d_G(1, h) + d_G(2, h)) \quad (1b)$$

The first mapping tell us that home final demand in each sector is a fixed share of total sectorial production, while the second mapping stipulates that value-added in each sector is a fixed share of the total amount of home final demand. Putting the two mappings together we obtain:

$$v_G(j, h) = v_G(d_G(x_G(j))) = \beta(j) \cdot (\alpha(1) \cdot x_G(1) + \alpha(2) \cdot x_G(2)) \quad (2)$$

Notice that the value-added in sector j depends on a fraction, $\beta(j)$, of the weighted sum of the total production in each sector. The sum of those fractions must be equal to one.

The specific mapping for the extended Leontief model is logically defined, for $j=1, 2$, as the symmetric counterpart of the extended Ghosh equations:

$$v_L(j, h) = v_L(x_L(j)) = \alpha'(j) \cdot x_L(j) \quad (3a)$$

$$d_L(j, h) = d_L(v_L(j, h)) = \beta'(j) \cdot (v_L(1, h) + v_L(2, h)) \quad (3b)$$

Substituting the first expression into the second one we obtain:

$$d_L(j, h) = d_L(v_L(j, h)) = \beta'(j) \cdot (\alpha'(1) \cdot x_L(1) + \alpha'(2) \cdot x_L(2)) \quad (4)$$

Comparing equations (2) and (3a) we observe that in the extended L model the only variable affecting, in a positive relation, value-added in sector j is production in sector j . In the extended G model, as we said before, the value-added of a sector is affected by the production of the two sectors. This is a key difference between the two extended models concerning the relationship between production and value-added. The other key difference between the two models concerns the relationship between the production of a sector and its domestic demand. In the extended L model, we see in expression (4) that domestic demand for sector j depends on the production levels of the two sectors, while in the extended G model, equation (1a) shows that it depends exclusively on the production of sector j . The two different ways of endogenizing the relevant variables discussed above gives rise to the different anomalies in the equilibrium behavior of the two models. We have seen some of these anomalies in the simulation runs described in Table 2a-b above.

This is not, however, the end of this story. When we compare the behavior of the two models in the assignment of final domestic demand, we can see in the extended L model

the symmetrical counterpart of what we have been saying about the relationship between sectorial value-added and total production in the extended G model.

In the extended G model we find that there will always be a positive relationship between increasing production, in a given sector, and increasing domestic final demand for the good of that sector. However, in the extended L model we may have an increase in production and value-added in a given sector with a lower domestic demand for the good of that sector. This unwanted property of the extended L model will happen as often as the unwanted property of the extended G model concerning value-added and production at the sectorial level.

It is nonetheless quite surprising that an increase in output and value-added in a given sector will not translate into additional domestic demand for the same good. In general, and under an income normality assumption, more available income should generate more demand for all goods, rather than less for some. Even the recourse to argue some consumption substitution will not do since in the standard linear models this is axiomatically ruled out.

Finally, we also want to comment on the unwanted possibility, shared by the two models, of having an increase in total aggregate production while total value-added decreases in the economy, or viceversa. By looking at equation (2), we can see that if the α coefficients in this equation are equal, then we have a positive relationship between aggregate output and aggregate value-added. Otherwise, as we have seen from Oosterhaven's previous numerical example, the unwanted property becomes indeed real. In fact, we will see from our 2×2 analysis that we also are able to construct examples with the undesirable property for the extended G model. Of course, the same happens for the extended L model.

By looking at expression (3a) for the extended L model, we can also conclude that if the α' coefficients have the same value, aggregate production is positively related to aggregate value-added, and so the unwanted property disappears. Outside this stringent equality condition, we can also show the presence of the corresponding anomaly in the extended L model by means of a numerical example. This example is quickly built using the 'transposed' input-output table appearing in Guerra and Sancho (2011), as we have seen in our Table 2a above.

3.2.2. Solving the extended models for the 2×2 case.

We start substituting expressions (2) and (4) into (G.3) and (L.3), respectively. This substitutions yield extended coefficient matrices $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$, which can then be used to solve for the values of output \mathbf{x} in both models. We obtain the following expressions, where we use again the markers L and G to distinguish the solutions:

$$\mathbf{x}_L = (\mathbf{I} - \hat{\mathbf{A}})^{-1} \cdot \mathbf{d}(\mathbf{f}) \tag{EL.1}$$

$$\mathbf{x}_G = (\mathbf{I} - \hat{\mathbf{B}})^{-1} \cdot \mathbf{v}(\mathbf{f}) \quad (\text{EG.1})$$

The inverse matrices $(\mathbf{I} - \hat{\mathbf{A}})^{-1} = [\hat{a}(i, j)]$ and $(\mathbf{I} - \hat{\mathbf{B}})^{-1} = [\hat{b}(i, j)]$ can be seen to have the following elements:

$$\hat{a}(1,1) = 1 - (a(2,2) + \beta'(2) \cdot \alpha'(2)) / \Delta_L$$

$$\hat{a}(1,2) = (a(1,2) + \beta'(1) \cdot \alpha'(2)) / \Delta_L$$

$$\hat{a}(2,1) = (a(2,1) + \beta'(2) \cdot \alpha'(1)) / \Delta_L$$

$$\hat{a}(2,2) = 1 - (a(1,1) + \beta'(1) \cdot \alpha'(1)) / \Delta_L$$

$$\hat{b}(1,1) = 1 - (b(2,2) + \beta(2) \cdot \alpha(2)) / \Delta_G$$

$$\hat{b}(1,2) = (b(2,1) + \beta(2) \cdot \alpha(2)) / \Delta_G$$

$$\hat{b}(2,1) = (b(1,2) + \beta(2) \cdot \alpha(1)) / \Delta_G$$

$$\hat{b}(2,2) = 1 - (b(1,1) + \beta(1) \cdot \alpha(1)) / \Delta_G,$$

where Δ_L and Δ_G are the determinants of matrices $(\mathbf{I} - \hat{\mathbf{A}})$ and $(\mathbf{I} - \hat{\mathbf{B}})$, respectively. Since the economies have to be productive, we may assume, invoking the Hawking-Simon conditions, that both determinants are positive.

We will assume now, as a way of normalizing the total value of the exogenous variables from both models, and introducing Oosterhaven's neutral shifts, that the total amount of foreign demand and foreign inputs are unitary:

$$d(1, f) + d(2, f) = v(1, f) + v(2, f) = 1$$

3.2.3. Sectorial analysis of equilibrium

It is now a matter of performing some algebra to verify that the equilibrium solution for production and value-added in the extended G model is as follows:

$$x_G^*(1) = \hat{b}(1) \cdot v(1, f) + \hat{b}(1, 2) \quad (4)$$

$$x_G^*(2) = \hat{b}(2) \cdot v(1, f) + \hat{b}(2, 2) \quad (5)$$

$$v_G^*(1, h) = \beta(1) \cdot \Omega(0) \cdot v(1, f) + \Omega(1) \quad (6)$$

$$v_G^*(2, h) = \beta(2) \cdot \Omega(0) \cdot v(1, f) + \Omega(1) \quad (7)$$

where it can be seen³ that:

$$\hat{b}(1) = (1 - \bar{b}(2) - \alpha(2)) / \Delta_G \quad (8)$$

$$\hat{b}(2) = -(1 - \bar{b}(1) - \alpha(1)) / \Delta_G \quad (9)$$

$$\Omega(0) = (\alpha(2) \cdot \bar{b}(1) - \alpha(1) \cdot \bar{b}(2) + \alpha(1) - \alpha(2)) / \Delta_G \quad (10)$$

and $\bar{b}(1)$ and $\bar{b}(2)$ are the sum of columns 1 and 2, respectively, in matrix **B**. A quick substitution also shows:

$$\Omega(0) = \alpha(1) \cdot \hat{b}(1) + \alpha(2) \cdot \hat{b}(2)$$

Similar algebra manipulations and notational conventions to the above ones would yield equilibrium output in the Extended L model as:

$$x_L^*(1) = \hat{a}(1) \cdot d(1, f) + \hat{a}(1, 2) \quad (11)$$

$$x_L^*(2) = \hat{a}(2) \cdot d(1, f) + \hat{a}(2, 2) \quad (12)$$

with home final demand being:

$$d_L^*(1, h) = \beta'(1) \cdot (\Omega'(0) \cdot d(1, f) + \Omega'(1)) \quad (13)$$

$$d_L^*(2, h) = \beta'(2) \cdot (\Omega'(0) \cdot d(1, f) + \Omega'(1)) \quad (14)$$

In these expressions we would have:

$$\hat{a}(1) = (1 - \bar{a}(2) - \alpha'(2)) / \Delta_L \quad (15)$$

$$\hat{a}(2) = -(1 - \bar{a}(1) - \alpha'(1)) / \Delta_L \quad (16)$$

$$\begin{aligned} \Omega'(0) &= (\alpha'(2) \cdot \bar{a}(1) - \alpha'(1) \cdot \bar{a}(2) + \alpha'(1) - \alpha'(2)) / \Delta_L = \\ &= \alpha'(1) \cdot \hat{a}(1) + \alpha'(2) \cdot \hat{a}(2) \end{aligned} \quad (17)$$

³ The specifics of $\Omega(1)$ in (6) and (7) are omitted since this magnitude plays no role in the subsequent analysis. The same applies later to $\Omega'(1)$.

We are now in a position to compare the equilibrium relationships for the behavior of sectorial production and sectorial value-added in the extended G model, and sectorial production and sectorial home final demand in the extended L model. We will see that these relationships run in parallel (or symmetric) fashion in both models.

In the extended G model we first check expressions (4) to (7). Notice that the exogenous variable affecting the two endogenous variables in sectors $j=1, 2$, output $x(j)$ and home value-added $v(j, h)$ is external value-added in sector 1, $v(1, f)$. Therefore any change in $v(1, f)$ will affect the endogenous variables this way:

$$\frac{\partial x_G(1)}{\partial v(1, f)} = \hat{b}(1), \dots, \frac{\partial x_G(2)}{\partial v(1, f)} = \hat{b}(2)$$

$$\frac{\partial v_G(1, h)}{\partial v(1, f)} = \beta(1) \cdot \Omega(0), \dots, \frac{\partial v_G(2, h)}{\partial v(1, f)} = \beta(2) \cdot \Omega(0)$$

and the signs of these derivatives become the key factor in the discussion. It can be readily proved that it is always the case that $\hat{b}(1) > 0$ and $\hat{b}(2) < 0$. From expression (8), since $\Delta_G > 0$ the sign of $\hat{b}(1)$ will be determined simply by the sign of $1 - \bar{b}(2) - \alpha(2)$. Take the second equation of (L.1) and divide it by $x(2)$ to obtain $b(2,1) + b(2,2) + \alpha(2) < 1$, provided external final demand is positive; from here we immediately find that $\Delta_G \cdot \hat{b}(1) = 1 - b(2,1) - b(2,2) - \alpha(2) = 1 - \bar{b}(2) - \alpha(2) > 0$. A similar argument would show that $\hat{b}(2) < 0$.

Endogenous home value-added, however, will always move in the *same* direction in both sectors. This is so because $\beta(j) > 0$ and thus it all rests on the sign of parameter $\Omega(0)$, which can be seen to have any sign, either positive or negative. If $\Omega(0) > 0$, home value-added will surely rise in both sectors at once. If $\Omega(0) < 0$ there would apparently be an ex-post input substitution between home and foreign inputs. This seems unlikely to be the case in practical terms since no substitution is allowed in the linear models. In any case we will *always* have one sector's value-added going in the opposite direction of its production. However, by the nature of the endogenizing rules, the direction of change in sectorial output, will always agree with the direction of change in sectorial home final demand.

This alleged inconsistent behavior of the extended G model is not possible in the extended L model and the reason is equation (3). However, and for exactly the same formal reasons, we may find in equilibrium a likewise anomalous relationship in the extended L model. In this case the anomaly is between the direction of change in sectorial output (or value-added) and home final demand for that sector.

That this is indeed the case can be ascertained if we look at the equations (11) through (14) and their corresponding derivatives:

$$\frac{\partial x_L(1)}{\partial d(1, f)} = \hat{a}(1), \dots, \frac{\partial x_L(2)}{\partial d(1, f)} = \hat{a}(2)$$

$$\frac{\partial d_L(1, h)}{\partial d(1, f)} = \beta'(1) \cdot \Omega'(0), \dots, \frac{\partial d_L(2, h)}{\partial d(1, f)} = \beta'(2) \cdot \Omega'(0)$$

The same line of reasoning that we took in the extended G model can also be made now for the extended L model. The signs of the parameter affecting equilibrium production in each sector will be once again different and it can be seen that it is always the case that $\hat{a}(1) \geq 0$ and $\hat{a}(2) \leq 0$. Since the proof is formally similar to the one given above for the extended G model, it is omitted here. We see from (13) and (14), however, that there is only one parameter affecting the sign of the derivatives of internal demand in both sectors, i.e. $\Omega'(0)$. Regardless of the sign of $\Omega'(0)$, it must be different from the sign of one of the two previous parameters affecting sectorial production (and value-added). Hence, we will observe in the extended L model increases in sectorial production (and value-added) along with a decrease in the internal (home) demand of that same sector. Notice the anomaly: an increase in final foreign demand for a given good may induce a reduction in the home demand for both goods, even if production goes up for some good.

3.2.4 Aggregate analysis of the equilibrium

We now consider the equilibrium relationships between aggregate output and aggregate exogenous changes.

Aggregate equilibrium output and value-added, in each of the G and L models, can be seen to be as follows:

$$x_G^* = \hat{b} \cdot v(1, f) + \hat{b}(1, 2) + \hat{b}(2, 2) \quad (18)$$

$$v_G^* = \Omega(0) \cdot v(1, f) + \Omega(1) \quad (19)$$

$$x_L^* = \hat{a} \cdot d(1, f) + \hat{a}(1, 2) + \hat{a}(2, 2) \quad (20)$$

$$v_L^* = \Omega'(0) \cdot d(1, f) + \Omega'(1) \quad (21)$$

where we would now have:

$$\hat{b} = (\bar{b}(1) - \bar{b}(2) + \alpha(1) - \alpha(2)) / \Delta_G = \hat{b}(1) + \hat{b}(2)$$

$$\hat{a} = (\bar{a}(1) - \bar{a}(2) + \alpha'(1) - \alpha'(2)) / \Delta_L = \hat{a}(1) + \hat{a}(2)$$

Again, for our purposes here, we do not need to specify the expressions for $\Omega(1)$ and $\Omega'(1)$. For the extended G model, as we can see from (18) and (19), a change in the same direction in total aggregate production and total value-added will require that the

parameters affecting $v(1, f)$ in (18) (i.e. \hat{b}) and in (19) (i.e. $\Omega(0)$) both have the same sign. We now show that this is not going to be the case always and why these parameters may have different signs.

Let us begin by considering the product of the two relevant parameters:

$$P = \hat{b} \cdot \Omega(0) = (\bar{b}(1) - \bar{b}(2) + \alpha(1) - \alpha(2)) \cdot (\alpha(2) \cdot \bar{b}(1) - \alpha(1) \cdot \bar{b}(2) + \alpha(1) - \alpha(2))$$

If the sign of this product P is positive, then we will have the same direction of change in the aggregate variables. It is easy to see that the following statements are true:

- a) If $\alpha(1) = \alpha(2)$ or $\bar{b}(1) = \bar{b}(2)$ then $P > 0$
- b) If $\alpha(1) = \alpha(2)$ and $\bar{b}(1) = \bar{b}(2)$ then $\hat{b} = 0$ and $\Omega(0) = 0$. Hence $P = 0$
- c) If $\alpha(2) = \bar{b}(1)$ and $\alpha(1) = \bar{b}(2)$ then $\hat{b} = 0$ and if $\alpha(1) > (<) \alpha(2)$ then $\Omega(0) > (<) 0$. Either way $P = 0$

We have to consider now the case $\hat{b} > 0$ and $\Omega(0) < 0$. Notice now:

- d) $\hat{b} > 0$ if and only if $\bar{b}(2) + \alpha(2) < \bar{b}(1) + \alpha(1)$
- e) $\Omega(0) < 0$ if and only if $\alpha(2) \cdot \bar{b}(1) + \alpha(1) < \alpha(1) \cdot \bar{b}(2) + \alpha(2)$

These two previous inequalities will be satisfied simultaneously whenever the following ones hold:

$$\alpha(2) \cdot \bar{b}(1) + \alpha(1) < \alpha(1) \cdot \bar{b}(2) + \alpha(2) < \bar{b}(2) + \alpha(2) < \bar{b}(1) + \alpha(1)$$

The following numerical example satisfies these inequalities:

$$\alpha(1) = 0.15 \quad \alpha(2) = 0.20 \quad \bar{b}(1) = 0.25 \quad \bar{b}(2) = 0.13$$

From these figures, it is easily checked that:

$$\hat{b}(1) = 2.713 > 0$$

$$\hat{b}(2) = -2.429 < 0$$

$$\hat{b} = \hat{b}(1) + \hat{b}(2) = 0.283 > 0$$

$$\Omega(0) = \alpha(1) \cdot \hat{b}(1) + \alpha(2) \cdot \hat{b}(2) = -0.079 < 0$$

$$\Delta_G = 0.247 > 0$$

with values for the Ghosh coefficient matrix \mathbf{B} as follows:

$$\mathbf{B} = \begin{bmatrix} 0.15 & 0.05 \\ 0.10 & 0.08 \end{bmatrix}$$

According to our analysis, and for the previous parameter values, an increase in $v(1, f)$ will increase production in sector one, decrease production in sector 2, and decrease value-added in both sectors. Total production will increase, but total value-added will decrease, and thus the anomalies in the extended G model are present. The same type of example can be found for the extended Leontief model: in this model, where unwanted property shows up when changes in total production (and total value-added) go in the opposite direction of changes in total domestic demand.

4. Alternative ways to make value-added and final demand endogenous.

We have discussed in the previous section some anomalous implications that may follow from the standard extension implementation of the Leontief and Ghosh models. As we have seen, both these models may have unwanted properties in the behavior of fundamental equilibrium variables, both at the sectorial and the aggregate level. Here we wish to consider some other specific function that relates those variables in a sensible economic way and, at the same time, avoids the inconsistencies found under the previous rules.

We may have the following proposal where for each sector its endogenous variables are related only to the other variable of the same sector.

New extended Ghosh model “rules”:

$$d(j, h) = \alpha(j) \cdot x(j), \quad j=1,2 \quad ; \quad v(j, h) = d(j, h), \quad j = 1,2.$$

New extended Leontief model “rules”:

$$v(j, h) = \alpha(j) \cdot x(j), \quad j=1,2 \quad ; \quad d(j, h) = v(j, h), \quad j = 1,2.$$

Under these two rules, both models will behave in a “proper” way. We will have by construction that in each sector value-added, final home demand and total production will all change in the same direction in response to an external shock. At the aggregate level and under the present rules, however, we may still have inconsistent behavior between the direction of the change in total production and total value-added (or total internal demand).

In effect, going through the same procedure as in section 2, we may solve for the equilibrium solution of the endogenous variables as a function of the exogenous ones. In particular, for the Ghosh model, after the usual algebra we find the following fundamental derivatives at the sectorial equilibrium values:

$$\frac{\partial x_G(1)}{\partial v(1, f)} = \hat{b}(1) > 0, \frac{\partial x_G(2)}{\partial v(1, f)} = \hat{b}(2) < 0$$

$$\frac{\partial v_G(1, h)}{\partial v(1, f)} = \alpha(1) \cdot \hat{b}(1) > 0, \frac{\partial v_G(2, h)}{\partial v(1, f)} = \alpha(2) \cdot \hat{b}(2) < 0$$

While at the aggregate level we find the following expressions:

$$\frac{\partial x_G}{\partial v(1, f)} = \hat{b} = \hat{b}(1) + \hat{b}(2)$$

$$\frac{\partial v_G}{\partial v(1, f)} = \Omega(0) = \alpha(1) \cdot \hat{b}(1) + \alpha(2) \cdot \hat{b}(2)$$

As can be seen, at the sectorial level, all derivatives have the correct sign. However, the sign of the derivatives, at the aggregate level, are ambiguous. Hence, we still have the same problem that we had under the previous rules, and so the same analysis made in section 2.2.4 applies here. In that analysis, we showed that one of the sufficient conditions to avoid the unwanted aggregate property, in both models, is to assume that the α 's are the same for all sectors. A parallel analysis, to the previous one, can be made for the new extended Leontief model, which would lead to the same type of equations and conclusions.

In both models, the new defined rules tell us that, the facto, the value-added of a sector is a positive constant fraction of the total production of that sector. Since the input-output coefficients are also a constant fraction of production, our rules have a sound economic property resembling the assumptions made empirically to define the production function of a sector. Now, both models share this property. We also see that value-added, in a given sector, is equal in both models to final demand. We interpret this assumption as a way to as measure the value-added of a sector in terms of the product produced by such sector. We see this very often in neoclassical economics when we measure the value of wages that a firm is paying to its labor input in terms of its final product. In the neoclassical analysis, the marginal product of the input determines its unit price (in production units). Here, in our framework, we just take a fraction of the production.

5. Conclusions

By using a 2x2 commodity framework, we have explored, the roots of the alleged anomalies in the extended Ghosh model first pointed out by Oosterhaven (2012) by means of some numerical examples. We find that systematically we may have a positive change in the production of one sector with a decrease in its value added. We have proved that some other anomalies also lurk in the extended Leontief model. Namely, we can have some sector where, systematically, a positive change in output goes along with a negative change in its final domestic consumption. Furthermore, both models, share the unwanted property that it is possible to have a negative relationship between aggregate total production and aggregate total values added.

In order to clarify the nature of Oosterhaven's (2012) numerical example, regarding anomalies in the extended Ghosh model, two aspects are worth commenting. One is, as we can see in section 3, that if the example would turn out to be a critic for the extended Ghosh model, then so would be for the extended Leontief model. They share the same mathematical structure and thus a simple transposing of the input output table of Guerra and Sancho (2011), transmits the incongruity among them. For the sake of clarification, the second comment has to do with Oosterhaven's shift exercise. We have seen that such a shift can be additively separated in two equilibrium states. In each of these states the sign of the equilibrium variables in the extended Ghosh model is the correct and logical one. Their combination, however, can have either sign, as we have shown, and this depends on the relative strength and interplay of direct, indirect and induced effects.

Finally, we should mention that a new set of rules to make endogenous the relation between value added and final demand is considered in section 4. Under these new rules, at the sectorial level we are able to overcome the previous criticisms for both models, but not at the aggregate level. Nevertheless, some simple hypothesis can also overcome the anomalous aggregate behavior of the two models both under the new as well as the old rules.

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Albalade, D. (PPRE-IREA)

"Shifting death to their Alternatives: The case of Toll Motorways"
(Octubre 2007)

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Segarra-Blasco, A. (GRIT); **García-Quevedo, J.** (IEB); **Teruel-Carrizosa, M.** (GRIT)

"Barriers to innovation and public policy in catalonia"
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Bel, G. (PPRE-IREA); **Foote, J.**

“Comparison of recent toll road concession transactions in the United States and France”
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XREAP2007-12

Segarra-Blasco, A. (GRIT);

“Innovation, R&D spillovers and productivity: the role of knowledge-intensive services”
(Novembre 2007)

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Bermúdez Morata, Ll. (RFA-IREA); **Guillén Estany, M.** (RFA-IREA), **Solé Auró, A.** (RFA-IREA)

“Impacto de la inmigración sobre la esperanza de vida en salud y en discapacidad de la población española”
(Novembre 2007)

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Calaeys, P. (AQR-IREA); **Ramos, R.** (AQR-IREA), **Suriñach, J.** (AQR-IREA)

“Fiscal sustainability across government tiers”
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Sánchez Hugalbe, A. (IEB)

“Influencia de la inmigración en la elección escolar”
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Durán Weitkamp, C. (GRIT); **Martín Bofarull, M.** (GRIT) ; **Pablo Martí, F.**

“Economic effects of road accessibility in the Pyrenees: User perspective”
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“The Causal Relationship between Individual’s Choice Behavior and Self-Reported Satisfaction: the Case of Residential Mobility in the EU”
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“Car ownership and access to jobs in Spain”
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Bel, G. (PPRE-IREA) ; **Fageda, X.** (PPRE-IREA)

“Privatization and competition in the delivery of local services: An empirical examination of the dual market hypothesis”
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Matas, A. (GEAP); **Raymond, J. L.** (GEAP); **Roig, J. L.** (GEAP)

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Bermúdez, Ll. (RFA-IREA)

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Calonge, S. (CREB); **Tejada, O.**

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Sanromá, E. (IEB); **Ramos, R.** (AQR-IREA), **Simon, H.**

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“Technology, Business Models and Network Structure in the Airline Industry”
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“Why do educated mothers matter? A model of parental help”
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Fageda, X. (GiM-IREA), **Perdiguero, J.** (GiM-IREA)

“An empirical analysis of a merger between a network and low-cost airlines”
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Miguélez, E. (AQR-IREA); **Gómez-Miguélez, I.**

“Singling out individual inventors from patent data”
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“¿Afecta la sobreeducación de los padres al rendimiento académico de sus hijos?”
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Pitt, D., Guillén, M. (RFA-IREA), **Bolancé, C.** (RFA-IREA)

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“How much risk is mitigated by LTC Insurance? A case study of the public system in Spain”
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“Mixture of bivariate Poisson regression models with an application to insurance”
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“Age effects, unobserved characteristics and hedonic price indexes: The Spanish car market in the 1990s”
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“A correlation sensitivity analysis of non-life underwriting risk in solvency capital requirement estimation”
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Guillén, M. (RFA-IREA), **Pérez-Marín, A.** (RFA-IREA), **Alcañiz, M.** (RFA-IREA)

“A logistic regression approach to estimating customer profit loss due to lapses in insurance”
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Jiménez, J. L., Perdiguero, J. (GiM-IREA), **García, C.**

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“The impact of agglomeration effects and accessibility on wages”
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Segarra, A. (GRIT)

“R&D cooperation between Spanish firms and scientific partners: what is the role of tertiary education?”
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García-Pérez, J. I.; Hidalgo-Hidalgo, M.; Robles-Zurita, J. A.

“Does grade retention affect achievement? Some evidence from PISA”
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Arespa, M. (CREB)

“Macroeconomics of extensive margins: a simple model”
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García-Quevedo, J. (IEB), **Pellegrino, G.** (IEB), **Vivarelli, M.**

“The determinants of YICs’ R&D activity”
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González-Val, R. (IEB), **Olmo, J.**

“Growth in a Cross-Section of Cities: Location, Increasing Returns or Random Growth?”
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“The Innovation and Imitation Dichotomy in Spanish firms: do absorptive capacity and the technological frontier matter?”
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Borrell, J. R. (GiM-IREA), **Jiménez, J. L., García, C.**

“Evaluating Antitrust Leniency Programs”
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XREAP2012-02

Ferri, A. (RFA-IREA), **Guillén, M.** (RFA-IREA), **Bermúdez, Ll.** (RFA-IREA)

“Solvency capital estimation and risk measures”
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Ferri, A. (RFA-IREA), **Bermúdez, Ll.** (RFA-IREA), **Guillén, M.** (RFA-IREA)

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“Quantitative analysis of image factors in a cultural heritage tourist destination”

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“What underlies localization and urbanization economies? Evidence from the location of new firms”

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Muñiz, I. (GEAP), **Calatayud, D.**, **Dobaño, R.**

“Los límites de la compacidad urbana como instrumento a favor de la sostenibilidad. La hipótesis de la compensación en Barcelona medida a través de la huella ecológica de la movilidad y la vivienda”

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Arqué-Castells, P. (GEAP), **Mohnen, P.**

“Sunk costs, extensive R&D subsidies and permanent inducement effects”

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“Local Distance-Based Generalized Linear Models using the dbstats package for R”

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“What about people in European Regional Science?”

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“Intermediary and structural determinants of early childhood health in Colombia: exploring the role of communities”

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“Do labour mobility and networks foster geographical knowledge diffusion? The case of European regions”

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Varela-Irimia, X-L. (GRIT)

“Profitability, uncertainty and multi-product firm product proliferation: The Spanish car industry”

(Setembre 2012)



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Duró, J. A. (GRIT), Teixidó-Figueras, J. (GRIT)

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Manresa, A. (CREB), Sancho, F.

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xreap@pcb.ub.es