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Growth, heterogeneous technological interdependence, and spatial externalities: Theory and Evidence*

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Abstract

We present a growth model with interdependencies in the heterogeneous technological progress, physical capital and stock of knowledge that yields a growth-initial equation that can be taken to the data. We then use data on EU-NUTS2 regions and a correlated random effects specification to estimate the resulting spatial Durbin dynamic panel model with spatially weighted individual effects. QML estimates support our model against simpler alternatives that impose a homogeneous technology and limit the sources of spatial externalities. Also, our results indicate that rich regions tend to have higher "unobserved productivity" and are likely to stay rich because of the strong time and spatial dependence of the GDP per capita. Poor regions, on the other hand, tend to enjoy "unobserved productivity" spillovers but are like to stay poor unless they increase their saving rates.

Keywords: correlated random effects, Durbin model, economic growth, spatial panel data

JEL Classification: C23, O47

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1 Introduction

Historically, the empirical economic growth literature consisted mostly of "aspatial empirical analyses that have ignored the influence of spatial location on the process of growth" (De Long and Summers, 1991; Fingleton and López-Bazo, 2006, p. 178). In the last two decades, however, a number of studies seek to incorporate "spatial effects" in the standard (i.e., non-spatial) economic growth models. In particular, the idea that the spatial location of an economy may drive its economic growth has been developed using models of absolute location, which account for the location of one economy in the geographical space, and models of relative location, which account for the location of one economy with respect to the others. Econometrically, these two types of models are closely related to the concepts of spatial heterogeneity and spatial dependence (Abreu et al., 2005).

Although spatial heterogeneity is usually associated with parameter heterogeneity (see e.g. Ertur and Koch, 2007; Basile, 2008), the most common approach in the literature is to allow for unobserved differences using panel data (Islam, 1995; Elhorst et al., 2010). Also, knowledge spillovers are the main mechanism employed to incorporate interactions between economies into the Solow-Swan neoclassical growth model (López-Bazo et al., 2004; Egger and Pfaffermayr, 2006; Pfaffermayr, 2009, 2012). It is interesting to note, however, that these two streams of the literature have developed rather separately. Notable exceptions include Elhorst et al. (2010), who consider the extension of the model proposed by Ertur and Koch (2007) to panel data; Ho et al. (2013), who consider an ad-hoc extension of the model proposed by Mankiw et al. (1992) that includes a spatial autoregressive term and a spatial time lag term; and Yu and Lee (2012), who, using a simplified version of the technology assumed by Ertur and Koch (2007), derive a growth model with spatial externalities based on the model of Mankiw et al. (1992). This paper aims to contribute to this limited literature by considering a growth model with spatial heterogeneity and spatial externalities that nests the models introduced by Islam (1995), López-Bazo et al. (2004) and Ertur and Koch (2007).

To be precise, we present a growth model with interdependencies in the (heterogeneous) technological progress, physical capital and stock of knowledge.[1] The basic framework is similar to that of Ertur and Koch (2007), but we consider additional sources of externalities across economies. While they assume that the technological progress depends on the own stock of physical capital and the stock of knowledge of the other economies, we also consider the role of both the physical capital (López-Bazo et al., 2004; Egger and Pfaffermayr, 2006) and the (unobserved) initial level of technology (De Long and Summers, 1991; LeSage and Fischer, 2012) of the other economies. Moreover, we do not assume a common exogenous technological progress but account for heterogeneity in the initial level of technology, which here is interpreted as a proxy for total factor productivity (Islam, 1995).

Having presented our model, we then derive the steady-state equation and a growth-initial equation that can be taken to the data. This is where the generality of our model comes at a cost, since not all the parameters of interest are identified (a limitation that also arises in the benchmark model of Ertur and Koch 2007). To be precise, the model contains a set of theoretical constraints that, if valid, allows us to identify most of the implied parameters (in contrast, the constrained version of Ertur and Koch's (2007), which is derived from a subset of our constraints, is fully identified). This means that, provided that the constrained model is valid, finding evidence of global technological interdependence between the economies would lead us to reject the models of Islam (1995) and López-Bazo et al. (2004), whereas, if no such evidence was found, we would reject the model of Ertur and Koch (2007). In any case, finding evidence of heterogeneity in the initial level of technology would support our model against these alternatives.

Ultimately, the identification problem arises because, even in the constrained model, we cannot separate: *i)* the (direct) effect that, as an input of the production function, the stock of physical capital has on the output from the (indirect) effect that it has as a driver of the technology; and *ii)* the local) effect that the stock of physical capital of the neighbouring economies has on the technology and, subsequently, the output from the (global) effect that, due to the presence of technological interdependencies, the stock of physical capital of the

[1]It is worth noting that the model can easily be extended to incorporate the role of human capital (López-Bazo et al., 2004; Fingleton and López-Bazo, 2006). We leave this issue for future research.

neighbouring economies has on the output.[2] Still, we argue that simple changes in the unconstrained model specification (e.g., introducing the stock of physical capital lagged one period in the technological progress, rather than using its current value) and/or additional restrictions on the parameters of the constrained model (e.g., the equality of the effects that the stock of physical capital has on the output as a production input and as a technology input) may address this limitation. To illustrate our argument, we consider imposing the additional constraint(s) that the remaining unidentified technological parameters are consistent with the model of López-Bazo et al. (2004) and/or that of Ertur and Koch (2007).

The econometric specification of the resulting growth-initial equation corresponds to the spatial Durbin dynamic panel model (see also Elhorst et al., 2010; Yu and Lee, 2012; Ho et al., 2013), but with spatially weighted individual-specific effects. Thus, given the obvious interest in distinguishing the individual effects from their spatial spillovers, we resort to a correlated random effects specification (Miranda et al., 2017a,b). In particular, we estimate our growth-initial equation by Quasi-Maximum Likelihood (see also Lee and Yu, 2016) using EU-NUTS2 regional data from Cambridge Econometrics. We use regional data because, as López-Bazo et al. (2004, p. 43) argue, once it is taken on board that "[e]conomies interact with each other (...), linkages are [likely] to be stronger [between close-by regions] than across heterogeneous countries". We provide results for both the constrained and unconstrained versions of our model.

We find evidence of technological interdependence in the output per capita of the EU regions, that is, a positive and significant impact of the level of technology of the neighbouring regions. However, there is also evidence of "unobserved" technological interdependence in the EU regions (i.e., local spatial contagion of the "unobserved productivity"), which supports our assumed technology. Also, the constrained model specification produces estimates of the implied parameters that statistically reject the models of Islam (1995) and López-Bazo et al. (2004). Lastly, results from our identification strategy do not support the role that López-Bazo et al. (2004) and Ertur and Koch (2007) assume capital plays in shaping the technological

[2] **?**, p. 154 is generally credited for "distinguishing between a *global* and a *local* range of dependence and the way in which this translates into the incorporation in a regression specification of spatially lagged dependent variables (Wy) [and] spatially lagged explanatory variables (WX)".

progress.

The rest of the paper is organised as follows. In section 2 we present the model.[3] In section 3 we discuss the data and the estimation results. Section 4 concludes.

2 The Model

2.1 Technological interdependencies in growth

Our starting point is the Solow growth model originally proposed by Mankiw et al. (1992) using cross-section data and extended later by Islam (1995) to panel data (see also Barro and Sala-i-Martin, 2003). Let us then consider a Cobb-Douglas production function for region $i = 1, \ldots, N$ in time $t = 1, \ldots, T$:

$$Y_{it} = A_{it} K_{it}^{\alpha} L_{it}^{1-\alpha}, \tag{2.1}$$

where Y_{it} denotes output, K_{it} physical capital (α is thus the capital share or output elasticity parameter), L_{it} labour, and A_{it} technology. All the variales are in levels and there are constant returns to scale in production. Also, while output, capital and labour are typically assumed to be observable, technology is assumed to be (partially) unobservable. Mankiw et al. (1992), for example, assume that $\ln A = a + \varepsilon$, where a is a constant term and ε is the standard i.i.d error.

For the purposes of this paper, a major feature of this model is that technology is assumed to grow exogenously and at the same rate in all regions. This rules out the existence of knowledge spillovers arising from technological interdependence between the regional economies. However, accounting for technological interdependence and knowledge spillovers is critical when analysing how "the relative location of an economy affects economic growth" (Elhorst et al., 2010, p. 338). In the literature, depending on whether knowledge spillovers turn out to be "local" or "global" (**?**), we find two main approaches to the introduction of spatial externalities in the Solow growth model.

[3]To facilitate the reading, we moved the derivation of some results (the balanced growth rate, a Taylor approximation to the marginal productivity of capital, the speed of convergence, and the solution to the steady state differential equation) to the appendix.

On the one hand, López-Bazo et al. (2004) and Egger and Pfaffermayr (2006) consider growth models where the knowledge spillovers are local in nature, in the sense that they are limited to the neighbouring regions (at least initially).[4] To be precise, in López-Bazo et al. (2004) technology is assumed to depend on both the physical and human capital of the neighbouring regions, whereas in Egger and Pfaffermayr (2006) is assumed to grow exogenously and at the same rate in all regions (as in Mankiw et al. 1992 and Islam 1995), so that the externalities arise from the assumption that total factor productivity depends on the capital-labour ratio of the region and the spatially weighted capital-labour of the other regions. Ertur and Koch (2007), on the other hand, assume that the technological progress of an economy depends on the stock of physical capital per worker in that economy as well as the stock of knowledge of the other economies. More specifically, they assume that the technology of an economy is a geometrically weighted average of the technology of the other economies, thus making knowledge spillovers to spread over all the regions (and hence become "global"). However, it is still assumed that "some proportion of technological progress is exogenous and identical in all countries" [p. 1036].

In this paper, we extend the model of Ertur and Koch (2007) by introducing spatial dependence in the stock of capital, as well as heterogeneity and spatial dependence in the exogenous technological progress (while holding the assumption that the technological progress of an economy depends on the stock of knowledge of the other economies). In this vein, our assumed technology combines the alternative sources of spatial externalities considered in models of relative location with the unobserved heterogeneity that characterises the models of absolute location (Abreu et al., 2005). In particular, our model shares with that of Ertur and Koch (2007) the main source of parameter heterogeneity. Namely, the speed of convergence to the steady state, as discussed below. Yet we eventually estimate a constrained version in which the speed of convergence is identical for all economies (Elhorst et al., 2010; Yu and Lee, 2012). To be precise, the estimated econometric specification corresponds to a variant of the spatial Durbin dynamic panel model recently considered by Lee and Yu (2016) that includes not only

[4] See also Fingleton and López-Bazo (2006), Pfaffermayr (2009) and Pfaffermayr (2012).

individual-specific effects but also their spatial spillovers (Miranda et al., 2017a).[5]

Next we derive our empirical specification, which adopts the form of a growth-initial equation. To a large extent, our approach follows the steps of Ertur and Koch (2007). Thus, we first discuss and motivate the assumed technology, then we obtain the output per worker equation at the steady state, and finally the growth-initial equation.

2.2 Technology

Let us denote by Ω_{it} the exogenous technological progress and by $k_{it} = \dfrac{K_{it}}{L_{it}}$ the level of physical capital per worker (of region i in period t). Ertur and Koch (2007, p. 1036) assume that the technology of region i in period t is given by

$$A_{it} = \Omega_{it} k_{it}^{\phi} \prod_{j \neq i}^{N} A_{jt}^{\gamma w_{ij}}, \tag{2.2}$$

where "[t]he parameter ϕ describes the strength of home externalities generated by physical capital accumulation" and "the degree of [regional] technological interdependence generated by the level of spatial externalities is described by γ". Notice that the spatial relation between region i and its neighbouring regions is represented by a set of spatial weights or "exogenous friction terms" w_{ij}, with $j = 1, \ldots, N$, that are assumed to satisfy the following properties: $w_{ij} = 0$ if $i = j$, $0 \leq w_{ij} \leq 1$, and $\sum_j w_{ij} = 1$. Lastly, Ertur and Koch (2007) assume that $\Omega_{it} = \Omega_t = \Omega_0 \exp(\mu t)$, where μ is the constant rate of growth of the exogenous technological progress. Therefore, the technology eventually assumed is $A_{it} = \Omega_0 \exp(\mu t) k_{it}^{\phi} \prod_{j \neq i}^{N} A_{jt}^{\gamma w_{ij}}$.

However, as previously pointed out, there are alternative approaches to the inclusion of

[5] As Basile (2008, p. 532-533) points out, "the local Spatial Durbin Model (...) proposed by Ertur and Koch (2007) is a general and flexible specification, since it allows identification of both spatial-interaction effects and parameter heterogeneity (...). In essence, this is the model considered here. The global Spatial Durbin Model (...) represents a less general specification, because it imposes the restriction of parameter homogeneity". In essence, this is the model we estimate. Lastly, "[t]he model proposed by López-Bazo et al. (2004) (..) imposes a further restriction on the parameters since the spatial lags of the structural characteristics of the regions are not included" (this also applies to the model proposed by Egger and Pfaffermayr 2006). In essence, this model is nested in ours.

knowledge spillovers in the Solow model. In a series of papers, López-Bazo et al. (2004, p. 46), Egger and Pfaffermayr (2006), Fingleton and López-Bazo (2006) and Pfaffermayr (2009, 2012) argue that the physical (and human) capital may be an alternative source of externalities, "[t]he reasoning behind such spillovers [being] basically the diffusion of technology from other regions caused by investments in physical (...) capital". In mathematical terms, such a technology may adopt the following functional form:

$$A_{it} = \Omega_0 \exp(\mu t) \prod_{j \neq i}^{N} k_{jt}^{\gamma w_{ij}}, \tag{2.3}$$

where, for the sake of comparability, we have used the same notation as in 2.2. However, the interpretation of the parameter γ is different here, for it now "measures the [strength of the] externality across economies" originated from variations in physical capital (López-Bazo et al., 2004; Fingleton and López-Bazo, 2006, p. 46). It is also important to stress that these papers maintain the assumption of an homogeneous exogenous technological progress growing at a constant rate, i.e., $\Omega_{it} = \Omega_0 \exp(\mu t)$.

Our assumed technology features those displayed in 2.2 and 2.3. However, we depart from these studies in the assumptions they made with respect to the exogenous technological progress. First, they assume that it is homogeneous across regions. However, as Mankiw et al. (1992, p. 6) point out, the Ω_0 "term reflects not just technology but resource endowments, climate, institutions, and so on; it may therefore differ across countries". In line with this argument, we introduce regions' heterogeneity into the definition of the exogenous technological progress by assuming that $\Omega_{it} = \Omega_{i0} \exp(\mu t)$.[6]

Second, as Islam (1995, p. 1149) points out, Ω_{i0} "is an important source of parametric difference in the aggregate production function across [regions]". Econometrically, it can be interpreted as an individual-specific effect (possibly correlated with some of the covariates in the initial-growth specification eventually derived). Economically, it is "a measure of efficiency with

[6]Alternative ways of modelling the exogenous technological progress are $\Omega_{it} = \Omega_0 \exp(\mu_i t)$ and $\Omega_{it} = \Omega_{i0} \exp(\mu_i t)$. However, these proposals would considerably increase the number of parameters of the model (by more than N, since it can be shown that the balanced growth rate becomes heterogeneous too) and make identification difficult, if not impossible (Lee and Yu, 2016).

which the [regions] are transforming their capital and labor resources into output and hence is very close to the conventional concept of total factor productivity" [p. 1155-1156]. These arguments are behind the second twist we introduce with respect to the models of López-Bazo et al. (2004), Ertur and Koch (2007) and others, since it opens the door to considering productivity spillovers as an additional source of spatial externalities (LeSage and Fischer, 2012; Miranda et al., 2017b). As De Long and Summers (1991, p. 487) point out, "it is difficult to believe that Belgian and Dutch or US and Canadian economic growth would ever significantly diverge, or that substantial productivity gaps would appear within Scandinavia".

All in all, a production technology that may account for these alternative sources of spatial dependence is the following:

$$A_{it} = \Omega_{it} \prod_{j \neq i}^{N} \Omega_{jt}^{\gamma_1 w_{ij}} k_{it}^{\phi} \prod_{j \neq i}^{N} k_{jt}^{\gamma_2 w_{ij}} \prod_{j \neq i}^{N} A_{jt}^{\gamma_3 w_{ij}} \tag{2.4}$$

with $\Omega_{it} = \Omega_{i0} \exp(\mu t)$ and Ω_{i0} non-observable (which is why Ω_{it} does not have a coefficient in 2.4). Notice that γ_3 and γ_2 play the same role as γ in 2.2 and 2.3, respectively, whereas γ_1 can be interpreted as the degree of technological interdependence generated from the (unobserved) productivity spillovers. In particular, $\gamma_1 = \phi = \gamma_2 = \gamma_3 = 0$ would lead us to the model proposed by Islam (1995), $\gamma_1 = \phi = \gamma_3 = 0$ (with $\gamma_2 \neq 0$) to the model proposed by López-Bazo et al. (2004), and $\gamma_1 = \gamma_2 = 0$ (with $\phi \neq 0$ and $\gamma_3 \neq 0$) to the model proposed by Ertur and Koch (2007). Notice also that, in contrast to the local contagion models of López-Bazo et al. (2004) and Egger and Pfaffermayr (2006), both ours and that of Ertur and Koch (2007) are models of global contagion (?). We differ, however, in that whereas in their case there are no (global) spatial externalities in the stock of knowledge unless $\gamma_3 \neq 0$, there still are here if either $\gamma_1 \neq 0$ or $\gamma_2 \neq 0$ (albeit of a local nature). This is because our model accounts for both global and local contagion. Lastly, it is interesting to note that there are no capital externalities in our model if $\phi = \gamma_2 = 0$ (neither global nor local). This is because our model accounts for both the role of the own physical capital (Ertur and Koch, 2007) and that of the other economies (López-Bazo et al., 2004) in the technological progress.

2.3 The production function

In order to obtain the explicit form of the Cobb-Douglas production function in 2.1 given our assumed technology, let us consider 2.4 expressed in logs and matrix form:

$$\begin{aligned} A &= \Omega + \gamma_1 W\Omega + \phi k + \gamma_2 W k + \gamma_3 W A \\ &= (I-\gamma_3 W)^{-1}\Omega + \gamma_1 (I-\gamma_3 W)^{-1} W\Omega + \phi (I-\gamma_3 W)^{-1} k + \gamma_2 (I-\gamma_3 W)^{-1} W k \qquad (2.5) \end{aligned}$$

where the parameters γ_1, γ_2 and γ_3 have been previously described (in particular, it is now assumed that $1/\gamma_3$ is not an eigenvalue of W when $\gamma_3 \neq 0$), A is the $N \times 1$ vector of logarithms of the technology, I is the $N \times N$ identity matrix, $\Omega = \Omega_0 + \iota_N \mu t$ is the $N \times 1$ vector of logarithms of the exogenous technological progress with $\Omega_0 = (\ln \Omega_{10}, \ldots, \ln \Omega_{N0})'$ and ι_N being a $N \times 1$ vector of ones, k is the $N \times 1$ vector of of logarithms of the capital per worker, and W is the $N \times N$ spatial weight matrix that describes the spatial arrangement of the regions.

Le us now denote by $w_{ij}^{(r)}$ the row i and column j element of matrix W^r. Notice that, since W is assumed to be row-normalized, if all the eigenvalues of W lie in the interval $(-1, 1)$ and $|\gamma_3| < 1$, then $(I - \gamma_3 W)^{-1} = \sum_{r=0}^{\infty} \gamma_3^r W^r$. Thus,

$$\begin{aligned} \ln A_{it} &= \sum_{j=1}^{N}\sum_{r=0}^{\infty} \gamma_3^r w_{ij}^{(r)} \ln \Omega_{jt} + \gamma_1 \sum_{j=1}^{N}\sum_{r=0}^{\infty} \gamma_3^r w_{ij}^{(r+1)} \ln \Omega_{jt} + \phi \sum_{j=1}^{N}\sum_{r=0}^{\infty} \gamma_3^r w_{ij}^{(r)} \ln k_{jt} \\ &+ \gamma_2 \sum_{j=1}^{N}\sum_{r=0}^{\infty} \gamma_3^r w_{ij}^{(r+1)} \ln k_{jt} \\ &= \sum_{j=1}^{N} \ln \Omega_{jt}^{\sum_{r=0}^{\infty} \gamma_3^r w_{ij}^{(r)}} + \sum_{j=1}^{N} \ln \Omega_{jt}^{\frac{\gamma_1}{\gamma_3}\sum_{r=1}^{\infty} \gamma_3^r w_{ij}^{(r)}} + \sum_{j=1}^{N} \ln k_{jt}^{\phi \sum_{r=0}^{\infty} \gamma_3^r w_{ij}^{(r)}} + \sum_{j=1}^{N} \ln k_{jt}^{\frac{\gamma_2}{\gamma_3}\sum_{r=1}^{\infty} \gamma_3^r w_{ij}^{(r)}}, \end{aligned}$$

so that we may rewrite 2.5 as

$$
\begin{aligned}
A_{it} &= \prod_{j=1}^{N} \Omega_{jt}^{\sum\limits_{r=0}^{\infty}\gamma_3^r w_{ij}^{(r)}} \prod_{j=1}^{N} \Omega_{jt}^{\frac{\gamma_1}{\gamma_3}\sum\limits_{r=1}^{\infty}\gamma_3^r w_{ij}^{(r)}} \prod_{j=1}^{N} k_{jt}^{\phi\sum\limits_{r=0}^{\infty}\gamma_3^r w_{ij}^{(r)}} \prod_{j=1}^{N} k_{jt}^{\frac{\gamma_2}{\gamma_3}\sum\limits_{r=1}^{\infty}\gamma_3^r w_{ij}^{(r)}} \\
&= \Omega_{it}^{1+\left(\frac{\gamma_3+\gamma_1}{\gamma_3}\right)\sum\limits_{r=1}^{\infty}\gamma_3^r w_{ii}^{(r)}} \prod_{j\neq i}^{N} \Omega_{jt}^{\left(\frac{\gamma_3+\gamma_1}{\gamma_3}\right)\sum_{r=1}^{\infty}\gamma_3^r w_{ij}^{(r)}} k_{it}^{\phi+\left(\frac{\phi\gamma_3+\gamma_2}{\gamma_3}\right)\sum_{r=1}^{\infty}\gamma_3^r w_{ii}^{(r)}} \prod_{j\neq i}^{N} k_{jt}^{\left(\frac{\phi\gamma_3+\gamma_2}{\gamma_3}\right)\sum_{r=1}^{\infty}\gamma_3^r w_{ij}^{(r)}}
\end{aligned}
$$

by using $\prod_{j=1}^{N} \Omega_{jt}^{w_{ij}^{(0)}} = \Omega_{it}$ and $\prod_{j=1}^{N} k_{jt}^{\phi w_{ij}^{(0)}} = k_{it}^{\phi}$.

Also, let us now define $u_{ii} = \alpha+\phi+\left(\frac{\phi\gamma_3+\gamma_2}{\gamma_3}\right)\sum_{r=1}^{\infty}\gamma_3^r w_{ii}^{(r)}$ and $u_{ij} = \left(\frac{\phi\gamma_3+\gamma_2}{\gamma_3}\right)\sum_{r=1}^{\infty}\gamma_3^r w_{ij}^{(r)}$, with $u_{ii} + \sum_{j\neq i}^{N} u_{ij} = \sum_{j=1}^{N} u_{ij} = \alpha+\phi+\frac{\phi\gamma_3+\gamma_2}{1-\gamma_3} = \alpha + \frac{\phi+\gamma_2}{1-\gamma_3}$. Then, given that $y_{it} = A_{it}k_{it}^{\alpha}$,

$$
y_{it} = \Omega_{it}^{1+\left(\frac{(\gamma_3+\gamma_1)(u_{ii}-\alpha-\phi)}{(\phi\gamma_3+\gamma_2)}\right)} \prod_{j\neq i}^{N} \Omega_{jt}^{\frac{(\gamma_3+\gamma_1)u_{ij}}{\phi\gamma_3+\gamma_2}} k_{it}^{u_{ii}} \prod_{j\neq i}^{N} k_{jt}^{u_{ij}} \tag{2.6}
$$

Notice that "this model implies spatial heterogeneity in the parameters of the production function", a feature shared with that of Ertur and Koch (2007, p. 1037). We differ, however, in that it is no longer the case that "if there are no physical capital externalities, i.e., $\phi = 0$, we have $u_{ii} = \alpha$ and $u_{ij} = 0$, (...) then the production function is written in the usual form" (as in e.g. Mankiw et al. 1992 and Islam 1995). As previously pointed out, here we further require that $\gamma_1 = \gamma_2 = \gamma_3 = 0$. Put it differently, there are no physical capital externalities in the model of Ertur and Koch (2007) if $\phi = 0$, *regardless of* γ_3. In our model, however, we further require that $\gamma_2 = 0$. That is, there are capital externalities to the extent that $\gamma_3 \neq 0$ and either $\phi \neq 0$ or $\gamma_2 \neq 0$. This is because, following López-Bazo et al. (2004), we account for the local role of the capital in the technology through the parameter γ_2.

2.4 The Steady State equation

To derive the equation describing the output per worker of region i at the steady state, we proceed in the following way. First we rewrite the production function in matrix form, $y = A + \alpha k$, and substitute the technology by its expression in 2.5. We then pre-multiply both sides of the resulting equation by $I - \gamma_3 W$ to obtain

$$y = \Omega + \gamma_1 W\Omega + (\alpha + \phi)k + (\gamma_2 - \alpha\gamma_3)Wk + \gamma_3 Wy \tag{2.7}$$

Lastly, we replace in 2.7 the log of the capital per worker in region i by its log value at the steady state, $\ln k_{it}^*$. To this end, we start by noting that the evolution of capital is governed by the following dynamic equation:

$$\dot{k}_{it} = s_i y_{it} - (n_i + \delta)k_{it} \tag{2.8}$$

where the dot over a variable denotes its derivative with respect to time, s_i is the fraction of output saved, n_i is the growth rate of labour, and δ is the annual rate of depreciation of capital (common to all regions). Given that production shows decreasing returns to scale, equation 2.8 implies that the capital-output ratio is constant and converges to a balanced growth rate g defined by $\frac{\dot{k}_{it}}{k_{it}} = ln\,\dot{y}_{it} = ln\,\dot{k}_{it} = g = \frac{\mu\,(1+\gamma_1)}{(1-\gamma_3)(1-\alpha)-\phi-\gamma_2}$ (see appendix *A*). Also, it can be shown that, given a balanced growth rate g and 2.8 (see e.g. Barro and Sala-i-Martin, 2003), $\frac{k_{it}^*}{y_{it}^*} = \frac{s_i}{n_i+\delta+g}$ and $\ln k_{it}^* = \ln y_{it}^* + \ln\left(\frac{s_i}{n_i+\delta+g}\right)$.[7]

What is thus left is to introduce in 2.7 (rewritten for economy i rather than in matrix form) the expression obtained for the log of the capital per worker in region i at the steady state. In doing so, we obtain the equation describing the output per worker of region i at the steady

[7] It is also interesting to note that, if we compute the marginal productivity of capital, $\frac{\dot{k}_{it}}{k_{it}} = s_i\frac{y_{it}}{k_{it}} - (n_i+\delta)$, using the expression defining y_{it} in 2.6, we obtain $\frac{\dot{k}_{it}}{k_{it}} = s_i\Omega_{it}^{1+\left(\frac{(\gamma_3+\gamma_1)(u_{ii}-\alpha-\phi)}{(\phi\gamma_3+\gamma_2)}\right)}\prod_{j\neq i}^{N}\Omega_{jt}^{\frac{(\gamma_3+\gamma_1)u_{ij}}{\phi\gamma_3+\gamma_2}}k_{it}^{u_{ii}-1}\prod_{j\neq i}^{N}k_{jt}^{u_{ij}} - (n_i+g)$. Therefore, provided that $\alpha + \frac{\phi+\gamma_2}{1-\gamma_3} < 1$, there are diminishing returns to the capital, as in the model of Ertur and Koch (2007).

state:

$$\ln y_{it}^* = \frac{\ln \Omega_{it}}{1-\alpha-\phi} + \frac{\gamma_1}{1-\alpha-\phi}\sum_{j=1}^{N} w_{ij} \ln \Omega_{jt} + \frac{\alpha+\phi}{1-\alpha-\phi} \ln\left(\frac{s_i}{n_i+\delta+g}\right)$$
$$+ \frac{\gamma_2-\alpha\gamma_3}{1-\alpha-\phi}\sum_{j=1}^{N} w_{ij} \left(\frac{s_j}{n_j+\delta+g}\right) + \frac{(1-\alpha)\gamma_3+\gamma_2}{1-\alpha-\phi}\sum_{j=1}^{N} w_{ij} \ln y_{jt}^* \tag{2.9}$$

Notice that this equation differs from that obtained by Ertur and Koch (2007) in two main features, reflecting ultimately differences in the assumed technology. First, the heterogeneous exogenous technological progress, since Ω_{it} is assumed to be Ω_t in Ertur and Koch (2007) and, consequently, no exogenous technological interdependences are considered. In particular, the term $\frac{\gamma_1}{(1-\alpha-\phi)}\sum_{j=1}^{N} w_{ij} \ln \Omega_{jt}$ is missing in their steady state equation. Second, the relation between the output per worker of an economy at the steady state and that of its neighbours, $\frac{(1-\alpha)\gamma_3+\gamma_2}{1-\alpha-\phi}\sum_{j=1}^{N} w_{ij} \ln y_{jt}^*$. Whereas in the model of Ertur and Koch (2007) there is no global contagion in the output unless $\gamma_3 \neq 0$ (provided of course that $\alpha \neq 1$), here we may still have such contagion to the extent that $\gamma_2 \neq 0$ (as in López-Bazo et al. 2004), *even if* $\gamma_3 = 0$. More generally, these features of our model are also absent in the above mentioned growth studies (López-Bazo et al., 2004; Egger and Pfaffermayr, 2006; Fingleton and López-Bazo, 2006; Pfaffermayr, 2009, 2012).

2.5 The growth-initial equation

In the standard, non-spatial growth models (see e.g. Barro and Sala-i-Martin, 2003), the analog of equation 2.9 gives an expression for the output per worker in the steady state that does not depend on the output per worker in the steady state of the other economies (i.e., the term $\frac{(1-\alpha)\gamma_3+\gamma_2}{1-\alpha-\phi}\sum_{j=1}^{N} w_{ij} \ln y_{jt}^*$ is missing). Thus, a log-linear approximation to the dynamics around the steady state using a Taylor expansion produces a growth-initial regression equation that can be estimated using the appropriate method. In our case, however, this approach would produce a rather complex system of first-order differential linear equations whose solution is not directly estimable due to the presence of variables at the steady state (Egger and Pfaffermayr, 2006, for example, approximate them using a set of exogenous variables). In particular, a log linearisation of the marginal productivity of capital, $\frac{\dot{k}_{it}}{k_{it}}$, around the steady state yields (see appendix *B*)

$$\frac{\dot{k}_{it}}{k_{it}} = g + (u_{ii} - 1)(n_i + \delta + g)\left(\ln k_{it} - \ln k_{it}^*\right) + \sum_{j \neq i}^{N} u_{ij}(n_i + \delta + g)\left(\ln k_{jt} - \ln k_{jt}^*\right) \tag{2.10}$$

Notice that this result coincides with the one obtained by Ertur and Koch (2007).

To tackle this issue, Ertur and Koch (2007) hypothesise that the differences between the observed and the steady state values of the capital and output per worker across regions correspond to the following expressions:

$$\begin{aligned} \ln y_{it} - \ln y_{it}^* &= \Theta_j \left(\ln y_{jt} - \ln y_{jt}^*\right) \\ \ln k_{it} - \ln k_{it}^* &= \Phi_j \left(\ln k_{jt} - \ln k_{jt}^*\right) \end{aligned} \tag{2.11}$$

This yields the following speed of convergence (see appendix *C*):

$$\frac{d \ln y_{it}}{dt} = g - \lambda_i \left(\ln y_{it} - \ln y_{it}^*\right) \tag{2.12}$$

with

$$\lambda_i = \frac{\sum_{j=1}^{N} u_{ij} \frac{1}{\Phi_j} (n_j + g + \delta)}{\sum_{j=1}^{N} u_{ij} \frac{1}{\Phi_j}} - \sum_{j=1}^{N} u_{ij}(n_j + \delta + g) \frac{1}{\Theta_j} \tag{2.13}$$

Solving the differential equation in 2.12 for $\ln y_{it}$ (see appendix *D*) and evaluating the solution at $t = t_2$:

$$\ln y_{it_2} = g\left(t_2 - t_1 e^{-\lambda_i \tau}\right) - e^{-\lambda_i \tau} \ln y_{it_1} + \left(1 - e^{-\lambda_i \tau}\right) \ln y_{i0}^* \tag{2.14}$$

with $\tau = t_2 - t_1$. In particular, under the assumption that the speed of convergence is homogeneous across regions ($\lambda_i = \lambda$ for $i = 1, \cdots, N$):

$$\ln y_{it_2} = g\left(t_2 - t_1 e^{-\lambda\tau}\right) + e^{-\lambda\tau} \ln y_{it_1} + (1 - e^{-\lambda\tau}) \ln y_{i0}^* \tag{2.15}$$

At this point it is convenient to write the previous expression in matrix form:

$$y(t_2) = g\left(t_2 - t_1 e^{-\lambda\tau}\right) \iota_N + e^{-\lambda\tau} y(t_1) + \left(1 - e^{-\lambda\tau}\right) y^*(0) \tag{2.16}$$

where $y(t_2)$ is a $N \times 1$ vector containing the log of the outcome per worker at t_2, ι_N is a $N \times 1$ vector of ones, $y(t_1)$ is a $N \times 1$ vector containing the log of the outcome per worker at t_1, and $y^*(0)$ is a $N \times 1$ vector containing the log of the initial level of output per worker at the steady state. The reason for this is that facilitates replacing $y^*(0)$ by 2.9 at $t = 0$, which, in matrix form, is:

$$y^*(0) = (I - \rho W)^{-1} \left[\frac{1}{1-\alpha-\phi}\Omega(0) + \frac{\gamma_1}{1-\alpha-\phi} W\Omega(0) + \frac{\alpha+\phi}{1-\alpha-\phi} S + \frac{\gamma_2 - \alpha\gamma_3}{1-\alpha-\phi} WS\right] \tag{2.17}$$

where $\rho = \dfrac{(1-\alpha)\gamma_3 + \gamma_2}{1-\alpha-\phi}$ (it is assumed that $1/\rho$ is not an eigenvalue of W when $\rho \neq 0$) and $S = \left\{\ln\left(\dfrac{s_i}{n_i + \delta + g}\right)\right\}_{i=1,\ldots,N}$.

Thus, we introduce 2.17 in 2.16 and pre-multiply both sides of the resulting equation by $I - \rho W$ to obtain:

$$\begin{aligned} y(t_2) = {} & g(1-\rho)\left(t_2 - t_1 e^{-\lambda\tau}\right) \iota_N + e^{-\lambda\tau} \left(I - \rho W\right) y(t_1) + \rho W y(t_2) \\ & + \left(1 - e^{-\lambda\tau}\right) \left[\frac{1}{1-\alpha-\phi}\Omega(0) + \frac{\gamma_1}{1-\alpha-\phi} W\Omega(0) + \frac{\alpha+\phi}{1-\alpha-\phi} S + \frac{\gamma_2 - \alpha\gamma_3}{1-\alpha-\phi} WS\right] \end{aligned} \tag{2.18}$$

Alternatively, we can rewrite this equation for country i as

$$\begin{aligned}\ln y_{it_2} &= e^{-\lambda\tau}\ln y_{it_1} - \rho e^{-\lambda\tau}\sum_{j=1}^{N} w_{ij}\ln y_{jt_1} + \rho\sum_{j=1}^{N} w_{ij}\ln y_{jt_2} \\ &+ \frac{\left(1-e^{-\lambda\tau}\right)(\alpha+\phi)}{1-\alpha-\phi}\ln s_i - \frac{\left(1-e^{-\lambda\tau}\right)(\alpha+\phi)}{1-\alpha-\phi}\ln(n_i+\delta+g) \\ &+ \frac{\left(1-e^{-\lambda\tau}\right)(\gamma_2-\alpha\gamma_3)}{1-\alpha-\phi}\sum_{j=1}^{N} w_{ij}\ln s_j - \frac{\left(1-e^{-\lambda\tau}\right)(\gamma_2-\alpha\gamma_3)}{1-\alpha-\phi}\sum_{j=1}^{N} w_{ij}\ln(n_j+\delta+g) \\ &+ \left(\frac{\left(1-e^{-\lambda\tau}\right)}{1-\alpha-\phi}\ln\Omega_{i0}\right) + \left(\frac{\left(1-e^{-\lambda\tau}\right)\gamma_1}{1-\alpha-\phi}\sum_{j=1}^{N} w_{ij}\ln\Omega_{j0}\right) \\ &+ g(1-\rho)\left(t_2 - t_1 e^{-\lambda\tau}\right)\end{aligned} \tag{2.19}$$

3 Empirical results

3.1 Model specification and identification strategies

To derive our econometric specification, notice that equation 2.19 (plus an i.i.d. shock ε) corresponds to the spatial Durbin dynamic panel model with individual-specific effects and their spatial spillovers:

$$\begin{aligned}z_{it} &= \overline{\gamma}_1 z_{i,t-1} + \overline{\gamma}_2\sum_{j=1}^{N} w_{ij} z_{j,t-1} + \rho\sum_{j=1}^{N} w_{ij} z_{jt} + \beta_1 x_{1_{it}} + \beta_2 x_{2_{it}} + \theta_1\sum_{j=1}^{N} w_{ij} x_{1_{jt}} + \theta_2\sum_{j=1}^{N} w_{ij} x_{2_{jt}} \\ &+ \mu_i + \sum_{j=1}^{N} w_{ij}\alpha_j + f_t + \varepsilon_{it}\end{aligned} \tag{3.1}$$

where $z_{it} = \ln y_{it_2}$, $z_{i,t-1} = \ln y_{it_1}$, $x_{1_{it}} = \ln s_{it}$, $x_{2_{it}} = \ln(n_{it}+\delta+g)$, $\overline{\gamma}_1 = e^{-\lambda\tau}$, $\overline{\gamma}_2 = -\rho e^{-\lambda\tau}$, $\beta_1 = \frac{\left(1-e^{-\lambda\tau}\right)(\alpha+\phi)}{1-\alpha-\phi}$, $\beta_2 = -\frac{\left(1-e^{-\lambda\tau}\right)(\alpha+\phi)}{1-\alpha-\phi}$, $\theta_1 = \frac{\left(1-e^{-\lambda\tau}\right)(\gamma_2-\alpha\gamma_3)}{1-\alpha-\phi}$, $\theta_2 = -\frac{\left(1-e^{-\lambda\tau}\right)(\gamma_2-\alpha\gamma_3)}{1-\alpha-\phi}$, $\mu_i = \frac{\left(1-e^{-\lambda\tau}\right)}{1-\alpha-\phi}\ln\Omega_{i0}$, $\alpha_i = \frac{\left(1-e^{-\lambda\tau}\right)\gamma_1}{1-\alpha-\phi}\ln\Omega_{i0}$ and $f_t = g(1-\rho)\left(t_2 - t_1 e^{-\lambda\tau}\right)$.

This means that equation 3.1 corresponds to the model specification discussed by Lee and Yu

(2016), except that their model does not distinguishes the spatial counterparts of the individual effects ($\sum_{j=1}^{N} w_{ij}\alpha_j$). In other words, their individual effects correspond to $\mu_i + \sum_{j=1}^{N} w_{ij}\alpha_j$ in 3.1. In fact, in our model the individual effects and their spatial counterparts are proportional (by a rate γ_1). This is therefore a particular case of the more general specification proposed by Miranda et al. (2017a).

To distinguish the individual effects from their spatial spillovers, we assume a correlated random effects specification for the individual effects (μ_i) and their spatial spillovers (α_i). This means making use of the following correlation functions (Mundlak, 1978; Chamberlain, 1982):

$$\begin{aligned} \mu_i &= \pi_{\mu_1}\left(\frac{1}{T}\sum_{t=1}^{T} x_{1_{it}}\right) + \pi_{\mu_2}\left(\frac{1}{T}\sum_{t=1}^{T} x_{2_{it}}\right) + \upsilon_{\mu i} \\ \alpha_i &= \pi_{\alpha_1}\left(\frac{1}{T}\sum_{t=1}^{T} x_{1_{it}}\right) + \pi_{\alpha_2}\left(\frac{1}{T}\sum_{t=1}^{T} x_{2_{it}}\right) + \upsilon_{\alpha i}, \end{aligned} \tag{3.2}$$

where π_{μ_1}, π_{μ_2}, π_{α_1} and π_{α_2} are the parameters associated with the period-means of the regressors, and $\upsilon_{\mu i}$ and $\upsilon_{\alpha i}$ are random error terms with $E(\upsilon_{\mu i}) = 0 = E(\upsilon_{\alpha i})$, $Var(\upsilon_{\mu i}) = \sigma^2_\mu$, $Var(\upsilon_{\alpha i}) = \sigma^2_\alpha$ and $Cov(\upsilon_{\mu i}, \upsilon_{\alpha i}) = \sigma_{\mu\alpha}$.

The last thing to notice about our econometric specification is that the implied parameters (α, ϕ and γ_2, on the one hand; γ_1, λ, γ_3, and $\ln\Omega_{i0}$, on the other) are not identified. In particular, we cannot obtain a single estimate of γ_1 (since this can be obtained from each (α_i, μ_i) pair, but also from either π_{μ_1} and π_{α_1} or π_{μ_2} and π_{α_2}), λ (since this can be obtained from $\overline{\gamma}_1$, but also from $\overline{\gamma}_2$ and ρ), γ_3 (since this requires ρ, $\overline{\gamma}_1$, either β_1 or β_2, and either θ_1 or θ_2, respectively) and $\ln\Omega_{i0}$ (since this requires either μ_i, $\overline{\gamma}_1$ and either β_1 or β_2, or α_i, $\overline{\gamma}_1$, γ_1 and either β_1 or β_2) because in principle these parameters are overidentified. However, it is easy to see that equations 3.1 and 3.2 three sets of constraints on the parameters: *i)* $\beta_1 = -\beta_2$ and $\theta_1 = -\theta_2$ (arising from the assumption that the production function is homogeneous of degree one, thus making the output per capita to depend only on the stock of physical capital); *ii)* $\overline{\gamma}_2 = -\rho\overline{\gamma}_1$ (arising from the assumed spatial-time dynamics of the technology); and *iii)* $\alpha_i = \gamma_1\mu_i$ (i.e., $\pi_\alpha = \gamma_1\pi_\mu$, $\sigma^2_\alpha = \gamma_1^2\sigma^2_\mu$ and $\sigma_{\mu,\alpha} = \gamma_1\sigma^2_\mu$, which arise from the assumed spatial contagion in the heterogeneous exogenous technology and unobserved productivity).[8] By imposing these six constraints on 3.1 and 3.2 (i.e., the "unconstrained model"), we obtain a constrained version of our model in which γ_1, λ, γ_3, and $\ln \Omega_{i0}$ are identified.

To this end, we start by replacing 3.2 into 3.1, which in matrix form yields:

$$Z_t = \overline{\gamma}_1 Z_{t-1} + \overline{\gamma}_2 W Z_{t-1} + \rho W Z_t + X_t \beta + W_t X \theta + \overline{X} \Pi_\mu + W \overline{X} \Pi_\alpha + f_t + \eta_t \tag{3.3}$$

where $X_t = \left(x_{1_t} \,\vdots\, x_{2_t} \right)$, $\overline{X}$ denote period-means of X_t, $\beta = (\beta_1, \beta_2)'$, $\theta = (\theta_1, \theta_2)'$, $\Pi_\mu = (\pi_{\mu_1}, \pi_{\mu_2})'$, $\Pi_\alpha = (\pi_{\alpha_1}, \pi_{\alpha_2})'$, and the error term is $\eta_t = \upsilon_\mu + W\upsilon_\alpha + \varepsilon_t$, with variance-covariance matrix given by $J_T \otimes (\sigma_\mu^2 I + \sigma_{\mu\alpha}(W + W') + \sigma_\alpha^2 WW') + \sigma_\varepsilon^2 I_{NT}$, J_T being a $T \times T$ matrix of ones and I_{NT} being the $NT \times NT$ identity matrix. This is the unconstrained version of our econometric model.

Let us now define $S_1 = I - \rho W$, $S_2 = I + \gamma_1 W$ and $X_{it}^* = \ln \left(\frac{s_{it}}{n_{it} + \delta + g} \right) = \ln (S_{it})$. Then, the constrained model is given by

$$S_1 Z_t = \overline{\gamma}_1^c S_1 Z_{t-1} + \beta^c X^* + \theta^c W X^* + S_2 \overline{X} \Pi_\mu^c + f_t + \eta_t^c \tag{3.4}$$

with $\overline{\gamma}_2 = -\rho \overline{\gamma}_1^c$, $\beta_1 = -\beta_2 = \beta^c$, $\theta_2 = -\theta_1 = \theta^c$ and $\Pi_\alpha = \gamma_1 \Pi_\mu^c$ and $\eta_t^c = \varepsilon_t + S_2 \upsilon_\mu$, with variance-covariance matrix given by $J_T \otimes (\sigma_\mu^2 S_2 S_2') + \sigma_\varepsilon^2 I_{NT}$. Notice that, in contrast to 3.3, the estimation of the constrained version of our econometric model in 3.4 (see e.g. Lee and Yu, 2016; Miranda et al., 2017a) allows us to obtain an estimate of: *i)* the degree of technological interdependence between the unobserved productivity, γ_1 (directly from S_2); *ii)* the speed of convergence, λ (from $\overline{\gamma}_1^c$); *iii)* the degree of technological interdependence between the economies, γ_3 (from $\overline{\gamma}_1^c$, β^c and θ^c); and *iv)* the unobserved productivity, $\ln \Omega_{i0}$ (from μ_i, β^c and $\overline{\gamma}_1^c$). In particular, obtaining a statistically significant estimate of γ_1 should be interpreted as supportive evidence for our model. Also, obtaining a statistically significant estimate of γ_3 would lead us to reject the models of Islam (1995) and López-Bazo et al. (2004).

[8]While *i)* also arises in the model of Ertur and Koch (2007), *ii)* and *iii)* are specific to our model specification. In this respect, notice that Elhorst et al. (2010, p. 343) also consider the constraint $\overline{\gamma}_2 = -\rho \overline{\gamma}_1$. However, while in our case it arises directly from the derivation of our model specification, they argue that this "constraint is unnecessarily restrictive because no theoretical or empirical reason exists to impose it".

The problem, of course, is that we still cannot identify α, ϕ and γ_2 (only $\alpha + \phi$ and $\gamma_2 - \alpha\gamma_3$), although in this case it is because these parameters are under-identified. Since both the own stock of physical capital and that of the neighbouring economies are arguments of the technology, we cannot separate the effect that, as an input of the production function, the stock of physical capital has on the output (i.e., α) from the effect that it has as a driver of the technology (i.e., ϕ). Neither can we separate the local effect that the stock of physical capital of the neighbouring economies has on the technology and, subsequently, the output (i.e., γ_2), from the global effect that the stock of physical capital of the neighbouring economies has on the technology and, subsequently, the output (i.e., $\alpha\gamma_3$). Still, there are ways to circumvent this identification problem.

One way is to modify the specification of the model. There are no identification problems, for example, if we are willing to assume that the stock of physical capital enters the technological progress lagged one period. That is, if we are willing to assume that $A_{it} = \Omega_{it} \prod_{j \neq i}^{N} \Omega_{jt}^{\gamma_1 w_{ij}} k_{it-1}^{\phi} \prod_{j \neq i}^{N} k_{jt-1}^{\gamma_2 w_{ij}} \prod_{j \neq i}^{N} A_{jt}^{\gamma_3 w_{ij}}$ (see, in contrast, equation 2.4). Neither there are if we argue that different arguments of the technology require different weight matrices. In mathematical terms, this means assuming that $A_{it} = \Omega_{it} \prod_{j \neq i}^{N} \Omega_{jt}^{\gamma_1 w_{ij}^{\Omega}} k_{it}^{\phi} \prod_{j \neq i}^{N} k_{jt1}^{\gamma_2 w_{ij}^{k}} \prod_{j \neq i}^{N} A_{jt}^{\gamma_3 w_{ij}^{A}}$, where, in obvious notation, w_{ij}^{Ω}, w_{ij}^{k} and w_{ij}^{A} denote different weight matrices (see e.g. Lee and Yu, 2016). These approaches, however, involve the derivation of a new model (the steady state equation and the speed of convergence, for example, would surely be altered) and/or require additional data to construct the weight matrices (in our empirical application, we may for example need data on bilateral trade flows and geographical distances between the EU regions). We thus leave these approaches for future research.

In this paper, we simply notice that some of the remaining implied parameters would be identified if an additional appropriate constrain was available. If we were willing to assume, for example, that the impact of the own physical stock and that of the other economies in the level of technology is the same (i.e., $\phi = \gamma_2$), then we may obtain an estimate of α and $\phi = \gamma_2$ from γ_3, β^c and θ^c. However, since our assumed technology encompasses that of López-Bazo et al. (2004) and Ertur and Koch (2007), we find that it is of greater interest to constrain the under-identified implied parameters of the technology (i.e., ϕ and γ_2) to be consistent with the technology these papers assume. Thus, under the assumption that the technology of López-Bazo et al. (2004) is the appropriate (i.e., $\phi = 0$), we can obtain an estimate of α and γ_2 from 3.4, whereas under the assumption that the technology of Ertur and Koch (2007) is the appropriate (i.e., $\gamma_2 = 0$), we can obtain an estimate of α and ϕ from 3.4. Further, under the assumption that neither the own capital nor that of the neighbouring economies have a role in shaping the technology (i.e., $\phi = \gamma_2 = 0$), we can obtain an estimate of α from the following constrained model:[9]

$$S_1 Z_t = \overline{\gamma}_1^c S_1 Z_{t-1} + \beta^c S_1 X^* + S_2 \overline{X} \Pi_\mu^c + f_t + \eta_t^c \tag{3.5}$$

However, these estimates should be interpreted with care. If $\gamma_3 = 0$, then we can interpret an statistically significant estimate of γ_2 that is obtained under $\phi = 0$ as supportive evidence for the model of López-Bazo et al. (2004). That is, there exist local spatial externalities in the technology associated with the stock of capital (but not global, since the data supports that $\gamma_3 = 0$). If $\gamma_3 \neq 0$, however, the evidence is consistent with the model of Ertur and Koch (2007, p. 1036), and then the question that we can address by imposing the constraint $\gamma_2 = 0$ (but not that $\phi = 0$) is whether there are indeed "home externalities generated by physical capital accumulation". In particular, only if $\phi \neq 0$ we may conclude that there exist global spatial externalities associated with the stock of capital (although we remain uncertain about whether there are local externalities in capital because we have imposed that $\gamma_2 = 0$). Lastly, if $\phi = \gamma_2 = 0$, we may conclude that, regardless of the value taken by γ_3, there is no spatial contagion of the stock of capital (neither in the technology nor in the output). However, if the data rejects the validity of these constraints, then we remain uncertain about what is the nature of these spatial externalities: local (i.e., $\phi \neq 0$), global (i.e., $\gamma_2 \neq 0$), or both local and global (i.e., $\phi \neq 0$ and $\gamma_2 \neq 0$).

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[9]Notice that, while imposing the assumptions that either $\phi = 0$ or $\gamma_2 = 0$ in 3.4 does not yield a new model specification, imposing both assumptions simultaneously does yield a new model specification (3.5) in which $\theta^c = \beta^c \gamma_3 = \beta^c \rho$ (see also Ertur and Koch 2007 and Elhorst et al. 2010).

3.2 Estimates from EU-NUTS2 regions

We estimate the model given by 3.3 using the approach and model specifications of Lee and Yu (2016) and Miranda et al. (2017a). We use the first as a benchmark for our basic parameters ($\overline{\gamma}_1$, $\overline{\gamma}_2$, ρ, β_1, β_2, θ_1 and θ_2, which, since all the variables are in logs, can be interpreted as elasticities) and the second to obtain the whole set of estimates (i.e., the basic ones plus those appearing in the correlation functions: π_{μ_1}, π_{μ_2}, π_{α_1} and π_{α_2}), test the validity of the constrained version of the model (using a Likelihood Ratio test), and estimate the implied parameters (using the constrained version of the model). We also follow this scheme in the discussion of the results. This means that we will start with an analysis of the estimates of the basic and correlation functions parameters in the unconstrained and constrained models, then will go on with the estimates of the implied parameters, and we will conclude with a description of the geographical distribution of the estimated "unobserved productivity" of the EU regions ($\ln \hat{\Omega}_{i0}$) and its estimated spatial spillover ($\hat{\gamma}_1 \sum_{j=1}^{N} w_{ij} \ln \hat{\Omega}_{j0}$).

First, however, a word about the data. We use EU NUTS2 regional data from Cambridge Econometrics to estimate our model. In particular, our initial sample is analogous to the one analysed by Elhorst et al. (2010), so that we can use their results as a benchmark to which ours will be compared. Thus, we initially consider 189 regions across 14 EU countries (Austria, Belgium, Germany, Denmark, Greece, Finland, France, Ireland, Italy, the Netherlands, Portugal, Spain, Sweden and the United Kingdom) using time intervals of five years (see also Ho et al., 2013; Lee and Yu, 2016) over the period 1982 to 2002. This results in a balanced panel dataset with 4 time periods (1982-1987, 1987-1992, 1992-1997, 1997-2002).[10]

It is worth noting, however, that we have explored alternative samples to check the robustness of our results. First, we extended our initial sample to cover the years of the recent global crisis (the time intervals 2002-2007 and 2007-2012). Second, we considered different

[10] To be precise, the (small) differences between our sample and that of Elhorst et al. (2010) are the following. First, they have data on Luxembourg and the period 1977-1982. Second, in their sample "the islands (such as those associated with southern European countries) are assumed to be connected to the mainland, so that each region has at least one neighbour" (p. 353). Here we only consider continental regions, which means that our sample does not include the Spanish cities of Ceuta and Melilla, the French's "Départements d'outre mer", and the Greek, Finish, French, Italian and Spanish islands.

time intervals in a wider time period (1980 to 2015, with observations for 1980-1985, 1985-1990, and up to 2010-2015, which was the last available period at the moment of writing this paper). Third, we considered alternative groups of countries (e.g., including Norway, which is a non-EU country, and/or dropping Portugal, Ireland, Italy, Spain and/or Greece, which are countries that have faced –severe– economic growth problems over the last decade). In all those cases, the estimates we obtained for the (un)constrained model remained largely unaltered. We illustrate this by reporting results from these alternative sampling schemes: the period 2002 to 2012, the period 1980 to 2015, the period 1982 to 2002 without including Portugal, Ireland, Italy, Spain and Greece (the so-called "PIIGS") and the period 1982 to 2002 without including Greece (since in all these cases results when including Norway were not substantially different).

All these estimates were obtained using real GDP per capita as the dependent variable (i.e., y_{it} is real GDP at 2005 constant prices over total population, in thousands of people). As for the explanatory variables, s_{it} is the ratio between investment expenditures and gross value-added (at 2005 constant prices and as a percentage) and n_{it} is the growth rate of the population over time (computed as in Islam 1995). As it is common in the literature (see e.g. Mankiw et al., 1992; Islam, 1995; Ertur and Koch, 2007), we assume that $\delta + g = 0.05$ to compute the depreciation rate. Note also that time dummies and a constant term were included in the set of explanatory variables to account for f_t.

[Insert Table 1 about here]

Table 1 provides descriptive statistics for the dependent and main explanatory variables (i.e., y_{it}, s_{it} and n_{it}). In particular, we report the statistics for the five samples considered and the periods effectively used in estimation in each case (notice that we lose one observation due to the inclusion of the lagged dependent variable in the model). The differences in the values of the statistics across the samples considered are of small magnitude, particularly between the original sample and the same sample without Greece. In fact, the observed differences arise in the GDP and the saving rate, whereas the distribution of the depreciation rate remains almost unaltered across samples. It is also interesting to note that the recent economic crisis seems to have increased the levels of GDP and savings, but mostly for those regions that were already in the top of the distribution (i.e., the centre of the distribution of these variables has shifted to the right and the upper tail has increased, thus making differences between the extremes larger). The effect is similar when dropping the PIIGS from the original sample, except that now it it is the lower tail of the distribution the one that increases (i.e., we are dropping regions with levels of GDP and savings that are lower than those of the rest of the sample).

[Insert Table 2 about here]

We move now to the analysis of the estimates of the model and, as previously pointed out, start by considering the estimates of the unconstrained version of the model. These are reported in Table 2. In particular, the first reported estimates (in column two) were obtained using the approach and model specification of Lee and Yu (2016), whereas the rest (columns three to seven) were obtained using that of Miranda et al. (2017a). We report results for the initial sample (period 1982 to 2002) in columns two and three and, subsequently, for the other samples considered (periods 1982 to 2012, 1980 to 2015, 1982 to 2002 without the PIIGS, and 1982 to 2002 without Greece).

We find a remarkable regularity in both the values and the statistical significance of the coefficients across the samples and estimation approaches considered. Perhaps the only differences worth mentioning are: *i)* the slightly lower value of the coefficient associated with the time-lagged dependent variable ($\overline{\gamma}_1$) when estimating the model using the approach of Lee and Yu (2016); and *ii)* the lack of statistical significance of the coefficient associated with the saving rate (β_1) when considering the years of the recent crisis (i.e., the samples covering the periods 2002 to 2012 and 1980 to 2015). This caveat aside, all sets of estimates provide essentially the same picture.

In particular, the basic parameters are all statistically significant (except for θ_2) and have the predicted signs (see Ertur and Koch, 2007).[11] Consistent with the constraint $\overline{\gamma}_2 = -\rho\overline{\gamma}_1$,

[11] Our estimates of the basic parameters are largely consistent with those reported by Basile (2008) using an analogous sample of regions and the period 1988 to 2000. They also concur with those reported in panel data studies analysing countries rather than regions (see e.g. Ho et al., 2013; Lee and Yu, 2016). In contrast, we find some differences with those reported by Pfaffermayr (2009), who consider an analogous period of analysis but whose sample includes Norway's and Switzerland's regions.

the spatial and time lagged dependent variables have a high and positive coefficient, whereas the spatially weighted lagged dependent variable has a negative and smaller coefficient in absolute value (see also Ho et al., 2013; Lee and Yu, 2016). Thus, the level of GDP per capita of the European regions is largely determined by its past GDP per capita and the current and past GDP per capita of their neighbours. Further, the saving rate of an economy contributes positively to its GDP per capita, but its depreciation rate and the saving rate of the neighbouring regions both contribute negatively. All in all, these results indicate that richest areas are likely to stay rich (more so they if are geographically close to rich areas, like e.g. in the so-called "blue banana") while poorer areas can only (partially) catch up if they increase their saving rates and/or are geographically close to rich areas.

As for the correlation functions parameters, there is evidence of *i)* correlation between the individual effects and the covariates (since both the –mean– saving and depreciation rates are statistically significant) and *ii)* spatial contagion in the individual effects (since the spatially weighted –mean– saving rate is generally statistically significant). In addition, two of the variance components, σ_μ^2 and σ_ε^2, are statistically significant. This supports our correlated random effects model specification. In particular, results are consistent with the constraint $\alpha_i = \gamma_1 \mu_i$, which implies a "fixed effects" error term model with proportional spatial contagion (Miranda et al., 2017a).

[Insert Table 3 about here]

Next we consider the results for the constrained version of the model, which are reported in Table 3. Before discussing the estimates, however, it is important to assess the validity of equation 3.4 in the different samples considered. To this end, we used a Likelihood Ratio test. We found that the "fully" constrained version of the model (i.e., the model resulting from imposing the constraints $\beta_1 = -\beta_2$, $\theta_1 = -\theta_2$, $\overline{\gamma}_2 = -\rho\overline{\gamma}_1$ and $\alpha_i = \gamma_1 \mu_i$) was statistically supported only in the last two samples (i.e., the period 1982 to 2002 without the PIIGS and without Greece).[12] Estimates from this fully constrained version of the model are reported

[12]In particular, the Likelihood Ratio test statistics we obtained in the first three samples were 18.42 (period 1982 to 2002), 42.06 (period 1982 to 2012) and 27.26 (period 1980 to 2015), all statistically significant at

in Table 3b. Still, after testing the validity of each constraint individually, we found that a "partially" constrained version of the model in which only the constraint $\alpha_i = \gamma_1 \mu_i$ was imposed was not rejected in the first three samples (periods 1982 to 2002, 1982 to 2012, and 1980 to 2015). Estimates from this partially constrained version of the model are reported in Table 3a.[13]

At first sight, there is very little to comment on the results reported in Table 3a since, as expected, they are very similar to the ones obtained from the unconstrained model (see Table 2). Yet two things are worth mentioning. First, the correlation functions parameters and the variance components parameters are all statistically significant. This again supports our correlated random effects specification. Second, the coefficient reflecting the degree of technological interdependence generated from the productivity spillovers, γ_1, shows a negative and (at least in two of the samples considered) statistically significant value. Also, the estimates we obtain for γ_1 are similar across the samples considered. This indicates, given the imposed constraint $\alpha_i = \gamma_1 \mu_i$, that there exists a negatively proportional relation between the individual effects of the EU regions and their spatial spillovers. We will return to this point when we analyse the geographical distribution of $\ln \hat{\Omega}_{i0}$ and $\hat{\gamma}_1 \sum_{j=1}^{N} w_{ij} \ln \hat{\Omega}_{j0}$.

As for the estimates of the "fully" constrained version of the model, the first thing to notice is that they are similar in the two samples considered (except for the lack of statistical significance of θ^c in the sample without the PIIGS). In particular, the basic parameters are all statistically significant and have the predicted signs (see Ertur and Koch, 2007). Also, if we compare our results with those obtained by Elhorst et al. (2010), our estimates of the difference in the logs of the saving and depreciation rates, as well as that of its spatial counterpart, are both larger (and statistically significant, whereas only the former is in their case). The estimated coefficient of the spatially lagged dependent variable, on the other hand, is analogous to the one reported

standard levels. The Likelihood Ratio test statistics of the other samples (the period 1982 to 2002 without the PIIGS and without Greece) are reported in the last row of Table 3.

[13]Notice that, since γ_1 is an implied parameter, it is reported in Table 4 (along with the rest of the implied parameters obtained from the fully constrained model). However, since γ_1 is identified in both the fully and partially constrained models, for the sake of comparability we have also included its estimates among the results reported in Table 3.

by Elhorst et al. (2010). Lastly, the rest of parameters have estimated values in line with those obtained for the "partially" constrained version of the model (including γ_1, as previously pointed out).

We then use these "fully constrained" estimates to obtain the implied parameters of the theoretical model. These are reported in Table 4. In particular, the first block of Table 4 contains the estimates of the parameters that are directly identified (γ_1, λ and γ_3), the second block the estimates of α and γ_2 obtained under the assumption that the technology considered by López-Bazo et al. (2004) is the appropriate (i.e., imposing the additional constraint $\phi = 0$), and the third block the estimates of α and ϕ obtained under the assumption that the technology considered by Ertur and Koch (2007) is the appropriate (i.e., imposing the additional constraint $\gamma_2 = 0$).

[Insert Table 4 about here]

Firstly, the statistical significance of the degree of technological interdependence generated from the (unobserved) productivity spillovers, γ_1, supports our assumed technology (against the related alternatives of Islam 1995, López-Bazo et al. 2004 and Ertur and Koch 2007). Secondly, the estimated speed of convergence, as measured by λ, is around 2% and statistically significant, which is a standard result in the literature (Barro and Sala-i-Martin, 2003; López-Bazo et al., 2004; Ertur and Koch, 2007; Lee and Yu, 2016). Thirdly, the statistical significance of the degree of technological interdependence, as measured by γ_3, supports the model of Ertur and Koch (2007) and contradicts the models of Islam (1995) and López-Bazo et al. (2004). Moreover, its value is similar to the one found by Ertur and Koch (2007) and Elhorst et al. (2010), somewhere in between them. Fourthly, the estimates of the capital share, as measured by α, obtained when imposing the additional constraint(s) that $\phi = 0$ (López-Bazo et al., 2004) and/or $\gamma_2 = 0$ (Ertur and Koch, 2007) are in line with those obtained in the literature (Barro and Sala-i-Martin, 2003; Ertur and Koch, 2007; Elhorst et al., 2010). Fifthly, the parameter capturing capital externalities at the local level (γ_2) and that allowing for capital externalities at the global level (ϕ, through γ_3), obtained when imposing the additional constraint that either $\phi = 0$ (López-Bazo et al., 2004) or $\gamma_2 = 0$ (Ertur and Koch, 2007), respectively, are not statistically significant. In fact, we cannot reject the null hypothesis that both parameters are zero. The LR test statistic obtained from the models 3.4 and 3.5 is 0.53 for the sample 1982 to 2002 without the PIIGS and 0.53 for the sample 1982 to 2002 without Greece, none of them being statistically significant at standard levels.[14]

All in all, these results point to the the existence of spatial spillovers in the unobserved productivity and the level of technology. That is, we find evidence supporting the existence of both local and global spillovers in the stock of knowledge. In contrast, there is no sign of the capital externalities in technology found by either López-Bazo et al. (2004) or Ertur and Koch (2007). That is, we do not find evidence of spatial externalities in the stock of capital. Lastly, our estimates support our model specification against that of Islam (1995), López-Bazo et al. (2004) and Ertur and Koch (2007).

[Insert Figure 1 about here]

To conclude our empirical analysis, we report the geographical distribution of the estimated "unobserved productivity" and its spatial spillover (to reiterate, obtained from the constrained model in 3.4) in Figure 1. More precisely, Figure 1 presents a map of the European regions considered and the values of these statistics grouped by quantiles: Figure 1*a* reports $\ln \hat{\Omega}_{i0}$ (the "unobserved productivity") whereas Figure 1*b* reports $\hat{\gamma}_1 \sum_{j=1}^{N} w_{ij} \ln \hat{\Omega}_{j0}$ (the spatial spillover of the "unobserved productivity", that is, the impact on the technology of unit i of all the units neighbouring i having their "unobserved productivity"). Notice that we have opted for using the estimates from the 1982-2002 sample without Greece to construct Figure 1 because this allows us to analyse a larger number of regions. It is important to stress, however, that results were not substantially different when using the 1982-2002 sample without the PIIGS. Notice also that, given the negative and statistically significant value found for γ_1, there is a negatively proportional relation between the unobserved productivity of each EU region and the spatial contagion of this unobserved productivity on its neighbouring regions.[15]

[14]More generally, the other four findings largely hold when imposing on the "fully constrained" model in 3.4 the additional constraint that $\phi = \gamma_2 = 0$, that is, when estimating the constrained model in 3.5.

[15]These spillovers correspond to the (local) spill-in effects proposed by Miranda et al. (2017b). We do not

With this in mind, we start by noting the considerable heterogeneity that Figure 1a displays, which contradicts the standard assumption of homogeneous exogenous technological progress. In particular, results indicate that the regions with the lowest estimated "unobserved productivity" are mostly located in Scandinavia (Finland and Sweden), Scotland and North of England, Northern Ireland, Central-South of France, South-Est of Germany, Austria, Central and North-West of Spain, and North-West and South of Italy. Figure 1a also shows that the geographical distribution of the higher estimated "unobserved productivity" covers the so-called "blue banana" (from the South of the UK to the South-West of Germany, thus including the North of France, Belgium and the Netherlands), plus Denmark and the Mediterranean regions of the South-West of France and Central Italy.

What is also interesting to note is that about half of the regions in the high productivity group can be qualified as "rich", meaning here that their average GDP per capita over the periods considered is in the upper quantile of the distribution. On the other hand, the same criterion would lead us to qualify about half of the regions with low estimated productivities as "poor". Thus, it seems that many of the richer/poorer regions tend to have higher/lower (unobserved) productivities. In fact, the Spearman rank correlation between $\ln \hat{\Omega}_{i0}$ and the average GDP per capita is 0.36 and statistically significant.

As for the spillovers associated with the "unobserved productivity", Figure 1b reveals that the pattern tends to be opposite to the one found for the estimated "unobserved productivity". In particular, the largest values are found in the Northern regions (i.e., Scandinavia, East of Ireland, the UK Midlands and South of Scotland), but also in the East (i.e., Austria) and South (South-West of France, North and West of Spain, and South of Italy) of Europe. This means that these are (often poor) regions whose "unobserved productivity" is more impacted by the "unobserved productivity" of its neighbours. South of England and Ireland, Belgium, the Netherlands, and West Germany, on the other hand, stand as the areas with the lowest spillovers. This means that these are (mostly rich) regions whose output per capita is barely affected by the "unobserved productivity" of its neighbours.

report the spill-out effects because, given the proportional relation that imposes the constraint $\alpha_i = \gamma_1 \mu_i$, its geographical distribution is no more informative than that of $\ln \hat{\Omega}_{i0}$ (in fact, since both $\ln \hat{\Omega}_{i0}$ and γ_1 take negative values, the spill-out effects take positive values and are larger/smaller the smaller/larger $\ln \hat{\Omega}_{i0}$ is).

4 Conclusions

We present a growth model that extends previous knowledge-spillovers models in several directions. First, we do not assume a common exogenous technological progress but account for heterogeneity in the initial level of technology. Second, we assume that the technological progress depends not only on the stock of physical capital and the stock of knowledge of the other economies, but also on the physical capital and the (unobserved) initial level of technology of the other economies. Thus, our assumed technology combines the alternative sources of (global and local) spatial externalities considered in previous models of relative location with the unobserved heterogeneity that characterises previous models of absolute location.

We use EU-NUTS2 regional information from Cambridge Econometrics to test whether the data supports the main features of our growth model. In particular, our econometric specification is derived from the growth-initial equation of the model and takes the form of a spatial Durbin dynamic panel model with spatially weighted individual effects. As a downside, some of the implied parameters of the model are not identified. We discuss alternative ways to circumvent this limitation.

We estimate the model by QML using a correlated random effects specification for the individual effects and their spatial spillovers. Our results support our model specification. Also, they are largely *i)* consistent with other studies using analogous data; and *ii)* robust to the use of alternative specifications, samples and estimation approaches. In particular, we find evidence of the existence of (global) spatial spillovers arising from the level of technology, but not from the investment in capital (neither global nor local). Also, our estimates indicate that the level of GDP per capita of the European regions is largely determined by their past GDP per capita and the current and past GDP per capita of their neighbours, their saving rate and that of their neighbours, and their depreciation rate. However, the role of unobservable characteristics is worth noting: richest areas (e.g., the "blue banana") are so partially because of their higher "unobserved productivity" and a number of poor regions benefit from "unobserved productivity" spillovers.

Table 1: **Descriptive statistics**

(a) *Sample I: 1982-2002*

Variable	Mean	St. Dev.	Min	P25	Median	P75	Max
GDP	23,393	9,961	6,321	18,554	22,307	26,227	133,452
s	23.39	4.50	9.98	20.65	23.08	25.77	46.08
$n+\delta+g$	0.05	0.00	0.04	0.05	0.05	0.06	0.07

(b) *Sample II: 1982-2012*

Variable	Mean	St. Dev.	Min	P25	Median	P75	Max
GDP	25,355	11,536	6,321	19,698	23,997	28,934	176,529
s	23.69	4.76	9.98	20.64	23.47	26.17	48.84
$n+\delta+g$	0.05	0.01	0.04	0.05	0.05	0.06	0.08

(c) *Sample III: 1980-2015*

Variable	Mean	St. Dev.	Min	P25	Median	P75	Max
GDP	25,322	11,842	5,798	19,567	24,020	28,829	191,016
s	23.51	4.81	9.39	20.50	23.26	25.83	46.31
$n+\delta+g$	0.05	0.01	0.04	0.05	0.05	0.06	0.07

(d) *Sample IV: 1982-2002 w/o PIIGS (Portugal, Ireland, Italy, Spain and Greece)*

Variable	Mean	St. Dev.	Min	P25	Median	P75	Max
GDP	25,317	10,247	12,208	20,464	23,397	27,307	133,452
s	23.28	4.44	10.82	20.65	23.00	25.41	46.08
$n+\delta+g$	0.05	0.00	0.04	0.05	0.05	0.06	0.07

(e) *Sample V: 1982-2002 w/o EL (Greece)*

Variable	Mean	St. Dev.	Min	P25	Median	P75	Max
GDP	23,936	9,881	6,321	19,188	22,620	26,525	133,452
s	23.34	4.41	9.98	20.65	23.08	25.70	46.08
$n+\delta+g$	0.05	0.00	0.04	0.05	0.05	0.06	0.07

Note: Number of observations: $189\times4=756$ (Sample I), $189\times6=1,134$ (Sample II), $189\times7=1,323$ (Sample III), $139\times4=556$ (Sample IV), and $180\times4=720$ (Sample V). GDP is real GDP (at 2005 constant prices, in Euros) per capita (using total population, in thousands of people). s is the ratio between investment expenditures and gross value-added (as a percentage and at 2005 constant prices, in Euros). n is is the working-age population growth rate (computed as in Islam 1995) and $\delta+g=0.05$ (as in e.g. Mankiw et al., 1992; Islam, 1995; Ertur and Koch, 2007).

Table 2: **QML estimates (unconstrained model)**

	Sample I (1982-2002)	Sample I (1982-2002)	Sample II (1982-2012)	Sample III (1980-2015)	Sample I (w/o PIIGS)	Sample I (w/o EL)
$\overline{\gamma}_1$	0.6291*** (0.0304)	0.9049*** (0.0145)	0.9177*** (0.0160)	0.8520*** (0.0294)	0.8681*** (0.0221)	0.8980*** (0.0157)
$\overline{\gamma}_2$	−0.3202*** (0.0556)	−0.4317*** (0.0366)	−0.4746*** (0.0290)	−0.3934*** (0.0338)	−0.4757*** (0.0412)	−0.4706*** (0.0362)
ρ	0.5281*** (0.0432)	0.5047*** (0.0380)	0.5603*** (0.0277)	0.5463*** (0.0273)	0.5587*** (0.0513)	0.5357*** (0.0383)
β_1	0.1149*** (0.0283)	0.0774** (0.0354)	0.0124 (0.0187)	−0.0053 (0.0149)	0.0604 (0.0405)	0.1031*** (0.0349)
β_2	−0.1624*** (0.0434)	−0.1952*** (0.0542)	−0.1742*** (0.0370)	−0.1045*** (0.0320)	−0.1564*** (0.0506)	−0.1536*** (0.0529)
θ_1	−0.0944*** (0.0339)	−0.0907** (0.0419)	−0.0526** (0.0259)	0.0018 (0.0187)	−0.1090*** (0.0506)	−0.1154*** (0.0410)
θ_2	0.0553 (0.0577)	0.0528 (0.0714)	0.0337 (0.0482)	0.0317 (0.0404)	0.1085 (0.0703)	0.0446 (0.0697)
π_{μ_1}		−0.1131*** (0.0397)	−0.0526** (0.0259)	−0.0606** (0.0306)	−0.1185** (0.0482)	−0.1432*** (0.0403)
π_{μ_2}		0.3486*** (0.0728)	0.3321*** (0.0596)	0.3310*** (0.0752)	0.3358*** (0.0888)	0.3037*** (0.0737)
π_{α_1}		0.0954* (0.0502)	0.0829** (0.0337)	0.0613 (0.0393)	0.1223* (0.0637)	0.1189* (0.0508)
π_{α_2}		−0.1637 (0.1112)	−0.1244 (0.0846)	−0.1453 (0.1011)	−0.2721 (0.1360)	−0.0975 (0.1137)
σ^2_μ		0.0006*** (0.0002)	0.0004** (0.0001)	0.0010** (0.0003)	0.0013*** (0.0003)	0.0007*** (0.0002)
σ^2_α		1.7×10^{-5} (0.0004)	0.0002 (0.0003)	0.0008 (0.0005)	0.0001 (0.0008)	1.2×10^{-5} (0.0004)
$\sigma_{\mu\alpha}$		0.0001 (0.0002)	−0.0002 (0.0002)	−0.0007* (0.0004)	−0.0003 (0.0006)	−0.0001 (0.0003)
σ^2_ε		0.0035*** (0.0002)	0.0030*** (0.0002)	0.0028*** (0.0002)	0.0019*** (0.0002)	0.0028*** (0.0002)

Note: All estimates obtained using the approach proposed by Miranda et al. (2017a), except for those in column two, which were obtained using the approach proposed by Lee and Yu (2016). Time dummies included but not reported. The symbol * indicates statistically significant at the 10% level, ** at the 5% level and *** at the 1% level.

Table 3: **QML estimates (constrained model)**

(a) Partially constrained model

	Sample I (1982-2002)	Sample II (1982-2012)	Sample III (1980-2015)
$\overline{\gamma}_1$	0.9028*** (0.0144)	0.9217*** (0.0147)	0.8674*** (0.0271)
$\overline{\gamma}_2$	−0.4455*** (0.0373)	−0.4853*** (0.0281)	−0.4099*** (0.0319)
ρ	0.5253*** (0.0384)	0.5608*** (0.0271)	0.5434*** (0.0273)
β_1	0.0595 (0.0394)	0.0019 (0.0196)	−0.0086 (0.0151)
β_2	−0.1891*** (0.0562)	−0.1717*** (0.0372)	−0.1044*** (0.0321)
θ_1	−0.0453 (0.0377)	−0.0116 (0.0174)	0.0142 (0.0153)
θ_2	0.0323 (0.0790)	0.0283 (0.0487)	0.0361 (0.0391)
$\pi_{\mu_1^c}$	−0.0903*** (0.0446)	−0.0336 (0.0261)	−0.0467* (0.0270)
$\pi_{\mu_2^c}$	0.3423*** (0.0781)	0.3309*** (0.0597)	0.3320*** (0.0707)
$\sigma_\mu^{2^c}$	0.0005** (0.0002)	0.0004*** (0.0001)	0.0009*** (0.0003)
$\sigma_\varepsilon^{2^c}$	0.0035*** (0.0002)	0.0031*** (0.0002)	0.0029*** (0.0001)
γ_1	−0.3803 (0.3303)	−0.3773* (0.2267)	−0.4644** (0.1903)
LR-test	4.82	5.35	3.02

(b) Fully constrained model

	Sample I (w/o PIIGS)	Sample I (w/o EL)
$\overline{\gamma_1^c}$	0.8700*** (0.0195)	0.9026*** (0.0131)
ρ^c	0.5747*** (0.0405)	0.5496*** (0.0361)
β^c	0.0797** (0.0323)	0.1083*** (0.0333)
θ^c	−0.0575 (0.0318)	−0.0780** (0.0383)
$\pi_{\mu_1^c}$	−0.1257*** (0.0412)	−0.1445*** (0.0392)
$\pi_{\mu_2^c}$	0.2195*** (0.07530)	0.2507*** (0.0617)
σ_μ^c	0.0013*** (0.0003)	0.0006*** (0.0002)
$\sigma_\varepsilon^{2^c}$	0.0020*** (0.0002)	0.0031*** (0.0002)
γ_1	−0.3854* (0.2108)	−0.4432* (0.2459)
LR-test	9.88	7.65

Note: The upper letter c denotes constrained parameters (see section 3.2 for details). All estimates obtained using the approach proposed by Miranda et al. (2017a). Time dummies included but not reported. The symbol * indicates statistically significant at the 10% level, ** at the 5% level and *** at the 1% level. LR-test is the Likelihood Ratio test statistic of the hypothesis that the constraint $\alpha_i = \gamma_1 \mu_i$ is valid (Table 3*a*) and the constraints $\beta_1 = -\beta_2$, $\theta_1 = -\theta_2$, $\overline{\gamma}_2 = -\rho\overline{\gamma}_1$ and $\alpha_i = \gamma_1 \mu_i$ are valid (Table 3*b*).

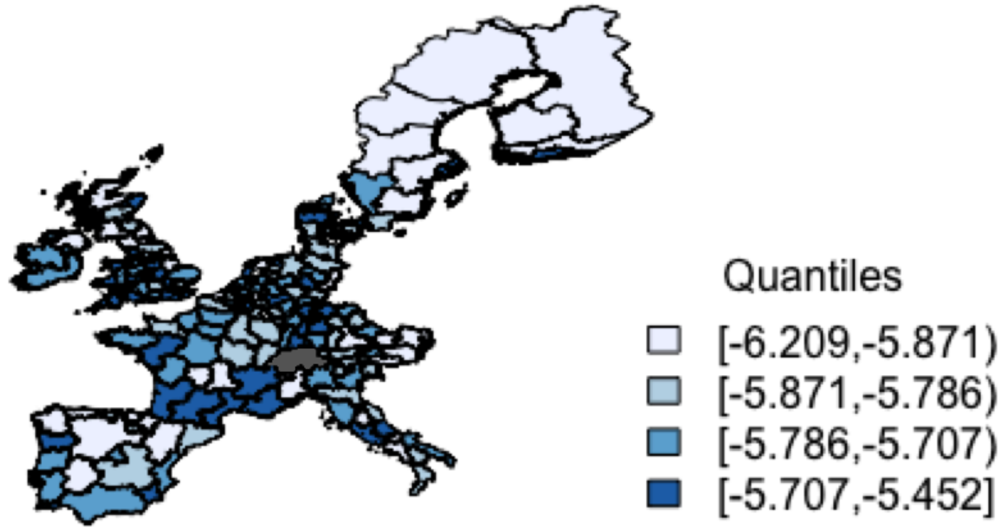
Table 4: **Implied Parameters**

	Sample I (w/o PIIGS)	Sample I (w/o EL)
γ_1	-0.3854^{*}	-0.4432^{*}
	(0.2108)	(0.2459)
λ	0.0205^{***}	0.0279^{***}
	(0.0029)	(0.0045)
γ_3	0.6394^{***}	0.6305^{***}
	(0.1346)	(0.1162)
Assumed technology: López-Bazo et al. (2004)		
α	0.5265^{***}	0.3801^{***}
	(0.0843)	(0.1021)
γ_2	-0.0425	-0.0346
	(0.0561)	(0.0679)
Assumed technology: Ertur and Koch (2007)		
α	0.5930^{***}	0.4349^{***}
	(0.1216)	(0.1374)
ϕ	-0.0665	-0.0548
	(0.0744)	(0.0983)

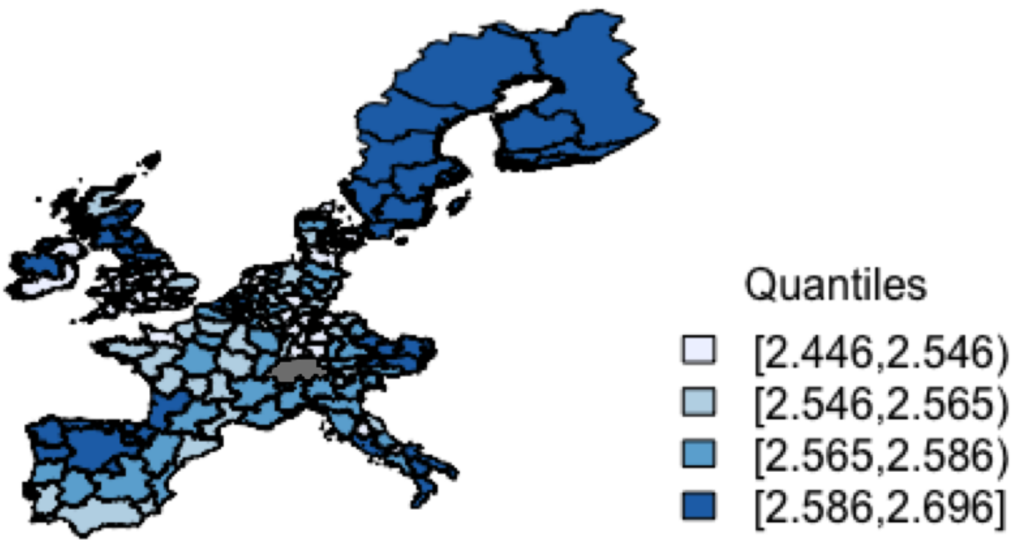
Note: * indicates statistically significant at the 10% level, ** at the 5% level and *** at the 1% level. Except for γ_1, standard errors were obtained using the delta method.

Figure 1: **Estimated individual effects and their spatial spillovers**

(a) Geographical distribution of $ln\hat{\Omega}_{i0}$



(b) Geographical distribution of $\hat{\gamma}_1 \sum_{j=1}^{N} w_{ij} \ln \hat{\Omega}_{j0}$



A The balanced growth rate

From equation 2.6:

$$\ln y_{it} = \left[1 + \left(\frac{(\gamma_3 + \gamma_1)(u_{ii} - \alpha - \phi)}{(\phi\gamma_3 + \gamma_2)}\right)\right] \ln \Omega_{it} + \frac{(\gamma_3 + \gamma_1)}{\phi\gamma_3 + \gamma_2} \sum_{j \neq i}^{N} u_{ij} \ln \Omega_{jt} + u_{ii} \ln k_{it} + \sum_{j \neq i}^{N} u_{ij} \ln k_{jt}$$

Since $\ln \Omega_{it} = \ln \Omega_{i0} + \mu t$, then:

$$\frac{d \ln y_{it}}{dt} = \left[1 + \left(\frac{(\gamma_3 + \gamma_1)(u_{ii} - \alpha - \phi)}{(\phi\gamma_3 + \gamma_2)}\right)\right] \mu + \frac{(\gamma_3 + \gamma_1)}{\phi\gamma_3 + \gamma_2} \sum_{j \neq i}^{N} u_{ij}\mu + u_{ii} g + \sum_{j \neq i}^{N} u_{ij} g$$

Also, using $u_{ii} + \sum_{j \neq i}^{N} u_{ij} = \sum_{j=1}^{N} u_{ij} = \alpha + \frac{\phi + \gamma_2}{1 - \gamma_3}$,

$$\frac{d \ln y_{it}}{dt} = \left(1 - \frac{(\gamma_3 + \gamma_1)(\alpha + \phi)}{(\phi\gamma_3 + \gamma_2)} + \frac{(\gamma_3 + \gamma_1)}{(\phi\gamma_3 + \gamma_2)} \left(\frac{\alpha(1 - \gamma_3) + \phi + \gamma_2}{1 - \gamma_3}\right)\right) \mu + \sum_{j=1}^{N} u_{ij} g = g,$$

which after some algebra becomes:

$$\left(\frac{1 + \gamma_1}{1 - \gamma_3}\right) \mu + \left(\frac{\alpha(1 - \gamma_3) + \phi + \gamma_2}{1 - \gamma_3}\right) g = g$$

Therefore,

$$g = \frac{\mu\,(1 + \gamma_1)}{(1 - \gamma_3)(1 - \alpha) - \phi - \gamma_2}$$

B Taylor approximation to the marginal productivity of capital

The Taylor approximation of $\frac{\dot{k}_{it}}{k_{it}}$ around the steady state $(k_{1t}^*, \cdots, k_{Nt}^*)$ is

$$\begin{aligned}
\frac{\dot{k}_{it}}{k_{it}} &= \frac{\dot{k}_{it}^*}{k_{it}^*} + \sum_{j=1}^{N} \left\{ \left. \frac{\partial \left(\dot{k}_{it}/k_{it} \right)}{\partial \ln k_{jt}} \right|_{k_{jt}^*} (\ln k_{jt} - \ln k_{jt}^*) \right\} \\
&= g + \left. \frac{\partial \left(\dot{k}_{it}/k_{it} \right)}{\partial \ln k_{it}} \right|_{k_{it}^*} (\ln k_{it} - \ln k_{it}^*) + \sum_{j \neq i}^{N} \left\{ \left. \frac{\partial \left(\dot{k}_{it}/k_{it} \right)}{\partial \ln k_{jt}} \right|_{k_{jt}^*} (\ln k_{jt} - \ln k_{jt}^*) \right\}
\end{aligned}$$

Next we calculate the two derivatives involved. First, let us rewrite the marginal productivity of capital (see footnote 7) as

$$\frac{\dot{k}_{it}}{k_{it}} = s_i \Omega_{it}^{c_{ii}} \prod_{j \neq i}^{N} \Omega_{jt}^{c_{ij}} e^{(u_{ii}-1)\ln k_{it}} \prod_{j \neq i}^{N} e^{u_{ij} \ln k_{jt}} - (n_i + \delta)$$

with $k_{it}^{u_{ii}-1} = e^{(u_{ii}-1)\ln k_{it}}$, $c_{ii} = 1 + \left(\frac{(\gamma_3 + \gamma_1)(u_{ii} - \alpha - \phi)}{(\phi\gamma_3 + \gamma_2)} \right)$ and $c_{ij} = \frac{(\gamma_3 + \gamma_1)u_{ij}}{\phi\gamma_3 + \gamma_2}$. Thus,

$$\left. \frac{\partial \left(\dot{k}_{it}/k_{it} \right)}{\partial \ln k_{it}} \right|_{k_{it}^*} = s_i \Omega_{it}^{c_{ii}} \prod_{j \neq i}^{N} \Omega_{jt}^{c_{ij}} (u_{ii} - 1) e^{(u_{ii}-1)\ln k_{it}^*} \prod_{j \neq i}^{N} e^{u_{ij} \ln k_{jt}^*}$$

Also, given that $s_i \left[\frac{y_{it}^*}{k_{it}^*} \right] - (n_i + \delta) - g = 0$, replacing y_{it}^* by 2.6 at the steady state we obtain

$$s_i \Omega_{it}^{c_{ii}} \prod_{j \neq i}^{N} \Omega_{jt}^{c_{ij}} \prod_{j \neq i}^{N} {k_{jt}^*}^{u_{ij}} = (n_i + \delta + g) \, {k_{it}^*}^{1-u_{ii}} \qquad \text{(B.1)}$$

Consequently,

$$\left.\frac{\partial\left(\dot{k}_{it}/k_{it}\right)}{\partial \ln k_{it}}\right|_{k_{it}^*} = (u_{ii}-1)(n_i+\delta+g)$$

Lastly, bearing in mind that $\prod_{j\neq i}^{N} e^{u_{ij}\ln k_{jt}^*} = e^{\sum_{j\neq i}^{N} u_{ij}\ln k_{jt}^*}$,

$$\left.\frac{\partial\left(\dot{k}_{it}/k_{it}\right)}{\partial \ln k_{jt}}\right|_{k_{jt}^*} = s_i\Omega_{it}^{c_{ii}}\prod_{j\neq i}^{N}\Omega_{jt}^{c_{ij}} e^{u_{ij}\ln k_{jt}^*}u_{ij} = u_{ij}(n_i+\delta+g)$$

Therefore:

$$\frac{\dot{k}_{it}}{k_{it}} = \frac{d\ln k_i(t)}{dt} = g + (u_{ii}-1)(n_i+\delta+g)\left(\ln k_{it}-\ln k_i^*\right) + \sum_{j\neq i}^{N} u_{ij}(n_i+\delta+g)\left(\ln k_{jt}-\ln k_{jt}^*\right)$$

C Speed of convergence

Let us take the total derivative of 2.6:

$$\frac{d\ln y_{it}}{dt} = \left[1+\left(\frac{(\gamma_1+\gamma_2)(u_{ii}-\alpha-\phi)}{\phi\gamma_3+\gamma_2}\right)\right]\frac{d\ln\Omega_{it}}{dt} + \frac{(\gamma_3+\gamma_1)}{\phi\gamma_3+\gamma_2}\sum_{j\neq i}^{N} u_{ij}\frac{d\ln\Omega_{jt}}{dt}$$
$$+ u_{ii}\frac{d\ln k_{it}}{dt} + \sum_{j\neq i}^{N} u_{ij}\frac{d\ln k_{jt}}{dt}$$

Given that $\dfrac{d \ln \Omega_{it}}{dt} = \dfrac{d \ln \Omega_{jt}}{dt} = \mu$, we concentrate on the derivatives with respect to k. To this end, let us consider the final result of appendix B:

$$\begin{aligned}\frac{d \ln k_{it}}{dt} &= g + (u_{ii} - 1)(n_i + \delta + g)\left(\ln k_{it} - \ln k_{it}^*\right) + \sum_{j \neq i}^{N} u_{ij}(n_i + \delta + g)\left(\ln k_{jt} - \ln k_{jt}^*\right) \\ &= g - (n_i + \delta + g)\left(\ln k_{it} - \ln k_{it}^*\right) + u_{ii}(n_i + \delta + g)\left(\ln k_{it} - \ln k_{it}^*\right) \\ &+ \sum_{j \neq i}^{N} u_{ij}(n_i + \delta + g)\left(\ln k_{jt} - \ln k_{jt}^*\right)\end{aligned}$$

Then, using equation 2.6

$$\ln y_{it} = \left[1 + \left(\frac{(\gamma_1 + \gamma_2)(u_{ii} - \alpha - \phi)}{\phi\gamma_3 + \gamma_2}\right)\right] \ln \Omega_{it} + \frac{(\gamma_3 + \gamma_1)}{\phi\gamma_3 + \gamma_2} \sum_{j \neq i}^{N} u_{ij} \ln \Omega_{jt} + u_{ii} \ln k_{it} + \sum_{j \neq i}^{N} u_{ij} \ln k_{jt}$$

and its value at the steady state

$$\ln y_{it}^* = \left[1 + \left(\frac{(\gamma_1 + \gamma_2)(u_{ii} - \alpha - \phi)}{\phi\gamma_3 + \gamma_2}\right)\right] \ln \Omega_{it} + \frac{(\gamma_3 + \gamma_1)}{\phi\gamma_3 + \gamma_2} \sum_{j \neq i}^{N} u_{ij} \ln \Omega_{jt} + u_{ii} \ln k_{it}^* + \sum_{j \neq i}^{N} u_{ij} \ln k_{jt}^*$$

we obtain

$$\ln y_{it} - \ln y_{it}^* = u_{ii}(\ln k_{it} - \ln k_{it}^*) + \sum_{j \neq i}^{N} u_{ij}(\ln k_{jt} - \ln k_{jt}^*) \tag{C.1}$$

Therefore,

$$\frac{d \ln k_{it}}{dt} = g - (n_i + \delta + g)\left(\ln k_{it} - \ln k_{it}^*\right) + (n_i + \delta + g)(\ln y_{it} - \ln y_{it}^*)$$

Plugging the previous result into the total derivative of 2.6:

$$\frac{d\ln y_{it}}{dt} = \left[1 + \left(\frac{(\gamma_1+\gamma_2)(u_{ii}-\alpha-\phi)}{\phi\gamma_3+\gamma_2}\right)\right]\mu + \frac{(\gamma_3+\gamma_1)}{\phi\gamma_3+\gamma_2}\sum_{j\neq i}^{N} u_{ij}\mu$$

$$+ u_{ii}\left(g - (n_i+\delta+g)\left(\ln k_i(t) - \ln k_i^*\right) + (n_i+\delta+g)(\ln y_i - \ln y_i^*)\right)$$

$$+ \sum_{j\neq i}^{N} u_{ij}\left(g - (n_j+\delta+g)\left(\ln k_j(t) - \ln k_j^*\right) + (n_j+\delta+g)(\ln y_j - \ln y_j^*)\right)$$

$$= \left[1 + \left(\frac{(\gamma_1+\gamma_2)(u_{ii}-\alpha-\phi)}{\phi\gamma_3+\gamma_2}\right)\right]\mu + \frac{(\gamma_3+\gamma_1)}{\phi\gamma_3+\gamma_2}\sum_{j\neq i}^{N} u_{ij}\mu + u_{ii}g + \sum_{j\neq i}^{N} u_{ij}g$$

$$- \left(u_{ii}(n_i+\delta+g)\left(\ln k_{it} - \ln k_{it}^*\right) + \sum_{j\neq i}^{N} u_{ij}(n_j+\delta+g))\left(\ln k_{jt} - \ln k_{jt}^*\right)\right)$$

$$+ \left(u_{ii}(n_i+\delta+g)\left(\ln y_{it} - \ln y_{it}^*\right) + \sum_{j\neq i}^{N} u_{ij}(n_j+\delta+g))\left(\ln y_{jt} - \ln y_{jt}^*\right)\right)$$

The first term in the previous expression corresponds to the balanced growth rate g (see appendix A). As for the second term, let us assume that, for each economy i, there exists Λ_i such that:

$$\sum_{j=1}^{N} u_{ij}(n_j+g+\delta)(\ln k_{jt} - \ln k_{jt}^*) = \Lambda_i\left(u_{ii}(\ln k_{it} - \ln k_{it}^*) + \sum_{j\neq i}^{N} u_{ij}(\ln k_{jt} - \ln k_{jt}^*)\right)$$

Thus,

$$\frac{d\ln y_{it}}{dt} = g - \Lambda_i\left(u_{ii}(\ln k_{it} - \ln k_{it}^*) + \sum_{j\neq i}^{N} u_{ij}(\ln k_{jt} - \ln k_{jt}^*)\right)$$

$$+ u_{ii}(n_i+\delta+g)\left(\ln y_{it} - \ln y_{it}^*\right) + \sum_{j\neq i}^{N} u_{ij}(n_j+\delta+g)\left(\ln y_{jt} - \ln y_{jt}^*\right)$$

$$= g - \Lambda_i(\ln y_{it} - \ln y_{it}^*) + u_{ii}(n_i+\delta+g)\left(\ln y_{it} - \ln y_{it}^*\right) + \sum_{j\neq i}^{N} u_{ij}(n_j+\delta+g)\left(\ln y_{jt} - \ln y_{jt}^*\right)$$

where the second expression is obtained by using C.1.

Finally, from the first hypothesis in 2.11 we have that $\left(\ln y_{it} - \ln y_{it}^*\right)\Theta_j^{-1} = \ln y_{jt} - \ln y_{jt}^*$.

This allows us to obtain the speed of convergence to the steady state:

$$\frac{d\ln y_{it}}{dt} = g - \left(\Lambda_i - u_{ii}(n_i+\delta+g) - \sum_{j\neq i}^{N} u_{ij}(n_j+\delta+g)\Theta_j^{-1}\right)(\ln y_{it} - \ln y_{it}^*)$$
$$= g - \lambda_i\left(\ln y_{it} - \ln y_{it}^*\right)$$

What is left is to derive the expressions defining Λ_i and λ_i. First, by plugging the second hypothesis in 2.11, $(\ln k_{it} - \ln k_{it}^*)\,\Phi_j^{-1} = \ln k_{jt} - \ln k_{jt}^*$, into our assumption on the existence of Λ_i:

$$\sum_{j=1}^{N} u_{ij}(n_j+g+\delta)(\ln k_{jt} - \ln k_{jt}^*) = \Lambda_i\left(u_{ii}(\ln k_{it} - \ln k_{it}^*) + \sum_{j\neq i}^{N} u_{ij}(\ln k_{jt} - \ln k_{jt}^*)\right)$$
$$\sum_{j=1}^{N} u_{ij}(n_j+g+\delta)(\ln k_j - \ln k_j^*) = \Lambda_i \sum_{j=1}^{N} u_{ij}(\ln k_j(t) - \ln k_j^*)$$
$$\sum_{j=1}^{N} u_{ij}(n_j+g+\delta)\left(\ln k_i(t) - \ln k_i^*\right)\Phi_j^{-1} = \Lambda_i \sum_{j=1}^{N} u_{ij}\left(\ln k_i(t) - \ln k_i^*\right)\Phi_j^{-1}$$
$$\Lambda_i = \frac{\sum_{j=1}^{N} u_{ij}\frac{1}{\Phi_j}(n_j+g+\delta)}{\sum_{j=1}^{N} u_{ij}\frac{1}{\Phi_j}} \tag{C.2}$$

Second, plugging the previous result into $\lambda_i = \Lambda_i - u_{ii}(n_i+\delta+g) - \sum_{j\neq i}^{N} u_{ij}(n_j+\delta+g)\Theta_j^{-1}$ and assuming that $\Theta_i^{-1} = 1$:

$$\lambda_i = \Lambda_i - u_{ii}(n_i+\delta+g)\Theta_i^{-1} - \sum_{j\neq i}^{N} u_{ij}(n_j+\delta+g)\Theta_j^{-1}$$
$$\lambda_i = \Lambda_i - \sum_{j=1}^{N} u_{ij}(n_j+\delta+g)\Theta_j^{-1}$$
$$\lambda_i = \frac{\sum_{j=1}^{N} u_{ij}\frac{1}{\Phi_j}(n_j+g+\delta)}{\sum_{j=1}^{N} u_{ij}\frac{1}{\Phi_j}} - \sum_{j=1}^{N} u_{ij}(n_j+\delta+g)\frac{1}{\Theta_j}$$

D Differential equation solution

We start by noticing that the steady state in 2.9 can be written as

$$\ln y_{it}^* = \frac{1}{1-\alpha-\phi}\sum_{j=1}^{N}\sum_{r=0}^{\infty}\rho^r w_{ij}^{(r)}\ln\Omega_{jt} + \frac{\gamma_1}{1-\alpha-\phi}\sum_{j=1}^{N}\sum_{r=0}^{\infty}\rho^r w_{ij}^{(r+1)}\ln\Omega_{jt}$$
$$+\left(\frac{\alpha+\phi}{1-\alpha-\phi}\right)\sum_{j=1}^{N}\sum_{r=0}^{\infty}\rho^r w_{ij}^{(r)}\ln\left(\frac{s_j}{n_j+\delta+g}\right)+\frac{\gamma_2-\alpha\gamma_3}{1-\alpha-\phi}\sum_{j=1}^{N}\sum_{r=0}^{\infty}\rho^r w_{ij}^{(r+1)}\ln\left(\frac{s_j}{n_j+\delta+g}\right)$$

with $\rho = \frac{\gamma_2-\alpha\gamma_3}{1-\alpha-\phi}$. Using this, we can see that $\frac{d\ln y_{it}^*}{dt} = \frac{(1+\gamma_1)\mu}{1-\alpha-\phi}\left(\frac{1}{1-\rho}\right)$. In fact, since $\frac{1}{1-\rho} = \frac{1-\alpha-\phi}{(1-\alpha)(1-\gamma_3)-\phi-\gamma_2}$,

$$\frac{d\ln y_{it}^*}{dt} = g \tag{D.1}$$

Notice that *D*.1 can be seen as another differential equation, which have a particular solution on $\ln y_{i0}^*$:

$$\ln y_{it}^* = gt + \ln y_{i0}^* \tag{D.2}$$

Plugging equation *D*.2 and 2.12 we obtain:

$$\frac{d\ln y_{it}}{dt} = g - \lambda_i\left(\ln y_{it} - gt - \ln y_{i0}^*\right) \tag{D.3}$$

We use the integrating factor method to solve the differential equation in *D*.3. We first reorder terms and then multiply the equation by the integrating factor $e^{\int \lambda_i dt} = e^{\lambda_i t}$ to obtain

$$\frac{d}{dt}\left(e^{\lambda_i t}\ln y_{it}\right) = e^{\lambda_i t}g + \lambda_i e^{\lambda_i t}\left(gt + \ln y_{i0}^*\right)$$

By integrating on both sides, we obtain the general solution:

$$\ln y_{it} = gt + \ln y_{i0}^* + Ce^{-\lambda_i t}$$

The particular solution for $t = t_1$ implies that $C = (\ln y_{it_1} - gt_1 - \ln y_{i0}^*)\, e^{\lambda_i t_1}$. Thus, for any t we have:

$$\ln y_{it} = g\left(t - t_1 e^{-\lambda_i(t-t_1)}\right) + \ln y_{it_1} e^{-\lambda_i(t-t_1)} + \left(1 - e^{-\lambda_i(t-t_1)}\right) \ln y_{i0}^*$$

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