PROBABILITY: A GRADUATE COURSE

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Springer, New York, 2005
603 pages

This is a graduate level text in probability theory. The book is clearly written and the emphasis is placed on theory, while there is mention of some applications. The reader needs to have a moderate background of measure theory.

Chapters 1 to 8 make up the body of a graduate probability course. It is a little surprising, but very useful, that there is a chapter, Chapter 3, dedicated exclusively to probabilistic inequalities. Also included is Chapter 9, which is of a more specialized nature, and Chapter 10 which deals with martingale theory.

The book begins with basics from the measure theory, such as σ-algebras, set theory, measurability and Lebesgue integration. Later it goes to the Borel-Cantelli lemmas, inequalities, characteristic functions and the classical limit theorems: the Law of Large Numbers, the Central Limit Theorem and the Law of the Iterated Logarithm. After an introduction to the generalizations and extensions of these three classical limit theorems, the book concludes with a chapter devoted to martingales, one of the most important tools in probability theory.

A list of notations and symbols precedes the main body of the text and an appendix with some mathematical tools and facts, a bibliography and an index conclude the book. At the end of each chapter there is a list of problems with some comments.

In the first chapter, *Introductory Measure Theory*, we find the basis concepts of measure theory necessary to read the book.

The second chapter is called *Random Variables.* Here, the author introduces random variables and presents the basis concepts (distributions, expectation, moments, independence, conditional distributions, Borel-Cantelli lemmas...) and concrete applications of probability models.

The third chapter is a collect of different inequalities. Inequalities play an important role in probability theory because very often one needs to estimate: certain probabilities by others, moments of sums by sums of moments, etc. . . In this chapter we find, among other topics, tail probabilities estimated via moments (Markov's inequality, Chebyshev's inequality, Kolmogorov's inequality...), moment inequalities, Jensen's inequality and probability inequalities for maxima.

Chapter 4 is devoted to characteristic functions. In this chapter the author defines characteristic functions, proves some basic facts as uniqueness, inversion and the multiplication property and introduces the cumulant generating function, the probability generating function and the moment generating function.

The title of Chapter 5 is *Convergence*. This is an introductory chapter to the four following ones where the classical limit theorems are studied. In this chapter the author defines the various modes of convergence, proves the uniqueness of limits and the relations between them. The author also presents some results that are useful when one needs to investigate the convergence of a sequence of random variables.

In Chapter 6 the author presents the Weak and the Strong Laws of Large Numbers as well as some of their applications.

Chapter 7 is devoted to the Central Limit Theorem. In this chapter the Lindeberg-Lévy-Feller theorem, where the summands are independent but not identically distributed, and Lyapounov's version of the result are also proved. After this, the author presents the Berry-Esseen theorem, which is a convergence rate result for the central limit theorem. The remaining part of the chapter contains some rate results for tail probabilities and some comments on local limit theorems for discrete random variables and the concept of large deviations.

Continuing with the classical limit theorems, Chapter 8 is devoted to the Law of the Iterated Logarithm. In this chapter the main focus is on the Hartman-Wintner-Strassen theorem, which deals with the independent and identically distributed case.

The results given in Chapters 6, 7 and 8 are basically for sums of independent, identically distributed random variables. In Chapters 7 and 8 finite variance was an additional assumption. In Chapter 9 the author provides an introduction to some more general limit theorems.

Finally, Chapter 10 is devoted to martingales. Following some introductory material on conditional expectations and the definition of a martingale, the author presents us with some examples, convergence results, results for stopped martingales, regular martingales, uniformly integrable martingales, stopped random walks, reversed martingales and submartingales.

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