Jonathan Ayebakuro Orama

STUDY ABOUT AGGREGATION OPERATORS FOR THE ASSESSMENT OF CONCORDANCE LEVEL IN OUTRANKING-BASED MULTI-CRITERIA DECISION AIDING METHODS

MASTER THESIS

Directed by Dr. Aida Valls Mateu

COMPUTER SECURITY ENGINEERING AND ARTIFICIAL INTELLIGENCE

UNIVERSITAT ROVIRA I VIRGILI

Tarragona

2018

Acknowledgements

I would firstly like to thank the URV for the opportunity to study and obtain a master degree, and for the challenging curriculum put together to properly educate. Also, I want to thank all staff of the computer security engineering and artificial intelligence master course, for their help and guidance through the program.

In particular, I will like to acknowledge the director of this thesis work, Dr. Aida Valls Mateu. For her help and continued guidance, she really put the effort to assist me through the whole work even through mail correspondence at odd hours. I can say without a doubt she was the best choice for my TFM director.

I will also like to appreciate my mum for her love and support through this period, and also my siblings for their encouragement.

Finally, I want to give all glory to the Almighty GOD for the life, knowledge and wisdom He gave to me to enable me through the period of the master course and TFM.

Table of Contents

List of Figures

List of Tables

Abstract

In Multi-Criteria Decision Aiding (MCDA), aggregation operators are used to merge the information provided by different criteria about a set of alternatives. This overall score could then be used to construct a rank order or ordered classification of alternatives. The ELECTRE outranking method is inspired in voting procedures. It employs the Weighted Average operator (WA) to calculate the overall concordance index when constructing a pairwise outranking relation that evaluates the degree of truth of the assertion "option *a* is at least as good as option *b*". The use of the weighted average operator has been observed to have an undesired compensative effect between opposite values. In this master thesis, we propose modifying the concordance calculation in ELECTRE by substituting the weighted average operator for operators from the Ordered Weighted Average (OWA) family of operators to give more flexibility to the definition of concordance as "a sufficient majority in favor of the outranking relation". In particular, we study these three operators: OWAWA (Ordered Weighted Average Weighted Average), WOWA (Weighted Ordered Weighted Average), IOWA (Induced Ordered Weighted Average). They are appropriate operators because they are designed to combine the advantages of the Weighted Average operator and Ordered Weighted Average operator. That is, they retain the importance given to each criterion while adding also a conjunctive/disjunctive character to the definition of majority. The three proposed OWA-based outranking constructs are implemented as Web services for integration into the European Diviz workbench, so they are based on the ELECTRE concordance module already deployed in Diviz. Three different experiments were carried out with various parameterization of each proposed method, which were compared to the conventional method used in ELECTRE. These experiments show that OWAWA and WOWA are suitable to combine Weighted Averaging and Ordered Weighted Averaging. On the contrary, an undesired behaviour is observed in the IOWA operator which makes it unsuitable for use in ELECTRE.

1 Introduction

Decisions are an integral part of several aspects of life. In various engineering, science and business fields, individuals are required to make informed decisions based on imprecise and uncertain data. Making an informed decision involves choosing the best option out of a set of alternatives considering their performances in certain criteria. Some decisions may be simple with a clear cut optimal choice, while others could be complex and require trade-offs between criteria to decide. Multiple Criteria Decision Aiding (MCDA) provides methods of analysis to help in decision making.

The MCDA discipline studies systematic methods for complex decision problems concerning diverse and often contradictory criteria, by analyzing a set of possible alternatives in order to find the best one [4]. There are three approaches to MCDA: Multi-Attribute Utility Theory (MAUT) also known as the English model, Outranking methods also known as the French model and DRSA decision rules. Outranking methods is one of the most successful approaches to decision aiding. It is based on social choice models that copy the human reasoning procedure [8].

MCDA methods take a set of alternatives (i.e. potential solutions) and generate a ranking of the alternatives according to a set of criteria. Criteria are tools constructed for the evaluation of alternatives compared in terms of suitability based on the decision maker's needs. Each criterion corresponds to a point of view considered in the decision process, they are assigned weights which signifies their importance in the decision. Outranking methods are characterized by being based on constructing preference relations between the alternatives by means of pairwise comparisons, instead of aggregating directly the values given by the criteria (as done in MAUT). The aim is to build a binary outranking relation *aSb*, which means "*a* is at least as good as *b*" [4]. Each criterion is asked about its contribution to this outranking assertion and it provides a vote in favor or against to *aSb*. Votes must be aggregated in order to associate a value to *aSb* for all possible pairs of alternatives. There are two main methods known as PROMETHEE and ELECTRE. In this study, we focus on ELECTRE method as it strictly applies the concept of veto. The veto concept enforces the respect of minority principle, in calculating discordance it is necessary to ensure no criterion totally refutes the assertion (*aSb*) in question. Moreover, ELECTRE method has been widely acknowledged as an efficient decision aiding tool with successful applications in many domains [8].

ELECTRE uses a weighted average to merge all the votes supporting *aSb* and then it includes the opposite votes by using a veto procedure. Once the valued outranking relation is constructed, different exploitation procedures exist in order to derive a ranking from it [4]. The contribution of this master thesis is the use of other aggregation operators for merging the votes in favor of the outranking relation. In particular, we propose the use of OWA-like operators because they enable the definition of conjunctive/disjunctive policies of aggregation that may be more appropriate in some decision problems. The compensation problem of classic weighted average may be solved with the possibility of establishing a more appropriate and-like aggregation (to model simultaneity) or or-like operator (for replaceability). As we do not want to suppress the weights representing the voting power for each criterion, we propose the use of Weighted OWA operators like OWAWA, WOWA and IOWA.

These family of Weighted OWA operators also serve another purpose in their usage. They are not solely OWA operators as they combine OWA and WA in their operation. Each operator can be parameterized to either be an OWA or WA operator, as such can be used in cases where both OWA and WA are suitable. Furthermore, as combinations of OWA and WA they take advantages from both sides and can be used in cases that would benefit from this combination.

1.1 Aims of the Master Thesis

The main goal of this master thesis is to propose a new approach to the aggregation of partial concordances in ELECTRE, in an attempt to solve some shortcomings of the conventional aggregation method and give more flexibility to the method. In particular we would like to address the OWA family of operators as suitable aggregation replacements to the Weighted Average operator used in ELECTRE.

The main goal of this Thesis can be divided into the following tasks:

- Study the outranking methods, focusing on ELECTRE.
- Study the OWA family of aggregation (OWAWA, WOWA & IOWA) operators and the parameters that permit to define different models and behaviours of the same operator.
- Define new measures of concordance and discordance to construct outranking relations that make use of OWA operators.
- Create a software that is able to easily compare the results and performance of different parameterizations of these measures.
- Test the systems with different datasets and study the performances of the measures defined.

1.2 Structure of the thesis

The master thesis is structured as follows. Section 1 provides a brief description of the thesis work, highlighting the purpose of the work, the goal to be achieved and basic definition of terms, concepts and techniques seen in the work.

Section 2 provides a more detailed explanation of concepts and techniques required in the implementation of the work. Firstly, a review of the ELECTRE technique, including an overview of outranking relations in the ELECTRE methodology, and the different versions of the technique. Finally, the section concludes with an explanation of the concordance index, its formulation, concepts and some extensions to its operation.

Section 3 presents an overview of aggregation operators, outlining different popular aggregation operators. Furthermore, it presents the different aggregation operators based on OWA that will be used in this study.

Section 4 defines the new procedure for calculating the overall concordance. The software implementation of the family of weighted OWA operators, the tools need for the implementation, and the algorithms implemented.

Section 5 makes an empirical analysis and comparison of the concordance values obtained from the three weighted OWA operators. The concordance values are exploited to obtain a ranking of alternatives, and compared using concordance values obtained from WA and OWA as reference. Finally, the similarity between operators is observed and documented.

Finally, section 6 summarizes the work, discusses the main conclusions of the work and possible future work to be done.

2 Definition of concepts

In this work we focus on the ELECTRE methodology an MCDA technique. In this chapter, definitions of basic terminology, concepts and techniques relevant to this master thesis are presented.

2.1 Multiple Criteria Decision Analysis (MCDA)

Due to the importance of decision making in business, engineering and various aspects of human life, there is an ever growing popularity of systems that aid in the decision making process. Decision aiding is defined in [4] as follows: Decision aiding is the activity of the person who, through the use of explicit but not necessarily completely formalized models, helps obtain elements of responses to the question posed by the stakeholder in a decision process. By this definition, we understand that any model built to analyze a question in an attempt to provide an accurate response is a decision support system. The elements mentioned in the definition are points that help in clarifying the decision.

The MCDA discipline seeks to provide models or structures that aid in decision making. More often in decision analysis, decisions are multiple criteria based. This is because criteria can be seen as points of view considered by the decision maker, therefore it is not rational to have a single point of view in decision making. The *monocriterion* approach could lead to wrongly neglecting realistic points of view, on the other hand the multiple criteria approach allows for debates between criteria in the decision aiding process.

The MCDA problem is characterized by the following:

- A set of actions or alternatives, denoted by $A = \{a, b, c\}$.
- A set of criteria to evaluate a, denoted by $G = \{g_1, g_2, \dots, g_p\}$.

And the goal is to: **Find** the best alternative or a subset of the best alternatives, or **Rank** the alternatives from the best alternative to the worst, or **Classify** each alternative into predefined groups.

2.1.1 MCDA Approaches

MCDA methods can be divided into three main approaches with different outlook to decision analysis. They include:

- **Multi-Attribute Utility Theory (MAUT):** It is also known as the American model, or Functional model. A utility score is given to each criteria representing the performance of an alternative with respect to that criteria. These utility scores are aggregated to find the global utility score, the best alternative is one with the best global utility score. Utility scores are compensable i.e. a low performance on one criteria is compensated by a high performance on another.
- **Outranking approach:** It is also known as the French model, or Relational model. In this approach, all pairs of alternatives are compared expressing their relative preference on each criterion. This means that two alternatives are compared to ascertain which outranks the other, and criteria, vote or express their preference of either alternative. This approach allows for indifference and incomparability between alternatives. The most popular outranking methods are: PROMETHEE and ELECTRE.
- **Decision rule approach:** It is also known as the Logical model. This approach is based on preference modelling in terms of *"if...then..."* decision rules. What this means is that the decision maker seeks to establish logical rules that justify preferential information. Once a set of decision rules are established, the decision maker may then apply them to actions or alternatives in other to obtain preference relations. An example of the decision rule approach is Dominance-based Rough Set Approach (DRSA), here rules are established using the principle of dominance.

2.1.2 Basic MCDA concepts

There are three main concepts that are very important in the decision aiding process. In this sub-section, we give a brief explanation of these concepts and how they are used in the decision aiding process.

2.1.2.1 Alternative or potential Action

An alternative is an option or an action to be taken in the decision making process. For example, a decision maker wishing to make hotel lodging arrangements, will have a set of hotels as alternatives. Alternatives are best seen as potential alternatives or potential actions. The word *'potential'* is key here as it necessary to have a set of alternatives that are useful in the decision to be made. This means from the previous example, a decision maker should not be selecting a hotel to lodge in from a list of churches or schools. This is to say that alternatives should be chosen properly to aid the decision making process. An alternative is deemed potential, if it is feasible, and of some interest in the decision making process [4].

Sometimes, alternatives may not be finite as they evolve during decision aiding process, but it is necessary to look at alternatives this way as most MCDA methods model them as finite and mutually exclusive.

2.1.2.2 Criterion

A criterion is a tool constructed for the comparison of alternatives. The performance of alternatives on a criterion or family of criteria determine their preference. As said before criteria are well defined points of view, and it should be possible to score each alternative after considering all pertinent attributes linked to these points of view. This score is called the *performance* and it is denoted by $g(a)$, where g represents the criterion and a the alternative. The performance is usually a real number, and must adhere to a certain scale. Scale here is the manner in which performance scores are represented, there are various kinds of scales including: numerical scales, verbal scales etc. Numerical scales represent performance scores in numbers, while verbal scales represent them in words.

In choosing preference, an alternative a is preferred to another alternative b, considering criteria g_i if $g_i(a) \geq g_i(b)$. Going back to the example of making lodging arrangements, some criteria could be as follows: the hotel's class (is it a 5 star hotel?), the distance of the hotel to a meeting location, the services offered in the hotel (wifi, gym etc.), security of the hotel etc.

There are two ways to treat criteria introduced in ELECTRE which are as follows:

- **True-criteria:** Introduced in ELECTRE I. It assumes that any slight difference in the performance score of an alternative over another indicates a strict preference. This means that the understanding of criteria is clear and performance scores are very accurate. This is not true in real world application, as data is imprecise, uncertain and not clear-cut. This problem lead to the introduction of pseudo-criteria.
- **Pseudo-criteria:** Introduced in ELECTRE III to handle imprecise and uncertain data. The pseudo-criteria model establishes preference by introducing the preference (p) and indifference (q) thresholds. If an alternative exceeds another by the amount assigned as p, it is strictly preferred to the other. While, an alternative is considered indifferent to another if its performance falls within the range of indifference. These can be expressed in equations as follows:
    - *aPb* a is strictly preferred to b iff: $g(a) - g(b) > p$
    - *aIb* a is indifferent to b iff: $|g(a) - g(b)| \leq q$
    - *aQb* a is weakly preferred to b iff: $q < g(a) - g(b) \leq p$

2.1.2.3 Problematic

The problematic in this sense is the type of problem, or formulation of the problem to be tackled by the decision maker. The description of the problematic answers the following questions: What is the purpose of this problem?, what results do we wish to achieve?, how do we pose the problem? Etc. After the problematic is established, the process can be carried out and the results analyzed.

There are three common problematic in MCDA which are as follows:

- **The choice problematic:** This is the most common problematic found in the decision making process. It is simply a problem of choice, the decision maker wishes to choose the best alternative or set of alternatives from the potential alternatives. The problem of making lodging arrangement mentioned before is an example of this problematic, as the decision maker wishes to choose a hotel to lodge.
- **The sorting problematic:** The goal of this problematic is classification of alternatives into pre-defined categories. Sorting is a popular problem in real world situations, this problematic can be applied to many relevant fields, including business, artificial intelligence, networking etc.
- **The ranking problematic:** This is another very popular problematic. The goal is a complete or partial preorder of alternatives, a ranking of alternatives. The order could be ascending or descending order.

2.2 ELECTRE

ELECTRE is an MCDA (Multi-criteria Decision Analysis) technique that was originated in an attempt to solve some drawbacks seen in weighted-sum based aggregation techniques. ELECTRE means **El**imination and **C**hoice **E**xpressing the **Re**ality [14]. ELECTRE falls into the outranking category, and was among the earliest techniques to be in this category popularly known as the French school.

In 1965 a team of consultants brought together by SEMA, a European consultancy company were tasked to model a multi-criteria decision support system to provide decision aiding for firms choosing new activities. MARSAN a weighted sum technique was first developed, but presented some drawbacks that needed to be fixed. Bernard Roy a mathematician and computer scientist renowned for his work on graph theory was consulted to fix the drawbacks seen in MARSAN. His solution would become ELECTRE. ELECTRE built by Bernard Roy focused on the choice problematic, this means it was built to select the best action or set of actions. It was later renamed ELECTRE I. ELECTRE I was further developed by various researchers applying it to other problematic, this brought about various versions including: ELECTRE Is, ELECTRE Iv, ELECTRE II, ELECTRE III.

2.2.1 Outranking relations in the ELECTRE methodology

The so-called outranking methods in the MCDA literature are based on conducting a pairwise comparison of alternatives with regards to each criterion. The goal of the comparison is to find out if it satisfies an outranking relation aSb meaning that alternative a is "at least as good as" alternative b.

In comparing two alternatives a and b based on an outranking relation S, one of the four options below would define the preference of the decision maker.

- aSb and not bSa - aPb; a is strictly preferred to b.
- bSa and not aSb - bPa; b is strictly preferred to a.

- aSb and bSa - aIb; a is indifferent to b.
- Not aSb and not bSa - aRb; a is incomparable to b.

The outranking relation S may be binary or valued. In this work we study the case of valued outranking relations, having then $S = A \rightarrow [0,1]$. The value assigned to S is usually denoted as credibility (on the outranking relation).

In the ELECTRE method, to calculate the credibility of aSb, two conditions must hold:

- **Concordance condition:** After pairwise comparison of a and b for each criterion, a majority of the criteria must support aSb. It accounts for the majority opinion.
- **Non-discordance condition**: Ensures that among the minority no criteria strongly refutes aSb. It permits the right to veto (i.e. "respect to minorities").

The outranking concept explained above is inspired in voting models used in different theories of social election. It is similar to voting procedures applied United Nations Security Council, where some countries have the right to veto the majority opinion. Following this idea, in ELECTRE methodology, to calculate the credibility value of the outranking relation $\rho(a,b) \in [0,1]$, the following steps are applied [4]:

1) Calculation of a partial concordance index for each criterion $c_j(a,b) \in [0,1]$. In each criterion, two discrimination thresholds may be used to model the uncertainty of the decision maker: the indifference and the preference threshold.
2) Calculation of the overall concordance $c(a,b) \in [0,1]$. It is calculated as a weighted average of $c_j(a,b)$ using as weights the voting power of each criterion. The resulting value represent the strength of the coalition of criteria being in favor of the outranking relation aSb.

3) Calculation of a partial discordance index for each criterion $d_j(a,b) \in [0,1]$. The DM can give to some criteria the right to veto the majority opinion if there are essential reasons to refute it. In this case, the criteria has an associated veto threshold, such that larger differences of this threshold in favor of b will eliminate the possibility that option a outranks option b.

4) Calculation of the final credibility as:

$$\rho(a,b) = \begin{cases} c(a,b) & if\ \forall_j\ \ d_j(a,b) \leq c(a,b) \\ c(a,b). \prod_{j \in J(a,b)} \frac{1-d_j(a,b)}{1-c(a,b)} & otherwise \end{cases} \qquad (1)$$

, where $J(a,b)$ is the set of criteria for which the discordance is larger than the overall concordance. The $\rho(a,b)$ follows these rules:

- When there is no discordant criterion, the credibility outranking relation is the same as in the concordance index.
- When a discordant criterion invokes veto power, the relation is not credible.
- In cases where the concordance index is lower than the discordance index, the credibility index becomes lower than the concordance index.

Once the credibility matrix is obtained, an exploitation procedure is applied in order to establish a preference-based order among the alternatives

In the next sub-sections we give an overview of the versions of ELECTRE, their procedure, formulations and the problematic they are designed to solve.

2.2.2 ELECTRE I

ELECTRE I is the earliest version of ELECTRE. It is actually the official name given to ELECTRE originally devised by Bernard Roy [14]. The successes of ELECTRE I when applied to problems instigated the evolution of ELECTRE, and brought about the various versions seen today. [4].

ELECTRE I uses true criteria values. This means that the criteria values must have clear numeric scales and the comparison is made in a Boolean way, without considering any uncertainty. Imprecise values for criteria cannot be handled by ELECTRE I. Due to the nature of criteria seen in real-world applications, ELECTRE I is not practical. To solve this problem ELECTRE IS, ELECTRE III, ELECTRE IV and ELECTRE TRI make use of pseudo-criteria to build concordance and discordance indexes.

ELECTRE methods main steps includes:

- Build the outranking relation S.
- Exploit the outranking relation depending of the problematic to be solved.

2.2.2.1 *Building the outranking relation*

This is the first step of any ELECTRE technique. The method of building the outranking relation varies based on the nature of the criteria (i.e. is it True-Criteria or Pseudo-Criteria). As mentioned before, the outranking relation determines the preference of one alternative over another.

In order to build the outranking relation (S), the concordance and discordance of a pairwise comparison must be computed. The concordance condition and non discordance condition are checked and a credibility index is calculated as said before. This credibility index is the outranking relation.

Concordance:

To compute the concordance, the partial concordance of each criterion must be computed. ELECTRE I uses true-criteria so computing partial concordance is straight forward. The process is shown below:

Given a set of criteria ($g_1, g_2, g_3, \ldots g_j$), the partial concordance for each criteria is obtained as follows:

$c_j(a, b) = 1\ iff\ \ g_j\ fully\ supports\ the\ relation\ aSb$

$c_j(a, b) = 0\ iff\ \ g_j\ doesn't\ support\ the\ relation\ aSb$

Finally, using the partial concordance the measure of the concordance coalition is computed. The measure of the concordance coalition is the strength of the criteria that support the relation aSb. The concordance is given as:

$$c(a, b) = \frac{\sum_{j=1} c_j(a,b) w_j}{\sum_{j=1}^{n} w_j} \qquad (2)$$

, where w_j is the weight of $criterion\ g_j$.

The numerator of the equation above is the sum of the weights of all criteria that support the relation aSb, while the denominator is the total sum of all weights. $c(a, b) : within\ range$ [0,1]. The equation above means as more criteria with high importance support the relation aSb, $c(a, b)$: $tends\ to$ 1. When $c(a, b) = 1$, it means all criteria support the relation aSb.

Discordance:

To ensure the respect of minorities principle, criteria that refutes the relation must be checked. The introduction of the veto threshold in ELECTRE Iv aids in the discordance calculation.

The veto (v_j) is the threshold once exceeded for any discordant criterion, no matter how the large number concordant criteria the relation cannot be accepted. To measure discordance the maximum differential value of the set of discordant criteria is compared with the veto. If found to exceed the veto, the relation in question is thrown out.

The discordance $d(a,b)$ is as shown below:

$$\left[d(a,b) = \max_{\{j:g_j(a)<g_j(b)\}}\{g_j(b) - g_j(a)\}\right] > v_j \qquad (3)$$

If the equation above is true, the relation in question is dismissed.

2.2.2.2 ***Exploiting the outranking relation***

Exploiting the outranking relation requires using the data gotten from concordance, discordance and credibility index calculation to solve the problematic. The problematic can be choice, ranking or sorting. In the case of ELECTRE I the problematic to be solved is choice. Which means the technique seeks to find the best alternative/action. A method called *graph kernel* is used to solve the choice problematic in ELECTRE I.

In graph kernel concept, a full-directed graph is built using the credibility matrix. The nodes in the graph represent alternatives while edges are the relation between alternatives. After the complete digraph is built showing outranking, indifference and incomparability a *kernel* is then selected. The kernel is the solution to the choice problematic, it is a subset of the alternatives containing the best alternatives.

The kernel is built using the following rules:

- Any alternative not in the kernel must be outranked by at least one alternative in the kernel.
- All alternatives in the kernel are incomparable.
- When choosing the kernel, alternatives in a cycle are considered as one alternative.

2.2.3 ELECTRE II

ELECTRE II is a modification of ELECTRE I, It is the first ELECTRE method developed to handle the ranking problematic. In ranking, the decision maker seeks to arrange all alternatives in an order of preference. ELECTE II is similar to ELECTRE I in the sense that both methods make use of true-criteria. The discordance calculation is the same as in ELECTRE I with no veto, but it deviates in the calculation of concordance.

ELECTRE implements embedded outranking relations. Instead of one outranking relation like in ELECTRE I, there are two embedded outranking relations. These outranking relations are as follows:

- ***Strong outranking relation***, based on the concordance level s^1. For a relation to be a strong outranking relation, the strength of the concordant coalition must be equal to or exceed the concordance level s^1.
- ***Weak outranking relation***, based on the concordance level s^2. Like in the strong outranking relation above, the coalition strength must equal or exceed the concordance level s^2.

2.2.3.1 *Building the outranking relation*

The concordance is calculated as in ELECTRE I as a weighted average of binary partial concordances. In addition, the condition for a “outranks” b has a minimum threshold to be considered true (i.e. credibility threshold):

$$c(aSb) \geq s^r \; and \; c(aSb) \geq c(bSa), for \; r = 1,2 \qquad (4)$$

This means for a "outranks" b to be true, the concordance for aSb must be greater that the concordance level $s^1 \; or \; s^2$(for both weak and strong outranking). Also, the concordance for $aS^r b$ must be greater than that of $bS^r a$. Once this statement is true, the concordance condition is met. The discordance condition is performed just like in ELECTRE I.

2.2.3.2 *Exploiting the outranking relation*

Graph kernel:

The graph kernel method is also used in ELECTRE II for the ranking of alternatives. To perform the ranking the following steps are carried out:

- Like in ELECTRE I, a digraph is built but using the strong outranking relation S^1. Cycles in the digraph are replaced with an alternative.
- The kernel is found for this digraph using the rules stated in ELECTRE I. If the kernel contains more than one alternative, a tie-breaker step is carried out. In the tie-breaker, the weak outranking relation S^2 is used and a new kernel formed.
- The final kernel gotten in the steps above is at the top of the ranking. All alternatives in the kernel are eliminated from the initial digraph and the previous steps are carried out again.
- These steps are carried out iteratively until the complete ranking is formed.

Net flow score (NFS):

Net flow score is another method of ranking in ELECTRE II. The steps are as follows:

- First, a digraph is built (like in ELECTRE I) with the strong outranking relation S^1.
- Strength and weakness of each alternative are calculated. They are measured in the digraph. The strength of alternative a is defined as the sum of the credibility values of the output edges

to the node a. The weakness of alternative a is defined as the sum of the credibility values of the input edges to the node a. In terms of outranking relations, the net flow score of an alternative a is defined in Eq. 5.

$$NFS\ (a) \ = \ |b \ \in \ A : \ aSb| - |b \ \in \ A : \ bSa| \qquad (5)$$

- A total ranking can be derived from the NFS, being the higher the score, the better.

If two or more alternatives have the same net flow score, a digraph of those alternatives is built using the weak outranking relation S^2 and the net flow score of the alternatives is calculated. The alternatives according to their net flow score.

2.2.4 ELECTRE III

ELECTRE III was developed as improvement on ELECTRE II. It was built to deal with imprecise and uncertain data. It is the most used ELECTRE method and has seen many successes in application to real-world problems, a shortcoming seen in ELECTRE I.

ELECTRE III introduced Pseudo-criteria and two new thresholds in concordance and discordance calculation.

- ***Indifference threshold*** *(q_j)*: if the difference between the performance scores of alternatives being compared with respect to a criterion g_j doesn't exceed the threshold q_j, those alternatives are considered to be indifferent ($g_j(a) \leq g_j(b) - q_j$).
- ***Preference threshold*** *(p_j)*: in pairwise comparison, *a* is *strictly preferred* to *b* if *a* has performance greater than the sum of the performance score of *b* and the value of the preference threshold ($g_j(a) \geq g_j(b) + p_j$).

The thresholds above also help to define an outranking relation as strict preference or weak preference. I.e. an alternative's performance score that exceeds another by the amount of the preference threshold is strictly preferred, while an alternative that exceeds the indifference threshold but doesn't meet up to the preference threshold is weakly preferred.

2.2.4.1 *Building the outranking relation*

Concordance:

The partial concordances are assigned as shown below:

$$c_j(a,b) = \begin{cases} 1 & if\ g_j(a) \geq\ g_j(b) - q_j \\ 0 & if\ g_j(a) \leq\ g_j(b) - p_j \\ \frac{p_j - \left(g_j(b) - g_j(a)\right)}{p_j - q_j} & if\ g_j(b) - p_j \leq g_j(a) \leq g_j(b) - q_j \end{cases} \qquad (6)$$

Where: $g_j(a)$ is the performance score of alternative a with respect to criterion g_j.

$g_j(b)$ is the performance score of alternative b with respect to criterion g_j.

p_j is the preference threshold of criterion g_j.

q_j is the indifference threshold of criterion g_j.

Once partial concordance is defined, then the overall concordance index is calculated as shown below:

$$c(a,b) = \frac{\sum w_j c_j(a,b)}{\sum w_j} \qquad (7)$$

, where

$c_j(a,b)$ is the partial concordance of aSb with respect to the criterion g_j.

w_j is the weight or importance assigned to criterion g_j.

$\sum w_j$ is the total importance or the sum of the weights of all criteria.

Discordance:

Partial discordances are assigned as shown below:

$$d_j(a,b) = \begin{cases} 0 & if\ g_j(b) \leq\ g_j(a) + p_j \\ 1 & if\ g_j(b) \geq\ g_j(a) + v_j \\ \frac{g_j(b)-g_j(a)-p_j}{v_j-p_j} & otherwise \end{cases} \tag{8}$$

, where

$d_j(a,b)$ is the partial concordance of aSb with respect to the criterion g_j.

v_j is the veto value assigned to criterion g_j.

In the case of the discordance as shown above, the veto threshold is used. If 99% of criteria support the relation aSb, and just one criterion has a performance that exceeds its veto threshold the relation is unacceptable.

2.2.4.2 *Exploiting the outranking relation*

The problematic focused on in ELECTRE III is the ranking problematic. ELECTRE uses the distillation procedure to rank alternatives. The distillation procedure is basically applying net flow score recursively.

The distillation procedure works by finding two orders, the descending and ascending order. Then the intersection of both orders becomes the final ranking solution.

Descending pre-order:

1. The credibility index is made crisp by setting a max value for preference. This means above the set max the outranking relation is accepted, and below that value it is discarded.

2. The Net flow score (NFS) for each alternative is calculated. The set of all alternatives is labelled A.
3. The set of alternatives with the highest NFS is the first distillate D_1. If D_1 is a singleton then D_1 is removed from A and steps 2 and 3 are carried out on A-D_1 . Else, if D_1 contains two or more alternatives carry out steps 2 and 3 on D_1 recursively to find a distillate with only one alternative.
4. Carry out the previous steps till all alternatives are classified.

Ascending pre-order:

The ascending pre-order is performed just as in the descending pre-order above with a slight difference. In the ascending pre-order, the set of lowest NFS is the first distillate. This means, the worst alternatives are ordered first.

Final order:

The intersection of the ascending and descending pre-order is found. Noting the following situations:

- Indifference; where *a* and *b* are indifferent in both orders.
- Preference; where *a* 'outranks' *b* in at least one pre-order.
- Incomparability; where *a* 'outranks' *b* in one pre-order and *b* 'outranks' *a* in the other.

ELECTRE IV was built as an extension to ELECTRE III. Choosing of appropriate weights to assign to different criteria can prove difficult. Sometimes, all criteria are given the same value for the weight. This is not optimal, a better solution would be to check all combination of weights and find an intersection of the pre-order created by each combination. This process will be tasking. ELECTRE IV works without assigning weights to criteria. The method uses a family of pseudo-criteria and defines up to five embedded outranking relations. I.e. $(S^1, \ldots, S^5)$, the relation S^{r+1} must be less credible than S^r.

2.2.5 ELECTRE TRI

ELECTRE TRI method was developed to solve the sorting problematic. The method classifies actions, objects or alternatives into different categories. The ELECTRE TRI method was proposed by B. Roy and Bouyssou.

Assuming there are k categories as follows: $(C^1, C^2, \dots, C^h, \dots, C^k)$, where C^k is the best category and C^1 the worst. A set of actions or alternatives A will be sorted into these categories.

To sort alternatives in A, each individual alternative must be compared to upper and lower profiles defined for each category. The alternative is then placed in the category it belongs.

Profile definition:

A category C^h is defined by two profiles: the lower profile b^{h-1} and the upper profile b^h. Both profiles are defined separately on each criterion. The profiles are considered as imaginary alternatives.

Sorting:

The credibility index of the assertions aSb^h and b^hSa are defined just as in ELECTRE III, and a cut crisp value is used to create a crisp outranking relation. The following situations can be observed of the assertions:

- a "is indifferent to" b^h (aIb^h) if and only if aSb^h and b^hSa.
- a "is strictly preferred to" b^h (aPb^h) if and only if aSb^h and not b^hSa.
- b^h "is strictly preferred to" a (b^hPa) if and only if not aSb^h and b^hSa.
- a "is incomparable to" b^h (aRb^h) if and only if not aSb^h and not b^hSa.

The pessimistic and optimistic procedures are used for the actual sorting process exploiting the relations shown above. After the procedures are carried out, if their results coincide we have the final result.

In the *pessimistic view*, the alternative a is put in a category C^h in which a 'outranks' its lower profile b^h for all criteria. In other words, an action/alternative is placed in a category in which on each criterion it meets the lowest cut-off.

In the *optimistic view*, the alternative a is put in a category C^f in which a 'outranks' its lower profile b^f for at least one criterion. In other words, an action/alternative is placed in a category in which for at least one criterion it meets the lowest cut-off.

2.3 Extensions to the concordance index

In this section we explain some works that have modified the original definition of concordance in ELECTRE to deal with different situations.

2.3.1 Concordance index with criterion interactions

[Figueira et al.], in ELECTRE METHODS WITH INTERACTION [5] seek to extend the concordance index as seen in [4] to accommodate interactions between criteria. In conventional ELECTRE methods, criteria are modelled to have no interaction with each other. The assumption is made that for ELECTRE to be applied no evaluation criterion is directly connected to another criterion. In real-world applications this may not always be true.

The concordance index is modified to take into consideration three possible interactions: mutual strengthening, mutual weakening and antagonistic.

2.3.1.1 Mutual strengthening

Mutual strengthening is observed when the importance of a criteria pair (g_i, g_j) are considered to be more when both criteria are concordant simultaneously for an assertion aSb. This can be expressed as follows:

g_i is assigned the weight k_i, and g_j is assigned the weight k_j. If g_i and g_j support an assertion aSb, their combined voting power will be $k_i + k_j$ for that assertion. The criteria (g_i, g_j) are mutual strengthening if the DM considers the combined importance $k_i + k_j$ to be insufficient. I.e. their contribution should be larger than $k_i + k_j$. This interaction can be modelled by deciding on a mutual strengthening coefficient $k_{ij} > 0$ which is added to the weights of criteria g_i and g_j. The coefficient is only added to the weights when both g_i and g_j are concordant on a particular assertion.

2.3.1.2 Mutual weakening

Mutual weakening is observed when the importance of pair (g_i, g_j) are considered to be less when both criteria are concordant for an assertion aSb because there is some relation of replaceability. This can be expressed as follows:

g_i is assigned the weight k_i, and g_j is assigned the weight k_j. If g_i and g_j support an assertion aSb, their combine voting power will be $k_i + k_j$ for that assertion. The criteria (g_i, g_j) are mutual weakening if the DM considers the combined importance $k_i + k_j$ to be excessive. I.e. their contribution should be smaller than $k_i + k_j$. This interaction can be modelled by deciding on a mutual weakening coefficient $k_{ij} < 0$ which is added to the weights of criteria g_i and g_j. The coefficient is only added to the weights when both g_i and g_j are concordant on a particular assertion. The mutual weakening coefficient must be negative.

2.3.1.3 Antagonistic

Antagonistic interaction is observed if a criterion g_i strongly, or weakly supports an assertion aSb and another criterion g_h strongly opposes that assertion. This means criteria are considered to be antagonistic if one belongs to $C(aPb\,, aQb)$ while the other belongs to $C(bPa)$. The degree of interaction should be established by the DM by stating if the contribution of g_i to the concordance index when $g_i \in C(aPb\,, aQb)$ needs to be smaller than its weight k_i, in cases where $g_h \in C(bPa)$.

The antagonistic interaction can be modelled by introducing an antagonistic $k'_{ih} > 0$. Which is applied as follows:

- In the cases where $g_i \in C(aPb\,, aQb)$, $k'_{ih} = 0$ and the contribution of g_i is reflected in the weight k_i.
- In the cases where $g_h \in C(bPa)$, $k'_{ih} > 0$ and the contribution of g_i is reflected in the weight $k_i - k'_{ih}$.
- Also, if g_h is reverse antagonistic then in cases where $g_i \in C(aPb\,, aQb)$, the contribution of g_h is reflected in its weight $k_h - k'_{hi}$. $k'_{hi} > 0$ and $k'_{hi} = k'_{ih}$.

To model these interactions in a real-world application of ELECTRE certain steps must be followed:

- *Step 1*: The intrinsic weights $k_i, i = 1, \dots, n$ of the set of criteria $F : (g_1, g_2, \dots, g_i)$ must be assigned.
- *Step 2*: The DM must decide on the interactions between the pairwise combinations of all criteria. E.g. $g_i\ is\ mutually\ strengthens\ g_h$. A criterion g_i could have multiple interactions with different criteria.
- *Step 3*: Numerical values are assigned to interaction coefficients associated with criteria pairs found in step 2. The DM considers each pair assigns appropriate values to coefficients to represent the interactions.

- *Step 4*: In this step the balance conditions are checked. All coefficients provided in the previous step must be checked to satisfy the following conditions:

  In the case of mutual weakening the coefficient k_{ij} must satisfy the condition: $k_i - |k_{ij}| \geq 0$. This is to ensure the contribution of a criterion is never below zero.

  In the case of an antagonistic interaction, the coefficient k'_{ih} must satisfy the condition: $k_i - k'_{ih} \geq 0$. Also for the same reason as in mutual weakening. Suppose mutual weakening and antagonistic interactions exists between two criteria, the coefficients must satisfy the condition: $k_i - k'_{ih} - |k_{ij}| \geq 0$. It must be positive.

Finally, the net balance of all interactions of a criterion is given by: $For\ all\ i \in F,$

$$k_i - \left(\sum_{\{i,j\}:k_{ij}<0} \left|k_{ij}\right| + \sum_h k'_{ih} \right) > 0 \qquad (9)$$

This ensures that after all interactions of a criterion have been numerically applied, the weight of the criterion does not fall below zero.

2.3.1.4 Concordance computation with applied interactions

The concordance equations for quasi and pseudo criteria is given below.

The quasi criterion model:

The quasi-criterion model describes a criterion where the indifference threshold is equal to the preference threshold. I.e. $q_i\big(g_i(a)\big) = p_i\big(g_i(a)\big), for\ all\ a \in A$; A is the set of all action/alternatives. This eliminates the weak preference situation found in the conventional pseudo criteria.

$c(a,b)$ for quasi criteria is given as:

$$c(a,b) = \frac{1}{K(a,b)}\left(\sum_{i\in C(bPa)} k_i + \sum_{\{i,j\}\in L(a,b)} k_{ij} - \sum_{(i,h)\in O(a,b)} k'_{ih}\right) \tag{10}$$

, where

$$K(a,b) = \sum_{i\in F} k_i + \sum_{\{i,j\}\in L(a,b)} k_{ij} - \sum_{(i,h)\in O(a,b)} k'_{ih} \tag{11}$$

$L(a,b)$ denotes the set of all pairs $\{i,j\}$ such that $i,j \in \overline{C}(bPa)$;

$O(a,b)$ denotes the set of all ordered pairs $\{i,h\}$ such that $i \in \overline{C}(bPa)\ and\ h \in C(bPa)$.

The pseudo criterion:

$c(a,b)$ for pseudo criteria is given as:

$$c(a,b) = \frac{1}{K(a,b)}\left(\sum_{i\in C(bPa)} c_i(a,b)k_i + \sum_{\{i,j\}\in L(a,b)} Z(c_i(a,b),c_j(a,b))k_{ij} - \right.$$
$$\left.\sum_{(i,h)\in O(a,b)} Z(c_i(a,b),c_j(b,a))k'_{ih}\right) \tag{12}$$

, where

$$K(a,b) = \sum_{i\in F} k_i + \sum_{\{i,j\}\in L(a,b)} Z\left(c_i(a,b),c_j(a,b)\right)k_{ij} - \sum_{(i,h)\in O(a,b)} Z(c_i(a,b),c_j(b,a))k'_{ih} \tag{13}$$

The function $Z()$ is used to represent the ambiguity of weak preference. This extension to the concordance index was proven to embody mutual strengthening, mutual weakening and antagonistic interactions.

2.3.2 Concordance as positive and negative reasons

[22] seeks to model the outranking relation in terms of positive and negative reasons. This approach states that for the assertion 'a is at least as good as b' to accepted, there exists sufficient (majority) positive reasons that support the assertion and there are no negative reasons that strongly disagree with the assertion. From [22]:

Alternative x is better than alternative y iff there is a majority of reasons supporting x wrt to y and there is no strong opposition to x wrt to y.

The outranking relation 'a is at least as good as b' $S(a,b)$ is denoted as shown below:

$$S(a,b) \Leftrightarrow C(a,b) \ \wedge \ \neg D(a,b) \qquad (14)$$

, where

$C(a,b)$ means there is a majority of reasons supporting a wrt to b

$D(a,b)$ means there is a strong opposition to a wrt to b

$\neg D(a,b)$ is the negation of $D(a,b)$, meaning there is no opposition to a wrt to b

$\wedge$ is conjunction, means both the majority and minority opinions must be considered to form the relation.

The concordance evaluated in terms of the positive reasons that support the assertion.

2.3.2.1 *Preference modelling according to positive and negative reasons*

To model preference of pairwise comparisons of alternatives, [22] describes a novelty logic representation (DDT Logic). DDT Logic has the following representation below:

$\neg\ \alpha\ (not\ \alpha, the\ negation)$

$\nsim\ \alpha\ (the\ complement\ of\ \alpha, \sim \alpha \equiv \neg \nsim \neg \nsim\ \alpha)$

$\Delta\alpha\ (presence\ of\ truth\ \alpha)$

$\Delta\neg\alpha\ (presence\ of\ truth \neg\ \alpha)$

$\mathbf{T}\alpha\ (the\ true\ extension\ of\ \alpha)$

$\mathbf{K}\alpha\ (the\ contradictory\ extension\ of\ \alpha)$

$\mathbf{U}\alpha\ (the\ unknown\ extension\ of\ \alpha)$

$\mathbf{F}\alpha\ (the\ false\ extension\ of\ \alpha)$

The preference situations for an assertion 'x is at least as good as y' is modelled using these representations as shown below:

$\mathbf{T}S(x,y) \Leftrightarrow \Delta S(x,y) \land\ \neg\Delta\neg S(x,y)$

$\mathbf{K}S(x,y) \Leftrightarrow \Delta S(x,y) \land\ \Delta\neg S(x,y)$

$\mathbf{U}S(x,y) \Leftrightarrow \neg\Delta S(x,y) \land\ \neg\Delta\neg S(x,y)$

$\mathbf{F}S(x,y) \Leftrightarrow \neg\Delta S(x,y) \land\ \Delta\neg S(x,y)$

Where the following means,

$\Delta S(x,y)$: there is (presence of) truth in claiming that x is at least as good as y.

$\Delta\neg S(x,y)$: there is (presence of) truth in claiming that x is not at least as good as y.

$\neg\Delta S(x,y)$: there is no (presence of) truth in claiming that x is at least as good as y.

$\neg\Delta\neg S(x,y)$: there is no (presence of) truth in claiming that x is not at least as good as y.

The relations $\mathbf{T}S(x,y), \mathbf{K}S(x,y), \mathbf{U}S(x,y), \mathbf{F}S(x,y)$ maybe combined to conventional preference relations. The equivalent to the conventional strict preference (P) and Indifference (I) is shown below:

$\mathbf{T}P(x,y) \equiv \mathbf{T}S(x,y) \wedge \mathbf{F}S^{-1}(x,y)$ – Shows strict preference where S^{-1} is the inverse of the outranking relation.

$\mathbf{T}I(x,y) \equiv \mathbf{T}S(x,y) \wedge \mathbf{T}S^{-1}(x,y)$ – Shows indifference.

2.3.2.2 Concordance applying positive and negative reasons

Alternatives are compared using intervals, an alternative x score will fall within interval $[l(x), r(x)]$ called the left and right interval respectively. The right interval is the upper bound while the left is the lower bound. Comparing two alternatives x and y with intervals $[l(x), r(x)]$ and $[l(y), r(y)]$ respectively, could show strict preference $P(x,y)\ if\ r(x) > l(x) > r(y) > l(y)$, weak preference $Q(x,y)\ if\ r(x) > r(y) > l(x) > l(y)$, and indifference $I(x,y)\ if\ r(x) > r(y) > l(y) > l(x)$.

As such the concordance of a pairwise comparison of (x,y) where $x, y \in A$ is given by [12]:

$$C(x,y) \equiv \Delta S(x,y) = r(x) \geq l(y) \wedge [r(x) < r(y) \vee l(x) \geq l(y)] \qquad (15)$$

And for a non-conventional preference structure,

$$C(x,y) \equiv \Delta S(x,y) = r(x) \geq l(y) \qquad (16)$$

, where $[l(y), r(y)]$ represents the interval of plausible values for $g(x)$, of a given criterion g.

This approach of positive and negative reasons deviates from numerical values to embody more of the meaning of outranking. It was also shown to handle imprecise criterion values.

2.3.3 Concordance index with redefined preference threshold

The paper [17] looks at a situation where the extent to which a criterion surpasses the preference threshold can be reflected in the importance of that criterion in the concordance coalition and the credibility index.

Considering an outranking relation aSb, a criterion g_j having performance values $g_j(a)\ and\ g_j(b)$. If the difference $g_j(a) - g_j(b)$ is greater than the preference threshold, the extent to which it is greater should be taken into consideration when defining the credibility of a over b.

To implement this model, [17] proposes two ways:

***Reinforced preference threshold*:**

This refers to the value of the difference $g_j(a) - g_j(b)$ where the criterion g_j is considered important and its weight is increased. This reinforced preference threshold is greater than the preference threshold.

The reinforced preference threshold for criterion g_j is represented as $rp_j(g_j(a))$. When $rp_j(g_j(a))$ is crossed the weight coefficient w_j of criterion g_j is replaced in the concordance index $c(a,b)$ by $\omega_j w_j$, where $\omega_j > 1$. ω_j is called the reinforcement factor.

The concordance index can be represented as shown below:

$$\hat{C}(a,b) = \frac{\sum_{\{j \in C^{RP}(a,b)\}} \omega_j w_j + \sum_{\{j \in C^{S}(a,b) \setminus C^{RP}(a,b)\}} w_j + \sum_{\{j \in C^{Q}(a,b)\}} w_j \varphi_j}{\sum_{\{j \in C^{RP}(a,b)\}} \omega_j w_j + \sum_{\{j \in F \setminus C^{RP}(a,b)\}} w_j} \qquad (17)$$

, where

$C^{RP}(a,b)$ denotes the set of criteria for which $g_j(a) > g_j(b) + rp_j(g_j(a))$

***Counter-veto threshold*:**

This second threshold corresponds to the value of the difference of performances $g_j(a) - g_j(b)$ that is judged meaningful for the weakening the veto against the credibility of the assertion $c(a, b)$. The counter-veto threshold targets this approach from the point of the discordance. The credibility index is then computed by combining the modified concordance and discordance.

2.4 Summary

This chapter presented MCDA, its approaches and its fundamental concepts. Specially, we focused on the ELECTRE method, giving detailed explanations of its formulation and various versions. In ELECTRE, partial concordances can be defined in two ways according to the types of criteria (true and pseudo criteria). Also, overall concordance is usually calculated by performing a weighted mean of partial concordance values.

Furthermore, we looked at other works that present extensions to the concordance definition and the thresholds. None of these works propose the use of other aggregation operators in aggregating the partial concordance values as proposed in this work.

3 Weighted OWA operators

In this chapter we study in detail the aggregation operators, since they are in the core of this work. In particular, we explain operators that allow the assignment of relevance weights to the sources of information. In addition, as we want to have the possibility of modelling the degree of simultaneity of the aggregation procedure, the aggregation operators studied are from the family of OWA operators. The operators that allow this double weighting are known as "weighted OWA".

3.1 Aggregation operators

Aggregation operators are mathematical formulations that map a set of *n* values R^n to a single value R and must satisfy certain properties (Idempotency, Monotonicity etc.). The most popular aggregation operators are averaging operators. The simplest is the arithmetic averaging operator also known as arithmetic mean:

In arithmetic mean (AM) set of *n* arguments $A: (a_1, \dots, a_N)$ is mapped to a single value $AM(A)$ using the formula:

$$AM(A) = \frac{1}{n}\sum_{i=1}^{n} a_i \qquad (18)$$

This means $AM(A)$ is a summation of the set of arguments $A: (a_1, \dots, a_n)$ divided by the number of elements in the set *n*.

Other averaging operators includes: weighted averaging (WA) operator and ordered weighted averaging (OWA). WA and OWA are especially useful in decision support systems.

3.1.1 Weighted averaging operator (WA)

Given a set of arguments $A = (a_1, \dots, a_n)$ and a weighting vector V with weights $v_j \in [0,1]$ associated with each argument, such that $\sum_{j=1}^{n} v_j = 1$. The weighted average is defined as:

$$WA(A) = \sum_{i,j=1}^{n} v_j a_i \qquad (19)$$

In weighted averaging as seen above, arguments are assigned weights to give them influence over the aggregate. These weights are related to the importance of the sources of the values aggregated. More important sources are given higher weights and therefore have greater influence on the aggregate. The introduction of weights makes this operator useful in decision support systems.

It is also important to note that the weighted averaging (WA) operator can be used as the arithmetic mean (AM), if the all weights in the weighting vector V are set to $v_j = {}^{1}\!/_{n}$. Where n is the total number of weights.

3.1.2 Ordered weighted averaging operator (OWA)

This operator was introduced by Ronald R. Yager in 1988 [24]. It provides a parameterized family of mean type aggregation operators. The novel feature of this operator is the reordering of arguments according to their values before weights are assigned.

Given a set of arguments $A = (a_1, \dots, a_n)$ and a weighting vector W with weights $w_j \in [0,1]$, such that $\sum_{j=1}^{n} w_j = 1$. The ordered weighted average is defined as:

$$OWA(A) = \sum_{j=1}^{n} w_j b_j \qquad (20)$$

, where b_j is the jth largest of the a_i.

An interesting fact about OWA is that weights are not fixed to arguments, they are rather distributed in certain ways to perform different aggregation policies (disjunctive or conjunctive) decided upon by the decision maker (DM). For example, the DM could assign weights in such a way that extreme arguments are regarded less than central arguments.

A good way to look at the OWA and WA weighting systems is in the terms of various sensors providing information. Let us say we have n sensors, i.e. n information sources providing n arguments. The weighting system in WA assigns importance to information sources based on reliability of sensors, this is represented as associated weights. While, the weighting system in OWA rather considers the positioning of arguments after the reordering. For example,

- For the maximum; $w_1 = 1, and\ w_j = 0\ for\ all\ j \neq 1,\ j = (1, \dots, n)$
- For the minimum; $w_n = 1, and\ w_j = 0\ for\ all\ j \neq n,\ j = (1, \dots, n)$
- Disregarding extremes; $w_1 = w_n = 0,\ j = (1, \dots, n)$

In summary, the weights of OWA shows the importance of arguments in relation to the ordering of the arguments. The WA and OWA operators are fundamental parts of the OWAWA, IOWA and WOWA operators. In some decision problems the DM may be interested in using OWA weights to define the andness/orness of the aggregation together with assigning WA weights to the different criteria. Next subsections introduce three different ways of combining them in OWA-like operators that exploit the advantages of both OWA and WA approaches.

3.2 Ordered weighted averaging – weighted averaging operator (OWAWA)

The paper [11] introduces a new averaging operator the OWAWA operator which is a generalization of the weighted averaging operator (WA) and the ordered weighted averaging operator (OWA). It unifies the WA operator and the OWA operator, this unification makes the OWAWA operator applicable in situations where either the WA or OWA are suitable.

The novel feature of the OWAWA operator is the ability to account for the degree of importance of WA and OWA in specific situations. This means that when a problem or situation is better solved with WA than OWA, more importance is given to WA than OWA.

An OWAWA operator is a mapping: $A = (a_1, \ldots, a_n) \longrightarrow R$ of dimension n, having an associated weighting vector V (WA), with $\sum_{j=1}^{n} v_i = 1$ and $v_i \in [0,1]$ and a weighting vector W (OWA), with $\sum_{i=1}^{n} w_j = 1$ and $w_j \in [0,1]$, such that:

$$OWAWA(A) = \beta \sum_{j=1}^{n} w_j b_j + (1-\beta) \sum_{i=1}^{n} v_i a_i \qquad (21)$$

, where b_j is the jth largest of the a_i and $\beta \in [0,1]$.

The value of β defines the degree of importance assigned OWA or WA in the situation. As $\beta \rightarrow 1$, the importance of OWA increases while as $\beta \rightarrow 0$, the importance of WA increases. To perform the conventional OWA approach β is set to 1 and to perform the conventional WA approach β is set to 0.

The OWAWA operator is monotonic, idempotent, commutative and bounded. The OWAWA operator is applicable in all situations where WA and OWA operators are applicable, but it used optimally in a situation where we want to consider the degree of importance of each case and the attitudinal character of the decision maker.

3.3 Weighted ordered weighted averaging (WOWA)

The WOWA operator introduced in [19] as a combination of the WA operator and the OWA operator. It seeks to combine the advantages of the OWA and WA approaches. This combination is performed by means of constructing a different weight that integrates the associated weighting system seen in WA, V, with the weighting according to ordering of OWA, W. These two weighting systems display advantages in different situations. The WOWA operator is modelled to combine these advantages. The definition of the WOWA operator is given as follows:

Let $V$and W be the weighting vectors $V = (v_1, \ldots, v_n)$, $W = (w_1, \ldots, w_n)$ such that:

$\sum_{i=1}^{n} v_i = 1$ and $v_i \in [0,1]$

$\sum_{i=1}^{n} w_i = 1$ and $w_i \in [0,1]$

A WOWA operator is a mapping: $A = (a_1, \ldots, a_n) \longrightarrow R$ of dimension n where,

$$WOWA(A) = \sum_{j=1}^{n} \omega_j b_j \qquad (22)$$

, where b_j is the jth largest of the a_i and the weight ω_i is defined taking into account the importance of the sources of the arguments and their position after the reordering step. The weight ω_i is defined as:

$$\omega_i = w^*\left(\sum_{j\leq i} v_{\sigma(j)}\right) - w^*\left(\sum_{j<i} v_{\sigma(j)}\right) \text{ with } \sum_{i=1}^{n} \omega_i = 1. \qquad (23)$$

w^* is a non-decreasing function that interpolates the points $\{(0,0)\} \cup \left\{(i/n\,, \sum_{j\leq i} w_j)\right\} \forall i = 1, \ldots, n$. $w^*$is required to be a straight line when the points can be interpolated in this way. w^* maybe replaced by regular monotonically non-decreasing quantifier $Q(x)$. Where $Q(0) = 0, Q(1) = 1$ and if $x > y$ then $Q(x) \geq Q(y)$.

In [26] and [27], regular monotonically non-decreasing quantifiers are used as fuzzy quantifiers to make weight extraction possible. They can be used here if they satisfy the requirements of w^*, where w^* is a montone increasing function within [0, 1] interval with $w^*(0) = 0$ and $w^*(1) = 1$. Some common examples of these quantifiers include: $Q(x) = x$, $Q(x) = x^2$, $Q(x) = \sqrt{x}$.

The WOWA operator is defined in such a way that it reduces to the OWA operator when $v_i = 1/n$ and reduces to the WA operator when $w_i = 1/n$. This shows that OWA and WA are special cases of the generalized WOWA operator.

In the implementation of this operator for this work, regular monotonically non-decreasing quantifiers will be used instead of interpolating a given weighting vector w. This means policies would be established using functions rather than a user-defined weighting vector.

3.4 Induced ordered weighted averaging (IOWA)

The induced ordered weighted averaging operator (IOWA) is introduced in the paper [25], it is modelled as an expansion to the OWA operator. The IOWA operator approaches the ordering step of the OWA operator in a new way, rather ordering arguments by their numeric values an ordered inducing variable is used to order the arguments. In other words, the order of arguments is induced by the order inducing variable. This order inducing variable could be defined to handle ordinal, numeric arguments or mixed arguments.

A novel feature of the IOWA operator is the representation of arguments to be aggregated as a two-tuple called an OWA pair. The OWA pair contains the order inducing variable and the argument variable. The IOWA operator is described as follows:

Given n arguments to be aggregated denoted as $(a_1, \dots, a_n)$, W^T the OWA weighting vector and B the ordered argument vector gotten by ordering the arguments. The OWA operator can be expressed as:

$$OWA(a_1, \dots, a_n) = W^T B \qquad (24)$$

The IOWA operator modifies this expression by representing arguments as OWA pair $\langle u_i, a_i \rangle$, where u_i is the order inducing variable of the ith argument and a_i is the argument variable of the ith argument. In the reordering step a_i is not used but u_i.

This means the ordered argument vector B is now based on u_i values and is written as B_u. The IOWA operator can then be defined as:

$$IOWA(\langle u_1, a_1 \rangle, \dots, \langle u_1, a_1 \rangle) = W^T B_u \qquad (25)$$

The ordered argument vector B_u is formed such that b_j is the a value of the OWA pair having the jth largest u value. The IOWA operator is idempotent, communicative, monotonic and bounded.

The IOWA operator may also be defined in the same terms as seen in the previous operators, this is shown below:

$$IOWA(\langle u_1, a_1 \rangle, \dots, \langle u_1, a_1 \rangle) = \sum_{j=1}^{n} \omega_j b_j^u \qquad (26)$$

In aggregating OWA pair ties may occurs. A tie occurs when two OWA pairs $\langle u_j, a_j\rangle, \langle u_k, a_k\rangle$ have equal order inducing variables, i.e. $u_j = u_k$. In cases like this, each OWA pair is replaced with an OWA pair having the same order inducing variable u but an argument variable which is an average of the previous argument variables. This means:

$\langle u_j, a_j\rangle$ and $\langle u_k, a_k\rangle$ $where\ u_j = u_k$ is replaced by $\langle (u_j = u_{k)}, (a_j + a_k/2)\rangle$ in the aggregation process.

OWA and WA operators are special cases of the IOWA operator. As such the OWA operator is obtained from the IOWA operator in a case where for OWA pair $\langle u_i, a_i\rangle$, $u_i = a_i\ \{i{:}\ 1, \dots, n\}$. This means that b_j is the jth largest $u_i = a_i$ value.

In the case WA it may obtained from IOWA when OWA pairs are defined as $\langle -i, a_i\rangle$. This means the order inducing variable u_i is the negation of the index of an argument.

For this thesis work, to combine WA and OWA the WA weighting vector would be used as the order inducing variable. This means partial concordance values of criteria are ordered according to the criteria weights before applying the OWA weights in aggregating.

3.5 Summary and discussion

This chapter provided an insight into aggregation operators and their use in the decision aiding process. We highlight the formulation of a popular group of aggregation operators called averaging operators, focusing on weighted averaging operators.

Furthermore, we present the family of weighted OWA operators needed in this thesis work. We point out the reason for choosing IOWA, OWAWA and WOWA and show how they combine the weighted averaging (WA) operator and the ordered weighted averaging (OWA) operator. Finally, we describe in detail the formulation of IOWA, OWAWA, and WOWA operators.

4 Proposed aggregation method for concordance in ELECTRE and implementation

The focus of this thesis work is the modification of the overall concordance calculation in the ELECTRE decision aiding technique. There are three variations of this modification, which include changing the aggregation of partial concordances by substituting the WA operator for three operators from the OWA family of operators. OWAWA, IOWA and WOWA as explained in the previous chapter are the candidates for this modification. In this chapter we formalize the proposed aggregation methods, its implementation and the tools used in the implementation.

4.1 Using weighted OWA in overall concordance calculation

Some previous works have considered a modification of the way that overall concordance is calculated in ELECTRE in different situations. The paper [17] looks at a situation where the extent to which a criterion surpasses the preference threshold can be reflected in a change in the importance of that criterion in the concordance calculation. In [5] the concordance index is modified to take into consideration three possible interactions between the criteria that modify each joint importance: mutual strengthening, mutual weakening and antagonistic. In both cases, the weights are modified but the overall *concordance index* for each pair a, b is calculated as the weighted average of the partial concordances indices.

Having $C = \{c_j(a,b)\}, j = 1..n$:

$$c\ (a,b) = WA(C) \qquad (27)$$

In this master thesis we propose the substitution of the WA operator by a weighted OWA operator, presented in chapter 3. The first proposal is using OWAWA operator that linearly combines both the result of WA and the result of OWA. In this case, the parameter beta must be defined by the user. This parameter allows to base the result most on the criteria importance weights or on the and/or weights.

$$c\ (a, b) = OWAWA_{\beta}(C) \qquad (28)$$

The second proposal consists in using IOWA operator with the criteria importance V used as order-inducing variable. In this case, the values provided by the most important criteria will the ones assigned to the first weights of the OWA vector W.

$$c\ (a, b) = IOWA(< V, C >) \qquad (29)$$

The third approach uses the WOWA operator which generates a new weighting vector from the V and W.

$$c\ (a, b) = WOWA(C) \qquad (30)$$

The use of these combined aggregation operators opens a new view of the concept of Overall Concordance in the ELECTRE method. Partial concordances represent the positive vote of each criterion in favor of the assertion "alternative a outranks alternative b". The overall concordance has to give a value to the assertion "a majority of criteria is voting positively and with high strength". WA operator permits to give different voting power to each criterion but the notion of simultaneity and conjunctiveness is not possible to be represented with WA. Therefore, these new proposals open the possibility of establishing the degree of simultaneity in the definition of "majority of criteria with high strength".

4.2 Tools

This sub-section contains few tools used in implementing and testing the proposed system described in the previous sub-section. The work is implemented by modification of the ELECTRE concordance program provided by the diviz workbench that is being developed by the European MCDA community.

4.2.1 Diviz workbench

Diviz is a free open source software tool used to design, execute and share complex MCDA algorithms and experiments. It was derived from the open source BioSide software, and is aimed to be a useful tool for researchers with interest in building workflows for decision aiding experiments, but also for academics who can use the system in their lectures for teaching. Diviz is a brainchild of the decision deck project, a French nonprofit association aimed at developing open source programs implementing MCDA algorithms. Decision deck was formally established on November 2008, and also developed XMCDA, XMCDA web services and GIS.

XMCDA is an important part of the diviz workbench. All input and output files are written in XMCDA. XMCDA is a data standard which allows to represent Multi-Criteria Decision Analysis (MCDA) data elements in XML according to a clearly defined grammar. XMCDA is an instance of UMCDA-ML, which is the Universal Multi-Criteria Decision Analysis Modelling Language and which is one of the scientific initiatives inside the Decision Deck project. UMCDA-ML is intended to be a universal modelling language to express MCDA concepts and generic decision aid processes.

XCMDA goals are to ease:

- The interaction of different MCDA algorithms.
- The execution of various algorithms on the same problem instance.

- The visual representation of MCDA concepts and data structures via standard tools like web browsers.

There are several algorithms implemented in the diviz workbench provided by several researching bodies including; ITAKA research group at the URV, kappalab etc. Diviz tools are aimed to help practitioners who use MCDA tools to support decision makers in real world problems, teachers with need for MCDA tools in coursework, researchers who want to test, compare or develop methods. Some features of diviz workbench are:

- It possesses a workspace where modules and input files can be interconnected.
- Programs are represented as drag and drop modules which can be place on the workspace and linked to other modules to provide unique results.
- It may have several active workspaces with different unique workflows, and several execution instances of variations of these workflows.
- All computations are made on remote servers over the web and output files are saved on the local computer.
- Constructed workflows may be deployed as new web services which is available to all diviz users.

Information and latest releases of the diviz workbench and other decision deck products can be found at: https://www.diviz.org

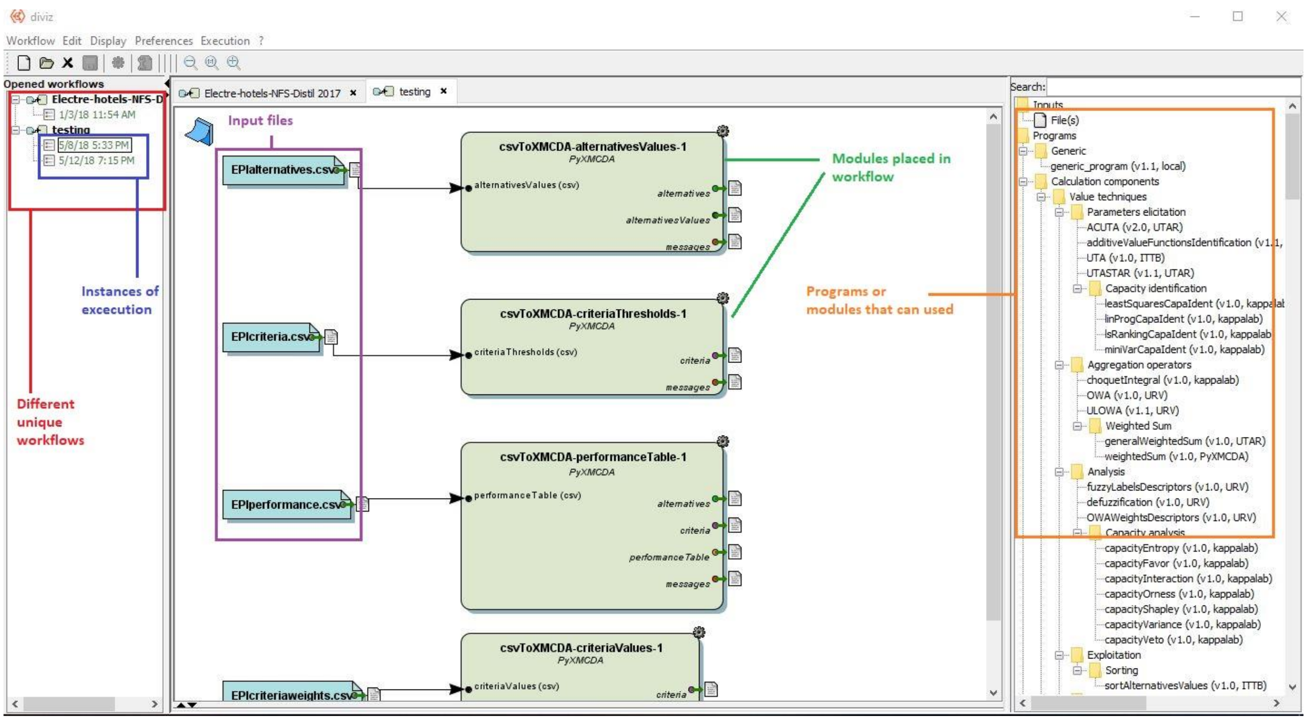

Figure 1: Diviz workbench interface

As said, it is possible to upload new modules to diviz platform, but they must be firstly approved by the consortium. They usually require that the methods implemented in the modules have been previously published in specialized journals of high impact. Therefore, for this master thesis purposes, the diviz user interface will not be used because we cannot upload our new modules as they are just being designed and tested. Although diviz will be used for some data preparation purposes, we will run our programs locally through a command line interface (CLI).

4.2.2 Python IDLE

The programs are written in python version 2. It is necessary to use an editor to organize code when programming. The python IDLE is python's stock editor. IDLE stands for integrated development and learning environment. It was initially released on December 22, 1998 and is packaged for both Linux and Windows platforms.

The IDLE has a simple interface, is beginner friendly and suitable for educational environments. The main features of IDLE includes:

- Multi-window text editor with syntax highlighting, auto-completion, smart indent and other.
- Python shell with syntax highlighting.
- Integrated debugger with stepping, persistent breakpoints, and call stack visibility.

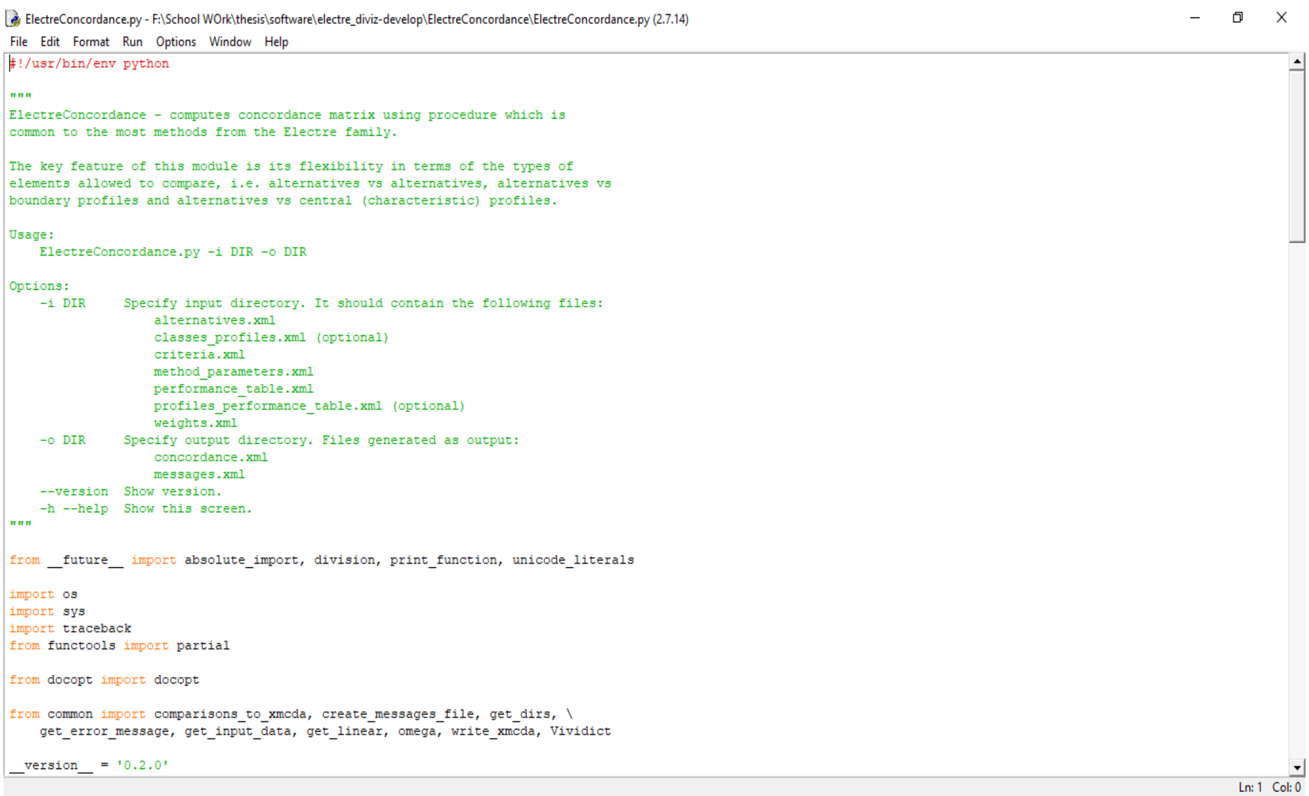

Figure 2: Python IDLE interface

4.2.3 Windows PowerShell

In this thesis work the programs are scripts and need to be run from a command line interface (CLI). Windows PowerShell is task automation and configuration management framework, it consists a command line shell and a scripting language. It was first released on November 14, 2006 and is package in the latest versions of the windows operating system. The popular command prompt (cmd) can be used instead of the PowerShell.

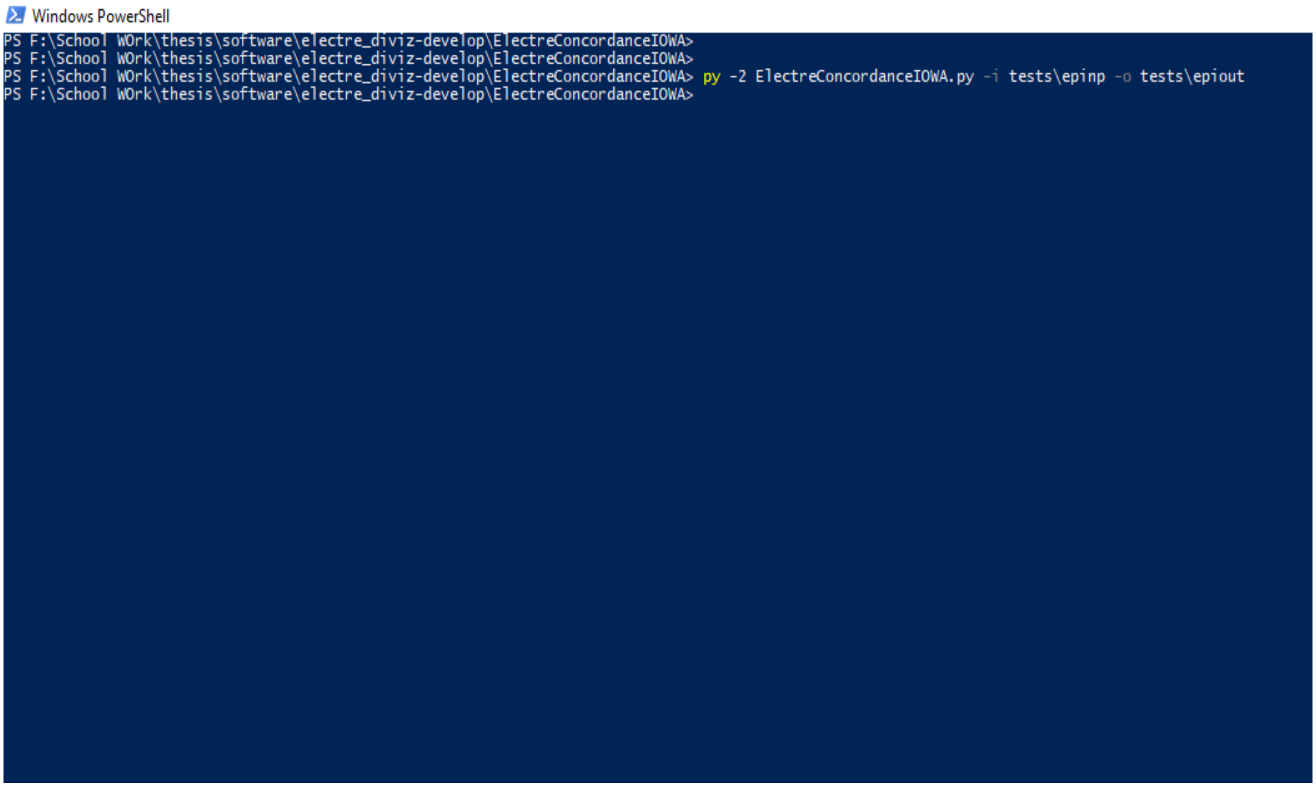

Figure 3: PowerShell – Command Line Interface (CLI)

4.3 Implementation

The implementation done is prepared to be integrated as new modules into the diviz workbench in the future. Therefore, they are implemented as webservices using the XMCDA language for the input and output data. In that way, the new modules can work together with the rest of components of diviz.

4.3.1 ELECTRE workflow in diviz

The ELECTRE workflow in diviz is built my adding modules to calculate concordance, discordance and credibility. Figure 4 below shows a sample ELECTRE workflow with a cutRelationCrisp module to provide a stricter outranking relation and NFS exploitation. Only the concordance module is substituted in this workflow with the implementation of weighted OWA operators.

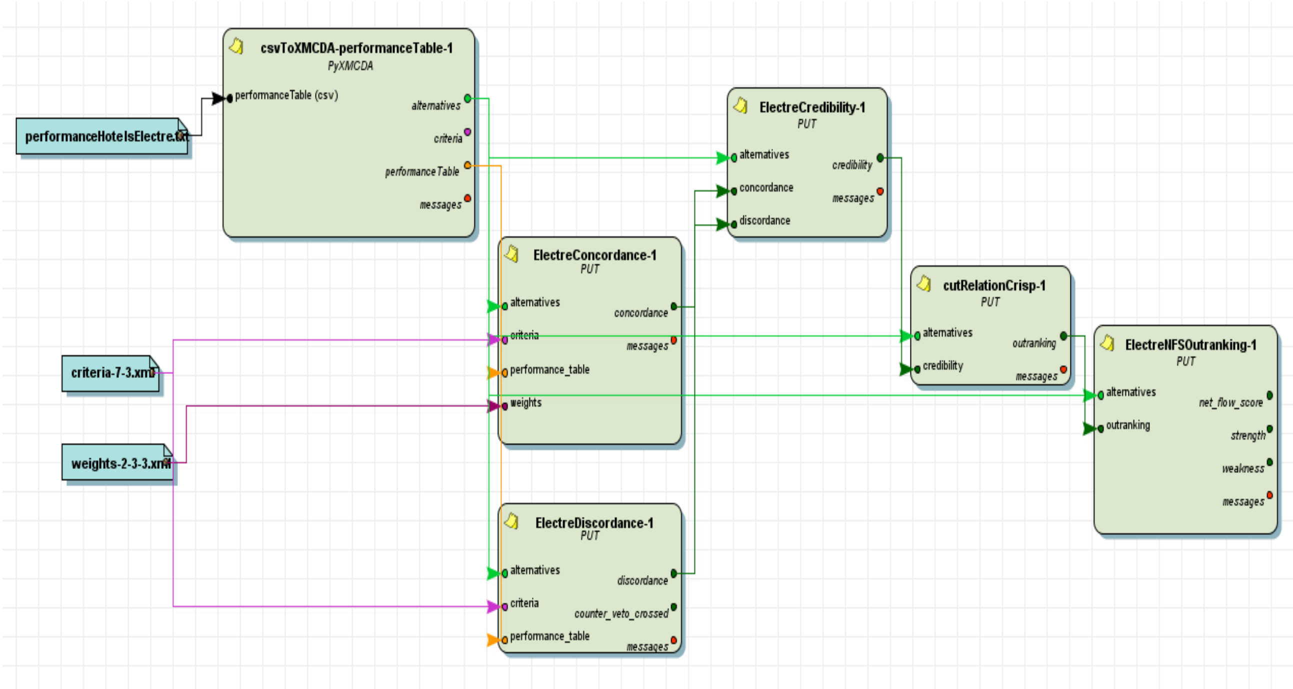

Figure 4: ELECTRE workflow with NFS

The software implementation of the proposed aggregation methods, is based on the ELECTRE concordance program already in use in the diviz workbench. As calculation of partial concordances is the same in the conventional ELECTRE method and in the proposed method, the programs written in this work only modify the ELECTRE concordance calculation in use in diviz.

The ELECTRE concordance source files are obtained from the github address of its author(s): https://github.com/xor-xor/electre_diviz

4.3.2 Software structure and data files

The software structure in this thesis work follows the ElectreConcordance program in the diviz workbench with exception of the method parameters input file which is integrated in the module and not visible in the workflow, also the OWA weights input file needed by the weighted OWA operators.

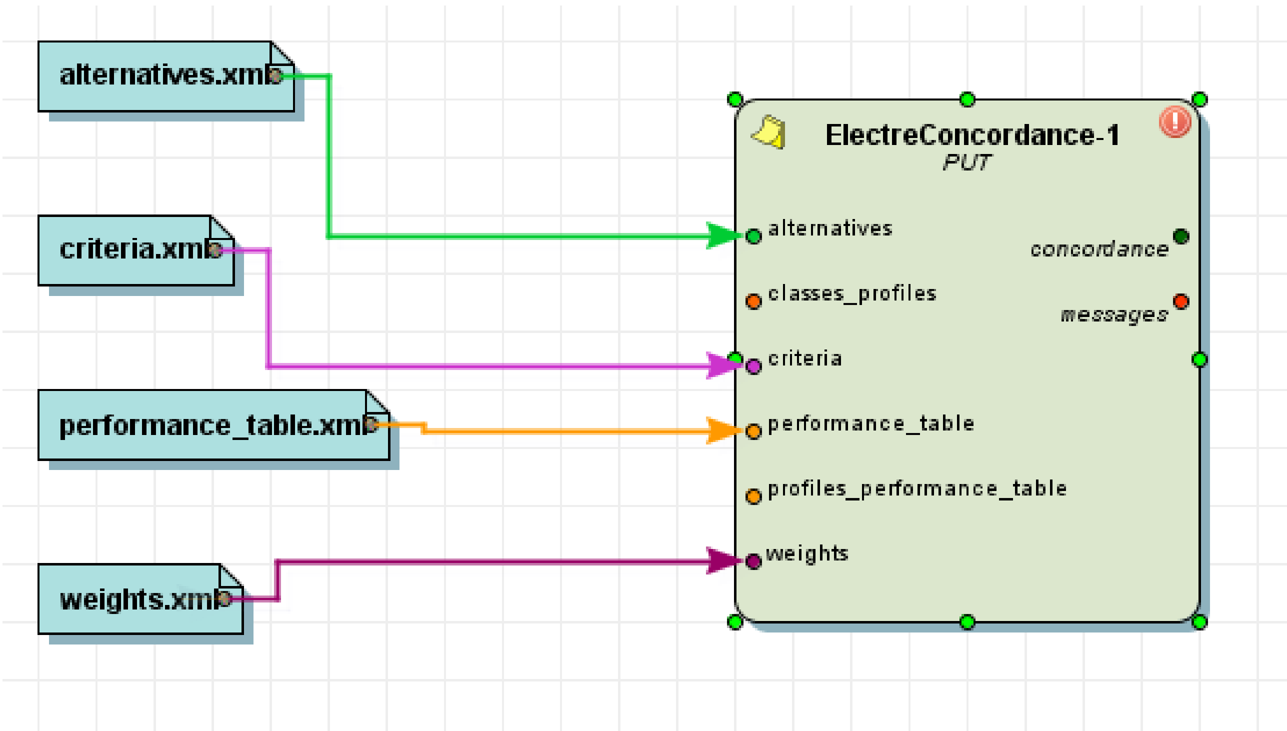

Figure 5: ELECTRE concordance program structure

The structure consists of 4 input files and 2 output files, and the main goal is the overall concordance index. All files are written in XMCDA and provide information for the program or results obtained after calculations.

Physically typing input files in XMCDA could present some errors and mistakes. Unique tag names and information must be used properly, also experiments with large number of alternatives, criteria and performance scores would take up a lot of time when typing. For this purpose, some author(s) developed some diviz programs to convert input files written in comma separated values (.csv) to XMCDA files (.xml).

These csv files can be easily created with MS Excel and then convert to XMCDA. Some of these programs relevant to this work are listed below:

- ***csvToXMCDA-alternativesValues*:** This program creates XMCDA file containing a list of alternatives to be used in the decision making process provided with a proper csv file.
- ***csvToXMCDA-criteriaThresholds*:** Creates XMCDA file containing criteria and their thresholds values. Including indifference, preference and veto thresholds.
- ***csvToXMCDA-criteriaValues*:** Creates XMCDA file containing criteria and their weight values.
- ***csvToXMCDA-performanceTable*:** Creates XMCDA files for criteria, alternatives and performance table. Performance table file contains alternatives and their performance with respect to criteria involved in the decision making process.

These programs listed above are used to create the input files used in this thesis work. To implement the proposed aggregation operators, we need to introduce some more input files with additional information. Three different programs are built for each aggregation operator, and they have varying input files. Figure 6, 7 and 8 below shows the new input files. OWAWA and IOWA require identical input files, while WOWA does not require an OWA weights file, because it supplies the OWA weights used in the other implementations.

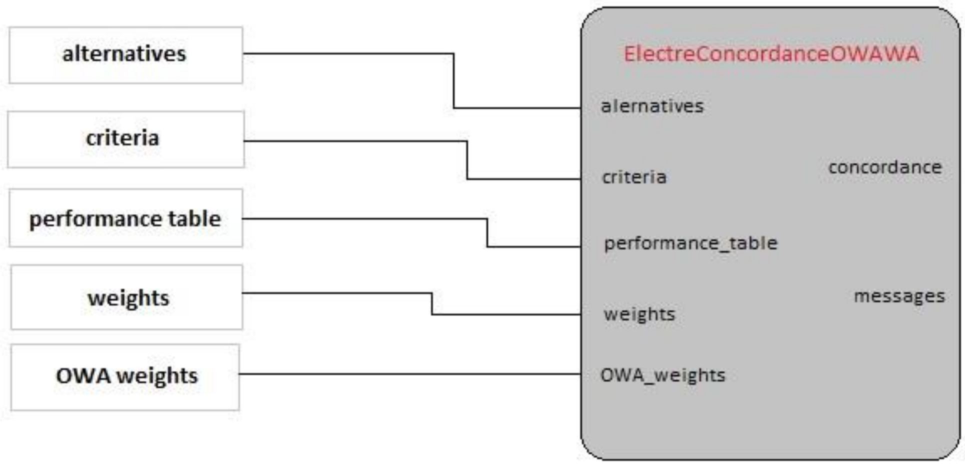

Figure 6: Proposed OWAWA program structure

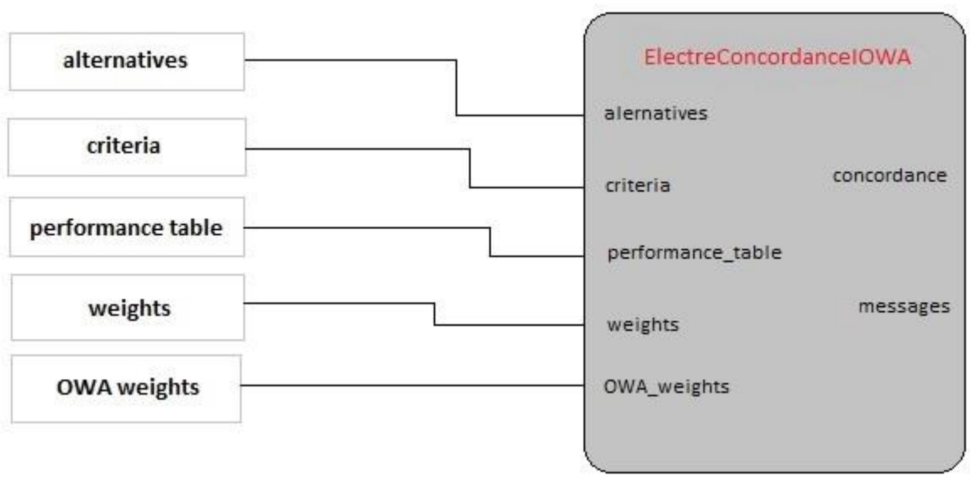

Figure 7: Proposed IOWA program structure

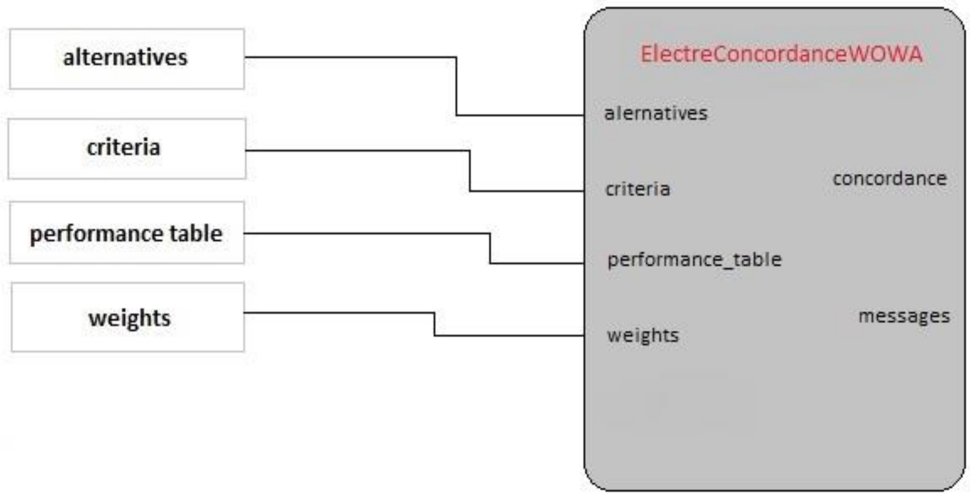

Figure 8: Proposed WOWA program structure

4.3.2.1 Input files

alternatives.xml: In this file alternatives to be analyzed are listed and defined. Below we have a sample of 3 countries defined as alternatives.

```
<alternatives>
  <alternative id="Afghanistan">
    <active>true</active>
  </alternative>
  <alternative id="Laos">
    <active>true</active>
  </alternative>
  <alternative id="Mongolia">
    <active>true</active>
  </alternative>
   ...
</alternatives>
```

criteria.xml: This contains information about criteria needed in the ELECTRE technique. It contains the criteria ID, preference direction of each criteria (if the DM wishes to maximize or minimize), preference threshold value, indifference threshold value and veto value. Below we have a sample of a criteria.xml file.

```xml
<criteria>
   <criterion id="AirQuality">
     <scale>
       <quantitative>
         <preferenceDirection>max</preferenceDirection>
       </quantitative>
     </scale>
     <thresholds>
       <threshold mcdaConcept="ind">
         <constant>
           <real>0.0</real>
         </constant>
       </threshold>
       <threshold mcdaConcept="pref">
         <constant>
           <real>5.0</real>
         </constant>
       </threshold>
       <threshold mcdaConcept="veto">
         <constant>
           <real>30.0</real>
         </constant>
       </threshold>
     </thresholds>
   </criterion>
   ...
</criteria>
```

method_parameters.xml: The method parameters file holds some parameter values required by the method, these values are not provided as input files but as input boxes, dropdowns or check boxes among the settings of a method module. The parameters differ with each method. In the case of this work, parameters like the OWAWA beta value, the WOWA policies, and the IOWA operator selection are all provided in the method parameter files. Below we have samples of OWAWA, IOWA and WOWA method parameters.

OWAWA: The comparison_with parameter indicates what is being compared alternatives or boundary profiles. The beta value is a real number in range [0, 1].

```xml
<methodParameters>
    <parameter name="comparison_with">
      <value>
        <label>alternatives</label>
      </value>
    </parameter>
    <parameter name="beta_value">
      <value>
        <real>1.0</real>
      </value>
    </parameter>
```

```
</methodParameters>
```

WOWA: The aggregation type here chooses from options WA, OWA, and WOWA. For aggregation policy the following options are possible:

- Low argument high importance (conjunctive), this means the lower the partial concordance the higher the weight.
- High argument high importance (disjunctive), this means the higher the partial concordance the higher the weight.
- High importance to centrals, this means after ordering, partial concordances in the center have higher weights than extremes.
- High importance to extremes, this means after ordering, partial concordances in the center have lesser weights than extremes.

```
<methodParameters>
       <parameter name="comparison_with">
         <value>
           <label>alternatives</label>
         </value>
       </parameter>
       <parameter name="aggregation_type">
         <value>
           <label>WOWA</label>
         </value>
       </parameter>
       <parameter name="aggregation_policy">
         <value>
           <label>low_argument_high_importance</label>
         </value>
       </parameter>
     </methodParameters>
```

IOWA: Just as in WOWA aggregation type include WA, OWA and IOWA.

```
<methodParameters>
  <parameter name="comparison_with">
    <value>
      <label>alternatives</label>
    </value>
  </parameter>
  <parameter name="aggregation_type">
    <value>
      <label>IOWA</label>
```

```
        </value>
      </parameter>
    </methodParameters>
```

performance_table.xml: This file holds performance values of each alternative with respect to each criterion. Below is a sample of the performance table where, Afghanistan is an alternative and AirQuality, WaterSanitation and HeavyMetals are criteria.

```
<performanceTable>
    <alternativePerformances>
      <alternativeID>Afghanistan</alternativeID>
      <performance>
        <criterionID>AirQuality</criterionID>
        <value>
          <real>44.67</real>
        </value>
      </performance>
      <performance>
        <criterionID>WaterSanitation</criterionID>
        <value>
          <real>25.75</real>
        </value>
      </performance>
      <performance>
        <criterionID>HeavyMetals</criterionID>
        <value>
          <real>0.0</real>
        </value>
      </performance>
      ...
    </alternativePerformances>
  </performanceTable>
```

weights.xml: This file contains the criteria weights assigned by the DM to represent importance or voting power of different criteria. Below is a sample of the weights input file.

```
 <criteriaValues mcdaConcept="Importance" name="significance">
    <criterionValue>
      <criterionID>AirQuality</criterionID>
      <value>
        <real>0.275</real>
      </value>
    </criterionValue>
    <criterionValue>
      <criterionID>WaterSanitation</criterionID>
      <value>
        <real>0.135</real>
      </value>
    </criterionValue>
   </criterionValue>
```

```
</criteriaValues>
```

weightsOWA.xml: This file contains OWA weights generated by the WOWA operator, establishing certain weighting policies. Below is a sample of this input file.

```
<alternativesValues>
      <alternativeValue>
        <values>
            <value>
                <real>0.66</real>
            </value>
            <value>
                <real>0.10</real>
            </value>
            <value>
                <real>0.06</real>
            </value>
            <value>
                <real>0.05</real>
            </value>
            <value>
                <real>0.04</real>
            </value>
            <value>
                <real>0.03</real>
            </value>
            <value>
                <real>0.03</real>
            </value>
            <value>
                <real>0.03</real>
            </value>
        </values>
    </alternativeValue>
</alternativesValues>
```

4.3.2.2 Output files

concordance.xml: This is the main output file, and it contains the results of concordance calculations. The file is outlined with pairwise comparisons of all alternatives and the respective concordance value. Below is a sample of the concordance file.

```
<alternativesComparisons>
  <pairs>
    <pair>
      <initial>
        <alternativeID>Afghanistan</alternativeID>
      </initial>
      <terminal>
```

```
          <alternativeID>Afghanistan</alternativeID>
        </terminal>
        <value>
          <real>1.0</real>
        </value>
      </pair>
      <pair>
        <initial>
          <alternativeID>Afghanistan</alternativeID>
        </initial>
        <terminal>
          <alternativeID>Laos</alternativeID>
        </terminal>
        <value>
          <real>0.49264</real>
        </value>
      </pair>
      <pair>
        <initial>
          <alternativeID>Laos</alternativeID>
        </initial>
        <terminal>
          <alternativeID>Afghanistan</alternativeID>
        </terminal>
        <value>
          <real>0.62</real>
        </value>
      </pair>
      <pair>
        <initial>
          <alternativeID>Laos</alternativeID>
        </initial>
        <terminal>
          <alternativeID>Laos</alternativeID>
        </terminal>
        <value>
          <real>1.0</real>
        </value>
      </pair>
    </pairs>
  </alternativesComparisons>
```

messages.xml: This acts an error log file, it has nothing to do with the concordance calculations. It carries information on the status of the execution, if it was a good execution or if an error occurred. Below is a sample of the messages file after a successful execution.

```
  <xmcda:XMCDA xmlns:xmcda='http://www.decision-deck.org/2012/XMCDA-2.2.0'
    xmlns:xsi='http://www.w3.org/2001/XMLSchema-instance'
    xsi:schemaLocation='http://www.decision-deck.org/2012/XMCDA-2.2.0
http://www.decision-deck.org/xmcda/_downloads/XMCDA-2.2.0.xsd'>
  <methodMessages>
    <logMessage>
      <text><![CDATA[Everything OK.]]></text>
```

```
  &lt;/logMessage&gt;
&lt;/methodMessages&gt;
&lt;/xmcda:XMCDA&gt;
```

4.3.3 Algorithms

In this sub-section we present the program algorithms implemented in this work. The algorithms follow the explanations of the aggregation operators provided in chapter 3 with a few changes discussed below.

- The OWA weights are defined by the policies established in the WOWA operator implementation, this is necessary to allow comparison of all three operators. Instead of defining a specific OWA weighting vector, the weights are extracted from the WOWA implementation and use to create the OWA weights input file needed by IOWA and OWAWA.
- In the case of WOWA, this work implements regular monotonically non-decreasing quantifiers $Q(x)$ instead of interpolating between the OWA weights and criteria weights to create the new weighting vector. This means that only the criteria weighting vector is defined, and the quantifiers are used to established OWA policies in particular conjunctive and disjunctive polices. In experiment 1 and 2: $Q(x) = x^2$ is used for conjunctive and $Q(x) = \sqrt{x}$ for disjunctive, while in experiment 3: $Q(x) = x^5$ is used for conjunctive and $Q(x) = \sqrt[5]{x} = x^{1/5}$ is used for disjunctive.
- Finally, in the case of IOWA the criteria weights are used as the order inducing variable thereby combining WA and OWA in the implementation.

4.3.3.1 *Algorithm for concordance calculation with the OWAWA operator*

// Initialize elements:

Get partial concordances $c_i(a, b)$;

Get criteria weights (*v*);

Get OWA weights (*w*);

Get beta value;

// Calculate overall concordance using weighted averaging with *v*:

For all partial concordances $c_i(a, b)$ do

Multiply partial concordance by respective criterion weight;

Overall concordance (using *v*) = Sum all multiplications in the previous step;

// Calculate overall concordance using ordered weighted averaging with *w*:

Sort all partial concordances $c_i(a, b)$ in ascending/decreasing order;

Multiply partial concordance by OWA weight according their order in both array;

Overall concordance (using *w*) = Sum of all multiplications in the previous step;

// Calculate OWAWA overall concordance:

OWAWA overall concordance = [Overall concordance (using *v*) * (1 – beta value)] +

[Overall concordance (using *w*) * beta value]

4.3.3.2 Algorithm for concordance calculation with the IOWA operator

```
// Initialize elements:
    Get partial concordances c_i(a,b);
    Get criteria weights (v);
    Get OWA weights (w);
    Get aggregation type;
// Calculate overall concordance:
    If aggregation type is WA then
        Set order inducing variable (u) to the negative value of respective indices of each partial
      concordance;
        Create OWA pairs < u, c_i(a,b) >;
        Sort OWA pairs using u in ascending/decreasing order;
        Multiply sorted partial concordance by criteria weight according their order;
        Overall concordance = Sum of all multiplications in the previous step;
    Else if aggregation type is OWA then
        Set order inducing variable (u) to the partial concordance values;
        Create OWA pairs < u, c_i(a,b) >;
        Sort OWA pairs using u in ascending/decreasing order;
        Multiply sorted partial concordance by OWA weight according their order;
        Overall concordance = Sum of all multiplications in the previous step;
    Else if aggregation type is IOWA then
        Set order inducing variable (u) to criteria weights (v);
        Create OWA pairs < u, c_i(a,b) >;
        Sort OWA pairs using u in ascending/decreasing order;
        If a tie occurs between two or more order inducing variables in OWA pairs then
            Replace the partial concordances c_i(a,b) of each tied value with their average (i.e.
          [c_i(a,b) + c_j(a,b) + c_k(a,b) + ...]/n where n is the number of tied values);
        End if
        Multiply sorted partial concordance by OWA weight according their order;
        Overall concordance = Sum of all multiplications in the previous step;
    End if
```

4.3.3.3 Algorithm for concordance calculation with WOWA operator

// Initialize elements:

Get partial concordances $c_i(a, b)$;

Get criteria weights (v);

Get aggregation type;

Get aggregation policy;

// Calculate overall concordance:

<u>If</u> aggregation type is WA <u>then</u>

Calculate new weighting vector (ω) using the equation

$\omega_i = w^*\left(\sum_{j\leq i} v_{\sigma(j)}\right) - w^*\left(\sum_{j<i} v_{\sigma(j)}\right)$ {In this case w^* is a regular monotonically non-decreasing quantifier $Q(x)$ where $Q(x) = x$};

Sort partial concordances $c_i(a, b)$ in ascending/decreasing order;

Multiply sorted partial concordance by new weighting vector (ω) according their order;

Overall concordance = Sum of all multiplications in the previous step;

<u>Else if</u> aggregation type is OWA <u>then</u>

Set each criteria weight (v_i) to $v_i = 1/n$ where n is the total number of criteria weights;

Calculate new weighting vector (ω) using the equation

$\omega_i = w^*\left(\sum_{j\leq i} v_{\sigma(j)}\right) - w^*\left(\sum_{j<i} v_{\sigma(j)}\right)$ {In this case w^* is a regular monotonically non-decreasing quantifier $Q(x)$ where $Q(x)$ depends on the policy};

Sort partial concordances $c_i(a, b)$ in ascending/decreasing order;

Multiply sorted partial concordance by new weighting vector (ω) according their order;

Overall concordance = Sum of all multiplications in the previous step;

<u>Else if</u> aggregation type is OWA <u>then</u>

Calculate new weighting vector (ω) using the equation

$\omega_i = w^*\left(\sum_{j\leq i} v_{\sigma(j)}\right) - w^*\left(\sum_{j<i} v_{\sigma(j)}\right)$ {In this case w^* is a regular monotonically non-decreasing quantifier $Q(x)$ where $Q(x)$ depends on the policy};

Sort partial concordances $c_i(a, b)$ in ascending/decreasing order;

Multiply sorted partial concordance by new weighting vector (ω) according their order;

Overall concordance = Sum of all multiplications in the previous step;

<u>End if</u>

4.4 Summary and discussion

In this chapter we described the proposed methodology that seeks to add flexibility to the ELECTRE technique by changing the way overall concordance is calculated. Also, we showed how the family of weighted OWA operators are implemented and integrated into the ELECTRE technique.

Furthermore, we discussed the tools needed to build the proposed system highlighting the diviz workbench and its ELECTRE concordance program modified in this work. Then, we looked at the software structure of proposed system, its input and output files. Finally, we presented the algorithms for calculating concordances using the three weighted OWA operators in focus.

The main characteristics of the proposed system are as follows:

- Input files include: alternatives, criteria, criteria weights, OWA weights, method parameters, performance table.
- Result of the system is overall concordance index; i.e. concordance values of pairwise assertions of all possible alternative combinations.
- Operators IOWA, OWAWA, WOWA are substituted for the conventional aggregation method WA when calculating overall concordance.

5 Experiments

The OWA-based outranking construction proposed has been applied to three different case studies. To evaluate the differences produced by the different operators, we compare the ranking obtained using the Net Flow Score. A minimum credibility of 0.8 in the outranking relation is used in this procedure. The correlation between different rankings is calculated to see how new proposals are able to integrate both sets of weights.

5.1 Finding a hotel

The first case study poses the problem of making lodging arrangements to attend a congress in Jyväskylä (Finland). The DM wishes to make a choice from six hotel alternatives all in proximity to the congress site. The choice will be made based on the criteria and weights listed below.

The data used in this case study is given in Table 1 and Table 2. Six hotels have been evaluated using 6 criteria: **C01-** Distance to the congress site, **C02-** Distance to the city center, **C03-** Sports facilities, **C04-** Restaurants available, **C05-** Category and **C06-** Services provided (wifi, laundry, etc.). The first two criteria are minimized (-) and the rest are maximized (+).

	C01-	**C02-**	**C03+**	**C04+**	**C05+**	**C06+**
Alexandra	1600.0	300.0	2.0	3.0	4.0	5.0
Sokos	1700.0	400.0	2.0	2.0	4.0	5.0
Cumulus	1700.0	550.0	4.0	0.0	3.0	3.0
Scandic	600.0	350.0	3.0	2.0	4.0	2.0
Kampus	1550.0	610.0	4.0	0.0	3.0	2.0
Alba	110.0	1300.0	1.0	1.0	3.0	4.0

Table 1: Hotels performance table

	C01	C02	C03	C04	C05	C06
Indifference	200.0	100.0	0.0	0.0	0.0	1.0
Preference	700.0	300.0	1.0	1.0	0.0	1.0
Weight	0.1	0.3	0.3	0.05	0.15	0.1

Table 2: Criteria parameters

Two sets of OWA weights have been considered for this study: a disjunctive policy with w_d = (0.408, 0.169, 0.130, 0.109, 0.096, 0.088) and a conjunctive policy with weights w_c = (0.028, 0.083, 0.139, 0.194, 0.25, 0.306). These weights were obtained from the use of a regular monotonic non-decreasing quantifier, as proposed in [17]. To establish the disjunctive policy, the quantifier $Q(x) = \sqrt{x}$ is used, while for the conjunctive policy $Q(x) = x^2$. The OWA weights calculation is done by using the quantifiers listed above as the monotonically increasing function w^* as shown in Eq. 23.

Next tables show the overall concordance values obtained with the three combined operators proposed in this paper to merge the partial concordance indices. Also, concordance values of weighted averaging and ordered weighted averaging operators are included as reference.

	Alexandra	Sokos	Cumulus	Scandic	Kampus	Alba
Alexandra	1.0	1.0	0.7	0.6	0.7	0.9
Sokos	0.95	1.0	0.7	0.6	0.7	0.9
Cumulus	0.475	0.625	1.0	0.55	1.0	0.85
Scandic	0.85	0.9	0.7	1.0	0.7	0.842
Kampus	0.4	0.535	1.0	0.46	1.0	0.75
Alba	0.2	0.2	0.4	0.2	0.4	1.0

Table 3: Outranking values with weighted average (WA)

For OWAWA, three values of β have been tested, $\beta = 0.3,\ \beta = 0.5,\ \beta = 0.7$. OWAWA is a linear combination of WA and OWA so lower β values should have high correlation with WA and vice versa. In orange, concordances higher than 0.8 are highlighted, as they are the ones used in the NFS ranking procedure. For conjunctive policies, we have observed that the values of the outranking matrix are much lower than with the rest, finding very few values above the threshold of 0.8 and leading to rankings with many ties.

	Alexandra	**Sokos**	**Cumulus**	**Scandic**	**Kampus**	**Alba**
Alexandra	1.0	1.0	0.912	0.816	0.912	0.912
Sokos	0.912	1.0	0.912	0.816	0.912	0.912
Cumulus	0.6095	0.6745	1.0	0.642	1.0	0.816
Scandic	0.816	0.912	0.912	1.0	0.912	0.85632
Kampus	0.577	0.6355	1.0	0.603	1.0	0.707
Alba	0.577	0.577	0.816	0.577	0.816	1.0

Table 4: Outranking values with OWA disjunctive (OWAd)

	Alexandra	**Sokos**	**Cumulus**	**Scandic**	**Kampus**	**Alba**
Alexandra	1.0	1.0	0.694	0.444	0.694	0.694
Sokos	0.694	1.0	0.694	0.444	0.694	0.694
Cumulus	0.14575	0.21525	1.0	0.1805	1.0	0.444
Scandic	0.444	0.694	0.694	1.0	0.694	0.549
Kampus	0.111	0.17355	1.0	0.1388	1.0	0.25
Alba	0.111	0.111	0.444	0.111	0.444	1.0

Table 5: Outranking values with OWA conjunctive (OWAc)

	Alexandra	**Sokos**	**Cumulus**	**Scandic**	**Kampus**	**Alba**
Alexandra	1.0	1.0	0.7636	0.6648	0.7636	0.9036
Sokos	0.9386	1.0	0.7636	0.6648	0.7636	0.9036
Cumulus	0.51535	0.63985	1.0	0.5776	1.0	0.8398
Scandic	0.8398	0.9036	0.7636	1.0	0.7636	0.846296
Kampus	0.4531	0.56515	1.0	0.5029	1.0	0.7371
Alba	0.3131	0.3131	0.5248	0.3131	0.5248	1.0

Table 6: Outranking values with OWAWA beta=0.3 disjunctive (OWAWA.3d)

	Alexandra	**Sokos**	**Cumulus**	**Scandic**	**Kampus**	**Alba**
Alexandra	1.0	1.0	0.6982	0.5532	0.6982	0.8382
Sokos	0.8732	1.0	0.6982	0.5532	0.6982	0.8382
Cumulus	0.376225	0.502075	1.0	0.43915	1.0	0.7282
Scandic	0.7282	0.8382	0.6982	1.0	0.6982	0.7541
Kampus	0.3133	0.426565	1.0	0.36364	1.0	0.6
Alba	0.1733	0.1733	0.4132	0.1733	0.4132	1.0

Table 7: Outranking values with OWAWA beta=0.3 conjunctive (OWAWA.3c)

	Alexandra	Sokos	Cumulus	Scandic	Kampus	Alba
Alexandra	1.0	1.0	0.806	0.708	0.806	0.906
Sokos	0.936	1.0	0.806	0.708	0.806	0.906
Cumulus	0.54225	0.64975	1.0	0.596	1.0	0.833
Scandic	0.833	0.906	0.806	1.0	0.806	0.84916
Kampus	0.4885	0.58525	1.0	0.5315	1.0	0.7285
Alba	0.3885	0.3885	0.608	0.3885	0.608	1.0

Table 8: Outranking values with OWAWA beta=0.5 disjunctive (OWAWA.5d)

	Alexandra	Sokos	Cumulus	Scandic	Kampus	Alba
Alexandra	1.0	1.0	0.697	0.522	0.697	0.797
Sokos	0.822	1.0	0.697	0.522	0.697	0.797
Cumulus	0.310375	0.420125	1.0	0.36525	1.0	0.647
Scandic	0.647	0.797	0.697	1.0	0.697	0.6955
Kampus	0.2555	0.354275	1.0	0.2994	1.0	0.5
Alba	0.1555	0.1555	0.422	0.1555	0.422	1.0

Table 9: Outranking values with OWAWA beta=0.5 conjunctive (OWAWA.5c)

	Alexandra	Sokos	Cumulus	Scandic	Kampus	Alba
Alexandra	1.0	1.0	0.8484	0.7512	0.8484	0.9084
Sokos	0.9234	1.0	0.8484	0.7512	0.8484	0.9084
Cumulus	0.56915	0.65965	1.0	0.6144	1.0	0.8262
Scandic	0.8262	0.9084	0.8484	1.0	0.8484	0.852024
Kampus	0.5239	0.60535	1.0	0.5601	1.0	0.7199
Alba	0.4639	0.4639	0.6912	0.4639	0.6912	1.0

Table 10: Outranking values with OWAWA beta=0.7 disjunctive (OWAWA.7d)

	Alexandra	Sokos	Cumulus	Scandic	Kampus	Alba
Alexandra	1.0	1.0	0.6958	0.4908	0.6958	0.7558
Sokos	0.7708	1.0	0.6958	0.4908	0.6958	0.7558
Cumulus	0.244525	0.338175	1.0	0.29135	1.0	0.5658
Scandic	0.5658	0.7558	0.6958	1.0	0.6958	0.6369
Kampus	0.1977	0.281985	1.0	0.23516	1.0	0.4
Alba	0.1377	0.1377	0.4308	0.1377	0.4308	1.0

Table 11: Outranking values with OWAWA beta=0.7 conjunctive (OWAWA.7c)

	Alexandra	Sokos	Cumulus	Scandic	Kampus	Alba
Alexandra	1.0	1.0	0.7115	0.609	0.7115	0.8975
Sokos	0.912	1.0	0.7115	0.609	0.7115	0.8975
Cumulus	0.463125	0.607375	1.0	0.53525	1.0	0.8095
Scandic	0.8095	0.8975	0.7115	1.0	0.7115	0.83805
Kampus	0.391	0.520825	1.0	0.4487	1.0	0.707
Alba	0.205	0.205	0.423	0.205	0.423	1.0

Table 12: Outranking values with IOWA disjunctive (IOWAd)

	Alexandra	Sokos	Cumulus	Scandic	Kampus	Alba
Alexandra	1.0	1.0	0.9445	0.7225	0.9445	0.778
Sokos	0.694	1.0	0.9445	0.7225	0.9445	0.778
Cumulus	0.291375	0.319125	1.0	0.30525	1.0	0.472
Scandic	0.472	0.778	0.9445	1.0	0.9445	0.64924
Kampus	0.2775	0.302475	1.0	0.2886	1.0	0.25
Alba	0.444	0.444	0.889	0.444	0.889	1.0

Table 13: Outranking values with IOWA conjunctive (IOWAc)

	Alexandra	Sokos	Cumulus	Scandic	Kampus	Alba
Alexandra	1.0	1.0	0.83666	0.7746	0.83666	0.94868
Sokos	0.97467	1.0	0.83666	0.7746	0.83666	0.94868
Cumulus	0.68351	0.78561	1.0	0.73455	1.0	0.92195
Scandic	0.92195	0.94869	0.83666	1.0	0.83666	0.9172092
Kampus	0.63246	0.72435	1.0	0.67329	1.0	0.86602
Alba	0.44722	0.44722	0.63245	0.44722	0.63245	1.0

Table 14. Outranking values with WOWA disjunctive (WOWAd)

The NFS values (Eq. 5) given to each hotel are shown in Table 15. Their ranking positions (1..6) are given in Table 16,with the hotels in the best positions highlighted.

	Alexandra	Sokos	Cumulus	Scandic	Kampus	Alba
WA	0	0	1	3	0	-4
OWAd	3	3	-3	3	-4	-2
OWAc	1	-1	0	0	0	0
OWAWA.3d	0	0	1	3	0	-4
OWAWA.3c	1	0	0	1	0	-2
OWAWA.5d	2	2	-2	5	-3	-4
OWAWA.5c	1	1	0	0	0	-2
OWAWA.7d	2	2	-2	5	-3	-4
OWAWA.7c	1	-1	0	0	0	0
IOWAd	0	0	1	3	0	-4
IOWAc	3	0	-4	3	-4	2
WOWAd	2	2	-2	5	-2	-5
WOWAc	1	0	0	1	0	-2

Table 15: Net Flow Score for each hotel and each method

	Alexandra	Sokos	Cumulus	Scandic	Kampus	Alba
WA	4	4	2	1	4	6
OWAd	2	2	5	2	6	4
OWAc	1	6	3,5	3,5	3,5	3,5
OWAWA.3d	4	4	2	1	4	6
OWAWA.3c	1,5	4	4	1,5	4	6
OWAWA.5d	2,5	2,5	4	1	5	6
OWAWA.5c	1,5	1,5	4	4	4	6
OWAWA.7d	2,5	2,5	4	1	5	6
OWAWA.7c	1	6	3,5	3,5	3,5	3,5
IOWAd	4	4	2	1	4	6
IOWAc	1,5	4	5,5	1,5	5,5	3
WOWAd	2,5	2,5	4,5	1	4,5	6
WOWAc	1,5	4	4	1,5	4	6

Table 16: Rank position of each hotel obtained from NFS

We can see that in most of the cases Alba is in the worst position, although with a conjunctive policy the worst is Sokos. The best position is given to Scandic or Alexandra, sometimes in a tie. Sokos is also in the best position when a disjunctive policy is used. The case of Sokos hotel is quite interesting because its position is the least stable of all hotels.

A look at the criteria weights show criteria C02 and C03 to have a combined 60% of the total importance assigned to criteria, as such hotels like Scandic with good performance values on C02 and C03 have better positions in WA. It can also be observed that Alexandra hotel has no low score in any criterion and some high values, thus it is the winner in case of conjunctive policies. This is because conjunctive policies give more importance to lower scores. It is also worth to notice that Sokos is the worst when using OWA disjunctive, but when including the criteria weights it improves its position, as it is good in C02 and C03, as said before. Thus, the operators balance both weighting vectors. Kampus is in an intermediate position with WA because it has bad scores in non-relevant criteria, but when including the and/or weights, it goes to worst positions because of its low score in C04 and C06.

In order to measure the similarity between the rankings, Table 17 gives the Spearman rho correlation between the 3 results obtained using a single set of weights (WA, OWAd and OWAc) with respect to the use of the two sets of weights together. Table 17 also indicates the operator that gives a highest correlation (most similar ranking, with correlation higher than 0.95) for each of the proposed methods.

	WA	**OWAd**	**OWAc**	**Closest** (>=0.95)
WA	**1,00**	**0,16**	**0,00**	
OWAd	**0,16**	**1,00**	**0,00**	
OWAc	**0,00**	**0,00**	**1,00**	
OWAWA.3d	1,00	0,16	0,00	IOWAd
OWAWA.3c	0,66	0,56	0,46	WOWAc
OWAWA.5d	0,71	0,77	0,00	OWAWA.7d, WOWAd
OWAWA.5c	0,16	0,56	0,00	-
OWAWA.7d	0,71	0,77	0,00	OWAWA.5c, WOWAd
OWAWA.7c	0,00	0,00	1,00	-
IOWAd	1,00	0,16	0,00	OWAWA.3d
IOWAc	0,06	0,81	0,44	-
WOWAd	0,66	0,75	0,00	OWAWA.5d,OWAWA.7d
WOWAc	0,66	0,56	0,46	OWAWA.3c

Table 17: Correlation between the different rankings obtained in dataset Hotels

From the correlations, we can see that the disjunctive policies with OWAWA with low beta and IOWA give similar results to the WA. Rankings similar to OWA with disjunctive weights are obtained with OWAWA also disjunctive and high beta (OWA-like), as expected. Also OWAWA with high beta reproduces the ranking of OWA for the conjunctive case. An interesting observation is that WOWA seems to give a significantly different ranking to all the three basic ones.

5.2 Generating a ranking of universities

The second case study comes from paper [10], with data about British universities from https://www.thecompleteuniversityguide.co.uk/league-tables/rankings a ranking is built. The paper [10] is concerned with applying ELECTRE methods to ranking of universities, allowing the involvement of the user in the ranking process through user defined weights. The large dataset makes it a good case study for this thesis work.

We use the same weights and thresholds than paper [10], but we increased the number of alternatives to 20 universities in order to observe the behavior of the methods in a larger dataset. Five criteria are taken: **C01-** Academic services spend, **C02-** Completion, **C03-** Entry standards, **C04-** Facilities spend, **C05-** Good honors. Again $Q(x) = \sqrt{x}$ and $Q(x) = x^2$ were used to establish the OWA weights, which are disjunctive w_d = (0.447, 0.185, 0.070, 0.053, 0.044), conjunctive w_c = (0.04, 0.12, 0.20, 0.28, 0.36). All criteria in this case are maximized denoted by (+). Table 18 and 19 show the dataset selected for testing.

	C01+	**C02+**	**C03+**	**C04+**	**C05+**
Indifference	140.6	9	40	43.7	3.485
Preference	281.2	18	80	87.4	13.94
Weights	0.1	0.3	0.3	0.2	0.1

Table 18: Criteria parameters for Universities

In Table 19 actual university names are substituted for abbreviated indices of the universities. U1 meaning university 1, going up to U20 to represent all 20 universities.

	C01+	C02+	C03+	C04+	C05+
U1	947	79	400	228	69.7
U2	1406	64	350	204	47.6
U3	677	90	300	349	61.8
U4	561	65	247	188	52.3
U5	1006	88	352	437	65.8
U6	765	77	280	198	55.6
U7	1556	85.9	432	679	82.6
U8	1065	77	379	797	69.9
U9	1409	85	296	531	68.5
U10	1151	78.9	257	460	69.4
U11	1038	88.2	376	283	65.5
U12	1408	90.4	342	555	75.8
U13	1107	85.4	338	509	66.7
U14	1703	94.8	479	468	85.9
U15	1273	85.0	306	294	73.2
U16	1633	75.4	244	613	65.8
U17	800	68.0	260	85	58.9
U18	1929	94.4	421	568	84.9
U19	1198	81.2	312	573	67.3
U20	543	72.5	276	141	56.9

Table 19: Universities performance table

The concordance values of the three weighted OWA aggregation operators will not be added here for this case study, due to the large nature of the data set. They are provided in the Annex, Pg. 95 – Pg. 107.

Table 20 shows the NFS values of each university and operator including WA and OWA reference operators. Also, the university ranking positions as observed from their NFS values is shown in Table 21. For a graphical representation, Figure 9 provides a visual representation of the rank positions of the 20 universities. The horizontal axis of Figure 9 shows the identifier of each method, where **1** – WA, **2** – OWAd, **3** – OWAc, **4** – OWAWA.3d, **5** – OWAWA.3c, **6** – OWAWA.5d, **7** – OWAWA.5c, **8** – OWAWA.7d, **9** – OWAWA7c, **10** – IOWAd, **11** – IOWAc, **12** – WOWAd, **13** – WOWAc.

	U1	U2	U3	U4	U5	U6	U7	U8	U9	U10	U11	U12	U13	U14	U15	U16	U17	U18	U19	U20
WA	0	-9	-6	-16	1	-16	17	10	3	-5	1	9	4	16	-1	0	-16	18	6	-16
OWAd	-1	-11	-9	-16	0	-14	17	7	5	-1	0	8	2	17	0	7	-16	17	4	-16
OWAc	-3	-6	-8	-17	0	-12	17	12	6	-4	-2	11	4	11	-3	4	-15	17	5	-17
OWAWA.3d	2	-9	-6	-16	2	-16	17	7	2	-5	3	9	4	16	-2	2	-16	18	4	-16
OWAWA.3c	-2	-9	-8	-15	2	-14	18	15	4	-4	-1	11	5	11	-4	1	-18	16	8	-16
OWAWA.5d	3	-9	-7	-16	2	-16	17	7	3	-5	3	9	4	16	-1	1	-16	18	3	-16
OWAWA.5c	-3	-4	-8	-17	1	-12	17	12	6	-4	-3	11	3	11	-3	4	-17	17	6	-17
OWAWA.7d	-1	-10	-8	-16	1	-16	17	7	5	-1	1	9	2	16	0	5	-16	18	3	-16
OWAWA.7c	-3	-5	-8	-17	0	-12	17	12	6	-4	-2	11	4	11	-3	4	-15	17	5	-18
IOWAd	3	-9	-6	-16	2	-16	17	7	3	-6	4	9	4	16	-1	-1	-16	18	4	-16
IOWAc	-6	-5	-9	-17	-3	-15	16	7	7	-1	-5	10	-1	16	3	11	-13	19	4	-18
WOWAd	2	-11	-3	-16	3	-15	16	9	1	-1	3	8	2	17	1	-4	-15	17	1	-15
WOWAc	0	-9	-7	-16	2	-12	18	14	3	-4	0	9	5	12	-4	2	-18	15	7	-17

Table 20: Net flow score for each university and method

	U1	U2	U3	U4	U5	U6	U7	U8	U9	U10	U11	U12	U13	U14	U15	U16	U17	U18	U19	U20
WA	11.5	16	15	18.5	9.5	18.5	2	4	8	14	9.5	5	7	3	13	11.5	18.5	1	6	18.5
OWAd	13.5	16	15	19	11	17	2	5.5	7	13.5	11	4	9	2	11	5.5	19	2	8	19
OWAc	12.5	15	16	19.5	10	17	1.5	3	6	14	11	4.5	8.5	4.5	12.5	8.5	18	1.5	7	19.5
OWAWA.3d	10.5	16	15	18.5	10.5	18.5	2	5	10.5	14	8	4	6.5	3	13	10.5	18.5	1	6.5	18.5
OWAWA.3c	12	16	15	18	9	17	1	3	8	13.5	11	4.5	7	4.5	13.5	10	20	2	6	19
OWAWA.5d	8.5	16	15	18.5	11	18.5	2	5	8.5	14	8.5	4	6	3	13	12	18.5	1	8.5	18.5
OWAWA.5c	12	14.5	16	19	10	17	1.5	3	6.5	14.5	12	4.5	9	4.5	12	8	19	1.5	6.5	19
OWAWA.7d	13.5	16	15	18.5	10.5	18.5	2	5	6.5	13.5	10.5	4	9	3	12	6.5	18.5	1	8	18.5
OWAWA.7c	12.5	15	16	19	10	17	1.5	3	6	14	11	4.5	8.5	4.5	12.5	8.5	18	1.5	7	20
IOWAd	9.5	16	14.5	18.5	11	18.5	2	5	9.5	14.5	7	4	7	3	12.5	12.5	18.5	1	7	18.5
IOWAc	15	13.5	16	19	12	18	2.5	6.5	6.5	10.5	13.5	5	10.5	2.5	9	4	17	1	8	20
WOWAd	8.5	16	14	20	6.5	18	3	4	11	13	6.5	5	8.5	1.5	11	15	18	1.5	11	18
WOWAc	11.5	16	15	18	9.5	17	1	3	8	13.5	11.5	5	7	4	13.5	9.5	20	2	6	19

Table 21: Rank position of each university obtained from NFS

From the Table 21 we can see that university 18 is best in most methods and second best only when university 7 is best. This consistency puts university 18 and 7 as best options. It is also worth noting that university 14 is tied for best in OWAd and WOWAd. As for the least performance, it is between university 20, 17 or 4.

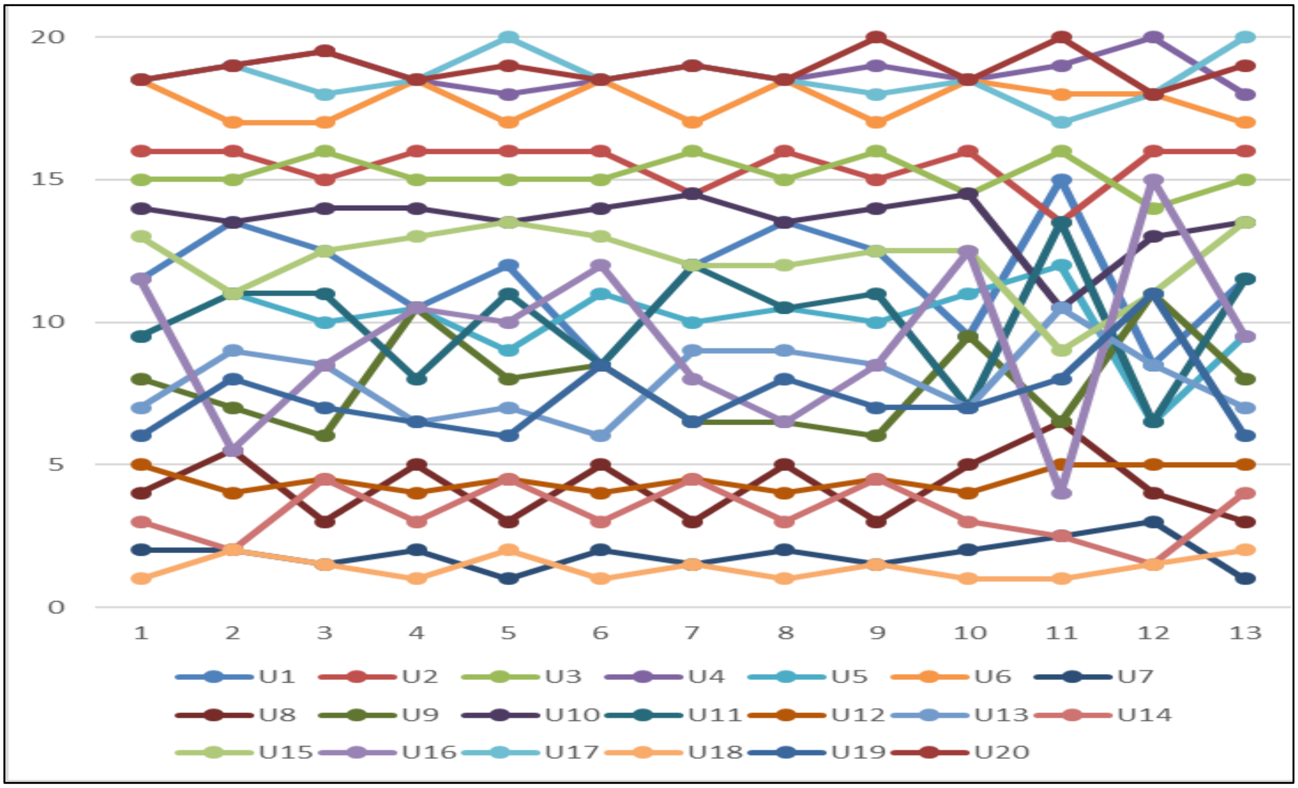

Figure 9: Rank positions of the 20 universities values with weighted average (WA)

The most interesting universities are those sensitive to methods and policies, this can be clearly seen in the rank position visualization shown in Figure 9. Aggregation with IOWA (10&11) and with WOWA (11&12) is able to change the position of some universities in this dataset. Although the ones in the best and worst positions are robust to the change of agregation operator. For example, U16 and U1 are universities that are sensitive to the aggregation policy. U16 is excellent in two criteria (w=0.2 and 0.1) and very bad in one (w=0.3). Therefore, when using WA it appears in at rank 11/20, with OWAd it goes to upper positions (6/20).

We can also observe that there are many rank reversals between universities in ranks 5 to 15 for IOWAc. These reversals happens in cases where a university performs excellently in less important sources and poorly in more important sources. As such when IOWAc is applied, high OWA weights are assigned to the less important sources and low OWA weights to the more imporant sources. This causes the fluctuations in rank observed, an example of this case is university 16 it has a rank of 4/20 in IOWAc and 12/20 in IOWAd. A deeper analysis of the IOWA operator reveals that using the criteria importance weights V as order inducing variable leads to strange results in some cases.

Correlations table (Table 22) shows that in this case study WA and OWA are initially highly correlated, therefore their combination also leads to high correlation values in most methods. WOWAd is the one that differentiates a bit from the rest. IOWAc is suprisingly similar to OWAd.

	WA	**OWAd**	**OWAc**	**Closest** (>=0.99)
WA	**1,00**	**0,95**	**0,98**	
OWAd	**0,95**	**1,00**	**0,98**	
OWAc	**0,98**	**0,98**	**1,00**	
OWAWA.3d	0,99	0,94	0,96	OWAWA.5d, IOWAd
OWAWA.3c	0,99	0,96	0,98	WOWAc
OWAWA.5d	0,98	0,93	0,95	OWAWA.3d, IOWAd
OWAWA.5c	0,97	0,98	1,00	OWAWA.7c, WOWAc
OWAWA.7d	0.97	0,99	0,98	-
OWAWA.7c	0,98	0,98	1,00	OWAWA5c
IOWAd	0,99	0,92	0,95	OWAWA.3d, OWAWA.5d
IOWAc	0,88	0,97	0,93	-
WOWAd	0,93	0,86	0,88	-
WOWAc	0,99	0,96	0,98	OWAWA3c, OWAWA.5c

Table 22: Correlation between the different rankings obtained in dataset Universities

5.3 Environmental ranking of countries

The third case study comes from the 2018 Environmental Performance Index (EPI) [29], an annual measurement of environmental performance of state's policies. The 2018 EPI featured 180 countries as participants, and 10 environment issues which serve as criteria. The goal of the EPI is the ranking of all countries.

We use a small subset of 24 countries from 180 presented in the 2018 EPI. The subset is selected focusing on diversity in the countries' profile in order to avoid high correlations between methods. Out of 10 environmental issues, 2 attributes were removed as they do not apply to all 180 countries and provide missing performance scores in some cases. The weights of these 2 attributes have been redistributed equally among the remaining 8 issues, in order to maintain the weighting proportions.

The criteria selected for this experiment are as follows: **C01** – Air Quality, **C02** – Water Sanitation, **C03** **–** Heavy Metals, **C04 –** Biodiversity Habitat, **C05** – Climate Energy, **C06 –** Air Pollution, **C07 –** Water Resources, **C08 –** Agriculture. In this case all criteria are maximized (+), and the OWA weights are disjunctive: w_d = (0.66, 0.10, 0.06, 0.05, 0.04, 0.03, 0.03, 0.03) and conjunctive: w_c = (0.00, 0.00, 0.01, 0.02, 0.06, 0.14, 0.28, 0.49). These weights are established using the quantifiers; conjunctive, $Q(x) = x^5$ and disjunctive, $Q(x) = \sqrt[5]{x} = x^{1/5}$.

Table 23 and 24 below show the criteria parameters and performance score of this case study.

	C01+	**C02+**	**C03+**	**C04+**	**C05+**	**C06+**	**C07+**	**C08+**
Indifference	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Preference	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00
weights	0.275	0.135	0.035	0.165	0.195	0.075	0.075	0.045

Table 23: Criteria parameters for Countries dataset

	C01+	C02+	C03+	C04+	C05+	C06+	C07+	C08+
A - Afghanistan	44.67	25.75	0.00	13.44	33.04	91.44	0.00	44.77
B - Laos	23.37	27.59	33.69	76.77	77.39	52.56	0.00	33.06
C - Mongolia	64.30	60.54	40.31	75.28	53.98	41.66	63.00	14.35
D - Philippines	62.96	42.86	37.10	78.08	54.54	66.85	60.34	33.45
E - Sri_Lanka	69.91	53.09	66.58	80.38	70.88	51.73	0.00	22.05
F - Thailand	37.90	59.37	75.36	74.91	45.75	48.79	77.05	27.17
G - Barbados	100.00	55.64	60.69	27.14	34.57	40.21	64.44	10.23
H - Cuba	86.47	57.50	46.03	74.37	49.15	75.32	72.52	13.59
I - Trinidad_and_Tobago	93.91	56.29	70.35	88.93	50.02	50.27	70.83	0.00
J - Georgia	54.93	59.93	68.28	62.33	48.41	48.55	81.51	5.70
K - Moldova	59.92	61.02	60.77	31.05	47.90	59.02	83.06	46.51
L - Poland	53.04	69.23	69.29	96.37	64.33	72.30	92.35	43.37
M - Germany	84.09	96.74	100.00	96.92	55.47	93.30	99.65	61.21
N - Argentina	84.57	73.07	59.78	55.64	46.65	19.51	72.13	70.69
O - Chile	14.39	68.24	38.02	72.57	68.62	57.08	80.20	34.64
P - Panama	78.88	43.45	53.10	86.00	54.32	46.87	78.12	11.23
Q - Venezuela	90.39	50.39	37.32	96.21	37.80	27.99	80.74	17.60
R - Kuwait	75.17	65.79	39.00	86.33	31.78	22.66	96.88	30.75
S - Saudi_Arabia	79.91	62.38	43.17	48.33	40.47	25.40	87.79	29.41
T - Kiribati	61.85	20.89	40.21	85.71	43.71	54.35	58.45	12.38
U - Samoa	64.07	29.32	45.76	75.47	55.13	41.99	26.32	11.81
V- Rwanda	37.87	14.96	38.53	63.24	69.91	68.44	0.00	32.14
W - Seychelles	88.03	58.28	60.41	63.49	93.25	2.63	26.32	14.01
X - Zambia	41.66	9.08	35.26	98.75	51.67	77.09	65.61	28.20

Table 24: Performance scores of countries

As in the case of generating a ranking of universities, the concordance values for this case will be included in the Annex Pg. 108 – Pg. 121.

The next tables provide the NFS and rank positions of each country in the test dataset. To fit Table 25 and 26, the country names are substituted with alphabets as indicators shown in Table 24.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
WA	-17	-3	1	1	3	-3	1	1	4	-3	1	6	19	0	-2	4	1	0	-1	-5	-4	-4	2	-2
OWAd	-14	-3	-2	-1	0	-1	-6	1	1	0	3	20	23	2	-1	0	-2	-2	-2	-6	-4	-3	-1	-2
OWAc	-1	0	-1	-1	0	-1	0	0	0	-2	-1	1	12	0	0	0	0	-1	-1	-1	-2	0	0	-1
OWAWA.3d	-17	-4	2	3	6	-7	1	5	6	-8	1	6	19	1	-2	4	1	0	-1	-7	-6	-6	4	-1
OWAWA.3c	-3	0	-1	-1	1	-2	0	-1	0	-2	0	2	13	1	0	0	0	-1	-1	-1	-3	0	0	-1
OWAWA.5d	-18	-8	-2	1	5	-10	-7	9	15	-9	-1	17	22	2	0	5	2	5	-4	-11	-9	-7	5	-2
OWAWA.5c	-1	0	-1	-1	0	-1	0	-1	0	-2	-1	1	13	0	0	0	0	-1	-1	-1	-2	0	0	-1
OWAWA.7d	-20	-14	-2	-1	8	-7	-13	11	18	-9	-2	21	23	7	4	8	4	6	0	-14	-8	-20	10	-10
OWAWA.7c	-1	0	-1	-1	0	-1	0	0	0	-2	-1	1	12	0	0	0	0	-1	-1	-1	-2	0	0	-1
IOWAd	-17	-16	0	-1	8	-14	4	12	19	-8	-6	5	18	9	-7	8	9	5	0	-9	-2	-17	13	-13
IOWAc	-4	-7	-8	-2	4	11	-5	-2	-3	-3	14	19	22	13	2	-2	-10	-3	1	-10	-10	-4	-5	-8
WOWAd	-18	-7	-2	-1	4	-4	-3	3	7	-7	-1	16	20	2	-1	4	5	3	-4	-7	-4	-7	4	-2
WOWAc	-1	0	-1	-1	0	-2	0	0	0	-1	-1	1	11	0	0	0	0	-1	-1	-1	-2	0	0	0

Table 25: Net flow score for each country and method

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
WA	24	19	9.5	9.5	5	19	9.5	9.5	3.5	19	9.5	2	1	13.5	16.5	3.5	9.5	13.5	15	23	21.5	21.5	6	16.5
OWAd	24	19.5	16	11.5	8	11.5	22.5	5.5	5.5	8	3	2	1	4	11.5	8	16	16	16	22.5	21	19.5	11.5	16
OWAc	18	8	18	18	8	18	8	8	8	23.5	18	2	1	8	8	8	8	18	18	18	23.5	8	8	18
OWAWA.3d	24	18	9	8	3	21.5	11.5	5	3	23	11.5	3	1	11.5	17	6.5	11.5	14	15.5	21.5	19.5	19.5	6.5	15.5
OWAWA.3c	23.5	9	17	17	3.5	21.5	9	17	9	21.5	9	2	1	3.5	9	9	9	17	17	17	23.5	9	9	17
OWAWA.5d	24	19	14.5	11	6.5	22	17.5	4	3	20.5	13	2	1	9.5	12	6.5	9.5	6.5	16	23	20.5	17.5	6.5	14.5
OWAWA.5c	17.5	7.5	17.5	17.5	7.5	17.5	7.5	17.5	7.5	23.5	17.5	2	1	7.5	7.5	7.5	7.5	17.5	17.5	17.5	23.5	7.5	7.5	17.5
OWAWA.7d	23.5	21.5	14.5	13	6.5	16	20	4	3	18	14.5	2	1	8	10.5	6.5	10.5	9	12	21.5	17	23.5	5	19
OWAWA.7c	18	8	18	18	8	18	8	8	8	23.5	18	2	1	8	8	8	8	18	18	18	23.5	8	8	18
IOWAd	23.5	22	12.5	14	7.5	21	11	4	1	18	16	9.5	2	5.5	17	7.5	5.5	9.5	12.5	19	15	23.5	3	20
IOWAc	15.5	19	20.5	10	6	5	17.5	10	13	13	3	2	1	4	7	10	23	13	8	23	23	15.5	17.5	20.5
WOWAd	24	21.5	14.5	12	6	18	16	8.5	3	21.5	12	2	1	10	12	6	4	8.5	18	21.5	18	21.5	6	14.5
WOWAc	18.5	8.5	18.5	18.5	8.5	23.5	8.5	8.5	8.5	18.5	18.5	2	1	8.5	8.5	8.5	8.5	18.5	18.5	18.5	23.5	8.5	8.5	8.5

Table 26: Rank position of each country obtained from NFS

Studying the rank positions provided by Table 26, we can see that Germany (M) holds the best position in all methods except IOWAd. In IOWAd Trinidad and Tobago (I) holds the best position, this is because it performs best in C01 (Air Quality). C01 has a high importance, and this importance is reinforced by IOWAd assigning the highest OWA weight to C01, this greatly increases the influence of C01 in the ranking process. This situation might be corrected by considering discordance, as this thesis work only considers the concordance index. Another example is Poland (L), it averages second and third position in all methods except IOWAd, where it is ranked 9.5/24. This is because it has a good performance generally but scores poorly in C01.

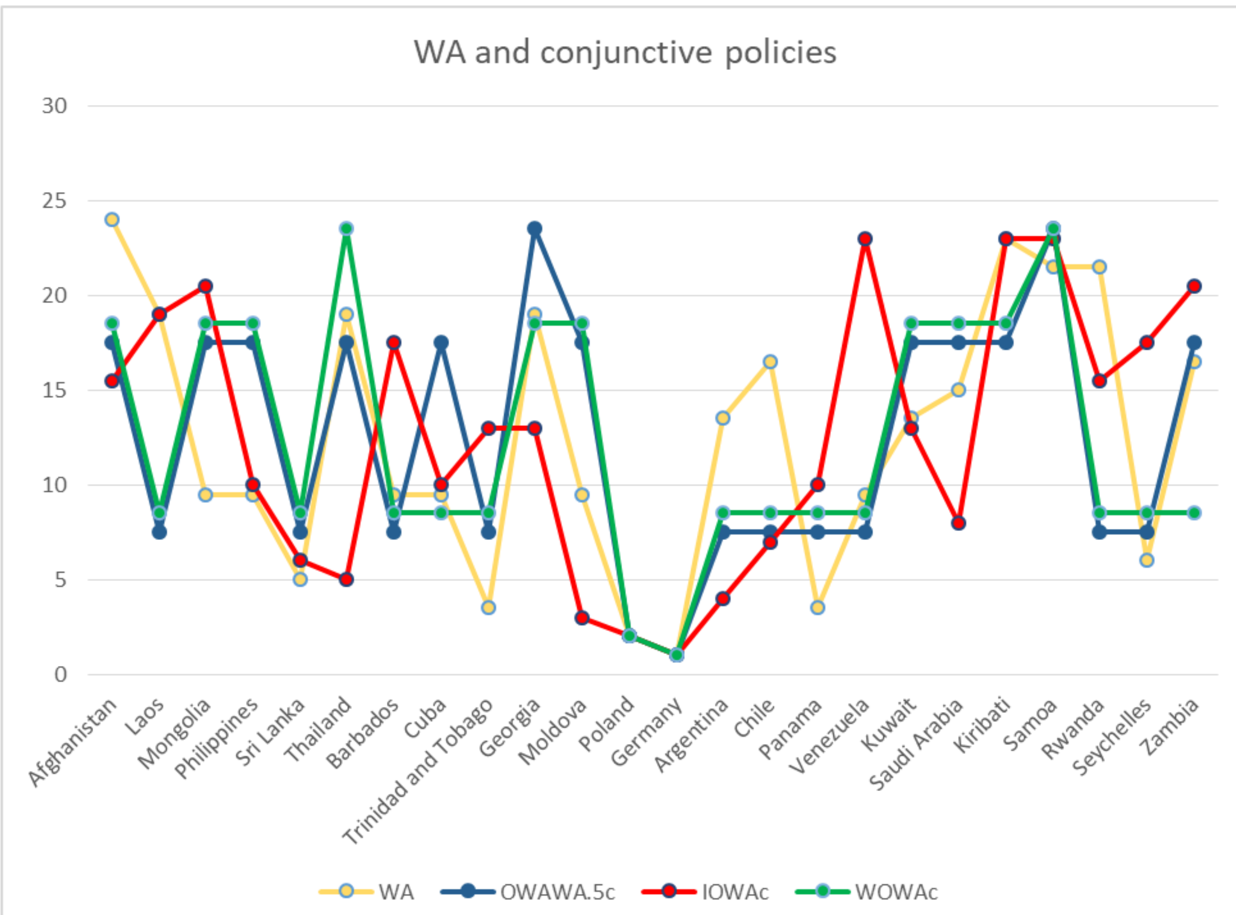

Figure 10: Rank positions of the 24 countries with weighted average (WA) and conjunctive polices

It is also worth noting that when using the conjunctive policy, a lot of ties occur in the rank positions of countries except in the case of IOWAc. These ties occur because of the strict nature of the policy, for a country to be considered better than another it must perform better in all criteria. This strict nature maybe valuable in situations where the DM wants a more thorough ranking.

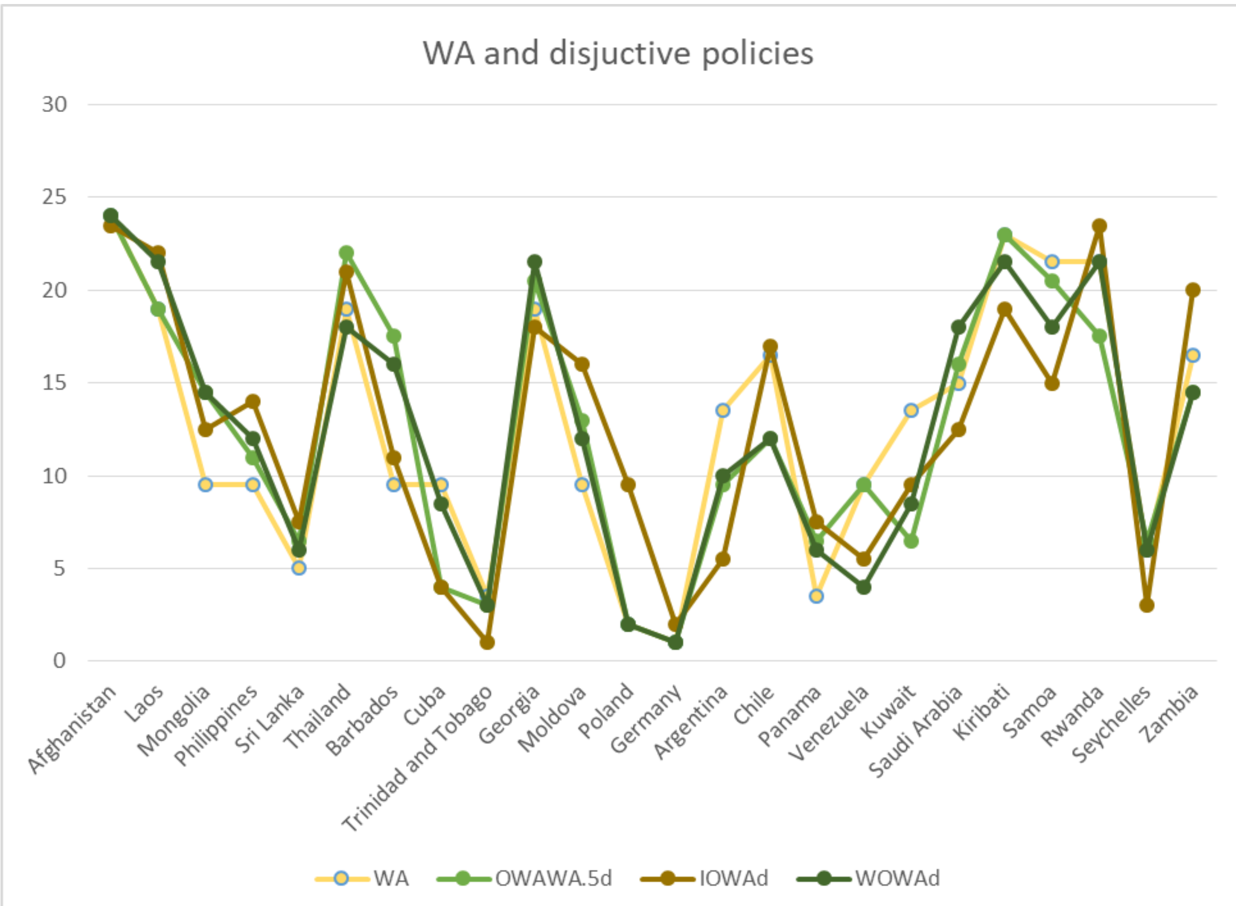

Figure 11: Rank positions of the 24 countries with weighted average (WA) and disjunctive polices

Figure 10 and 11 shows how the methods and policies compared to the standard operator used in ELECTRE, the weighted average (WA). It can be seen that conjunctive approaches are more different from WA than disjunctive approaches. This is because disjunctive approaches mainly reinforce WA. Conjunctive approaches seem more beneficial as the ranking is strict, also OWAWA.5c and WOWAc seem like the best at combining WA and OWA.

	WA	**OWAd**	**OWAc**	**Closest** (>=0.90)
WA	**1,00**	**0,70**	**0,62**	
OWAd	**0,70**	**1,00**	**0,42**	
OWAc	**0,62**	**0,42**	**1,00**	
OWAWA.3d	0,96	0,67	0,65	OWAWA.5d
OWAWA.3c	0,67	0,54	0,87	OWAWA.5c
OWAWA.5d	0,88	0,74	0,67	OWAWA.3d, OWAWA.7d, WOWAd
OWAWA.5c	0,59	0,35	0,95	OWAWA.3c, OWAWA.7c
OWAWA.7d	0.84	0,79	0,57	OWAWA.5d, WOWAd
OWAWA.7c	0,62	0,42	1,00	OWAWA5c, WOWAc
IOWAd	0,83	0,58	0,54	-
IOWAc	0,44	0,78	0,35	-
WOWAd	0,90	0,71	0,64	OWAWA.5d, OWAWA.7d
WOWAc	0,59	0,42	0,92	OWAWA.7c

Table 27: Correlation between the different rankings obtained in dataset Countries

In Table 27, we have the spearman rho correlation of all methods including WA and OWA as reference. We can confirm what Figure 10 and 11 showed, all disjunctive approaches have high (>0.80) correlation with WA while conjunctive approaches have low (<70) correlation. It is also observed that IOWA is not close to any other method, its highest correlation is with WA (0.83). This might be due to the fact that its method of combining WA and OWA greatly differs from the rest.

5.4 Summary and discussion

In this chapter, the proposed weighted OWA aggregation operators were tested. They are tested using three unique case studies including; finding a hotel, university rankings, and environment performance index of countries. The concordance index of each dataset is calculated and exploited using the Net flow score method. A ranking of alternatives is obtained respective to each method, visualizations, and correlation tables of each method is created for analysis.

It has been observed that the OWA policy in use has an effect on the correlation of the methods, disjunctive approaches are more correlated to WA than conjunctive. Also, conjunctive policy is more strict in accepting the concordance, as such produces more ties than the disjunctive policy.

It is can also observed for the OWAWA method that results from lower beta values are more correlated with WA, while those with higher beta values are more correlated with OWA. In most cases, IOWAd is very similar to WA, on the other hand IOWAc produces the most fluctuations in ranking observed. Finally, from all the methods proposed, WOWA conjunctive (WOWAc) and OWAWA beta = 0.5 (OWAWA.5c) are the best ones to model the concept of “majority” in ELECTRE methods.

6 Conclusions and future work

This master thesis work presents three new approaches to the aggregation of partial concordances in the ELECTRE outranking method. Using weighted averaging may sometimes have an undesired compensative effect between opposite values, as such we look to a family of OWA operators to avoid this effect. In this work we have proposed the use of OWAWA, WOWA and IOWA, which combine WA and OWA, to substitute WA, introducing a new way of weighting values. This new approach has been submitted as a research paper in two international conferences:

- EURO: 29th European Conference on Operational Research (CORE B): http://euro2018valencia.com/. The paper has been accepted.

  Title of the paper: Redefining the concordance index in ELECTRE by means of OWA aggregation operators.

  Authors: Aida Valls, Jonathan A. Orama

- MDAI: 15th International Conference on Modeling Decisions for Artificial Intelligence (CORE B): http://www.mdai.cat/mdai2018. Answer about acceptance should be received on June 6th.

  Title of the paper: Constructing an outranking relation with weighted OWA for multi-criteria decision analysis.

  Authors: Jonathan A. Orama, Aida Valls

For the purpose of constructing and formalizing these new approaches, the following goals were formulated and successfully carried out.

1. Outranking methods has been studied extensively with special focus on the ELECTRE methodology, as ELECTRE serves as the base for the proposed methods.

2. Aggregation operators has also been studied. Mainly the three aggregation operators implemented in this work (OWAWA, IOWA, WOWA). They have also been fine tuned to suit the purpose of this thesis work.

3. We extended the concordance index in three different ways by redefining the aggregation of partial concordances in overall concordance calculation. These methods combine the WA and OWA operators. The first method substitutes the weighted average (WA) operator for the ordered weighted averaging weighted averaging operator (OWAWA) in the aggregation of partial concordances. The OWAWA operator is a linear combination of WA and OWA, it uses a variable (beta β) to regulate the degree of OWA and WA in a particular scenario. The second method similar to the first, substitutes the WA operator for the induced ordered weighted averaging operator (IOWA). The IOWA operator performs the OWA method on the partial concordances to be aggregated, but instead of ordering the partial concordance according to their values they are ordered according to their criteria weights. The third method like the first two substitutes the WA operator for the weighted ordered weighted averaging operator (WOWA). The WOWA operator performs the OWA method on the partial concordances to be aggregated, but it changes the OWA weights for a new string of weights obtained using regular monotonically non-decreasing quantifiers to established OWA policies. All three methods are exploited using the Net flow score procedure to obtain a ranking of alternatives.

4. Three different software implementations are built for each method, using the already existing ELECTRE concordance module from the diviz workbench. The source files for the ELECTRE concordance module written in python is gotten from the github address referred by the official

diviz website. The source files are modified replace WA for the three aggregation methods proposed in this work.

5. Finally, we tested the proposed methods with three unique case studies. The first case study poses the problem of finding a hotel for lodging to attend a congress. The DM wishes to choose a hotel from 6 hotels in proximity to the congress meeting. The DM must consider the performances of these hotels according to certain criteria. The second is about making a ranking of 20 British universities, using user defined criteria weights. The third case study is the 2018 environmental performance index, it is concerned with making a ranking of countries according to their performances in certain environmental issues. This case study selects a small subset of 24 countries out of 180 participating countries in the index. In all cases, the concordance values of each method included WA and OWA are calculated, and exploited using the Net flow score procedure to make a ranking. Then results of methods are analyzed and compared.

The analysis and research carried out in this thesis work produces the following conclusions about the proposed methods.

- All three proposed methods show convincing signs of successfully combining the WA operator and the OWA operator when used in the ELECTRE technique. This can become beneficial in cases where there is a need to combine the advantages of WA and OWA. Also, all three methods model the combination of OWA and WA in different ways.

- The methods with conjunctive policy are proven to be more strict in accepting the concordance than methods with the disjunctive policy. This means that a change of policy affects the chance of concordance acceptance. The conjunctive policy would be useful to model the concept of "majority of criteria in agreement with *aSb*" that is one of the key requirements when constructing an outranking relation in ELECTRE. A conjunctive operator makes the evaluation of this assertion more strict than using a weighted average

- The proposed methods can function in cases that benefit from the conventional ELECTRE concordance calculations. In most cases the results obtained from the proposed methods does not deviate too much from the results of WA, and reveals some hidden points missed by the conventional ELECTRE method.

- The IOWA operator produces the strangest results with several reversals in ranking. This is due to the fact that criteria weights are used as order inducing variables. IOWA disjunctive (IOWAd) method presents as a reinforcement of WA, due to the fact that high OWA weights are assigned to the most important sources. Also, IOWA conjunctive has an undesired behaviour observed in a case where a low partial concordance is given by criteria with high importance weight, and they are placed in the first positions during aggregation, so they will receive a low W weight and their contribution is minimized. This seems to go in contrary to common sense. Therefore, we do not recommend IOWA to be used in ELECTRE.

The OWAWA operator and WOWA operators seem like the most stable choices, producing reasonable results according to what one could expect when doing a manual analysis by common sense. In particular, the WOWA conjunctive (WOWAc) and the OWAWA beta = 0.5 conjunctive (OWAWA.5c) methods seem the best methods in combining the advantages of OWA and WA, because they provide suitable ranking in all case studies.

In future work and further research, the following areas maybe be of interest.

- A research of best cases and scenarios in which WOWA or OWAWA methods will be most useful, that is to say which scenarios will draw the desired results from each of these two methods.

- Deeper analysis of the proposed methods could be carried out while considering discordance of criteria as this could solve some issues found in this work. Also, applying them to other datasets to find new results.

- Research could be carried out to determine optimal beta values required in various scenarios when applying the OWAWA operator.

7 References

1. Bouyssou, D.: Building Criteria: A Prerequisite for MCDA, In: Bana e Costa C. A. (eds). Readings in Multiple Criteria Decision Aid. Springer, Berlin, Heidelberg, (1990).

2. Clua, S.B.: Ontology-based construction of outranking relations for decision support, Master Thesis, URV-ETSE, (2017).

3. Del Vasto-Terrientes, L., Valls, A., Zielniewicz, P., Slowinski, R.: Extending Concordance and Discordance Relations to Hierarchical Sets of Criteria in ELECTRE-III Method, MDAI (Modeling Decisions on Artificial Intelligence), Girona, (2012).

4. Figueira, J.R., Greco, S., Ehrgott, M.: Multiple criteria decision analysis: State of the art surveys. Boston: Springer, (2005).

5. Figueira, J.R., Greco, S., Roy, B.: Electre methods with interaction between criteria: An extension of the concordance index, European J. Operational Research, 119, 479–495, (2014).

6. Figueira, J.R., Greco, S., Roy, B., Słowiński, R.: Electre methods: main features and recent developments, Applied Optimization, 103(2), 51–89, (2011)

7. Figueira, J.R., Greco, S., Roy, B., Słowiński, R.: An overview of ELECTRE methods and their recent extensions, Journal of Multi-Criteria Decision Analysis, 20, 61–85, (2013).

8. Govindan, K., Jepsen, M. B.: ELECTRE: A comprehensive literature review on methodologies and applications, European J. Operational Research, 250, 1–29, (2015).

9. Greco, S., Matarazzo, B., Słowiński, R.: Decision rule approach, In: Figueira, J.R., Greco, S., Ehrgott, M., (eds) Multiple criteria decision analysis: State of the art surveys. Boston: Springer, 507– 555, (2005).

10. Ishizaka, A., Giannoulis, C.: A web-based decision support system with ELECTRE III for a personalised ranking of British universities, Decision Support Systems, 48, 488–497, (2010).

11. Merigo, J. M.: On the use of the OWA operator in the weighted average and its application in decision making, In: Proceedings of the World Congress on Engineering. WCE, London, U.K., (2009).

12. Ngo The, A., Tsoukiàs, A.: A new axiomatisation of PQI interval orders, Submitted, (2001).

13. Perny, P., Roy, B.: The use of fuzzy outranking relations in preference modelling, Fuzzy Sets and Systems, 49, 33–53, (1992).

14. Roy, B.: Classement et choix en présence de point de vue multiples: Le méthode ELECTRE, Revue Francaise d'Informatique et de Recherche Opérationnelle, 8, 57-75, (1968).

15. Roy, B.: The outranking approach and the foundations of ELECTRE methods, Theory and Decision, 31, (1991).

16. Roy, B., Bouyssou, D.: Decision Aid: An elementary introduction with emphasis on multiple criteria, Investigación Operativa, 2, 95–110, (1991).

17. Roy, B., Słowiński, R.: Handling effects of reinforced preference and counter-veto in credibility of outranking, European Journal of Operational Research, 188(1), 185–190, (2008).

18. Soniran Shofade J.O.: Considering hierarchical structure of criteria in ELECTRE decision aiding methods, Master Thesis, URV-ETSE, (2011).

19. Torra, V.: The Weighted OWA Operator, International Journal of Intelligent Systems, 12, 153–166, (1997).

20. Torra, V.: The WOWA operator and the interpolation function W*: Chen and Otto's interpolation method revisited, Fuzzy Sets and Systems, 113(3), 389–396, (2000).

21. Torra, V., Narukawa, Y.: Modeling decisions: Information fusion and aggregation operators, Springer, (2007).

22. Tsoukiàs A., Perny P., Vincke P.: From Concordance / Discordance to the Modelling of Positive and Negative Reasons in Decision Aiding. In: Bouyssou D., Jacquet-Lagrèze E., Perny P., Słowiński R., Vanderpooten D., Vincke P. (eds) Aiding Decisions with Multiple Criteria. International Series in Operations Research & Management Science, 44, Springer, Boston, MA, (2002).

23. Valls, A., Orama, J.: Redefining the concordance index in ELECTRE by means of OWA aggregation operators, European Conference on Operational Research, 9, accepted, (2018).

24. Yager, R. R.: On ordered weighted averaging aggregation operators in multi-criteria decision making, IEEE Transactions on Systems, Man, and Cybernetics, 18, 183–190, (1988).

25. Yager, R. R.: Induced Ordered Weighted Averaging Operators, IEEE Transactions on Systems, Man, and Cybernetics-Part B: Cybernetics, 29, 141–150, (1999).

26. Yager R. R.: Families of OWA operators, Fuzzy Sets and Systems, 59, 125–148 (1993).

27. Yager R. R.: Quantifier guided aggregation using OWA operators, International Journal of Intelligent Systems, 11, 49–73 (1996).

28. Yager, R., Kacprzyk, J., Beliakov, G.: Recent Developments in the Ordered Weighted Averaging Operators: Theory and Practice, Springer, (2011).

29. Yale Center for Environmental Law & Policy, and Center for International Earth Science Information Network at Columbia University: Environmental Performance Index, https://epi.envirocenter.yale.edu, Accessed 2018.

8 Annex

8.1 Concordance values for the universities dataset

	U1	U2	U3	U4	U5	U6	U7	U8	U9	U10	U11	U12	U13	U14	U15	U16	U17	U18	U19	U20
U1	1.000	0.900	0.730	1.000	0.800	1.000	0.610	0.800	0.700	0.750	0.940	0.590	0.780	0.080	0.798	0.700	1.000	0.390	0.720	1.000
U2	0.620	1.000	0.400	0.990	0.400	0.820	0.090	0.570	0.400	0.500	0.440	0.400	0.400	0.000	0.400	0.560	0.920	0.070	0.430	0.940
U3	0.560	0.820	1.000	1.000	0.600	1.000	0.300	0.360	0.670	0.660	0.630	0.580	0.680	0.300	0.820	0.690	1.000	0.300	0.680	1.000
U4	0.330	0.600	0.340	1.000	0.004	0.855	0.000	0.200	0.232	0.437	0.007	0.000	0.000	0.000	0.157	0.557	0.900	0.000	0.172	0.990
U5	0.936	0.900	1.000	1.000	1.000	1.000	0.300	0.794	0.700	0.996	1.000	0.637	0.870	0.500	0.872	0.700	1.000	0.382	0.763	1.000
U6	0.570	0.675	0.640	1.000	0.357	1.000	0.300	0.300	0.609	0.601	0.282	0.288	0.492	0.006	0.600	0.636	1.000	0.020	0.621	1.000
U7	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.800	1.000	1.000	1.000	1.000	1.000	0.942	1.000	1.000	1.000	0.900	1.000	1.000
U8	1.000	0.900	0.867	1.000	0.933	1.000	0.714	1.000	0.900	1.000	0.927	0.730	1.000	0.207	0.952	0.900	1.000	0.505	1.000	1.000
U9	0.700	0.895	1.000	1.000	0.880	1.000	0.395	0.500	1.000	1.000	0.700	0.918	0.985	0.473	0.988	0.765	1.000	0.487	1.000	1.000
U10	0.700	0.618	0.907	1.000	0.697	1.000	0.307	0.500	0.791	1.000	0.690	0.305	0.676	0.270	0.929	0.700	1.000	0.083	0.687	1.000
U11	0.993	0.900	0.898	1.000	0.800	1.000	0.480	0.791	0.700	0.796	1.000	0.634	0.800	0.300	0.892	0.700	1.000	0.562	0.786	1.000
U12	0.865	1.000	1.000	1.000	1.000	1.000	0.463	0.800	1.000	1.000	1.000	1.000	1.000	0.537	1.000	0.874	1.000	0.553	1.000	1.000
U13	0.835	0.900	1.000	1.000	1.000	1.000	0.300	0.792	0.900	1.000	1.000	0.836	1.000	0.487	0.953	0.700	1.000	0.430	0.907	1.000
U14	1.000	1.000	1.000	1.000	1.000	1.000	0.800	0.800	0.911	1.000	1.000	0.801	1.000	1.000	1.000	0.800	1.000	0.739	0.800	1.000
U15	0.700	0.970	0.948	1.000	0.755	1.000	0.343	0.552	0.800	0.800	0.775	0.800	0.800	0.285	1.000	0.700	1.000	0.308	0.800	1.000
U16	0.696	0.700	0.693	1.000	0.580	1.000	0.448	0.494	0.890	0.998	0.573	0.437	0.667	0.300	0.777	1.000	1.000	0.200	0.790	1.000
U17	0.358	0.400	0.500	0.800	0.121	0.800	0.003	0.339	0.375	0.569	0.101	0.000	0.093	0.000	0.288	0.667	1.000	0.000	0.423	0.943
U18	1.000	1.000	1.000	1.000	1.000	1.000	0.800	0.800	1.000	1.000	1.000	1.000	1.000	0.865	1.000	0.994	1.000	1.000	1.000	1.000
U19	0.700	0.952	1.000	1.000	1.000	1.000	0.300	0.597	0.950	1.000	0.820	0.896	1.000	0.347	0.977	0.900	1.000	0.360	1.000	1.000
U20	0.313	0.556	0.503	0.984	0.161	0.881	0.153	0.309	0.505	0.613	0.127	0.108	0.344	0.000	0.483	0.648	0.917	0.000	0.634	1.000

Table 28: Outranking values with weighted average (**WA**) for universities

	U1	U2	U3	U4	U5	U6	U7	U8	U9	U10	U11	U12	U13	U14	U15	U16	U17	U18	U19	U20
U1	1.000	0.894	0.867	1.000	0.894	1.000	0.646	0.894	0.774	0.840	0.970	0.689	0.877	0.114	0.832	0.774	1.000	0.500	0.799	1.000
U2	0.778	1.000	0.632	0.987	0.632	0.901	0.417	0.711	0.632	0.680	0.659	0.632	0.632	0.000	0.632	0.637	0.921	0.100	0.644	0.941
U3	0.723	0.864	1.000	1.000	0.722	1.000	0.447	0.553	0.730	0.718	0.783	0.622	0.755	0.447	0.803	0.767	1.000	0.447	0.746	1.000
U4	0.529	0.774	0.632	1.000	0.018	0.912	0.000	0.298	0.346	0.531	0.031	0.000	0.000	0.000	0.234	0.609	0.890	0.000	0.204	0.988
U5	0.974	0.894	1.000	1.000	1.000	1.000	0.447	0.886	0.774	0.995	1.000	0.685	0.931	0.632	0.859	0.774	1.000	0.497	0.850	1.000
U6	0.732	0.804	0.804	1.000	0.478	1.000	0.447	0.447	0.646	0.633	0.424	0.311	0.587	0.009	0.632	0.682	1.000	0.030	0.662	1.000
U7	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.894	1.000	1.000	1.000	1.000	1.000	0.975	1.000	1.000	1.000	0.894	1.000	1.000
U8	1.000	0.894	0.953	1.000	0.976	1.000	0.742	1.000	0.894	1.000	0.974	0.802	1.000	0.451	0.949	0.894	1.000	0.632	1.000	1.000
U9	0.894	0.963	1.000	1.000	0.957	1.000	0.623	0.774	1.000	1.000	0.894	0.943	0.995	0.616	0.987	0.836	1.000	0.624	1.000	1.000
U10	0.894	0.796	0.966	1.000	0.893	1.000	0.460	0.774	0.836	1.000	0.890	0.480	0.879	0.490	0.972	0.774	1.000	0.124	0.849	1.000
U11	0.993	0.894	0.946	1.000	0.894	1.000	0.558	0.883	0.774	0.889	1.000	0.681	0.894	0.447	0.880	0.774	1.000	0.608	0.877	1.000
U12	0.952	1.000	1.000	1.000	1.000	1.000	0.719	0.894	1.000	1.000	1.000	1.000	1.000	0.684	1.000	0.897	1.000	0.700	1.000	1.000
U13	0.942	0.894	1.000	1.000	1.000	1.000	0.447	0.891	0.894	1.000	1.000	0.822	1.000	0.624	0.950	0.774	1.000	0.567	0.950	1.000
U14	1.000	1.000	1.000	1.000	1.000	1.000	0.894	0.894	0.953	1.000	1.000	0.895	1.000	1.000	1.000	0.894	1.000	0.821	0.894	1.000
U15	0.894	0.989	0.972	1.000	0.876	1.000	0.527	0.795	0.894	0.894	0.920	0.894	0.894	0.429	1.000	0.774	1.000	0.467	0.894	1.000
U16	0.889	0.894	0.886	1.000	0.846	1.000	0.670	0.765	0.960	0.998	0.843	0.725	0.881	0.632	0.887	1.000	1.000	0.447	0.926	1.000
U17	0.613	0.632	0.774	0.894	0.400	0.894	0.005	0.515	0.539	0.640	0.370	0.000	0.282	0.000	0.400	0.727	1.000	0.000	0.487	0.970
U18	1.000	1.000	1.000	1.000	1.000	1.000	0.894	0.894	1.000	1.000	1.000	1.000	1.000	0.952	1.000	0.997	1.000	1.000	1.000	1.000
U19	0.894	0.949	1.000	1.000	1.000	1.000	0.447	0.813	0.947	1.000	0.936	0.887	1.000	0.537	0.975	0.894	1.000	0.546	1.000	1.000
U20	0.468	0.729	0.761	0.992	0.281	0.902	0.228	0.463	0.591	0.651	0.275	0.158	0.393	0.000	0.560	0.700	0.912	0.000	0.680	1.000

Table 29: Outranking values with OWA disjunctive **(OWAd)** for universities

	U1	**U2**	**U3**	**U4**	**U5**	**U6**	**U7**	**U8**	**U9**	**U10**	**U11**	**U12**	**U13**	**U14**	**U15**	**U16**	**U17**	**U18**	**U19**	**U20**
U1	1.000	0.640	0.578	1.000	0.640	1.000	0.180	0.640	0.360	0.514	0.901	0.276	0.601	0.013	0.497	0.360	1.000	0.075	0.420	1.000
U2	0.403	1.000	0.160	0.958	0.160	0.719	0.037	0.271	0.160	0.229	0.198	0.160	0.160	0.000	0.160	0.205	0.731	0.009	0.178	0.799
U3	0.298	0.570	1.000	1.000	0.294	1.000	0.040	0.112	0.298	0.281	0.384	0.154	0.333	0.040	0.428	0.350	1.000	0.040	0.321	1.000
U4	0.093	0.360	0.205	1.000	0.002	0.744	0.000	0.027	0.031	0.095	0.003	0.000	0.000	0.000	0.021	0.150	0.664	0.000	0.039	0.961
U5	0.916	0.640	1.000	1.000	1.000	1.000	0.040	0.623	0.360	0.986	1.000	0.235	0.767	0.160	0.571	0.360	1.000	0.073	0.538	1.000
U6	0.301	0.430	0.463	1.000	0.187	1.000	0.040	0.040	0.180	0.162	0.103	0.074	0.160	0.001	0.160	0.231	1.000	0.003	0.203	1.000
U7	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.640	1.000	1.000	1.000	1.000	1.000	0.924	1.000	1.000	1.000	0.640	1.000	1.000
U8	1.000	0.640	0.840	1.000	0.920	1.000	0.328	1.000	0.640	1.000	0.912	0.457	1.000	0.043	0.827	0.640	1.000	0.167	1.000	1.000
U9	0.640	0.874	1.000	1.000	0.856	1.000	0.154	0.360	1.000	1.000	0.640	0.826	0.982	0.149	0.958	0.518	1.000	0.155	1.000	1.000
U10	0.640	0.412	0.895	1.000	0.637	1.000	0.048	0.360	0.524	1.000	0.631	0.150	0.606	0.068	0.910	0.360	1.000	0.011	0.535	1.000
U11	0.975	0.640	0.816	1.000	0.640	1.000	0.112	0.615	0.360	0.629	1.000	0.229	0.640	0.040	0.645	0.360	1.000	0.145	0.601	1.000
U12	0.839	1.000	1.000	1.000	1.000	1.000	0.290	0.640	1.000	1.000	1.000	1.000	1.000	0.233	1.000	0.692	1.000	0.259	1.000	1.000
U13	0.802	0.640	1.000	1.000	1.000	1.000	0.040	0.633	0.640	1.000	1.000	0.479	1.000	0.155	0.845	0.360	1.000	0.118	0.832	1.000
U14	1.000	1.000	1.000	1.000	1.000	1.000	0.640	0.640	0.841	1.000	1.000	0.643	1.000	1.000	1.000	0.640	1.000	0.470	0.640	1.000
U15	0.640	0.964	0.906	1.000	0.598	1.000	0.092	0.409	0.640	0.640	0.730	0.640	0.640	0.050	1.000	0.360	1.000	0.064	0.640	1.000
U16	0.628	0.640	0.664	1.000	0.528	1.000	0.238	0.348	0.873	0.996	0.522	0.329	0.608	0.160	0.684	1.000	1.000	0.040	0.748	1.000
U17	0.191	0.160	0.360	0.640	0.091	0.640	0.000	0.097	0.112	0.200	0.065	0.000	0.041	0.000	0.047	0.294	1.000	0.000	0.198	0.898
U18	1.000	1.000	1.000	1.000	1.000	1.000	0.640	0.640	1.000	1.000	1.000	1.000	1.000	0.838	1.000	0.989	1.000	1.000	1.000	1.000
U19	0.640	0.827	1.000	1.000	1.000	1.000	0.040	0.451	0.819	1.000	0.784	0.683	1.000	0.099	0.916	0.640	1.000	0.104	1.000	1.000
U20	0.055	0.313	0.348	0.972	0.072	0.706	0.020	0.051	0.158	0.187	0.051	0.015	0.156	0.000	0.113	0.256	0.701	0.000	0.228	1.000

Table 30: Outranking values with OWA conjunctive (**OWAc**) for universities

	U1	U2	U3	U4	U5	U6	U7	U8	U9	U10	U11	U12	U13	U14	U15	U16	U17	U18	U19	U20
U1	1.000	0.898	0.773	1.000	0.828	1.000	0.620	0.828	0.722	0.780	0.950	0.623	0.813	0.090	0.808	0.722	1.000	0.421	0.745	1.000
U2	0.671	1.000	0.469	0.988	0.469	0.846	0.190	0.609	0.469	0.556	0.504	0.469	0.469	0.000	0.469	0.582	0.923	0.077	0.492	0.943
U3	0.613	0.837	1.000	1.000	0.640	1.000	0.344	0.420	0.687	0.677	0.674	0.596	0.706	0.344	0.817	0.716	1.000	0.344	0.700	1.000
U4	0.392	0.652	0.431	1.000	0.008	0.872	0.000	0.229	0.267	0.465	0.014	0.000	0.000	0.000	0.180	0.573	0.897	0.000	0.182	0.989
U5	0.947	0.898	1.000	1.000	1.000	1.000	0.344	0.821	0.722	0.995	1.000	0.652	0.889	0.539	0.869	0.722	1.000	0.417	0.789	1.000
U6	0.619	0.714	0.689	1.000	0.394	1.000	0.344	0.344	0.620	0.611	0.324	0.295	0.520	0.008	0.609	0.649	1.000	0.023	0.634	1.000
U7	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.828	1.000	1.000	1.000	1.000	1.000	0.953	1.000	1.000	1.000	0.898	1.000	1.000
U8	1.000	0.898	0.892	1.000	0.946	1.000	0.723	1.000	0.898	1.000	0.941	0.752	1.000	0.280	0.951	0.898	1.000	0.543	1.000	1.000
U9	0.758	0.915	1.000	1.000	0.903	1.000	0.464	0.582	1.000	1.000	0.758	0.926	0.989	0.516	0.988	0.786	1.000	0.528	1.000	1.000
U10	0.758	0.672	0.925	1.000	0.775	1.000	0.353	0.582	0.805	1.000	0.750	0.358	0.737	0.336	0.942	0.722	1.000	0.095	0.736	1.000
U11	0.993	0.898	0.912	1.000	0.828	1.000	0.503	0.819	0.722	0.823	1.000	0.649	0.828	0.344	0.889	0.722	1.000	0.576	0.813	1.000
U12	0.891	1.000	1.000	1.000	1.000	1.000	0.540	0.828	1.000	1.000	1.000	1.000	1.000	0.581	1.000	0.881	1.000	0.598	1.000	1.000
U13	0.867	0.898	1.000	1.000	1.000	1.000	0.344	0.822	0.898	1.000	1.000	0.832	1.000	0.528	0.951	0.722	1.000	0.471	0.920	1.000
U14	1.000	1.000	1.000	1.000	1.000	1.000	0.828	0.828	0.924	1.000	1.000	0.830	1.000	1.000	1.000	0.828	1.000	0.764	0.828	1.000
U15	0.758	0.976	0.955	1.000	0.791	1.000	0.398	0.625	0.828	0.828	0.819	0.828	0.828	0.328	1.000	0.722	1.000	0.355	0.828	1.000
U16	0.754	0.758	0.751	1.000	0.660	1.000	0.515	0.575	0.911	0.998	0.654	0.524	0.731	0.399	0.810	1.000	1.000	0.274	0.830	1.000
U17	0.435	0.469	0.582	0.828	0.204	0.828	0.004	0.392	0.424	0.590	0.182	0.000	0.150	0.000	0.322	0.685	1.000	0.000	0.422	0.951
U18	1.000	1.000	1.000	1.000	1.000	1.000	0.828	0.828	1.000	1.000	1.000	1.000	1.000	0.891	1.000	0.995	1.000	1.000	1.000	1.000
U19	0.758	0.951	1.000	1.000	1.000	1.000	0.344	0.662	0.949	1.000	0.855	0.893	1.000	0.404	0.976	0.898	1.000	0.416	1.000	1.000
U20	0.359	0.608	0.580	0.987	0.197	0.887	0.176	0.355	0.531	0.625	0.172	0.123	0.359	0.000	0.506	0.664	0.916	0.000	0.648	1.000

Table 31: Outranking values with OWAWA beta=0.3 disjunctive (**OWAWA.3d**) for universities

	U1	U2	U3	U4	U5	U6	U7	U8	U9	U10	U11	U12	U13	U14	U15	U16	U17	U18	U19	U20
U1	1.000	0.822	0.687	1.000	0.752	1.000	0.481	0.752	0.598	0.682	0.929	0.499	0.730	0.060	0.707	0.598	1.000	0.293	0.631	1.000
U2	0.558	1.000	0.328	0.979	0.328	0.792	0.076	0.478	0.328	0.421	0.366	0.328	0.328	0.000	0.328	0.452	0.867	0.050	0.352	0.901
U3	0.485	0.748	1.000	1.000	0.512	1.000	0.222	0.288	0.558	0.547	0.555	0.456	0.580	0.222	0.705	0.591	1.000	0.222	0.573	1.000
U4	0.261	0.528	0.303	1.000	0.003	0.821	0.000	0.148	0.172	0.334	0.006	0.000	0.000	0.000	0.116	0.435	0.829	0.000	0.132	0.981
U5	0.930	0.822	1.000	1.000	1.000	1.000	0.222	0.743	0.598	0.993	1.000	0.517	0.839	0.398	0.782	0.598	1.000	0.290	0.696	1.000
U6	0.490	0.601	0.587	1.000	0.306	1.000	0.222	0.222	0.481	0.470	0.228	0.224	0.392	0.005	0.468	0.514	1.000	0.015	0.496	1.000
U7	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.752	1.000	1.000	1.000	1.000	1.000	0.937	1.000	1.000	1.000	0.822	1.000	1.000
U8	1.000	0.822	0.859	1.000	0.929	1.000	0.598	1.000	0.822	1.000	0.922	0.648	1.000	0.157	0.915	0.822	1.000	0.404	1.000	1.000
U9	0.682	0.889	1.000	1.000	0.873	1.000	0.323	0.458	1.000	1.000	0.682	0.891	0.984	0.376	0.979	0.691	1.000	0.387	1.000	1.000
U10	0.682	0.556	0.904	1.000	0.679	1.000	0.229	0.458	0.711	1.000	0.672	0.259	0.655	0.209	0.924	0.598	1.000	0.062	0.642	1.000
U11	0.988	0.822	0.873	1.000	0.752	1.000	0.369	0.738	0.598	0.746	1.000	0.513	0.752	0.222	0.818	0.598	1.000	0.437	0.731	1.000
U12	0.857	1.000	1.000	1.000	1.000	1.000	0.411	0.752	1.000	1.000	1.000	1.000	1.000	0.448	1.000	0.820	1.000	0.465	1.000	1.000
U13	0.825	0.822	1.000	1.000	1.000	1.000	0.222	0.745	0.822	1.000	1.000	0.729	1.000	0.387	0.921	0.598	1.000	0.336	0.885	1.000
U14	1.000	1.000	1.000	1.000	1.000	1.000	0.752	0.752	0.890	1.000	1.000	0.754	1.000	1.000	1.000	0.752	1.000	0.658	0.752	1.000
U15	0.682	0.968	0.936	1.000	0.708	1.000	0.268	0.509	0.752	0.752	0.761	0.752	0.752	0.215	1.000	0.598	1.000	0.235	0.752	1.000
U16	0.676	0.682	0.684	1.000	0.564	1.000	0.385	0.450	0.885	0.998	0.558	0.405	0.649	0.258	0.749	1.000	1.000	0.152	0.777	1.000
U17	0.308	0.328	0.458	0.752	0.112	0.752	0.002	0.267	0.296	0.459	0.090	0.000	0.078	0.000	0.216	0.555	1.000	0.000	0.355	0.930
U18	1.000	1.000	1.000	1.000	1.000	1.000	0.752	0.752	1.000	1.000	1.000	1.000	1.000	0.857	1.000	0.993	1.000	1.000	1.000	1.000
U19	0.682	0.915	1.000	1.000	1.000	1.000	0.222	0.553	0.911	1.000	0.809	0.832	1.000	0.272	0.959	0.822	1.000	0.283	1.000	1.000
U20	0.235	0.484	0.457	0.981	0.135	0.829	0.113	0.231	0.401	0.486	0.105	0.080	0.288	0.000	0.372	0.531	0.853	0.000	0.512	1.000

Table 32: Outranking values with OWAWA beta=0.3 conjunctive (**OWAWA.3c**) for universities

	U1	U2	U3	U4	U5	U6	U7	U8	U9	U10	U11	U12	U13	U14	U15	U16	U17	U18	U19	U20
U1	1.000	0.897	0.800	1.000	0.847	1.000	0.628	0.847	0.737	0.797	0.956	0.642	0.832	0.097	0.815	0.737	1.000	0.443	0.760	1.000
U2	0.702	1.000	0.516	0.988	0.516	0.862	0.255	0.639	0.516	0.592	0.549	0.516	0.516	0.000	0.516	0.598	0.923	0.084	0.535	0.943
U3	0.645	0.844	1.000	1.000	0.664	1.000	0.373	0.459	0.699	0.689	0.705	0.604	0.721	0.373	0.813	0.731	1.000	0.373	0.714	1.000
U4	0.431	0.687	0.488	1.000	0.011	0.883	0.000	0.249	0.289	0.484	0.019	0.000	0.000	0.000	0.196	0.583	0.895	0.000	0.188	0.989
U5	0.955	0.897	1.000	1.000	1.000	1.000	0.373	0.840	0.737	0.995	1.000	0.661	0.901	0.566	0.866	0.737	1.000	0.440	0.807	1.000
U6	0.651	0.739	0.722	1.000	0.418	1.000	0.373	0.373	0.628	0.617	0.353	0.300	0.540	0.008	0.616	0.659	1.000	0.025	0.642	1.000
U7	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.847	1.000	1.000	1.000	1.000	1.000	0.959	1.000	1.000	1.000	0.897	1.000	1.000
U8	1.000	0.897	0.910	1.000	0.955	1.000	0.728	1.000	0.897	1.000	0.950	0.766	1.000	0.329	0.951	0.897	1.000	0.569	1.000	1.000
U9	0.797	0.930	1.000	1.000	0.919	1.000	0.509	0.637	1.000	1.000	0.797	0.931	0.989	0.544	0.988	0.800	1.000	0.555	1.000	1.000
U10	0.797	0.707	0.937	1.000	0.795	1.000	0.383	0.637	0.814	1.000	0.790	0.393	0.777	0.380	0.951	0.737	1.000	0.104	0.768	1.000
U11	0.993	0.897	0.923	1.000	0.847	1.000	0.519	0.837	0.737	0.842	1.000	0.658	0.847	0.373	0.886	0.737	1.000	0.586	0.832	1.000
U12	0.909	1.000	1.000	1.000	1.000	1.000	0.591	0.847	1.000	1.000	1.000	1.000	1.000	0.610	1.000	0.886	1.000	0.627	1.000	1.000
U13	0.888	0.897	1.000	1.000	1.000	1.000	0.373	0.842	0.897	1.000	1.000	0.829	1.000	0.555	0.950	0.737	1.000	0.498	0.930	1.000
U14	1.000	1.000	1.000	1.000	1.000	1.000	0.847	0.847	0.932	1.000	1.000	0.848	1.000	1.000	1.000	0.847	1.000	0.780	0.847	1.000
U15	0.797	0.980	0.960	1.000	0.815	1.000	0.435	0.674	0.847	0.847	0.848	0.847	0.847	0.357	1.000	0.737	1.000	0.387	0.847	1.000
U16	0.792	0.797	0.790	1.000	0.713	1.000	0.559	0.630	0.925	0.998	0.708	0.581	0.774	0.466	0.832	1.000	1.000	0.323	0.858	1.000
U17	0.486	0.516	0.637	0.847	0.260	0.847	0.004	0.427	0.457	0.605	0.236	0.000	0.188	0.000	0.344	0.697	1.000	0.000	0.455	0.957
U18	1.000	1.000	1.000	1.000	1.000	1.000	0.847	0.847	1.000	1.000	1.000	1.000	1.000	0.909	1.000	0.995	1.000	1.000	1.000	1.000
U19	0.797	0.950	1.000	1.000	1.000	1.000	0.373	0.705	0.948	1.000	0.878	0.891	1.000	0.442	0.976	0.897	1.000	0.453	1.000	1.000
U20	0.390	0.643	0.632	0.988	0.221	0.892	0.191	0.386	0.549	0.632	0.202	0.133	0.369	0.000	0.522	0.674	0.915	0.000	0.657	1.000

Table 33: Outranking values with OWAWA beta=0.5 disjunctive (**OWAWA.5d**) for universities

	U1	U2	U3	U4	U5	U6	U7	U8	U9	U10	U11	U12	U13	U14	U15	U16	U17	U18	U19	U20
U1	1.000	0.770	0.656	1.000	0.720	1.000	0.395	0.720	0.530	0.634	0.921	0.436	0.694	0.047	0.647	0.530	1.000	0.231	0.571	1.000
U2	0.514	1.000	0.280	0.973	0.280	0.771	0.065	0.419	0.280	0.366	0.318	0.280	0.280	0.000	0.280	0.382	0.828	0.038	0.302	0.872
U3	0.432	0.697	1.000	1.000	0.449	1.000	0.170	0.238	0.484	0.471	0.506	0.369	0.510	0.170	0.626	0.522	1.000	0.170	0.501	1.000
U4	0.213	0.480	0.275	1.000	0.003	0.799	0.000	0.113	0.132	0.266	0.005	0.000	0.000	0.000	0.089	0.353	0.782	0.000	0.106	0.975
U5	0.926	0.770	1.000	1.000	1.000	1.000	0.170	0.709	0.530	0.991	1.000	0.436	0.819	0.330	0.722	0.530	1.000	0.228	0.650	1.000
U6	0.436	0.553	0.552	1.000	0.272	1.000	0.170	0.170	0.395	0.382	0.193	0.181	0.326	0.004	0.380	0.434	1.000	0.011	0.412	1.000
U7	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.720	1.000	1.000	1.000	1.000	1.000	0.934	1.000	1.000	1.000	0.770	1.000	1.000
U8	1.000	0.770	0.853	1.000	0.927	1.000	0.521	1.000	0.770	1.000	0.919	0.593	1.000	0.125	0.890	0.770	1.000	0.336	1.000	1.000
U9	0.670	0.884	1.000	1.000	0.868	1.000	0.275	0.430	1.000	1.000	0.670	0.873	0.983	0.311	0.973	0.642	1.000	0.321	1.000	1.000
U10	0.670	0.515	0.901	1.000	0.667	1.000	0.178	0.430	0.658	1.000	0.660	0.228	0.641	0.169	0.920	0.530	1.000	0.047	0.611	1.000
U11	0.984	0.770	0.857	1.000	0.720	1.000	0.296	0.703	0.530	0.712	1.000	0.432	0.720	0.170	0.769	0.530	1.000	0.354	0.694	1.000
U12	0.851	1.000	1.000	1.000	1.000	1.000	0.377	0.720	1.000	1.000	1.000	1.000	1.000	0.385	1.000	0.783	1.000	0.407	1.000	1.000
U13	0.818	0.770	1.000	1.000	1.000	1.000	0.170	0.713	0.770	1.000	1.000	0.657	1.000	0.321	0.899	0.530	1.000	0.274	0.870	1.000
U14	1.000	1.000	1.000	1.000	1.000	1.000	0.720	0.720	0.876	1.000	1.000	0.722	1.000	1.000	1.000	0.720	1.000	0.605	0.720	1.000
U15	0.670	0.967	0.928	1.000	0.676	1.000	0.218	0.481	0.720	0.720	0.752	0.720	0.720	0.168	1.000	0.530	1.000	0.186	0.720	1.000
U16	0.662	0.670	0.679	1.000	0.554	1.000	0.343	0.421	0.882	0.997	0.546	0.383	0.638	0.230	0.731	1.000	1.000	0.120	0.769	1.000
U17	0.275	0.280	0.430	0.720	0.106	0.720	0.002	0.218	0.243	0.385	0.083	0.000	0.068	0.000	0.168	0.481	1.000	0.000	0.310	0.921
U18	1.000	1.000	1.000	1.000	1.000	1.000	0.720	0.720	1.000	1.000	1.000	1.000	1.000	0.851	1.000	0.992	1.000	1.000	1.000	1.000
U19	0.670	0.890	1.000	1.000	1.000	1.000	0.170	0.524	0.885	1.000	0.802	0.790	1.000	0.223	0.950	0.770	1.000	0.232	1.000	1.000
U20	0.184	0.435	0.426	0.980	0.117	0.794	0.087	0.180	0.332	0.401	0.089	0.062	0.250	0.000	0.298	0.452	0.809	0.000	0.431	1.000

Table 34: Outranking values with OWAWA beta=0.5 conjunctive (**OWAWA.5c**) for universities

	U1	U2	U3	U4	U5	U6	U7	U8	U9	U10	U11	U12	U13	U14	U15	U16	U17	U18	U19	U20
U1	1.000	0.896	0.827	1.000	0.866	1.000	0.635	0.866	0.752	0.814	0.961	0.661	0.850	0.104	0.822	0.752	1.000	0.466	0.776	1.000
U2	0.732	1.000	0.562	0.988	0.562	0.878	0.320	0.668	0.562	0.628	0.593	0.562	0.562	0.000	0.562	0.613	0.922	0.091	0.579	0.942
U3	0.676	0.852	1.000	1.000	0.687	1.000	0.403	0.497	0.712	0.701	0.736	0.611	0.734	0.403	0.809	0.745	1.000	0.403	0.727	1.000
U4	0.470	0.722	0.546	1.000	0.014	0.895	0.000	0.269	0.312	0.503	0.024	0.000	0.000	0.000	0.211	0.594	0.893	0.000	0.195	0.989
U5	0.963	0.896	1.000	1.000	1.000	1.000	0.403	0.859	0.752	0.995	1.000	0.671	0.913	0.592	0.864	0.752	1.000	0.463	0.824	1.000
U6	0.684	0.765	0.755	1.000	0.442	1.000	0.403	0.403	0.635	0.624	0.381	0.305	0.559	0.009	0.622	0.669	1.000	0.027	0.650	1.000
U7	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.866	1.000	1.000	1.000	1.000	1.000	0.966	1.000	1.000	1.000	0.896	1.000	1.000
U8	1.000	0.896	0.927	1.000	0.963	1.000	0.734	1.000	0.896	1.000	0.960	0.781	1.000	0.378	0.950	0.896	1.000	0.594	1.000	1.000
U9	0.836	0.942	1.000	1.000	0.934	1.000	0.555	0.692	1.000	1.000	0.836	0.936	0.992	0.573	0.988	0.815	1.000	0.583	1.000	1.000
U10	0.836	0.743	0.949	1.000	0.834	1.000	0.414	0.692	0.823	1.000	0.830	0.428	0.818	0.424	0.959	0.752	1.000	0.112	0.800	1.000
U11	0.993	0.896	0.931	1.000	0.866	1.000	0.535	0.856	0.752	0.861	1.000	0.668	0.866	0.403	0.884	0.752	1.000	0.595	0.850	1.000
U12	0.926	1.000	1.000	1.000	1.000	1.000	0.642	0.866	1.000	1.000	1.000	1.000	1.000	0.640	1.000	0.890	1.000	0.657	1.000	1.000
U13	0.910	0.896	1.000	1.000	1.000	1.000	0.403	0.861	0.896	1.000	1.000	0.826	1.000	0.583	0.949	0.752	1.000	0.526	0.938	1.000
U14	1.000	1.000	1.000	1.000	1.000	1.000	0.866	0.866	0.941	1.000	1.000	0.867	1.000	1.000	1.000	0.866	1.000	0.796	0.866	1.000
U15	0.836	0.986	0.965	1.000	0.840	1.000	0.472	0.722	0.866	0.866	0.877	0.866	0.866	0.386	1.000	0.752	1.000	0.419	0.866	1.000
U16	0.831	0.836	0.828	1.000	0.766	1.000	0.604	0.684	0.939	0.998	0.762	0.639	0.816	0.532	0.854	1.000	1.000	0.373	0.885	1.000
U17	0.537	0.562	0.692	0.866	0.316	0.866	0.004	0.463	0.490	0.619	0.290	0.000	0.225	0.000	0.367	0.709	1.000	0.000	0.468	0.962
U18	1.000	1.000	1.000	1.000	1.000	1.000	0.866	0.866	1.000	1.000	1.000	1.000	1.000	0.926	1.000	0.996	1.000	1.000	1.000	1.000
U19	0.836	0.950	1.000	1.000	1.000	1.000	0.403	0.748	0.948	1.000	0.901	0.889	1.000	0.480	0.976	0.896	1.000	0.490	1.000	1.000
U20	0.423	0.677	0.684	0.990	0.245	0.896	0.206	0.417	0.566	0.640	0.231	0.143	0.378	0.000	0.537	0.685	0.914	0.000	0.666	1.000

Table 35: Outranking values with OWAWA beta=0.7 disjunctive (**OWAWA.7d**) for universities

	U1	U2	U3	U4	U5	U6	U7	U8	U9	U10	U11	U12	U13	U14	U15	U16	U17	U18	U19	U20
U1	1.000	0.718	0.624	1.000	0.688	1.000	0.309	0.688	0.462	0.586	0.913	0.372	0.657	0.033	0.587	0.462	1.000	0.168	0.510	1.000
U2	0.470	1.000	0.232	0.967	0.232	0.750	0.054	0.360	0.232	0.311	0.270	0.232	0.232	0.000	0.232	0.311	0.789	0.027	0.252	0.843
U3	0.378	0.646	1.000	1.000	0.387	1.000	0.118	0.187	0.409	0.395	0.457	0.283	0.439	0.118	0.547	0.453	1.000	0.118	0.429	1.000
U4	0.165	0.432	0.248	1.000	0.002	0.777	0.000	0.079	0.091	0.197	0.004	0.000	0.000	0.000	0.062	0.272	0.735	0.000	0.079	0.970
U5	0.923	0.718	1.000	1.000	1.000	1.000	0.118	0.675	0.462	0.989	1.000	0.356	0.798	0.262	0.662	0.462	1.000	0.166	0.605	1.000
U6	0.382	0.503	0.517	1.000	0.238	1.000	0.118	0.118	0.309	0.294	0.157	0.139	0.260	0.003	0.292	0.353	1.000	0.008	0.328	1.000
U7	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.688	1.000	1.000	1.000	1.000	1.000	0.930	1.000	1.000	1.000	0.718	1.000	1.000
U8	1.000	0.718	0.848	1.000	0.924	1.000	0.444	1.000	0.718	1.000	0.916	0.539	1.000	0.092	0.865	0.718	1.000	0.267	1.000	1.000
U9	0.658	0.880	1.000	1.000	0.863	1.000	0.227	0.402	1.000	1.000	0.658	0.854	0.983	0.246	0.967	0.592	1.000	0.254	1.000	1.000
U10	0.658	0.474	0.899	1.000	0.655	1.000	0.126	0.402	0.604	1.000	0.648	0.197	0.627	0.129	0.916	0.462	1.000	0.033	0.581	1.000
U11	0.981	0.718	0.841	1.000	0.688	1.000	0.222	0.668	0.462	0.679	1.000	0.351	0.688	0.118	0.719	0.462	1.000	0.270	0.657	1.000
U12	0.846	1.000	1.000	1.000	1.000	1.000	0.342	0.688	1.000	1.000	1.000	1.000	1.000	0.324	1.000	0.747	1.000	0.348	1.000	1.000
U13	0.812	0.718	1.000	1.000	1.000	1.000	0.118	0.681	0.718	1.000	1.000	0.586	1.000	0.254	0.878	0.462	1.000	0.211	0.855	1.000
U14	1.000	1.000	1.000	1.000	1.000	1.000	0.688	0.688	0.862	1.000	1.000	0.691	1.000	1.000	1.000	0.688	1.000	0.551	0.688	1.000
U15	0.658	0.967	0.919	1.000	0.645	1.000	0.167	0.452	0.688	0.688	0.743	0.688	0.688	0.121	1.000	0.462	1.000	0.137	0.688	1.000
U16	0.649	0.658	0.673	1.000	0.544	1.000	0.301	0.392	0.878	0.997	0.537	0.361	0.626	0.202	0.712	1.000	1.000	0.088	0.760	1.000
U17	0.242	0.232	0.402	0.688	0.100	0.688	0.001	0.169	0.191	0.311	0.076	0.000	0.057	0.000	0.119	0.406	1.000	0.000	0.265	0.912
U18	1.000	1.000	1.000	1.000	1.000	1.000	0.688	0.688	1.000	1.000	1.000	1.000	1.000	0.846	1.000	0.991	1.000	1.000	1.000	1.000
U19	0.658	0.865	1.000	1.000	1.000	1.000	0.118	0.495	0.859	1.000	0.795	0.747	1.000	0.173	0.935	0.718	1.000	0.181	1.000	1.000
U20	0.132	0.387	0.395	0.976	0.099	0.759	0.060	0.128	0.262	0.315	0.074	0.043	0.212	0.000	0.224	0.374	0.766	0.000	0.349	1.000

Table 36: Outranking values with OWAWA beta=0.7 conjunctive (**OWAWA.7c**) for universities

	U1	**U2**	**U3**	**U4**	**U5**	**U6**	**U7**	**U8**	**U9**	**U10**	**U11**	**U12**	**U13**	**U14**	**U15**	**U16**	**U17**	**U18**	**U19**	**U20**
U1	1.000	0.887	0.788	1.000	0.858	1.000	0.643	0.858	0.745	0.807	0.956	0.632	0.842	0.085	0.814	0.745	1.000	0.407	0.769	1.000
U2	0.597	1.000	0.429	0.987	0.429	0.811	0.105	0.604	0.429	0.538	0.456	0.429	0.429	0.000	0.429	0.591	0.915	0.071	0.457	0.937
U3	0.532	0.808	1.000	1.000	0.644	1.000	0.316	0.387	0.710	0.700	0.600	0.616	0.730	0.316	0.801	0.739	1.000	0.316	0.723	1.000
U4	0.282	0.571	0.374	1.000	0.005	0.844	0.000	0.211	0.245	0.460	0.008	0.000	0.000	0.000	0.166	0.588	0.887	0.000	0.182	0.988
U5	0.932	0.887	1.000	1.000	1.000	1.000	0.316	0.851	0.745	0.995	1.000	0.674	0.908	0.458	0.856	0.745	1.000	0.403	0.817	1.000
U6	0.538	0.650	0.688	1.000	0.382	1.000	0.316	0.316	0.643	0.633	0.297	0.304	0.520	0.007	0.632	0.672	1.000	0.021	0.656	1.000
U7	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.858	1.000	1.000	1.000	1.000	1.000	0.939	1.000	1.000	1.000	0.887	1.000	1.000
U8	1.000	0.887	0.859	1.000	0.930	1.000	0.685	1.000	0.887	1.000	0.923	0.706	1.000	0.149	0.946	0.887	1.000	0.463	1.000	1.000
U9	0.684	0.889	1.000	1.000	0.874	1.000	0.424	0.542	1.000	1.000	0.684	0.911	0.984	0.430	0.987	0.808	1.000	0.444	1.000	1.000
U10	0.684	0.592	0.902	1.000	0.680	1.000	0.324	0.542	0.817	1.000	0.673	0.329	0.667	0.215	0.925	0.745	1.000	0.088	0.739	1.000
U11	0.992	0.887	0.927	1.000	0.858	1.000	0.506	0.848	0.745	0.853	1.000	0.671	0.858	0.316	0.878	0.745	1.000	0.592	0.842	1.000
U12	0.858	1.000	1.000	1.000	1.000	1.000	0.500	0.858	1.000	1.000	1.000	1.000	1.000	0.499	1.000	0.885	1.000	0.518	1.000	1.000
U13	0.826	0.887	1.000	1.000	1.000	1.000	0.316	0.850	0.887	1.000	1.000	0.819	1.000	0.444	0.947	0.745	1.000	0.408	0.934	1.000
U14	1.000	1.000	1.000	1.000	1.000	1.000	0.858	0.858	0.937	1.000	1.000	0.859	1.000	1.000	1.000	0.858	1.000	0.789	0.858	1.000
U15	0.684	0.968	0.963	1.000	0.810	1.000	0.365	0.597	0.858	0.858	0.763	0.858	0.858	0.301	1.000	0.745	1.000	0.326	0.858	1.000
U16	0.679	0.684	0.677	1.000	0.558	1.000	0.446	0.535	0.884	0.998	0.550	0.403	0.649	0.255	0.763	1.000	1.000	0.142	0.779	1.000
U17	0.387	0.429	0.542	0.858	0.136	0.858	0.003	0.361	0.398	0.602	0.114	0.000	0.103	0.000	0.304	0.708	1.000	0.000	0.449	0.960
U18	1.000	1.000	1.000	1.000	1.000	1.000	0.858	0.858	1.000	1.000	1.000	1.000	1.000	0.858	1.000	0.996	1.000	1.000	1.000	1.000
U19	0.684	0.946	1.000	1.000	1.000	1.000	0.316	0.645	0.943	1.000	0.810	0.883	1.000	0.296	0.974	0.887	1.000	0.310	1.000	1.000
U20	0.329	0.556	0.544	0.989	0.174	0.891	0.161	0.326	0.534	0.647	0.138	0.114	0.366	0.000	0.509	0.686	0.906	0.000	0.670	1.000

Table 37: Outranking values with IOWA disjunctive (**IOWAd**) for universities

	U1	U2	U3	U4	U5	U6	U7	U8	U9	U10	U11	U12	U13	U14	U15	U16	U17	U18	U19	U20
U1	1.000	0.680	0.782	1.000	0.800	1.000	0.192	0.800	0.480	0.656	0.946	0.378	0.756	0.021	0.577	0.480	1.000	0.103	0.549	1.000
U2	0.607	1.000	0.400	0.963	0.400	0.826	0.298	0.444	0.400	0.427	0.438	0.400	0.400	0.000	0.400	0.262	0.761	0.018	0.407	0.822
U3	0.490	0.660	1.000	1.000	0.440	1.000	0.080	0.261	0.381	0.354	0.601	0.156	0.437	0.080	0.438	0.464	1.000	0.080	0.418	1.000
U4	0.236	0.600	0.510	1.000	0.013	0.829	0.000	0.053	0.062	0.116	0.022	0.000	0.000	0.000	0.042	0.161	0.681	0.000	0.046	0.966
U5	0.971	0.680	1.000	1.000	1.000	1.000	0.080	0.781	0.480	0.986	1.000	0.280	0.870	0.200	0.592	0.480	1.000	0.102	0.683	1.000
U6	0.506	0.620	0.681	1.000	0.284	1.000	0.080	0.080	0.192	0.164	0.214	0.077	0.211	0.002	0.160	0.274	1.000	0.005	0.228	1.000
U7	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.800	1.000	1.000	1.000	1.000	1.000	0.971	1.000	1.000	1.000	0.680	1.000	1.000
U8	1.000	0.680	0.964	1.000	0.982	1.000	0.372	1.000	0.680	1.000	0.980	0.567	1.000	0.202	0.847	0.680	1.000	0.281	1.000	1.000
U9	0.920	0.972	1.000	1.000	0.968	1.000	0.385	0.720	1.000	1.000	0.920	0.871	0.996	0.273	0.963	0.635	1.000	0.276	1.000	1.000
U10	0.920	0.660	0.975	1.000	0.919	1.000	0.103	0.720	0.608	1.000	0.917	0.344	0.896	0.219	0.972	0.480	1.000	0.022	0.770	1.000
U11	0.978	0.680	0.898	1.000	0.800	1.000	0.128	0.772	0.480	0.787	1.000	0.271	0.800	0.080	0.656	0.480	1.000	0.150	0.756	1.000
U12	0.964	1.000	1.000	1.000	1.000	1.000	0.602	0.800	1.000	1.000	1.000	1.000	1.000	0.397	1.000	0.742	1.000	0.430	1.000	1.000
U13	0.956	0.680	1.000	1.000	1.000	1.000	0.080	0.798	0.680	1.000	1.000	0.498	1.000	0.276	0.850	0.480	1.000	0.210	0.907	1.000
U14	1.000	1.000	1.000	1.000	1.000	1.000	0.800	0.800	0.912	1.000	1.000	0.802	1.000	1.000	1.000	0.800	1.000	0.606	0.800	1.000
U15	0.920	0.992	0.948	1.000	0.788	1.000	0.219	0.734	0.800	0.800	0.940	0.800	0.800	0.111	1.000	0.480	1.000	0.145	0.800	1.000
U16	0.907	0.920	0.918	1.000	0.888	1.000	0.485	0.701	0.971	0.996	0.886	0.667	0.911	0.520	0.831	1.000	1.000	0.200	0.944	1.000
U17	0.464	0.400	0.720	0.800	0.387	0.800	0.001	0.207	0.222	0.248	0.323	0.000	0.197	0.000	0.077	0.375	1.000	0.000	0.268	0.944
U18	1.000	1.000	1.000	1.000	1.000	1.000	0.800	0.800	1.000	1.000	1.000	1.000	1.000	0.964	1.000	0.994	1.000	1.000	1.000	1.000
U19	0.920	0.847	1.000	1.000	1.000	1.000	0.080	0.746	0.840	1.000	0.952	0.687	1.000	0.239	0.926	0.680	1.000	0.243	1.000	1.000
U20	0.117	0.524	0.681	0.985	0.184	0.754	0.041	0.109	0.200	0.204	0.184	0.029	0.208	0.000	0.129	0.314	0.735	0.000	0.268	1.000

Table 38: Outranking values with IOWA conjunctive (**IOWAc**) for universities

	U1	U2	U3	U4	U5	U6	U7	U8	U9	U10	U11	U12	U13	U14	U15	U16	U17	U18	U19	U20
U1	1.000	0.949	0.853	1.000	0.894	1.000	0.781	0.894	0.837	0.868	0.968	0.761	0.886	0.139	0.891	0.837	1.000	0.613	0.849	1.000
U2	0.776	1.000	0.632	0.994	0.632	0.900	0.295	0.746	0.632	0.703	0.660	0.632	0.632	0.000	0.632	0.738	0.961	0.123	0.650	0.971
U3	0.751	0.905	1.000	1.000	0.771	1.000	0.548	0.600	0.817	0.812	0.790	0.763	0.828	0.548	0.908	0.833	1.000	0.548	0.825	1.000
U4	0.562	0.774	0.561	1.000	0.013	0.919	0.000	0.365	0.424	0.651	0.022	0.000	0.000	0.000	0.287	0.742	0.948	0.000	0.251	0.994
U5	0.965	0.949	1.000	1.000	1.000	1.000	0.548	0.891	0.837	0.998	1.000	0.798	0.931	0.707	0.933	0.837	1.000	0.610	0.873	1.000
U6	0.755	0.818	0.792	1.000	0.515	1.000	0.548	0.548	0.781	0.775	0.458	0.382	0.689	0.012	0.774	0.797	1.000	0.036	0.788	1.000
U7	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.894	1.000	1.000	1.000	1.000	1.000	0.968	1.000	1.000	1.000	0.949	1.000	1.000
U8	1.000	0.949	0.927	1.000	0.964	1.000	0.840	1.000	0.949	1.000	0.960	0.848	1.000	0.453	0.975	0.949	1.000	0.706	1.000	1.000
U9	0.837	0.943	1.000	1.000	0.935	1.000	0.629	0.707	1.000	1.000	0.837	0.955	0.992	0.684	0.994	0.873	1.000	0.695	1.000	1.000
U10	0.837	0.786	0.946	1.000	0.834	1.000	0.554	0.707	0.887	1.000	0.830	0.469	0.821	0.508	0.961	0.837	1.000	0.152	0.824	1.000
U11	0.996	0.949	0.946	1.000	0.894	1.000	0.684	0.889	0.837	0.892	1.000	0.796	0.894	0.548	0.943	0.837	1.000	0.746	0.886	1.000
U12	0.926	1.000	1.000	1.000	1.000	1.000	0.679	0.894	1.000	1.000	1.000	1.000	1.000	0.732	1.000	0.932	1.000	0.743	1.000	1.000
U13	0.910	0.949	1.000	1.000	1.000	1.000	0.548	0.889	0.949	1.000	1.000	0.913	1.000	0.695	0.975	0.837	1.000	0.651	0.951	1.000
U14	1.000	1.000	1.000	1.000	1.000	1.000	0.894	0.894	0.953	1.000	1.000	0.895	1.000	1.000	1.000	0.894	1.000	0.859	0.894	1.000
U15	0.837	0.984	0.972	1.000	0.866	1.000	0.584	0.740	0.894	0.894	0.877	0.894	0.894	0.509	1.000	0.837	1.000	0.541	0.894	1.000
U16	0.834	0.837	0.817	1.000	0.755	1.000	0.649	0.703	0.937	0.999	0.750	0.648	0.814	0.548	0.872	1.000	1.000	0.447	0.886	1.000
U17	0.570	0.632	0.707	0.894	0.283	0.894	0.006	0.580	0.605	0.747	0.262	0.000	0.217	0.000	0.491	0.816	1.000	0.000	0.537	0.970
U18	1.000	1.000	1.000	1.000	1.000	1.000	0.894	0.894	1.000	1.000	1.000	1.000	1.000	0.926	1.000	0.997	1.000	1.000	1.000	1.000
U19	0.837	0.975	1.000	1.000	1.000	1.000	0.548	0.768	0.974	1.000	0.902	0.944	1.000	0.574	0.988	0.949	1.000	0.586	1.000	1.000
U20	0.558	0.738	0.707	0.992	0.261	0.936	0.280	0.555	0.700	0.783	0.242	0.194	0.437	0.000	0.686	0.804	0.957	0.000	0.796	1.000

Table 39: Outranking values with WOWA disjunctive (**WOWAd**) for universities

	U1	U2	U3	U4	U5	U6	U7	U8	U9	U10	U11	U12	U13	U14	U15	U16	U17	U18	U19	U20
U1	1.000	0.810	0.553	1.000	0.640	1.000	0.373	0.640	0.490	0.572	0.898	0.384	0.619	0.029	0.646	0.490	1.000	0.168	0.522	1.000
U2	0.442	1.000	0.160	0.978	0.160	0.717	0.009	0.343	0.160	0.274	0.198	0.160	0.160	0.000	0.160	0.338	0.858	0.020	0.189	0.894
U3	0.324	0.698	1.000	1.000	0.387	1.000	0.090	0.137	0.450	0.439	0.403	0.346	0.472	0.090	0.681	0.483	1.000	0.090	0.465	1.000
U4	0.133	0.360	0.149	1.000	0.000	0.764	0.000	0.060	0.069	0.213	0.001	0.000	0.000	0.000	0.047	0.323	0.816	0.000	0.088	0.980
U5	0.892	0.810	1.000	1.000	1.000	1.000	0.090	0.631	0.490	0.992	1.000	0.409	0.767	0.250	0.765	0.490	1.000	0.164	0.585	1.000
U6	0.328	0.472	0.443	1.000	0.199	1.000	0.090	0.090	0.373	0.362	0.113	0.167	0.274	0.002	0.360	0.406	1.000	0.006	0.388	1.000
U7	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.640	1.000	1.000	1.000	1.000	1.000	0.905	1.000	1.000	1.000	0.810	1.000	1.000
U8	1.000	0.810	0.773	1.000	0.887	1.000	0.533	1.000	0.810	1.000	0.875	0.564	1.000	0.045	0.909	0.810	1.000	0.265	1.000	1.000
U9	0.490	0.821	1.000	1.000	0.796	1.000	0.157	0.250	1.000	1.000	0.490	0.863	0.974	0.231	0.978	0.595	1.000	0.241	1.000	1.000
U10	0.490	0.384	0.856	1.000	0.486	1.000	0.095	0.250	0.641	1.000	0.479	0.131	0.461	0.089	0.881	0.490	1.000	0.025	0.494	1.000
U11	0.987	0.810	0.816	1.000	0.640	1.000	0.252	0.627	0.490	0.634	1.000	0.405	0.640	0.090	0.804	0.490	1.000	0.326	0.619	1.000
U12	0.770	1.000	1.000	1.000	1.000	1.000	0.218	0.640	1.000	1.000	1.000	1.000	1.000	0.290	1.000	0.781	1.000	0.312	1.000	1.000
U13	0.719	0.810	1.000	1.000	1.000	1.000	0.090	0.630	0.810	1.000	1.000	0.704	1.000	0.241	0.914	0.490	1.000	0.194	0.833	1.000
U14	1.000	1.000	1.000	1.000	1.000	1.000	0.640	0.640	0.841	1.000	1.000	0.643	1.000	1.000	1.000	0.640	1.000	0.549	0.640	1.000
U15	0.490	0.949	0.907	1.000	0.581	1.000	0.120	0.318	0.640	0.640	0.617	0.640	0.640	0.090	1.000	0.490	1.000	0.101	0.640	1.000
U16	0.485	0.490	0.551	1.000	0.358	1.000	0.233	0.244	0.825	0.998	0.351	0.226	0.453	0.090	0.653	1.000	1.000	0.040	0.643	1.000
U17	0.153	0.160	0.250	0.640	0.023	0.640	0.001	0.120	0.156	0.346	0.016	0.000	0.032	0.000	0.106	0.447	1.000	0.000	0.276	0.898
U18	1.000	1.000	1.000	1.000	1.000	1.000	0.640	0.640	1.000	1.000	1.000	1.000	1.000	0.770	1.000	0.989	1.000	1.000	1.000	1.000
U19	0.490	0.909	1.000	1.000	1.000	1.000	0.090	0.377	0.905	1.000	0.694	0.816	1.000	0.143	0.956	0.810	1.000	0.152	1.000	1.000
U20	0.099	0.339	0.259	0.973	0.079	0.793	0.046	0.096	0.284	0.378	0.043	0.034	0.224	0.000	0.255	0.423	0.843	0.000	0.404	1.000

Table 40: Outranking values with WOWA conjunctive (**WOWAc**) for universities

8.2 Concordance values for the countries dataset

IDENTFIERS
A - Afghanistan
B - Laos
C - Mongolia
D - Philippines
E - Sri_Lanka
F - Thailand
G - Barbados
H - Cuba
I - Trinidad_and_Tobago
J - Georgia
K - Moldova
L - Poland
M - Germany
N - Argentina
O - Chile
P - Panama
Q - Venezuela
R - Kuwait
S - Saudi_Arabia
T - Kiribati
U - Samoa
V- Rwanda
W - Seychelles
X - Zambia

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
A	1.000	0.555	0.120	0.120	0.195	0.395	0.255	0.120	0.120	0.120	0.104	0.120	0.047	0.075	0.395	0.120	0.129	0.315	0.120	0.255	0.159	0.605	0.120	0.530
B	0.605	1.000	0.480	0.369	0.436	0.480	0.480	0.405	0.315	0.480	0.360	0.195	0.195	0.435	0.678	0.315	0.325	0.315	0.480	0.423	0.568	0.616	0.285	0.399
C	0.880	0.636	1.000	0.766	0.210	0.770	0.668	0.540	0.375	0.815	0.757	0.275	0.137	0.435	0.475	0.362	0.456	0.305	0.535	0.760	0.909	0.685	0.495	0.676
D	0.880	0.805	0.729	1.000	0.284	0.755	0.494	0.405	0.315	0.755	0.710	0.275	0.159	0.435	0.578	0.434	0.348	0.337	0.480	0.813	0.881	0.771	0.360	0.685
E	0.880	0.748	0.790	0.805	1.000	0.710	0.581	0.456	0.372	0.778	0.670	0.486	0.195	0.470	0.670	0.485	0.485	0.305	0.470	0.721	0.925	0.880	0.320	0.640
F	0.605	0.642	0.486	0.305	0.321	1.000	0.725	0.517	0.371	0.539	0.402	0.035	0.000	0.510	0.503	0.349	0.505	0.318	0.549	0.485	0.512	0.685	0.530	0.349
G	0.880	0.520	0.449	0.520	0.485	0.309	1.000	0.410	0.437	0.339	0.345	0.275	0.275	0.385	0.310	0.481	0.589	0.580	0.385	0.546	0.599	0.520	0.535	0.502
H	0.880	0.681	0.693	0.638	0.560	0.784	0.690	1.000	0.491	0.824	0.750	0.350	0.275	0.785	0.550	0.530	0.508	0.580	0.748	0.835	0.769	0.760	0.659	0.665
I	0.880	0.726	0.686	0.704	0.738	0.762	0.680	0.822	1.000	0.782	0.677	0.310	0.275	0.801	0.475	0.712	0.715	0.745	0.745	0.894	0.760	0.685	0.706	0.651
J	0.880	0.535	0.304	0.245	0.272	0.751	0.684	0.411	0.457	1.000	0.553	0.303	0.000	0.545	0.385	0.320	0.515	0.305	0.539	0.440	0.320	0.655	0.447	0.588
K	0.925	0.640	0.399	0.398	0.330	0.800	0.725	0.436	0.442	0.780	1.000	0.320	0.000	0.380	0.505	0.365	0.560	0.356	0.452	0.729	0.412	0.565	0.365	0.613
L	0.912	0.805	0.725	0.725	0.530	0.965	0.725	0.680	0.718	0.896	0.697	1.000	0.342	0.576	0.833	0.725	0.725	0.657	0.725	0.725	0.725	0.805	0.530	0.850
M	1.000	0.805	1.000	1.000	0.805	1.000	0.725	0.869	0.725	1.000	1.000	0.805	1.000	0.929	0.805	1.000	0.725	1.000	1.000	1.000	1.000	0.805	0.588	0.940
N	0.925	0.565	0.565	0.565	0.530	0.651	0.644	0.552	0.319	0.581	0.794	0.455	0.320	1.000	0.490	0.490	0.410	0.713	0.850	0.760	0.565	0.565	0.445	0.565
O	0.605	0.391	0.620	0.485	0.437	0.613	0.690	0.556	0.525	0.670	0.573	0.303	0.195	0.515	1.000	0.525	0.552	0.478	0.615	0.545	0.594	0.596	0.495	0.485
P	0.880	0.685	0.837	0.871	0.517	0.756	0.555	0.494	0.407	0.754	0.636	0.275	0.150	0.510	0.519	1.000	0.341	0.734	0.688	0.915	0.963	0.685	0.335	0.715
Q	0.880	0.685	0.574	0.685	0.582	0.515	0.480	0.560	0.366	0.548	0.480	0.435	0.417	0.590	0.545	0.695	1.000	0.733	0.606	0.710	0.695	0.677	0.635	0.601
R	0.831	0.709	0.721	0.706	0.695	0.695	0.506	0.420	0.334	0.695	0.650	0.392	0.033	0.315	0.629	0.491	0.290	1.000	0.474	0.722	0.695	0.717	0.495	0.565
S	0.880	0.532	0.565	0.529	0.530	0.530	0.615	0.270	0.255	0.530	0.650	0.282	0.045	0.169	0.385	0.530	0.521	0.656	1.000	0.634	0.547	0.540	0.330	0.565
T	0.749	0.625	0.449	0.461	0.315	0.630	0.480	0.199	0.179	0.572	0.477	0.275	0.000	0.320	0.509	0.275	0.305	0.450	0.449	1.000	0.513	0.685	0.345	0.445
U	0.880	0.642	0.754	0.584	0.078	0.635	0.480	0.422	0.240	0.680	0.635	0.275	0.182	0.435	0.475	0.242	0.305	0.305	0.470	0.680	1.000	0.685	0.340	0.640
V	0.470	0.497	0.338	0.338	0.352	0.588	0.480	0.240	0.315	0.480	0.435	0.212	0.195	0.435	0.603	0.315	0.350	0.347	0.483	0.338	0.315	1.000	0.277	0.477
W	0.880	0.715	0.621	0.640	0.680	0.576	0.573	0.685	0.375	0.770	0.729	0.470	0.470	0.670	0.505	0.685	0.523	0.505	0.694	0.685	0.760	0.880	1.000	0.640
X	0.579	0.626	0.465	0.420	0.360	0.755	0.555	0.480	0.480	0.480	0.435	0.240	0.212	0.435	0.531	0.377	0.501	0.466	0.469	0.555	0.420	0.612	0.360	1.000

Table 41: Outranking values with weighted average (**WA**) for countries dataset

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
A	1.000	0.895	0.760	0.760	0.820	0.820	0.802	0.760	0.760	0.760	0.725	0.760	0.414	0.660	0.820	0.760	0.763	0.820	0.760	0.820	0.777	0.910	0.760	0.870
B	0.910	1.000	0.870	0.812	0.881	0.870	0.870	0.820	0.820	0.870	0.760	0.660	0.660	0.820	0.862	0.820	0.834	0.820	0.870	0.852	0.896	0.911	0.820	0.854
C	0.940	0.898	1.000	0.919	0.760	0.870	0.931	0.870	0.820	0.910	0.865	0.660	0.463	0.820	0.820	0.816	0.884	0.820	0.869	0.940	0.960	0.910	0.910	0.889
D	0.940	0.970	0.924	1.000	0.847	0.910	0.877	0.820	0.820	0.910	0.870	0.660	0.537	0.820	0.891	0.864	0.868	0.851	0.870	0.951	0.960	0.952	0.870	0.910
E	0.940	0.935	0.940	0.910	1.000	0.870	0.925	0.875	0.848	0.930	0.870	0.787	0.660	0.870	0.870	0.910	0.910	0.820	0.870	0.924	0.970	0.940	0.870	0.870
F	0.910	0.903	0.930	0.838	0.886	1.000	0.970	0.920	0.903	0.923	0.829	0.660	0.000	0.903	0.839	0.901	0.918	0.834	0.904	0.910	0.937	0.910	0.940	0.870
G	0.940	0.870	0.863	0.870	0.820	0.685	1.000	0.814	0.812	0.769	0.771	0.660	0.660	0.820	0.760	0.860	0.884	0.870	0.820	0.893	0.917	0.870	0.896	0.858
H	0.940	0.926	0.941	0.918	0.870	0.892	0.940	1.000	0.903	0.925	0.882	0.760	0.660	0.910	0.870	0.870	0.885	0.870	0.911	0.970	0.963	0.940	0.920	0.911
I	0.940	0.926	0.921	0.913	0.931	0.885	0.940	0.920	1.000	0.918	0.872	0.760	0.660	0.932	0.820	0.914	0.910	0.910	0.910	0.946	0.940	0.910	0.928	0.897
J	0.940	0.878	0.864	0.820	0.838	0.908	0.943	0.863	0.898	1.000	0.887	0.740	0.000	0.910	0.820	0.870	0.910	0.820	0.890	0.870	0.870	0.903	0.901	0.884
K	0.970	0.940	0.914	0.886	0.870	0.940	0.970	0.900	0.893	0.937	1.000	0.760	0.000	0.870	0.910	0.910	0.940	0.872	0.901	0.958	0.915	0.910	0.910	0.917
L	0.962	0.970	0.970	0.970	0.940	0.970	0.970	0.952	0.964	0.989	0.951	1.000	0.749	0.917	0.974	0.970	0.970	0.943	0.970	0.970	0.970	0.970	0.940	0.957
M	1.000	0.970	1.000	1.000	0.970	1.000	0.970	0.986	0.970	1.000	1.000	0.970	1.000	0.967	0.970	1.000	0.970	1.000	1.000	1.000	1.000	0.970	0.946	0.989
N	0.970	0.910	0.910	0.910	0.870	0.871	0.935	0.906	0.836	0.852	0.925	0.820	0.760	1.000	0.870	0.870	0.870	0.921	0.940	0.940	0.910	0.910	0.914	0.910
O	0.910	0.915	0.940	0.910	0.892	0.926	0.940	0.896	0.910	0.932	0.868	0.740	0.660	0.871	1.000	0.910	0.937	0.902	0.910	0.927	0.923	0.928	0.910	0.910
P	0.940	0.910	0.951	0.939	0.821	0.895	0.910	0.891	0.854	0.906	0.821	0.660	0.508	0.870	0.849	1.000	0.844	0.907	0.902	0.963	0.992	0.910	0.842	0.910
Q	0.940	0.910	0.886	0.910	0.847	0.820	0.870	0.870	0.835	0.862	0.792	0.757	0.746	0.870	0.863	0.910	1.000	0.897	0.843	0.923	0.910	0.900	0.910	0.890
R	0.932	0.926	0.932	0.924	0.910	0.910	0.888	0.870	0.844	0.910	0.870	0.779	0.294	0.820	0.897	0.880	0.870	1.000	0.895	0.933	0.910	0.932	0.910	0.910
S	0.940	0.881	0.910	0.878	0.870	0.870	0.910	0.841	0.820	0.870	0.870	0.669	0.108	0.764	0.820	0.870	0.924	0.909	1.000	0.921	0.889	0.888	0.870	0.910
T	0.911	0.910	0.872	0.838	0.820	0.850	0.870	0.736	0.781	0.872	0.773	0.660	0.000	0.785	0.843	0.817	0.820	0.864	0.840	1.000	0.892	0.910	0.854	0.820
U	0.940	0.900	0.923	0.844	0.662	0.820	0.870	0.849	0.760	0.870	0.820	0.660	0.615	0.820	0.820	0.761	0.820	0.820	0.870	0.905	1.000	0.910	0.848	0.870
V	0.870	0.903	0.852	0.857	0.860	0.870	0.870	0.760	0.820	0.870	0.820	0.683	0.660	0.820	0.890	0.820	0.870	0.865	0.873	0.853	0.820	1.000	0.817	0.880
W	0.940	0.910	0.889	0.870	0.870	0.807	0.908	0.910	0.820	0.897	0.884	0.760	0.760	0.870	0.820	0.910	0.858	0.820	0.877	0.910	0.940	0.940	1.000	0.870
X	0.886	0.911	0.892	0.869	0.870	0.910	0.910	0.870	0.870	0.870	0.820	0.760	0.684	0.820	0.842	0.844	0.894	0.855	0.858	0.910	0.882	0.890	0.870	1.000

Table 42: Outranking values with OWA disjunctive (OWAd) for countries dataset

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
A	1.000	0.068	0.000	0.000	0.010	0.010	0.007	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.010	0.000	0.000	0.010	0.000	0.010	0.003	0.090	0.000	0.030
B	0.090	1.000	0.030	0.014	0.047	0.030	0.030	0.010	0.010	0.030	0.000	0.000	0.000	0.010	0.045	0.010	0.015	0.010	0.030	0.023	0.069	0.094	0.010	0.024
C	0.230	0.072	1.000	0.145	0.000	0.030	0.190	0.030	0.010	0.090	0.028	0.000	0.000	0.010	0.010	0.009	0.051	0.010	0.048	0.230	0.434	0.090	0.090	0.059
D	0.230	0.510	0.240	1.000	0.021	0.090	0.041	0.010	0.010	0.090	0.030	0.000	0.000	0.010	0.072	0.028	0.029	0.022	0.030	0.336	0.431	0.381	0.030	0.090
E	0.230	0.207	0.230	0.090	1.000	0.030	0.159	0.037	0.032	0.182	0.030	0.005	0.000	0.030	0.030	0.090	0.090	0.010	0.030	0.157	0.510	0.230	0.030	0.030
F	0.090	0.102	0.193	0.017	0.055	1.000	0.510	0.135	0.093	0.179	0.018	0.000	0.000	0.079	0.017	0.077	0.127	0.016	0.119	0.090	0.214	0.091	0.230	0.041
G	0.230	0.030	0.038	0.030	0.010	0.000	1.000	0.013	0.009	0.001	0.002	0.000	0.000	0.010	0.000	0.026	0.051	0.030	0.010	0.064	0.161	0.030	0.092	0.025
H	0.230	0.163	0.322	0.126	0.030	0.079	0.230	1.000	0.080	0.162	0.048	0.000	0.000	0.090	0.030	0.030	0.071	0.030	0.093	0.510	0.448	0.230	0.175	0.138
I	0.230	0.166	0.161	0.103	0.189	0.053	0.230	0.168	1.000	0.128	0.033	0.000	0.000	0.194	0.010	0.110	0.090	0.090	0.090	0.282	0.230	0.090	0.174	0.070
J	0.230	0.042	0.028	0.010	0.017	0.087	0.256	0.027	0.145	1.000	0.067	0.000	0.000	0.090	0.010	0.030	0.090	0.010	0.061	0.030	0.030	0.079	0.076	0.051
K	0.510	0.230	0.107	0.054	0.030	0.230	0.510	0.075	0.065	0.216	1.000	0.000	0.000	0.030	0.090	0.090	0.230	0.033	0.081	0.402	0.114	0.090	0.090	0.124
L	0.432	0.510	0.510	0.510	0.230	0.510	0.510	0.341	0.451	0.815	0.334	1.000	0.000	0.122	0.580	0.510	0.510	0.256	0.510	0.510	0.510	0.510	0.230	0.397
M	1.000	0.510	1.000	1.000	0.510	1.000	0.510	0.767	0.510	1.000	1.000	0.510	1.000	0.483	0.510	1.000	0.510	1.000	1.000	1.000	1.000	0.510	0.289	0.821
N	0.510	0.090	0.090	0.090	0.030	0.031	0.205	0.136	0.017	0.023	0.183	0.010	0.000	1.000	0.030	0.030	0.030	0.142	0.230	0.230	0.090	0.090	0.126	0.090
O	0.090	0.112	0.294	0.090	0.063	0.164	0.230	0.068	0.090	0.193	0.048	0.000	0.000	0.032	1.000	0.090	0.215	0.078	0.090	0.169	0.149	0.188	0.090	0.090
P	0.230	0.090	0.335	0.224	0.011	0.067	0.090	0.062	0.037	0.115	0.010	0.000	0.000	0.030	0.022	1.000	0.020	0.086	0.078	0.446	0.888	0.090	0.019	0.090
Q	0.230	0.090	0.054	0.090	0.026	0.010	0.030	0.030	0.016	0.027	0.005	0.000	0.000	0.030	0.027	0.090	1.000	0.070	0.019	0.149	0.090	0.075	0.090	0.060
R	0.195	0.165	0.193	0.154	0.090	0.090	0.057	0.030	0.020	0.090	0.030	0.003	0.000	0.010	0.092	0.045	0.030	1.000	0.095	0.196	0.090	0.191	0.090	0.090
S	0.230	0.046	0.090	0.042	0.030	0.030	0.090	0.019	0.010	0.030	0.030	0.000	0.000	0.001	0.010	0.030	0.157	0.118	1.000	0.139	0.059	0.057	0.030	0.090
T	0.094	0.090	0.065	0.020	0.010	0.022	0.030	0.000	0.004	0.034	0.003	0.000	0.000	0.004	0.019	0.009	0.010	0.028	0.018	1.000	0.063	0.090	0.023	0.010
U	0.230	0.074	0.156	0.020	0.000	0.010	0.030	0.022	0.000	0.030	0.010	0.000	0.000	0.010	0.010	0.000	0.010	0.010	0.030	0.083	1.000	0.090	0.021	0.030
V	0.030	0.079	0.023	0.025	0.026	0.030	0.030	0.000	0.010	0.030	0.010	0.000	0.000	0.010	0.060	0.010	0.030	0.028	0.034	0.023	0.010	1.000	0.010	0.045
W	0.230	0.090	0.062	0.030	0.030	0.008	0.087	0.090	0.010	0.070	0.056	0.000	0.000	0.030	0.010	0.090	0.037	0.010	0.041	0.090	0.230	0.230	1.000	0.030
X	0.054	0.094	0.062	0.048	0.030	0.090	0.090	0.030	0.030	0.030	0.010	0.000	0.000	0.010	0.019	0.019	0.065	0.035	0.025	0.091	0.048	0.080	0.030	1.000

Table 43: Outranking values with OWA conjunctive **(OWAc**) for countries dataset

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
A	1.000	0.657	0.312	0.312	0.383	0.523	0.419	0.312	0.312	0.312	0.291	0.312	0.157	0.251	0.523	0.312	0.319	0.467	0.312	0.425	0.344	0.697	0.312	0.632
B	0.697	1.000	0.597	0.502	0.569	0.597	0.597	0.530	0.467	0.597	0.480	0.335	0.335	0.551	0.733	0.467	0.477	0.467	0.597	0.552	0.667	0.705	0.446	0.536
C	0.898	0.715	1.000	0.812	0.375	0.800	0.747	0.639	0.509	0.844	0.789	0.391	0.235	0.551	0.579	0.498	0.584	0.460	0.635	0.814	0.924	0.753	0.620	0.740
D	0.898	0.855	0.787	1.000	0.453	0.802	0.609	0.530	0.467	0.802	0.758	0.391	0.272	0.551	0.672	0.563	0.504	0.491	0.597	0.855	0.905	0.825	0.513	0.753
E	0.898	0.804	0.835	0.837	1.000	0.758	0.684	0.582	0.515	0.824	0.730	0.576	0.335	0.590	0.730	0.613	0.613	0.460	0.590	0.782	0.939	0.898	0.485	0.709
F	0.697	0.720	0.619	0.465	0.491	1.000	0.799	0.638	0.531	0.654	0.530	0.223	0.000	0.628	0.603	0.515	0.629	0.473	0.655	0.613	0.639	0.753	0.653	0.505
G	0.898	0.625	0.573	0.625	0.586	0.422	1.000	0.531	0.550	0.468	0.473	0.391	0.391	0.516	0.445	0.595	0.678	0.667	0.516	0.650	0.694	0.625	0.643	0.609
H	0.898	0.754	0.767	0.722	0.653	0.816	0.765	1.000	0.615	0.855	0.790	0.473	0.391	0.823	0.646	0.632	0.621	0.667	0.797	0.876	0.827	0.814	0.738	0.739
I	0.898	0.786	0.756	0.766	0.796	0.799	0.758	0.851	1.000	0.823	0.736	0.445	0.391	0.840	0.579	0.773	0.774	0.795	0.795	0.909	0.814	0.753	0.773	0.724
J	0.898	0.638	0.472	0.418	0.442	0.798	0.762	0.547	0.589	1.000	0.653	0.434	0.000	0.655	0.516	0.485	0.634	0.460	0.644	0.569	0.485	0.729	0.583	0.677
K	0.939	0.730	0.553	0.544	0.492	0.842	0.799	0.575	0.578	0.827	1.000	0.452	0.000	0.527	0.627	0.529	0.674	0.511	0.587	0.798	0.563	0.669	0.529	0.704
L	0.927	0.855	0.799	0.799	0.653	0.967	0.799	0.761	0.791	0.924	0.773	1.000	0.464	0.679	0.875	0.799	0.799	0.743	0.799	0.799	0.799	0.855	0.653	0.882
M	1.000	0.855	1.000	1.000	0.855	1.000	0.799	0.904	0.799	1.000	1.000	0.855	1.000	0.940	0.855	1.000	0.799	1.000	1.000	1.000	1.000	0.855	0.696	0.954
N	0.939	0.669	0.669	0.669	0.632	0.717	0.731	0.658	0.474	0.663	0.833	0.565	0.452	1.000	0.604	0.604	0.548	0.775	0.877	0.814	0.669	0.669	0.586	0.669
O	0.697	0.548	0.716	0.613	0.573	0.707	0.765	0.658	0.641	0.749	0.661	0.434	0.335	0.622	1.000	0.641	0.667	0.605	0.704	0.659	0.693	0.696	0.620	0.613
P	0.898	0.753	0.871	0.892	0.608	0.798	0.662	0.613	0.541	0.800	0.691	0.391	0.258	0.618	0.618	1.000	0.492	0.786	0.752	0.929	0.972	0.753	0.487	0.774
Q	0.898	0.753	0.668	0.753	0.662	0.607	0.597	0.653	0.507	0.643	0.574	0.531	0.515	0.674	0.640	0.760	1.000	0.782	0.677	0.774	0.760	0.744	0.718	0.688
R	0.861	0.774	0.784	0.771	0.760	0.760	0.621	0.555	0.487	0.760	0.716	0.508	0.112	0.467	0.709	0.608	0.464	1.000	0.600	0.785	0.760	0.782	0.620	0.669
S	0.898	0.637	0.669	0.633	0.632	0.632	0.704	0.441	0.425	0.632	0.716	0.398	0.064	0.347	0.516	0.632	0.642	0.732	1.000	0.720	0.650	0.645	0.492	0.669
T	0.797	0.711	0.576	0.574	0.467	0.696	0.597	0.360	0.360	0.662	0.565	0.391	0.000	0.460	0.609	0.438	0.460	0.574	0.567	1.000	0.627	0.753	0.498	0.558
U	0.898	0.719	0.805	0.662	0.253	0.691	0.597	0.550	0.396	0.737	0.691	0.391	0.312	0.551	0.579	0.398	0.460	0.460	0.590	0.748	1.000	0.753	0.493	0.709
V	0.590	0.618	0.492	0.494	0.505	0.673	0.597	0.396	0.467	0.597	0.551	0.353	0.335	0.551	0.689	0.467	0.506	0.502	0.600	0.493	0.467	1.000	0.439	0.597
W	0.898	0.774	0.701	0.709	0.737	0.645	0.673	0.753	0.509	0.808	0.775	0.557	0.557	0.730	0.600	0.753	0.623	0.600	0.749	0.753	0.814	0.898	1.000	0.709
X	0.671	0.712	0.593	0.555	0.513	0.802	0.662	0.597	0.597	0.597	0.551	0.396	0.353	0.551	0.624	0.517	0.618	0.582	0.586	0.662	0.559	0.695	0.513	1.000

Table 44: Outranking values with OWAWA beta=0.3 disjunctive (**OWAWA.3d**) for countries dataset

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
A	1.000	0.409	0.084	0.084	0.140	0.280	0.181	0.084	0.084	0.084	0.073	0.084	0.033	0.053	0.280	0.084	0.091	0.224	0.084	0.182	0.112	0.451	0.084	0.380
B	0.451	1.000	0.345	0.263	0.319	0.345	0.345	0.287	0.224	0.345	0.252	0.137	0.137	0.308	0.488	0.224	0.232	0.224	0.345	0.303	0.419	0.460	0.203	0.286
C	0.685	0.467	1.000	0.579	0.147	0.548	0.525	0.387	0.266	0.598	0.538	0.193	0.096	0.308	0.336	0.256	0.334	0.217	0.389	0.601	0.766	0.507	0.374	0.491
D	0.685	0.717	0.582	1.000	0.205	0.556	0.358	0.287	0.224	0.556	0.506	0.193	0.111	0.308	0.426	0.312	0.253	0.242	0.345	0.670	0.746	0.654	0.261	0.507
E	0.685	0.585	0.622	0.591	1.000	0.506	0.454	0.330	0.270	0.599	0.478	0.342	0.137	0.338	0.478	0.367	0.367	0.217	0.338	0.551	0.801	0.685	0.233	0.457
F	0.451	0.480	0.398	0.219	0.241	1.000	0.661	0.403	0.288	0.431	0.287	0.025	0.000	0.381	0.357	0.267	0.391	0.227	0.420	0.367	0.422	0.507	0.440	0.256
G	0.685	0.373	0.325	0.373	0.343	0.217	1.000	0.291	0.309	0.238	0.242	0.193	0.193	0.273	0.217	0.345	0.428	0.415	0.273	0.401	0.468	0.373	0.402	0.359
H	0.685	0.525	0.581	0.484	0.401	0.572	0.552	1.000	0.368	0.626	0.539	0.245	0.193	0.577	0.394	0.380	0.377	0.415	0.552	0.738	0.673	0.601	0.514	0.507
I	0.685	0.558	0.528	0.524	0.573	0.549	0.545	0.626	1.000	0.586	0.484	0.217	0.193	0.618	0.336	0.531	0.528	0.549	0.549	0.710	0.601	0.507	0.547	0.477
J	0.685	0.387	0.221	0.175	0.196	0.552	0.556	0.296	0.363	1.000	0.407	0.212	0.000	0.409	0.273	0.233	0.388	0.217	0.395	0.317	0.233	0.482	0.336	0.427
K	0.801	0.517	0.312	0.295	0.240	0.629	0.661	0.328	0.329	0.611	1.000	0.224	0.000	0.275	0.381	0.283	0.461	0.259	0.341	0.631	0.322	0.423	0.283	0.466
L	0.768	0.717	0.661	0.661	0.440	0.829	0.661	0.578	0.637	0.872	0.588	1.000	0.239	0.440	0.757	0.661	0.661	0.537	0.661	0.661	0.661	0.717	0.440	0.714
M	1.000	0.717	1.000	1.000	0.717	1.000	0.661	0.838	0.661	1.000	1.000	0.717	1.000	0.795	0.717	1.000	0.661	1.000	1.000	1.000	1.000	0.717	0.499	0.904
N	0.801	0.423	0.423	0.423	0.380	0.465	0.512	0.427	0.228	0.414	0.611	0.322	0.224	1.000	0.352	0.352	0.296	0.541	0.664	0.601	0.423	0.423	0.349	0.423
O	0.451	0.308	0.522	0.367	0.325	0.478	0.552	0.409	0.395	0.527	0.415	0.212	0.137	0.370	1.000	0.395	0.451	0.358	0.458	0.432	0.461	0.474	0.374	0.367
P	0.685	0.507	0.686	0.677	0.365	0.549	0.416	0.364	0.296	0.562	0.448	0.193	0.105	0.366	0.370	1.000	0.244	0.540	0.505	0.774	0.941	0.507	0.240	0.528
Q	0.685	0.507	0.418	0.507	0.415	0.364	0.345	0.401	0.261	0.392	0.338	0.304	0.292	0.422	0.390	0.514	1.000	0.534	0.430	0.542	0.514	0.496	0.472	0.439
R	0.640	0.546	0.563	0.540	0.514	0.514	0.371	0.303	0.240	0.514	0.464	0.275	0.023	0.224	0.468	0.357	0.212	1.000	0.360	0.564	0.514	0.560	0.374	0.423
S	0.685	0.386	0.423	0.383	0.380	0.380	0.458	0.195	0.182	0.380	0.464	0.197	0.032	0.118	0.273	0.380	0.412	0.495	1.000	0.485	0.400	0.395	0.240	0.423
T	0.552	0.465	0.334	0.328	0.224	0.448	0.345	0.139	0.126	0.410	0.334	0.193	0.000	0.225	0.362	0.196	0.217	0.323	0.320	1.000	0.378	0.507	0.249	0.315
U	0.685	0.472	0.575	0.415	0.055	0.448	0.345	0.302	0.168	0.485	0.448	0.193	0.127	0.308	0.336	0.169	0.217	0.217	0.338	0.501	1.000	0.507	0.245	0.457
V	0.338	0.371	0.243	0.244	0.254	0.421	0.345	0.168	0.224	0.345	0.308	0.148	0.137	0.308	0.440	0.224	0.254	0.251	0.348	0.244	0.224	1.000	0.197	0.347
W	0.685	0.528	0.453	0.457	0.485	0.405	0.427	0.507	0.266	0.560	0.527	0.329	0.329	0.478	0.357	0.507	0.377	0.357	0.498	0.507	0.601	0.685	1.000	0.457
X	0.422	0.467	0.344	0.309	0.261	0.556	0.416	0.345	0.345	0.345	0.308	0.168	0.148	0.308	0.377	0.269	0.370	0.337	0.336	0.416	0.309	0.452	0.261	1.000

Table 45: Outranking values with OWAWA beta=0.3 conjunctive (**OWAWA.3c**) for countries dataset

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
A	1.000	0.725	0.440	0.440	0.508	0.608	0.528	0.440	0.440	0.440	0.415	0.440	0.231	0.368	0.608	0.440	0.446	0.568	0.440	0.538	0.468	0.758	0.440	0.700
B	0.758	1.000	0.675	0.591	0.658	0.675	0.675	0.613	0.568	0.675	0.560	0.428	0.428	0.628	0.770	0.568	0.579	0.568	0.675	0.638	0.732	0.764	0.553	0.627
C	0.910	0.767	1.000	0.842	0.485	0.820	0.800	0.705	0.598	0.863	0.811	0.468	0.300	0.628	0.648	0.589	0.670	0.563	0.702	0.850	0.934	0.798	0.703	0.782
D	0.910	0.888	0.826	1.000	0.566	0.833	0.685	0.613	0.568	0.833	0.790	0.468	0.348	0.628	0.735	0.649	0.608	0.594	0.675	0.882	0.920	0.862	0.615	0.798
E	0.910	0.841	0.865	0.858	1.000	0.790	0.753	0.665	0.610	0.854	0.770	0.637	0.428	0.670	0.770	0.698	0.698	0.563	0.670	0.822	0.948	0.910	0.595	0.755
F	0.758	0.772	0.708	0.572	0.604	1.000	0.848	0.719	0.637	0.731	0.615	0.348	0.000	0.706	0.671	0.625	0.711	0.576	0.726	0.698	0.724	0.798	0.735	0.609
G	0.910	0.695	0.656	0.695	0.653	0.497	1.000	0.612	0.625	0.554	0.558	0.468	0.468	0.603	0.535	0.671	0.737	0.725	0.603	0.719	0.758	0.695	0.715	0.680
H	0.910	0.803	0.817	0.778	0.715	0.838	0.815	1.000	0.697	0.875	0.816	0.555	0.468	0.848	0.710	0.700	0.696	0.725	0.829	0.903	0.866	0.850	0.790	0.788
I	0.910	0.826	0.803	0.808	0.835	0.824	0.810	0.871	1.000	0.850	0.775	0.535	0.468	0.866	0.648	0.813	0.813	0.828	0.828	0.920	0.850	0.798	0.817	0.774
J	0.910	0.706	0.584	0.533	0.555	0.830	0.814	0.637	0.677	1.000	0.720	0.521	0.000	0.728	0.603	0.595	0.713	0.563	0.715	0.655	0.595	0.779	0.674	0.736
K	0.948	0.790	0.656	0.642	0.600	0.870	0.848	0.668	0.668	0.859	1.000	0.540	0.000	0.625	0.708	0.638	0.750	0.614	0.677	0.844	0.663	0.738	0.638	0.765
L	0.937	0.888	0.848	0.848	0.735	0.968	0.848	0.816	0.841	0.942	0.824	1.000	0.545	0.747	0.903	0.848	0.848	0.800	0.848	0.848	0.848	0.888	0.735	0.903
M	1.000	0.888	1.000	1.000	0.888	1.000	0.848	0.927	0.848	1.000	1.000	0.888	1.000	0.948	0.888	1.000	0.848	1.000	1.000	1.000	1.000	0.888	0.767	0.964
N	0.948	0.738	0.738	0.738	0.700	0.761	0.789	0.729	0.577	0.717	0.859	0.638	0.540	1.000	0.680	0.680	0.640	0.817	0.895	0.850	0.738	0.738	0.680	0.738
O	0.758	0.653	0.780	0.698	0.664	0.769	0.815	0.726	0.718	0.801	0.720	0.522	0.428	0.693	1.000	0.718	0.744	0.690	0.763	0.736	0.758	0.762	0.703	0.698
P	0.910	0.798	0.894	0.905	0.669	0.825	0.733	0.692	0.630	0.830	0.728	0.468	0.329	0.690	0.684	1.000	0.592	0.821	0.795	0.939	0.977	0.798	0.589	0.813
Q	0.910	0.798	0.730	0.798	0.715	0.668	0.675	0.715	0.601	0.705	0.636	0.596	0.581	0.730	0.704	0.803	1.000	0.815	0.725	0.816	0.803	0.788	0.773	0.745
R	0.882	0.818	0.826	0.815	0.803	0.803	0.697	0.645	0.589	0.803	0.760	0.585	0.164	0.568	0.763	0.686	0.580	1.000	0.684	0.827	0.803	0.825	0.703	0.738
S	0.910	0.706	0.738	0.703	0.700	0.700	0.763	0.556	0.538	0.700	0.760	0.475	0.077	0.466	0.603	0.700	0.723	0.782	1.000	0.777	0.718	0.714	0.600	0.738
T	0.830	0.768	0.660	0.649	0.568	0.740	0.675	0.467	0.480	0.722	0.625	0.468	0.000	0.553	0.676	0.546	0.563	0.657	0.645	1.000	0.703	0.798	0.600	0.633
U	0.910	0.771	0.839	0.714	0.370	0.728	0.675	0.636	0.500	0.775	0.728	0.468	0.398	0.628	0.648	0.502	0.563	0.563	0.670	0.793	1.000	0.798	0.594	0.755
V	0.670	0.700	0.595	0.598	0.606	0.729	0.675	0.500	0.568	0.675	0.628	0.447	0.428	0.628	0.746	0.568	0.610	0.606	0.678	0.596	0.568	1.000	0.547	0.678
W	0.910	0.813	0.755	0.755	0.775	0.691	0.740	0.798	0.598	0.834	0.806	0.615	0.615	0.770	0.663	0.798	0.690	0.663	0.786	0.798	0.850	0.910	1.000	0.755
X	0.733	0.769	0.678	0.644	0.615	0.833	0.733	0.675	0.675	0.675	0.628	0.500	0.448	0.628	0.687	0.610	0.697	0.660	0.664	0.733	0.651	0.751	0.615	1.000

Table 46: Outranking values with OWAWA beta=0.5 disjunctive (**OWAWA.5d**) for countries dataset

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
A	1.000	0.312	0.060	0.060	0.103	0.203	0.131	0.060	0.060	0.060	0.052	0.060	0.024	0.038	0.203	0.060	0.065	0.163	0.060	0.133	0.081	0.348	0.060	0.280
B	0.348	1.000	0.255	0.192	0.241	0.255	0.255	0.208	0.163	0.255	0.180	0.098	0.098	0.223	0.361	0.163	0.170	0.163	0.255	0.223	0.319	0.355	0.148	0.211
C	0.555	0.354	1.000	0.455	0.105	0.400	0.429	0.285	0.193	0.453	0.393	0.138	0.068	0.223	0.243	0.186	0.253	0.158	0.292	0.495	0.672	0.388	0.293	0.367
D	0.555	0.658	0.484	1.000	0.152	0.423	0.267	0.208	0.163	0.423	0.370	0.138	0.079	0.223	0.325	0.231	0.189	0.180	0.255	0.575	0.656	0.576	0.195	0.388
E	0.555	0.477	0.510	0.448	1.000	0.370	0.370	0.247	0.202	0.480	0.350	0.245	0.098	0.250	0.350	0.288	0.288	0.158	0.250	0.439	0.718	0.555	0.175	0.335
F	0.348	0.372	0.340	0.161	0.188	1.000	0.618	0.326	0.232	0.359	0.210	0.018	0.000	0.295	0.260	0.213	0.316	0.167	0.334	0.288	0.363	0.388	0.380	0.195
G	0.555	0.275	0.243	0.275	0.248	0.155	1.000	0.211	0.223	0.170	0.174	0.138	0.138	0.198	0.155	0.254	0.320	0.305	0.198	0.305	0.380	0.275	0.314	0.264
H	0.555	0.422	0.507	0.382	0.295	0.431	0.460	1.000	0.285	0.493	0.399	0.175	0.138	0.438	0.290	0.280	0.289	0.305	0.421	0.673	0.609	0.495	0.417	0.402
I	0.555	0.446	0.423	0.404	0.464	0.407	0.455	0.495	1.000	0.455	0.355	0.155	0.138	0.497	0.243	0.411	0.403	0.418	0.418	0.588	0.495	0.388	0.440	0.360
J	0.555	0.288	0.166	0.128	0.145	0.419	0.470	0.219	0.301	1.000	0.310	0.151	0.000	0.318	0.198	0.175	0.303	0.158	0.300	0.235	0.175	0.367	0.261	0.319
K	0.718	0.435	0.253	0.226	0.180	0.515	0.618	0.256	0.253	0.498	1.000	0.160	0.000	0.205	0.298	0.228	0.395	0.194	0.267	0.565	0.263	0.328	0.228	0.369
L	0.672	0.658	0.618	0.618	0.380	0.738	0.618	0.510	0.584	0.855	0.515	1.000	0.171	0.349	0.706	0.618	0.618	0.457	0.618	0.618	0.618	0.658	0.380	0.623
M	1.000	0.658	1.000	1.000	0.658	1.000	0.618	0.818	0.618	1.000	1.000	0.658	1.000	0.706	0.658	1.000	0.618	1.000	1.000	1.000	1.000	0.658	0.439	0.880
N	0.718	0.328	0.328	0.328	0.280	0.341	0.424	0.344	0.168	0.302	0.489	0.233	0.160	1.000	0.260	0.260	0.220	0.427	0.540	0.495	0.328	0.328	0.285	0.328
O	0.348	0.252	0.457	0.288	0.250	0.389	0.460	0.312	0.308	0.432	0.310	0.152	0.098	0.273	1.000	0.308	0.383	0.278	0.353	0.357	0.372	0.392	0.293	0.288
P	0.555	0.388	0.586	0.548	0.264	0.412	0.323	0.278	0.222	0.434	0.323	0.138	0.075	0.270	0.270	1.000	0.180	0.410	0.383	0.680	0.926	0.388	0.177	0.403
Q	0.555	0.388	0.314	0.388	0.304	0.263	0.255	0.295	0.191	0.288	0.243	0.217	0.208	0.310	0.286	0.393	1.000	0.402	0.313	0.429	0.393	0.376	0.363	0.330
R	0.513	0.437	0.457	0.430	0.393	0.393	0.281	0.225	0.177	0.393	0.340	0.198	0.017	0.163	0.360	0.268	0.160	1.000	0.284	0.459	0.393	0.454	0.293	0.328
S	0.555	0.289	0.328	0.285	0.280	0.280	0.353	0.144	0.133	0.280	0.340	0.141	0.023	0.085	0.198	0.280	0.339	0.387	1.000	0.386	0.303	0.299	0.180	0.328
T	0.421	0.358	0.257	0.240	0.163	0.326	0.255	0.100	0.091	0.303	0.240	0.138	0.000	0.162	0.264	0.142	0.158	0.239	0.234	1.000	0.288	0.388	0.184	0.228
U	0.555	0.358	0.455	0.302	0.039	0.323	0.255	0.222	0.120	0.355	0.323	0.138	0.091	0.223	0.243	0.121	0.158	0.158	0.250	0.382	1.000	0.388	0.181	0.335
V	0.250	0.288	0.180	0.181	0.189	0.309	0.255	0.120	0.163	0.255	0.223	0.106	0.098	0.223	0.331	0.163	0.190	0.187	0.258	0.181	0.163	1.000	0.143	0.261
W	0.555	0.403	0.341	0.335	0.355	0.292	0.330	0.388	0.193	0.420	0.392	0.235	0.235	0.350	0.258	0.388	0.280	0.258	0.368	0.388	0.495	0.555	1.000	0.335
X	0.317	0.360	0.264	0.234	0.195	0.423	0.323	0.255	0.255	0.255	0.223	0.120	0.106	0.223	0.275	0.198	0.283	0.250	0.247	0.323	0.234	0.346	0.195	1.000

Table 47: Outranking values with OWAWA beta=0.5 conjunctive (**OWAWA.5c**) for countries dataset

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
A	1.000	0.793	0.568	0.568	0.633	0.693	0.638	0.568	0.568	0.568	0.539	0.568	0.304	0.485	0.693	0.568	0.573	0.669	0.568	0.651	0.592	0.819	0.568	0.768
B	0.819	1.000	0.753	0.679	0.748	0.753	0.753	0.696	0.669	0.753	0.640	0.521	0.521	0.705	0.807	0.669	0.681	0.669	0.753	0.723	0.798	0.823	0.660	0.718
C	0.922	0.819	1.000	0.873	0.595	0.840	0.852	0.771	0.687	0.882	0.833	0.545	0.365	0.705	0.717	0.680	0.756	0.666	0.769	0.886	0.944	0.843	0.786	0.825
D	0.922	0.921	0.866	1.000	0.678	0.864	0.762	0.696	0.669	0.864	0.822	0.545	0.424	0.705	0.797	0.735	0.712	0.697	0.753	0.910	0.936	0.898	0.717	0.843
E	0.922	0.879	0.895	0.879	1.000	0.822	0.822	0.749	0.705	0.884	0.810	0.697	0.521	0.750	0.810	0.783	0.783	0.666	0.750	0.863	0.957	0.922	0.705	0.801
F	0.819	0.824	0.797	0.678	0.717	1.000	0.897	0.799	0.743	0.808	0.701	0.473	0.000	0.785	0.738	0.736	0.794	0.679	0.797	0.783	0.809	0.843	0.817	0.713
G	0.922	0.765	0.739	0.765	0.720	0.573	1.000	0.693	0.700	0.640	0.644	0.545	0.545	0.690	0.625	0.746	0.796	0.783	0.690	0.789	0.821	0.765	0.788	0.752
H	0.922	0.852	0.867	0.834	0.777	0.860	0.865	1.000	0.779	0.895	0.842	0.637	0.545	0.873	0.774	0.768	0.772	0.783	0.862	0.930	0.905	0.886	0.842	0.837
I	0.922	0.866	0.850	0.850	0.873	0.848	0.862	0.891	1.000	0.877	0.814	0.625	0.545	0.893	0.717	0.854	0.852	0.861	0.861	0.930	0.886	0.843	0.862	0.823
J	0.922	0.775	0.696	0.648	0.668	0.861	0.865	0.727	0.765	1.000	0.787	0.609	0.000	0.801	0.690	0.705	0.792	0.666	0.785	0.741	0.705	0.828	0.765	0.795
K	0.957	0.850	0.759	0.739	0.708	0.898	0.897	0.761	0.758	0.890	1.000	0.628	0.000	0.723	0.789	0.747	0.826	0.717	0.766	0.890	0.764	0.807	0.747	0.826
L	0.947	0.921	0.897	0.897	0.817	0.969	0.897	0.870	0.890	0.961	0.875	1.000	0.627	0.815	0.932	0.897	0.897	0.857	0.897	0.897	0.897	0.921	0.817	0.925
M	1.000	0.921	1.000	1.000	0.921	1.000	0.897	0.951	0.897	1.000	1.000	0.921	1.000	0.956	0.921	1.000	0.897	1.000	1.000	1.000	1.000	0.921	0.839	0.974
N	0.957	0.807	0.807	0.807	0.768	0.805	0.847	0.800	0.681	0.771	0.886	0.711	0.628	1.000	0.756	0.756	0.732	0.859	0.913	0.886	0.807	0.807	0.774	0.807
O	0.819	0.758	0.844	0.783	0.755	0.832	0.865	0.794	0.795	0.854	0.779	0.609	0.521	0.764	1.000	0.795	0.821	0.775	0.822	0.812	0.824	0.829	0.786	0.783
P	0.922	0.843	0.917	0.919	0.730	0.853	0.804	0.772	0.720	0.861	0.765	0.545	0.401	0.762	0.750	1.000	0.693	0.855	0.838	0.949	0.983	0.843	0.690	0.852
Q	0.922	0.843	0.792	0.843	0.768	0.729	0.753	0.777	0.694	0.768	0.699	0.660	0.647	0.786	0.768	0.846	1.000	0.848	0.772	0.859	0.846	0.833	0.828	0.803
R	0.902	0.861	0.869	0.858	0.846	0.846	0.773	0.735	0.691	0.846	0.804	0.663	0.216	0.669	0.817	0.764	0.696	1.000	0.768	0.869	0.846	0.867	0.786	0.807
S	0.922	0.776	0.807	0.773	0.768	0.768	0.822	0.670	0.651	0.768	0.804	0.553	0.089	0.585	0.690	0.768	0.803	0.833	1.000	0.834	0.787	0.784	0.708	0.807
T	0.862	0.825	0.745	0.725	0.669	0.784	0.753	0.575	0.601	0.782	0.684	0.545	0.000	0.645	0.743	0.654	0.666	0.740	0.723	1.000	0.778	0.843	0.701	0.708
U	0.922	0.822	0.872	0.766	0.487	0.765	0.753	0.721	0.604	0.813	0.765	0.545	0.485	0.705	0.717	0.606	0.666	0.666	0.750	0.838	1.000	0.843	0.696	0.801
V	0.750	0.781	0.698	0.701	0.708	0.785	0.753	0.604	0.669	0.753	0.705	0.542	0.521	0.705	0.804	0.669	0.714	0.710	0.756	0.699	0.669	1.000	0.655	0.759
W	0.922	0.852	0.808	0.801	0.813	0.738	0.807	0.843	0.687	0.859	0.838	0.673	0.673	0.810	0.726	0.843	0.757	0.726	0.822	0.843	0.886	0.922	1.000	0.801
X	0.794	0.825	0.764	0.734	0.717	0.864	0.804	0.753	0.753	0.753	0.705	0.604	0.542	0.705	0.749	0.703	0.776	0.738	0.741	0.804	0.744	0.807	0.717	1.000

Table 48: Outranking values with OWAWA beta=0.7 disjunctive (**OWAWA.7d**) for countries dataset

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
A	1.000	0.214	0.036	0.036	0.066	0.126	0.081	0.036	0.036	0.036	0.031	0.036	0.014	0.023	0.126	0.036	0.039	0.102	0.036	0.084	0.050	0.245	0.036	0.180
B	0.245	1.000	0.165	0.120	0.163	0.165	0.165	0.129	0.102	0.165	0.108	0.059	0.059	0.138	0.235	0.102	0.108	0.102	0.165	0.143	0.219	0.251	0.093	0.136
C	0.425	0.241	1.000	0.331	0.063	0.252	0.333	0.183	0.120	0.308	0.247	0.083	0.041	0.138	0.150	0.115	0.172	0.099	0.194	0.389	0.577	0.269	0.212	0.244
D	0.425	0.599	0.386	1.000	0.100	0.290	0.177	0.129	0.102	0.290	0.234	0.083	0.048	0.138	0.224	0.150	0.125	0.117	0.165	0.479	0.566	0.498	0.129	0.269
E	0.425	0.369	0.398	0.305	1.000	0.234	0.285	0.163	0.134	0.361	0.222	0.149	0.059	0.162	0.222	0.209	0.209	0.099	0.162	0.326	0.635	0.425	0.117	0.213
F	0.245	0.264	0.281	0.104	0.135	1.000	0.575	0.250	0.176	0.287	0.133	0.011	0.000	0.208	0.163	0.159	0.240	0.106	0.248	0.209	0.303	0.269	0.320	0.133
G	0.425	0.177	0.161	0.177	0.153	0.093	1.000	0.132	0.137	0.103	0.105	0.083	0.083	0.123	0.093	0.163	0.213	0.195	0.123	0.209	0.293	0.177	0.225	0.168
H	0.425	0.318	0.433	0.280	0.189	0.290	0.368	1.000	0.203	0.361	0.258	0.105	0.083	0.299	0.186	0.180	0.202	0.195	0.290	0.608	0.544	0.389	0.321	0.296
I	0.425	0.334	0.319	0.284	0.354	0.266	0.365	0.364	1.000	0.324	0.226	0.093	0.083	0.376	0.150	0.290	0.278	0.287	0.287	0.465	0.389	0.269	0.334	0.244
J	0.425	0.190	0.110	0.081	0.094	0.286	0.385	0.142	0.239	1.000	0.213	0.091	0.000	0.227	0.123	0.117	0.218	0.099	0.204	0.153	0.117	0.252	0.187	0.212
K	0.635	0.353	0.195	0.157	0.120	0.401	0.575	0.183	0.178	0.385	1.000	0.096	0.000	0.135	0.215	0.173	0.329	0.130	0.193	0.500	0.203	0.233	0.173	0.271
L	0.576	0.599	0.575	0.575	0.320	0.647	0.575	0.443	0.531	0.839	0.443	1.000	0.103	0.259	0.656	0.575	0.575	0.377	0.575	0.575	0.575	0.599	0.320	0.533
M	1.000	0.599	1.000	1.000	0.599	1.000	0.575	0.797	0.575	1.000	1.000	0.599	1.000	0.617	0.599	1.000	0.575	1.000	1.000	1.000	1.000	0.599	0.379	0.856
N	0.635	0.233	0.233	0.233	0.180	0.217	0.336	0.261	0.107	0.190	0.366	0.144	0.096	1.000	0.168	0.168	0.144	0.313	0.416	0.389	0.233	0.233	0.221	0.233
O	0.245	0.196	0.392	0.209	0.175	0.299	0.368	0.215	0.221	0.336	0.205	0.091	0.059	0.177	1.000	0.221	0.316	0.198	0.248	0.281	0.282	0.310	0.212	0.209
P	0.425	0.269	0.486	0.418	0.163	0.274	0.230	0.191	0.148	0.307	0.198	0.083	0.045	0.174	0.171	1.000	0.116	0.280	0.261	0.586	0.911	0.269	0.114	0.278
Q	0.425	0.269	0.210	0.269	0.193	0.162	0.165	0.189	0.121	0.183	0.148	0.130	0.125	0.198	0.183	0.272	1.000	0.269	0.195	0.317	0.272	0.256	0.254	0.222
R	0.386	0.328	0.352	0.320	0.272	0.272	0.191	0.147	0.114	0.272	0.216	0.120	0.010	0.102	0.253	0.179	0.108	1.000	0.209	0.354	0.272	0.349	0.212	0.233
S	0.425	0.192	0.233	0.188	0.180	0.180	0.248	0.094	0.084	0.180	0.216	0.084	0.014	0.051	0.123	0.180	0.267	0.280	1.000	0.288	0.205	0.202	0.120	0.233
T	0.290	0.251	0.180	0.152	0.102	0.204	0.165	0.060	0.056	0.195	0.145	0.083	0.000	0.099	0.166	0.089	0.099	0.154	0.147	1.000	0.198	0.269	0.120	0.141
U	0.425	0.245	0.336	0.189	0.023	0.198	0.165	0.142	0.072	0.225	0.198	0.083	0.055	0.138	0.150	0.073	0.099	0.099	0.162	0.262	1.000	0.269	0.117	0.213
V	0.162	0.204	0.117	0.119	0.124	0.197	0.165	0.072	0.102	0.165	0.138	0.064	0.059	0.138	0.223	0.102	0.126	0.124	0.169	0.118	0.102	1.000	0.090	0.174
W	0.425	0.278	0.229	0.213	0.225	0.178	0.233	0.269	0.120	0.280	0.258	0.141	0.141	0.222	0.159	0.269	0.183	0.159	0.237	0.269	0.389	0.425	1.000	0.213
X	0.212	0.254	0.183	0.160	0.129	0.290	0.230	0.165	0.165	0.165	0.138	0.072	0.064	0.138	0.172	0.127	0.196	0.164	0.158	0.231	0.160	0.240	0.129	1.000

Table 49: Outranking values with OWAWA beta=0.7 conjunctive (**OWAWA.7c**) for countries dataset

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
A	1.000	0.792	0.065	0.065	0.100	0.725	0.134	0.065	0.065	0.065	0.055	0.065	0.022	0.035	0.725	0.065	0.070	0.165	0.065	0.115	0.079	0.810	0.065	0.775
B	0.275	1.000	0.225	0.181	0.217	0.225	0.225	0.190	0.165	0.225	0.160	0.100	0.100	0.195	0.848	0.165	0.173	0.165	0.225	0.202	0.258	0.276	0.125	0.201
C	0.935	0.817	1.000	0.890	0.085	0.870	0.300	0.240	0.180	0.900	0.865	0.660	0.070	0.195	0.750	0.173	0.226	0.165	0.239	0.905	0.942	0.835	0.210	0.857
D	0.935	0.900	0.735	1.000	0.132	0.885	0.231	0.190	0.165	0.885	0.855	0.660	0.081	0.195	0.802	0.209	0.194	0.184	0.225	0.921	0.812	0.880	0.160	0.870
E	0.935	0.864	0.915	0.900	1.000	0.855	0.280	0.226	0.190	0.905	0.850	0.774	0.100	0.225	0.850	0.245	0.245	0.165	0.225	0.887	0.965	0.935	0.155	0.840
F	0.275	0.821	0.224	0.137	0.159	1.000	0.340	0.237	0.184	0.250	0.181	0.030	0.000	0.242	0.763	0.173	0.254	0.174	0.261	0.245	0.233	0.835	0.240	0.303
G	0.935	0.775	0.756	0.775	0.745	0.673	1.000	0.731	0.734	0.697	0.703	0.660	0.660	0.725	0.690	0.764	0.810	0.825	0.725	0.792	0.818	0.775	0.791	0.767
H	0.935	0.841	0.858	0.825	0.780	0.883	0.310	1.000	0.233	0.911	0.870	0.695	0.660	0.890	0.785	0.775	0.364	0.825	0.886	0.940	0.887	0.870	0.654	0.847
I	0.935	0.854	0.848	0.845	0.860	0.874	0.310	0.911	1.000	0.899	0.853	0.690	0.660	0.911	0.750	0.849	0.875	0.885	0.885	0.941	0.870	0.835	0.850	0.842
J	0.935	0.782	0.144	0.115	0.128	0.878	0.313	0.200	0.223	1.000	0.255	0.684	0.000	0.260	0.725	0.150	0.250	0.165	0.251	0.215	0.150	0.824	0.196	0.810
K	0.965	0.840	0.262	0.404	0.150	0.910	0.340	0.220	0.208	0.900	1.000	0.690	0.000	0.200	0.790	0.180	0.280	0.197	0.233	0.685	0.292	0.805	0.180	0.830
L	0.957	0.900	0.340	0.340	0.240	0.970	0.340	0.319	0.334	0.751	0.321	1.000	0.153	0.272	0.914	0.340	0.340	0.308	0.340	0.340	0.340	0.900	0.240	0.938
M	1.000	0.900	1.000	1.000	0.900	1.000	0.340	0.686	0.340	1.000	1.000	0.900	1.000	0.907	0.900	1.000	0.340	1.000	1.000	1.000	1.000	0.900	0.380	0.978
N	0.965	0.805	0.805	0.805	0.775	0.841	0.300	0.601	0.148	0.805	0.899	0.740	0.690	1.000	0.770	0.770	0.210	0.883	0.930	0.905	0.805	0.805	0.380	0.805
O	0.275	0.190	0.294	0.245	0.205	0.282	0.310	0.253	0.250	0.301	0.246	0.140	0.100	0.232	1.000	0.250	0.276	0.239	0.275	0.267	0.275	0.276	0.210	0.245
P	0.935	0.835	0.931	0.931	0.756	0.877	0.260	0.241	0.201	0.885	0.820	0.660	0.077	0.230	0.770	1.000	0.182	0.881	0.749	0.958	0.980	0.835	0.143	0.875
Q	0.935	0.835	0.797	0.835	0.781	0.755	0.225	0.785	0.320	0.780	0.739	0.718	0.711	0.790	0.781	0.835	1.000	0.875	0.802	0.848	0.835	0.828	0.820	0.805
R	0.910	0.851	0.857	0.849	0.835	0.835	0.219	0.175	0.144	0.835	0.805	0.711	0.016	0.130	0.817	0.345	0.145	1.000	0.230	0.858	0.835	0.857	0.210	0.805
S	0.935	0.783	0.805	0.781	0.775	0.775	0.275	0.128	0.115	0.775	0.805	0.663	0.108	0.115	0.725	0.775	0.262	0.863	1.000	0.840	0.789	0.789	0.150	0.805
T	0.886	0.820	0.482	0.625	0.130	0.814	0.225	0.083	0.086	0.791	0.739	0.660	0.000	0.136	0.766	0.122	0.165	0.218	0.207	1.000	0.527	0.835	0.150	0.740
U	0.935	0.819	0.869	0.819	0.036	0.820	0.225	0.208	0.130	0.850	0.820	0.660	0.093	0.195	0.750	0.131	0.165	0.165	0.225	0.867	1.000	0.835	0.147	0.840
V	0.225	0.784	0.184	0.187	0.181	0.821	0.225	0.130	0.165	0.225	0.195	0.108	0.100	0.195	0.840	0.165	0.195	0.192	0.227	0.185	0.165	1.000	0.122	0.370
W	0.935	0.875	0.845	0.840	0.845	0.799	0.268	0.870	0.180	0.884	0.870	0.760	0.760	0.850	0.790	0.870	0.537	0.790	0.859	0.870	0.905	0.935	1.000	0.840
X	0.488	0.821	0.214	0.192	0.160	0.885	0.260	0.225	0.225	0.225	0.195	0.095	0.084	0.195	0.768	0.172	0.243	0.217	0.218	0.260	0.191	0.807	0.160	1.000

Table 50: Outranking values with IOWA disjunctive (**IOWAd**) for countries dataset

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
A	1.000	0.493	0.380	0.380	0.480	0.380	0.380	0.380	0.380	0.380	0.283	0.380	0.063	0.100	0.380	0.380	0.380	0.380	0.380	0.400	0.386	0.500	0.380	0.400
B	0.620	1.000	0.390	0.421	0.483	0.390	0.390	0.290	0.380	0.390	0.010	0.000	0.000	0.110	0.277	0.380	0.514	0.380	0.390	0.364	0.403	0.426	0.390	0.636
C	0.620	0.617	1.000	0.614	0.120	0.030	0.481	0.310	0.300	0.310	0.028	0.000	0.000	0.110	0.500	0.300	0.708	0.590	0.332	0.890	0.503	0.620	0.510	0.558
D	0.620	1.000	0.612	1.000	0.485	0.390	0.408	0.290	0.380	0.390	0.110	0.000	0.000	0.110	0.723	0.398	0.848	0.684	0.390	0.685	0.510	0.828	0.490	0.790
E	0.620	0.703	0.880	0.520	1.000	0.110	0.890	0.782	0.508	0.713	0.500	0.224	0.000	0.600	0.500	0.890	0.890	0.590	0.600	0.838	0.900	0.620	0.880	0.510
F	0.620	0.641	0.995	0.614	0.931	1.000	1.000	0.900	0.960	0.909	0.513	0.490	0.000	0.700	0.537	0.969	0.916	0.670	0.763	0.890	0.999	0.622	1.000	0.832
G	0.620	0.610	0.711	0.610	0.120	0.005	1.000	0.594	0.297	0.283	0.484	0.000	0.000	0.590	0.490	0.734	0.610	0.590	0.590	0.770	0.866	0.610	0.768	0.587
H	0.620	0.715	0.943	0.713	0.220	0.131	0.510	1.000	0.500	0.400	0.116	0.100	0.000	0.210	0.600	0.400	0.665	0.590	0.600	0.990	0.998	0.720	0.483	0.675
I	0.620	0.674	0.703	0.620	0.691	0.118	0.720	0.581	1.000	0.605	0.501	0.490	0.000	0.674	0.500	0.620	0.610	0.600	0.600	0.638	0.720	0.620	0.712	0.610
J	0.620	0.630	0.708	0.610	0.646	0.215	0.746	0.610	0.753	1.000	0.585	0.391	0.000	0.700	0.590	0.710	0.710	0.590	0.610	0.610	0.710	0.618	0.718	0.610
K	0.900	0.990	0.990	0.890	0.500	0.500	1.000	0.890	0.500	0.500	1.000	0.280	0.000	0.690	0.970	0.990	0.990	0.871	0.890	0.990	0.990	0.890	0.990	0.890
L	0.822	1.000	1.000	1.000	1.000	0.510	1.000	0.940	0.896	1.000	0.824	1.000	0.009	0.705	1.000	1.000	1.000	0.909	1.000	1.000	1.000	1.000	1.000	0.899
M	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.720	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.996
N	0.900	0.890	0.890	0.890	0.400	0.302	0.811	0.882	0.400	0.300	0.703	0.300	0.280	1.000	0.790	0.790	0.790	0.827	0.800	0.890	0.890	0.890	0.928	0.890
O	0.620	0.992	0.770	0.890	0.500	0.505	0.510	0.406	0.500	0.484	0.134	0.016	0.000	0.211	1.000	0.500	0.979	0.794	0.410	0.775	0.504	0.850	0.510	0.890
P	0.620	0.620	0.805	0.620	0.113	0.172	0.490	0.748	0.416	0.389	0.011	0.000	0.000	0.210	0.558	1.000	0.638	0.599	0.600	0.836	0.968	0.620	0.334	0.610
Q	0.620	0.620	0.587	0.620	0.150	0.110	0.390	0.390	0.390	0.375	0.064	0.010	0.009	0.210	0.531	0.410	1.000	0.435	0.110	0.617	0.410	0.501	0.490	0.615
R	0.620	0.771	0.772	0.749	0.410	0.410	0.410	0.410	0.405	0.410	0.130	0.106	0.045	0.210	0.672	0.410	0.890	1.000	0.537	0.781	0.410	0.822	0.510	0.890
S	0.620	0.686	0.890	0.664	0.400	0.400	0.410	0.610	0.400	0.400	0.130	0.009	0.000	0.200	0.590	0.400	0.938	0.801	1.000	0.890	0.636	0.737	0.500	0.890
T	0.601	0.700	0.769	0.562	0.210	0.110	0.390	0.222	0.384	0.390	0.017	0.000	0.000	0.110	0.545	0.389	0.590	0.599	0.310	1.000	0.490	0.620	0.399	0.510
U	0.620	0.617	0.738	0.495	0.100	0.010	0.390	0.654	0.280	0.290	0.010	0.000	0.000	0.110	0.500	0.282	0.590	0.590	0.600	0.758	1.000	0.620	0.367	0.510
V	0.600	0.918	0.696	0.797	0.480	0.380	0.390	0.280	0.380	0.390	0.110	0.023	0.000	0.110	0.730	0.380	0.870	0.824	0.425	0.705	0.380	1.000	0.390	0.790
W	0.620	0.610	0.762	0.510	0.120	0.016	0.773	0.790	0.300	0.303	0.474	0.000	0.000	0.500	0.490	0.790	0.589	0.490	0.504	0.790	0.890	0.620	1.000	0.510
X	0.600	0.708	0.490	0.520	0.490	0.390	0.490	0.390	0.390	0.390	0.110	0.110	0.010	0.110	0.330	0.390	0.678	0.371	0.322	0.495	0.490	0.439	0.490	1.000

Table 51: Outranking values with IOWA conjunctive (**IOWAc**) for countries dataset

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
A	1.000	0.888	0.654	0.654	0.721	0.830	0.751	0.654	0.654	0.654	0.634	0.654	0.374	0.596	0.830	0.654	0.661	0.794	0.654	0.761	0.685	0.904	0.654	0.881
B	0.904	1.000	0.863	0.815	0.845	0.863	0.863	0.835	0.794	0.863	0.815	0.721	0.721	0.847	0.925	0.794	0.798	0.794	0.863	0.841	0.892	0.908	0.778	0.832
C	0.975	0.912	1.000	0.946	0.732	0.949	0.922	0.884	0.822	0.960	0.946	0.772	0.506	0.847	0.862	0.814	0.854	0.789	0.881	0.947	0.980	0.927	0.869	0.924
D	0.975	0.958	0.935	1.000	0.772	0.945	0.868	0.835	0.794	0.945	0.934	0.772	0.587	0.847	0.896	0.845	0.810	0.804	0.863	0.959	0.972	0.949	0.815	0.927
E	0.975	0.943	0.954	0.958	1.000	0.934	0.896	0.854	0.818	0.951	0.923	0.866	0.721	0.860	0.923	0.865	0.865	0.789	0.860	0.936	0.985	0.975	0.796	0.915
F	0.904	0.913	0.863	0.785	0.796	1.000	0.938	0.874	0.817	0.880	0.822	0.511	0.000	0.872	0.871	0.810	0.872	0.795	0.885	0.865	0.874	0.927	0.881	0.803
G	0.975	0.877	0.851	0.877	0.865	0.789	1.000	0.834	0.847	0.804	0.806	0.772	0.772	0.826	0.791	0.864	0.898	0.897	0.826	0.886	0.902	0.877	0.881	0.871
H	0.975	0.925	0.926	0.913	0.891	0.951	0.928	1.000	0.866	0.962	0.944	0.811	0.772	0.953	0.887	0.881	0.870	0.897	0.944	0.965	0.948	0.947	0.914	0.919
I	0.975	0.938	0.925	0.932	0.941	0.946	0.926	0.961	1.000	0.952	0.925	0.791	0.772	0.956	0.862	0.934	0.935	0.943	0.943	0.978	0.947	0.927	0.932	0.916
J	0.975	0.882	0.786	0.755	0.770	0.944	0.927	0.835	0.847	1.000	0.886	0.787	0.000	0.886	0.826	0.796	0.876	0.789	0.883	0.849	0.796	0.918	0.849	0.897
K	0.985	0.915	0.829	0.824	0.801	0.956	0.938	0.844	0.846	0.951	1.000	0.796	0.000	0.824	0.872	0.817	0.891	0.813	0.852	0.936	0.834	0.892	0.817	0.905
L	0.982	0.958	0.938	0.938	0.881	0.993	0.938	0.925	0.936	0.976	0.930	1.000	0.805	0.895	0.964	0.938	0.938	0.919	0.938	0.938	0.938	0.958	0.881	0.967
M	1.000	0.958	1.000	1.000	0.958	1.000	0.938	0.970	0.938	1.000	1.000	0.958	1.000	0.985	0.958	1.000	0.938	1.000	1.000	1.000	1.000	0.958	0.897	0.987
N	0.985	0.892	0.892	0.892	0.881	0.918	0.916	0.873	0.791	0.895	0.954	0.854	0.796	1.000	0.867	0.867	0.837	0.934	0.968	0.947	0.892	0.892	0.845	0.892
O	0.904	0.828	0.907	0.865	0.844	0.905	0.928	0.888	0.879	0.923	0.894	0.785	0.721	0.875	1.000	0.879	0.888	0.863	0.907	0.886	0.900	0.900	0.869	0.865
P	0.975	0.927	0.965	0.973	0.876	0.945	0.889	0.868	0.831	0.945	0.913	0.772	0.555	0.874	0.877	1.000	0.805	0.940	0.926	0.982	0.992	0.927	0.803	0.935
Q	0.975	0.927	0.895	0.927	0.896	0.876	0.863	0.891	0.811	0.887	0.863	0.846	0.838	0.900	0.886	0.930	1.000	0.940	0.903	0.934	0.930	0.925	0.913	0.902
R	0.963	0.934	0.937	0.933	0.930	0.930	0.870	0.841	0.799	0.930	0.917	0.828	0.266	0.794	0.910	0.864	0.781	1.000	0.859	0.937	0.930	0.936	0.869	0.892
S	0.975	0.881	0.892	0.880	0.881	0.881	0.907	0.769	0.761	0.881	0.917	0.776	0.127	0.695	0.826	0.881	0.877	0.918	1.000	0.911	0.886	0.884	0.801	0.892
T	0.944	0.910	0.842	0.847	0.794	0.910	0.863	0.724	0.698	0.894	0.860	0.772	0.000	0.791	0.873	0.771	0.789	0.851	0.852	1.000	0.870	0.927	0.808	0.850
U	0.975	0.914	0.944	0.897	0.598	0.913	0.863	0.841	0.752	0.926	0.913	0.772	0.672	0.847	0.862	0.753	0.789	0.789	0.860	0.926	1.000	0.927	0.806	0.915
V	0.860	0.869	0.805	0.805	0.808	0.899	0.863	0.752	0.794	0.863	0.847	0.732	0.721	0.847	0.904	0.794	0.811	0.809	0.864	0.805	0.794	1.000	0.772	0.859
W	0.975	0.935	0.908	0.915	0.926	0.895	0.895	0.927	0.822	0.949	0.938	0.860	0.860	0.923	0.872	0.927	0.872	0.872	0.929	0.927	0.947	0.975	1.000	0.915
X	0.893	0.911	0.855	0.836	0.815	0.945	0.889	0.863	0.863	0.863	0.847	0.752	0.726	0.847	0.881	0.818	0.871	0.858	0.859	0.889	0.838	0.906	0.815	1.000

Table 52: Outranking values with WOWA disjunctive (**WOWAd**) for countries dataset

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
A	1.000	0.060	0.000	0.000	0.000	0.010	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.010	0.000	0.000	0.003	0.000	0.001	0.000	0.081	0.000	0.042
B	0.081	1.000	0.025	0.010	0.021	0.025	0.025	0.011	0.003	0.025	0.006	0.000	0.000	0.016	0.149	0.003	0.004	0.003	0.025	0.015	0.066	0.089	0.002	0.010
C	0.528	0.117	1.000	0.320	0.000	0.271	0.137	0.046	0.007	0.360	0.255	0.002	0.000	0.016	0.024	0.007	0.020	0.003	0.053	0.254	0.695	0.151	0.030	0.145
D	0.528	0.338	0.288	1.000	0.003	0.245	0.030	0.011	0.003	0.245	0.180	0.002	0.000	0.016	0.066	0.017	0.005	0.004	0.025	0.357	0.671	0.284	0.006	0.151
E	0.528	0.236	0.308	0.338	1.000	0.180	0.075	0.022	0.011	0.286	0.135	0.027	0.000	0.023	0.135	0.027	0.027	0.003	0.023	0.200	0.677	0.528	0.003	0.107
F	0.081	0.134	0.034	0.005	0.004	1.000	0.200	0.050	0.012	0.072	0.024	0.000	0.000	0.040	0.034	0.006	0.034	0.003	0.062	0.027	0.038	0.151	0.042	0.016
G	0.528	0.038	0.021	0.038	0.027	0.004	1.000	0.016	0.017	0.006	0.007	0.002	0.002	0.008	0.003	0.026	0.091	0.066	0.008	0.049	0.083	0.038	0.054	0.033
H	0.528	0.168	0.217	0.121	0.055	0.344	0.156	1.000	0.034	0.406	0.255	0.005	0.002	0.298	0.050	0.042	0.066	0.066	0.237	0.406	0.287	0.254	0.192	0.177
I	0.528	0.207	0.216	0.187	0.224	0.277	0.145	0.414	1.000	0.311	0.146	0.003	0.002	0.334	0.024	0.204	0.187	0.229	0.229	0.577	0.254	0.151	0.191	0.138
J	0.528	0.045	0.003	0.001	0.002	0.241	0.151	0.014	0.036	1.000	0.063	0.003	0.000	0.048	0.008	0.003	0.036	0.003	0.053	0.016	0.003	0.130	0.021	0.090
K	0.677	0.107	0.019	0.024	0.004	0.328	0.200	0.021	0.025	0.303	1.000	0.003	0.000	0.008	0.033	0.006	0.055	0.006	0.022	0.270	0.024	0.058	0.006	0.106
L	0.635	0.338	0.200	0.200	0.042	0.837	0.200	0.149	0.191	0.698	0.166	1.000	0.005	0.071	0.432	0.200	0.200	0.124	0.200	0.200	0.200	0.338	0.042	0.489
M	1.000	0.338	1.000	1.000	0.338	1.000	0.200	0.619	0.200	1.000	1.000	0.338	1.000	0.732	0.338	1.000	0.200	1.000	1.000	1.000	1.000	0.338	0.105	0.783
N	0.677	0.058	0.058	0.058	0.042	0.117	0.111	0.134	0.007	0.082	0.357	0.020	0.003	1.000	0.028	0.028	0.012	0.189	0.444	0.254	0.058	0.058	0.037	0.058
O	0.081	0.012	0.115	0.027	0.024	0.102	0.156	0.063	0.040	0.138	0.070	0.003	0.000	0.037	1.000	0.040	0.052	0.025	0.088	0.048	0.089	0.090	0.030	0.027
P	0.528	0.151	0.413	0.511	0.037	0.253	0.053	0.030	0.021	0.260	0.104	0.002	0.000	0.035	0.039	1.000	0.005	0.219	0.187	0.643	0.883	0.151	0.004	0.187
Q	0.528	0.151	0.063	0.151	0.078	0.036	0.025	0.055	0.018	0.051	0.027	0.016	0.014	0.071	0.048	0.162	1.000	0.213	0.103	0.181	0.162	0.142	0.103	0.094
R	0.433	0.181	0.195	0.177	0.162	0.162	0.046	0.013	0.007	0.162	0.116	0.012	0.000	0.003	0.114	0.052	0.002	1.000	0.038	0.196	0.162	0.192	0.030	0.058
S	0.528	0.043	0.058	0.042	0.042	0.042	0.088	0.001	0.001	0.042	0.116	0.002	0.000	0.001	0.008	0.042	0.040	0.138	1.000	0.127	0.049	0.047	0.004	0.058
T	0.238	0.095	0.044	0.035	0.003	0.122	0.025	0.000	0.001	0.066	0.036	0.002	0.000	0.007	0.036	0.002	0.003	0.020	0.019	1.000	0.060	0.151	0.005	0.017
U	0.528	0.121	0.258	0.082	0.000	0.103	0.025	0.014	0.001	0.145	0.103	0.002	0.000	0.016	0.024	0.001	0.003	0.003	0.023	0.146	1.000	0.151	0.005	0.107
V	0.023	0.031	0.004	0.005	0.007	0.071	0.025	0.001	0.003	0.025	0.016	0.001	0.000	0.016	0.081	0.003	0.005	0.005	0.026	0.005	0.003	1.000	0.002	0.045
W	0.528	0.187	0.104	0.107	0.145	0.068	0.062	0.151	0.007	0.289	0.224	0.023	0.023	0.135	0.033	0.151	0.072	0.033	0.172	0.151	0.254	0.528	1.000	0.107
X	0.105	0.096	0.031	0.023	0.006	0.245	0.053	0.025	0.025	0.025	0.016	0.001	0.002	0.016	0.043	0.013	0.032	0.023	0.023	0.053	0.020	0.088	0.006	1.000

Table 53: Outranking values with WOWA conjunctive (**WOWAc**) for countries dataset