A unified approach to the analysis of incoherent Doppler lidars

Antonio Lázaro

Department of Electronics, Electrics and Automatic Engineering, Rovira Virgili University, 43007 Tarragona, Spain
antonioramon.lazaro@urv.net

Aniceto Belmonte

Department of Signal Theory and Communications, Technical University of Catalonia, 08034 Barcelona, Spain
belmonte@tsc.upc.edu

Abstract: This work describes the use of a generalized modal scattering matrix theory as a fast, efficient approach to the analysis of incoherent Doppler lidars. The new technique uses Bessel beams, a type of optical vortices, as the basic modal expansion characterizing optical signals. The tactic allows solving both multilayered reflections problems and spatial diffraction phenomena using scattering parameters associated with the transmitted and reflected spectrum of vortices. Here, we will show the capabilities of the technique by considering realistic incoherent Doppler systems based on Fabry-Perot etalons.

OCIS codes: (350.5500) Propagation; (120.3180) Interferometry; (120.6200) Spectrometers and spectroscopic instrumentation; (050.1970) Diffractive optics; (280.3640) Lidar;

References and links

1. J. A. Dobrowolski, *Introduction to Computer Methods for Microwave Circuit Analysis and Design* (Artech House, Boston, 1991).
2. R. E. Collin, *Foundations for Microwave Engineering* (McGraw-Hill, New York, 1966).
3. K. Kurokawa, "Power waves and the scattering matrix," IEEE Trans.Microwave Theory Tech. **13**, 194-202 (1965).
4. J. W. Goodman, *Introduction to Fourier Optics* (McGraw-Hill, Boston, 1996).
5. J. Durnin, "Exact solutions for nondiffracting beams. I. The scalar theory," J. Opt. Soc. Am. A **4,** 651-654 (1986).
6. J. Durnin, J.J. Miceli, Jr. and J.H. Eberly, "Diffraction-Free Beams," Phys. Rev. Lett. **58**, 1499-1501 (1987).
7. A. Yariv, *Introduction to Optical Electronics* (Holt, Reinhart and Winston, New York, 1985).
8. M. Nieto-Vesperinas, *Scattering and Diffraction in Physical Optics* (John Wiley & Sons, Inc., New York, 1991).
9. V.A. Monaco, P.Tiberio, "Computer-Aided Analysis of Microwave Circuits," IEEE Trans. Microwave Theory Tech. **22**, 249-263 (1974).
10. D. M. Pozart, *Microwave Engineering* (Wiley, New York, 2004).
11. G.Hernandez, *Fabry-Perot Interferometers* (Cambridge U.Press, Cambridge, UK, 1988).
12. J.A.McKay, "Single and tandem Fabry-Perot Etalons as solar background filters for lidar," Appl. Opt. **38**, 5851-5857 (1999).

1. Introduction

The scattering matrix is the most common used technique to study propagation waves in transmission lines and microwave systems [1-3]. However, the application of scattering theory in optics has usually been limited to multilayer propagation of plane waves, and the analysis of spatial diffraction, the most characteristic effect of any optical element, has had to be considered through techniques much less competent. One of the most used approaches, the angular spectrum propagation method, analyzes the propagation of optical perturbations through systems consisting of optical elements, apertures, and free space regions using two-dimensional fast Fourier transforms [4]. By using Fourier techniques, the incident beams are decomposed in plane waves that propagate through the optical system. Unfortunately, although this method requires a larger number of plane waves to represent typical optical perturbances, it deals very poorly with optical interferometric systems (e.g. Fabry-Perot and Fizeau filters) where multiple reflections need to be accurately considered to describe correctly their behavior. Consequently, methods other than those based on angular spectra are required to solve this set of problems.

All in one, a unified approach for solving interferometric optical problems are not yet became readily available because of the difficulties to put together a systematic able to deal simultaneously with multiple reflections and diffraction. We aim to address these difficulties and use a generalized modal scattering matrix as a efficient approach to the analysis of incoherent interferometric optical systems. In contrast with other methods, the new technique uses Bessel beams [5, 6], a type of optical vortices, as the basic modal expansion characterizing optical signals. Bessel beams are free-space, exact solutions of the wave equation that are not subject to transverse spreading (diffraction). The tactic allows solving both multilayered reflections problems and spatial diffraction phenomena using scattering parameters (like in microwave systems) associated with the transmitted and reflected spectrum of optical vortices. Although this rigorous approach may have interest in many areas of optics, our main motivation in developing the technique has been the study of spectrometry problems on remote sensing systems. Intrinsically complicated and extremely sensitive, incoherent Doppler lidars are being considered as ideal tools to comprehend the atmospheric winds behavior.

In Section 2, we describe the principles of the technique and define the main parameters needed to model both basic optical elements and more complex optical systems. Section 3 describe the scattering matrix associated to a Fabry-Perot etalon and consider its use in the modeling of a complex, realistic incoherent interferometric system based on Fabry-Perot etalons. Section 3 recapitulates the main conclusions of this research.

2. A generalized scattering matrix representation

In this work we suggest a novel approach to fast and accurate analysis of large-scale incoherent optical lidar systems based on interferometric devices. We show that such systems may be described by scattering matrices (S matrices) and are connected by free-space propagations with corresponding S matrices. The properties of the entire optical interferoemeter are then accurately described by recursive combination of the individual S matrices of the functional optical components and the free-space propagations into a total S matrix.

The scattering parameters (transmission and reflection coefficients) are determined from the scattered fields. Classically, these transmitted and reflected fields are well typify just when canonical illuminations (i.e., plane) are considered [7]. It results more difficult to define scattering parameters and to consider scattering matrix representations of optical systems using realistic illuminations. Fourier modal methods have been developed to overcome these difficulties [8]. In those, to define complex optical illuminations into the considered systems, it has been customary to decompose the incoming fields into its wave spectrum. Using Fourier transforms, the angular spectrum method express the propagated field as the sum of plane waves with different phase delays depending on the plane wave propagation angle [4]. Unfortunately, any Fourier decomposition of realistically complex 2-D optical illuminations needs to use a considerable number of modes. As the amount of wave planes required to properly describe the optical system is in the order of thousands, these spectral techniques are used to be of little practical interests.

This work extends the S matrix theory based on the angular spectrum for propagating waves by using Bessel beams, a type of optical vortices, as the basic modal expansion characterizing optical signals. Bessel beams are free-space, exact solutions of the wave equation that are not subject to transverse spreading (diffraction). They are non-diffractive

#71197 - $15.00 USD Received 19 May 2006; revised 2 August 2006; accepted 3 August 2006

waves which, as plane waves, are easily propagated in the transform domain [5, 6]. In general, any optical field can be decomposed in term of Bessel modes. The main advantage is that, in general, just a few Bessel beams are needed to describe any practical optical situation.

To be specifics, using this new base we decompose any arbitrary wave U as:

$$U(r,\varphi) = \sum_{m=-M}^{M} \sum_{n=0}^{N} f_{mn} J_m\left(\nu_{mn} r\right) \mathrm{e}^{-jm\varphi} , \tag{1}$$

where J_m is the *m-th* order Bessel function of the first kind, and r and φ denotes the cylindrical coordinates. The product of these Bessel functions with the harmonic functions in Eq. (1) defines the so-called Bessel beams. The spatial frequencies ν_{mn} distinguish any of the optical modes used to describe the complex amplitude U. A number of $(2M+1)\times(N+1)$ modes have been considered. Once the modal basis has been set, the wave U can be explicitly described as a vector with components f_{mn}. The number of modes needed depend on the illumination beam studied. For example, for a typical gaussian beam we need to consider just a few dozen modes. For more complex illuminations, such as those considering optical aberrations, a higher number of modes may be required.

To illustrate our approach, we consider a interferometric system that consists of linear two-port components, including free-space propagations. In the most general case, each component may be a diffractive optical device. For the arbitrary diffractive optical device shown in Fig.1, each incident Bessel beam with order l at port i, $a_{i,l}$, produces reflected Bessel modes at port i, $b_{i,l}$ and transmitted Bessel modes at port j, $b_{j,l}$. As the device is diffractive, each input Bessel beam generates a complete set of output modes. The generalized scattering parameters can be defined at input port i with input mode m and output port j and mode n:

$$S_{ji,mn} = \left.\frac{b_{j,n}}{a_{i,m}}\right|_{a_{k,v}=0,k\neq i,v\neq m} \tag{2}$$

For a simple two-port component, this matrix can be written as:

$$\begin{pmatrix} \boldsymbol{b}_1 \\ \boldsymbol{b}_2 \end{pmatrix} = \boldsymbol{S} \begin{pmatrix} \boldsymbol{a}_1 \\ \boldsymbol{a}_2 \end{pmatrix} = \begin{pmatrix} \boldsymbol{R}_{11} & \boldsymbol{T}_{12} \\ \boldsymbol{T}_{21} & \boldsymbol{R}_{22} \end{pmatrix} \begin{pmatrix} \boldsymbol{a}_1 \\ \boldsymbol{a}_2 \end{pmatrix} \tag{3}$$

Here, the choice of the transmission $\boldsymbol{T}$ and reflection $\boldsymbol{R}$ matrices makes explicit the physical meaning of the four submatrices of $\boldsymbol{S_j}$. Arbitrary lidar systems can be analyzed using this generalized scattering parameters. Each subsystem is connected to other subsystems in the same way as microwave circuits. With this modal method, reflection between subsystems are taken in count with the S-parameter formulism. This formalism allows to consider diffractive optics by defining non-diagonal S-matrix. The computationally expensive 2-D convolution of plane waves in classical beam propagation methods is here a simple product of full modal S-matrix. If the optical device is not diffractive, the S-matrix is diagonal. Non-diffractive optical elements have diagonal S-matrix, as it has the non-diffractive propagation of our Bessel beams. To obtain the S parameter of a lidar system from the S parameters of the composed subsystem the same analysis techniques than in microwave networks can be used [9, 10].

#71197 - $15.00 USD Received 19 May 2006; revised 2 August 2006; accepted 3 August 2006

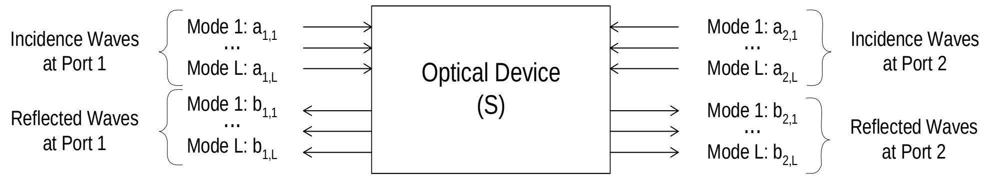

Fig. 1 Outgoing and incoming waves in two-port optical network. We denote the incoming and outgoing waves at ports i by the L=(2M+1)x(N+1)-component vectors ai, and bi, respectively

3. Modal parameters for Fabry-Perot etalons

Our generalized modal scattering matrix approach permits considering multilayered reflections problems and spatial diffraction phenomena using scattering parameters associated with the transmitted and reflected vortical spectrum. In general, the formalism reduces the analysis of a general interferometric system to the definition of S matrices for any of its constituent optical components. As it was indicated before, our main motivation in developing the technique has been the study of spectrometry problems on remote sensing systems. Intrinsically complicated and extremely sensitive, incoherent Doppler lidars based on Fabry-Perot etalons are being considered as ideal tools to comprehend the atmospheric winds behavior.

It is straightforward to apply our analysis method to the study of Fabry-Perot interferometers. Figure 2 shows a simple, schematic Fabry-Perot etalon composed by two mirrors separated a distance *d*. In our technique framework, we describe the etalon as a three linear two-port elements, i.e., two reflective layers and one free-space propagation. Basically, any mirror can be represented by a field scattering matrix ***S*** with $S_{11} = S_{22} = \sqrt{\mathcal{R}}$ and $S_{21} = S_{12} = j\sqrt{1-\mathcal{R}}$, where $\mathcal{R}$ is the complex reflectivity of the mirror surfaces and $j=\sqrt{-1}$ takes into consideration the $\pi/2$ phase change introduced to the transmitted waves by the reflective layer. Obviously $\mathcal{R}$ and, consequently, the scattering parameters S_{11}, S_{21}, S_{12}, and S_{22}, have no dependency with the spatial frequencies ν_{mn} of the illumination wave.

On the other hand, the free-space propagation scattering description needs to consider an implicit dependency on the spatial frequencies ν_{mn} used in Eq. (1) to describe the waves traveling the etalon cavity. Since each Bessel beam component has a different associated spatial frequency ν_{mn}, each travels a differently the distance between the two etalon parallel planes, and relative phase delays are thus introduced. The well-known direction cosine $(1-(\lambda\nu_{mn})^2)^{1/2}$ along the propagation axis must be regarded as the factor describing the phase delay associated to the Bessel-wave component with spatial frequency ν_{mn}. Now, the required free-space propagation can be analyzed through a scattering matrix ***S*** with $S_{11} = S_{22} = 0$ and $S_{21} = S_{12} = exp\,[-j\,k\,d\,(1-(\lambda\nu_{mn})^2)^{1/2}]$, where λ is the illumination wavelength and $k= 2\pi/\lambda$ is the wavenumber. Certainly, these results consider the foundations of scalar diffraction theory. In fact, as with the angular spectrum of plane waves, we are formulating the scalar diffraction in a linear framework where the basic optical disturbances are our Bessel beams: If a complex field distribution is analyzed across any plane, the various spatial components can be identified as Bessel waves traveling away from that plane so that the field amplitude across any other plane can be calculated by adding the contributions of these Bessel waves taking due account of the phase shifts they undergone during propagation. Using these propagation terms, we make sure that our estimations collect any diffraction effect due to diffracting structures (per example, apertures limiting the incoming light) or finite illumination sources (such as Gaussian field illuminations). Also, we use scalar diffraction in its most general forms, i.e., we do not need to consider Fresnel and Fraunhofer approximations used to reduce

#71197 - $15.00 USD Received 19 May 2006; revised 2 August 2006; accepted 3 August 2006

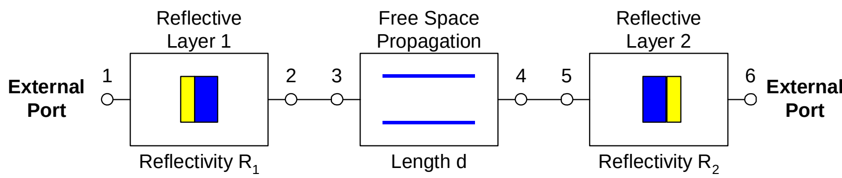

Fig. 2. A Fabry-Perot interferometer can be analyzed as a multiport optical system composed of three different two-port elements. Waves $\boldsymbol{b_2}$ transmitted by the first reflective layer propagated a distance d to reach the second reflective layer as incoming waves $\boldsymbol{a_5}$. By properly defining the scattering matrix of any of these three elements, our modal approach allows to describes any interference and diffractive problem characterizing the Fabry-Perot etalon behavior.

the mathematical manipulations. Our estimations are valid for any possible range making possible to consider both near and far fields of the illumination.

Finally, connecting the three S matrices needed to build a Fabry-Perot scattering model, we have a full, exact, electromagnetic description of the interferometric device. Well beyond the scope of paraxial and Fourier approaches, with our technique any possible consideration of electromagnetic optics is implicit in the analysis. Furthermore, by defining new S matrices for any other possible component of the instrument, this modal scattering method allow us readily to define realistic system configurations with diverse complexity and without compromise the accuracy of the analysis. We can extent the technique to consider multiple-etalon interferometric sytems in a reliable way by simply adding new Fabry-Perot etalons separated by free space propagations. Any diffraction, interference, and coupling effects are automatically considered.

So far, we have defined Fabry-Perot etalons by using three different two-port elements. However, it is worth to comment that is possible to follow a different approach. In fact, if we are interested in the analysis of multi-etalon interferometers, the analysis may be simpler if we could define an S matrix for the etalon itself. Per example, if we were able to define this basic etalon scattering matrix, and without considering any secondary optics, the analysis of a three-etalon system would need to consider just three S matrices instead of the nine matrices requiring the previous approach implementation. By considering any Fabry-Perot etalon as a three-element optical network, we can steadily obtain its S matrix by applying the so-called Mason's gain rules [10]. The representation of our optical resonator by a block diagram showing the basic connectivity of its components (Fig. 3), allows finding the transfer function of the system by working out the forward and backward paths and their gains. In our case, the paths are those connecting the three optical components (two mirrors and the propagation) and the gains are just the transmissivities S_{21}, S_{12} and reflectivities S_{11}, S_{22} we have defined for them. It is easy to apply Mason´s rule to obtain the S-matrix components of a Fabry-Perot resonator:

$$\begin{aligned} S_{11}(\nu_{mn}) = S_{22}(\nu_{mn}) &= \frac{\sqrt{\mathcal{R}} + \sqrt{\mathcal{R}}\ \exp\left[-jkd\sqrt{1-(\lambda\nu_{mn})^2}\right]}{1-\ \mathcal{R}\ \exp\left[-jkd\sqrt{1-(\lambda\nu_{mn})^2}\right]} \\ S_{21}(\nu_{mn}) = S_{12}(\nu_{mn}) &= \frac{\sqrt{1-\mathcal{R}^2}\ \exp\left[-jkd\sqrt{1-(\lambda\nu_{mn})^2}\right]}{1-\ \mathcal{R}\ \exp\left[-jkd\sqrt{1-(\lambda\nu_{mn})^2}\right]} \end{aligned} \tag{4}$$

Obviously, this synthetic approach must lead to the same results that the genuine one considering a matrix for any component on every etalon. We have tested and confirmed this result. Also, it is worth to note that results in (4) are similar ─but not identical─ to the well known Airy formulas for the transmission of an ideal Fabry-Perot resonator [11]. Here, we consider not just the interference of multiple resonator modes traveling back and forth

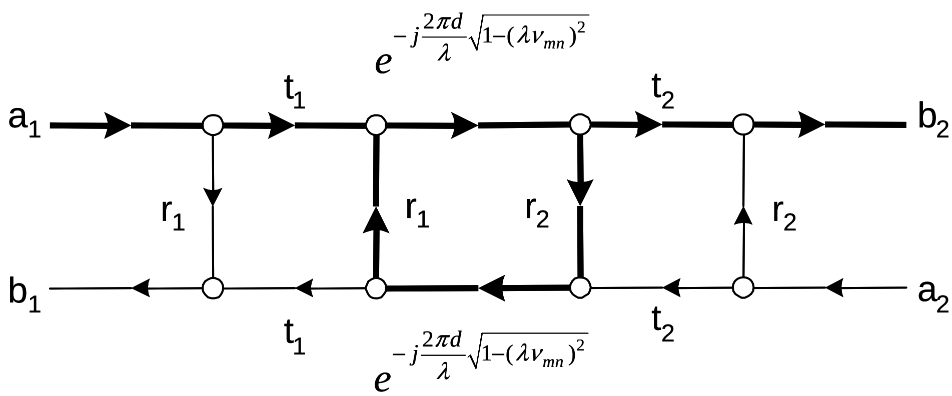

Fig. 3. Flow chart for a Fabry-Perot interferometer. In bold line, the signal path for an input wave that propagates to the output of the etalon. To find the transfer function of the etalon system represented by this block diagram we use Mason's gain rule.

between the two mirrors, but any diffraction effect and illumination divergence not contemplated in the basic Airy expressions.

Next, we show the result of applying our techniques to a tandem of two Fabry-Perot resonators acting as an incoherent Doppler interferometric system. Direct detection Doppler lidars probing atmospheric winds need to consider both Mie scattering from aerosols in the lower troposphere and Rayleigh scattering form the lower stratosphere. As the spectral widths of aerosol (MHz) and molecular (GHz) backscattered signals are orders of magnitude different, two different interferometric channels must be defined in any reliable working lidar system. In an effort to increase the overall system efficiency, usual instrument configurations capture the light reflected by a high-spectral-resolution aerosol Fabry-Perot etalon into another wider passband molecular etalon. Certainly, the analysis and optimization of such complex instruments has to consider in a realistic way the limitations caused by any cross-talk between light channels and any interference and diffractive effect.

Fig.6 (left) shows the flowchart representing two etalons in cascade. Using Mason rules, the two-etalon transmittance for a given Bessel mode *l* (element of the S matrix diagonal as this network is not dispersive) is obtained:

$$T(\rho_l) = S_{21}(\rho_l) = \frac{T_1\tau_{21}T_2}{1-\tau_{21}R_2\tau_{12}R_1} \tag{5}$$

where T_1 and T_2 are the two etalon transmittances, R_1 and R_2 are the etalon reflectances, and τ_{21} is the transmittance (attenuation) between the two etalons and τ_{12} is the transmittance (isolation) in the reflected signal between the two etalons.

Note, that expression (5) has the same form of the well-known Fabry-Perot Airy function. Alternatively, this result an be obtained by summing the multiple wave reflections between etalons [12]. However, using the proposed flow chart method, compact analytical expressions for triple or other optical circuits can be easily obtained. Distinctively, expressions (4) and (5) give the transmittance for each Bessel mode *l*, and they depend on the the radial angular spectrum frequency associated to this wave ($\rho_l=\nu_{mn}$). Thus, expressions (4) and (5) takes into account the angular dispersion of the light at the input. As a simple example, Fig.6 shows the transmittances function of the two-etalon combination (reflectivity R=90%, separations of 0.24 mm and 2.12 mm). This figure shows the ideal case without interreflections simulated sing an isolator between the two etalons (τ_{12}=0, τ_{21}=1), and without attenuator (τ_{12}= τ_{21}=1), and with attenuator of 5% (τ_{12}= τ_{21}=0.95). This figure shows that interreflections can be reduced using an attenuator, but the cost is a reduction in the peak transmittance. This fact is well

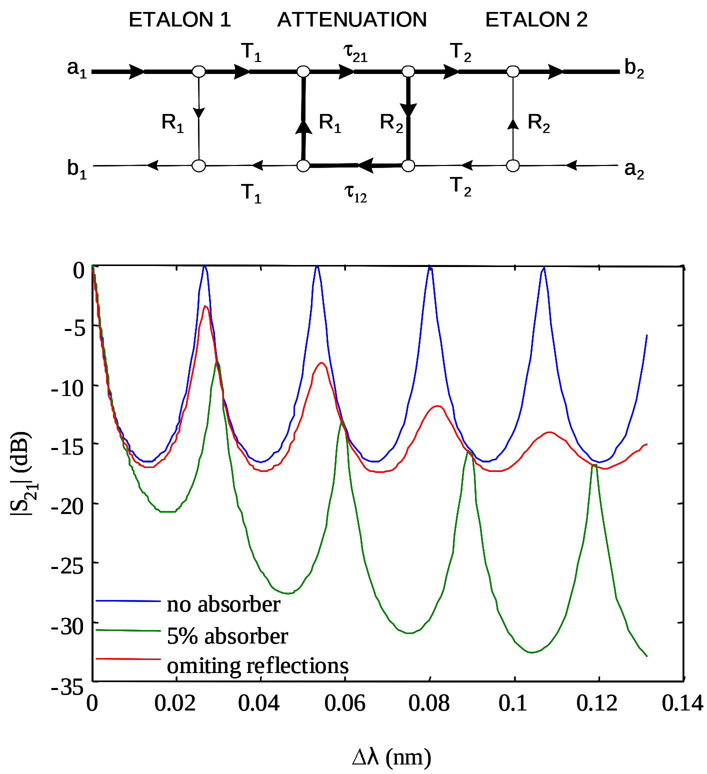

Fig. 4. Flow chart (up) for two etalons with a attenuator used to reduce reflections between etalons. In bold line, it is market the signal path for an input wave that propagate to the output of the second etalon. Transmittance for the dual-etalon solar filter without absorber, with 5% absorber, and isolated etalons omitting reflections (down).

known in microwave area, where attenuators are common connected between mismatched devices to improve the return loss of the second device. Multiple etalon systems improve the background rejection with a modest cost to signal transmittance due the attenuation losses [12].

4. Conclusion

The behavior of an incoherent Doppler lidar can be efficiently analyzed in terms of normalized wave variables at the ports of optical component multiports described by generalized scattering matrices. The new transfer matrix uses vortical Bessel beam as the basic modal expansion characterizing optical signals. The tactic allows solving both multilayered reflections problems and spatial diffraction phenomena using scattering parameters associated with the transmitted and reflected vortical spectrum. Although a wide variety of matrix representations for optical elements are known, the transfer scattering matrix representation shows the characteristic needed to properly address most of the electromagnetic problems of concern in optical interferometry. We are able to define useful relationships among different multiport matrix representations suitable to describe the complexity of the interactions in interferometric systems.

These research has been devoted to the definition and development of this new, rigorous systematic allowing the analysis and design of realistic incoherent lidar setups. All the results shown in this work have been carried out in the framework of *VBS* (Vortical Beam Spectra), a self-contained software package implementing our new, generalized modal scattering matrix theory and including a library of blocks to model the behavior of optical elements and systems. *VBS* tools are helping us to implement commercial and research optical interferometric systems and their components. The software serves a broad range of tasks across a variety of optical problems from analysis and design to optimization and modeling.

Interferometers used for atmospheric wind studies need to produce narrow and smooth interference fringes. By using our generalized scattering matrix formalism, we have analyzed the details of mode formation in multiple Fabry-Perot incoherent Doppler lidar systems and we have extended these results to the apparently more complex situation of incoherent Doppler lidars based on Fizeau wedge interferometers. The unique capabilities of the technique have been used to address the optimization of the interferometric device parameters, those producing the sharpest fringes in the detection plane. We have chosen to show the sensitivity of the method by studying the wind measurement uncertainty inherent to the lidar instrumentation as an indication of the performance of systems based on multiple Fabry-Perot etalons and Fizeau wedges. We intend to present these results in a companion paper.

#71197 - $15.00 USD Received 19 May 2006; revised 2 August 2006; accepted 3 August 2006