IDENTIFYING TECHNOLOGY SHOCKS AT THE BUSINESS CYCLE VIA SPECTRAL VARIANCE DECOMPOSITIONS*

YULIYA LOVCHA

Universitat Rovira-i-Virgili and CREIP.

Phone: (+34) 977759847

Fax: (+34) 977758907

e_mail: yuliya.lovcha@gmail.com

ALEJANDRO PEREZ-LABORDA

Universitat Rovira-i-Virgili and CREIP

Phone: (+34) 977758912

Fax: (+34) 977758907

e_mail: alejandro.perez@urv.cat

*This paper has benefited from numerous comments and suggestions from two anonymous Referees. Perez-Laborda acknowledges financial support from the Spanish Ministry of Economy and Competitiveness through AEI/FEDER-EU (ECO2016-75410-P) grant. The usual disclaimers apply. Address correspondence to Yuliya Lovcha, Department of Economics, Universitat Rovira-i-Virgili. Av. Universitat 1, 43204 Reus, Spain, or e-mail to yuliya.lovcha@gmail.com

SPECTRAL VARIANCE IDENTIFICATION

Yuliya Lovcha. Department of Economics, Universitat Rovira-i-Virgili. Av. Universitat 1, 43204 Reus, Spain. Fax: (+34) 977758907. *Phone:* (+34) 977759847. *e*mail:yuliya.lovcha@gmail.com.

Abstract: In this paper, we identify the technology shock at business cycle frequencies to improve the performance of the SVAR in small samples. To this end, we propose a new identification method based on the spectral decomposition of the variance, which targets the contributions of the shock in theoretical models. Results from a Monte-Carlo assessment show that the proposed method can deliver a precise estimate of the response of hours in small samples. We illustrate the application of our methodology using U.S. data and a standard RBC. We find a positive response of hours in the short-run following a non-significant, near-zero impact. This result is robust to a large set of credible parameterizations of the theoretical model.

Keywords: SVAR; frequency domain; RBC model; hours worked; technology shock

JEL Classification: C1, E3

1. INTRODUCTION

Structural vector autoregressions (SVARs) are widely employed in the analysis of economic fluctuations. The SVAR provides a flexible representation of relationships among variables, allowing for a quantitative assessment of their responses to shocks. However, it is necessary to impose additional restrictions to identify the shock of interest from the reduced-form VAR estimation. These restrictions are generally built on the grounds of economic theory to ensure the identified shock has the correct economic interpretation.

In this regard, the long-run (LR) restriction has become the standard scheme for the technology shock since it was employed by Gali (1999).[1] This restriction imposes technology to be the only shock contributing to the long-run productivity variance, a condition satisfied in many theoretical models. Given its relative simplicity, the scheme has become popular among researchers and is currently employed in many SVAR studies.

However, there are also strong arguments that advise against the use of the LR restriction to identify the technology shock. As noted in Uhlig (2004), changes in social attitude towards workplace or capital taxation may equally contribute to moving productivity in the long-run, invalidating the assumptions on which the scheme is constructed. More importantly, even if the LR restriction is met, its implementation requires a precise estimation of the low-frequency variance, something that is unfeasible from the short data spans available [Faust and Leeper (1997)]. As a result, the SVAR performs poorly in small-samples, and the response of hours is recovered with significant bias and considerable uncertainty [Christiano et al. (2003); Erceg et al. (2005); Chari et al. (2008)].

In this paper, we propose an alternative identification method aimed to improve the small-sample performance of the SVAR. Our approach is based on the spectral decomposition of the variance [Geweke (1977)] and minimizes the distance between the empirical contributions to the business cycle fluctuations of the variables and a set of targets for these percentages. The proposed Spectral Variance (SV) method does not impose the technology shock alone to explain all productivity fluctuations at the business cycle, nor does select the maximal contribution admissible in the SVAR, as many other shocks may move productivity at this range of frequencies. We assess the performance of our methodology in samples of simulated data from a Real Business Cycle (RBC) model, showing it can deliver a precise estimate of the response of hours in small samples.

In empirical applications, the SV method may target shares derived from Dynamic Stochastic General Equilibrium (DSGE) models to identify the shock in the SVAR. Resorting to an economic model is necessary for this context because it cannot be expected that technology alone explains all fluctuations in labor productivity at business cycle frequencies. However, the SV method does not impose the complete structure of the theoretical model; for example, the dynamic relationships between variables are estimated from the data. Furthermore, the particular targets taken from the DSGE are generally non-admissible in the SVAR and cannot be exactly matched. As a result, the empirical response of hours usually differs from the response in the theoretical model, not only in magnitude and persistence but sometimes even in sign.

We apply the proposed methodology to U.S. data using the contributions implied by a standard RBC. The SV method recovers a positive response of hours in the short-run, following a non-significant, near-zero impact. We generalize this finding by establishing a robust interval for the estimated response of hours. To do so, we base on a large set of credible parameterizations of the theoretical model, following an approach similar to the one Dedola and Neri (2007), Pappa (2009), and Canova and Paustian (2011) employed to establish robust sign restrictions. Overall, the robust interval for the response of hours offers considerable support for the baseline impulse response.

Our proposed methodology is not the first identification scheme placing the identification of the technology shock at a finite horizon. Ramey (2016) does an excellent review of the existing methods. Uhlig (2004), for example, restricts the forecast revision of productivity variance at four years to be driven by the technology shock only, with the acknowledgment, however, that the actual contribution at this horizon is not total. Francis et al. (2014) propose to maximize the share of the identified shock in the productivity forecast error variance at ten years. In this way, the contribution of the identified shock to the future uncertainty of productivity at the selected horizon is not necessarily 100%, but the maximum admissible in the SVAR at this horizon. DiCecio and Owyang (2010) develop a frequency-domain alternative of the previous identification method in which the authors maximize the contribution of the technology shock to the fluctuations of productivity at low-frequencies. Overall, these existent schemes still impose large shares for the contribution of the technology shock to productivity fluctuations; therefore, they have to be employed at relatively long horizons so as not to incur a significant identification bias. As a result, although all of them outperform the LR restriction, they are still associated with substantial uncertainty, making it difficult to establish robust conclusions on the magnitude and sign of the response of hours.

Likewise, our work is related to studies using theoretical models to identify the technology shock. Uhlig (2004), for example, selects the number of horizons at which to restrict the contribution of the technology shock based on a parametrized RBC model. Evans and Marshall (2009) employed an aggregate measure of technology borrowed from the Basu et al. (2001) model, which they modified after to ensure orthogonality with the other shocks in the system. More generally, the SV method is also linked to approaches using external information for the identification of the SVAR, such as Blanchard and Perotti (2002), Mertens and Ravn (2013), or Kilian and Lee (2014). For example, Kilian and Lee (2014) identify the oil shock, bringing the oil-price elasticity in the SVAR close to an external estimate.

The remainder of this paper is organized as follows: Section 2 contains the econometric framework. In this section, we define our proposed method and place it in the literature. In Section 3, we assess our identification scheme on simulated data jointly with other identification methods proposed in the literature. The empirical implementation is carried out in Section 4, where we recover the response of hours in U.S. data, generalize the empirical findings, and show how the SV method can be employed to study the compatibility between the model and the data. Section 5 discusses some extensions of the empirical section. Finally, Section 6 offers some concluding remarks. There is also a separate Supplement containing additional results.

2. ECONOMETRIC FRAMEWORK

2.1 The Structural VAR

Consider productivity growth as the first variable of the model, followed by the logarithm of hours worked and other possible variables summarized by the vector Z_t. This specification is standard across the literature and is consistent with the persistence of productivity and hours in theoretical models [Christiano et al. (2004); Chari et al. (2008)].[2]

Let the $N \times 1$ vector $X_t = \left(\Delta \ln p_t, \ln l_t, Z_t\right)'$ collect all the model variables. If $N = 2$, the model includes productivity growth and (log-) hours only. This simple specification, which is very popular in the literature, is considered as a baseline in our paper. A reduced-form VAR(p) for X_t can be written in MA(∞) form as:

$$X_t = \left[I - F\left(L\right)\right]^{-1} u_t = C\left(L\right) u_t . \tag{1}$$

where I is the identity matrix, $F(L)$ is a lag-polynomial matrix of order p, and u_t is a zero-mean error vector with Σ_u variance-covariance matrix. The structural MA representation is given by:

$$X_t = C(L) A\varepsilon_t \,, \tag{2}$$

where ε_t is a vector of uncorrelated structural errors with identity variance-covariance matrix. We further assume the normality of these shocks to ensure their statistical independence [Lanne et al. (2017)].

The elements of the matrix $C(L)A$ in (2) are infinite order polynomials whose coefficients are the impulse responses (IRF) of the variables to the structural shocks. The matrix A, which maps structural shocks into reduced form shocks, satisfies:

$$\Sigma_u = AA' \,.$$

Since Σ_u is symmetric, the previous system is undetermined and requires $N(N-1)/2$ additional restrictions.[3] Often, just the first column of A is of interest, as it is the only component needed to compute the responses of the variables to the technology shock given the assumed ordering. However, the system as a whole would remain just partially identified.

2.1.1 Set for candidate solutions

Let $u_t = H\eta_t$, where H is the lower-triangular Cholesky decomposition of Σ_u . By construction, the Cholesky shocks η_t have unit variance and are mutually uncorrelated, but they rarely correspond to the shocks of interest. However, one may find ε_t from a large set of candidates resulting from linear combinations of the Cholesky shocks as:

$$\varepsilon_t = P'\eta_t ,$$

where P_t is an orthonormal squared matrix satisfying $P'P = PP' = I$. Therefore, $u_t = HPP'\eta_t = HP\varepsilon_t$, implying that the structural matrix is identified as $A = HP$.

For $N = 2$, orthonormal matrices can be derived from the Givens rotation matrix:

$$P(\phi) = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix},$$

for $\phi = [-\pi/2, \pi/2]$. Thus, one can obtain a broad set of candidates by randomly drawing ϕ from its interval (or using a finite-dimensional grid). Givens rotation matrices can be extended to $N = 3$; however, for larger dimensions, computationally more efficient methods are recommended, such as in Rubio-Ramirez et al. (2010).

2.1.2 The Decomposition of the Variance in the Frequency Domain

The spectral decomposition of the variance measures the shares of the structural disturbances in the fluctuations of the variables at a given frequency or range of frequencies. It is preferred here over the popular forecast error variance decomposition because the business cycle is naturally defined in frequency-domain concepts (usually cycles with a periodicity between eight to 32 quarters). On the contrary, each h-step-ahead forecast error contributes to the variance at all frequencies, albeit with different proportions.

Consider the structural MA representation of the model in (2). The spectral density matrix of X_t at a given frequency ω is given by:

$$f_X(\omega) = \frac{1}{2\pi} C\left(e^{i\omega}\right) \Sigma_u C\left(e^{-i\omega}\right)', \tag{3}$$

where i denotes the imaginary unit and $C\left(e^{-i\omega}\right)$ the complex conjugate of $C\left(e^{i\omega}\right)$. The main diagonal of the spectral density matrix contains the univariate spectrums of the variables in X_t ; the off-diagonal elements are the cross-spectral densities. Given that the spectrum of a process can be interpreted as the decomposition of the variance into a set of uncorrelated components at each frequency, the share of the k-th shock in the fluctuations of the n-th variable at the frequency ω is given by:

$$w_{nk}^{\omega} = \frac{s_n{}'C\left(e^{i\omega}\right)A_k A_k{}'C\left(e^{-i\omega}\right)' s_n}{s_n{}'C\left(e^{i\omega}\right)\Sigma_u C\left(e^{-i\omega}\right)' s_n}, \tag{4}$$

where A_k is the k-th column of the structural matrix A and s_n is a selector vector with an n-th element equal to one and other elements equal to zero. The expression (4) has to be integrated over the range of interest to assess the shares of structural shocks in fluctuations at a range of frequencies. Let $R=\left[\omega_1,\omega_2\right]$ be the selected range. The percentage of the k-th disturbance in the fluctuations of the nth variable at the range R is given by:

$$w_{nk}^{R} = \frac{s_n{}'\left[\int_R C\left(e^{i\omega}\right)A_k A_k{}'C\left(e^{-i\omega}\right)' d\omega\right]s_n}{s_n{}'\left[\int_R C\left(e^{i\omega}\right)\Omega C\left(e^{-i\omega}\right)' d\omega\right]s_n}, \tag{5}$$

where the definite integrals can be approximated by summations for the Fourier frequencies $\omega_j = 2\pi j/T$ for $j=1,2,...T/2$ belonging to the selected range (Sargent, 1987). As can be deduced from the last expression, the contribution of the different structural shocks critically depends on the entries of A.

2.2 The Spectral Variance Method (SV)

In this section, we define our proposed identification method. After that, we briefly review the other identification strategies employed in the literature, comparing our approach with the existing methods.[4]

2.2.1 Definition of the SV method

The Spectral Variance (SV) method brings the contributions at business cycle frequencies of the shock identified in the SVAR as close as possible to a given set of targets. Like with the other schemes in the literature, the method identifies the technology shock only (the first column of the matrix *A* given the assumed ordering), but the system as a whole remains partially identified if $N > 2$.

Let an orthogonal matrix *P* defines an alternative representation of the SVAR model in terms of the structural errors:

$$u_t = HP\varepsilon_t ,$$

where H is the lower-triangular Cholesky decomposition of the variance-covariance matrix of the reduced-form errors Σ_u.

Denote the k-th column of the matrix *P* by α, and let T_{nk}^R be the targeted percentage for the contribution of the n-th shock to the fluctuations of the k-th variable at the frequency range $R = [\omega_1, \omega_2]$. The SV method solves the following optimization problem:

$$\min_{\alpha} D_k^R(\alpha) = \sum_n \left(w_{nk}^R(\alpha) - T_{nk}^R \right)^2 . \qquad (6)$$

The term $w_{nk}^R(\alpha)$ in (6) is the contribution of the k-th shock to the fluctuations of the n-th variable at R in the SVAR, which can be computed for $A_k = H\alpha$ from Equation (5).

Therefore, the SV method minimizes the Euclidian distance between the contributions of the k-th shock to the fluctuations of all variables in the system at the selected frequency range and the given targets. Since productivity growth is ordered first, the technology shock is identified at business cycle frequencies by setting $k=1$ and defining R as being formed by cycles with periodicity ranging from eight to 32 quarters, which is the standard definition of the business cycle in the literature.[5]

In empirical applications, the proposed scheme may target shares derived from parametrized DSGE models. Targeting the contribution of the technology shock in a theoretical model makes the identification of the technology shock model-based. However, the complete structure of the DSGE model is not imposed in the SVAR; for example, the dynamic relationships between the variables are estimated from the data. As a result, the targeted percentages are generally not feasible and cannot be matched exactly, and the response of hours in the SVAR does not coincide with the response of this variable in the DSGE. The two responses may differ substantially in some cases, not only in magnitude but in sign as well.

Nevertheless, we propose to generalize the empirical findings targeting a large set of contributions based on many credible parameterizations of the theoretical model. To do so, we assume all the parameters in the DSGE to be uniformly and independently distributed over sufficiently wide intervals, like in Dedola and Neri (2007), Pappa (2009), or Canova and Paustian (2011). More specifically, we employ the following procedure to generalize the empirical findings of the SV method:

1. Select ranges for all parameters in the DSGE, chosen to be large enough to include all theoretically reasonable values in the literature. The intervals for the model parameters can be alternatively estimated via Bayesian methods (Herbst & Schorfheide, 2016). The

advantage of specifying sufficiently wide intervals is that the range of outcomes the model can produce is obtained without relying on any particular dataset.

2. Draw values for all DSGE parameters within their credible intervals from independent uniform distributions to generate a credible parameterization.
3. Compute the business cycle contributions of technology shock in the DSGE model using this parameterization.
4. Use the resulting contributions as targets for the SV method to identify the estimated SVAR, and find the empirical response of hours.
5. Return to step 2 until the desired number "P" of credible parameterizations of the DSGE has been obtained. After that, selected percentiles can be used to construct robust ranges for the impulse response of hours in the SVAR.

2.2.2 Relationship with other identification methods

The long-run (LR) identification [Shapiro and Watson (1988); Blanchard and Quah (1989)] is the most popular scheme for identifying the technology shock in the SVAR. It restricts the (log-)productivity variance to be driven in the long-run by the technology shock only. This condition is equivalent to setting the shock contribution to productivity growth (i.e., log-productivity difference) to 100% at zero frequency. Therefore, to impose the restriction, the estimate of the spectral matrix of the VAR process at zero frequency is usually decomposed as the product of a triangular matrix by its transpose:

$$S_Y(0) = C(1)\Sigma_u C(1)' = DD' , \qquad (7)$$

where $C(1) = \left[I - F(1)\right]^{-1}$, being $F(1)$ the sum of autoregressive matrices. Therefore, the matrix A is determined by solving $C(1)A = D$.

Christiano et al. (2003) and Erceg et al. (2005) assess the LR restriction showing that the response of hours to a shock identified with this method is recovered with significant bias and substantial uncertainty. The underlying reason is that the application of the LR scheme requires a precise estimate of the spectrum at zero frequency, which is not feasible in small samples. As can be seen from Equation (7), the bias and uncertainty of the estimate of $F(1)$ percolate into the spectrum estimated at zero frequency magnified by the non-linear transformation $C(1) = \left[I - F(1)\right]^{-1}$, and thus to the structural matrix as well.

Difficulties in estimating the long-run variance have been used to justify identification restrictions at finite horizons, such as in the methodologies in Uhlig (2004), DiCeccio and Owyang (2010), and Francis et al. (2014). The separate Supplement contains a simple example showing this asymmetric pass-through.

Uhlig's (2004) medium-run (MR) scheme restricts the shock to explain all the variance of the forecast revision of productivity at a finite horizon $0 \leq h < \infty$. The author selects a horizon of four years ($h = 16$ for quarterly data) based on the inspection of a parameterized RBC model acknowledging, however, that other shocks may contribute to moving productivity at this horizon as well. Notice that the LR restriction can be obtained as the limit of the MR when $h \to \infty$, as this scheme imposes a 100% contribution in the long-run.

The Max-Share (MS) method (Francis et al., 2014) restricts the contribution of the technology shock to the forecast error variance of (log-)productivity at a finite horizon h. Thus, the MS method is related to Uhlig's MR identification, as the forecast error variance is the sum of the variance of the forecast revision up to (and including) h. Also, given that a 100% contribution of the technology shock to the forecast error variance of productivity may not be feasible in the SVAR at the selected horizon, the authors employ a maximization routine that identifies the shock with the maximum contribution possible. In this way, the MS imposes a 100% contribution only if it is feasible in the SVAR. As for the selected horizon, Francis et al. (2014) fix it to 10 years ($h = 40$ quarters), moving the restriction further into the long-run than the MR method. Thomet and Wegmueller (in press) recently use the MS scheme to assess the response of hours worked in a set of 14 OECD countries.

DiCecio and Owyang (2010) propose a version of the MS method in which the identification is produced in the frequency domain. Their Frequency-Domain (FD) identification is based on maximizing the contribution of the identified shock in the fluctuations of productivity over a low-frequency range. The authors employ two different frequency ranges, "low" frequencies (cycles from eight to 20 years) and "very low" frequencies (frequencies with a period longer than 20 years, excluding the zero frequency). Like the MS identification, the FD method does not necessarily impose the shock to explain all productivity fluctuations at the selected range (which may not be feasible), but the maximum admissible in the SVAR.

A detailed description of the MR, MS, and FD schemes is provided in the Supplement. These schemes may perform better than the LR restriction in small samples, mostly because the bias and uncertainty transmitted to the estimate of the productivity variance at a finite horizon are smaller than in the long-run. However, they still impose the identified shock to explain a large share of productivity fluctuations at the selected horizon (either 100% or the maximum possible in the SVAR). As shocks other than technology may contribute to moving productivity at short horizons, a large misidentification bias may appear using these methods if the horizon is set too short.

The proposed SV approach is related to the MR, MS, and FD schemes, as all try to move the restrictions away from the long-run to improve the small-sample performance of the SVAR. However, it departs from these three schemes in many important aspects. Unlike the MR and MS, the SV is defined in the frequency domain, which allows for a precise characterization of the business cycle. It uses a complete frequency range for setting the restriction instead of a single frequency (or specific horizon, as the MR), thus not generating undesirable kinks in the spectrum, as noted in Francis et al. (2014). In this sense, our scheme is closely related to the FD identification of DiCecio and Owyang (2010). However, the SV method is aimed to identify the shock at the business cycle. Consequently, it does not impose the shock in the SVAR to explain all productivity fluctuations at this range, nor the scheme selects the maximum possible contribution in the SVAR either, as these conditions, although palatable at low-frequencies, are not expected to be satisfied at the business cycle.

In this sense, the SV identification generalizes the MR, MS, and FD schemes allowing for any possible contribution to productivity growth fluctuations. However, as it is shown in Section 4, targeting only the shock share in productivity growth fluctuations does not generally lead to a unique solution to the problem in (6). Although set identification may be valuable, especially in combination with other schemes, the penalty function in Equation (6) includes the contribution to the fluctuations of all variables, not just productivity growth, which helps to discriminate between candidate solutions.

Finally, note that the SV is defined in terms of productivity *growth* (productivity log difference), while the FD is defined in terms of (log-) productivity. Targeting productivity growth fluctuations places the method closer to the LR restriction. As explained above, the LR identification can be alternatively defined in the frequency domain by restricting the shock contribution to *productivity growth* to be 100% at zero frequency. Thus, the FD identification does not nest the LR restriction when the maximization is produced at zero frequency. However, the LR can be obtained in the SV method defining the frequency range as to be formed by the zero frequency and using 100% for the contribution to productivity growth fluctuations as a sole target (it is, leaving the targets for the other variables free in the minimization algorithm). Besides, defining the scheme in terms of productivity growth is computationally more convenient, as it is how the variable enters the VAR.

3. THE SIMULATION STUDY

Before recovering the response of hours in the data, we assess the SV identification in a simulation study, together with the other existing identification approaches. The objective here is to compare the different methods in a common framework, investigating whether they offer actual advantages over the LR in small samples. Concerning this, notice from Equation (2) that any difference in performance must come from the identified structural matrix A, as $C(L)$ is the same for all methods.

To do this, we follow Sims (1984) and evaluate the schemes in recovering the response of hours from data simulated from an RBC, which has become the standard benchmark for SVAR assessment [Christiano et al. (2003, 2007); Erceg et al. (2005); Uhlig (2005); Chari et al (2008); Francis et al. (2014)]. As noted in Chari et al. (2008), simulating data from an RBC may generate an additional bias in the response of hours recovered in the SVAR, as the true RBC model's VAR has an infinite number of lags.

3.1. The RBC Model

We employ the two-shock RBC of Christiano et al. (2007) to simulate the data, which provides a good fit to the data compared to other RBC specifications. Details on the transformation of variables, log-linearization, model solution, and state-space representation can be found in the Supplement to Lovcha and Perez-Laborda (2015).

3.1.1 RBC description

The representative agent maximizes the expected utility of consumption per capita c_t and hours worked l_t:

$$\max_{c_t, l_t} E_0 \sum_{t=0}^{\infty} \left(\beta\left(1+\gamma\right)\right)^t \left[\ln c_t + \varphi \frac{\left(1-l_t\right)^{1-\sigma} - 1}{1-\sigma}\right],$$

subject to the household budget constraint: $c_t + (1-\tau_x)i_t \leq (1-\tau_{l,t})w_t l_t + r_t k_t + T_t$. Here, w_t and r_t are the wage and the rental rate on capital, τ_x and $\tau_{l,t}$ are investment and labor income tax rates, and T_t is a lump-sum tax. The parameter σ controls the curvature of the utility function, φ is the time allocation parameter, γ is the growth rate of the population, and β is the discount factor. Capital accumulation is given by: $i_t = (1+\gamma)k_{t+1} - (1-\delta)k_t$, where k_t is the per capita capital stock at the beginning of the period and δ is the depreciation rate. Finally, the total resource constraint is given by: $c_t + i_t \leq y_t$.

The representative firm's production function is:

$$y_t = k_t^{\alpha}(Z_t l_t)^{1-\alpha}.$$

Z_t in the previous function is the state of technology and $\alpha \in (0,1)$. The technology process is represented by a random walk: $\ln Z_t = \mu_z + \ln Z_{t-1} + \sigma_z \varepsilon_t^z$, and the process for the labor tax shock by stationary but persistent AR(1): $\tau_{l,t} - \tau_l = \rho_l(\tau_{l,t-1} - \tau_l) + \sigma_l \varepsilon_t^l$. The shocks ε_t^z and ε_t^l are i.i.d. random variables, orthogonal to each other, with zero mean and unit variance. The constants μ_z and τ_l are the mean growth rate of technology and the mean labor tax rate, respectively. The magnitude of the autoregressive coefficient ρ_l is restricted to be less than one to guarantee stationarity.

As standard in the literature, time *t* decisions are made after the realization of the shocks. In this way, the technology shock alone moves productivity in the long-run. Therefore, the model satisfies the LR restriction.

3.1.2 Baseline parametrization

The baseline parameterization of the RBC model is the same employed by Christiano et al. (2007), which is summarized in Table 1.

[Table 1 around here]

The contributions of the technology shock to the business cycle fluctuations of the variables implied by this parameterization are 80.36% and 7.48%, for productivity growth and hours worked, respectively where, as standard in the literature, we have defined the business cycle as being formed by frequency components with periodicities from eight to 32 quarters. These percentages can be assessed by applying Equation (5) to the state-space representation of the RBC, and thus, are intrinsic characteristics of the parameterized model. See the Appendix of Lovcha and Perez-Laborda (2015) for details.

3.2 Simulation Set-Up

We simulate 1,000 quarterly artificial bivariate datasets of T=240 observations each (60 years of data) from the baseline parameterization of the RBC. For each simulated dataset, we estimate a VAR(4), which is the standard lag-length in the literature, and identify the technology shock using our and the other referenced schemes.[6] We then compute the response of hours and compare this response with that in the model.

For the MR and MS methods, the horizon required for their implementation is set at four ($h=16$) and ten ($h=40$) years, respectively, as in Uhlig (2004) and Francis et al. (2014). For the FD, we maximize the shock contribution to the productivity fluctuations with a period longer than 32 quarters, excluding the zero frequency.[7] Finally, for the proposed SV scheme, we target the contributions in the baseline RBC, making the targets satisfied in the model used as DGP.

3.3 Monte-Carlo Results

Selected simulation results are provided in Figure 1 and Table 2. Figure 1 collects the mean of the estimated responses of hours across simulations (solid line). The dashed lines and the shadowed area highlight the 16^{th} and 84^{th} percentiles and the 5^{th} and 95^{th} percentiles, respectively (i.e., the 68^{th} and 90^{th} percentile bands). We also include the response in the RBC (red line with dots) to assess the bias visually. Table 2 reports the mean contribution of the identified shock at business cycle frequencies together with their 68^{th} percentile intervals across simulations. The results in Table 2 allow us to investigate if, regardless of the horizon at which the restriction is placed, the shock identified in the SVAR and the true shock in the RBC have similar business cycle properties, as this range is usually the horizon of interest.

[Figure 1 around here]

As Figure 1 shows, the results of the LR method in our simulations reproduce the findings of Christiano et al. (2003) and Erceg et al. (2005). The LR returns a significantly upward biased response of hours recovered with huge uncertainty, with the percentile bands containing the zero-response at all lags. Also, results in Table 2 indicate that, although the shock in the SVAR explains the same share of the productivity growth fluctuations at zero frequency as in the RBC (100%), it severely underestimates the percentage at business cycle frequencies. The contribution to hours worked at this range is also biased, with a mean value across simulations larger than the actual percentage in the theoretical model.

[Table 2 around here]

The MR, MS, and FD schemes improve over the LR restriction, especially in terms of the bias. As Figure 1 shows, there is no apparent bias in the response of hours recovered with the MR and FD schemes. The response estimated with the MS method presents some small upward bias. Note, however, that the MS sets the identification at a slightly longer horizon than the other two methods, and the bias can be reduced by setting the restriction a tad closer (for example, at the eight-year horizon). In any case, the bias found using the MS method is very small, with the mean response across simulations being very close to the actual response in the RBC. Overall, although these methods impose a large contribution of the shock identified in the SVAR to the productivity fluctuations, there is no evidence of significant misidentification bias in the estimated response of hours. The absence of bias is mostly explained by the fact that the actual contribution of the technology shock in the RBC at the horizons selected by the authors is also large; therefore, the restrictions imposed by these methods are satisfied in the theoretical model to a great extent.[8] Furthermore, results in Table 2 show that, on average, the shock identified by these schemes explains percentages of the business cycle fluctuations of the variables that are close to the actual shares in the RBC, especially for productivity growth. Overall, our simulation exercise shows that the MR, MS, and FD methods overcome the problem of the magnification of the bias in the long-term variance using the LR restriction.

However, the results of the simulation exercise also reveal that the uncertainty associated with these methods is still huge, making it difficult to establish robust conclusions on the magnitude and sign of the response of hours. As Figure 1 shows, the percentile bands for this response recovered with the MR, MS, and FD methods are wide. In particular, none of these methods yield a significant response of hours in the short-run, as even the 68th percentile bands include the zero response at all horizons. Results in Table 2 also reflect the uncertainty associated with these methods, as the percentile intervals for the recovered contributions are wide. Thus, the business cycle properties of the shock identified in the SVAR very often differ substantially from the actual properties the technology shock has in the RBC.

Can the uncertainty associated with these methods be reduced by setting the identification of the technology shock at shorter horizons, such as the business cycle? [9] The answer is that uncertainty can be reduced but at the cost of introducing a significant misidentification bias. The reason is that the contribution of the technology shock in RBC models falls substantially at shorter horizons, as it is expected that other shocks move productivity at these ranges. Consequently, restricting the SVAR to deliver the maximum share possible (or even 100%, for the MR) induces a significant bias in the estimate of the response of hours to the technology shock.

To see this result more clearly, let us study the FD method further. As noted above, if one employs low frequencies for identification, the response of hours is recovered without bias but with high uncertainty. For illustrative purposes, we move the restriction to the business cycle and use the same FD method to identify the shock (from now on called FD-BC method). The simulation results obtained with the FD-BC method are also reported in Figure 1 and Table 2.

Moving the restriction to the business cycle reduces the uncertainty associated with the FD method; the percentile bands for the response of hours recovered with the FD-BC are significantly narrower than with the standard FD. However, the estimated response is strongly biased, with the actual response in the theoretical model lying outside the percentile bands. The reason is that the contribution of the technology shock to productivity fluctuations in the RBC is significantly smaller at business cycle frequencies (80.1%) and the maximum admissible in the SVAR at this range becomes too large.[10] Notice that the differences between the FD and the FD-BC can only be attributed to the selected horizon, as the imposed restriction is the same.

Finally, the first panel in Figure 1 reports the simulation results obtained with the SV method, which also sets the restriction at the business cycle. As with the MR, MS, and FD methods, there is no evidence of bias in the response recovered with the proposed SV scheme. However, the identification is set at a shorter horizon than with these methods; therefore, the percentile bands are narrow enough to return a positive and significant response of hours in the short-run, close to the response in the RBC.

We cannot judge the SV based on the percentages recovered in Table 2, as the scheme tries to minimize their distance to the percentages in the RBC. Notice, however, that the actual values in the RBC are not matched exactly in the SVAR across simulations. The reason is that the estimated VAR is different from the RBC model's VAR used to simulate the data due to both sampling uncertainty and model misspecification, making the actual percentages in the RBC generally unfeasible.

In summary, the LR restriction has poor small sample properties. Schemes placing similar restrictions at finite, but relatively long horizons outperform the LR restriction but are still associated with considerable uncertainty. In particular, none of these methods recovers a significant response of hours from data simulated from an RBC, even using 68% percentile bands. Our results show that moving the identification of the technology shock to the business cycle improves the precision of the estimated response of hours, making it possible to derive robust conclusions on its magnitude and sign. However, it is necessary to resort to economic models, as it cannot be expected the contribution of the technology shock to be the maximum admissible in the SVAR at this range, and it is not like that in theoretical models either.

4. EMPIRICAL IMPLEMENTATION

After assessing the SV method in simulations, we recover the response of hours to a shock identified with this scheme in U.S. data. To start, we employ the contributions implied by the baseline parameterization of the RBC as targets for the method. After that, we generalize the empirical findings using a large set of targets, based on many credible parameterizations of the RBC model. Before discussing these issues, we will briefly describe the dataset employed.

4.1 Data Description

We construct a bivariate quarterly dataset on hours worked and labor productivity collecting data from the Federal Reserve Bank of St. Louis (FRED). The dataset is similar to Gali (1999) except in the definition of hours. Gali (1999) employs the log of total employee hours in non-agricultural establishments. Given that hours in RBC models are usually defined in per capita terms, we perform the analysis using hours per capita. More specifically, the "non-agricultural business sector productivity" is the natural logarithm of the OPHNFB series in the FRED dataset. The hours series was derived by subtracting the civilian non-institutional population over the age of 16 years (CNP16OV) from the non-farming business sector hours of all persons (HOANBS), both in natural logarithms. Except for population, all series were seasonally adjusted and ran from 1948:1 to 2009:4, covering a slightly more extended period than other datasets in the literature.[11]

4.2 The Response of Hours to Technology Shock

We employ a four-order VAR for estimation, which is the standard lag-length in the literature. After reduced-form VAR estimation, we identify the technology shock with the SV method by targeting the shares implied by the baseline parameterization of the RBC (7.48% and 80.36%, for hours and productivity growth, respectively). Finally, we compute the response of hours and the contributions of the identified shock at the business cycle in the SVAR.

Figure 2 shows how the proposed SV scheme works. The gray curve is formed by the scatter plot of all contributions to the fluctuations of the variables at business cycle frequencies that can be reached by the shock identified in the SVAR. This admissible set is found by computing the contributions in 10,000 candidate SVAR identifications generated by Givens rotation matrices randomly drawing ϕ from $[-\pi/2, \pi/2]$ using a uniform distribution, as explained in Section 2. Consequently, any candidate identification of the shock in the SVAR has a business cycle contribution to hours and productivity growth represented by a point in the curve.

[Figure 2 around here]

The SV method identifies the shock having contributions in the admissible set with the smaller distance (in modulus) from the point targeted. Point “A” signals the technology shock contributions in the baseline parameterization of the RBC. Since this point is not far from the set of admissible percentages, the resulting contributions in the SVAR do not differ much from the targeted values but do not match the RBC contributions. Specifically, the SV identifies a shock with a 74.34% and 10.76% contribution to productivity growth and hours worked fluctuations, respectively, signaled in the figure with a “B.”

Finally, Figure 2 also shows why the method targets the contribution to both productivity growth and hours worked. Notice that any possible contribution to productivity growth in the SVAR is associated with two admissible percentages for hours; only the maximum (or minimum) admissible is associated with a single percentage. Therefore, if the method only targets the contribution to productivity growth fluctuations, the set of candidate identifications is not singleton unless the value targeted is the maximum the SVAR can admit (or exceeds this percentage). The two solutions match the contribution aimed, but differ in the share in hours.

By targeting the contributions to the fluctuations of the two variables (or in general, the contributions to all variables in the system), one forces the identification method to deliver a single solution for the vast majority of targets. Targeting only the contribution to productivity growth may still be an interesting option, especially if one selects the candidate identifications resulting from targeting a credible interval for this contribution instead of a single percentage. In any case, the technology shock contribution to productivity growth in the baseline parameterization of the RBC is higher than the maximum possible in the SVAR in this dataset, and one obtains a single solution targeting productivity growth only. We have highlighted this alternative solution with a “C” in figure.[12]

[Figure 3 around here]

The response of hours recovered by the SV method in the SVAR is depicted in the first panel of Figure 3, together with the 68^{th} and 90^{th} error bands computed by the nonparametric bootstrap method. As the figure shows, the SV scheme recovers a positive and statistically significant response in the short-run. The magnitude is similar to the response of hours in the RBC but differs substantially on impact. While the contemporaneous response is positive and strong in the RBC model (see Figure 1), it is virtually zero in the SVAR, statistically non-significant. The figure also reports the response of hours using the whole set of schemes assessed in the simulation study (i.e., the LR, MR, MS, FD, and FD-BC) to compare the empirical findings with those obtained with other identification methods. As can be observed in the figure, the responses are, in general, not much different from those produced by the SV. Only the frequency-domain method of DiCecio and Owyang (2010) generates a different response when maximization is carried out at the business cycle (FD-BC), which is negative on impact and non-significant at all other horizons. The responses returned by other methods are similar to the SV response but presenting wider error bands, consistent with the results of the Monte-Carlo simulation. Specifically, the responses recovered with the MR, MS, and FD schemes are only significant using 68^{th} bootstrapped error bands.

[Table 3 around here]

Finally, Table 3 reports the contributions at the business cycle of the shocks identified with the different methods. As this table shows, all schemes identify a shock with a similar contribution to productivity growth fluctuations, not far from the percentages in the baseline RBC. The exception is the FD-BC, where this percentage is smaller than in RBC. The estimated shares in hours are more heterogeneous, although none of the methods yield a large value, consistent with the consensus view that technology is not an essential driver of the fluctuations of hours worked at the business cycle [Christiano et al. (2007); Francis et al. (2014)]. The results in Table 3 reflect once more the uncertainty associated with identification methods other than the SV; the wide confidence intervals signal that the properties of the technology shock at business cycle frequencies are estimated with significant variance.

4.3 Generalizing the Empirical Findings

In our baseline parametrization of the RBC, the technology shock has a substantial contribution to productivity growth fluctuations at the business cycle but a small share in hours. One may question, however, if these contributions are robust implications of the model, and if not, how sensible our results are to the particular parametrization employed.

To generalize the empirical findings, and untighten them from the percentages in the baseline parametrization, we compute the technology shock contributions in many credible parametrizations of the RBC and use all these percentages to construct a robust interval for the response of hours in the SVAR recovered with the SV method.

4.3.1 The contribution of the technology shock in a large set of RBC parametrizations

Like in Dedola and Neri (2007), Pappa (2009) and Canova and Pausitan (2011), our first step consists of selecting intervals for all RBC parameters, chosen to be large enough to include all theoretically reasonable values.

Credible intervals for model parameters are provided in the last part of Table 1 and are constructed to include the values for the parameters given in the literature. After selecting these intervals, we generate P=10,000 different parameterizations of the RBC, drawing from independent uniforms for all parameters inside their corresponding intervals. For each resulting parameter vector, we compute the technology shock contributions at the business cycle and the response of hours in the RBC.

[Figure 4 around here]

A scatter plot of all (pairs of) RBC contributions implied by all credible parameterizations is depicted in gray in Figure 4. The technology shock contributions implied by all these parameterizations are concentrated in the upper left corner of the figure, suggesting the theoretical model robustly delivers a large percentage for productivity growth and a small percentage for hours. However, we find parameterizations implying different shares, but these unusual percentages never come together; that is, no parameterization simultaneously delivers a small contribution to productivity growth and large contribution to hours worked.

In addition to technology shock contributions, we assess the response of hours at all parameterizations of the RBC model, which are summarized in Figure 5 (left panel). The total range at each horizon is shadowed in gray, while the mean response is signaled with a solid black line. We also delineate a robust interval for the response of hours in the RBC model, which contains the 16^{th} and 84^{th} percentile. This robust interval for RBC responses is marked with red dashed lines in the figure.

[Figure 5 around here]

As expected, no matter the parameterization, the response of hours in the RBC is positive at all horizons and persistent, decaying to zero slowly in the long-run. Although the total range of responses obtained in all the parametrizations of the RBC is relatively wide, most of them are concentrated in a narrow interval around the mean, very close to the response implied by the baseline parameterization.

4.3.2 Deriving a robust interval for the response of hours in the SVAR

As a second step, we employ the contributions in all RBC parameterizations as targets for the SV, therefore producing 10,000 different identifications of technology shock in the data. In each, we compute the contribution of the identified shock at business cycle frequencies in the SVAR and the corresponding response of hours.

The resulting set of SVAR contributions is also depicted in Figure 4. We have highlighted with blue circles those associated with a positive response of hours. The remaining contributions, associated with a non-positive response at least at one horizon, are represented with red squares.

As Figure 4 shows, the resulting collection of circles and squares form a subset of all admissible shares (depicted in Figure 2). The contributions in the SVAR, taken as a whole, are not far from the percentages implied by all RBC parametrizations, although the share of the technology shock in productivity growth fluctuations in the theoretical model is often too large to be admissible in the SVAR. Given that the percentages in the RBC pool in the upper left corner of the figure, most of the resulting contributions in the SVAR are associated with a positive response at all horizons. Still, in around 20% of the identifications, the estimated response of hours is negative at some horizon.

The left panel of Figure 5 collects the set of responses recovered in the SVAR. The shadowed area represents the full range of responses, and the solid line is the mean. As with the RBC, we derive a robust interval for the response of hours recovered by the SV method by selecting the responses between the 16th and 84th percentiles. This robust interval for the empirical response is also depicted with a dashed red line in the figure.

As can be observed in Figure 5, the total range of responses in the RBC and the SVAR are quite different. While responses in the RBC are always positive, no matter the parameterization, the SVAR sometimes returns a negative response in the data, even though the contributions targeted are implied by RBC parametrizations. Specifically, the contemporaneous response is negative (albeit not necessarily significant) when the percentage in productivity growth fluctuations falls below 70%. If this percentage is below 40%, the response of hours never turns positive. Negative responses are associated with RBC targets situated in the bottom left corner of Figure 5, characterized by low contributions to both productivity growth and hours.[13]

However, when we focus on where most of the responses are placed, the results are different. The robust sets in the RBC and the SVAR are similar, with an average response across the sets that are virtually identical in the short-run. Nonetheless, the responses in these sets still differ on impact. While in the RBC are always positive and strong, the responses in the robust set for the SVAR are concentrated on impact in a narrow interval around zero.

Overall, the results of this exercise provide substantial support to empirical findings obtained with the baseline parameterization; the responses inside the robust set for the SVAR are positive in the short-run, departing from a near-zero contemporaneous value.

4.3.3 Is the RBC compatible with the data?

Results in Figures 4 and 5 can also be employed to study the consistency between the RBC and the data. Recall that the proposed identification method brings the SVAR as close as possible to the targeted shares. Thus, if the model and the data are compatible, the resulting contributions of the shock identified by the SV method should not be too far from the percentages in the RBC. Likewise, the responses of hours in the SVAR and the RBC are expected to have a similar shape or, at least, the same sign. If not, the theoretical model and the data are just not compatible.

It is important to stress that compatibility does not necessarily imply “validity,” as the procedure gives the RBC model from which the targets are taken the best chance. Finding no evidence of compatibility, however, does undermine the validity of the model.

Overall, we find substantial compatibility between the RBC model and the data for most of the parameterizations of the model. This result, however, must be qualified. First, many parameterizations imply a contribution to productivity growth fluctuations too high to be admissible. Second, although the responses inside the robust interval for the SVAR are similar to the responses in the RBC in the short-run, their contemporaneous impact widely differs. Finally, there is still a non-negligible set of credible parameterizations of the RBC that are not compatible with a positive response in the SVAR.

5. Extensions

We extend our empirical section in several directions. First, we assess the response of hours in a different dataset, also popular in the literature. After that, we include the investment-output ratio to determine the stability of the empirical findings to the inclusion of more variables. The third extension considers an alternative version of the SV method, which employs the forecast error variance. Finally, we use Sobol sequences to study whether uniformly and independently drawing P=10,000 different parametrizations of the RBC is enough to cover the entire parameter space. We briefly discuss the results of these extensions in the main text, but the corresponding figures and tables are reported in the Supplement to save space.

As the first extension, we consider an alternative dataset similar to Christiano et al. (2003). It contains data from all sectors (including the farming) and has also been collected from the FRED database. The total business productivity is measured as the log-output per hour of all persons (OPHPBS), and hours worked as the log-ratio of the business hours of all persons (HOABS) to the civilian non-institutional population over the age of 16 years (CNP16OV). The "Civilian non-institutional population over the age of 16" is converted to quarterly by taking simple averages of monthly observations.

Overall, the SV method delivers similar results in the two datasets. The response of hours recovered by the SV method in the second dataset is also positive and significant in the short-run, departing from a small, non-significant, negative contemporaneous impact.

The second extension investigates whether the results change in response to the specification of the empirical model. To do this, we expand the dataset with the investment-output ratio. This variable is computed as the natural logarithm of the ratio of real gross private domestic investment (GPDIC96 in FRED) to the corresponding series of output. According to Chari et al. (2008), the inclusion of a capital-like variable may change the results of the empirical model, and the investment-output ratio is the only capital-like variable producing an invertible moving average representation of the RBC model in the state space form. The technology shock contribution to the business cycle variance in the three-shocks extension of the Christiano et al. (2007) RBC model is 73.34%, 5.34%, and 27.90% for productivity growth, hours, and the investment-output ratio, respectively. We employ these percentages as a target for the SV, finding a response of hours in the SVAR that is once more non-significant on impact, but turning positive and significant in the short-run, similar to the response obtained with a bivariate specification.

We also study the response of hours obtained with an alternative definition of the identification method based on targeting the technology shock contribution to the forecast error variance. As noted before, our scheme employs the spectral decomposition of the variance since it permits a more precise characterization of the business cycle. The results are not expected to differ much if instead, one uses the forecast error variance at middle-run horizons. The contributions of the technology shock to the forecast error variance at six years in RBC is 82% for productivity growth and 5% for hours, respectively. We employ this alternative version in actual data finding a similar, albeit slightly smaller response. However, the associated uncertainty is larger, with wider error everywhere. In particular, the response of hours to the shock recovered using the forecast error variance alternative is not significant at any horizon, not even using the 68th percentile bootstrapped error bands.

As the last extension, we ensure covering the parameter space more evenly using Sobol sequences. In Section 4.3, we draw 10,000 times from independent uniforms for all parameters inside their interval to obtain a broad set of parametrizations of the RBC. Although the number of parametrizations of the model is large compared to other studies, we still could have left empty some parts of the parameter space. A Sobol sequence (Sobol, 1977) is a low-discrepancy, quasi-random sequence, which covers the unit hypercube with lower discrepancy than random sampling, filling the space of possibilities more evenly. The results obtained parametrizing the RBC with Sobol sequences are virtually identical to the results obtained by uniformly drawing, and are provided in the Supplement to conserve space.

6. Conclusion

In this work, we have identified technology shock at the business cycle to improve the small-sample properties of the SVAR. To do so, we have proposed a novel identification scheme that brings the identified shock as close as possible to the contributions the technology shock has in DSGE models at this range of frequencies. The proposed scheme is model-based, midway placed between the parameterized DSGE model and a typical SVAR scheme. The loss of generality, however, is associated with a substantial improvement of the SVAR in small samples and offers an interesting view of the compatibility between the theoretical model and the data that cannot be obtained using other schemes. We have shown how to generalize the empirical findings by establishing a robust interval for the estimated response of hours, based on a broad set of parameterizations of the theoretical model.

Throughout this paper, we have employed the hours-productivity debate as a connecting thread. However, the SV method can also be applied to the study of other questions addressed with SVARs, such as in the analysis of monetary or fiscal policy issues. Besides, an attractive version of the SV method may select candidate identifications satisfying intervals for shock contributions instead of targeting specific values, thus providing set identification. Finally, a remaining open question is whether one can use the contributions at the business cycle (or other frequency range) of a shock identified in the SVAR to parameterize a DSGE model, similar as it is done with the impulse-responses (see, e.g., Hall et al. 2012). In this case, the shock of interest can be identified in the SVAR as the econometrician ponders the safest. We consider these subjects exciting avenues for future research.

Notes

1. Gali (1999) finds a negative response of hours, contrary to what RBC theory predicts. However, the sign of the response depends crucially on the treatment of hours in the SVAR. If the series is differenced, as in Gali (1999), the response is negative. However, if hours enter in levels, as in Christiano et al. (2003, 2004), the response is positive using the same LR restriction. Besides, see Fout and Francis (2014) for a modification of RBC models to accommodate the contractionary effect on hours, and Guiuli and Tancioni (2017), for SVAR evidence of contractionary effects on other variables, such as investment.

2. Differencing hours is justified arguing that the series contains external fluctuations at low-frequencies [see, e.g., Francis and Ramey (2009)]. However, Christiano et al. (2003, 2004) provide statistical arguments in favor of the level specification. Moreover, Gospodinov et al. (2011) and Lovcha and Perez-Laborda (in press) show that differencing (or filtering)

biases the response of hours, invalidating posterior inference. See also Pesavento and Rosi (2005) or Lovcha and Perez-Laborda (2015) for alternative treatments of hours worked in the SVAR.

3. Restrictions to the signs of the responses of hours and productivity growth to own shocks are also required. As common in the literature, we assume both positive.

4. Christiano et al. (2007) show that a zero restriction on the contemporaneous response hours to the technology shock works well if the condition is satisfied. However, this short-run restriction is not met in standard macroeconomic models and is rarely employed in practice. Consequently, we do not consider this scheme in the main text, but its description and some empirical results are available in the Supplement.

5. The SV method cannot distinguish technology from another shock with the same contributions to all variables. Keeping the notation for the contribution of the kth shock to the fluctuations of the nth variable and the assumed order in the system, an additional condition to be met is that $\forall k \neq 1$, $\exists n$ s.t. $T_{nk}^{R} \neq T_{n1}^{R}$. The concern disappears if the contribution of the technology shock to at least one variable exceeds 50%. Large contributions to productivity fluctuations at the business cycle (of around 75%) are consistent both with standard RBC parametrizations and other identification schemes, as shown in Section 4.

6. Four lags are also the order typically selected by the Bayesian Information Criteria in actual data.

7. DiCecio and Owyang (2010) consider a "low" band (between eight to 20 years period) and a "very low" band (periods longer than 20 years, excluding the zero frequency). However, with 60 years of quarterly data, the last band exists only on a couple of Fourier frequencies. Consequently, we merge the two bands into a single "low-frequency" band.

8. In particular, the MR imposes 100% contribution to the forecast revision of productivity at four years, and the true value in the RBC model is as close as 99.2%. The

possibility of misidentification bias is, in principle, less for the MS and FD, as these methods determine the maximum admissible in the SVAR. Besides, the contribution of the technology shock to the productivity forecast error variance at ten years in the RBC is also huge (97.9%), and the same applies to the contribution to productivity fluctuations at low frequencies (93.4%). As a result, the maximization procedures in the MS and FD methods identify a shock with a share in productivity fluctuations at the selected horizon that is virtually identical to those the technology shock has in the RBC.

9. Recall that we define the business cycle as being formed by fluctuations between two and eight years period, as standard in the literature. Consequently, the range is largely dominated by relatively high Fourier frequencies with a period smaller than four years. Note that in T=240 quarterly observations, there are only seven Fourier frequencies between periods four and eight years, but 15 between periods two to four years. As a result, the MR, MS, and FD methods set the restriction at longer horizons than the business cycle.

10. The contributions given in the text do not exactly coincide with those in Table 2, as the table shows the contribution to productivity growth, not to productivity log-level.

11. We have not included the last years of the data to eliminate a big break associated with the post-2009 period. In this way, our results are directly comparable to Lovcha and Perez-Laborda (2015), who used the same dataset to assess the response of hours in a long-memory SVAR.

12. Although this solution achieves the maximum admissible, it does not coincide with the solution of the FD-BC, as this method is defined in terms of the productivity log-level.

13. Shock contributions with small percentages in both hours and productivity growth fluctuations at the business cycle, although rare for the RBC, are normal in standard parameterizations of new Keynesian models, such as in Ireland (2004).

REFERENCES

Basu, Susanto, John G. Fernald, and Matthew Shapiro (2001) Productivity growth in the 1990s: technology, utilization, or adjustment? *Carnegie-Rochester Conference Series on Public Policy* 55, 117-65.

Blanchard, Olivier and Roberto Perotti (2002) An empirical characterization of the dynamic effects of changes in government spending and taxes on output. *The Quarterly Journal of Economics 117*, 1329-1368.

Blanchard, Oliver and Daniel Quah (1989) The dynamic effects of aggregate demand and supply disturbances. *American Economic Review* 79, 655-673.

Canova, Favio and Matthias Paustian (2011) Business cycle measurement with some theory. *Journal of Monetary Economics* 58, 345–361.

Chari V.V., Patrick J. Kehoe, and Ellen R. McGrattan (2008) Are structural VARs with long-run restrictions useful in developing Business Cycle Theory? *Journal of Monetary Economics* 55, 1337-1352.

Christiano, Lawrence J., Martin Eichenbaum, and Robert Vigfusson (2003) What happens after a technology shock? NBER Working Paper No. 9819.

Christiano, Lawrence J., Martin Eichenbaum, and Robert Vigfusson (2004) The Response of hours to a technology shock: evidence based on direct measures of technology. *Journal of the European Economic Association* 2, 381-395.

Christiano Lawrence J., Martin Eichenbaum, and Robert Vigfusson (2007) Assessing structural VARs. In Daron Acemoglu, Kenneth Rogoff, and Michael Woodford (eds.), *NBER Macroeconomics Annual 2006*, Vol. 21, pp. 1-106. National Bureau of Economic Research.

Dedola, Luca and Stefano Neri (2007) What does a technology shock do? A VAR analysis with model-based sign restrictions. *Journal of Monetary Economics* 54, 512-549.

DiCecio, Ricardo and Michael T. Owyang (2010) Identifying technology shocks in the frequency domain. WP 2010-025A, Federal Reserve Bank of St.Louis.

Evans, Charles L. and David A. Marshall (2009) Fundamental economic shocks, and the macroeconomy. *Journal of Money, Credit and Banking* 41, 1515-1555.

Erceg, Christopher J., Luca Guerrieri, and Christopher Gust (2005) Can long-run restrictions identify technology shocks? *Journal of the European Economic Association* 3, 1237-1278.

Faust, Jon and Eric Leeper (1997) When do long-run identifying restrictions give reliable results? *Journal of Business and Economic Statistics* 15, 345-353.

Fout, Hamilton B. and Neville Francis (2014). Imperfect transmission of technology shocks and the business cycle consequences. *Macroeconomic Dynamics* 18, 418-437.

Francis, Neville, Michael T. Owyang, Jennifer E. Roush, and Ricardo DiCecio (2014) A flexible finite-horizon alternative to long-run restrictions with an application to technology shocks. *The Review of Economics and Statistics* 96, 38-647.

Francis, Neville and Valery A. Ramey (2009) Measures of per capita hours and their implications for the technology-hours debate. *Journal of Money, Credit and Banking* 41, 1071-1097.

Gali, Jordi (1999) Technology, employment, and the business cycle: Do technology shocks explain aggregate fluctuations? *American Economic Review* 89, 249-271.

Geweke, John (1977) The Dynamic Factor Analysis of Economic Time Series. In Dennis J. Aigner and Arthur Goldberger (eds.), *Latent Variables in Socio-Economic Models*, pp. 365-383. North-Holland: Amsterdam.

Gospodinov, Nicolay, Alex Maynard, and Elena Pesavento (2010) Sensitivity of impulse-responses to small low-frequency co-movements: reconciling the evidence on the effect of technology shocks. *Journal of Business and Economic Statistics* 28, 1-12.

Giuli, Francesco and Massimiliano Tancioni (2017) Contractionary technology shocks. Macroeconomic Dynamics 21, 1752-1789.

Hall, Alastair R., Atsushi Inoue, James M. Nason, and Barbara Rossi (2012) Information criteria for impulse-response function matching estimation of DSGE models. *Journal of Econometrics* 170, 499-518.

Herbst, Edward P. and Frank Schorfheide (2016) *Bayesian Estimation of DSGE Models*. Princeton; Oxford: Princeton University Press.

Ireland, Peter N. (2004) Technology Shocks in the New Keynesian Model. *The Review of Economics and Statistics* 86, 923-936.

Lanne, Markus, Mikka Meitz, and Pentti Saikkonen (2017) Identification and estimation of non-Gaussian structural vector autoregressions. *Journal of Econometrics* 196, 288-304.

Kilian, Lutz and Thomas K. Lee (2014) Quantifying the speculative component in the real price of oil: The role of global oil inventories. *Journal of International Money and Finance* 42, 71-87.

Lovcha, Yuliya and Alejandro Perez-Laborda (2015) Hours worked-productivity puzzle: identification in fractional integration settings. *Macroeconomic Dynamics* 19, 1593-1621.

Lovcha, Yuliya and Alejandro Perez-Laborda (in press) Trimmed Witthle estimation of the SVAR vs. filtering low-frequency fluctuations: applications to technology shocks. *Studies in Nonlinear Dynamics and Econometrics*. doi: 10.1515/snde-2018-0030.

Mertens, Karel and Morten O. Ravn (2013) The Dynamic Effects of Personal and Corporate Income Tax Changes in the United States. *American Economic Review* 103, 1212-47.

Pappa, Evi (2009) The effect of fiscal shocks on employment and the real wage. *International Economic Review* 50, 217-253.

Pesavento, Elena and Barbara Rossi (2005) Do technology shocks drive hours up or down? A little evidence from an agnostic procedure. *Macroeconomic Dynamics* 9, 478-488.

Prescott, Edward C. (1986) Theory Ahead of Business Cycle Measurement. *Federal Reserve Bank of Minneapolis Quarterly Review* 10, 9-22.

Ramey, Valery A. (2016) Macroeconomic shocks and their propagation. In John B. Taylor and Harald Uhlig (eds.), *Handbook of Macroeconomics* Vol. 2, pp. 71-172. Elsevier.

Rubio-Ramirez, Juan F., Daniel F. Waggoner, and Tao Zha (2010) Structural vector autoregressions. Theory of identification and algorithms for inference. *The Review of Economic Studies* 77, 665-696.

Sargent, Thomas J. (1987) *Macroeconomic Theory*. Academic Press, 2nd edition.

Sims, Christopher (1989) Models, and their uses. *American Journal of Agricultural Economics* 71, 489-494.

Shapiro, Matthew, and Mark W. Watson (1988) Sources of business cycles fluctuations. In Stanley Fisher (ed.), *NBER Macroeconomics Annual 1988*, Vol. 3, pp. 11-156. National Bureau of Economic Research.

Sobol, Ilya (1977) Distribution of points in a cube and approximate evaluation of integrals. *USSR Computational Mathematics and Mathematical Physics* 16, 236-242.

Thomet, Jaqueline and Philipp Wegmueller (in press) Technology shocks and hours worked: a cross country analysis. *Macroeconomic Dynamics*. doi: 10.1017/S1365100519000658.

Uhlig, Harald (2004) Do technology shocks lead to a fall in total hours worked*? Journal of the European Economic Association* 2: 361–371.

FIGURES AND TABLES

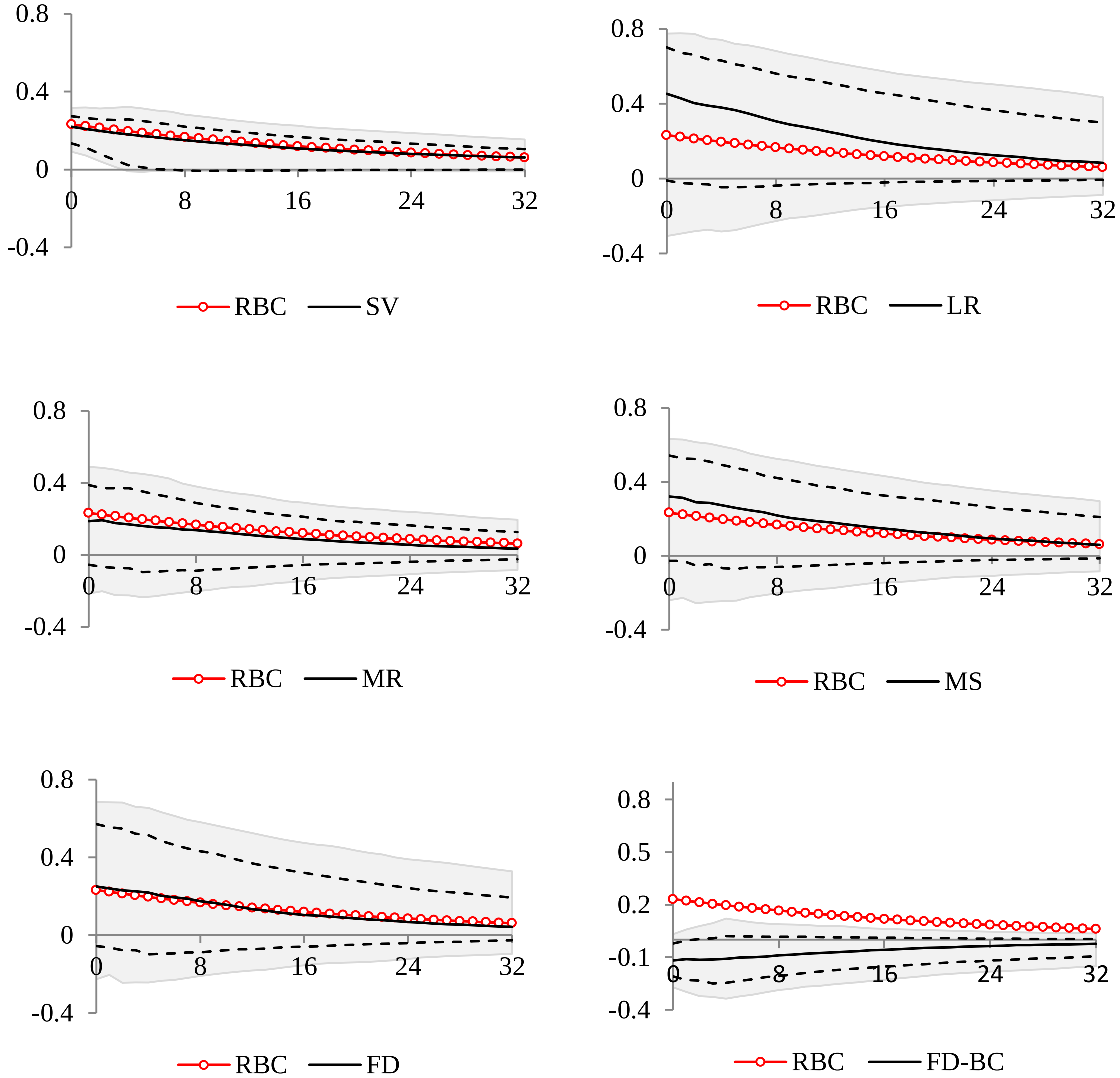

FIGURE 1. Simulation results: response of hours to the identified shock. The solid line is the mean response across simulations. The dashed and shadowed areas correspond to the 68th and 90th percentile bands, respectively. The red line marked with circles is the response in the baseline RBC.

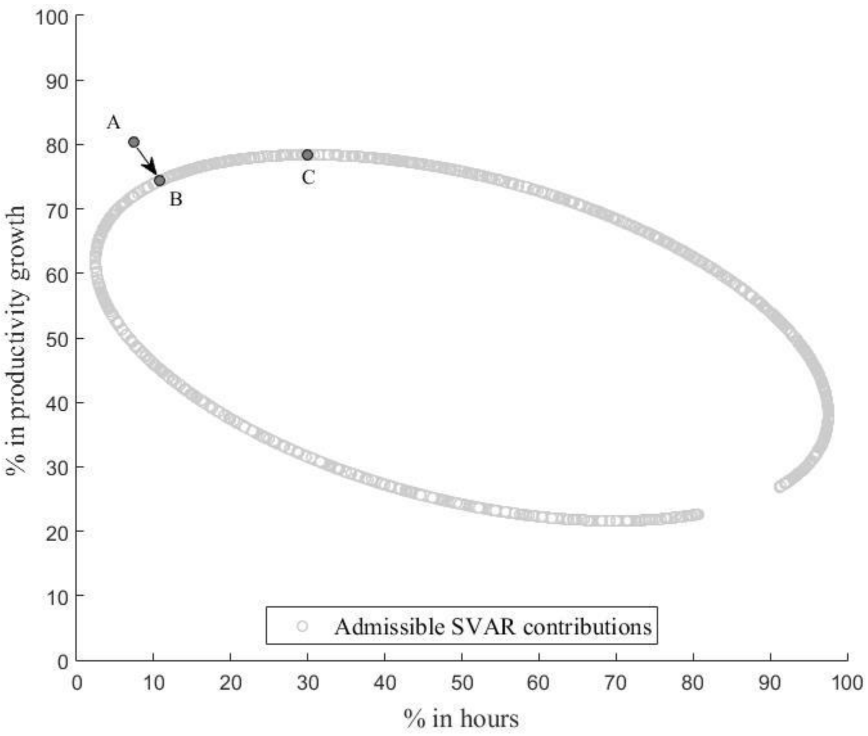

FIGURE 2. Set of shock contributions to the business cycle fluctuations of the variables that are admissible in the SVAR. The "A" point signals the contributions implied by the baseline RBC. The "B" point signals the contributions recovered by the SV method using "A" as a target. The "C" point signals the contributions that one would obtain targeting the RBC shock share in productivity growth only.

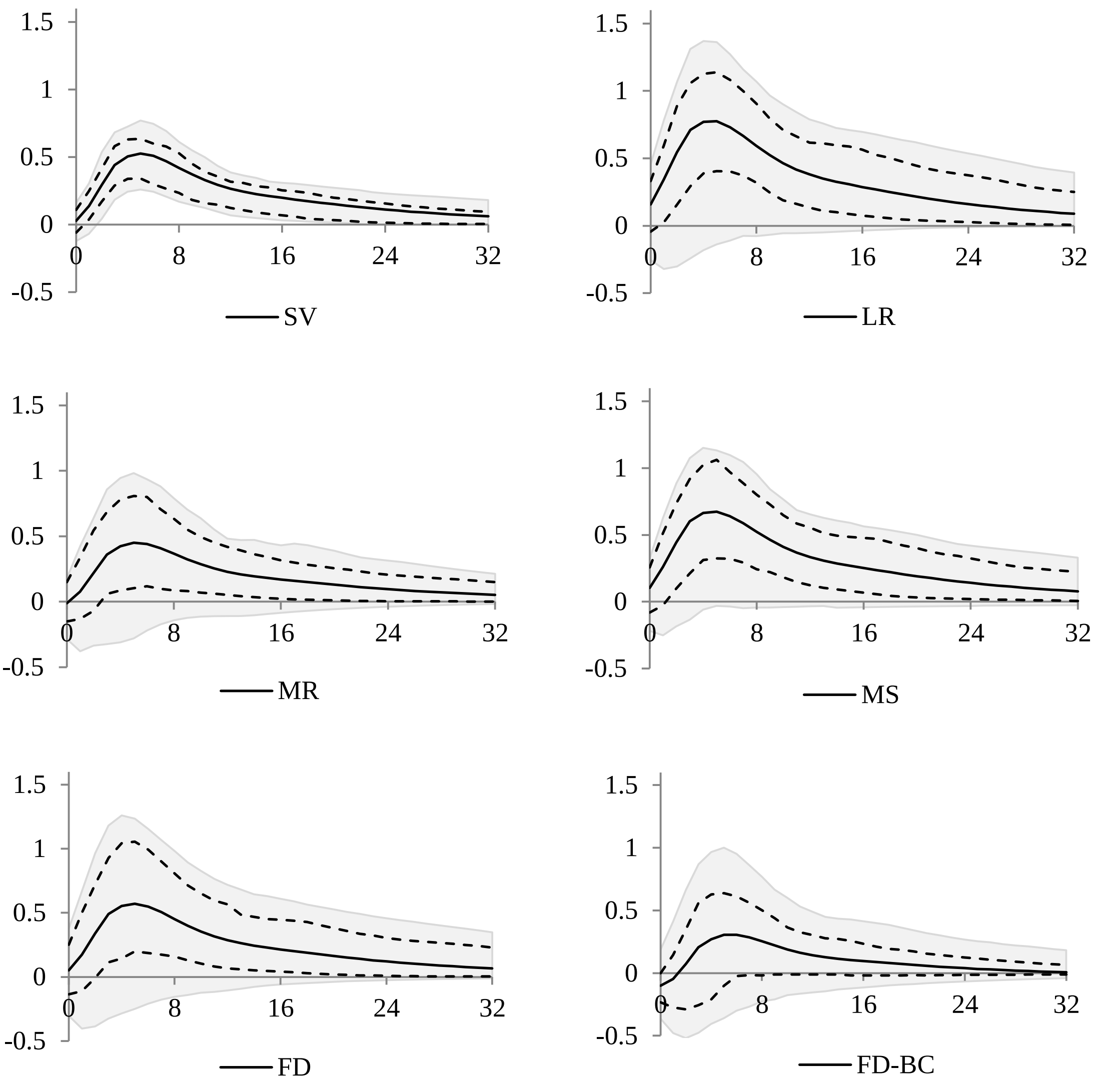

FIGURE 3. The response of hours worked to the identified technology shock in the SVAR. The dashed lines and shadowed area correspond to the 68th and 90th bootstrapped error bands, respectively.

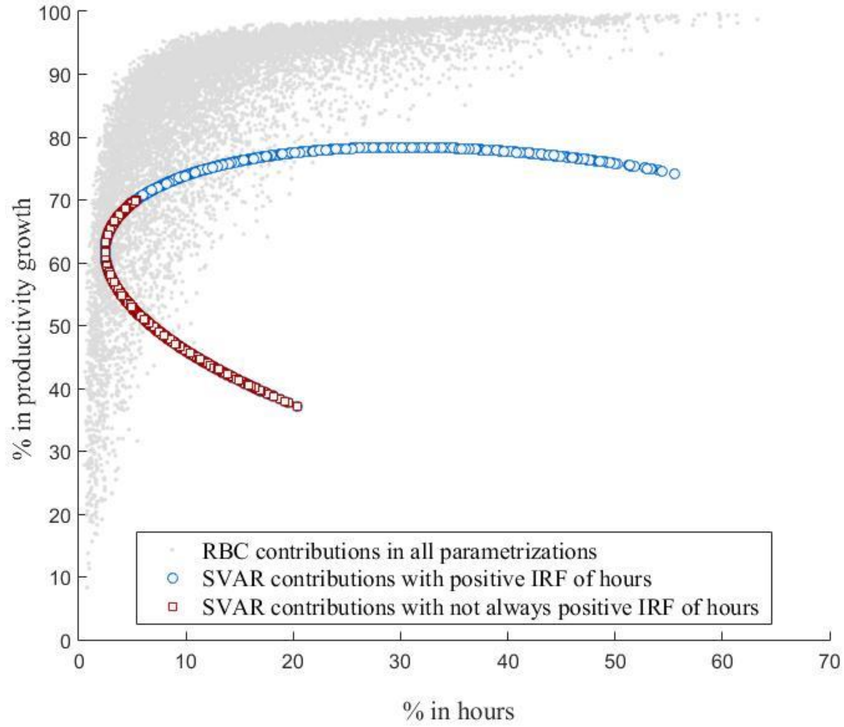

FIGURE 4. Business cycle contributions of the technology shock across RBC parametrizations and corresponding percentages in the SVAR.

RBC SVAR

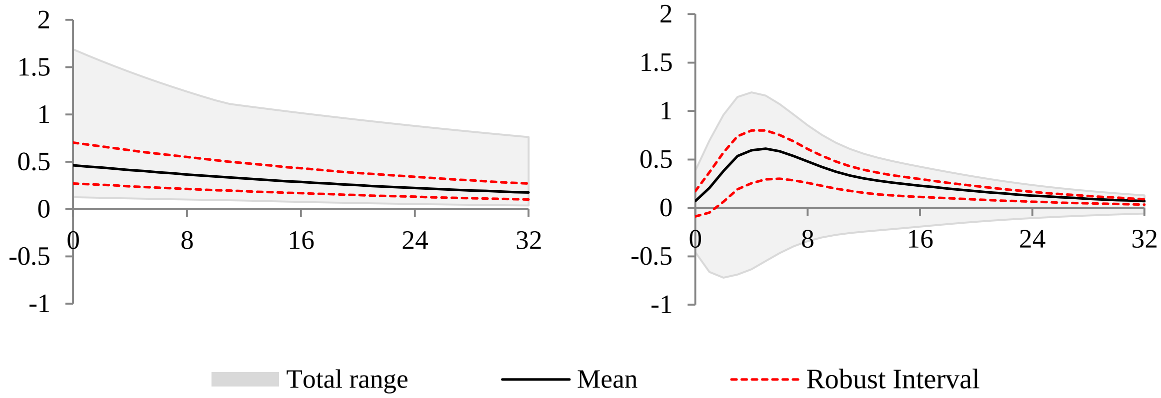

FIGURE 5. The left panel plots the response of hours across RBC parameterizations. The corresponding responses recovered by the SV method are depicted on the right. The shadowed grey area highlights the total range at each horizon. The solid line is the mean response. The dashed red lines highlight the robust interval formed by the responses within the 16th and 84th percentile (68th percentile band).

TABLE 1. The RBC model

The RBC:	
$E_0 \sum_{t=0}^{\infty} \left(\beta(1+\gamma)\right)^t \left[\ln c_t + \psi \frac{(1-l_t)^{1-\sigma} - 1}{1-\sigma}\right]$	$y_t = k_t^{\alpha} (Z_t l_t)^{1-\alpha}$
$c_t + (1-\tau_x) i_t \leq (1-\tau_{l,t}) w_t l_t + r_t k_t + T_t$	$\ln Z_t = \ln Z_{t-1} + \mu_z + \sigma_z \varepsilon_t^z, \quad \varepsilon_t^l \sim iidN(0,1)$
$i_t = (1+\gamma) k_{t+1} - (1-\delta) k_t$	$\tau_{l,t} - \tau_l = \rho_l (\tau_{l,t-1} - \tau_l) + \sigma_l \varepsilon_t^l, \quad \varepsilon_t^z \sim iidN(0,1)$
$c_t + i_t \leq y_t$	$E(\varepsilon_t^l \varepsilon_t^z) = 0$
Baseline parameterization	
$\beta = 0.98$	$\tau_x = 0.3$
$\gamma = 1.01^{1/4} - 1$	$\tau_l = 0.242$
$\psi = 2.5$	$\mu_z = 1 - 016^{1/4}$
$\sigma = 1$	$\rho_l = 0.986$
$\delta = 1 - (1-0.06)^{1/4}$	$\sigma_z = 0.00953$
$\alpha = 0.33$	$\sigma_l = 0.0056$
Credible intervals for RBC parameters	
$\beta \in [0.950;\ 0.995]$	$\tau_x \in [0.2;\ 0.4]$
$\gamma \in [1.01^{1/4} - 1;\ 1.02^{1/4} - 1]$	$\tau_l \in [0.2;\ 0.4]$
$\psi \in [1.6;\ 2.5]$	$\mu_z \in [1.005^{1/4} - 1;\ 1.03^{1/4} - 1]$
$\sigma \in [0;\ 10]$	$\rho_l \in [0.952;\ 0.9994]$
$\delta \in [1-(1-0.06)^{1/4};\ 1-(1-0.01)^{1/4}]$	$\sigma_z \in [0.00953;\ 0.0388]$
$\alpha \in [0.3;\ 0.4]$	$\sigma_l \in [0.0050;\ 0.016]$

The credible intervals include the values in all the parameterizations considered in Christiano et al. (2007), Chari et al. (2008), Dedola and Neri (2007), and Canova and Paustian (2011). For the parameters governing the process of shocks, we have constructed the intervals based on Christiano et al. (2007), Chari et al. (2008), Prescott (1986), Erceg et al. (2005), and Canova and Pausitan (2011). The values in other RBCs in the literature generally fall in the reported intervals.

TABLE 2. Simulation results: contribution of the identified shock at the business cycle

Share in:	$\ln l_t$	$\Delta \ln y_t$
DGP: Baseline RBC	7.48	80.36
SV	7.78 [3.89; 12.53]	80.65 [78.97; 84.01]
LR	30.18 [3.47; 67.73]	56.86 [18.40; 93.85]
MR	7.72 [1.69; 21.45]	85.44 [65.00; 97.69]
MS	16.96 [2.51; 40.69]	73.42 [40.20; 95.75]
FD	11.80 [1.96; 44.95]	80.39 [37.48; 97.53]
FD-BC	3.53 [0.77; 9.64]	98.57 [95.82; 99.58]

The numbers in the table are the mean shares of identified technology shock in the fluctuations of (log) hours and productivity growth at business cycle frequencies across simulations. The numbers inside parentheses correspond to the 68% percentile interval. The DGP is the baseline two-shock RBC of Christiano et al. (2007).

TABLE 3. Empirical results: contribution of the identified shock at the business cycle

%	$\ln l_t$	$\Delta \ln y_t$
SV	10.76 [7.18; 14.73]	74.34 [65.85; 80.31]
LR	23.42 [8.46; 53.91]	78.02 [62.02; 82.48]
MR	8.08 [3.88; 24.91]	72.60 [57.60; 8285]
MS	17.62 [6.20; 39.92]	76.96 [62.13; 83.30]
FD	12.63 [3.95; 38.87]	75.25 [59.20; 82.87]
FD-BC	5.06 [3.04; 23.03]	69.61 [57.78; 82.47]

The numbers in the table are the shock shares of the identified shock in the fluctuations of (log) hours and productivity growth at business cycle frequencies. The numbers inside parentheses correspond to the 68th percentile interval computed by the nonparametric bootstrap method. The SV targets the percentage business cycle contribution of technology shock in the baseline parameterization of the RBC at 7.48% and 80.36% for hours and productivity growth, respectively.