THE MOORE'S CLOSURE FOR ANALYZING RELATIONSHIPS BETWEEN AGENTS IN INDUSTRIAL CLUSTERS

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Industrial clusters try to exploit the effect of external economies and joint actions that come from the collaboration between their agents. But in order that these effects arise it is needed close cooperation partnerships between the agents in the industrial cluster, which could improve competitiveness. It’s obvious, therefore, that analyzing which are the relationships between the agents in the industrial cluster is critical to make strategic decisions that promote and improve the competitiveness of the industrial cluster. This paper proposes a methodology based on obtaining a fuzzy relation from which, applying Moore’s closure in an uncertain situation, we can identify subrelations that group industrial cluster agents depending on their degree of affinity based on the intensity of their relationships.

Keywords: *affinity, moore’s closure, fuzzy relations, industrial cluster*

JEL Classification: *C69, L14*

1. INTRODUCTION: INDUSTRIAL CLUSTER CONCEPT

The emergence of a flexible specialization production model has brought industrial dispersal or diffuse industrialization strategies, based on decentralized production models characterized by the concentration of companies in an industry in a environment geographically delimited, to acquire an increasing role, increasingly more intensely [3]. The different currents of thought that have analyzed this phenomenon have called it using different names (industrial districts, innovative environments, local production systems, etc.), but, nowadays, [20] proposal of industrial cluster has become the most used to refer to groups of companies in the same sector located in the same geographic area to share resources and capabilities and increase their competitiveness, both individually and globally.

Industrial clusters allow companies to improve their competitiveness because they take advantage of agglomeration economies, obtaining benefit from their proximity, from the existence of certain infrastructure and equipment in the territory, from diversified customer markets and labour markets, from a better

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access to information and knowledge, and from a social, cultural and institutional environment focused on the development of the main industrial cluster activity. Within the industrial cluster appear productive relations of cooperation of a certain intensity and consistency, based on the complementarity of the different production processes carried out by various companies in the same sector. When we analyze industrial clusters in a dynamic perspective we can see how their performance is a result of the integration of multiple different actions where many actors are involved, both individually and collectively [1]. The fact that this integration requires physical proximity relations, involves the configuration of unique spatial units in production, social, cultural, technological, political and institutional terms [15]. In that sense, [20] introduces the concept of industrial cluster, as the natural union of the companies in a particular sector, and with other related industries in a given territory. These companies develop connections with a large number of support services to generate synergies, externalities, cooperation and dissemination of technology; characteristics that give the industry cluster competitive advantages.

An industrial cluster is a group of companies and institutions geographically close, and related to a particular field, linked by common and complementary features. In other words, the specialization of human capital, the flow of information, the innovation processes and the diffusion of technology, and the relations between suppliers and customers, provide the ideal framework for the emergence of external economies to the firm but internal to the territories. Geographic proximity facilitates communication, technological externalities, leads to efficient delivery of intermediate inputs at lower costs, and allows a greater market share of inputs and outputs, as well as a reserve of qualified local labour. These externalities produce effects on the territories, and affect the efficiency and the competitiveness of the companies in the industrial cluster.

In addition to that, the development of joint actions in a deliberated way by all companies in the industrial cluster allow to take greater advantage of the benefits of external economies offered by the territory and to generate, therefore, a greater collective efficiency [22]. The collective efficiency view emphasizes the strengthening of relations between actors in the industrial cluster (competitors, buyers, suppliers, institutions, etc.) to achieve more efficiency and to increase innovation. That is, he believes that companies in the industrial cluster set a structural situation where, in a relatively small geographic area with clearly defined limits, live a multitude of private and public economic agents involved in a high density network of contracts and formal and informal agreements for the coordination of production complementarities.

According to [21], the existence of the industrial cluster facilitates the implementation of cooperation agreements that permits to exploit complementarities, economies of scale and scope as well as increase flexibility and the speed of reaction of firms to changes in the environment. Therefore, joint actions become a critical element for the correct work of industrial clusters, as they are closely related to the notion of inter-company cooperation as a result of trust and social capital ([11], [18], [19]). In short, the industrial cluster and inter-organizational networks that are established in a territory with several players from the same production chain contribute to enhance the competitiveness of the companies that make up and, therefore, attach great importance to the network of relationships between them in order to improve their individual and collective performance [21].

Thus, the essence of any industrial cluster is based on a close network of agreements, formal and informal, between all agents which are part of it, and that are maintained continuously over time. That is, we can define an industrial cluster as a compact network of relationships developed between actors within a specific geographical area. Consequently, the identification of key relationships between industrial cluster agents, who make a difference in terms of competitiveness, can become a basic part for the development of public and private policies, focused at enhancing competitiveness of companies in the industrial cluster. The argument is that both the quantity and quality of the network of structural relationships between the actors of a territory determine the competitiveness of companies located on it. If the quality of relational contracts between the agents in the industrial cluster is based on the generation of added value for the whole industrial cluster, the analysis of the structural relations network that exists is the basis for formulating management actions which permit to improve the performance of companies in the industrial cluster.

Consequently, it would be highly interesting to know the relationships between the various agents in the industrial cluster, and the degree of intensity. The problem lies in the difficulty to capture and measure the intensity of relations between agents and, therefore, to analyze how it affects the competitiveness of companies in the industrial cluster. In order to cover this analysis, and given the lack of tools for it, it has recently been developed a methodology based on the identification of strong subrelations within an industrial cluster, which shows the structural network of relationships between their agents [16], [17], [2].

This is an analytical tool used to study the relationships within the industrial cluster among their main agents, for a number of critical issues (technology, innovation, training, etc.) that determine his more or less synergistic performance, and that facilitates the development of a strategy that can improve the competitive conditions of the industrial cluster [16]. This tool, called Matrix Structural Relations, allows to establish relationships between agents in a particular industry cluster, analyzing the relationships and links between its elements, so that you can see the set of relationships between the agents in an industrial cluster, and consequently determine the type and the quality of these relationships. However, the methodology used by this matrix consists of the subjective allocation, by researchers, of a fixed previously values depending on how we estimate the relationship between agents (based on information obtained through questionnaires and interviews).

This procedure tries to approximate to the density of the network of relationships within the industrial cluster. For this reason we consider that, because the evaluation of relations between agents is based on highly subjective perceptions of the actors involved, their analysis would be more relevant using the tools they have developed the Theory of Fuzzy sub-Sets.

Therefore, this paper presents a methodology for analyzing the relationships between industrial cluster agents based on obtaining a fuzzy relation from which, applying Moore's closure in a uncertainly situation, to identify subrelations that join industrial cluster agents depending on their degree of affinity based on the intensity of their relationships. As [2] pointed, any industrial cluster can have internal cores within the relations between agents are more intense, that is, where the links are stronger related to other subsets of relations. The aim is to identify these strong cores or maximum subrelations between industrial cluster agents.

Before describing, in the third section, the proposed methodology through an example, the second section presents the axiomatic that allows his application. The work concludes with the presentation of the conclusions.

2. OBTAINING SUBRELATIONS: MOORE'S CLOSURE

Topology studies the properties of topological spaces, being interested in the comparison of objects and their classification. In general, topology refers to a family of subsets of a given set, which meet certain rules. This supposes that for a structure induced by a binary relationship, these rules are equivalent to the transitivity of the ratio. Given the case, as it occurs with the relations between the agents of an industrial cluster, that relations were not transitive, then we must use poorer mathematical structures, but more adaptable to economic reality, as they are pretopologies.

Specifically, given a set E, the obtained the set of parts or power set P(E). Given a functional application Γ from P(E) to P(E), we will say that Γ is a pretopology of E, if and only if [5], [6].

1) $\emptyset \in P(E)$ 2) $E \in P(E)$
3) $\Gamma \emptyset = \emptyset$
4) $\forall A_j \in P(E) : A_j \subset \Gamma A_j$; also here, the fourth axiom forces to:
5) $\Gamma E = E$

In its application to a management phenomenon, it must be interpreted the application functional Γ in the sense that you can include the notion of relationship. Then, it is said that the pretopology requires, according to the fourth axiom, that a grouping of elements of the set E is related to another which comprises at least the same elements [9].

As in ordinary pretopology, the functional application Γ takes the name "adherent application" or simply "adherence". Also, now we can associate Γ to the "interior application" or simply "interior", δ, that we obtain in the following way [7]:

$$\forall A_j \in P(E) : \delta A_j = \overline{\Gamma \overline{A_j}}$$

where $\overline{A_j}$ is the complement of A_j .

It is straightforward, then defining the "closed" and the "opened" [10]. In fact, an element of the power set A_j is a closed when:

$$A_j = \Gamma A_j$$

And element of the power set is an opened if:

$$A_j = \delta A_j$$

Since:

$$(A_j = \Gamma A_j) \quad => \quad (\overline{A}_j = \Gamma \overline{A}_j)$$

$$(A_j = \delta A_j) \quad => \quad (\overline{A}_j = \delta \overline{A}_j)$$

it necessary follows:

$$A_j \in P(E) \quad => \quad \overline{A}_j \in P(E)$$

Then, we conclude that, if the set of closed and opened contains A_j, it also must contain $\overline{A}_j$.

Given a pretopology, it will be said to be isotonous if:

$$\forall A_j , A_k \in P(E) : (A_j \subset A_k) => (\Gamma A_j \subset \Gamma A_k)$$

or in another way:

$$\forall A_J , A_K \in P(E) : (A_J \subset A_K) => (\Delta A_J \subset \Delta A_K)$$

This implies, in our case, that, when a grouping is formed by some people and another group with the same people plus others, the grouping related to the second will be made up of the same people of the relate done to first and possibly to others.

One of the concepts more used for the treatment of the groupings is the Moore's closure, base of the theory of the affinities. So it exists a closure of

Moore if the functional application Γ of a set of parts P(E) on P(E), has the properties of extension, idempotency and isotony [13]. Then, since all isotonous pretopology fulfils the conditions of extension and isotony, if it also had the one of idempotency, we would be in presence of a Moore's closure. Axiomatic of Moore's closure is [5], [6]:

1) $\emptyset \in P(E)$

2) $E \in P(E)$

3) $\forall A_j \in P(E) : A_j \subset \Gamma A_j$, extension

4) $\forall A_j \in P(E) : \Gamma (\Gamma A_j) \subset \Gamma A_j$, idempotency

5) $\forall A_j , A_k \in P(E) : (A_j \subset A_k) => (\Gamma A_j \subset \Gamma A_k)$, isotony

In the Moore's closure $\Gamma E = E$, as a result of the extension. Nevertheless it is not necessary condition that $\Gamma \emptyset = \emptyset$, as it happens in the pretopologies. Thus, not all Moore's closures are pretopologies. But, when a Moore's closure satisfy:

$\emptyset \in P(E) : \Gamma \emptyset = \emptyset$

then, this Moore's closure is a isotonous pretopology with idempotency.

Moore's closure occupies an important place in the process of developing the algorithm for the treatment of maximum grouping problems, which forces to establish a way that allows to find Moore's closures from a concept affordable enough [8], [9]. To this end, the graph theory is particularly useful for developing schemes in which the relationship between elements play an important role. A graph can be represented in matrix form as well as sagittately, limiting ourselves here to the first of these forms.

From a fuzzy relation $[R_\alpha]$ between the elements of a reference E, with $E = \{ E_i / i = 1 .... n\}$, we have that $[R_\alpha] = E \times E$, constituting a regular graph of level α, where [14], [13][1]:

$[R_\alpha] = \{(E_i , E_i) \in E \times E / \mu_R (E_i , E_i) \geq \alpha \}$, with $\alpha \in [0,1]$

The matrix corresponding to this graph, at the level α, shows by the valuations in the interval [0, 1], the degree of intensity in the relationship between each pair of elements of the set of reference E. In this regard, the predecessors and successors of an element E_i can be defined as:

E_j it's a successor of E_i at level α if $(E_i , E_j) \in [R_\alpha]$

[1] The fuzzy relation $[R_\alpha]$ can also be configured between two different benchmarks, ie $[R_\alpha] = E \times D$, with $E = \{E_i / i = 1 .... n\}$ and $D = \{D_j / j = 1 .... m\}$. Although this is not the case of the problem we address in this paper.

E_i it's a predecessor of E_i at level α if $(E_i\ , E_j) \in [R_\alpha]$

According to [4] and [13], these definitions allow us to find, for a given level α, the connection to the right R_α^+, that is, the functional application R_α^+ of P(E) in P(E) such that, for all $A_j \in$ P(E), R_α^+ is the subset of elements of E that are successors to the α level of any element belonging to A_j. Stated another way, for each element A_j of P(E) we have all groups of elements with which it has affinity. This is usually expressed as:

$$R_\alpha^+ A_j = \{ E_i \in E \,/\, (E_i\ , E_j) \in [R_\alpha]\ ,\ \forall E_i \in A_j \},\ \text{with } R_\alpha^+ \varnothing = E$$

Similarly, there is the connection on the left R_α^- at a certain level α, as the functional application of R_α^- of P(E) to P(E) such that, for all $A_j \in$ P (E), R_α^- is the subset of elements of E which are predecessors of any element α level belonging to A_j. Thus, each group of elements A_j of P(E) is related to all group elements with which has affinity. This is usually expressed as:

$$R_\alpha^- A_j = \{ E_i \in E \,/\, (E_i\ , E_j) \in [R_\alpha]\ ,\ \forall E_i \in A_j \},\ \text{with } R_\alpha^- \varnothing = E$$

The connection on the right ($R_\alpha^+ A_j$) and the connection on the left ($R_\alpha^- A_j$) can be found directly by simply reading at the fuzzy relation $[R_\alpha]$ as follows:

$$\forall E_i \subset A_j \in P(E): \qquad R_\alpha^+ A_j = \bigcap R_\alpha^+ \{ E_i\},\ \text{with } E_i \subset A_j$$

$$R_\alpha^- A_j = \bigcap R_\alpha^- \{ E_i\},\ \text{with } E_i \subset A_j$$

This shows that it suffices to observe for each group of rows (columns) in the fuzzy relation $[R_\alpha]$ those columns (rows) where valuations are at or above the α-cut considered in all rows (columns) form the affinity group.

Finally, the max-min convolutions of R_α^- with R_α^+ and R_α^+ with R_α^- provide, at level α, the two Moore's closures (M) of P(E) corresponding to the fuzzy relation $[R_\alpha]$.Which is expressed as[2]:

$$M_\alpha^{(1)} = R_\alpha^- A_j \circ R_\alpha^+ A_j$$

$$M_\alpha^{(2)} = R_\alpha^+ A_j \circ R_\alpha^- A_j$$

Bringing the set of Moore's closures consist of those elements $A_j \in$ P(E) that, as a pretopology, meet that $A_j = \Gamma\ A_j$ and $A_j = \delta\ A_j$, and for which the properties of extension, idempotency and isotonic are verified [13].

Following [4], we denote by C(E, $M^{(2)}$) the closed subset of P(E) corresponding to Moore's closure $M^{(2)}$. Since $R_\alpha^- A_j$ is a closed of P(E) to $M^{(2)}$, we can write:

$$C(E, M^{(2)}) = \bigcup R_\alpha^- A_j\ ,\ \text{with } A_j \in P(E)$$

[2] It uses the letter M (Moore) to denote respectively the adherent application (Γ) and the interior application (δ) within of pretopology.

and, similarly, to be $R_\alpha^+ A_j$ a closed for $M^{(1)}$, and designating by $C(E, M^{(1)})$ the closed subset of P(E) for Moore's closure $M^{(1)}$, we have:

$$C(E, M^{(1)}) = \bigcup R_\alpha^+ A_j \text{, with } A_j \in P(E)$$

The two families of closures $C(E, M^{(1)})$ and $C(E, M^{(2)})$ are isomorphic to each other and dual relative one to another, antitone nature. This is expressed as:

$$A \in C(E, M^{(1)}) \Rightarrow (B = R_\alpha^- A \in C(E, M^{(2)}) \text{ and } R_\alpha^+ B = A$$

$$B \in C(E, M^{(2)}) \Rightarrow (A = R_\alpha^+ B \in C(E, M^{(1)}) \text{ and } R_\alpha^- A = B$$

These families of closed can be associated with each other and are, each, a finite lattice. Furthermore, as pointed out, by the max-min convolution, both one and the other of these families of closed provide the groups with the greatest possible number of elements of the reference E. Thus, in all the vertices of each lattice place we put in one of them a group of elements of a family of closed and the other clusters of the other family of closed.

Is easy to verify that, when the two lattices are superimposed, one is obtained, which contains in each of the vertices the relation of the maximum groups of elements of the E set. When this happens it is said that there is an affinity. This lattice provides structured and ordered relations between maximum groups. When to this lattice are added, at its top and bottom, the edges (Ø, E) and (E, Ø), we are facing a Galois lattice, which sets the two sets of Moore's closures which have as upper and lower ends these relationships.

3. APPLICATION TO THE ANALYSIS OF INDUSTRIAL CLUSTER RELATIONS: AN EXAMPLE

For a better understanding of the procedure to follow in its application to relations between agents within an industrial cluster, we will follow a simplified example. Obviously, in an industrial cluster a lot of agents are involved, so that the number of potentially existing relationships grows exponentially, which is the reason why it would be enormously complex and repetitive to use a "real" example, since our aim is simply narrative and descriptive of the used method.[3] For this reason, as indicated by [2], although the true potential of the method is obtained when applied individually considering each and every one of the agents that form an industrial cluster, usually with reasons of simplification, grouping the agents in large types is presented because, if it is made at a

[3] In fact, in the application to the analysis of relationships in a given industrial cluster, given the large number of potential relationships that may exist, it should apply the method described by some computer program. Consider that the number of elements of the set P(E) or power set of the benchmark set is 2^n, where n is the number of items contained in the reference set E.

displeasure level, the great number of combinations that arise often render impractical the analysis.

First we should identify, in general terms, the agents involved in an industrial cluster by major types, which constitute the reference set E. In this respect, we synthesized, for example, as follows:

E_1 : Public organizations
E_2 : University and Technological and Scientific Centres
E_3 : Companies within the main activity sector of the industrial sector
E_4 : Supplier companies
E_5 : Customer companies

That means, $E = \{ E_1 , E_2 , E_3 , E_4 , E_5 \}$, so that all parts of E or its power set P(E), composed of 32 elements (E being composed of 5 elements, we have 2^5 combinations), is:

$$P(E) = \{ \{\emptyset\} , \{E_1\} , \{E_2\} , \{E_3\} , \{E_4\} , \{E_5\} , \{E_1 , E_2\}, \{E_1 , E_3\}, \{E_1 , E_4\}, \{E_1,E_5\}, \{E_2 , E_3\} , \{E_2 , E_4\} , \{E_2 , E_5\} , \{E_3 , E_4\} , \{E_3 , E_5\} , \{E_4 , E_5\} , \{E_1, E_2,E_3\}, \{E_1 , E_2 , E_4\} , \{E_1 , E_2 , E_5\} , \{E_1 , E_3 , E_4\} , \{E_1 , E_3 , E_5\} , \{E_1 , E_4, E_5\},\{E_2 , E_3 , E_4\} , \{E_2 , E_3 , E_5\} , \{E_2 , E_4 , E_5\} , \{E_3 , E_4 , E_5\} , \{E_1 , E_2, E_3 , E_4\} ,\{E_1 , E_2 , E_3 , E_5\} , \{E_1 , E_2 , E_4 , E_5\} , \{E_1 , E_3 , E_4 , E_5\} , \{E_2, E_3, E_4 , E_5\} ,\{E_1 ,E_2 , E_3 , E_4 , E_5\} \}$$

First, it analyzes the positive relationships between agents in the industrial cluster. That is, those where the relationship is “I win-you win”, and create value for the industrial cluster. The choice of positive collaboration is present when companies or institutions of the industrial cluster make a formal or informal attitude to help the parties to the relationship to achieve their goals. This attitude creates value to businesses and reduces both research and process development costs, etc., becoming a key element in an industrial cluster.

To obtain the fuzzy relation $[R_\alpha]$ it can be consulted, using questionnaires, a representative from each agent in the industrial cluster directly involved in the relationships with other agents, with the intention that each one of them express a valuation in the range [0, 1] about their relationships with each and every one of the other agents of the industrial cluster[4], where the maximum value 1 indicates a fully satisfactory and positive relationship with the other agent, that is, that it permits to achieve the objectives of the relationship (development of new technology, to conquer new markets, etc.) in an effectively and efficiently way, and in a climate of confidence and profit from the consulted agent point of view, whereas the minimum value 0 indicates no positive relationship with the

[4] Obviously, several representatives are also available for one participant in the industrial cluster. Under these circumstances, it would get half the valuation of all for every possibility of relationship with other agents. Furthermore, instead of using simple valuations may also be used fuzzy subsets (confidence intervals and triplets, triangular fuzzy numbers, etc.), see thereon [6].

corresponding agent. Thus, intermediate valuations in the interval [0, 1] indicate the degree of intensity in the satisfaction and performance obtained (depending on the objectives), from the consulted agent point of view, in the relationship. Given the nature of the problem addressed, based on the analysis of relationships between agents, the relationship of an agent with himself is ruled out. Let's suppose that the fuzzy relation arising from this consultation is the one shown in Table 1:

Table 1. Fuzzy relation of positive relationships between industrial cluster agents

		E_1	E_2	E_3	E_4	E_5
	E_1	___	.72	.82	.34	.48
	E_2	.94	___	.78	.81	.33
$[R_\alpha]$ =	E_3	.82	.78	___	.95	.76
	E_4	.58	.73	.74	___	.22
	E_5	.45	.52	.86	.18	___

Then, it comes to obtain the connection to the right and the connection to the left of this fuzzy relation to the desired level α to perform the analysis of the relationships. In our example, for simplicity, we follow the procedure only for α = .7, in this way, we rewrite the above fuzzy relation with a 1 in the boxes that have a value α ≥ .7, and with a 0 otherwise (the fuzzy matrix or relation $[R_{0.7}]$ becomes the following boolean matrix):

Table 2. Boolean matrix of positive relationships (α = .7)

		E_1	E_2	E_3	E_4	E_5
	E_1	___	1	1	0	0
	E_2	1	___	1	1	0
$[R_{0.7}]$ =	E_3	1	1	___	1	1
	E_4	0	1	1	___	0
	E_5	0	0	1	0	___

Now we are able to obtain the right connection, assigning each element of P(E) elements of the same P(E) in which there is a 1 at the corresponding rows:

$R^{+}_{0.7}\, Ø = E$
$R^{+}_{0.7}\, \{E_1\} = \{E_2, E_3\}$
$R^{+}_{0.7}\, \{E_2\} = \{E_1, E_3, E_4\}$
$R^{+}_{0.7}\, \{E_3\} = \{E_1, E_2, E_4, E_5\}$
$R^{+}_{0.7}\, \{E_4\} = \{E_2, E_3\}$
$R^{+}_{0.7}\, \{E_5\} = \{E_3\}$
$R^{+}_{0.7}\, \{E_1, E_2\} = \{E_3\}$
$R^{+}_{0.7}\, \{E_1, E_3\} = \{E_2\}$
$R^{+}_{0.7}\, \{E_1, E_4\} = \{E_2, E_3\}$
$R^{+}_{0.7}\, \{E_1, E_5\} = \{E_3\}$
$R^{+}_{0.7}\, \{E_2, E_3\} = \{E_1, E_4\}$

$R^{+}_{0.7}\{E_2, E_4\} = \{E_3\}$
$R^{+}_{0.7}\{E_2, E_5\} = \{E_3\}$
$R^{+}_{0.7}\{E_3, E_4\} = \{E_2\}$
$R^{+}_{0.7}\{E_3, E_5\} = \varnothing$
$R^{+}_{0.7}\{E_4, E_5\} = \{E_3\}$
$R^{+}_{0.7}\{E_1, E_2, E_3\} = \varnothing$
$R^{+}_{0.7}\{E_1, E_2, E_4\} = \{E_3\}$
$R^{+}_{0.7}\{E_1, E_2, E_5\} = \{E_3\}$,
$R^{+}_{0.7}\{E_1, E_3, E_4\} = \{E_2\}$
$R^{+}_{0.7}\{E_1, E_3, E_5\} = \varnothing$
$R^{+}_{0.7}\{E_1, E_4, E_5\} = \{E_3\}$
$R^{+}_{0.7}\{E_2, E_3, E_4\} = \varnothing$
$R^{+}_{0.7}\{E_2, E_3, E_5\} = \varnothing$
$R^{+}_{0.7}\{E_2, E_4, E_5\} = \{E_2\}$
$R^{+}_{0.7}\{E_3, E_4, E_5\} = \varnothing$
$R^{+}_{0.7}\{E_1, E_2, E_3, E_4\} = \varnothing$
$R^{+}_{0.7}\{E_1, E_2, E_3, E_5\} = \varnothing$
$R^{+}_{0.7}\{E_1, E_2, E_4, E_5\} = \{E_3\}$
$R^{+}_{0.7}\{E_1, E_3, E_4, E_5\} = \varnothing$,
$R^{+}_{0.7}\{E_2, E_3, E_4, E_5\} = \varnothing$
$R^{+}_{0.7}\{E_1, E_2, E_3, E_4, E_5\} = R^{+}_{0.7}\, E = \varnothing$

Also, the connection is obtained to the left, assigning to each element of P(E) the same elements of P(E) in which a value exists in the corresponding columns:

$R^{-}_{0.7}\,\varnothing = E$
$R^{-}_{0.7}\{E_1\} = \{E_2, E_3\}$
$R^{-}_{0.7}\{E_2\} = \{E_1, E_3, E_4\}$
$R^{-}_{0.7}\{E_3\} = \{E_1, E_2, E_4, E_5\}$
$R^{-}_{0.7}\{E_4\} = \{E_2, E_3\}$
$R^{-}_{0.7}\{E_5\} = \{E_3\}$
$R^{-}_{0.7}\{E_1, E_2\} = \{E_3\}$
$R^{-}_{0.7}\{E_1, E_3\} = \{E_2\}$
$R^{-}_{0.7}\{E_1, E_4\} = \{E_2, E_3\}$
$R^{-}_{0.7}\{E_1, E_5\} = \{E_3\}$
$R^{-}_{0.7}\{E_2, E_3\} = \{E_1, E_4\}$
$R^{-}_{0.7}\{E_2, E_4\} = \{E_3\}$
$R^{-}_{0.7}\{E_2, E_5\} = \{E_3\}$
$R^{-}_{0.7}\{E_3, E_4\} = \{E_2\}$
$R^{-}_{0.7}\{E_3, E_5\} = \varnothing$,
$R^{-}_{0.7}\{E_4, E_5\} = \{E_3\}$
$R^{-}_{0.7}\{E_1, E_2, E_3\} = \varnothing$
$R^{-}_{0.7}\{E_1, E_2, E_4\} = \{E_3\}$
$R^{-}_{0.7}\{E_1, E_2, E_5\} = \{E_3\}$
$R^{-}_{0.7}\{E_1, E_3, E_4\} = \{E_2\}$
$R^{-}_{0.7}\{E_1, E_3, E_5\} = \varnothing$

$R^{-}_{0.7}\{E_1, E_4, E_5\} = \{E_3\}$
$R^{-}_{0.7}\{E_2, E_3, E_4\} = \emptyset$
$R^{-}_{0.7}\{E_2, E_3, E_5\} = \emptyset$
$R^{-}_{0.7}\{E_2, E_4, E_5\} = \{E_3\}$
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$R^{-}_{0.7}\{E_1, E_2, E_3, E_4\} = \emptyset$
$R^{-}_{0.7}\{E_1, E_2, E_3, E_5\} = \emptyset$
$R^{-}_{0.7}\{E_1, E_2, E_4, E_5\} = \{E_3\}$
$R^{-}_{0.7}\{E_1, E_3, E_4, E_5\} = \emptyset$
$R^{-}_{0.7}\{E_2, E_3, E_4, E_5\} = \emptyset$
$R^{-}_{0.7}\{E_1, E_2, E_3, E_4, E_5\} = R^{-}_{0.7} E = \emptyset$

Applying the max-min convolution $R^{-}_{0.7} A_j \circ R^{+}_{0.7} A_j$, closed are obtained (for which it holds that $A_j = \Gamma A_j$, which are indicated by *), representing the first Moore's closure $M_{0.7}^{(2)}$ (adherent application Γ):[5]

$\emptyset \rightarrow E \rightarrow \emptyset$	*
$E_1 \rightarrow E_2, E_3 \rightarrow E_1, E_4$	
$E_2 \rightarrow E_1, E_3, E_4 \rightarrow E_2$	*
$E_3 \rightarrow E_1, E_2, E_4, E_5 \rightarrow E_3$	*
$E_4 \rightarrow E_2, E_3 \rightarrow E_1, E_4$	
$E_5 \rightarrow E_3 \rightarrow E_1, E_2, E_4, E_5$	
$E_1, E_2 \rightarrow E_3 \rightarrow E_1, E_2, E_4, E_5$	
$E_1, E_3 \rightarrow E_2 \rightarrow E_1, E_3, E_4$	
$E_1, E_4 \rightarrow E_2, E_3 \rightarrow E_1, E_4$	*
$E_1, E_5 \rightarrow E_3 \rightarrow E_1, E_2, E_4, E_5$	
$E_2, E_3 \rightarrow E_1, E_4 \rightarrow E_2, E_3$	*
$E_2, E_4 \rightarrow E_3 \rightarrow E_1, E_2, E_4, E_5$	
$E_2, E_5 \rightarrow E_3 \rightarrow E_1, E_2, E_4, E_5$	
$E_3, E_4 \rightarrow E_2 \rightarrow E_1, E_3, E_4$	
$E_4, E_5 \rightarrow E_3 \rightarrow E_1, E_2, E_4, E_5$	
$E_1, E_2, E_4 \rightarrow E_3 \rightarrow E_1, E_2, E_4, E_5$	
$E_1, E_2, E_5 \rightarrow E_3 \rightarrow E_1, E_2, E_4, E_5$	
$E_1, E_3, E_4 \rightarrow E_2 \rightarrow E_1, E_3, E_4$	*
$E_1, E_4, E_5 \rightarrow E_3 \rightarrow E_1, E_2, E_4, E_5$	
$E_2, E_4, E_5 \rightarrow E_3 \rightarrow E_1, E_2, E_4, E_5$	
$E_1, E_2, E_4, E_5 \rightarrow E_3 \rightarrow E_1, E_2, E_4, E_5$	*
$E_1, E_2, E_3, E_4, E_5 = E \rightarrow \emptyset \rightarrow E$	*

Accordingly, the family of closed corresponding to $C(E, M^{(2)})$, for α = .7, is:

$C(E, M^{(2)})=\{\{\emptyset\},\{E_2\},\{E_3\},\{E_1, E_4\},\{E_2, E_3\},\{E_1,E_3,E_4\},\{E_1,E_2,E_4,E_5\},\{E\}\}$

[5] Only we present no empty arches and elements.

Similarly, applying the max-min convolution $R^{+}_{0.7} A_j \circ R^{-}_{0.7} A_j$ are obtained closed (for which it holds that $A_j = \delta A_j$, which are indicated by *), representing the second Moore's closure $M_{0.7}^{(1)}$ (interior application δ):

$\varnothing \rightarrow E \rightarrow \varnothing$ *
$E_1 \rightarrow E_2, E_3 \rightarrow E_1, E_4$
$E_2 \rightarrow E_1, E_3, E_4 \rightarrow E_2$ *
$E_3 \rightarrow E_1, E_2, E_4, E_5 \rightarrow E_3$ *
$E_4 \rightarrow E_2, E_3 \rightarrow E_1, E_4$
$E_5 \rightarrow E_3 \rightarrow E_1, E_2, E_4, E_5$
$E_1, E_2 \rightarrow E_3 \rightarrow E_1, E_2, E_4, E_5$
$E_1, E_3 \rightarrow E_2 \rightarrow E_1, E_3, E_4$
$E_1, E_4 \rightarrow E_2, E_3 \rightarrow E_1, E_4$ *
$E_1, E_5 \rightarrow E_3 \rightarrow E_1, E_2, E_4, E_5$
$E_2, E_3 \rightarrow E_1, E_4 \rightarrow E_2, E_3$ *
$E_2, E_4 \rightarrow E_3 \rightarrow E_1, E_2, E_4, E_5$
$E_2, E_5 \rightarrow E_3 \rightarrow E_1, E_2, E_4, E_5$
$E_3, E_4 \rightarrow E_2 \rightarrow E_1, E_3, E_4$
$E_4, E_5 \rightarrow E_3 \rightarrow E_1, E_2, E_4, E_5$
$E_1, E_2, E_4 \rightarrow E_3 \rightarrow E_1, E_2, E_4, E_5$
$E_1, E_2, E_5 \rightarrow E_3 \rightarrow E_1, E_2, E_4, E_5$
$E_1, E_3, E_4 \rightarrow E_2 \rightarrow E_1, E_3, E_4$ *
$E_1, E_4, E_5 \rightarrow E_3 \rightarrow E_1, E_2, E_4, E_5$
$E_2, E_4, E_5 \rightarrow E_2 \rightarrow E_1, E_3, E_4$
$E_1, E_2, E_4, E_5 \rightarrow E_3 \rightarrow E_1, E_2, E_4, E_5$ *
$E_1, E_2, E_3, E_4, E_5 = E \rightarrow \varnothing \rightarrow E$ *

Thus the family of closed $C(E, M^{(1)})$, for α = .7, is:

$$C(E,M^{(1)})=\{\{\varnothing\},\{E_2\},\{E_3\},\{E_1,E_4\},\{E_2,E_3\},\{E_1,E_3,E_4\},\{E_1,E_2,E_4,E_5\},\{E\}\}$$

Both families of closed have the same cardinal, that means, they have the same number of elements (in particular, for our example, 8), and are dual in antitone nature. Furthermore, as shown in Figure 1, the lattices of both Moore's closures are isomorphic.

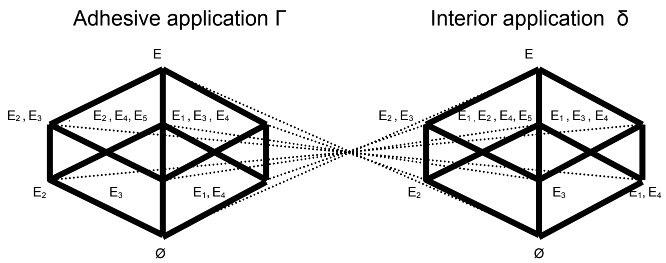

Figure 1. Moore's closures lattices for R (α = .7)

If one of the lattices is rotated 180º and then superimposed, we can verify the isomorphism and we can observe the affinities between agents of industrial cluster that they possess in common. This lattice then is a Galois lattice, which presents the affinities or maximum subrelations between the elements of reference E in an structured and ordered way, in our example for level α = .7, as it is showed in Figure 2.

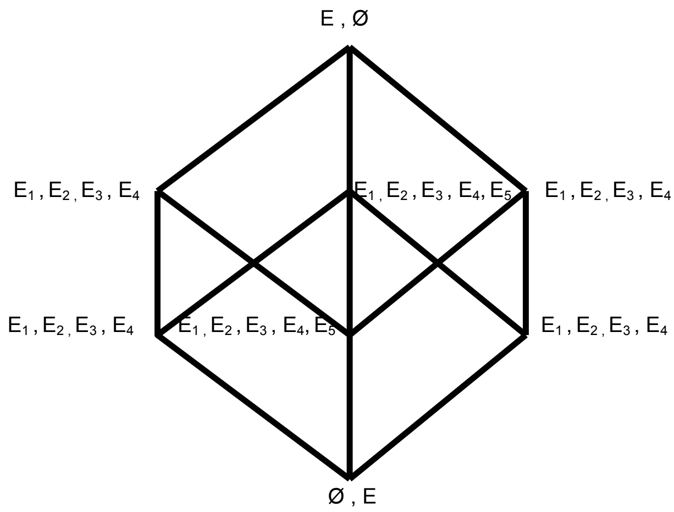

Figure 2. Positive affinities Galois' lattice for R (α = .7)

The isomorphism between Moore's lattices, for level α = .7, is specified into the following affinities:

$E_2 \rightarrow E_1, E_3, E_4 \quad : \quad E_1, E_2, E_3, E_4$
$E_3 \rightarrow E_1, E_2, E_4, E_5 \quad : \quad E_1, E_2, E_3, E_4, E_5$
$E_1, E_4 \rightarrow E_2, E_3 \quad : \quad E_1, E_2, E_3, E_4$

On the other hand, the relationship between industrial cluster agents are not necessarily positive for the parties but there may be relationships that don't allow to achieved the desired objectives and, therefore, could be perceived as negative for the asked agent. That is, in the industrial cluster negative relationships can also arise between the parties, those that are of the "I win-you lose", and which destroy value for the industrial cluster. Thus, there may be a negative collaboration in which companies or institutions of the industrial cluster adopt a selfish role and don't support the pursuit of common goals, taking an individualistic role in achieving its objectives. This attitude generally doesn't favour value and it is a challenge for the integration of an industrial cluster. In this sense, from the fuzzy relation $[R_α]$, which indicates the degree of intensity of positive relationships between the agents of industrial cluster, its complement can be constructed, which we will call $[S_α]$, where $[S_α] = [\bar{R}_α] = 1 - [R_α]$, indicating, therefore, the intensity of negative relationships between agents in the industrial cluster. In the fuzzy relation [S_α], the maximum value of 1 indicates a totally unsatisfactory and negative relationship with the other agent, that is, it doesn't allow to achieve the objectives set out in the relationship or is counterproductive or, if applicable (when R has the value 1), no negative relationship with the corresponding agent. Thus, intermediate valuations in the interval [0, 1] indicate the degree of intensity of dissatisfaction in the relationship from the standpoint of the agent asked. We return to rule the relation of an agent with himself out. From [R_α] we estimate [S_α]:

Table 3. Fuzzy relation of negative relationships between industrial cluster agents

		E_1	E_2	E_3	E_4	E_5
	E_1	___	.28	.18	.66	.52
	E_2	.06	___	.22	.19	.67
[S_α] =	E_3	.18	.22	___	.05	.24
	E_4	.42	.27	.26	___	.78
	E_5	.55	.48	.14	.82	___

We follow the procedure also only for α = .7, and rewrite [S_α] with a 1 in the boxes that have a value α ≥ .7, and a 0 otherwise (the fuzzy relation matrix or becomes the following boolean matrix):

Table 4. Boolean matrix of negative relationships (α = .7).

		E_1	E_2	E_3	E_4	E_5
	E_1	___	0	0	0	0
	E_2	0	___	0	0	0
[$S_{0.7}$] =	E_3	0	0	___	0	0
	E_4	0	0	0	___	1
	E_5	0	0	0	1	___

Thus the connection on the right is:

$S^+_{0.7}\ \emptyset = E$
$S^+_{0.7}\ \{E_1\} = \emptyset$
$S^+_{0.7}\ \{E_2\} = \emptyset$
$S^+_{0.7}\ \{E_3\} = \emptyset$
$S^+_{0.7}\ \{E_4\} = \{E_5\}$
$S^+_{0.7}\ \{E_5\} = \{E_4\}$
$S^+_{0.7}\ \{E_1 , E_2\} = \emptyset$
$S^+_{0.7}\ \{E_1 , E_3\} = \emptyset$
$S^+_{0.7}\ \{E_1 , E_4\} = \emptyset$
$S^+_{0.7}\ \{E_1 , E_5\} = \emptyset$
$S^+_{0.7}\ \{E_2 , E_3\} = \emptyset$
$S^+_{0.7}\ \{E_2 , E_4\} = \emptyset$
$S^+_{0.7}\ \{E_2 , E_5\} = \emptyset$

$S^{+}_{0.7}\{E_3 , E_4\} = \varnothing$
$S^{+}_{0.7}\{E_3 , E_5\} = \varnothing$
$S^{+}_{0.7}\{E_4 , E_5\} = \varnothing$
$S^{+}_{0.7}\{E_1 , E_2 , E_3\} = \varnothing$
$S^{+}_{0.7}\{E_1 , E_2 , E_4\} = \varnothing$
$S^{+}_{0.7}\{E_1 , E_2 , E_5\} = \varnothing$
$S^{+}_{0.7}\{E_1 , E_3 , E_4\} = \varnothing$
$S^{+}_{0.7}\{E_1 , E_3 , E_5\} = \varnothing$
$S^{+}_{0.7}\{E_1 , E_4 , E_5\} = \varnothing$
$S^{+}_{0.7}\{E_2 , E_3 , E_4\} = \varnothing$
$S^{+}_{0.7}\{E_2 , E_3 , E_5\} = \varnothing$
$S^{+}_{0.7}\{E_2 , E_4 , E_5\} = \varnothing$
$S^{+}_{0.7}\{E_3 , E_4 , E_5\} = \varnothing$
$S^{+}_{0.7}\{E_1 , E_2 , E_3 , E_4\} = \varnothing$
$S^{+}_{0.7}\{E_1 , E_2 , E_3 , E_5\} = \varnothing$
$S^{+}_{0.7}\{E_1 , E_2 , E_4 , E_5\} = \varnothing$
$S^{+}_{0.7}\{E_1 , E_3 , E_4 , E_5\} = \varnothing$
$S^{+}_{0.7}\{E_2 , E_3 , E_4 , E_5\} = \varnothing$
$S^{+}_{0.7}\{E_1 , E_2 , E_3 , E_4 , E_5\} = \varnothing$

And the connection on the left is:

$S^{-}_{0.7}\, \varnothing = E$
$S^{-}_{0.7}\{E_1\} = \varnothing$
$S^{-}_{0.7}\{E_2\} = \varnothing$
$S^{-}_{0.7}\{E_3\} = \varnothing$
$S^{-}_{0.7}\{E_4\} = \{E_5\}$
$S^{-}_{0.7}\{E_5\} = \{E_4\}$
$S^{-}_{0.7}\{E_1 , E_2\} = \varnothing$
$S^{-}_{0.7}\{E_1 , E_3\} = \varnothing$
$S^{-}_{0.7}\{E_1 , E_4\} = \varnothing$
$S^{-}_{0.7}\{E_1 , E_5\} = \varnothing$
$S^{-}_{0.7}\{E_2 , E_3\} = \varnothing$
$S^{-}_{0.7}\{E_2 , E_4\} = \varnothing$
$S^{-}_{0.7}\{E_2 , E_5\} = \varnothing$
$S^{-}_{0.7}\{E_3 , E_4\} = \varnothing$
$S^{-}_{0.7}\{E_3 , E_5\} = \varnothing$
$S^{-}_{0.7}\{E_4 , E_5\} = \varnothing$
$S^{-}_{0.7}\{E_1 , E_2 , E_3\} = \varnothing$
$S^{-}_{0.7}\{E_1 , E_2 , E_4\} = \varnothing$
$S^{-}_{0.7}\{E_1 , E_2 , E_5\} = \varnothing$
$S^{-}_{0.7}\{E_1 , E_3 , E_4\} = \varnothing$
$S^{-}_{0.7}\{E_1 , E_3 , E_5\} = \varnothing$
$S^{-}_{0.7}\{E_1 , E_4 , E_5\} = \varnothing$
$S^{-}_{0.7}\{E_2 , E_3 , E_4\} = \varnothing$
$S^{-}_{0.7}\{E_2 , E_3 , E_5\} = \varnothing$
$S^{-}_{0.7}\{E_2 , E_4 , E_5\} = \varnothing$

$S^{-}_{0.7}$ {E_3 , E_4 , E_5} = Ø
$S^{-}_{0.7}$ {E_1 , E_2 , E_3 , E_4} = Ø
$S^{-}_{0.7}$ {E_1 , E_2 , E_3 , E_5} = Ø
$S^{-}_{0.7}$ {E_1 , E_2 , E_4 , E_5} = Ø
$S^{-}_{0.7}$ {E_1 , E_3 , E_4 , E_5} = Ø
$S^{-}_{0.7}$ {E_2 , E_3 , E_4 , E_5} = Ø
$S^{-}_{0.7}$ {E_1 , E_2 , E_3 , E_4 , E_5} = Ø

Applying the max-min convolution $S^{-}_{0.7}\ A_j \circ S^{+}_{0.7}\ A_j$ the closed which represent the first Moore's closure $M_{0.7}^{(2)}$ (Γ adherent application) are obtained, for α = .7:

$E_4 \rightarrow E_5 \rightarrow E_4$
$E_5 \rightarrow E_4 \rightarrow E_5$

Accordingly, the family of closed corresponding to C(E, $M^{(2)}$), for α = .7, is:

C(E, $M^{(2)}$) = { {Ø} , {E_4} , {E_5} , {E} }

In the same way, applying $S^{+}_{0.7}\ A_j \circ S^{-}_{0.7}\ A_j$ the closed which represent the second Moore's closure $M_{0.7}^{(1)}$ (δ interior application) are obtained, for α = .7:

$E_4 \rightarrow E_5 \rightarrow E_4$
$E_5 \rightarrow E_4 \rightarrow E_5$

Accordingly, the family of closed corresponding to C(E, $M^{(1)}$), for α = .7, is:

C(E, $M^{(1)}$) = { {Ø} , {E_4} , {E_5} , {E} }

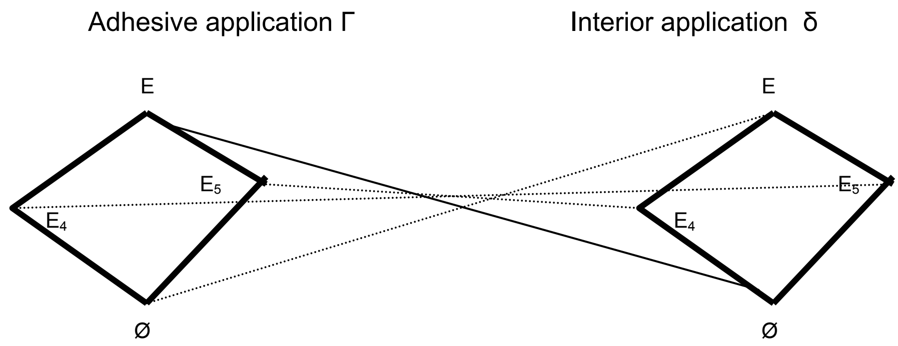

Figure 3. Moore's closures lattices for S (α = .7)

So we have the Galois' lattice shown in Figure 4, with a single negative affinity between agents in the industrial cluster:

$E_4 \rightarrow E_5 : E_4 , E_5$
$E_5 \rightarrow E_4 : E_4 , E_5$