TERM STRUCTURE OF INTEREST RATES ANALYSIS IN THE SPANISH MARKET

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The Term Structure of Interest Rates (TSIR) makes it possible to analyze investors' expectations of future interest rates. This study aims to make a comparative analysis of the TSIR to determine whether investors modify their expectations in such a turbulent financial scenario as the present one. The TSIR was estimated, in july 2007 and july 2008, using McCulloch's quadratic splines and fuzzy regressions.

Keywords: *term structure of interest rates, fuzzy regression, financial crisis*

1. INTRODUCTION

TSIR is defined as the relation between the maturity of a financial asset, which has a zero-coupon structure and is risk free, and the interest rate at which the market discounts this asset.

TSIR can be analyzed from two perspectives. Some authors attempt to explain TSIR by looking at the movements that are caused in the main variables of the economy: for example, inflation, product growth, etc. (see Shiller (1979); Fama & Bliss (1989); Campbell & Shiller (1991); Litterman & Scheinkman (1991); Chen & Scott (1993); Dai & Singleton (2000); Bedendo Cathcar & El-Jahe (2004), among others). Others, however, evaluate the ability of TSIR to predict future movements of real variables (Harvey (1988); Mishkin (1991); Estrella & Hardouvelis (1991); Pesaran & Timmermann (1995); Estrella (1997); and Fabozzi, Martellini & Priaulet (2005). In this paper, we evaluate the TSIR curve both before and during the financial crisis, in an attempt to determine whether the recent crisis has had repercussions on the TSIR in the particular case of the Spanish market.

In general, the TSIR cannot be directly observed in the long term, so it needs to be estimated. One of the most commonly used estimation methods is the fitting of the discount function using splines (McCulloch (1971,1975); Vasicek & Fong (1982); Nelson & Siegel (1987); and Svensson (1996)).

When we work with econometric models, we take a single value —mathematical expectation— as the price of the security on a particular date. Nevertheless, by using fuzzy regressions, we can work with all the information that we have on the market: that is to say, with all the prices of an asset in the same session.

We shall estimate the TSIR for the Spanish market using McCulloch's spline method (1971) and fuzzy regressions. We base the estimation on the studies by de Andrés (2000) and de Andrés & Terceño (2003, 2004).

The paper is structured in the following way. In section 2, we briefly discuss the economic and financial scenario of the recent months. In section 3, we present the methodology used. In section 4, we estimate the TSIR and present the results of the estimation. Finally, section 5 provides a summary of the conclusions.

2. ECONOMIC DEACCELERATION AND FINANCIAL CRISIS

In recent years, the Spanish economy has undergone considerable growth, above and beyond that of other countries in the euro zone. All this has been made possible by the boom of the property market and the driving force of private consumption. This expansion of internal demand has been encouraged by favourable credit conditions: that is to say, both easy access to finance and the reduction in the price of money. Between 2002 and 2006, for example, one of the main references of the mortgage market —the 1-year Euribor— had a mean maximum value of 3,493% and its lowest value, in 2004 was 2,274%. This combination of factors brought about the phenomenon of the property bubble, sustained, galloping consumption, and high rates of debts.

The present situation, however, is quite different. Economic deacceleration has taken over those sectors of the Spanish economy that had previously been its driving forces. Credits for housing are more expensive. The mean value of the 1-year Euribor for 2007 (4,450%) is double that of the lowest mean value of the previous years (2,274% in 2004). Those families that are most in debt have been obliged to check their consumption in order to be able to satisfy their financial commitments. At the same time, finance for new housing has not only been restricted but is also considerably more expensive. As a result, the demand for property has decreased in recent months, and the construction sector has entered a new recessive stage, and has severe problems of finance and liquidity. In short, when private consumption and the property sector began to feel the effects of the deacceleration, the rate of economic growth in Spain was no longer the same as during the recent years of economic bonanza.

From a more international perspective, the dynamism of the financial markets and, in particular, of the derivatives market has led to the proliferation of a certain type of bonds: securitized bonds. In markets such as the USA, these securities have been largely linked to such high-risk assets as subprime mortgages. The financial complexity inherent in such operations meant that, on occasion, the holders of these securitized bonds did not understand the nature of the real asset that backed the securities. In turn, the rating assigned to these assets did not match the real rating. The crisis that arose in the USA because of the subprime mortgages modified the financial scenario on a world scale. It was the first sign of a lack of confidence that finally took hold of the interbank market. It was this climate of lack of confidence and the ignorance of the eventual scope of the contagion is triggering a liquidity crisis in the international financial system which is having such severe consequences as restrictions on credit for companies and families, and the rise in the cost of money.

3. METHODOLOGY

As mentioned above, we shall estimate the TSIR by fitting the discount function. This method consists of fitting the discount function f(t), where $f(t) = (1+i_t)^{-t}$. Using this function, we obtain the spot interest rate for each maturity t, i_t, required to construct the TSIR.

To model the discount function, we use the paper by de Andrés (2000) which describes the method of McCulloch's quadratic splines (1971) using fuzzy regressions. Thus, the fuzzy modelling of the discount function, $\tilde{f}_t$, would be:

$$\tilde{f}_t = \sum_{j=0}^{m} \tilde{a}_j \cdot g_j(t)$$

where g (t) : piecewise quadratic function.

t: maturity.

m: number of pieces of the function g(t).

$\tilde{a}$: coefficients of the discount function, expressed as a Dubois-Prade L-R fuzzy number, where $\tilde{a} = (a_C, a_R)_L$, and a_C and a_R are the centre and the radius, respectively. Coefficient $\tilde{a}$ is taking as a triangular fuzzy number.

For the discount function $\tilde{f}_t$ to be equal to the certain value of 1 for the maturity t=0 —that is to say, $\tilde{f}_0 = (1,0)_L$ the following conditions are imposed:

$$\tilde{a}_0 = (a_{0C}, a_{0R})_L = (1,0)_L$$

$$g_0(t) = 1\,;$$

$$g_j(0) = 0,\quad j = 1, 2, ..., m$$

Therefore, the discount function $\tilde{f}_t$ can be rewritten as:

$$\tilde{f}_t = (1,0)_L + \sum_{j=1}^{m} \left(a_{jC}, a_{jR}\right)_L \cdot g_j(t) \qquad [1]$$

To determine it, we must calculate the number of splines, m, obtained from $m=k^{½}$, where k is the number of references in the sample. The higher m is, the better the fit will be. However, it may also have a negative effect on the smoothness of the final curve.

We define the price of the r-th bond, $\tilde{P}^r$, as the sum of each i-th payment generated by bond r, C_i^r, brought up to date by the discount factor $\tilde{f}_t$. Thus:

$$\tilde{P}^r = \sum_{i=1}^{n_r} C_i^r \tilde{f}_t \qquad r = 1, \ldots, k \qquad [2]$$

where k: the number of bonds in the sample.

n_r: number of flows of the r-th bond.

By replacing [1] in [2], we obtain the price estimated for the r-th bond, $\hat{\tilde{P}}^r$:

$$\hat{\tilde{P}}^r = (\hat{\tilde{P}}_C{}^r, \hat{\tilde{P}}_R{}^r)_L = \sum_{i=1}^{n_r} C_i^r \left[(1,0)_L + \sum_{j=1}^{m} \left(a_{jC}, a_{jR}\right)_L g_j(t) \right] =$$

$$= \sum_{i=1}^{n_r} C_i^r (1,0)_L + \sum_{i=1}^{n_r} C_i^r \sum_{j=1}^{m} \left(a_{jC}, a_{jR}\right)_L g_j(t) \qquad [3]$$

where $\hat{\tilde{P}}_C{}^r$: centre of the price estimated for the r-th bond.

$\hat{\tilde{P}}_R{}^r$: radius of the price estimated for the r-th bond.

From expression [3], we obtain the following equality:

$$(\hat{\tilde{P}}_C{}^r, \hat{\tilde{P}}_R{}^r)_L - \sum_{i=1}^{n_r} C_i^r (1,0)_L = \sum_{j=1}^{m} \left(a_{jC}, a_{jR}\right)_L \sum_{i=1}^{n_r} C_i^r g_j(t)$$

in such a way that the dependent variable is the fuzzy number, $\left(\hat{Y}_C^r, \hat{Y}_R^r\right)_L$,where:

$$\hat{Y}_c^r = \hat{P}_c^r - \sum_{i=1}^{n_r} C_i^r$$

$$\hat{Y}_R^r = \hat{P}_R^r$$

And the independent variable is the certain number, X_j^r, which takes the form:

$$X_j^r = \sum_{i=1}^{n_r} C_i^r g_j(t)$$

So the model to be estimated is: $\left(\hat{Y}_C^r, \hat{Y}_R^r\right)_L = \sum_{j=1}^{m} \left(a_{jC}, a_{jR}\right)_L X_j^r$

To obtain the coefficients $\tilde{a} = (a_C, a_R)_L$, we shall use Tanaka's possibilistic regression model (1987) to solve the following linear programming problem:

$$\text{Min } z = \sum_{r=1}^{k} \sum_{j=1}^{m} a'_{jR} \sum_{i=1}^{n_r} C_i^r \left|g_j(t_i^r)\right| = \sum_{j=1}^{m} a'_{jR} \sum_{r=1}^{k} \sum_{i=1}^{n_r} C_i^r \left|g_j(t_i^r)\right|$$

Subjet to :

$$\sum_{j=1}^{m} a'_{jC} \sum_{i=1}^{n_r} C_i^r g_j(t) - \sum_{j=1}^{m} a'_{jR} \sum_{i=1}^{n_r} C_i^r \left|g_j(t)\right| \le Y'^r_C - Y'^r_R$$

$$\sum_{j=1}^{m} a'_{jR} \sum_{i=1}^{n_r} C_i^r g_j(t) + \sum_{j=1}^{m} a'_{jR} \sum_{i=1}^{n_r} C_i^r \left|g_j(t)\right| \ge Y'^r_C - Y'^r_R$$

$$\sum_{j=1}^{m} a'_{jC} (g_j(sP) - g_j((s+1)P)) - \sum_{j=1}^{m} a'_{jR} \left(\left|g_j(sP)\right| - \left|g_j((s+1)P)\right|\right) \ge 0, s = 1,...,u-1$$

$$\sum_{j=1}^{m} a'_{jC} (g_j(sP) - g_j((s+1)P)) + \sum_{j=1}^{m} a'_{jR} \left(\left|g_j(sP)\right| - \left|g_j((s+1)P)\right|\right) \ge 0, s = 1,...,u-1$$

$$\sum_{j=1}^{m} a'_{jC} (g_j(uP) - \sum_{j=1}^{m} \frac{a'_{jR}}{L^{-1}(\alpha^*)} \left|g_j(uP)\right| \ge -1$$

$$\sum_{j=1}^{m} a'_{jC}(g_j(P)) + \sum_{j=1}^{m} \frac{a'_{jR}}{L^{-1}(\alpha^*)} \left|g_j(P)\right| \le 0$$

$$a'_{jR} \ge 0 \qquad j = 1,2,...m$$

where, $a'_{jC} = a_{jC}$

$a'_{jR} = a_{jR}\,(L^{-1}(\alpha^*))$

P: periodicity of the bond flows

u: period of the last flow of the longest maturity bond

Finally, once the coefficients $\tilde{a} = (a_C, a_R)_L$ have been estimated, we obtain the discount function $\tilde{f}_t = (1,0)_L + \sum_{j=1}^{m} (a_{jC}, a_{jR})_L \cdot g_j(t)$, on the basis of which, taking into account that $f(t) = \left[1 + (i_{tC}, i_{tR})_L\right]^{-t}$, we balance:

$$\left[1 + (i_{tC}, i_{tR})_L\right]^{-t} = (1,0)_L + \sum_{j=1}^{m} (a_{jC}, a_{jR})_L \cdot g_j(t)$$

and we isolate the spot interest rate, $(i_{tC}, i_{tR})_L$:

$$(i_{tC}, i_{tR})_L = \left[1 + \sum_{j=1}^{m} (a_{jC}, a_{jR})_L \cdot g_j(t)\right]^{-\frac{1}{t}} - 1$$

4. ESTIMATING THE TERM STRUCTURE OF INTEREST RATES

4.1 DATA

In most developed economies, the Public Debt Market tends to be highly efficient. Therefore, because TSIR must be constructed on the basis of interest rates that are free of the risk of debtor insolvency, the Public Debt Market provides information that is more suitable for determining TSIR.

In this paper, we select a sample of securities issued in the Spanish Public Debt Market. One of the main features of this market is that it is homogeneous, since only three kinds of well defined securities are issued: Treasury Bills, and State Bonds and Securities. In general terms, these instruments are designated in euros, the capital is redeemed on maturity and, in the case of bonds and securities, are issued as fixed, annual coupons.

For estimation purposes, two different dates were chosen: 19 July 2007 and 8 July 2008. The former is the start of the initial stage of the crisis, while the latter is one day in the period that the crisis was at its height. For each of the dates, we collected a sample of 13 instruments, all with a similar maturity and flow structure so they would not affect the results.

4.2 ESTIMATION USING FUZZY NUMBERS

Since we have selected 13 bonds (that is to say, k=13), 4 splines are defined for the function g(t). Table 1 shows the $\tilde{a}_j$ coefficients estimated for the discount function with a level of presumption of $\alpha^* = 0$.

Table 1. Estimation coefficients $\tilde{a}_j$

	July 2007	July 2008
$(\tilde{a}_{1C},\tilde{a}_{1R})$	(-0,02774857 ; 0,01223623)	(-0,02995581; 0,00257917)
$(\tilde{a}_{2C},\tilde{a}_{2R})$	(-0,05549714; 0)	(-0,05991163 ; 0)
$(\tilde{a}_{3C},\tilde{a}_{3R})$	(-0,09498728; 0)	(-0,11260741; 0)
$(\tilde{a}_{4C},\tilde{a}_{4R})$	(0,18649143; 0)	(0,00601976; 0)

Table 2 shows the values of the centres and radii of the discount function $\tilde{f}_t$ presented above in [1].

Table 2. Centres and radii of the discount function $\tilde{f}_t$

	July 2007		July 2008	
t	f_{tC}	f_{tR}	f_{tC}	f_{tR}
1	0,96762667	0,01019686	0,96487359	0,00932836
2	0,92600381	0,01631498	0,91971107	0,01492537
3	0,87513143	0,01835435	0,86451242	0,01679104
4	0,8130526	0,01835435	0,79468788	0,01679104

Using the data in Table 2, we can find the spot interest rates $\tilde{i}_t$:

$$(i_{tC},i_{tR})_L = \left[1+\sum_{j=1}^{m}\left(a_{jC},a_{jR}\right)_L \cdot g_j(t)\right]^{-\frac{1}{t}} - 1$$

Finally, the TSIR graph can be seen in Figure 1, which shows the function centres.

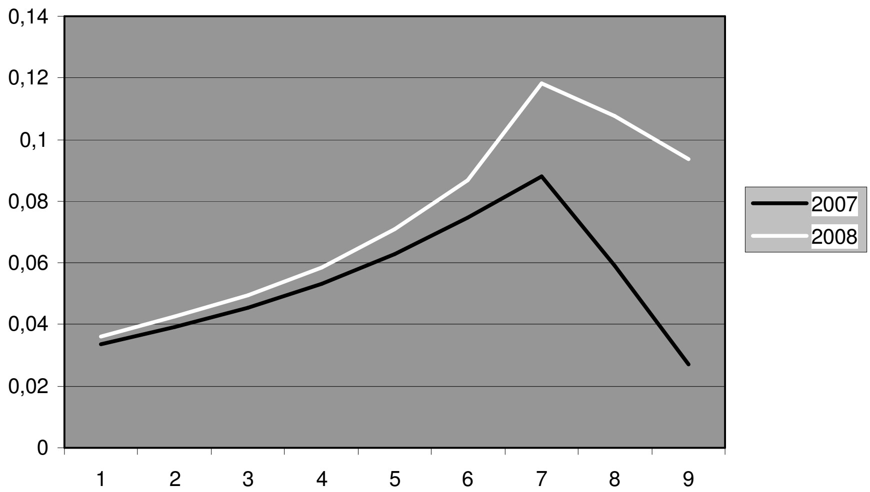

Figure 1. TSIR graph

4.3 RESULTS

The estimations made show that the TSIR behaves like a normal TSIR. As from year 7, there is a change in the slope of both curves. However, we do not consider this section because, first, the spline method is not stable for long periods[1] and, second, several authors believe that the TSIR loses predictive power in the long term[2].

The positive slope in the first years indicates that the economic agents expect the rates to increase in the future. In other words, the longer the term is, the more uncertain investors are that something may occur to affect their investments. The positive slope, then, is showing that economic agents require a premium to compensate for the risk of maintaining their investments over longer periods.

Our analysis shows that, despite the onset of the crisis, there are no differences in the expectations of investors because the TSIR behaves in the same way for the two dates.

Nevertheless, there is a margin between the two curves that can be explained by the increase in the liquidity premium demanded by the investors as a result of the liquidity crisis that the financial markets are experiencing. This lack of

[1] Deacon y Derry (1994)
[2] Mishkin (1991), Jardet (2004), Bedendo *et al*. (2007)

liquidity is reinforced with the data on the volumes negotiated on the dates selected. It can be seen that the negotiated total in cash buying and selling operations was 2.231,27 million euros in July 2007 and only 1.278,28 million euros in July 2008.

5. CONCLUSIONS

In this study, the TSIR has been estimated using McCulloch's quadratic spline method (1971) and fuzzy regressions on two significant dates in the genesis and development of the financial crisis.

The results of our estimation have two main points of interest. In the first place, it shows that the financial crisis has not led to economic agents changing their expectations of the evolution of interest rates, because the TSIR behaves in the same way before and during the crisis. In the second place, the curve shifts upwards, which we interpret as a demand by investors for a greater liquidity premium, because the financial crisis has been characterized by a worrying liquidity crisis.

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