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A permutation information theory tour through different interest rate maturities: the Libor case

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This paper analyses Libor interest rates for seven different maturities and referred to operations in British pounds, euros, Swiss francs and Japanese yen, during the period 2001–2015. The analysis is performed by means of two quantifiers derived from information theory: the permutation Shannon entropy and the permutation Fisher information measure. An anomalous behaviour in the Libor is detected in all currencies except euros during the years 2006–2012. The stochastic switch is more severe in one, two and three months maturities. Given the special mechanism of Libor setting, we conjecture that the behaviour could have been produced by the manipulation that was uncovered by financial authorities. We argue that our methodology is pertinent as a market overseeing instrument.

1. Introduction

Since the seminal work of Bachelier [1], prices in a competitive market have been modelled as a memoryless stochastic process. In fact, according to the efficient market hypothesis (EMH), prices fully reflect all available information [2]. This property was duly proved by Samuelson [3]. It is known that informational efficiency can vary over time. This may be due to several reasons. For example, Cajueiro & Tabak [4] studied the Chinese stock market and found that liquidity plays a role in explaining the evolution of the long-term memory; Bariviera [5] argued that informational efficiency in the Thai stock market is influenced by liquidity constraints, whereas Bariviera *et al.* [6] detailed the influence of the 2008 financial crisis on the memory endowment of the European fixed income market. However, changes in informational efficiency are essentially unpredictable, under the EMH framework.

The problem addressed in this paper stems from a newspaper. Mollenkamp & Whitehouse [7] published a disruptive article in the *Wall Street Journal*: they suggested that the Libor rate did not reflect what was expected, i.e. the cost of funding of prime banks. The British Bankers Association (BBA) defines Libor as '... the rate at which an individual Contributor Panel bank could borrow funds, were it to do so by asking for and then accepting inter-bank offers in reasonable market size, just prior to 11:00 [a.m.] London time'. Every London business day, each bank in the contributor panel (selected banks from BBA) makes a blind submission such that each banker does not know the quotes of the other bankers. A compiler, Thomson Reuters, then averages the second and third quartiles. This average is published and represents the Libor rate on a given day. In other words, *Libor is a trimmed average of the expected borrowing rates of leading banks.* Libor rates have been published for 10 currencies and 15 maturities. As it is defined, Libor is expected to be the best self-estimate of leading banks' borrowing cost at different maturities. Several publications in newspapers [8,9] casting doubts on Libor integrity triggered investigations by several surveillance authorities such as the US Department of Justice, the UK Financial Services Authority or the European Commission. All these offices found several traces of misconduct, which resulted in severe fines to leading banks.

There are a few papers dealing with this topic in academic journals. Most of them are focused, once the manipulation is known, on verifying or discarding its existence, by means of statistical tests. Taylor & Williams [10] documented the detachment of the Libor rate from other market rates such as overnight interest swap, effective Federal fund, certificate of deposits (CDs), credit default swaps and repo rates. Snider & Youle [11] studied individual quotes in the Libor bank panel and found that Libor quotes in the USA were not strongly related to other bank borrowing cost proxies. Abrantes-Metz *et al.* [12] analysed the distribution of the second digits of daily Libor rates between 1987 and 2008 and compared it with uniform and Benford's distributions. If we take into account the whole period, the null hypothesis that the empirical distribution follows either the uniform or Benford's distribution cannot be rejected. However, if we take into account only the period after the subprime crisis, the null hypothesis is rejected. This result calls into question the 'aseptic' setting of Libor. Monticini & Thornton [13] found evidence of Libor under-reporting after analysing the spread between one- and three-month Libor and the rate of CDs using the Bai & Perron [14] test for multiple structural breaks. For a historical overview of the Libor case from a regulator point of view, see the Federal Reserve Staff Report [15].

Recently, Bariviera *et al.* [16,17] presented preliminary results about three-month UK Libor manipulation. They performed a symbolic time-series (TS) analysis using the complexity entropy causality plane (CECP). Our approach goes deeper in two aspects. First, we study the behaviour of the Libor for several maturities and currencies. Second, we introduce a local information theory quantifier, which is able to detect tiny perturbations in the probability density function. Consequently, instead of analysing our results with global versus global quantifiers, as in the CECP, we study the TS by means of global versus local quantifiers, defined in the Shannon–Fisher plane.

Interest rates have broad and wide economic impact on our daily life. The lack of integrity of Libor as an information signal gives market participants a wrong proxy of borrowing costs, thus providing a bad rate for pricing financial products. Many mortgages as well as sovereign bonds have Libor-linked interest rates. Therefore, the wrong setting of the interest rates produces a cascade effect throughout the whole economy. Consequently, Libor reliability is crucial to private and public borrowers around the world.

Our approach is somewhat different from the previous literature. The aim of this paper is to propose a quantitative technique based on information theory quantifiers, in order to detect changes in the stochastic/chaotic underlying dynamics of the Libor TS. We claim that our methodology is able to capture changes in the process dynamics on an ongoing basis. In other words, our method allows periodic monitoring of interest TS. This market audit, done at regular intervals, can detect unusual mutation in the statistical properties of a TS. It is clear that not only manipulation can shift signal features. Other external forces, such as increasing noise or special economic situations, influence signal observation and measurement. Therefore, results should be read with care. A change in the stochastic dynamics should not be read exclusively as produced by manipulation. It could be due to spurious contamination of the signal or liquidity constraints, etc. However, our method acts as an early warning mechanism to detect some kind of 'market distress'. This study is relevant not only for researchers, but also for surveillance authorities who need an efficient market watch device in order to detect strange movements in key economic variables such as the Libor.

This paper is structured as follows. Section 2 describes the methodology. Section 3 details the data used in this paper. Section 4 discusses our empirical findings. Finally, §5 draws the main conclusions of our research.

2. Methodology

Financial markets are complex, dynamic systems in which unobservable hidden structures govern their behaviour. Usually, we can only observe an output, e.g. an equilibrium price, an interest rate fixing, etc. As a consequence, the researcher should study the behaviour of that output in order to infer the characteristics of the underlying dynamical phenomenon.

TS should be carefully analysed in order to extract relevant information for simulation and forecasting purposes. Information theory-derived quantifiers can be considered good candidates for this task because they are able to characterize some properties of the probability distribution associated with the observable or measurable quantity.

If the Libor rates were somewhat manipulated (as suggested in [7]), some change in the stochastic behaviour should appear. To be more precise, according to the EMH, interest rate TS should behave approximately as a standard Brownian motion. Manipulation is, by definition, the introduction of a strange deterministic device into the TS. According to Wold [18], a TS can be split into two components, one purely random and other deterministic. If manipulation is successful, the deterministic-induced behaviour should offset the random behaviour. This process results in a reduction of the natural stochastic character of the interest rates. We argue that the selected information theory quantifiers are able to detect such reduction and its temporal duration.

We use two specific quantifiers: permutation Shannon entropy and permutation Fisher information measure. These quantifiers are evaluated in pairs, displaying them in two-dimensional planar representation. This causal Shannon–Fisher plane gives an insight into global versus local perturbations on the behaviour of a TS of the underlying dynamics of a physical process under analysis.

(a) The Shannon entropy

The Shannon entropy is usually regarded as a natural measure of the quantity of information in a physical process. Given a continuous probability distribution function (PDF) $f(x)$ with $x \in \Delta \subset \mathbb{R}$ and $\int_\Delta f(x)\,\mathrm{d}x = 1$, its associated *Shannon entropy* S [19] is

$$S[f] = -\int_\Delta f \ln(f)\,\mathrm{d}x, \tag{2.1}$$

a measure of 'global character' that is not too sensitive to strong changes in the distribution taking place on a small-sized region. Let now $P = \{p_i;\ i = 1, \ldots, N\}$ be a discrete probability distribution, with N the number of possible states of the system under study. In the discrete case, we define a 'normalized' Shannon entropy, $0 \leq \mathcal{H} \leq 1$, as

$$\mathcal{H}[P] = \frac{S[P]}{S_{\max}} = \frac{\{-\sum_{i=1}^{N} p_i \ln(p_i)\}}{S_{\max}}, \tag{2.2}$$

where the denominator $S_{\max} = S[P_e] = \ln N$ is that attained by a uniform probability distribution $P_e = \{p_i = 1/N, \forall i = 1, \ldots, N\}$.

(b) The Fisher information measure

The *Fisher information measure* (FIM) $\mathcal{F}$ [20,21] constitutes a measure of the gradient content of the distribution $f(x)$, thus being quite sensitive even to tiny localized perturbations. It reads

$$\mathcal{F}[f] = \int_{\Delta} \frac{1}{f(x)} \left[\frac{\mathrm{d}f(x)}{\mathrm{d}x}\right]^2 \mathrm{d}x = 4 \int_{\Delta} \left[\frac{\mathrm{d}\psi(x)}{\mathrm{d}x}\right]^2. \tag{2.3}$$

FIM can be variously interpreted as a measure of the ability to estimate a parameter, as the amount of information that can be extracted from a set of measurements, and also as a measure of the state of disorder of a system or phenomenon [21]. In the previous definition of FIM (equation (2.3)), the division by $f(x)$ is not convenient if $f(x) \to 0$ at certain x-values. We avoid this if we work with real probability amplitudes $f(x) = \psi^2(x)$ [20,21], which is a simpler form (no divisors) and shows that $\mathcal{F}$ simply measures the gradient content in $\psi(x)$. The gradient operator significantly influences the contribution of minute local f-variations to FIM's value. Accordingly, this quantifier is called a 'local' one [21].

Let $P = \{p_i; i = 1, \ldots, N\}$ be a discrete probability distribution, with N the number of possible states of the system under study. The concomitant problem of information loss owing to discretization has been thoroughly studied and, in particular, it entails the loss of FIM's shift-invariance, which is of no importance for our present purposes [22,23]. For the FIM, we take the expression in terms of real probability amplitudes as starting point, then a discrete normalized FIM, $0 \leq \mathcal{F} \leq 1$, convenient for our present purposes, is given by

$$\mathcal{F}[P] = F_0 \sum_{i=1}^{N-1} [(p_{i+1})^{1/2} - (p_i)^{1/2}]^2. \tag{2.4}$$

It has been extensively discussed that this discretization is the best behaved in a discrete environment [24]. Here, the normalization constant F_0 reads

$$F_0 = \begin{cases} 1, & \text{if } p_{i^*} = 1 \text{ for } i^* = 1 \text{ or } i^* = N \text{ and } p_i = 0\ \forall i \neq i^* \\ \dfrac{1}{2}, & \text{otherwise.} \end{cases} \tag{2.5}$$

If our system lies in a very ordered state, which occurs when almost all the p_i-values are zeros except for a particular state $k \neq i$ with $p_k \cong 1$, we have a normalized Shannon entropy $\mathcal{H} \sim 0$ and a normalized FIM $\mathcal{F} \sim 1$. On the other hand, when the system under study is represented by a very disordered state, that is when all the p_i-values oscillate around the same value we obtain $\mathcal{H} \sim 1$, whereas $\mathcal{F} \sim 0$. One can state that the general FIM behaviour of the present discrete version (equation (2.4)) is opposite to that of the Shannon entropy, except for periodic motions [22,23]. The local sensitivity of FIM for discrete PDFs is reflected in the fact that the specific 'i-ordering' of the discrete values p_i must be seriously taken into account in evaluating the sum in equation (2.4). This point was extensively discussed by Rosso and co-workers [22,23] in previous works. The summands can be regarded as a kind of 'distance' between two contiguous probabilities. Thus, a different ordering of the pertinent summands would lead to a different FIM value, hereby its local nature. In this work, we follow the lexicographic order described by

Lehmer (http://www.keithschwarz.com/interesting/code/factoradicpermutation/FactoradicPermutation.hh.html) in the generation of Bandt–Pompe PDF (see §2c). Given the local character of FIM, when combined with a global quantifier as the Shannon entropy, conforms the Shannon–Fisher plane, $\mathcal{H} \times \mathcal{F}$, introduced by Vignat & Bercher [25]. These authors showed that this plane is able to characterize the non-stationary behaviour of a complex signal.

(c) The Bandt–Pompe method for probability distribution function evaluation

TS analysis, i.e. temporal measurements of variables, is a very important area of economic science. In particular, it is very important to extract information in order to unveil the underlying dynamics of irregular and apparently unpredictable behaviour in a given market. The starting point of TS analysis is to determine the most appropriate probability density function associated with the TS. Several methods compete for its proper estimation. For example, Rosso *et al.* [26] propose the frequency of occurrence; De Micco *et al.* [27] propose amplitude-based procedures; Mischaikow *et al.* [28] prefer binary symbolic dynamics; Powell *et al.* [29] recommend Fourier analysis; and Rosso *et al.* [30] introduce wavelets transformation, among others. The suitability of each of the proposed methodologies depends on the peculiarity of data, such as stationarity, length of the series, the variation of the parameters, the level of noise contamination, etc. In all these cases, global aspects of the dynamics can be somehow captured, but the different approaches are not equivalent in their ability to discern all relevant physical details. Bandt & Pompe [31] introduced a simple and robust symbolic method that takes into account time order of the TS.

The pertinent symbolic data are (i) created by ranking the values of the series and (ii) defined by reordering the embedded data in ascending order, which is tantamount to a phase space reconstruction with embedding dimension (pattern length) D and time lag τ. In this way, it is possible to quantify the diversity of the ordering symbols (patterns) derived from a scalar TS. Note that the appropriate symbol sequence arises naturally from the TS, and no model-based assumptions are needed. In fact, the necessary 'partitions' are devised by comparing the order of neighbouring relative values rather than by apportioning amplitudes according to different levels. This technique, as opposed to most of those in current practice, takes into account the temporal structure of the TS generated by the physical process under study. This feature allows us to uncover important details concerning the ordinal structure of the TS [23,32,33] and can also yield information about temporal correlation [34,35].

It is clear that this type of analysis of a TS entails losing some details of the original series' amplitude information. Nevertheless, by just referring to the series' intrinsic structure, a meaningful difficulty reduction has indeed been achieved by Bandt and Pompe with regard to the description of complex systems. The symbolic representation of TS by recourse to a comparison of consecutive ($\tau = 1$) or non-consecutive ($\tau > 1$) values allows for an accurate empirical reconstruction of the underlying phase space, even in the presence of weak (observational and dynamic) noise [31]. Furthermore, the ordinal patterns associated with the PDF are invariant with respect to nonlinear monotonic transformations. Accordingly, nonlinear drifts or scaling artificially introduced by a measurement device will not modify the estimation of quantifiers, a nice property if one deals with experimental data [36]. These advantages make the Bandt and Pompe methodology more convenient than conventional methods based on range partitioning (i.e. PDF based on histograms).

To use the Bandt & Pompe [31] methodology for evaluating the PDF, P, associated with the TS (dynamical system) under study, one starts by considering partitions of the pertinent D-dimensional space that will hopefully 'reveal' relevant details of the ordinal structure of a given one-dimensional TS $\mathcal{X}(t) = \{x_t; t = 1, \ldots, M\}$ with embedding dimension $D > 1$ ($D \in \mathbb{N}$) and embedding time delay τ ($\tau \in \mathbb{N}$). We are interested in 'ordinal patterns' of order (length) D generated by

$$(s) \mapsto (x_{s-(D-1)\tau}, x_{s-(D-2)\tau}, \ldots, x_{s-\tau}, x_s), \tag{2.6}$$

which assigns to each time s the D-dimensional vector of values at times $s, s-\tau, \ldots, s-(D-1)\tau$. Clearly, the greater the D-value, the more information on the past is incorporated into our vectors. By 'ordinal pattern' related to the time (s), we mean the permutation $\pi = (r_0, r_1, \ldots, r_{D-1})$ of $[0, 1, \ldots, D-1]$ defined by

$$x_{s-r_{D-1}\tau} \le x_{s-r_{D-2}\tau} \le \cdots \le x_{s-r_1\tau} \le x_{s-r_0\tau}. \tag{2.7}$$

In order to get a unique result, we set $r_i < r_{i-1}$ if $x_{s-r_i} = x_{s-r_{i-1}}$. This is justified if the values of x_t have a continuous distribution, so that equal values are very unusual.

For all the $D!$ possible orderings (permutations) π_i when embedding dimension is D, their associated relative frequencies can be naturally computed according to the number of times this particular order sequence is found in the TS, divided by the total number of sequences:

$$p(\pi_i) = \frac{\sharp\{s|s \le N-(D-1)\tau; (s)\ \texttt{has type}\ \pi_i\}}{N-(D-1)\tau}, \tag{2.8}$$

where the symbol $\sharp$ stands for 'number'. Thus, an ordinal pattern probability distribution $P = \{p(\pi_i), i = 1, \ldots, D!\}$ is obtained from the TS.

Consequently, it is possible to quantify the diversity of the ordering symbols (patterns of length D) derived from a scalar TS, by evaluating the so-called permutation entropy (Shannon entropy) and permutation FIM. Of course, the embedding dimension D plays an important role in the evaluation of the appropriate probability distribution, because D determines the number of accessible states $D!$ and also conditions the minimum acceptable length $M \gg D!$ of the TS that one needs in order to work with reliable statistics [32].

Regarding the selection of the parameters, Bandt and Pompe suggested working with $4 \le D \le 6$ and specifically considered an embedding delay $\tau = 1$ in their cornerstone paper [31]. Nevertheless, it is clear that other values of τ could provide additional information. It has been recently shown that this parameter is strongly related, if it is relevant, to the intrinsic TS of the system under analysis [37–39].

Additional advantages of the method reside in (i) its simplicity (we need few parameters: the pattern length/embedding dimension D and the embedding delay τ) and (ii) the extremely fast nature of the pertinent calculation process [40]. The Bandt and Pompe methodology can be applied not only to TS representative of low dimensional dynamical systems, but also to any type of TS (regular, chaotic, noisy or reality based). In fact, the existence of an attractor in the D-dimensional phase space is not assumed. The only condition for the applicability of the Bandt and Pompe method is a very weak stationary assumption: for $k \le D$, the probability for $x_t < x_{t+k}$ should not depend on t. For a review of Bandt and Pompe's methodology and its applications to physics, biomedical and econophysics signals, see Zanin *et al.* [41]. Rosso *et al.* [32] show that the above-mentioned quantifiers produce better descriptions of the process associated dynamics when the PDF is computed using the Bandt and Pompe rather than using the usual histogram methodology.

The Bandt and Pompe proposal for associating probability distributions to TS (of an underlying symbolic nature) constitutes a significant advance in the study of nonlinear dynamical systems [31]. The method provides univocal prescription for ordinary, global entropic quantifiers of the Shannon kind. However, as was shown by Rosso and co-workers [22,23], ambiguities arise in applying the Bandt and Pompe technique with reference to the permutation of ordinal patterns. This happens if one wishes to employ the Bandt and Pompe probability density to construct local entropic quantifiers, like the FIM, which would characterize TS generated by nonlinear dynamical systems.

The local sensitivity of the FIM for discrete PDFs is reflected in the fact that the specific 'i-ordering' of the discrete values p_i must be seriously taken into account in evaluating the sum in equation (2.4). The pertinent numerator can be regarded as a kind of 'distance' between two contiguous probabilities. Thus, a different ordering of the pertinent summands would lead to a different Fisher information value. In fact, if we have a discrete PDF given by $P = \{p_i, i = 1, \ldots, N\}$, we will have $N!$ possibilities for the i-ordering.

The question is, which is the arrangement that one could regard as the 'proper' ordering? The answer is straightforward in some cases, the histogram-based PDF constituting a conspicuous example. For such a procedure, one first divides the interval $[a, b]$ (with a and b the minimum and maximum amplitude values in the TS) into a finite number of non-overlapping subintervals (bins). Thus, the division procedure of the interval $[a, b]$ provides the natural order sequence for the evaluation of the PDF gradient involved in the FIM. In this paper, we chose for the Bandt–Pompe PDF the lexicographic ordering given by the algorithm of Lehmer (http://www.keithschwarz.com/interesting/code/factoradicpermutation/FactoradicPermutation.hh.html), among other possibilities, owing to it providing a better distinction of different dynamics in the Shannon–Fisher plane, $\mathcal{H} \times \mathcal{F}$ (see [22,23]).

3. Data

We analyse the Libor rates in British pound (GBP), euro (EUR), Swiss franc (CHF) and Japanese yen (JPY), for the following seven maturities: overnight (O/N), one week (1W), one month (1M), two months (2M), three months (3M), six months (6M) and 12 months (12M). The data coverage is from 2 January 2001 until 24 March 2015, for a total of 3711 data points. All data were retrieved from DataStream.

4. Results

We analyse each TS using the methodology described in §2. The PDF was computed following the Bandt and Pompe recipe, because it is the single method to introduce time-causality in PDF building. Although we have performed our analysis using embedding dimensions $D = 3, 4$ and 5 with time lag $\tau = 1$, we present the results only for $D = 4$, because it exhibits the best clarity for our explanations. The other embedding dimension results are similar.

We would like to know if the underlying generating process changes during the observation period. In order to evaluate such change, we construct sliding windows. The sliding windows work as follows: we consider the first $N = 300$ data points and evaluate the two quantifiers, $\mathcal{H}$ and $\mathcal{F}$. Then, we move $\delta = 20$ data points forward and compute the quantifiers of the new window with the following $N = 300$ data points. We continue this process until the end of the TS. After doing so, we obtain 170 estimation windows. Each window spans approximately one year, and the window step is approximately one month.

The initial and final dates of each estimation window are displayed in the electronic supplementary material, table S1. We also present in the electronic supplementary material the graphs corresponding to $\mathcal{H} \times \mathcal{F}$-plane for the Libor rates in GBP (electronic supplementary material, figure S1), EUR (electronic supplementary material, figure S2), CHF (electronic supplementary material, figure S3), JPY (electronic supplementary material, figure S4) for the seven considered maturities: O/N, 1W, 1M, 2M, 3M, 6M and 12M, when they are evaluated on the 170 estimation windows, respectively, given in this way a representation of the time evolution of them. From these figures, it can be observed that 1M, 2M and 3M maturities have similar shape. It can also be noted that 6M and 12M are very similar to each other and their localization points are more concentrated. We can summarize these results as: two distinct dynamics are detected. On the one side, 1M, 2M and 3M maturities scatter throughout the plane. On the other side, O/N, 1W, 6M and 12M graphs are less disperse and their points are more concentrated on the low right corner. In the financial economics vocabulary, we can say that the latter maturities are more informationally efficient: their TS are closer to a random walk. The middle maturities (1M, 2M, 3M) occupy places in the $\mathcal{H} \times \mathcal{F}$-plane that are consistent with fractional Brownian motion or with chaotic dynamics.

This is the first evidence that we are dealing with a single interest rate but with distinct generating processes. In order to highlight the unequal behaviour between the two groups of maturities, we count the number of points of the $\mathcal{H} \times \mathcal{F}$-plane that are closer to the low right corner and the number of points that are away from that corner. We divide the points into two

Table 1. Number of points in the inefficient ($\mathcal{H} < 0.75$, $\mathcal{F} > 0.3$) and efficient ($\mathcal{H} > 0.75$, $\mathcal{F} < 0.3$) regions for different maturities of Libor GBP. The information quantifiers were computed using Bandt–Pompe PDF with $D = 4$ and $\tau = 1$.

points within efficiency bounds	points outside efficiency bounds	percentage of efficient windows	series
107	63	*63******	GBP Libor O/N
103	67	*61******	GBP Libor 1W
67	103	39	GBP Libor 1M
56	114	33	GBP Libor 2M
58	112	34	GBP Libor 3M
93	77	*55*	GBP Libor 6M
106	64	*62******	GBP Libor 12M

***Significant at the 1% level.

Table 2. Number of points in the inefficient ($\mathcal{H} < 0.75$, $\mathcal{F} > 0.3$) and efficient ($\mathcal{H} > 0.75$, $\mathcal{F} < 0.3$) regions for different maturities of Libor EUR. The information quantifiers were computed using Bandt–Pompe PDF with $D = 4$ and $\tau = 1$.

points within efficiency bounds	points outside efficiency bounds	percentage of efficient windows	series
145	25	*85******	EUR Libor O/N
113	57	*66******	EUR Libor 1W
87	83	51	EUR Libor 1M
68	102	40	EUR Libor 2M
84	86	49	EUR Libor 3M
90	80	*53*	EUR Libor 6M
121	49	*71******	EUR Libor 12M

***Significant at the 1% level.

Table 3. Number of points in the inefficient ($\mathcal{H} < 0.75$, $\mathcal{F} > 0.3$) and efficient ($\mathcal{H} > 0.75$, $\mathcal{F} < 0.3$) regions for different maturities of Libor CHF. The information quantifiers were computed using Bandt–Pompe PDF with $D = 4$ and $\tau = 1$.

points within efficiency bounds	points outside efficiency bounds	percentage of efficient windows	series
107	63	*63******	CHF Libor O/N
95	75	*56**	CHF Libor 1W
59	111	35	CHF Libor 1M
66	104	39	CHF Libor 2M
58	112	34	CHF Libor 3M
78	92	*46*	CHF Libor 6M
102	68	*60**	CHF Libor 12M

Significant at *the 10% level and ***the 1% level.

regions. One, the most informational efficient region, comprises the points where $\mathcal{H} > 0.75$ and $\mathcal{F} < 0.3$. The other is the complement of the $\mathcal{H} \times \mathcal{F}$-plane. We display the results of the point counting in tables 1–4. Tables 1–4 confirm the ocular inspection of the $\mathcal{H} \times \mathcal{F}$-plane (see electronic supplementary material, figures S1–S4). Whereas for 1M, 2M and 3M maturities, only between 33% and 39% of the points are in the efficient area, the percentages for the other maturities are in the range 55–63%. We recall that each point reflects the estimation of the quantifiers at a given moving window.

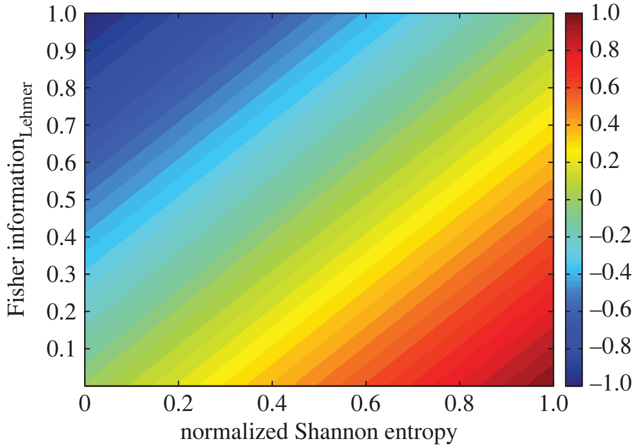

Figure 1. Colour map that reflects the different degrees of efficiency in the Shannon–Fisher plane according to equation (4.1). (Online version in colour.)

Table 4. Number of points in the inefficient ($\mathcal{H} < 0.75$, $\mathcal{F} > 0.3$) and efficient ($\mathcal{H} > 0.75$, $\mathcal{F} < 0.3$) regions for different maturities of Libor JPY. The information quantifiers were computed using Bandt–Pompe PDF with $D = 4$ and $\tau = 1$.

points within efficiency bounds	points outside efficiency bounds	percentage of efficient windows	series
49	121	29	JPY Libor O/N
50	120	29	JPY Libor 1W
59	111	*35******	JPY Libor 1M
64	106	*38******	JPY Libor 2M
64	106	*38******	JPY Libor 3M
82	88	*48*	JPY Libor 6M
68	102	*40******	JPY Libor 12M

***Significant at the 1% level.

We would like to enquire if the aforementioned characteristics changed in an ordered way through time. Taking into account that we are studying the changes across time in the degree of informational efficiency of financial TS, we propose the following efficiency index, $\mathcal{E}$:

$$\mathcal{E}[P] = \mathcal{H}[P] - \mathcal{F}[P]. \tag{4.1}$$

Given that both quantifiers, $\mathcal{H}$ and $\mathcal{F}$, are bounded between 0 and 1, and that their behaviour is expected opposite, the maximum value of our efficiency index is $\mathcal{E}[P] = 1$ when $\mathcal{H}[P] = 1$ and $\mathcal{F}[P] = 0$, and $\mathcal{E}[P] = -1$ when $\mathcal{H}[P] = 0$ and $\mathcal{F}[P] = 1$. The most informational efficient behaviour (i.e. the most random behaviour) is when Shannon entropy is maximized and Fisher information minimized. Consequently, our efficiency index $\mathcal{E}$ lies in the $[-1, 1]$ interval. Accordingly, our efficiency index can be represented in a colour map as in figure 1.

We aim to assess the evolution of the efficiency $\mathcal{E}$ through time. In order to do so, we display the result for each currency in a colour map where the x-axis is the temporal dimension (i.e. the estimation windows) and the y-axes are the different currencies. The colours of the map reflect a degree of efficiency according to the colour scale on the right of each graph and coincides with the colour scale of figure 1.

The map corresponding to GBP (figure 2*a*) clearly reflects two features. First that O/N, 1W, 6M and 12M maturities have been behaving better than the middle maturities. Even more, 6M and 12M have been globally the most efficient during all the period. On the contrary, 1M, 2M, 3M were informationally efficient until window 100 (5 August 2008–31 August 2009) and suffered

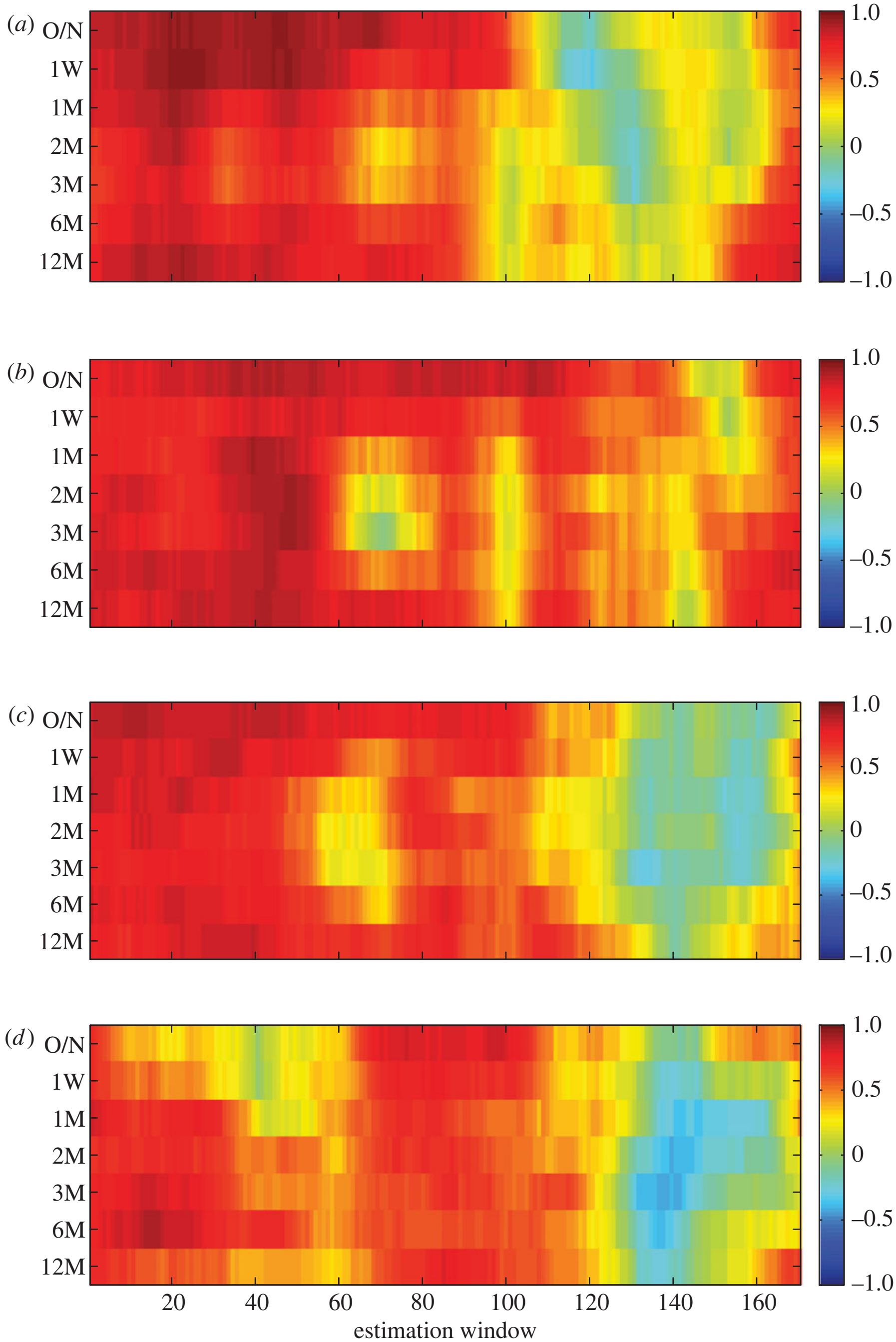

Figure 2. Colour map of the evolution of the degree of efficiency of different maturities and currencies of Libor. (*a*) GBP, (*b*) EUR, (*c*) CHF and (*d*) JPY. Efficiency is computed according to equation (4.1). The information quantifiers were computing using Bandt–Pompe PDF with $D = 4$ and $\tau = 1$. (Online version in colour.)

a sudden shift in stochastic behaviour, that returns to its previous path around window 160 (12 March 2013–5 May 2014). Additionally, we can observe a yellow area around windows 65–80, that roughly correspond to years 2006 and 2007. The behaviour detected by this colour map is consistent with the alleged manipulation of Libor rates reported in the press [7]. According to our results, Libor rates suffered from some kind of inefficient behaviour (coherent with manipulation) during the period 2006–2012. In 2013, the levels of informational efficiency recovered to levels similar to periods previous to the financial crisis.

The EUR market exhibits a similar pattern with respect to the two groups of maturities. However, the loss of efficiency is less severe. This can be observed in figure 2*b*, where there is a prevalence of orange and red colours.

The CHF market behaviour (figure 2*c*) is more similar to the GBP market (figure 2*a*). The region that comprises windows 120 until 160 reflects an eroded informational efficiency, that was slowly recovered at the final part of the observation period. Additionally, there is a yellow spot around windows 60–70 exclusively in 1M, 2M and 3M maturities.

The JPY market (figure 2*d*) was also affected in the degree of efficiency. In general, we can observe in the colour map that this is a less informational efficient market: light orange, yellow and light blue are dominant. There is also a severe erosion of the informational efficiency between windows 120 and 160, recovered at the end of the observation period mainly in O/N and 12M maturities.

5. Conclusion

The first finding that we can extract from our data analysis is that the Libor for all currencies and for maturities 1M, 2M and 3M is governed by a different process from the other maturities. The mentioned maturities usually reflect a lower level of informational efficiency than the other maturities. The second important finding is that during the years 2007–2012 (windows 120–160) there was a significant reduction in the informational efficiency of all markets, except the EUR market. This behaviour seems to be contemporary to the uncovered manipulation announced in the newspapers [7]. We would like to highlight that our method is not intended to find manipulation, but rather to uncover changes in the hidden stochastic structure of data. However, it is able to clearly discriminate areas of more deterministic behaviour in the TS, which could be consistent with the alleged interest rate rigging.

Data accessibility. Data were downloaded from DataStream from the university library of A.F.B.

Authors' contributions. A.F.B. and O.A.R. designed the study, collected and analysed the data; A.F.B., M.B.G., L.B.M. and O.A.R. discussed the results and contributed to the text of the manuscript. All authors gave final approval for publication.

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