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Abstract	<p>We propose to allocate the cost of a <i>minimum cost spanning tree</i> by defining a <i>claims problem</i> and using claims rules, then providing easy and intuitive ways to distribute this cost. Depending on the <i>starting point</i> that we consider, we define two models. On the one hand, the <i>benefit-sharing</i> model considers individuals' costs to the source as the starting point, and then the benefit of building the efficient tree is shared by the agents. On the other hand, the <i>costs-sharing</i> model starts from the individuals' minimum connection costs (the cheapest connection they can use), and the additional cost, if any, is then allocated. As we prove, both approaches provide the same family of allocations for every minimum cost spanning tree problem. These models can be understood as a central planner who decides the best way to connect the agents (the efficient tree) and also establishes the amount each agent has to pay. In so doing, the central planner takes into account the maximum and minimum amount they should pay and some equity criteria given by a particular (claims) rule. We analyze some properties of this family of cost allocations, specially focusing in coalitional stability (<i>core selection</i>), a central concern in the literature on cost allocation.</p>
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Keywords (separated by '-')	Minimum cost spanning tree problem - Claims problem - Cost sharing rules - Core selection
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Footnote Information	
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2 **A claims problem approach to the cost allocation**
3 **of a minimum cost spanning tree**

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7 **Abstract**

8 We propose to allocate the cost of a *minimum cost spanning tree* by defining a
9 *claims problem* and using claims rules, then providing easy and intuitive ways to
10 distribute this cost. Depending on the *starting point* that we consider, we define two
11 models. On the one hand, the *benefit-sharing* model considers individuals' costs to
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19 to pay. In so doing, the central planner takes into account the maximum and mini-
20 mum amount they should pay and some equity criteria given by a particular (claims)
21 rule. We analyze some properties of this family of cost allocations, specially focus-
22 ing in coalitional stability (*core selection*), a central concern in the literature on cost
23 allocation.

24 **Keywords** Minimum cost spanning tree problem · Claims problem · Cost sharing
25 rules · Core selection

26 **1 Introduction**

27 Consider a group of individuals who want to be connected to a water supply, or a
28 telephone or cable TV network. These individuals are located at different places,
29 and they have some (different) fixed costs of linking with any other individual or

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30 linking to the source. The purpose of the group is to be connected to the source
31 at the cheapest possible way (the *minimum cost spanning tree*). The allocation of
32 this cost among the individuals in the network, once the optimal spanning tree is
33 obtained, is an issue deeply studied in the literature, where different solutions have
34 been proposed: *Bird rule* (Bird 1976), *Kar* (Kar 2002), *Folk* (Feltkamp et al. 1994;
35 Bergantiños and Vidal-Puga 2007), *Cycle-complete* (Trudeau 2012), or the *Serial*
36 *cost sharing rule* (Moulin and Shenker 1992).

37 The present paper aims to define new methods of sharing the cost of the optimal
38 network by associating a *claims problem* to each minimum cost spanning tree situa-
39 tion and then using claims rules to allocate the total cost.

40 Claims problems are characterized by an endowment (to be distributed among the
41 agents) and a claim from each agent (the maximum amount to be allocated to this
42 agent). We propose two different approaches: the *benefit-sharing* and *costs-sharing*
43 models. In the first model the endowment is the benefit obtained from cooperation
44 when the minimum cost spanning tree is built and agents' claims are the difference
45 between their cheapest cost of connecting to the source and their cheapest connec-
46 tion cost. The alternative model establishes that individuals initially pay the cost of
47 their cheapest connection. Then, the endowment is the additional cost that must be
48 satisfied to cover the cost of the efficient tree, being the claims defined as in the pre-
49 vious model. Although both models provide different points of view, we will show
50 that no matter which view you choose, since both approaches provide the same fam-
51 ily of allocations for sharing the minimum cost of the network.

52 Even though both *mcs*t and *claims* problems involve a population of n agents,
53 their dimensionalities are very different. In a minimum cost spanning tree problem,
54 there is a source ω , and the problem is defined by the costs for connecting every
55 individual to the source; thus a minimum cost spanning tree problem is determined
56 by $(n + 1)n/2$ numbers. In a claims problem, there is an endowment and a claim for
57 each agent; thus a claims problem is determined by $n + 1$ numbers. Therefore, trans-
58 lating a minimum cost spanning tree problem into a claims problem involves some
59 "loss of information" and there are many ways to proceed.

60 On the other hand, this translation benefits from the simplicity and tradition of
61 claims rules (equal gains, equal losses, proportional gains/losses, etc.), that might
62 be found in the rich literature which originated with the seminal paper by (O'Neill
63 1982).

64 In real-world situations, when there is a conflict of interest in carrying out a joint
65 project, the simplicity of the solution is important for the agents to reach an agree-
66 ment. In this sense, our proposal has the appeal of an easy and intuitivemechanism
67 to convince the agents involved in the joint project about the equity of the solution.

68 Our proposal provides a bridge between the literature on claims problems and
69 that on sharing the cost in network problems. As far as we know, only Driessen
70 (1994) links both problems, although he analyzes the other way: transforms a claims
71 problem in a minimum cost spanning tree problem.

72 The paper is organized as follows. In the next section we present both the mini-
73 mum cost spanning tree problem and the claims problem. Section 3 introduces the
74 two mentioned approaches to associate a claims problem with a minimum cost
75 spanning tree situation and we prove that both models provide the same family of

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76 allocations. In Sect. 4 we discuss some properties of the allocations provided by our
 77 model. Section 5 analyzes the coalitional stability of the proposed allocations. Some
 78 final comments in Sect. 6 conclude the paper.

79 2 Preliminaries

80 2.1 Minimum cost spanning tree problem

81 A minimum cost spanning tree (hereafter *mcst*) problem involves a finite set of
 82 agents, $N = \{1, 2, \dots, n\}$, who need to be connected to a source ω . We denote by N_ω
 83 the set of agents and the source and the elements in N_ω are called nodes. There is an
 84 undirected complete graph connecting the nodes in N_ω . Any pair of nodes, $i, j \in N_\omega$,
 85 $i \neq j$, are connected by an edge $e_{ij} = (i, j)$ and $c_{ij} \in \mathbb{R}_+$ represents the cost of direct
 86 link, the arc e_{ij} , between any pair of nodes $i, j \in N_\omega$. We denote by $\mathbf{C} = [c_{ij}]$ the sym-
 87 metric cost matrix, where $c_{ii} = 0$, for all $i \in N_\omega$. The *mcst* problem is represented
 88 by the pair (N_ω, \mathbf{C}) , and the goal is to connect all the agents to the source (directly
 89 or through other agents) in the cheapest possible way. The solution to this problem,
 90 widely studied, is obtained by means of a spanning tree.

91 A network over N_ω is any subset of $N_\omega \times N_\omega$. A spanning tree over N_ω is a net-
 92 work p with no cycles that connects all elements of N_ω . We denote by $\mathcal{P}(N_\omega)$ the set
 93 of all spanning trees over N_ω . We can identify any spanning tree with a predecessor
 94 map $p : N \rightarrow N_\omega$ so that $j = p(i)$ is the agent (or the source) to whom i connects in
 95 her way towards the source. This map p defines the edges $e_i^p = (i, p(i))$ in the tree. In
 96 a spanning tree each agent is connected to the source ω , either directly, or indirectly
 97 through other agents. Moreover, given a spanning tree p , there is a unique path from
 98 any agent i to the source given by the arcs $(i, p(i)), (p(i), p^2(i)), \dots, (p^{t-1}(i), p^t(i) = \omega)$,
 99 for some $t \in \mathbb{N}$. The cost of building the spanning tree p is the sum of the cost of the
 100 arcs in this tree:

$$101 \quad C_p = \sum_{i=1}^n c_{ip(i)}.$$

102
 103 Prim (1957) provides an algorithm which solves the problem of connecting all the
 104 agents to the source such that the total cost of the network is minimum.¹ This opti-
 105 mal tree may not be unique. Denote by m a spanning tree with minimum cost and by
 106 $C_{m(N_\omega, \mathbf{C})}$ its cost (in what follows, when there is no confusion, we simply write C_m).
 107 That is, for all spanning tree $p \in \mathcal{P}(N_\omega)$

¹ This algorithm has n steps, as much as the number of agents. First, we select the agent i with smallest cost to the source, such that $c_{i\omega} \leq c_{j\omega}$, for all $j \in N$. In the second step, we select an agent in $N \setminus \{i\}$ with the smallest cost either to the source or to agent i , who is already connected. We continue until all agents are connected, at each step connecting an agent still not connected to a connected agent or to the source.

108

$$C_m = \sum_{i=1}^n c_{im(i)} \leq C_p = \sum_{i=1}^n c_{ip(i)}.$$

109

110 A game with transferable utility, *TU game*, is a pair (N, v) where N is the set of
 111 agents and $v : 2^N \rightarrow \mathbb{R}$ is known as the characteristic function and it satisfies
 112 $v(\emptyset) = 0$. $Sh(N, v)$ denotes the Shapley value (Shapley 1953) of (N, v) . Bird (1976)
 113 associated a *TU game* (N, v^-) to each *mcst* problem (N_ω, \mathbf{C}) defining for each coal-
 114 ition $S \subseteq N$, $v^-(S) = C_{m(S_\omega, \mathbf{C})}$; that is, the cost of the optimal spanning tree when only
 115 agents in S are involved. This is known as the *property rights* approach, because the
 116 agents in S assume that the rest of the players are not present, or that they cannot use
 117 the connections of agents outside S to lower the cost.

118 Through this work we will follow an alternative approach in which it is
 119 assumed that agents in a coalition S can connect the source through agents out-
 120 side this coalition. This context is known as *non-property rights*. In this case, the
 121 characteristic function is defined by $v^+(S) = \min \{v^-(T) : S \subseteq T\}$. As pointed out
 122 in Bogomolnaia and Moulin (2010), the core of the *non-property rights* coopera-
 123 tive game (N, v^+) is included in the corresponding core of the *TU game* (N, v^-) .
 124 Therefore, our approach is more demanding in terms of coalitional stability.

125 Once a minimum cost spanning tree m is selected, an important issue is how
 126 to allocate the cost C_m among the agents, that is defined by means of a *sharing*
 127 *rule* (or simply, a *solution*). In order to define a sharing rule it is important to
 128 decide if members of a coalition can freely connect the source through individu-
 129 als outside their coalition. In our *non-property rights* approach the non-nega-
 130 tivity in the agents' allocations is a natural requirement (see Bogomolnaia and
 131 Moulin (2010)). Then, a sharing rule α is a function that proposes for any *mcst*
 132 problem (N_ω, \mathbf{C}) an allocation

133

$$\alpha(N_\omega, \mathbf{C}) = (\alpha_1, \alpha_2, \dots, \alpha_n) \in \mathbb{R}_+^n, \text{ such that } \sum_{i=1}^n \alpha_i = C_m.$$

134

135 Among the mentioned sharing rules in *mcst* problems, *Bird*, *Folk* and *Serial* solu-
 136 tions are non-negative. We will compare our proposals with these solutions.

137 The *Bird* rule (Bird 1976) (B) is defined for each $i \in N$ as $B_i((N_\omega, \mathbf{C})) = c_{im(i)}$.
 138 As mentioned in Bergantiños and Vidal-Puga (2007) the idea of this rule is simple:
 139 agents connect sequentially to the source following Prim's algorithm and
 140 each agent pays the corresponding connection cost. The *Serial* rule (Moulin
 141 and Shenker 1992) (S) proposes to distribute the cost of each arc among the
 142 individuals that actually use it in her (unique) path joining the source. In both
 143 cases, if there are more than one spanning tree minimizing the total cost, then
 144 the solutions propose the average of the corresponding sharing in all these trees.
 145 Finally, the *Folk* rule (Feltkamp et al. 1994; Bergantiños and Vidal-Puga 2007)
 146 (F) assigns to each agent $i \in N$ the amount given by the Shapley value of the
 147 *non-property rights* cooperative game, $F_i((N_\omega, \mathbf{C})) = Sh(N, v^+)$.

148 2.2 Claims problems

149 Given a finite set of agents $N = \{1, 2, \dots, n\}$, a claims problem appears when
 150 some endowment should be distributed among these individuals, who demand
 151 more than what is available. It is formally defined by a vector $(E, d) \in \mathbb{R}_+ \times \mathbb{R}_+^n$,
 152 where E denotes the endowment and d is the vector of agents' demands, such
 153 that the agents' aggregate demand is greater than or equal to the endowment,
 154 $\sum_{i \in N} d_i \geq E$.

155 A *claims rule* φ is a function that associates with each claims problem (E, d)
 156 a distribution of the total endowment among the agents (*efficiency*), such that no
 157 agent is allocated neither a negative amount (*non-negativity*), nor more than their
 158 claim (*claim-boundedness*):

$$159 \quad 0 \leq \varphi_i(E, d) \leq d_i \forall i \in N, \quad \sum_{i=1}^n \varphi_i(E, d) = E.$$

160
 161 Many claims rules have been proposed in the literature (see Thomson (2019) for
 162 formal definitions, properties and references), among which it is worth mentioning
 163 the Proportional (*Pr*), the Constrained Equal Awards (*Cea*), the Constrained Equal
 164 Losses (*Cel*), or the Talmud (*T*). These solutions are defined as: for each claims
 165 problem (E, d) , let R denote the sum of the agents' claims, $R = \sum_{i \in N} d_i$. Then, for all
 166 $i \in N$, the above mentioned claims rules are defined as:

- 167 • $Pr_i(E, d) = \frac{E}{R} d_i$.
- 168 • $Cea_i(E, d) = \min \{d_i, \lambda\}$, where λ is selected such that $\sum_{i \in N} Cea_i(E, d) = E$.
- 169 • $Cel_i(E, d) = \max \{d_i - \mu, 0\}$, where μ is selected such that $\sum_{i \in N} Cel_i(E, d) = E$.
- 170 • $T_i(E, d) = Cea_i\left(\min \left\{E, \frac{1}{2} C\right\}, \frac{1}{2} c\right) + Cel_i\left(\max \left\{0, E - \frac{1}{2} C\right\}, \frac{1}{2} c\right)$.

171 A way to address this kind of situations is by analyzing the part of the individu-
 172 als' demand that is not satisfied. Specifically, given a claims problem (E, d) , the
 173 *dual problem* (L, d) is defined by focusing on the losses the agents have with
 174 respect to their claims, where L denotes the total loss the agents incur, $L = R - E$.
 175 Given a claims rule φ , its *dual rule* φ^D shares losses in the same way that φ shares
 176 gains (Aumann and Maschler 1985):

$$177 \quad \varphi_i^D(L, d) = d_i - \varphi_i(E, d), \quad i = 1, 2, \dots, n.$$

178
 179 The *Cea* and *Cel* rules are dual of each other. A claims rule φ is *self-dual* if $\varphi^D = \varphi$.
 180 The *Proportional* and *Talmud* rules are self-dual.

181 3 Mapping *mcst* problems into claims problems

182 As aforementioned, we aim to define a mapping \mathcal{M} that associates *mcst* situations
 183 with claims problems under two alternative approaches.

184 • The *benefit-sharing* approach considers that each individual is initially allocated
 185 her maximum possible *rational* cost, that is, fully paying her cheapest way to
 186 connect to the source (rational individuals would never pay more than this cost,
 187 since agents' goal is to connect the source at the minimum possible cost). Then,
 188 the savings obtained through cooperation are distributed among the individuals.
 189 Our argument is as follows:

190 *As individuals want to be connected to the source, they are willing*
 191 *to pay the cost of their connection to the source. In total, an amount that we*
 192 *denote by C_ω is contributed. But those funds are not yet used, and the net-*
 193 *work is not yet built. Then, as the network will be common owned, agents want*
 194 *their connections in the optimal network to be their cheapest ones and claim*
 195 *to reduce their contribution to this minimum amount, and demand the extra*
 196 *cost d_{i_*} to be returned. If agents agree to cooperate, then everybody can be*
 197 *connected with a total cost of C_m and a network might thus be built for this*
 198 *amount. The benefit of cooperation is $E = C_\omega - C_m$. Finally, if the agents*
 199 *agree on how the benefit of cooperation is shared, the minimum cost spanning*
 200 *tree is built.*

201 Then, the pair (E, d_*) clearly defines a claims problem.

202 • The *cost-sharing* approach proposes that individuals pay initially the cost of their
 203 cheapest connection. The remaining cost (whenever the cheapest connections do
 204 not define a spanning tree) is then distributed among the individuals. Under this
 205 approach the argument is as follows:

206 *In order to provide a common network, individuals are asked for*
 207 *an initial contribution that equals their minimum connection cost. But those*
 208 *funds, C^{\min} , are not enough to connect all individuals to the source, and the*
 209 *network is not yet built. If the agents agree to cooperate, then everybody can*
 210 *be connected with a total cost of C_m and a network might thus be built. The*
 211 *additional cost that remains to be distributed is the difference $E_o = C_m - C^{\min}$.*
 212 *Now agents may connect to the source, and their extra contribution cannot*
 213 *be greater than the difference between their connection cost to the source and*
 214 *their minimum connection cost, that we have denoted by d_{i_*} . Finally, if the*
 215 *agents agree on how the additional cost is distributed, the minimum cost span-*
 216 *ning tree is built.*

217 Then, the pair (E_o, d_*) clearly defines a claims problem.

218 In both models the claim of each individual is determined by

219
$$d_{i_*} \equiv c_{i\omega}^* - c_{i_*} \quad \text{for all } i \in N, \quad d_* = (d_{1_*}, d_{2_*}, \dots, d_{n_*}), \quad c_{i_*} = \min_{j \in N_o, j \neq i} \{c_{ij}\},$$

$$c_{i\omega}^* = \min_{P_{i\omega}} \left\{ \sum_{e \in P_{i\omega}} c(e) \right\} \quad P_{i\omega} : \text{path joining agent } i \text{ with the source } \omega.$$
 220

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221 Note that the actual cost for individuals to connect to the source is $c_{i\omega}^*$, since they can
 222 choose to use their direct connection edge (i, ω) or to use any path $P_{i\omega}$. We will refer
 223 to $c_{i\omega}^*$ as the individual i 's *rational connection cost to the source*.

224 3.1 Model 1: sharing the benefit of cooperation

225 We consider throughout this sub-section that the starting points are the rational con-
 226 nection cost to the source, $c_{i\omega}^*$, the most an individual is willing to pay. If a min-
 227 imum cost spanning tree with cost C_m is implemented, the benefit of cooperation
 228 $E = C_\omega - C_m$, $C_\omega = \sum_{i \in N} c_{i\omega}^*$, shall be returned. We assume that no individual will
 229 pay less than their minimum connection cost, so the claim d_{i*} represents the amount
 230 they request to be returned from their initial payment $c_{i\omega}^*$. Then, we define a map \mathcal{M}^1
 231 associating to any *mcst* problem (N_ω, \mathbf{C}) the claims problem $\mathcal{M}^1(N_\omega, \mathbf{C}) = (E, d_*)$,
 232 where $E = C_\omega - C_m$ and $d_{i*} = c_{i\omega}^* - c_{i*}$.

233 **Definition 1** For any claims rule φ the associated-1 sharing rule for *mcst* problems
 234 κ_1^φ is defined for any *mcst* problem (N_ω, \mathbf{C}) and all $i \in N$ by:

$$(235 \kappa_1^\varphi)_i(N_\omega, \mathbf{C}) = c_{i\omega}^* - \varphi_i(\mathcal{M}^1(N_\omega, \mathbf{C})).$$

236
 237 As previously mentioned, a claims rule fulfills *non-negativity*, which has a natu-
 238 ral interpretation in the *mcst* context: *no individual should be allocated an amount*
 239 *greater than their rational connection cost to the source*; and *claim-boundedness*
 240 meaning that *no individual should be allocated an amount below their cheapest con-*
 241 *nection cost*.

242 3.2 Model 2: sharing the extra cost

243 We now consider that individuals initially pay their corresponding minimum con-
 244 nection cost c_{i*} , so the total amount paid is $C^{\min} = \sum_{i \in N} c_{i*}$. If a minimum cost span-
 245 ning tree with cost C_m is implemented, there is an extra cost, $E_o = C_m - C^{\min}$, that
 246 must be distributed among the agents. As we assume that no individual can pay
 247 more than their rational connection cost to the source, the claim of individual i is
 248 $d_{i*} = c_{i\omega}^* - c_{i*}$. Obviously, this claims problem is well defined, since the aggregated
 249 claim exceeds the endowment, $\sum_{i=1}^n d_{i*} \geq E_o$. Then, we define a new map \mathcal{M}^2 asso-
 250 ciating to any *mcst* problem (N_ω, \mathbf{C}) the claims problem $\mathcal{M}^2(N_\omega, \mathbf{C}) = (E_o, d_*)$.

251 **Definition 2** For any claims rule φ the associated-2 sharing rule for *mcst* problems
 252 κ_2^φ is defined for any *mcst* problem (N_ω, \mathbf{C}) and all $i \in N$ by:

$$(253 \kappa_2^\varphi)_i(N_\omega, \mathbf{C}) = c_{i*} + \varphi_i(\mathcal{M}^2(N_\omega, \mathbf{C})).$$

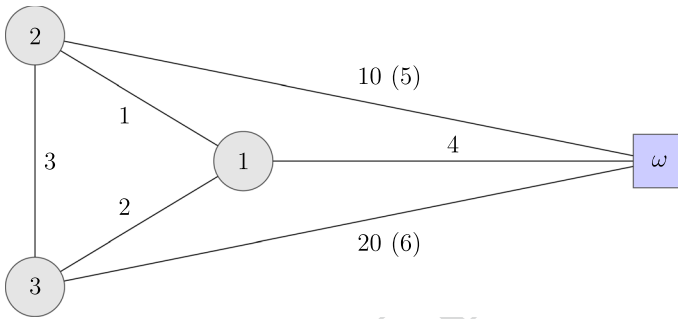
254
 255 In Example 1 we compute the allocations obtained by applying our models with
 256 different claims rules, and compare them with the ones provided by some *mcst* shar-
 257 ing rules.

Table 1 Proposals obtained by applying *mcst* solutions and claims rules in Example 1

	<i>Bird</i>	<i>Serial</i>	<i>Folk</i>	κ_1^{Pr}	κ_1^{Cea}	κ_1^{Cel}	κ_1^T	κ_2^{Pr}	κ_2^{Cea}	κ_2^{Cel}	κ_2^T
α_1	4	4/3	13/6	20/11	4/3	2	2	20/11	2	4/3	2
α_2	1	7/3	13/6	23/11	7/3	2	2	23/11	2	7/3	2
α_3	2	10/3	16/6	34/11	10/3	3	3	34/11	3	10/3	3

Author Proof

258 **Example 1** Let us consider the *mcst* problem defined by



259 **Remark 1** Although the direct cost of joining agent 2 to the source is 10 units, under
 260 our non-property rights approach the rational cost is 5 units through agent 1. Then,
 261 $c_{2\omega}^* = 5$. Analogously, the rational cost of joining agent 3 to the source ω is 6 units,
 262 $c_{3\omega}^* = 6$. The rational cost of each arc, when different from the direct cost, appears in
 263 brackets in the picture.

264 The minimum cost spanning tree is given by function m defined as:

265
$$m(1) = \omega \quad m(2) = 1 \quad m(3) = 1; \quad C_m = 7; \quad C_\omega = 15; \quad C^{\min} = 4.$$

266

267 In order to apply claims rules, the benefit of cooperation is $E = C_\omega - C_m = 8$. On
 268 the other hand, $c_* = (1, 1, 2)$, $c^* = (4, 5, 6)$, so the claims are $d_* = (3, 4, 4)$, and
 269 $E_o = 3$. Table 1 shows the obtained results.

270 We observe that the solutions defined by using the usual claims rules propose
 271 reasonable allocations of the total cost. The *Serial* solution is retrieved (in this
 272 example) through the *Cea* or *Cel* claims rules. We also note that κ_1 and κ_2 coin-
 273 cide when applied to Proportional or Talmud rules. This is a direct consequence
 274 of duality properties in claims rules, since these rules are self-dual, and it is for-
 275 mally established in the following result.

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276 **Proposition 1** For any *mcst* problem $(N_\omega, \mathbf{C}) \in \mathcal{N}_n$ and any claims rule φ ,

277
$$\kappa_1^\varphi(N_\omega, \mathbf{C}) = \kappa_2^{\varphi^D}(N_\omega, \mathbf{C}).$$

278 **Proof** Let us consider the associated claims problems

280
$$(E, d_*) = \mathcal{M}^1(N_\omega, \mathbf{C}), \quad (E_o, d_*) = \mathcal{M}^2(N_\omega, \mathbf{C}).$$

281 By definition of φ^D ,

282
$$\varphi_i(E, d_*) = d_{i_*} - \varphi_i^D\left(\sum_{i \in N} d_{i_*} - E, d_*\right) = d_{i_*} - \varphi_i^D(E_o, d_*).$$

283 Then,

284
$$\begin{aligned} (\kappa_1^\varphi)_i(N_\omega, \mathbf{C}) &= c_{ii}^* - \varphi_i(E, d_*) = c_{ii}^* - (d_{i_*} - \varphi_i^D(E_o, d_*)) \\ &= c_{i_*} + \varphi_i^D(E_o, d_*) = (\kappa_2^{\varphi^D})_i(N_\omega, \mathbf{C}) \end{aligned}$$

285 and duality is obtained. □

286 Consequently we obtain that if a claims rule φ is self dual, for any *mcst* problem
287 (N_ω, \mathbf{C}) both models propose the same distribution of the total cost.

288
$$\kappa_1^\varphi(N_\omega, \mathbf{C}) = \kappa_2^\varphi(N_\omega, \mathbf{C}).$$

289 In particular, the *Proportional* or *Talmud* rules provide the same allocation with the
290 pessimistic and the optimistic model.

291 Therefore, the two models propose the same family of cost allocations. Then,
292 hereinafter we will only analyze the model defined by \mathcal{M}^1 .

293 4 Properties

294 Bergantiños and Vidal-Puga (2007) provide a very exhaustive normative study
295 on the solutions of *mcst* problems. They present a list of properties that a solution
296 should satisfy and compare, among others, the *Bird* and *Folk* solutions in terms of
297 the properties that satisfy.²

298 In this section we analyze if some of these properties are fulfilled by the solutions
299 we have defined through claims rules. The property of coalitional stability (*core*
300 *selection*) is analyzed in the next section. We first briefly introduce the properties.

² They show that the *Folk* solution satisfies all properties we introduce, whereas the *Bird* solution fails to fulfill Continuity, Cost monotonicity and Population monotonicity. On the other hand, it is known that the *Serial* solution does not fulfill the crucial property of Individual rationality. Also, it can be shown that this solution does not fulfill Continuity, Cost monotonicity, nor Population monotonicity.

305 INDIVIDUAL RATIONALITY: A sharing rule α for *mcst* problems satisfies *Individual*
 306 *Rationality* if for each problem (N_ω, \mathbf{C}) , and all $i \in N$, $\alpha_i(N_\omega, \mathbf{C}) \leq c_{i\omega}^*$.

307 CONTINUITY: A solution α for *mcst* problems satisfies *Continuity* if α is continuous
 308 function of the cost matrix \mathbf{C} .

309 POSITIVITY: A solution α for *mcst* problems satisfies *Positivity* if for each problem
 310 (N_ω, \mathbf{C}) , and all $i \in N$, then $\alpha_i(N_\omega, \mathbf{C}) \geq 0$.

311 SYMMETRY: A solution α for *mcst* problems satisfies *Symmetry* if for each prob-
 312 lem (N_ω, \mathbf{C}) , whenever individuals $i, j \in N$ are such that $c_{ik} = c_{jk}$, for all $k \in N_\omega$, then
 313 $\alpha_i(N_\omega, \mathbf{C}) = \alpha_j(N_\omega, \mathbf{C})$.

314 COST MONOTONICITY: A solution α for *mcst* problems satisfies *Cost Monotonicity*
 315 if for any two problems $(N_\omega, \mathbf{C}), (N_\omega, \mathbf{C}')$, such that $c_{ij} < c'_{ij}$ for some $i \in N, j \in N_\omega$
 316 and $c_{kl} = c'_{kl}$ otherwise, $\alpha_i(N_\omega, \mathbf{C}) \leq \alpha_i(N_\omega, \mathbf{C}')$.

317 It is clear that, for any claims rule φ , our proposal fulfills these properties.

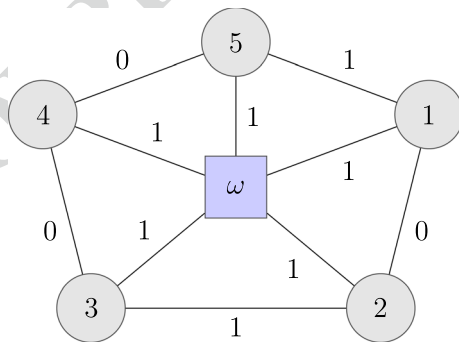
318 **Proposition 2** For any claims rule φ the solution κ_1^φ satisfies *Individual Rational-*
 319 *ity, Continuity and Positivity*. In addition, κ_1^φ satisfies *Symmetry* if the claims rule is
 320 *symmetric and satisfies Cost monotonicity if the claims rule is claims monotonic*.³

321 An additional property that has been considered for solutions of *mcst* problems is:

322 POPULATION MONOTONICITY: A solution α for *mcst* problems satisfies *Population*
 323 *Monotonicity* if for each problem (N_ω, \mathbf{C}) , and all $S \subset N$, $\alpha_i(N_\omega, \mathbf{C}) \leq \alpha_i(S_\omega, \mathbf{C})$ for
 324 all $i \in S$.

325 The following example shows that κ_1^φ does not fulfill this property.

326 **Example 2** Let us consider the *mcst* problem with $n = 5$ individuals depicted in the
 327 following figure (as the graph should be complete, we consider that the costs of the
 328 arcs not shown are all equal to 10).



329

³ A claims rule φ is *symmetric* if for any claims problem (E, d) , $d_i = d_j$ implies $\varphi_i(E, d) = \varphi_j(E, d)$. On the other hand, a claims rule is *claims monotonic* if an increase in an agent's claim does not harm her. Most of claims rules in the literature, and all we have introduced, satisfy these properties.

3FL01
 3FL02
 3FL03

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330 There are several trees with minimum cost. We consider

$$331 \quad m(1) = \omega \quad m(2) = 1 \quad m(3) = 4 \quad m(4) = 5 \quad m(5) = \omega$$

332
333 Throughout easy computations we obtain $C_m = 2$, $C_\omega = 5$, $E = 3$, and the claims
334 vector $d_* = (1, 1, 1, 1, 1)$. Therefore, for any (anonymous) claims rule φ

$$335 \quad (\kappa_1^\varphi)_i = c_{i\omega}^* - \frac{E}{5} = \frac{2}{5}, \quad i = 1, 2, 3, 4, 5$$

336
337 If we consider the coalition $S = \{3, 4, 5\}$ and the *mcst* problem (S_ω, \mathbf{C}) , our model
338 allocates $\frac{1}{3}$ to each agent in S (for any anonymous claims rule), contradicting Popula-
339 tion monotonicity.

340 5 Coalitional stability

341 In a *mcst* problem, cooperation is necessary in order to implement the optimal tree.
342 Then, coalitional stability is required to prevent that groups of individuals may have
343 incentives to build their own network and then a minimum cost spanning tree would
344 not be implemented. To this end, a cooperative game associated with a *mcst* problem
345 has been introduced so that, for each coalition $S \subseteq N$, the characteristic function
346 represents the cost of connecting all individuals in this coalition to the source. For-
347 mally, given the *mcst* problem (N_ω, \mathbf{C}) and a coalition $S \subseteq N$, the (*monotonic*) cost of
348 connecting this coalition to the source is (in our non-property context):

$$349 \quad v(S) = \min \{ C_m(T) : S \subseteq T \subseteq N \}$$

350
351 where $C_m(T)$ is the cost of the efficient tree of the problem $(T_\omega, \mathbf{C}|_T)$. The *core* asso-
352 ciated to a *mcst* problem is then defined by:

$$353 \quad co(N_\omega, \mathbf{C}) = \left\{ \alpha \in \mathbb{R}^n : \sum_{i \in S} \alpha_i \leq v(S), \quad \forall S \subseteq N, \quad \sum_{i \in N} \alpha_i = v(N) = C_m \right\}.$$

354
355 In Example 1 the characteristic function is:

$$356 \quad v(\{1\}) = 4; v(\{2\}) = v(\{1, 2\}) = 5; v(\{3\}) = v(\{1, 3\}) = 6; v(\{2, 3\}) = v(\{1, 2, 3\}) = 7.$$

357
358 Although all the allocations we obtained in this example (Table 1) belong to the
359 core, this is not true in general. In Example 2, the total amount allocated to coalition
360 S is $6/5$, which is greater than $v(S) = 1$. So, no allocation in the core can be obtained
361 in this example by using (anonymous) claims rules throughout our approach.

362 The following result shows a necessary and sufficient condition, in terms of the
363 *mcst* cost matrix, ensuring that the allocation provided by κ_1^φ belongs to the core
364 of the monotonic cooperative game, regardless of the claims rule φ used in its
365 definition.

366 **Theorem 1** Given a *mcs*t problem (N_ω, \mathbf{C}) , if

367
$$C_m - \sum_{i \notin S} c_{i*} \leq v(S) \quad \text{for all } S \subseteq N \text{ such that } v(S) \neq \sum_{i \in S} c_{i\omega}^* \quad (1)$$

368
 369 for any claims rule φ the allocation $(\kappa_1^\varphi)_i(N_\omega, \mathbf{C}) = c_{i\omega}^* - \varphi_i(E, d_*)$, $i \in N$, belongs
 370 to the core of the monotonic cooperative game associated with the *mcs*t problem.
 371 Conversely, if for any claims rule φ , the allocation $(\kappa_1^\varphi)(N_\omega, \mathbf{C})$ belongs to the core,
 372 Condition (1) is fulfilled.

373 **Proof** First we consider $S \subseteq N$ such that $v(S) \neq \sum_{i \in S} c_{i\omega}^*$. We need to prove that, for
 374 any claims rule φ ,

375
$$\sum_{i \in S} (\kappa_1^\varphi)_i(N_\omega, \mathbf{C}) \leq v(S).$$

376
 377 We know that any claims rule φ satisfies *non-negativity* and *claim-boundedness*,
 378 which implies that for all $S \subseteq N$,

379
$$\sum_{i \in S} \varphi_i(E, d_*) \geq \max \left\{ E - \sum_{i \notin S} d_{i*}, 0 \right\}.$$

380
 381 Note that $E - \sum_{i \notin S} d_{i*} = \sum_{i \in S} c_{ii}^* + c_{i*} - C_m$ and then

382
$$\begin{aligned} \sum_{i \in S} (\kappa_1^\varphi)_i(N_\omega, \mathbf{C}) &= \sum_{i \in S} c_{i\omega}^* - \sum_{i \in S} \varphi_i(E, d_*) \leq \sum_{i \in S} c_{i\omega}^* - \max \left\{ E - \sum_{i \notin S} d_{i*}, 0 \right\} \\ &\leq \sum_{i \in S} c_{i\omega}^* - \left(E - \sum_{i \notin S} d_{i*} \right) = C_m - \sum_{i \notin S} c_{i*} \leq v(S) \end{aligned}$$

383
 384 from Condition (1). Let us consider now a coalition $S \subseteq N$ such that $v(S) = \sum_{i \in S} c_{i\omega}^*$.
 385 As $(\kappa_1^\varphi)_i \leq c_{i\omega}^*$, obviously $\sum_{i \in S} (\kappa_1^\varphi)_i(N_\omega, \mathbf{C}) \leq v(S)$. So, for any claims rule φ , κ_1^φ is
 386 in the core of the monotonic cooperative game.

387 Conversely, let us suppose that for some coalition $S \subseteq N$, $v(S) \neq \sum_{i \in S} c_{i\omega}^*$ and
 388 $C_m - \sum_{i \notin S} c_{i*} > v(S)$. Consider the constrained dictatorial claims rule, φ^{CD} , in
 389 which the first agents to receive their claims are those outside S ; that is, we con-
 390 sider a permutation π such that $\pi(1), \pi(2), \dots, \pi(n-s) \notin S$, where s denotes the
 391 number of agents in S . Under our model, the claims rule provides the cost alloca-
 392 tion $\alpha_i = c_i^* - \varphi_i^{CD}(E, d_*)$, $\sum_{i \in N} \alpha_i = C_m$. If we analyze the endowment E and the
 393 demands of agents not in S , we obtain two possibilities:

394 (a) $E \geq \sum_{i \notin S} d_{i*}$; or (b) $E < \sum_{i \notin S} d_{i*}$

395 In the first case, $\varphi_i^{CD}(E, d_*) = d_{i*}$, so $\alpha_i = c_{i*}$ for all $i \notin S$. Then,

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396

$$\sum_{i \in S} \alpha_i = C_m - \sum_{i \notin S} c_{i*} > v(S)$$

397

398 and the allocation is not in the core.

399 The second case implies $\varphi_i^{CD}(E, d_*) = 0$, so $\alpha_i = c_{i\omega}^*$ for all $i \in S$. This allocation
 400 only can be in the core if $v(S) = \sum_{i \in S} c_{i\omega}^*$, a contradiction. So, the allocation is nei-
 401 ther in the core in this case. □

402 Checking Condition (1) may require as much calculus as directly testing that the
 403 allocation provided is in the core. Nevertheless, it is important to emphasize that
 404 this condition *only depends on the data of the mcst problem and once it is checked,*
 405 *it remains valid for any claims rule.* In order to interpret Condition (1), it says that,
 406 for any coalition S , there is some chance of obtaining benefits from cooperation even
 407 in the case that individuals outside S pay only their minimum connection cost (the
 408 minimum they can pay under our approach); or, all members in S pay her rational
 409 connection to the source, which is at the same time their minimum connection cost.

410 The sufficient and necessary condition obtained to guarantee coalitional stability
 411 may seem quite technical. However, it is useful from an operational point of view,
 412 since it allows us to identify sub-classes of *mcst* problems where the solution we
 413 propose is always a core selection, for every claims rule.

414 5.1 Some special classes of *mcst* problems

415 In this section we show some classes of *mcst* problems so that Condition (1) is
 416 always fulfilled and the allocation $\kappa_1^\varphi(N_\omega, \mathbf{C})$ belongs to the core of the monotonic
 417 cooperative game, for any claims rule φ .

418 5.1.1 2-*mcst* problems

419 Let us consider the so-called 2-*mcst* problems (Estévez-Fernández and Reijnierse
 420 2014; Subiza et al. 2016) in which the connection cost between two different indi-
 421 viduals (houses, villages, ...) can only take one of two possible values (low and high
 422 cost). Moreover, we consider problems (N_ω, \mathbf{C}) such that $c_{ij} = k_1$, $i, j \in N$, $i \neq j$,
 423 $c_{i\omega} = k_2$, with $0 \leq k_1 \leq k_2$. It is easy to check that Condition (1) is fulfilled. Our
 424 model proposes, for any claims rule φ , the allocation

$$425 (\kappa_1^\varphi)_i(N_\omega, \mathbf{C}) = k_2 - \frac{n-1}{n}(k_2 - k_1) \quad i = 1, 2, \dots, n,$$

426

427 which belongs to the core (it coincides with the *Folk* solution).

428 5.1.2 Information graph games

429 A related scenario appears when analyzing information graph games (Kuipers
 430 1993). This games can be formalized in the following way.

431 A set of customers N are all interested in a particular piece of information.
 432 A subset Z of N , called the informed set, already possesses this information.
 433 Other customers may purchase the information from a central supplier for a
 434 fixed price, say 1, or they may share the information with a friendly customer,
 435 who already has the information.

436 This situation can be represented by an undirected graph and the information graph
 437 game in a minimum cost spanning tree problem, where the cost of an arc is 0 or 1,
 438 by depending if one of the agents in the arc belong to Z . In this case, set N can be
 439 decomposed in disjoint components, $N = (\bigcup_{t=1}^r U_t) \cup (\bigcup_{t=1}^s C_t)$, such that:

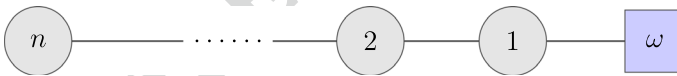
- 440 1. For each $i \in U_t, |U_t| = 1, c_{i*} = c_{i\omega}^* = 1$.
- 441 2. For each $i \in U_t, |U_t| > 1, c_{i*} = 0, c_{i\omega}^* = 1$.
- 442 3. For each $i \in C_t, c_{i*} = c_{i\omega}^* = 0$.

443 Now, for each coalition $S \subseteq N$, if S intersects k components of type $U_t, v(S) \geq k$ and
 444 $\sum_{i \notin S} c_{i*} \geq r - k$, whereas $C_m = r$. Therefore, condition (1) holds and, for any claims
 445 rule φ , the solution κ_1^φ is in the core of the cooperative game.

446 5.1.3 Linear *mcst* problems

447 Another focal class of *mcst* problems in which Condition (1) is always satisfied
 448 is given by *linear mcst* problems. Let us consider a group of individuals
 449 $N = \{1, 2, \dots, n\}$ situated in a row that wish to connect to a source ω . The cost of
 450 connecting one individual with the next one is 1 unit. If an individual wants to connect
 451 to the source, she must do it through all its neighbors on the way towards the
 452 source and pay all costs.

453



454 Formally, for each $i, j \in N, i \neq j$, the connection cost is $c_{ij} = |i - j|$. For each
 455 $i \in N$, the cost to the source is $c_{i\omega} = i$.

456 The minimum cost spanning tree connects each individual to the next, and the
 457 first one with the source, with a total cost $C_m = n$. It is easy to observe that Condi-
 458 tion (1) is fulfilled, since for all $S \subseteq N, |S| = s$,

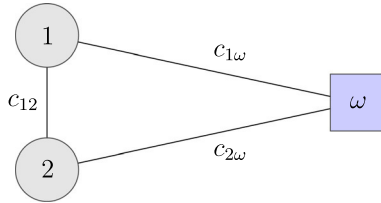
459
$$C_m = n, \quad \sum_{i \notin S} c_{i*} = n - s, \quad v(S) = \max \{i \in S\} \geq s.$$

460

461 5.1.4 Bipersonal *mcst* problems

462 If there are just $n = 2$ agents

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463 it is not hard to prove that Condition (1) is fulfilled. Moreover, in this case, it can be
 464 proved that the *Folk* solution is obtained with our model, if we use the *Talmud* claims
 465 rule; that is,

$$466 \quad \kappa_1^T(N_\omega, \mathbf{C}) = F(N_\omega, \mathbf{C}).$$

468 5.2 Modifying the claims vector

469 Up to this point, we have fixed an estate E , the benefit of cooperation, and a vector of
 470 claims d_* in order to apply our model. It is clear that other possibilities when defining
 471 the claim of each agent can be considered. The following result shows that the selection
 472 of the claims vector influences that the final allocation belongs to the core.

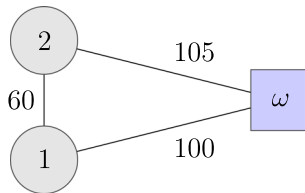
473 **Proposition 3** *Let us consider a mcst problem (N_ω, \mathbf{C}) . Let $E = C_\omega - C_m$ be the ben-*
 474 *efit of cooperation and let $c_\omega = (c_{1\omega}^*, c_{2\omega}^*, \dots, c_{n\omega}^*)$ be the vector of rational costs to*
 475 *the source. Then, there exists a claims vector d , such that for any claims rule φ ,*
 476 *$\kappa^\varphi(N_\omega, \mathbf{C}) = c_\omega - \varphi(E, \hat{d}) \in \text{co}(N_\omega, \mathbf{C})$.*

477 **Proof** To show the existence of the required claims vector, for each $i \in N$, consider
 478 $\hat{d}_i = c_{i\omega}^* - c_{im(i)}$. Then, $\sum_{i \in N} \hat{d}_i = E$, so (E, \hat{d}) is a degenerated claims problem and
 479 for any claims rule φ , $\varphi_i(E, \hat{d}) = \hat{d}_i$ and agent i is allocated the amount $\alpha_i = c_{im(i)}$,
 480 which is in the core of the cooperative game and coincides with the *Bird* solution if
 481 the minimum cost spanning tree is unique. \square

482 6 Final comments

483 The current paper explores a bridge between two independent problems that
 484 have been extensively analyzed in the literature: *minimum cost spanning tree* and
 485 *claims* problems. Specifically, we present new ways of allocating the cost of a
 486 network that are based on claims rules that share the benefit of cooperation. It is
 487 noteworthy that in our approach only two costs are used: the *rational cost to the*
 488 *source* and the *cost to the cheapest edge* (also the costs $c_{im(i)}$ are used in order to
 489 compute C_m , the cost of the efficient tree). The aforementioned feature (ignoring
 490 most of the available information) links our proposals with the so-called *reduc-*
 491 *tionism approach* (Bogomolnaia and Moulin 2010).

492 Our approach allows for easy and intuitive ways to distribute the cost of an
 493 optimal network among the involved agents. For instance, when using the pro-
 494 portional claims rule, our model proposes a proportional sharing of the benefit of
 495 cooperation, or a proportional distribution of the extra-cost. Analogously, when
 496 using egalitarian claims rules, we propose an equal sharing of the benefit of coop-
 497 eration, or an equal sharing of the extra-cost (subject that no agent pay more that
 498 their individual cost, nor a negative amount). Only the *Bird*, or *Serial* solutions
 499 are such easier methods. Nevertheless, the *Bird* solution can be seen as *unfair* and
 500 the *Serial* may propose for an agent a payment greater than its direct connection
 501 to the source. Let us observe the following example:
 502



503 Then, $C_m = 160$ and the *Bird* proposal is $B = (100, 60)$ (each agent pays their
 504 own connection); so, agent 1 does not obtain any gains from cooperation. The
 505 *Serial* solution is $S = (50, 110)$; so, agent 2 pays more than connecting directly to
 506 the source. The *Folk* solution proposes an equal sharing of the cost, $F = (80, 80)$.
 507 Our model proposes the following allocations, depending on the used claims
 508 rules:

$$509 \quad \kappa_1^{Pr} = (78.8, 81.2) \quad \kappa_1^{Cea} = (77.5, 77.5) \quad \kappa_1^{Cel} = \kappa_1^T = (80, 80)$$

511 As mentioned, a drawback of our proposal is that sometimes it fails to propose core
 512 allocations. A possible way to prevent coalitions leaving the group is to find the core
 513 allocation closest to our selected proposal (see Giménez-Gómez et al. (2020)). For
 514 instance, if the proportional criteria is assumed, and κ_1^{Pr} is not in the core, then we
 515 can obtain the allocation x in the core minimizing the distance $d(x, \kappa_1^{Pr})$, although
 516 we lose the simplicity and intuitive idea of the solution.

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