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CORRELATION BETWEEN CHARTRES CATHEDRAL AND THE CATASTERISM OF ANCIENT VIRGIN GODDESSES

Joan M. Seguí[1], Albert Samper[1] and Blas Herrera[2*]

[1]Unitat predepartamental d'Arquitectura, Universitat Rovira i Virgili, Avinguda de la Universitat, 1, 43204, Reus, Spain
[2]Departament d'Enginyeria Informàtica i Matemátiques, Universitat Rovira i Virgili, Avinguda Països catalans 26, 43007, Tarragona, Spain

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Abstract

The purpose of this paper is to project the astronomical daily orbits of stars through the centre of the main rose window of the Cathedral Basilica of Our Lady of Chartres, tracing their trajectories on the cathedral's floor plan. We geometrically show that in the astronomical epoch *J1200* (close to the period when the cathedral was built), the labyrinth, transept, apse, ambulatory, horizon and main rose window of the cathedral align accurately with the astronomical pattern of the stars constituting the catasterism of ancient virgin goddesses — *Elthor*, *Spica*, *Isis*, *Sirius* and *Adhara* — belonging to the *Taurus*, *Virgo* and *Canis Major* constellations. The choice of stars is not speculative but objective in nature. Given the slowness of the Earth's precessional motion and the stars' proper movement in the sky, this correlation occurred every day of every year during the construction of the cathedral.

Keywords: Chartres, cathedral, catasterism, Ancient Virgin Goddesses, constellations

1. Introduction and preliminary remarks

As stated in the title of the paper, we show the existing correlation between the architectural parameters of the Chartres Cathedral and certain celestial bodies associated with ancient virgin goddesses.

Katasterismoi is a Helenistic book written in Alexandria that some attribute to Eratosthenes or Pseudo-Eratosthenes [1]. The book describes the mythical origins of stars and constellations as they were interpreted in Hellenistic culture. However, in this paper, we use the word *catasterism* in a more general sense, irrespective of whether it refers to the Hellenistic culture or another culture. Therefore, we consider a catasterism to be the transformation of heroes or gods into stars, celestial objects, or star patterns in the celestial sphere. Creating catasterisms was a common praxis in several ancient civilizations. The interplay of civilizations from the Near East influenced the deities of several

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[*] E-mail: blas.herrera@urv.net

religions, such as Islamic, Christian, Jewish, Roman, Greek, Assyrian and Egyptian deities. The ancient Greek religion clearly created catasterisms and so did the ancient Egyptian religion, one of the most ancient religions.

One of the most important religious buildings of Christianity is the Cathedral Basilica of Our Lady of Chartres (Basilique Cathédrale Notre-Dame de Chartres), also known as the Cathedral Church of the Assumption of Our Lady (Cathédrale de l’Assomption de Notre-Dame). The Cathedral is devoted to the deity Notre-Dame, Virgin, Mother of God. The cult of Our Lady is not only characteristic of Christianity but also associated with the features of other ancient virgin goddesses of motherhood.

In fact, the origin of the cult to the virgin goddesses of motherhood dates back to the ancient Egyptian religion. In the Roman Catholic religion, Notre-Dame (Our Lady) has referred to the Virgin Mary since Roman times. The features of the Virgin Mary link her directly to Isis, the Egyptian cow-goddess of motherhood. Several authors associate one with the other, and they also attach great importance to the legacy and influence of the Egyptian goddess Isis on European civilization. For instance, in 1904, the prominent Egyptologist E.A. Wallis [2], described analogies between the Virgin Mary and Isis. Likewise, in 1964, mythologist J. Campbell commented on the similarities between the deities [3]. However, the similarities between Isis and the Virgin Mary and the influence of the cult of Isis on the Greco-Roman world or the Christian religion are outside the purview of this paper. We believe this matter has already been discussed thoroughly by the aforementioned researchers and other authors [4-6]. Besides, researchers such as architectural historian J.S. Curl have more recently supported the existence of such an influence. All of them agree that the two deities are the same or at least, they point out that “resemblances between Isis and the Virgin Mary = Our Lady = Notre-Dame are far too close and numerous to be accidental” [7].

Some of the main architectural parameters of the Chartres Cathedral are as follows: its longitude, latitude and orientation (the geographical location of the building); the positions of the ambulatory, the apse and the transept with respect to the main façade (the overall geometry); and the height of the main rose window and the position of the labyrinth (the architectural elements).

The architectural uniqueness of the main rose window comes from its impressive size, constructive complexity and important role in the composition of the main façade. All of these qualities are the reasons why the rose window has been the focus of multiple studies; for instance, the geometric analysis of its texture or construction and the study of solar rays entering the cathedral have been areas of interest. However, this rose window has never been examined as an astronomical instrument. We use the centre of the main rose window as the point through which a visual line passes pointing to the celestial sphere where the different parameters and unique locations in the cathedral determine the viewing direction. Using the centre of the rose window as the projection point of the stars, we project their astronomical orbits on the cathedral’s floor plan.

The purpose of this paper is to project the orbits of stars through the main rose window of the Chartres Cathedral (Figure 1).

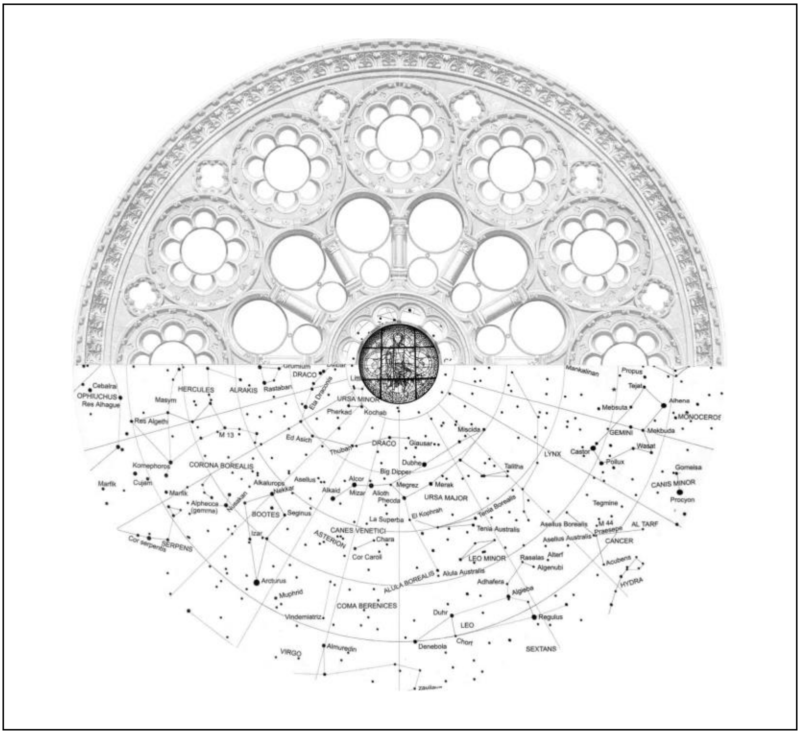

Figure 1. Graphical representation of the main rose window through which the astronomical orbits are projected.

This process of projecting star orbits through the main rose window of the Chartres Cathedral — in particular, the orbit of the star *Sirius* — is not an idea unique to us, nor is it attributable only to the Chartres Cathedral; it is an ancient process that was already used in some classical buildings. A relevant example is the Clementine Gnomon in the Basilica of Santa Maria degli Angeli in Rome [8, 9]. Pope Clement XI commissioned the astronomer and archaeologist Francesco Bianchini to create this gnomon. This gnomon is an ingenious device that allows a person to see the star orbits during the day and not only by night, projecting these daily orbits on the Basilica's floor in the same way as we do on the floor plan of the Chartres Cathedral. In fact, the Clementine Gnomon is a sundial synchronized with sidereal time. Using this sundial, it is possible to observe the meridian transits of the Sun and the stars *Sirius*, *Polaris* and *Arcturus* simultaneously (Figure 2) as ellipse and hyperbola arcs on the basilica's floor.

These projections were made through holes in the ceiling and through a hole in the centre of the Pope's coat of arms — showing a star on its emblem — placed on a wall of the basilica at a height of 20.34 m (Figure 3). In this paper, we repeat the process with Chartres Cathedral and its rose window, numerically calculating the orbits of the stars involved.

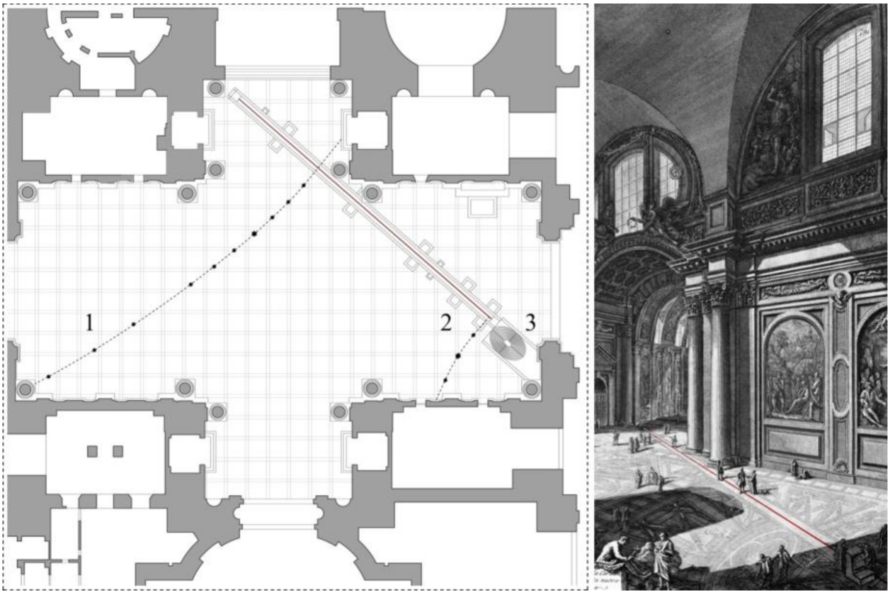

Figure 2. On the left, the graphical representation showing the projections of the orbits of *Sirius* (1), *Arcturus* (2) and *Polaris* (3) on the floor plan of Santa Maria degli Angeli e dei Martiri in Rome [8, p. 54]; on the right, part of a drawing by Giovanni Battista Piranesi showing the meridian line inside the church.

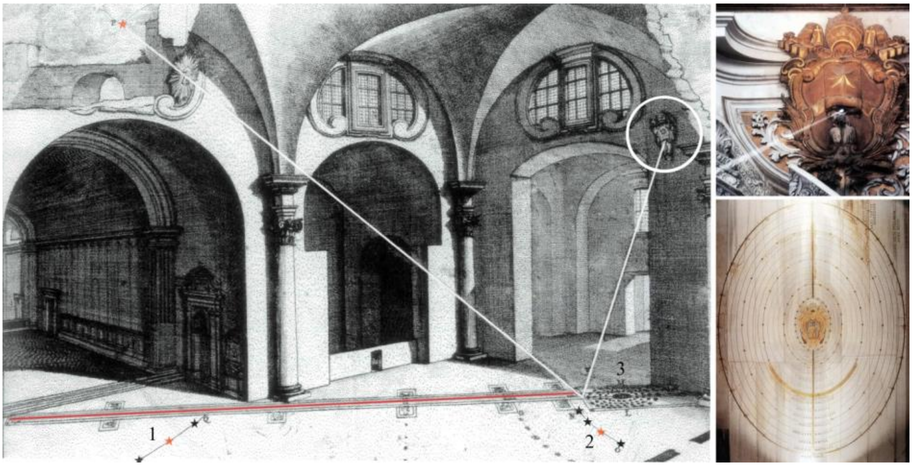

Figure 3. On the left, a partial reproduction of a drawing showing the star orbits of *Sirius* (1), *Arcturus* (2) and *Polaris* (3) projected on the floor plan of Santa Maria degli Angeli e dei Martiri in Rome [8, p. 15]; on the right, a photo of the coat of arms of Pope Clement XI with its hole and a photo of the orbit paths of *Polaris* on the floor plan of the church from astronomical epoch *J1700* to astronomical epoch *J2500*.

2. Celestial declinations as determined by the parameters of Chartres cathedral

2.1. Preliminary remarks

Among all the parameters of Chartres Cathedral, there is a set of 13 $\{\lambda,\varphi,\gamma,\mathcal{G},\mathcal{A},\mathcal{C},r,d,\mathcal{L},\bar{l},\bar{c},\bar{a},\bar{g}\}$ that can be considered the most important; we explain these parameters in this section.

With regard to the cathedral’s location, we consider the first three parameters which had to be decided by the master builders. Of the various events that took place during the construction, the cathedral’s longitude λ, latitude φ and orientation γ (the angle formed between the parallel where the cathedral lies and the ground line of the main façade, with west to north being the positive orientation) were the constants determining the building’s design (Figure 4 shows the time period of the construction of the cathedral).

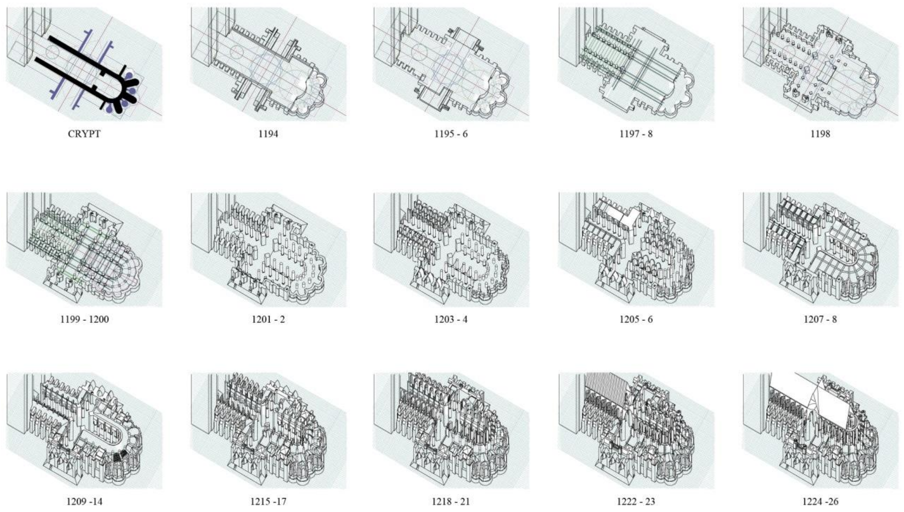

Figure 4. Chronology of Chartres Cathedral construction.

With regard to the overall geometry of the building, we consider three unique and relevant positions on the floor plan and their distances from the main façade: these positions are those of the ambulatory $\mathcal{G}$, the apse $\mathcal{A}$ and thetransept $\mathcal{C}$.

When examining the architecture and elements, we consider the height r of centre point of the main rose window (through which we project the astronomical orbits of celestial bodies) and the distance from the centre $\mathcal{L}$ of the labyrinth (on the cathedral’s floor) to the main façade. Besides, since the cathedral’s floor is not perfectly horizontal (it rises in height towards the ambulatory), for the above mentioned positions on the cathedral’s floor

$\{\mathcal{G},\mathcal{A},\mathcal{C},\mathcal{L}\}$ we must consider their topographic vertical elevations with respect to the lowest elevation (that of the access door of the main façade). Therefore, we consider the labyrinth's elevation $\bar{l}$, the transept's elevation $\bar{c}$, the apse's elevation $\bar{a}$ and the ambulatory's elevation $\bar{g}$. Ignoring this topographic feature could significantly alter all results; for instance, the difference in elevation between the cathedral's entrance and the ambulatory is more than 1 m (see Figures 5 and 6 and Table 1).

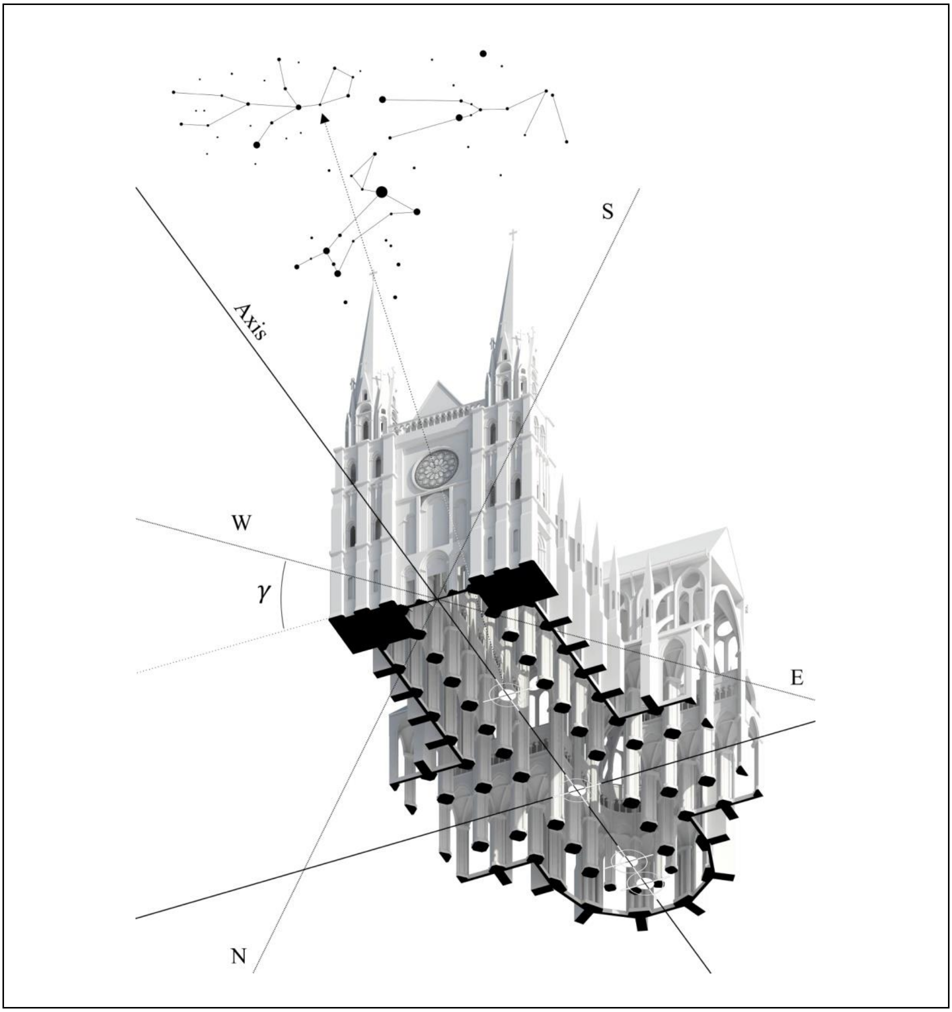

Figure 5. Architectural and location parameters determined by Chartres cathedral.

The parameters of location have been obtained through GPS coordinates, and the distance parameters have been measured in the field using a laser metre (Leica DISTO^{TM} D510). All measurements have been subsequently checked with a graphic survey (floor plan, elevation and cross-section) obtained from the Library of the Austrian Federal Ministry for Education, Arts and Culture and from Scanproject 'Heritage Monuments Board of Austria'. All parameters considered are shown in Figures 5 and 6 and Table 1.

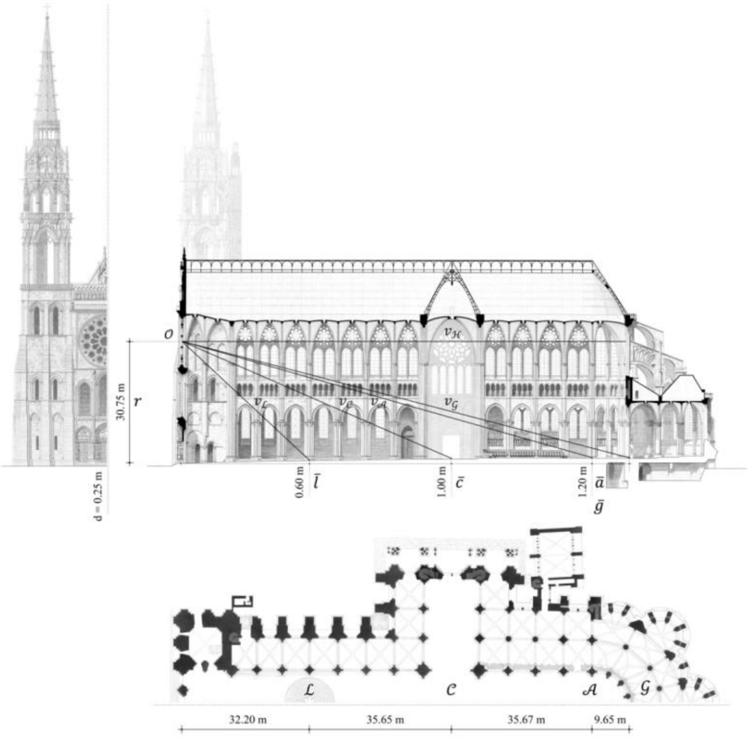

Figure 6. Geometric, architectural and elements parameters determined by the cathedral.

Table 1. Parameters intrinsic to Chartres cathedral.

<table>
<tr><th colspan="6">LOCATION</th></tr>
<tr><th colspan="2">Longitude λ</th><th colspan="2">Latitude φ</th><th colspan="2">Orientation γ</th></tr>
<tr><td colspan="2">−1°29'14"</td><td colspan="2">48°26'51"</td><td colspan="2">43°47'43"</td></tr>
<tr><th colspan="6">GEOMETRY</th></tr>
<tr><th colspan="2">Ambulatory $\mathcal{G}$</th><th colspan="2">Apse $\mathcal{A}$</th><th colspan="2">Transept $\mathcal{C}$</th></tr>
<tr><td colspan="2">$g = 113.17\,m$</td><td colspan="2">$a = 103.52\,m$</td><td colspan="2">$c = 67.85\,m$</td></tr>
<tr><td colspan="2">$\bar{g} = 1.20\,m$</td><td colspan="2">$\bar{a} = 1.20\,m$</td><td colspan="2">$\bar{c} = 1.00\,m$</td></tr>
<tr><th colspan="6">ELEMENTS</th></tr>
<tr><th colspan="3">Labyrinth $\mathcal{L}$</th><th colspan="3">Rose window $\mathcal{O}$</th></tr>
<tr><td colspan="3">$l = 32.20\,m$</td><td colspan="3">$r = 30.75\,m$</td></tr>
<tr><td colspan="3">$\bar{l} = 0.60\,m$</td><td colspan="3">$d = 0.25\,m$</td></tr>
</table>

In this paper, the geographic longitude parameter λ of Chartres cathedral is irrelevant for finding the positions on the celestial sphere.

During a full rotation of the earth around its axis (one day), the view from inside the Chartres cathedral through its rose window includes all the stars and constellations located within a circle $\mathcal{C}_\delta$ of the celestial sphere, which is only determined by the circle's declination δ. This declination δ depends on the direction of the line of sight passing through the centre $\mathcal{O}$ of the rose window. The parameters in Table 1 determine five architectural viewing directions, namely, the horizontal straight line $v_{\mathcal{H}}$ (corresponding to a visual straight line which runs parallel to the longitudinal axis of the cathedral at height r) and the four visual straight lines $v_{\mathcal{L}}$, $v_{\mathcal{C}}$, $v_{\mathcal{A}}$ and $v_{\mathcal{G}}$ passing through $\mathcal{O}$ and through the points $\mathcal{L}$, $\mathcal{C}$, $\mathcal{A}$ and $\mathcal{G}$, respectively (Figures 5 and 6).

It must be kept in mind that the orthogonal projection of $\mathcal{O}$ on the floor plan is off-center by exactly 0.25 m from the cathedral's longitudinal axis. This shift to the left ($d = 0.25\ m$) is shown in Table 1 and can be seen in the elevation view (Figure 6).

2.2. Calculating the celestial declination of the horizontal visual straight line $v_{\mathcal{H}}$

Even though the International Association of Geodesy approximates the Earth's geoid with the International Reference Ellipsoid 1924 — the Hayford ellipsoid — for the purposes of visual projection on the celestial sphere, we can consider that the geoid is approximated by sphere $\mathcal{E}$ since the longitude λ and latitude φ parameters will offer the same projection points on the celestial sphere.

Therefore, given the values of longitude λ and the latitude φ, considering spherical coordinates (ρ, θ, ϕ) (where $x = \rho cos\theta sin\phi$, $y = \rho sin\theta sin\phi$, $z = \rho cos\phi$) and the origin of the coordinate system at the centre of $\mathcal{E}$, we have the location of the Chartres cathedral at $(\theta_c, \phi_c) \simeq (0.03, 0.73)$; the radius ρ of the sphere $\mathcal{E}$ is irrelevant in our paper, so for our calculations we consider $\rho = 1$. As the starting point for measuring angle θ, we consider the intersection of the equator and the Greenwich Meridian. Throughout our paper, the angle amplitudes are in radians. Based on this data $(1, \theta_c, \phi_c)$, we obtain the Cartesian coordinates for the cathedral's position: $\vec{C} = (x_C, y_C, z_C) \simeq (0.66, 0.02, 0.75)$. If we now consider that Chartres Cathedral is on the meridian determined by $\theta = \frac{\pi}{2}$ (we can consider this because longitude λ is irrelevant due to the daily rotation of the celestial sphere), the horizon plan is directed by vectors $\vec{v} = (1,0,0)$, $\vec{u} = \left(0, sin\left(\frac{\pi}{2} + \phi_c\right), cos\left(\frac{\pi}{2} + \phi_c\right)\right) \simeq (0,0.75, -0.66)$. Therefore, the a direction vector of the longitudinal axis of the cathedral is $\vec{w} = cos\left(\frac{\pi}{2} - \xi\right)\vec{v} + sin\left(\frac{\pi}{2} - \xi\right)\vec{u}$, where $\xi \simeq 0.76$ is the amplitude in radians of the orientation angle γ, that is, $\vec{w} = (w_x, w_y, w_z) \simeq (0.69, 0.54, -0.48)$.

The visual straight line $v_{\mathcal{H}}$ parallel to the longitudinal axis of the cathedral and passing through the centre of the main rose window is $O + \mu\vec{w}$, such that $\mu \in \mathbb{R}$. This visual straight line $v_{\mathcal{H}}$ intersects the celestial sphere at point P, having right ascension and declination coordinates (α, δ_h) such that point P moves due to the Earth's rotation. Its right ascension α changes, thus generating a circle $\mathcal{C}_{\delta_h}$ of celestial points having the same declination δ_h. Therefore, $\mathcal{C}_{\delta_h}$ is the celestial circle determined by the horizontal visual straight line passing through the centre of the main rose window each day, and it is the daily astronomical orbit of the stars pointed at by $v_{\mathcal{H}}$ (Figure 7).

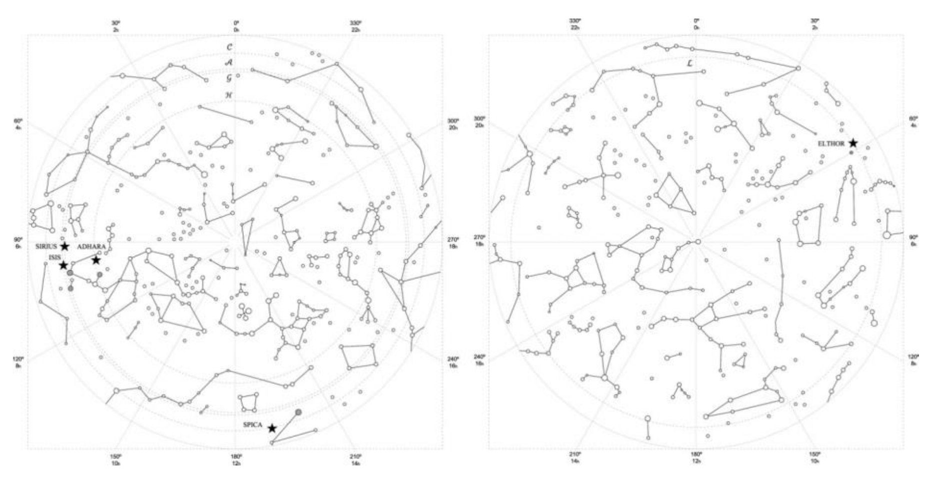

Figure 7. Celestial orbits $\mathcal{C}_{\delta_h}$, $\mathcal{C}_{\delta_l}$, $\mathcal{C}_{\delta_c}$, $\mathcal{C}_{\delta_a}$ and $\mathcal{C}_{\delta_g}$ for the positions $\mathcal{H}$, $\mathcal{L}$, $\mathcal{C}$, $\mathcal{A}$ and $\mathcal{G}$, respectively: (left) Austral hemisphere, (right) Boreal hemisphere. The black circles represent the stars of the ancient catasterism in the epoch *J2000*, while the black five-pointed marks represent the same stars in the epoch *J1200*.

Table 2. Celestial body declinations intrinsically determined by Chartres cathedral.

Ambulatory $\mathcal{G}$	$\delta_g \simeq -15^o 51' 19''$
Apse $\mathcal{A}$	$\delta_a \simeq -14^o 42' 34''$
Transept $\mathcal{C}$	$\delta_c \simeq -07^o 50' 29''$
Labyrinth $\mathcal{L}$	$\delta_l \simeq +09^o 28' 24''$
Horizon $\mathcal{H}$	$\delta_h \simeq -28^o 36' 22''$

To find the declination δ_h, which determines the orbit $\mathcal{C}_{\delta_h}$, we can assume for convenience that the celestial sphere has radius 1, so $\phi'_h = arcos(w_z)$ and $\delta_h = \frac{\pi}{2} - \phi'_h \simeq -0.50$ radians. In conclusion, every day the horizontal visual straight line $v_{\mathcal{H}}$ passing through the centre of the main rose window points at all stars in circle $\mathcal{C}_{\delta_h}$ having declination $\delta_h \simeq -28^o 36' 22''$ on the celestial sphere (Table 2).

2.3. Calculating the celestial declination of the labyrinth's visual straight line $v_{\mathcal{L}}$

With vectors $\vec{v}$ and $\vec{u}$, we calculate the unit vector $\vec{v} \times \vec{u} = \vec{z}$, where $\times$ is the cross product marking the zenith point of the celestial sphere vertically above the location of Chartres cathedral, so $\vec{z} \simeq (0{,}0.66{,}0.75)$. Thus, knowing $r = 30.75\,m$, $l = 32.20\,m$, $\bar{l} = 0.60\,m$ and the rose window's deviation $d = 0.25\,m$ (Table 1), we find that the visual straight line $v_{\mathcal{L}}$ passing through $\mathcal{L}$ and $\mathcal{O}$ is $\mathcal{O} + \mu\vec{l} = \mathcal{O} + \mu\left(\sqrt{l^2 + (r-\bar{l})^2}\,(cos(\omega_l)\vec{w} + sin(\omega_l)\vec{z}) + 0.25\vec{\zeta}\right)$ such that $\mu \in \mathbb{R}$, where ω_l is the amplitude of angle $\angle\left(\vec{w}, \overline{\mathcal{L}\mathcal{O}'}\right)$ in radians and $\vec{\zeta} = \vec{w} \times \vec{z}$ (Figure 4). $\mathcal{O}'$ would be the centre of the rose window if it was not deviated. Through calculations, we obtain $\vec{\zeta} \simeq (0.72, -0.52{,}0.46)$, $\omega_l = arctan\left(\frac{r-\bar{l}}{l}\right)$ and also $\vec{l} = (l_x, l_y, l_z) \simeq (22.47{,}37.26{,}7.26)$. The vector equation with standardized vector $\vec{l}_n$, i.e. $\|\vec{l}_n\| = 1$, of the visual straight line $v_{\mathcal{L}}$ is $\mathcal{O} + \mu\vec{l}_n$, where $\vec{l}_n = (l_{xn}, l_{yn}, l_{zn}) \simeq (0.51{,}0.85{,}0.16)$.

Similar to the case of the visual straight line $v_{\mathcal{H}}$, the visual straight line $v_{\mathcal{L}}$ intersects the celestial sphere at point P having right ascension and declination coordinates (α, δ_l) such that point P moves due to the earth's rotation; its right ascension α changes and thus generates a circle $\mathcal{C}_{\delta_l}$ of celestial points having the same declination δ_l. Therefore, $\mathcal{C}_{\delta_l}$ is the celestial circle determined by the visual straight line passing through the centre of the rose window and through the centre of the labyrinth each day, and it is the daily astronomical orbit of the stars pointed at by $v_{\mathcal{L}}$ (Figure 7).

To find the declination δ_l which determines this orbit $\mathcal{C}_{\delta_l}$, we can assume for convenience that the celestial sphere has radius 1, so $\phi'_l = arcos(l_{zn})$ and $\delta_l = \frac{\pi}{2} - \phi'_l \simeq 0.17$ radians. In conclusion, every day the visual straight line $v_{\mathcal{L}}$ passing through the centre of the labyrinth and the centre of the rose window points at all the stars in circle $\mathcal{C}_{\delta_l}$ having declination $\delta_l \simeq +09^{o}\,28'24''$ on the celestial sphere (Table 2).

2.4. Table of the celestial declinations as determined by the parameters of Chartres cathedral

Similar to the previous two subsections, we find the astronomical declinations as determined by the five architectural viewing directions $v_{\mathcal{H}}$, $v_{\mathcal{L}}$, $v_{\mathcal{C}}$, $v_{\mathcal{A}}$ and $v_{\mathcal{G}}$. As a summary, Table 2 shows the declination of the stars which can be viewed from each of the five positions intrinsically determined by Chartres cathedral.

3. Celestial declinations at astronomical epoch *J1200* as determined by the catasterism

3.1. Astronomical epoch J1200

To make objective claims, we have taken the celestial right ascension and declination coordinates for all astronomical positions on the celestial sphere.

The declination values stated in the previous section (Table 2) are intrinsic to Chartres cathedral, and they do not depend on the astronomical epoch; in other words, they depend only on the cathedral itself and they have remained invariant throughout the 800 years since the epoch *J1200*, close to the period when Chartres cathedral was built (Figure 4).

Nevertheless, an unavoidable problem arises: the positions of the stars on the celestial sphere are changing constantly. This variation is large because of the precessional motion of the Earth's axis (precession of equinoxes), and it is small due to the proper motion of each star. However, in any case, the combined variation caused by both movements must be taken into account. Therefore, the astronomical positions of the stars do vary over the 800 years. For this paper, considering only the mean equinox and not the true equinox is sufficient; in other words, it is sufficient to consider each star's proper motion and the Earth's precessional motion without taking into account the effects of nutation, annual aberration, annual parallax or gravitational deflection of light because the combined effects account for a difference of seven arcseconds at the most, which is irrelevant in this paper.

At the time when the cathedral was built, powerful optical instruments did not exist. Therefore, we only consider those stars which are visible to the naked eye; that is, those having visual magnitude $v \leq 6$ on the celestial sphere. The astronomical data of these stars can be found in the SIMBAD astronomical database [SIMBAD, http://simbad.u-strasbg.fr/simbad]. It is well known that Hipparchus first ranked stars in six magnitude classes according to their brightness, assigning magnitude $v = 6$ to the faintest stars which are barely visible to the naked eye. According to this scale, *Spica* is $\sqrt[5]{100^{6-0.97}} \simeq 102.8$ times brighter than the visual limit. Having visual magnitude $v = 0.97$, *Spica* $(\alpha\, Vir)$ has been typically used as a reference for visual magnitudes. For instance, in the *Piscis Austrinus* constellation, *Fomalhaut* $(\alpha\, PsA)$ has a visual magnitude $v = 1.16$ (its name is Arabic, meaning 'mouth of the fish'), and we find that *Spica* is $\sqrt[5]{100^{1.16-0.97}} \simeq 1.19$ times brighter tan *Fomalhaut*.

Regarding the star positions, there is very little difference between the present epoch (Figure 7) and the reference epoch *J2000*, which corresponds to the epoch of Julian day *JD(2451545.0)*. However, as a result of the Earth's precessional motion and each star's proper motion, we must consider the astronomical epoch *J1200*, which corresponds to the epoch of Julian day *JD(2159345.0)*.

Readers may turn to any book on Astronomy [10] for the definitions and calculations of CE Gregorian year, CE and BCE Julian year and Julian day number *JD*(#) according to the time measurement system proposed by Joseph Scaliger.

Taking into account all of the aforementioned points, to make the relevant calculations taking the celestial right ascension and declination coordinates of any star in astronomical epoch *J2000* and obtaining their corresponding celestial coordinates in astronomical epoch *J1200*, we have considered the mean equinox, the Earth's axis precessional motion and each star's proper motion; we have used the astronomical algorithms stated in [10, p. 134–135] and the data contained in the SIMBAD database [http://simbad.u-strasbg.fr/simbad].

3.2. Declination of the celestial bodies which make up the ancient catasterism

In the Middle Ages, the Western world only recognized the 48 constellations listed in Ptolemy's *Almagest* (85–165 CE) [11, 12]. Three of these 48 constellations represent virgins: the *Taurus* (Tau), the *Canis Major* (CMa) and the *Virgo* (Vir).

The three catasterism constellations (Tau representing a bull, CMa a dog and Vir a virgin) contain a total of 142 (Tau) + 87 (CMa) + 97 (Vir) = 326 stars visible to the naked eye without using optical instruments.

However, of 326 stars determined by these three constellations, only a few symbolize virgins: those which directly represent the global catasterisms of the three constellations in a single star and those directly representing a virgin goddess of motherhood. Therefore, we have as follows:

- **Taurus**: This is the catasterism of Isis [13] the Egyptian cow-goddess of motherhood [2, p. 202–221; 3, p. 42–91; 14]. It is also called Isis and *Bubulum caput* (cow's head) (see [15, *Tabula Vigesima Tertia* - *Tavrvs*]). Among the 142 visible stars in *Taurus*, none of them implicitly represents a virgin goddess of motherhood, but one directly represents the bull or cow: ***Elthor*** $(\lambda\, Tau)$. Its name is Arabic and means 'the bull'. Bayer also mentions it in his *Tabula Vigesima Tertia* from *Uranometria* [15] with the Latin name *Inpectore*, which means 'on the chest'; the star is located at the centre of the bull's chest. It has a visual magnitude of $v = 3.41$ (*Spica* is 9.46 times brighter than *Elthor*).
- **Canis Major**: This is the catasterism of Isis ([1, p. 59–60; 13, p. 117–124]). Among the 87 visible stars in *Canis Major* (Latin for 'greater dog'), three stars represent virgins, and one of the three directly represents the 'dog'. These stars are ***Isis*** $(\gamma\, CMa)$ (also known by its Arabic name, *Muliphein*, meaning 'the leader'), ***Adhara*** $(\varepsilon\, CMa)$ and ***Sirius*** $(\alpha\, CMa)$. *Tabula Trigesima Octava* - *Canis Maior* from *Uranometria* [15] locates *Isis* on the dog's forehead. *Isis* has a visual magnitude of $v = 4.12$ (*Spica* is 18.20 times brighter than *Isis*). *Adhara*, from the Arabic word $Adh\bar{a}r\bar{a}$, meaning 'virgins' [13, p. 130] – is located at the centre of the dog; it is the second

brightest star in the constellation with a visual magnitude of $v = 1.50$ (*Spica* is 1.63 times brighter than *Adhara*). *Sirius* is located at the centre of the dog's snout; it is also known as the 'Dog star' and also represents Isis [1, p. 59–60; 3, p. 339; 13, p. 123–124; 14]. It has a visual magnitude of $v = -1.46$ (*Sirius* is 9.38 times brighter than *Spica*).

- **Virgo**: Even though this constellation globally represents the goddesses Atargatis, Ceres, Isis, Erigone, Ishtar, Demeter, Dike, Tyche, Derceto [1, p. 40–41; 13, p. 460–466; 15, *Tabula Vigesima Septima - Virgo*], one of the 97 visible stars in *Virgo* (the virgin) individually represents a virgin. This star is *Erigone*, also known as ***Spica*** $(\alpha\ Vir)$, or the 'Virgin's spike' [13, p. 466–467; 15, *Tabula Vigesima Septima - Virgo*].

The above is summarized as follows:

Among the 48 constellations recognized in the Middle Ages, three represent ancient virgins: *Taurus*, *Canis Major* and *Virgo*. These three constellations include a total of 326 stars which are visible to the naked eye (without using optical instruments). Among this group of 326 stars, there are five which are related to ancient virgins; therefore, the choice of stars is not speculative but objective in nature. In order of celestial declination, these stars are: *Elthor*, *Spica*, *Isis*, *Sirius* and *Adhara* (Figure 7). These stars, determined by the ancient catasterism, are clearly visible to the naked eye because, as we have pointed out in subsection 3.1, all of them are bright enough relative to *Spica*.

To assign each star to an astronomical position on the celestial sphere, we used the SIMBAD database [http://simbad.u-strasbg.fr/simbad]. The obtained positions for the stars are listed in Table 3 (values in ICRS coordinates for reference epoch *J2000.0*, in order of celestial declination, see Figure 7). Taking into account the continuous movement of the stars across the celestial sphere as a result of the precession of equinoxes and the proper motion of each star, we can find the stars' true positions — and therefore their declinations — close to the period when Chartres Cathedral was built (*J1200*, see Figure 4). Through the relevant calculations [10, p. 134–135], we obtain the celestial positions for the reference epoch *J1200* (see the last two columns in Table 3).

Table 3. Coordinates of the celestial bodies which make up the catasterism.

	Right ascension α *J2000*	**Declination δ *J2000***	**Proper motion *mas/yr* in α**	**Proper motion *mas/yr* in δ**	**Right ascension α *J1200***	**Declination δ *J1200***
Elthor	$04^h00^m41^s$	$+12^\circ29'25''$	-8.02	-14.42	$03^h16^m53^s$	$+09^\circ55'02''$
Spica	$13^h25^m12^s$	$-11^\circ09'41''$	-42.35	-30.67	$12^h43^m32^s$	$-06^\circ52'23''$
Isis	$07^h03^m45^s$	$-15^\circ37'60''$	-0.14	-11.36	$06^h27^m35^s$	$-14^\circ44'56''$
Sirius	$06^h45^m09^s$	$-16^\circ42'58''$	-546.01	-1223.07	$06^h09^m51^s$	$-15^\circ54'24''$
Adhara	$06^h58^m38^s$	$-28^\circ58'20''$	$+3.24$	$+1.33$	$06^h27^m14^s$	$-28^\circ08'33''$

4. Astronomical relationship between the catasterism and the parameters of Chartres cathedral

Chartres Cathedral would be irrelevant if our analysis only took into account the cathedral’s location $\{\lambda, \varphi\}$. In that case, the latitude φ would be somewhat significant, but the longitude λ would be completely unimportant due to the daily rotation of the celestial sphere. Therefore, to achieve conclusions which are really related to the cathedral, the analysis must include the architectural parameters intrinsic to the $\{\gamma, \mathcal{G}, \mathcal{A}, \mathcal{C}, r, d, \mathcal{L}, \bar{l}, \bar{c}, \bar{a}, \bar{g}\}$.

4.1. Stars’ orbits as projected by the rose window on the cathedral’s floor plan

Each of the five stars which make up the catasterism, *Elthor*, *Spica*, *Isis*, *Sirius* and *Adhara*, has a daily circular orbit of $\mathcal{C}_{Elt}$, $\mathcal{C}_{Spi}$, $\mathcal{C}_{Isi}$, $\mathcal{C}_{Sir}$ and $\mathcal{C}_{Adh}$ as a result of the daily rotation of the celestial sphere (see Figure 7).

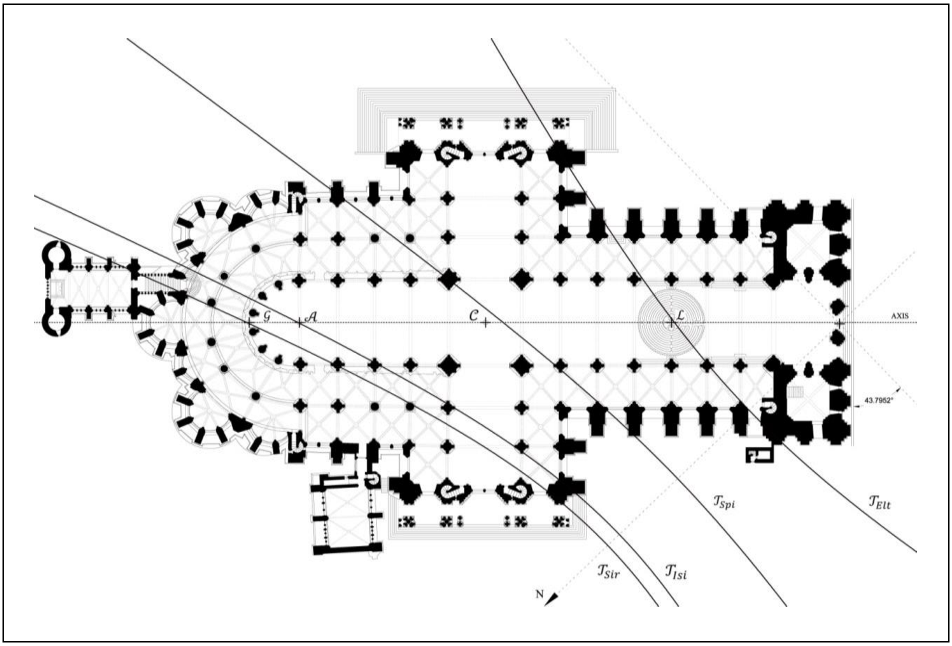

Figure 8. Paths $\mathcal{T}_i$ on the floor plan of Chartres cathedral.

Next, with the 13 parameters considered $\{\lambda, \varphi, \gamma, \mathcal{G}, \mathcal{A}, \mathcal{C}, r, d, \mathcal{L}, \bar{l}, \bar{c}, \bar{a}, \bar{g}\}$, the five daily orbits are projected on the different platforms inside Chartres Cathedral through the centre of the rose window. By means of these projections, we obtain four paths $\mathcal{T}_i$ on the different platforms (one path for each star: $\mathcal{T}_{Elt}$, $\mathcal{T}_{Spi}$, $\mathcal{T}_{Isi}$ and $\mathcal{T}_{Sir}$, respectively). In addition, we will obtain a fifth path $\mathcal{T}_{Adh}$ for *Adhara*. The latter path is not present on the cathedral's floor plan at any time during the daily rotation of the celestial sphere; to observe this star from inside the cathedral, we would have to place ourselves at the same height as the centre of the rose window. To plot the paths $\mathcal{T}_i$ in the astronomical epoch *J1200* with total control of the graphic process, we designed a software application. We entered all values in CAD vector format, and we precisely redrew the floor plan of the Chartres Cathedral. The obtained result is shown in Figure 8. We emphasize that the paths $\mathcal{T}_i$ of the four stars cut the central axis of the cathedral.

By simply observing Figure 8, it is evident that the paths $\mathcal{T}_{Elt}$, $\mathcal{T}_{Spi}$, $\mathcal{T}_{Isi}$ and $\mathcal{T}_{Sir}$ are graphically related to the points $\mathcal{G}$, $\mathcal{A}$, $\mathcal{C}$ and $\mathcal{L}$. The correlation in Figure 8 is not exact because the paths $\mathcal{T}_{Elt}$, $\mathcal{T}_{Isi}$ and $\mathcal{T}_{Sir}$ show minor deviations with respect to the points $\mathcal{L}$, $\mathcal{A}$ and $\mathcal{G}$, respectively, and the path $\mathcal{T}_{Spi}$ shows a larger deviation with respect to point $\mathcal{C}$. Such deviations are due to two geometric reasons: first, the angle formed between the platforms and the visual projection's straight lines $v_{\mathcal{L}}$, $v_{\mathcal{C}}$, $v_{\mathcal{A}}$ and $v_{\mathcal{G}}$; and second, the differences in declination between the visual straight lines and the catasterisms. Moreover, the deviations seen in Figure 8 contain error components that arise for reasons related to the building's construction, for instance decisions made by the architect during the construction process, building errors and inaccuracies, or even structural settlings of the building over 800 years.

In any case, we conducted a rigorous geometric/statistical analysis to measure and show the perfection of the correlation between the astronomical paths and the parameters of Chartres cathedral.

5. Discussion of the rigorous geometric/statistical analysis of correlation

The path $\mathcal{T}_i$ shown in Figure 8 apparently correlates with the architectural elements $\mathcal{G}$, $\mathcal{A}$, $\mathcal{C}$ and $\mathcal{L}$. However, the real correlation is the geometric correlation between the orbits $\mathcal{C}_{Elt}$, $\mathcal{C}_{Spi}$, $\mathcal{C}_{Isi}$, $\mathcal{C}_{Sir}$ and $\mathcal{C}_{Adh}$ and the elements $\mathcal{L}$, $\mathcal{C}$, $\mathcal{A}$, $\mathcal{G}$ and $\mathcal{H}$. In other words, the real correlation is the one which determines whether the positions of these elements of the cathedral are projected on the stars *Elthor*, *Spica*, *Isis*, *Sirius* and *Adhara* through the centre $\mathcal{O}$ of the main rose window. More precisely, we intend: a) to confirm whether or not there is a correlation between the celestial declinations obtained in section 2 (Table 2) and the celestial declinations obtained in section 3 (Table 3) and b) to calculate the geometric/statistical magnitude of that correlation.

5.1. Method to calculate the correlation between the catasterism and the parameters of Chartres cathedral

Next, we describe the method used in subsection 5.2 to confirm and calculate that correlation.

After rigorously calculating the celestial declination coordinates consistent with the problem posed in section 2 (Table 2) and in section 3 (Table 3), to establish a mathematical correlation between the declinations of elements $\mathcal{L}$, $\mathcal{C}$, $\mathcal{A}$, $\mathcal{G}$ and $\mathcal{H}$ of Chartres cathedral and the declinations of the stars forming the catasterism, we proceeded as follows:

1) We statistically calculated the probability of rejecting the null hypothesis h_0 = 'There is no linear correlation between the declinations determined by the orbits of the stars forming the ancient catasterism and the declinations determined by the elements of the cathedral', i.e. we calculated the significance level κ. The probability of such linear correlation is $1-\kappa$. The corresponding calculations used Student's *t*-test with n-2 degrees of freedom, where n = 5 is the number of coordinates.
2) We calculated the extent to which the declinations of the stars forming the catasterism explain the declinations of the elements of the cathedral using this correlation. For the necessary calculations, we used Pearson's adjusted coefficient of determination expressed as a percentage $\eta^2_{adj} \times 100$.
3) We calculated a measure of geometric similarity between the two sets of coordinates. This measure of geometric similarity is the geometric deviation D between the linear regression line and the first and third quadrants bisectrix within the coordinate range of the problem. More specifically, we calculated the deviation $\mathrm{D} = \max_{x\in[a,b]}|\rho + x\tan\theta - x|$, where $y = \rho + x\tan\theta$ is the regression line equation, $y = x$ is the bisectrix equation and a and b are the coordinates' minimum and maximum values, respectively. The measure θ of the regression line slope angle is determined using the classical variance and covariance's calculation, and parameter ρ is determined using the classical means and variances calculation.

5.2. Calculations and obtained measurements and results

In this subsection, we state the results from all processes and calculations detailed in the previous subsection.

Table 4 below shows the declinations of the stars constituting up the catasterism in the astronomical epoch *J1200* and the declinations given by the architectural elements of Chartres cathedral.

Table 4. Astronomical declinations of the stars constituting the catasterism in the astronomic epoch *J1200* and the declinations given by the cathedral's architectural elements.

Catasterism at *J1200.0*		**Chartres cathedral**	
Stars	**Declinations δ**	**Elements**	**Declinations δ**
Elthor	$+09^o55'02'' = p_1$	$\mathcal{L}$	$+09^o28'24'' = q_1$
Spica	$-06^o52'23'' = p_2$	$\mathcal{C}$	$-07^o50'29'' = q_2$
Isis	$-14^o44'56'' = p_3$	$\mathcal{A}$	$-14^o42'34'' = q_3$
Sirius	$-15^o54'24'' = p_4$	$\mathcal{G}$	$-15^o51'19'' = q_4$
Adhara	$-28^o08'33'' = p_5$	$\mathcal{H}$	$-28^o36'22'' = q_5$

From the data in Table 4, we find that the mean $m_p = \sum_{i=1}^{i=5}\frac{p_i}{5}$ of the p_i is $m_p \simeq -11.15$, and the mean m_q of the q_i is $m_q \simeq -11.51$. The standard deviation $\sigma_p = \sqrt{\sum_{i=1}^{i=5}\frac{(p_i-m)^2}{5}}$ of the p_i is $\sigma_p \simeq 12.54$, and the standard deviation σ_q of the q_i

is $\sigma_q \simeq 12.45$. The covariance $\sigma_{pq} = \sum_{i=1}^{i=5}\frac{(p_i-m_p)(q_i-m_q)}{5}$ between the p_i and the q_i is $\sigma_{pq} \simeq 156.09$. Pearson's correlation coefficient $R_{pq} = \frac{\sigma_{pq}}{\sigma_q \sigma_p}$ of the two sets is $R_{pq} \simeq 0.99$. Therefore, Pearson's adjusted coefficient of determination $\eta^2_{adj} = 1-\left(1-R^2_{pq}\right)\frac{5-1}{5-2-1}$ is $\eta^2_{adj} \simeq 0.98$.

Further, $t_{pq} = \left|\frac{R_{pq}\sqrt{5-2}}{\sqrt{1-R^2_{pq}}}\right| \simeq 12.16$. Thus, if we apply Student's *t*-test with $5-2$ degrees of freedom, the null hypothesis h_0 is rejected with significance level κ such that $\kappa \simeq 0.0005$, i.e. $1-\kappa \simeq 0.9994$.

The angular amplitude parameter $tan\ \theta = \frac{\sigma_{pq}}{\sigma_p^2}$ is $tan\ \theta \simeq 0.99$, the central parameter $\rho = m_q - \frac{\sigma_{pq}}{\sigma_p^2} m_p$ is $\rho \simeq -0.44^o$, the range $[a, b]$ of parameters p_i is $[-28.14, 9.92]$. Considering all of the above, the geometric deviation D is $D \simeq 0.52^o$. Table 5 summarizes the results of the above calculations: the probability of linear correlation, Pearson's coefficient of determination expressed as a percentage $\eta^2_{adj} \times 100$ and the geometric deviation D.

Table 5. Summary of the measurement values obtained as results of the geometric/statistical analysis.

Geometric/statistical parameters		
$1-\kappa, R_{pq}$	$\eta^2_{adj} \times 100$	D
$0.9994, 0.99$	98%	$0^o.52$

6. Conclusions

Table 5 summarizes the measurement values obtained as a result of the calculations described in section 5. The table provides the following information:

1) There is a linear correlation between the declinations of the stars constituting the catasterism of virgin goddesses in the astronomical epoch *J1200*, close to the period when Chartres cathedral was built (Figure 4) and the declinations given by the constructive design of the cathedral; the probability that this correlation exists is $1-\kappa = 0.9994$ with Pearson's coefficient $R_{pq} \simeq 0.99$.
2) Given the above linear correlation, the declinations of the stars constituting the catasterism almost completely (by $\eta^2_{adj} \times 100 \simeq 98\%$) explain the declinations generated by the constructive parameters $\{\lambda, \varphi, \gamma, \mathcal{G}, \mathcal{A}, \mathcal{C}, r, d, \mathcal{L}, \bar{l}, \bar{c}, \bar{a}, \bar{g}\}$ of the cathedral.
3) The geometric deviation D between both declination patterns is very small ($D \simeq 0^o.52$).

4) This correlation occurred not only for one day but for every day of every year during the construction of the cathedral, given the earth's slow precessional motion and the stars' proper movement (see Section 3.1).

Simply put, we have geometrically shown that, with no affirmation or refusal of any coincidence, at the astronomical epoch *J1200.0*: the labyrinth, the transept, the apse, the ambulatory and the main rose window of Chartres cathedral align accurately with the astronomical pattern of the stars constituting the ancient catasterism of virgin goddesses — *Elthor*, *Spica*, *Isis*, *Sirius* and *Adhara* — belonging to the *Taurus*, *Virgo* and *Canis Major* constellations. The choice of stars is not speculative in nature, but objective.

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