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# Definition and Calculation of an Eight-Centered Oval which is Quasi-Equivalent to the Ellipse

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**Abstract.** Let $\mathcal{E}_b$ be an ellipse ($b = \frac{minor\ axis}{major\ axis}$). In this paper we consider different approximations by ovals, which are composed from circular arcs and have also two axes of symmetry. We study a) three four-centered ovals (quadrarcs) $O^a_{4,b}$, $O^c_{4,b}$, and $O^l_{4,b}$, which share the vertices with the ellipse $\mathcal{E}_b$. In addition, $O^a_{4,b}$ has the same surface area, $O^c_{4,b}$ has the minimum error of curvature at the vertices, and $O^l_{4,b}$ has the same perimeter length. b) Further, we investigate three eight-centered ovals $O^c_{8,b}$, $O^{c-a}_{8,b}$ and $O^{c-l}_{8,b}$, which also share the vertices with $\mathcal{E}_b$. The ovals $O^c_{8,b}$ have the same curvature at the vertices, and in addition, $O^{c-a}_{8,b}$ has the same surface area, and $O^{c-l}_{8,b}$ has the same perimeter length as $\mathcal{E}_b$.
As a conclusion, the eight-centered oval $O^{c-l}_{8,b}$ seems to be optimal and can therefore be called 'quasi-equivalent' to $\mathcal{E}_b$. We show that the difference of surface areas $\mathcal{A}_b = \mathcal{A}(O^{c-l}_{8,b}) - \mathcal{A}(\mathcal{E}_b)$ is rather small; the maximum value $\mathcal{A}_{0.1969} = 0.007085$ is achieved at $b = 0.1969$. The deformation error $E_b = E(\mathcal{E}_b, O^{c-l}_{8,b})$ has the maximum value 0.008970 which is achieved at $b = 0.2379$.

*Key Words:* Eight-centered oval, quadrarc, quasi-equivalent oval, ellipse.
*MSC 2010:* 51M04, 51N20

## 1. The ellipse and the eight-centered oval

Approximating ellipses by circular arcs has been a classic subject of study by geometers. This has long been used for a wide range of applications, for instance in geometry, astronomy, art, architecture. The reader can easily find a great deal of classical literature on these topics, in special for eight-centered ovals and four-centered ovals (also named *quadrarcs*). This kind of approximation, using eight-centered ovals, has recently been used to analyze architectural constructions as amphitheaters and military forts [5, 6, 7, 15]; also in astronomy, for analyzing

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