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The strategic value of partial vertical integration

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Abstract

We investigate the strategic incentives for partial vertical integration, namely, partial ownership agreements between manufacturers and retailers, when retailers privately know their costs and engage in differentiated good price competition. The partial misalignment between the profit objectives within a partially integrated manufacturer-retailer hierarchy entails a higher retail price than under full integration. This 'information vertical effect' translates into an opposite 'competition horizontal effect': the partially integrated hierarchy's commitment to a higher price induces the competitor to increase its price, which strategically relaxes competition. Our analysis provides implications for vertical merger policy and theoretical support for the recently documented empirical evidence on partial vertical acquisitions.

Keywords: asymmetric information, partial vertical integration, vertical mergers, vertical restraints.

JEL Classification: D82, L13, L42.

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1. Introduction

Most of the practical and theoretical debate about the firms' organizational structure in vertically related markets has focused on two extreme alternatives: full vertical integration and separation. However, it is quite common to observe partial vertical integration, namely, partial ownership agreements in which a firm acquires less than 100% of shares in a vertically related firm (e.g., Allen and Phillips 2000; Fee et al. 2006; Reiffen 1998). Emphasizing the relevance of partial vertical integration, Riordan (2008) reports that in 2003 News Corp., a major owner of cable programming networks in the US, acquired 34% of shares in Hughes Electronics, which operates via its wholly-owned subsidiary Direct TV in the downstream market of direct broadcast satellite services. Gilo and Spiegel (2011) provide empirical evidence that partial vertical integration is much more common than full integration in telecommunications and media markets in Israel. For instance, Bezeq operates in the broadband Internet infrastructure market and holds a share of 49.77% in DBS Satellite Services that competes in the downstream multi-channel broadcast market.

Partial acquisitions have recently received great attention in antitrust control.[1] Despite the practical relevance of this phenomenon, relatively little theoretical research has been devoted so far to partial vertical acquisitions. The aim of this paper is to investigate the strategic incentives of vertically related firms to partially integrate and their competitive effects.

We address this question in a setting where two manufacturer-retailer hierarchies engage in differentiated good price competition and retailers are privately informed about their production costs. The economic literature has emphasized since Crocker's (1983) seminal contribution that a major problem within a supply hierarchy is that a firm can access privileged information about some relevant aspects of the market. In our framework, a manufacturer exclusively deals with its retailer, which is reasonable in the presence of product-specific investments that have to be sunk before production decisions take place.[2] Moreover, in line with the main literature on competing hierarchies under asymmetric information (e.g., Coughlan and Wernerfelt 1989; Katz 1991; Martimort 1996; Martimort and Piccolo 2010), bilateral contracting within a supply hierarchy is secret. This reflects the natural idea that the trading rules specified in a contractual relationship are not observed by competitors and therefore cannot be used for strategic purposes. Alternatively, these rules can be easily (secretly) renegotiated if both parties agree to do so.

In the benchmark case of full information within a supply hierarchy, a manufacturer that uses non-linear (secret) contracts is indifferent about the ownership stake in its retailer. This is because the manufacturer makes the retailer residual claimant for the hierarchy's profits and appropriates these profits through a fixed fee. The outcome of vertical integration is achieved irrespective of the ownership stake, and therefore vertical ownership arrangements are inconsequential.

This well-known 'neutrality result' (Coughlan and Wernerfelt 1989; Katz 1991) does not hold in the presence of asymmetric information. To begin with, consider a successive monopoly framework where a manufacturer-retailer pair operates in isolation and the retailer is privately

[1] In the sequel, we discuss the antitrust approach to partial acquisitions.

[2] For instance, bilateral exclusive relationships are common in the video rental market. Blockbuster provides each downstream retailer with the exclusive right to sell its brand in a geographical area where competing retailers distribute alternative brands.

informed about its costs. It is well established in the economic literature (e.g., Gal-Or 1991c) that asymmetric information within a supply hierarchy entails a higher retail price in order to curb the (costly) informational rents to the retailer. Full vertical integration guarantees the owner of the hierarchy complete control and removes the problem of asymmetric information, which improves the hierarchy's joint profits.

We show that the strict preference for full vertical integration does not carry over in a competitive environment. In a setting where two manufacturer-retailer pairs engage in differentiated good price competition, partial vertical integration can emerge in equilibrium. In line with the successive monopoly framework, a partial vertical ownership agreement entails an *information vertical effect*: the partial misalignment between the profit objectives of the manufacturer and the retailer leads to a higher retail price than under full integration in order to reduce the informational rents to the retailer. For a given price of the competitor, this form of double marginalization from asymmetric information reduces the hierarchy's profitability relative to full integration. In a competitive environment, however, the information vertical effect translates into an opposite *competition horizontal effect*: the partially integrated hierarchy's commitment to a higher price induces an accommodating behavior of the rival that increases its price as well. Therefore, partial vertical integration is profitable since it constitutes a strategic device to relax competition. The trade-off between the benefits of softer competition and the informational costs drives the equilibrium degree of vertical integration.

To better appreciate the rationale for our results, it is important to realize that, when a manufacturer is partially integrated with its retailer, it is common knowledge that there exists no contract which can 'solve' the problem of asymmetric information within the supply hierarchy. It follows from the seminal paper of Katz (1991) that, even when contracting is secret, this can affect the play of the continuation game. In our model, the rival firm — the 'outside party' in Katz's (1991) terminology — anticipates that the partially integrated hierarchy's retail price will be higher than under full integration, which induces the rival to increase its price in a game of strategic complementarity. Therefore, partial vertical integration exhibits a commitment value à la Katz (1991) that relaxes competition.

Partial ownership agreements have been examined by the US antitrust law since a long time. Section 7 of the Clayton Act of 1914 (currently, Section 18 of Title 15 of the US Code) provides that

> "no person engaged in commerce or in any activity affecting commerce shall acquire, directly or indirectly, the whole or *any part of the stock or other share capital* and no person subject to the jurisdiction of the Federal Trade Commission shall acquire the whole or any part of the assets of another person engaged also in commerce or in any activity affecting commerce, where [...] *the effect of such acquisition may be substantially to lessen competition*, or to tend to create a monopoly".[3]

In the Horizontal Merger Guidelines revised in 2010 a section has been introduced which is explicitly devoted to partial acquisitions. The seminal articles of Bresnahan and Salop (1986), Reynolds and Snapp (1986) and O'Brien and Salop (2000) provide a formal foundation for the

[3]The quotation (with emphasis added) is available at http://www.law.cornell.edu/uscode/text/15/18.

antitrust control of partial acquisitions between rival firms since they can entail a dampening of competition. More recently, Gilo et al. (2006) show the collusive effects of partial cross ownership. Foros et al. (2011) find that rival firms can prefer partial acquisitions to full mergers, which leads to softer competition. The main contribution of our paper is to unveil the strategic incentives for partial ownership agreements between firms that do not compete with each other but are vertically related. Our results indicate that partial vertical acquisitions can emerge in equilibrium and mitigate competition relative to full vertical mergers. As discussed in Section 8, our analysis extends to vertically related markets Foros et al. (2011)'s recommendation for antitrust investigations of partial divestitures. In the same vein, we also provide theoretical support for antitrust policies that favor full mergers over partial acquisitions.

In other countries, such as Austria, Germany, the UK, Australia, Canada, Japan and New Zealand, antitrust authorities are also entitled to scrutinize partial ownership agreements. For instance, the German Bundeskartellamt can investigate partial acquisitions that exhibit a competitively significant influence. However, the European Commission does not have any explicit competence in this area under the current merger control rules. Recent proposals aim at expanding the remit of the merger control function to enable the European Commission to examine partial acquisitions that entail non-controlling minority shareholdings. In the 2013 consultation document 'Towards more effective EU merger control' (p. 3), the European Commission recommends a reform of the current European merger control system in order to

> "extend the scope of the Merger Regulation to give the Commission the option to intervene in a limited number of problematic cases of *structural links* [i.e., partial acquisitions], in particular those creating structural links between competitors or in a *vertical relationship*".[4]

The results of our paper suggest that there is scope for antitrust intervention of the European Commission in this area. Our analysis is presented in a fairly general setting without making any particular assumption on functional forms. Remarkably, it also provides theoretical corroboration for the empirical evidence recently documented in Ouimet (2013) that partial equity stakes are more likely to be preferred to full integration in industries requiring relationship-specific investments, such as vertically related markets. The predictions of our model may serve as guidance for the empirical work on the competitive effects of partial vertical integration.

2. Related literature

As discussed in the introduction, the economic literature has extensively explored the private and social effects of partial acquisitions in horizontally related markets. Conversely, the literature on partial vertical acquisitions is still in its infancy. Dasgupta and Tao (2000) show that partial vertical ownership may perform better than take-or-pay contracts if upstream firms make investments that benefit downstream firms. More recent contributions are Greenlee and Raskovich (2006), Hunold et al. (2012) and Levy et al. (2016). In Section 8 we compare these contributions with our work when discussing the antitrust policy implications of our results.

[4] The document (including the quotation with emphasis added) can be found at http://ec.europa.eu/competition/consultations/2013_merger_control/merger_control_en.pdf.

Our analysis is also related to the literature on the strategic delegation in a competitive environment. The idea that, in a full information framework, delegation can act as a strategic commitment device to relax competition traces back to Fershtman and Judd (1987), Sklivas (1987), Bonanno and Vickers (1988) and more recently Jansen (2003). In particular, Bonanno and Vickers (1988) find that in a differentiated good price competition model a manufacturer prefers to sell its product through an independent retailer rather than directly to consumers if it can publicly commit to a wholesale price above marginal costs, which induces a more lenient behavior of rivals. However, as shown by Coughlan and Wernerfelt (1989) and Katz (1991), this result no longer holds when contracts cannot be observed by rivals. Katz (1991) specifies that the strategic value of delegation is restored if it is common knowledge that there exists no contract that can solve the agency problem. Caillaud and Rey (1995) provide an overview of the strategic use of vertical delegation. Gal-Or (1992, 1999) shows in alternative settings of asymmetric information that firms may follow different strategies of integration and separation. Along these lines, Barros (1997) demonstrates that in an oligopolistic industry some firms may profit from a commitment to face asymmetric information about their agents' operations, since they are prevented from extracting full surplus and can provide the agents with a credible incentive to invest. Contrary to the aforementioned contributions, we allow for a partial degree of vertical integration and show that partial vertical ownership agreements trade off the benefits of softer competition against the informational costs.

Our paper also belongs to the literature on vertical restraints. For our purposes, early relevant contributions are Rey and Stiglitz (1988, 1995) and Gal-Or (1991a, 1991b) that explore the impact of vertical restraints on competition. Martimort (1996) investigates the choice of competing manufacturers between a common and an exclusive retailer in a setting of adverse selection. Martimort and Piccolo (2007) qualify the results of Gal-Or (1991c) about the choice between resale price maintenance and quantity fixing contracts, according to the retailers' technology for providing services. In a model with competing manufacturer-retailer pairs, Martimort and Piccolo (2010) and Kastl et al. (2011) show that manufacturers may strategically prefer quantity fixing to resale price maintenance and explore the welfare consequences of these contractual relationships. Piccolo et al. (2014) investigate the allocation of residual claimancy in a setting with competing principal-agent hierarchies and demonstrate that a principal may find it optimal to retain a share of surplus from production with an inefficient agent because this reduces the mimicking incentives of the efficient agent. Our paper provides novel insights into the interaction between competition and the organizational structure of vertically related firms, and shows that a manufacturer can prefer to partially integrate with its retailer since the partial internalization of the retailer's rents leads to a dampening of competition.

The rest of the paper is structured as follows. Section 3 sets out the formal model. Section 4 considers the benchmark case of a manufacturer fully informed about the costs of its retailer. Section 5 shows that, in the presence of asymmetric information, partial vertical integration can emerge in equilibrium. Using explicit functions, Section 6 derives the equilibrium degree of vertical integration and performs a comparative statics analysis. Section 7 investigates alternative assumptions and the robustness of the results. Section 8 discusses some antitrust policy and empirical implications. Section 9 concludes. All formal proofs are provided in the Appendix.

3. The model

Setting We consider a vertically related market where two manufacturers, M_1 and M_2, provide symmetrically differentiated goods through two retailers, R_1 and R_2, which engage in price competition. As discussed in the introduction, we assume that each manufacturer is in an exclusive relationship with one retailer. In the spirit of Martimort and Piccolo (2010), we examine a setting where manufacturer M_1 and retailer R_1 exclusively deal with each other, while manufacturer M_2 is fully integrated with retailer R_2. As explained in Section 7.5, our results carry over in a more symmetric setting where both supply hierarchies decide on the degree of vertical integration.

We denote by $q_i\left(p_i, p_{-i}\right)$ the (direct) demand function for good $i = 1, 2$, which satisfies the following assumption.

Assumption 1 $-\frac{\partial q_i(p_i,p_{-i})}{\partial p_i} > \frac{\partial q_i(p_i,p_{-i})}{\partial p_{-i}} \geq 0$ *(product substitutability).*

Goods are imperfect substitutes (the second condition holds with equality for independent goods), and own-price effects are larger than cross-price effects.

Manufacturer M_1 offers retailer R_1 a contract that specifies a retail price p_1 for the good and a fixed franchise fee t_1 paid by the retailer to the manufacturer for the right to sell the good. The practice of dictating the final price to a retailer is commonly known as resale price maintenance. In Section 7.3 we show that our main results remain valid under a two-part tariff specifying a unit wholesale price and a fixed fee. Notably, resale price maintenance yields the manufacturer higher profits than a two-part tariff, and therefore our analysis does not depend on any restriction on the contract set that limits the manufacturer's profits.

Let $\theta_1 \in \{\theta_l, \theta_h\}$ be R_1's (constant) marginal costs, whose realization is R_1's private information at the time the contract is signed with M_1. Costs are θ_l with probability $\nu \in (0, 1)$ and θ_h with probability $1-\nu$. We define by $\Delta\theta \equiv \theta_h - \theta_l > 0$ the spread of the cost distribution. Retailer R_1's interim expected profits are

$$\pi_{R_1} = \left(p_1 - \theta_1\right) E_{\theta_2}\left[q_1\left(p_1, p_2\right)|\theta_1\right] - t_1, \tag{1}$$

where $E_{\theta_2}\left[q_1\left(p_1, p_2\right)|\theta_1\right]$ represents the expected quantity of R_1. The profits in (1) are interim expected, since they are evaluated at the contractual stage where R_1 is informed about its own costs θ_1 but does not know the costs θ_2 of the competitor $M_2 - R_2$. We allow for positive correlation between retail costs (e.g., Gal-Or 1991b, 1999; Martimort 1996), and therefore R_1's uncertainty on θ_2 depends on the realization of θ_1. This reflects the idea that in competitive markets the costs of rival firms are usually subject to common trends. Notably, our results fully apply with independent costs. In the example provided in Section 6, we consider the case of perfect cost correlation, which implies $\theta_1 = \theta_2$. In Section 7.4 we investigate the impact of cost correlation on the equilibrium ownership stake that the manufacturer holds in its retailer.

Manufacturer M_1's interim expected profits are

$$\pi_{M_1} = t_1 + \rho\left\{\left(p_1 - \theta_1\right) E_{\theta_2}\left[q_1\left(p_1, p_2\right)|\theta_1\right] - t_1\right\}, \tag{2}$$

which is a weighted sum of the upstream profits from the franchise fee t_1 (manufacturing costs are normalized to zero) and the downstream profits π_{R_1} in (1) from retail operations. When offering a contract to R_1, M_1 is concerned about the profits in (2). The parameter $\rho \in [0,1]$ denotes the ownership stake acquired by M_1 in R_1. Following O'Brien and Salop (2000), ρ captures the financial interest of the acquiring firm, which is entitled to receive a share of the profits of the acquired firm. If $\rho = 0$, the two firms are fully separated. If $\rho \in (0,1)$, M_1 has a partial ownership stake in R_1, and therefore the two firms are partially integrated. If $\rho = 1$, M_1 wholly owns R_1 and the two firms are fully integrated.

It is worth noting that the ownership stake ρ does not appear in R_1's profit function in (1). Therefore, R_1 maximizes the full profits arising from the retail activities irrespective of the ownership stake acquired by M_1. In other terms, all the shareholders of R_1 are treated equally. In line with some relevant contributions (e.g., Farrell and Shapiro 1990; Greenlee and Raskovich 2006; Hunold et al. 2012), this assumption can be justified on several grounds. The acquisition of a passive (non-controlling) ownership stake ensures the acquiring firm a participation in the acquired firm's profits but it does not entail any corporate control. This 'silent financial interest' does not lead to any change in the incentives of the acquired firm (e.g., Bresnahan and Salop 1986). Moreover, corporate law or antitrust law can impose a legal requirement — known as 'fiduciary obligation' —, which provides that the managers of the acquired firm must act in the interest of the firm as an independent, stand-alone entity. The main purpose of this requirement is the protection of the minority shareholders, in particular those with no other holdings. This implies that, even when the acquiring firm holds a large financial interest, the acquired firm continues to maximize its stand-alone profits. O'Brien and Salop (2000) provide an accurate discussion of how the requirement of fiduciary obligation can be implemented in practice.[5]

We wish to derive the equilibrium degree of vertical integration between manufacturer M_1 and retailer R_1, namely, the ownership stake ρ that M_1 decides to acquire in R_1. In line with the main literature on partial acquisitions (e.g., Foros et al. 2011; Greenlee and Raskovich 2006; Hunold et al. 2012), we assume that M_1 chooses the ownership stake ρ in R_1 that maximizes the (expected) joint profits of the two firms. This ensures that M_1 can design an offer to R_1 which makes the shareholders in both firms better off, so that they will find it mutually beneficial to sign such an agreement.[6] A joint profit maximizing ownership agreement does not leave any scope for mutually beneficial renegotiations and exhibits a commitment value. In Section 7.2 we consider the case in which the manufacturer maximizes its own profits when deciding on the ownership stake. In order to focus on the strategic effects of acquisition, we abstract from any cost saving that may arise from the ownership arrangement.

[5] We recognize that a sufficiently high ownership stake may influence the decisions of the acquired firm in favor of the acquiring firm. However, as long as the acquired firm does not fully internalize the objectives of the acquiring firm, the conflict of interests within the supply hierarchy and the problem of asymmetric information persist, so that our qualitative results apply. If the acquiring firm obtains full corporate control above a certain threshold that induces the acquired firm to maximize joint profits (or which allows the acquiring firm to access the acquired firm's relevant information), the problem of asymmetric information disappears above this threshold. The benefits of partial vertical integration still arise, but the equilibrium ownership stake depends on this threshold.

[6] In a similar vein, Farrell and Shapiro (1990) suggest the criterion of joint profits to derive the equilibrium ownership stake. Notably, this approach seems to reflect the practice of takeovers and acquisitions. For instance, in the US a bidder that makes an offer to purchase less than 100% of the shares of a firm must accept all shares tendered on a pro-rated basis. For further discussion on this point, we refer to Foros et al. (2011).

The interim expected profits of the fully integrated supply hierarchy $M_2 - R_2$ are

$$\pi_2 = (p_2 - \theta_2) E_{\theta_1} [q_2 (p_1, p_2) | \theta_2], \tag{3}$$

where $E_{\theta_1} [q_2 (p_1, p_2) | \theta_2]$ denotes the expected quantity of $M_2 - R_2$.[7] The two competing hierarchies do not know the costs of each other but, as discussed previously, their costs can be positively correlated. As it will become clear in the sequel, since $M_2 - R_2$ is fully integrated, our results are unaffected if M_2 does not know the costs of its downstream division R_2.

In order to characterize the equilibrium of the game, we impose the following conditions on the functional forms of profits π_i, $i = 1, 2$, where $\pi_1 \in \{\pi_{R_1}, \pi_{M_1}\}$ and π_2 are given by (1), (2), (3), and on the functional form of $M_1 - R_1$'s joint profits $\pi_{M_1 - R_1}$ (e.g., Vives 2001, Ch. 2).

Assumption 2 $\frac{\partial^2 \pi_i}{\partial p_i \partial p_{-i}} > 0$ *(strategic complementarity).*

Assumption 3 $\frac{\partial^2 \pi_i}{\partial p_i^2} < 0$, $\frac{\partial^2 \pi_{M_1 - R_1}}{\partial \rho^2} < 0$ *(concavity).*

Assumption 4 $\frac{\partial^2 \pi_i}{\partial p_i^2} + \frac{\partial^2 \pi_i}{\partial p_i \partial p_{-i}} < 0$ *(contraction).*

Assumption 2 indicates that the firms' best-response functions are positively sloped (Bulow et al. 1985).[8] Assumption 3 guarantees that the second-order optimality conditions are satisfied and ensures together with Assumption 4 that the equilibrium of the game is globally stable.

Contracting In line with relevant contributions on competing hierarchies under asymmetric information (e.g., Coughlan and Wernerfelt 1989; Gal-Or 1999; Kastl et al. 2011; Katz 1991; Martimort 1996; Martimort and Piccolo 2010), bilateral contracting within a hierarchy is secret. We invoke the revelation principle (e.g., Myerson 1982) in order to characterize the set of incentive feasible allocations. In our setting, this means that, for any strategy choice of $M_2 - R_2$, there is no loss of generality in deriving the best response of M_1 within the class of direct incentive compatible mechanisms. Specifically, M_1 offers R_1 a direct contract menu $\left\{t_1\left(\widehat{\theta}_1\right), p_1\left(\widehat{\theta}_1\right)\right\}_{\widehat{\theta}_1 \in \{\theta_l, \theta_h\}}$ that determines a fixed franchise fee $t_1(.)$ and a retail price $p_1(.)$ contingent on R_1's report $\widehat{\theta}_1 \in \{\theta_l, \theta_h\}$ about its costs θ_1. This contract menu must be incentive compatible, namely, it must induce R_1 to report truthfully its costs, which implies $\widehat{\theta}_1 = \theta_1$ in equilibrium.[9]

In our setting contracts are incomplete, since M_1 cannot contract upon either the retail price of the competitor $M_2 - R_2$ or any report of $M_2 - R_2$ about its costs. This assumption has

[7] In the baseline model we do not impose any particular restriction on θ_2 that can take values either within a discrete set or an interval. Moreover, we only require standard regularity conditions on the probability distribution function for θ_2.

[8] Sufficient (albeit not necessary) condition on the demand function for Assumption 2 is $\frac{\partial^2 q_i(p_i, p_{-i})}{\partial p_i \partial p_{-i}} \geq 0$. In the sequel, we sometimes make use of this condition.

[9] Since the manufacturer can obtain (a part of) the retailer's profits, it might infer the value of the retail costs and implement a penalty that extracts the full profits of the retailer arising from cost misreporting. However, this penalty is unfeasible in a range of reasonable circumstances. The profit realization may be affected by (independent) random shocks which, for instance, occur after the firms' decisions. In this case, retail costs cannot be directly inferred from the retailer's profits and, especially in the presence of limited liability, it would be unfeasible to design any penalty that deters cost misreporting. Furthermore, the fine implemented by the manufacturer would have the only effect of expropriating the profits of the other shareholders of the retailer. This would be interpreted as a violation of their rights and condemned by antitrust authorities.

a solid foundation in the literature (e.g., Gal-Or 1991a, 1991b, 1992, 1999; Kastl et al. 2011; Martimort 1996; Martimort and Piccolo 2010) and can be justified on several grounds. For instance, a contract contingent on the retail price of the competitor may be condemned as a collusive practice by antitrust authorities.[10]

Timing The sequence of events unfolds as follows.
(I) M_1 decides on which ownership stake $\rho \in [0,1]$ to acquire in R_1.
(II) R_1 and $M_2 - R_2$ privately learn their respective retail costs $\theta_1 \in \{\theta_l, \theta_h\}$ and θ_2.
(III) M_1 secretly makes an offer $\left\{t_1\left(\widehat{\theta}_1\right), p_1\left(\widehat{\theta}_1\right)\right\}_{\widehat{\theta}_1 \in \{\theta_l, \theta_h\}}$ to R_1. The offer can be either rejected or accepted by R_1.[11] If the offer is rejected, each firm obtains its outside option (normalized to zero), while $M_2 - R_2$ acts as a monopolist. If the offer is accepted, R_1 picks one element within the contract menu by sending a report $\widehat{\theta}_1 \in \{\theta_l, \theta_h\}$ about its costs.
(IV) Competition takes place in the downstream market and payments are made.

The manufacturer's decision on the ownership stake in the retailer is observable and takes place before the retailer learns its costs. This reflects the idea that the decisions on the firms' ownership rights — mainly when scrutinized by antitrust authorities — become public and are harder to alter than the (generally flexible) production activities that can be adjusted to the realization of costs. Observability and commitment value are standard features of the ownership stake in the literature on partial acquisitions. As Foros et al. (2011) emphasize, the use of the financial and corporate structure of a firm to affect competition is a widespread phenomenon. In Section 7.1 we consider the case in which the ownership stake can be made contingent on retail costs and is incorporated into the vertical contract between the manufacturer and the retailer.

The solution concept we adopt is Perfect Bayesian Equilibrium.[12] Proceeding backwards, we first compute the retail prices at the competition stage for a given ownership stake. Afterwards, we derive the equilibrium ownership stake.

4. Benchmark: full information within the supply hierarchy

To better appreciate how the strategic value of partial vertical integration follows from the presence of asymmetric information, we first consider the benchmark case in which M_1 is fully informed about R_1's costs.

We formalize the main results in the following remark.

Remark 1 *If M_1 is fully informed about R_1's costs $\theta_1 \in \{\theta_l, \theta_h\}$, the equilibrium retail price $\widehat{p}_i$ charged by $M_i - R_i$, $i = 1, 2$, satisfies*

$$E_{\theta_{-i}}\left[q_i\left(\widehat{p}_i, \widehat{p}_{-i}\right)|\theta_i\right] + \left(\widehat{p}_i - \theta_i\right)\frac{\partial E_{\theta_{-i}}\left[q_i\left(\widehat{p}_i, \widehat{p}_{-i}\right)|\theta_i\right]}{\partial p_i} = 0. \tag{4}$$

[10] Alternatively, the retail price charged by the rival can be hard to observe or verify because of the lack of proper auditing rights. We refer to Martimort (1996) for a discussion of this assumption.

[11] A take-it-or-leave-it offer is a standard assumption in the literature on competing hierarchies.

[12] As a standard equilibrium refinement, we require a 'no signaling what you do not know' condition (e.g., Martimort 1996). Whenever R_1 receives an unexpected offer from M_1, it does not change its beliefs about the equilibrium strategy of $M_2 - R_2$. This condition reflects the idea that a manufacturer cannot signal to its retailer information that it does not know about the competitor, since the supply hierarchies are independent and act simultaneously.

The equilibrium ownership stake that M_1 holds in R_1 is any $\widehat{\rho} \in [0,1]$.

The retail price of each supply hierarchy is set above marginal costs in order to equate (expected) marginal revenues with (expected) marginal costs from retail activities.[13] The problem of manufacturer M_1 coincides with the problem of the fully integrated hierarchy $M_2 - R_2$. Since contracting is secret and cannot be used for strategic purposes, a fully informed manufacturer using non-linear contracts finds it optimal to remove the double marginalization problem by making its retailer residual claimant for the hierarchy's profits, which are extracted via a fixed fee. Hence, the outcome of full integration is achieved irrespective of the ownership stake ρ, and the choice of the degree of vertical integration is inconsequential. This well-known 'neutrality result' (Coughlan and Wernerfelt 1989; Katz 1991) no longer holds in the presence of asymmetric information.

5. The case of asymmetric information

As discussed in Section 3, when R_1 privately knows its costs, M_1 can restrict attention to a direct incentive compatible contract menu $\{(t_{1l}, p_{1l}), (t_{1h}, p_{1h})\}$, where (t_{1l}, p_{1l}) and (t_{1h}, p_{1h}) are the contracts designed for the efficient and inefficient retailer, with costs θ_l and θ_h respectively.

5.1. Competition stage

We first derive the retail prices for a given ownership stake. In addition to the participation constraints $\pi_{R_{1l}} \geq 0$ and $\pi_{R_{1h}} \geq 0$ for the efficient and inefficient retailer, the contract offered by M_1 to R_1 must satisfy the following incentive compatibility constraints

$$\pi_{R_{1l}} = (p_{1l} - \theta_l) E_{\theta_2} [q_1 (p_{1l}, p_2) |\theta_l] - t_{1l} \geq (p_{1h} - \theta_l) E_{\theta_2} [q_1 (p_{1h}, p_2) |\theta_l] - t_{1h} \tag{5}$$

$$\pi_{R_{1h}} = (p_{1h} - \theta_h) E_{\theta_2} [q_1 (p_{1h}, p_2) |\theta_h] - t_{1h} \geq (p_{1l} - \theta_h) E_{\theta_2} [q_1 (p_{1l}, p_2) |\theta_h] - t_{1l}. \tag{6}$$

Conditions (5) and (6) ensure that R_1 does not benefit from misreporting its costs. As implied by the Spence-Mirrlees (single-crossing) property, the relevant incentive constraint is the one for the efficient retailer in (5), which is binding in equilibrium together with the participation constraint $\pi_{R_{1h}} \geq 0$ for the inefficient retailer.[14] Given the expression for $\pi_{R_{1h}}$ in (6) and $\pi_{R_{1h}} = 0$ in equilibrium, we can rewrite the binding constraint (5) after some manipulation as follows

$$\begin{aligned}
\pi_{R_{1l}} &= p_{1h} \{E_{\theta_2} [q_1 (p_{1h}, p_2) |\theta_l] - E_{\theta_2} [q_1 (p_{1h}, p_2) |\theta_h]\} \\
&\quad + \theta_h E_{\theta_2} [q_1 (p_{1h}, p_2) |\theta_h] - \theta_l E_{\theta_2} [q_1 (p_{1h}, p_2) |\theta_l] \\
&= \Delta\theta E_{\theta_2} [q_1 (p_{1h}, p_2) |\theta_h] - (p_{1h} - \theta_l) \\
&\quad \times \{E_{\theta_2} [q_1 (p_{1h}, p_2) |\theta_h] - E_{\theta_2} [q_1 (p_{1h}, p_2) |\theta_l]\},
\end{aligned} \tag{7}$$

which captures the informational rents that the efficient retailer commands to reveal truthfully its costs. Using $\pi_{R_{1h}} = 0$ and the fact that the constraint (5) is binding, M_1's problem of

[13] Throughout the analysis we assume interior solutions at the competition stage.

[14] Otherwise, M_1 could increase the franchise fee and be better off. For further technical details we refer to the proof of Lemma 1 in the Appendix.

maximizing its (expected) profits in (2) becomes

$$\max_{p_{1l},p_{1h}} \nu \left\{ (p_{1l} - \theta_l) E_{\theta_2} [q_1 (p_{1l}, p_2) | \theta_l] - (1-\rho) \pi_{R_{1l}} (p_{1h}) \right\}$$
$$+ (1-\nu) (p_{1h} - \theta_h) E_{\theta_2} [q_1 (p_{1h}, p_2) | \theta_h] . \tag{8}$$

The supply hierarchy $M_2 - R_2$ maximizes its profits in (3) as follows

$$\max_{p_2} (p_2 - \theta_2) E_{\theta_1} [q_2 (p_1, p_2) | \theta_2] . \tag{9}$$

After taking the derivative of $\pi_{R_{1l}}$ in (7) with respect to p_{1h}

$$\frac{\partial \pi_{R_{1l}}}{\partial p_{1h}} \equiv \Omega (p_{1h}) = \Delta\theta \frac{\partial E_{\theta_2} [q_1 (p_{1h}, p_2) | \theta_h]}{\partial p_{1h}} - E_{\theta_2} [q_1 (p_{1h}, p_2) | \theta_h] + E_{\theta_2} [q_1 (p_{1h}, p_2) | \theta_l]$$
$$- (p_{1h} - \theta_l) \left\{ \frac{\partial E_{\theta_2} [q_1 (p_{1h}, p_2) | \theta_h]}{\partial p_{1h}} - \frac{\partial E_{\theta_2} [q_1 (p_{1h}, p_2) | \theta_l]}{\partial p_{1h}} \right\} < 0, \tag{10}$$

we can formalize the equilibrium retail prices for a given ownership stake ρ.[15]

Lemma 1 *If R_1 is privately informed about its costs $\theta_1 \in \{\theta_l, \theta_h\}$, the retail price charged by $M_1 - R_1$ is $p_1^* \in \{p_{1l}^*, p_{1h}^*\}$, where p_{1l}^* and p_{1h}^* respectively satisfy*

$$E_{\theta_2} [q_1 (p_{1l}^*, p_2^*) | \theta_l] + (p_{1l}^* - \theta_l) \frac{\partial E_{\theta_2} [q_1 (p_{1l}^*, p_2^*) | \theta_l]}{\partial p_{1l}} = 0 \tag{11}$$

$$E_{\theta_2} [q_1 (p_{1h}^*, p_2^*) | \theta_h] + (p_{1h}^* - \theta_h) \frac{\partial E_{\theta_2} [q_1 (p_{1h}^*, p_2^*) | \theta_h]}{\partial p_{1h}} - \phi (\nu) (1-\rho) \Omega (p_{1h}^*) = 0, \tag{12}$$

with $\phi(\nu) \equiv \frac{\nu}{1-\nu}$. Furthermore, the retail price p_2^* charged by $M_2 - R_2$ satisfies

$$E_{\theta_1} [q_2 (p_1^*, p_2^*) | \theta_2] + (p_2^* - \theta_2) \frac{\partial E_{\theta_1} [q_2 (p_1^*, p_2^*) | \theta_2]}{\partial p_2} = 0. \tag{13}$$

Equipped with Lemma 1, we can show how the ownership stake affects the equilibrium retail prices. To simplify notation, we define $\widehat{p}_{1k}$ as the full information equilibrium price of R_1 with costs θ_k, $k = l, h$.

Proposition 1 *If $\rho = 1$, then $p_{1l}^* = \widehat{p}_{1l}$, $p_{1h}^* = \widehat{p}_{1h}$, $p_2^* = \widehat{p}_2$. Furthermore, $\frac{\partial p_{1l}^*}{\partial \rho} \leq 0$, $\frac{\partial p_{1h}^*}{\partial \rho} < 0$, $\frac{\partial p_2^*}{\partial \rho} \leq 0$, where equalities follow if consumer demands are independent.*

We illustrate the results of Lemma 1 and Proposition 1 with the help of Figure 1.[16] Note that M_1's asymmetric information best-response function r_{1l}^* for low costs ($\theta_1 = \theta_l$) coincides with the corresponding full information best-response function $\widehat{r}_{1l}$, regardless of the ownership stake. This is because R_1's informational rents in (7) are independent of p_{1l} and therefore M_1 does not find it profitable to implement any price distortion for the efficient retailer.

The ownership stake ρ affects the position of M_1's asymmetric information best-response function r_{1h}^* for high costs ($\theta_1 = \theta_h$), relative to the corresponding full information best-response function $\widehat{r}_{1h}$. Specifically, with a full acquisition of R_1 ($\rho = 1$), the two best-response functions

[15]The sign of (10) follows from Assumptions 1-2 and positively correlated (or independent) costs.
[16]The figure considers the case of linear demand.

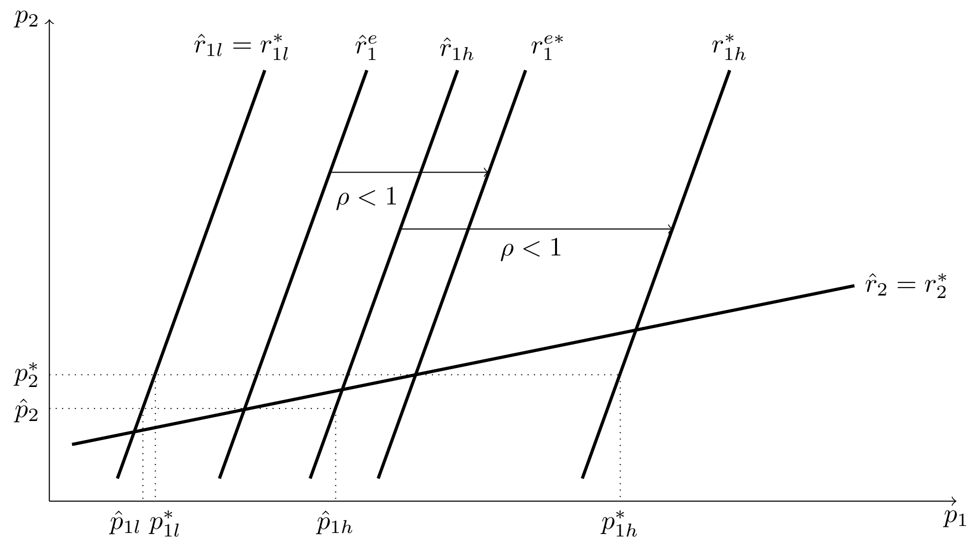


Figure 1: Best-response functions under full and asymmetric information

coincide. In this case, M_1 maximizes the hierarchy's joint profits in (8) and fully internalizes R_1's informational rents in (7). Since these rents are costless, M_1 does not want to implement any price distortion and, as Proposition 1 indicates, the asymmetric information prices in (11), (12) and (13) reflect the full information levels in (4).

An ownership stake ρ lower than 1 shifts M_1's asymmetric information best-response function r^*_{1h} outwards, and the price charged by the inefficient retailer is higher than under full information for any price of the competitor $M_2 - R_2$. A lower ρ makes the informational rents in (7) more costly for M_1, because they are internalized to a lesser extent in M_1's objective function in (8). As (10) reveals, in order to curb the informational rents M_1 can induce an upward price distortion for the inefficient retailer.[17] Since the ownership stake ρ measures the degree of M_1's internalization of R_1's rents, the magnitude of this form of double marginalization from asymmetric information increases when ρ declines. The outward shift in M_1's best-response function r^*_{1h} for $\rho < 1$ moves in the same direction M_1's expected best response (from $\widehat{r}^e_1$ to r^{e*}_1), while $M_2 - R_2$'s best response is clearly unaffected ($\widehat{r}_2 = r^*_2$).[18] Therefore, the two competing hierarchies can coordinate on higher prices. As Proposition 1 shows, a lower ownership stake ρ induces a higher equilibrium price p^*_{1h} of the inefficient retailer, which translates into a higher equilibrium price p^*_2 of the competitor $M_2 - R_2$ in the presence of strategic complementarity. In response, the equilibrium price p^*_{1l} of the efficient retailer increases as well.

It is worth noting that this latter result crucially depends on the fact that $M_2 - R_2$ cannot distinguish between the efficient and inefficient retailer, and it determines the price on the basis of the (conditional) distribution of R_1's costs. As we will see in Section 6, with perfect cost correlation the prices of the two supply hierarchies reflect the full information values in case of low costs, since $M_2 - R_2$ knows R_1's costs and anticipates that M_1's best-response function is unaffected for low costs.

[17]This result is reminiscent of the rent extraction-efficiency trade-off in optimal regulation (e.g., Baron and Myerson 1982).

[18]This holds even under asymmetric information within $M_2 - R_2$ since M_2 maximizes joint profits and does not find it profitable to distort the price of the privately informed division R_2. For the sake of clarity, the best-response function of $M_2 - R_2$ is depicted in Figure 1 for a given (and commonly known) θ_2.

5.2. Equilibrium ownership stake

Having derived the equilibrium retail prices for a given ownership stake, we can go back to the first stage of the game and determine the equilibrium ownership stake. Since M_1 chooses the ownership stake in R_1 in order to maximize the hierarchy's joint profits, M_1's problem is given by

$$\max_{\rho \in [0,1]} \nu \left[p_{1l}^* (\rho) - \theta_l\right] E_{\theta_2} \left[q_1 \left(p_{1l}^* (\rho), p_2^* (\rho)\right) |\theta_l\right] + (1-\nu) \left[p_{1h}^* (\rho) - \theta_h\right] E_{\theta_2} \left[q_1 \left(p_{1h}^* (\rho), p_2^* (\rho)\right) |\theta_h\right]. \tag{14}$$

We are now in a position to show our main results.

Proposition 2 *If R_1 is privately informed about its costs, partial vertical integration is more profitable for $M_1 - R_1$ than full vertical integration when consumer demands are interdependent. The equilibrium ownership stake that M_1 holds in R_1 is $\rho^* < 1$. If consumer demands are independent, full vertical integration arises in equilibrium, i.e., $\rho^* = 1$.*

Under asymmetric information M_1 is no longer indifferent about the ownership stake in R_1. If demands are independent and therefore each supply hierarchy acts as a monopolist, full integration is optimal because it removes any negative informational externality and maximizes the profits of the supply hierarchy taken in isolation. Notably, we obtain this result without the need to assume that the manufacturer exogenously acquires any relevant information about the retailer when they are fully integrated. As Proposition 1 indicates, under full integration the manufacturer completely internalizes the retailer's informational rents and does not find it optimal to implement any price distortion.

Proposition 2 shows that the strict preference for full vertical integration does not carry over in a competitive environment. In this case, partial vertical integration ensures the supply hierarchy $M_1 - R_1$ higher profits than full integration, and the equilibrium ownership stake that M_1 acquires in R_1 is $\rho^* < 1$. In order to substantiate the rationale for this result as provided in the introduction, it is helpful to recall from Proposition 1 that a partial misalignment between the profit objectives within a partially integrated hierarchy ($\rho < 1$) leads to an upward price distortion for the inefficient retailer to reduce the (costly) informational rents to the efficient retailer. This form of double marginalization from asymmetric information generates an *information vertical effect* that reduces ceteris paribus the profitability of the supply hierarchy. With price competition, the information vertical effect translates into an opposite *competition horizontal effect.* Since there exists no vertical contract that can 'solve' the problem of asymmetric information, it follows from Katz (1991) that even with secret contracting a partially integrated hierarchy can commit vis-à-vis the rival to a higher retail price than under full integration, which induces the rival to increase its price as well in the presence of strategic complementarity. Therefore, partial vertical integration constitutes a commitment device à la Katz (1991) to relax competition. The equilibrium degree of vertical integration trades off the benefits of softer competition against the informational costs.

To better appreciate the rationale for our results, it is convenient to write a second-order

Taylor approximation for $M_1 - R_1$'s joint profits around $\rho = 1$ as follows

$$\pi_{M_1-R_1}(\rho)\big|_{\rho=\widetilde{\rho}<1} \approx \pi_{M_1-R_1}(\rho)\big|_{\rho=1} - (1-\widetilde{\rho}) \left.\frac{\partial \pi_{M_1-R_1}(\rho)}{\partial \rho}\right|_{\rho=1} + \frac{(1-\widetilde{\rho})^2}{2} \left.\frac{\partial^2 \pi_{M_1-R_1}(\rho)}{\partial \rho^2}\right|_{\rho=1}.$$

As the proof of Proposition 2 shows, we have $\left.\frac{\partial \pi_{M_1-R_1}(\rho)}{\partial \rho}\right|_{\rho=1} < 0$, and therefore a departure from full vertical integration entails first-order benefits for the supply hierarchy $M_1 - R_1$. A reduction in ρ from $\rho = 1$ has no first-order information vertical effect associated with the costs of the double marginalization from asymmetric information (minimized at $\rho = 1$), but it induces a first-order competition horizontal effect which is beneficial for $M_1 - R_1$ in terms of coordination with $M_2 - R_2$ on higher prices. However, a lower ρ also entails second-order losses for $M_1 - R_1$, i.e., $\left.\frac{\partial^2 \pi_{M_1-R_1}(\rho)}{\partial \rho^2}\right|_{\rho=1} < 0$ (by Assumption 3), which stem from the informational costs. The resulting trade-off implies that the equilibrium ownership stake diverges from the full integration outcome until the level that equates the marginal benefits of relaxing competition with the marginal informational costs.

Since we know from Proposition 1 that a lower ownership stake leads to higher prices, in a competitive environment there exists a conflict of interests between consumers (and the society as a whole), whose welfare is maximized under full integration, and the supply hierarchy, which prefers to partially integrate. In Section 8 we discuss the antitrust policy implications of our results.

6. An illustrative example

Using explicit functions, we now derive the equilibrium degree of vertical integration and conduct a comparative statics analysis. The consumer demand for good $i = 1, 2$ takes the following form

$$q_i = \alpha - \beta p_i + \gamma p_{-i}, \tag{15}$$

where α and β are positive parameters, and $\gamma \in [0, \beta)$ denotes the degree of substitutability between goods.[19] The profits of R_1, M_1 and $M_2 - R_2$ are respectively given by (1), (2) and (3), with retail costs being now perfectly correlated, which implies $\theta_1 = \theta_2 \in \{\theta_l, \theta_h\}$. The assumption of perfect correlation between the retailers' types is relatively common in the literature on competing hierarchies (e.g., Kastl et al. 2011; Martimort 1996; Martimort and Piccolo 2010).

As discussed at the end of Section 5.1, with perfectly correlated costs $M_2 - R_2$ knows whether it faces an efficient retailer, whose price is not distorted for the purpose of reducing the informational rents. This implies that for $\theta_1 = \theta_2 = \theta_l$ the retail prices reflect the full information values, i.e., $p_{il}^* = \widehat{p}_{il} = \frac{\alpha+\beta\theta_l}{2\beta-\gamma}$, $i = 1, 2$. For $\theta_1 = \theta_2 = \theta_h$ the retail prices charged by $M_1 - R_1$ and $M_2 - R_2$ for a given ownership stake ρ are respectively

$$p_{1h}^* = \frac{(\alpha + \beta\theta_h)\left(4\beta^2 - \gamma^2\right) + \phi(\nu)(1-\rho)\left[4\beta^3\Delta\theta - \gamma^2(\alpha + \beta\theta_h)\right]}{(2\beta - \gamma)\left\{4\beta^2 - \gamma^2\left[1 + \phi(\nu)(1-\rho)\right]\right\}} \tag{16}$$

[19]The demand system in (15) follows from the optimization problem of a unit mass of identical consumers characterized by a quasi-linear utility function $y + U(q_1, q_2)$, where y is the composite good and $U(q_1, q_2) = a(q_1 + q_2) - \frac{1}{2}\left(bq_1^2 + bq_2^2 + 2gq_1q_2\right)$, with $a > 0$, $b > g \geq 0$, and $\alpha \equiv \frac{a(b-g)}{b^2-g^2}$, $\beta \equiv \frac{b}{b^2-g^2}$, $\gamma \equiv \frac{g}{b^2-g^2}$ (e.g., Vives 2001, Ch. 6).

$$p_{2h}^{*}=\frac{\left(\alpha+\beta\theta_{h}\right)\left(4\beta^{2}-\gamma^{2}\right)+\gamma\phi\left(\nu\right)\left(1-\rho\right)\left[2\beta^{2}\Delta\theta-\gamma\left(\alpha+\beta\theta_{h}\right)\right]}{\left(2\beta-\gamma\right)\left\{4\beta^{2}-\gamma^{2}\left[1+\phi\left(\nu\right)\left(1-\rho\right)\right]\right\}}. \tag{17}$$

These results illustrate with explicit solutions the main insights gleaned from Lemma 1 and Proposition 1.[20] The price in (16) of the inefficient retailer is inflated above the full information level when the amount of ownership stake ρ is lower than 1. Differentiating p_{1h}^{*} in (16) with respect to ρ yields

$$\frac{\partial p_{1h}^{*}}{\partial\rho}=-\frac{4\beta^{3}\left(2\beta-\gamma\right)\Delta\theta\phi\left(\nu\right)}{\left\{4\beta^{2}-\gamma^{2}\left[1+\phi\left(\nu\right)\left(1-\rho\right)\right]\right\}^{2}}<0, \tag{18}$$

which indicates that a lower ρ exacerbates the upward price distortion. As the discussion after Proposition 1 reveals, this is because M_1 internalizes to a lesser extent the informational rents in (7) and therefore it is more inclined to curb these rents with a price increase. For a given price charged by the competitor M_2-R_2, this form of double marginalization from asymmetric information reduces the profits of M_1-R_1 relative to full integration. However, in the presence of strategic complementarity in prices, M_2-R_2 reacts with an accommodating behavior. Differentiating p_{2h}^{*} in (17) with respect to ρ yields

$$\frac{\partial p_{2h}^{*}}{\partial\rho}=-\frac{4\beta^{2}\gamma\left(2\beta-\gamma\right)\Delta\theta\phi\left(\nu\right)}{\left\{4\beta^{2}-\gamma^{2}\left[1+\phi\left(\nu\right)\left(1-\rho\right)\right]\right\}^{2}}\leq 0, \tag{19}$$

where the equality holds for $\gamma=0$. A lower ρ allows the two competing hierarchies to coordinate on higher prices. Note from (18) and (19) that the price response of M_2-R_2 to a change in ρ is smoother than the price response of M_1-R_1, and it vanishes when consumer demands are independent ($\gamma=0$). Even though the two supply hierarchies share the same costs, M_1-R_1 sets a higher price than M_2-R_2 for $\rho<1$.

The following proposition illustrates the result of the trade-off between the benefits of softer competition and the informational costs.

Proposition 3 *If R_1 is privately informed about its costs $\theta_1\in\{\theta_l,\theta_h\}$, the equilibrium ownership stake that M_1 holds in R_1 is*

$$\rho^{*}=\max\left\{1-\frac{\gamma^{2}\left(4\beta^{2}-\gamma^{2}\right)\left[\alpha-\left(\beta-\gamma\right)\theta_{h}\right]}{\phi\left(\nu\right)\left\{8\beta^{3}\Delta\theta\left(2\beta^{2}-\gamma^{2}\right)+\gamma^{4}\left[\alpha-\left(\beta-\gamma\right)\theta_{h}\right]\right\}};0\right\}. \tag{20}$$

It holds $\rho^{*}<1$ when consumer demands are interdependent ($\gamma\neq 0$). In particular, we have

(i) partial vertical integration, i.e., $\rho^{*}\in(0,1)$, if $\phi\left(\nu\right)>\frac{\gamma^{2}\left(4\beta^{2}-\gamma^{2}\right)\left[\alpha-\left(\beta-\gamma\right)\theta_{h}\right]}{8\beta^{3}\Delta\theta\left(2\beta^{2}-\gamma^{2}\right)+\gamma^{4}\left[\alpha-\left(\beta-\gamma\right)\theta_{h}\right]}$;

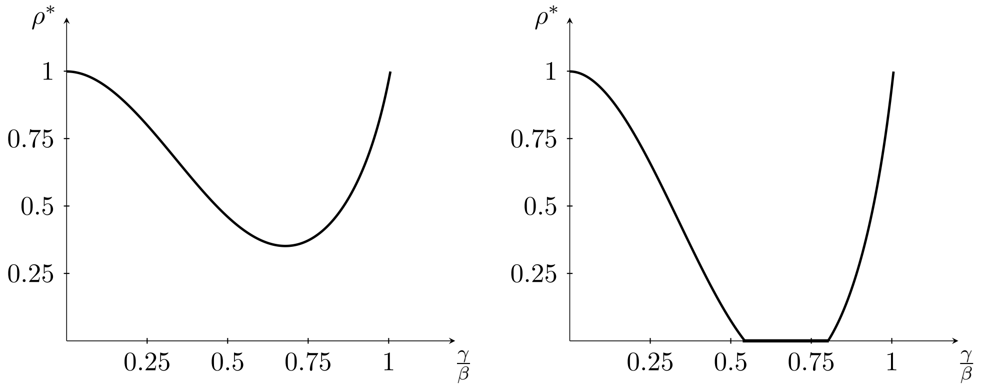
(ii) full vertical separation, i.e., $\rho^{*}=0$, otherwise.

Full vertical integration, i.e., $\rho^{*}=1$, is preferred if consumer demands are independent ($\gamma=0$).

Proposition 3 reveals that in a competitive environment partial vertical integration emerges in equilibrium if the probability ν of the efficient retailer is relatively high (recall that $\phi\left(\nu\right)\equiv\frac{\nu}{1-\nu}$).[21] In this case, a partial ownership stake optimally trades off the benefits of softer com-

[20] We refer to the proof of Proposition 3 in the Appendix for the formal derivation of these results.

[21] The condition $\alpha-\left(\beta-\gamma\right)\theta_{h}>0$, which ensures positive quantities under full information, implies that the ratio in (20) is positive for $\gamma\neq 0$.



(a) High probability of the efficient retailer (b) Intermediate probability of the efficient retailer

Figure 2: Product differentiation and ownership stake

petition against the costs of the double marginalization from asymmetric information.

When the probability ν of the efficient retailer is low enough, the hierarchy $M_1 - R_1$ prefers full separation. To understand the rationale for this result, we write a first-order Taylor approximation for $\frac{\partial \pi_{M_1-R_1}}{\partial \rho}$ around $\nu = 0$ as follows

$$\left.\frac{\partial \pi_{M_1-R_1}}{\partial \rho}\right|_{\nu=\widetilde{\nu}>0} \approx \left.\frac{\partial \pi_{M_1-R_1}}{\partial \rho}\right|_{\nu=0} + \widetilde{\nu} \left.\frac{\partial^2 \pi_{M_1-R_1}}{\partial \rho \partial \nu}\right|_{\nu=0} = -\widetilde{\nu}\frac{2\beta^2\gamma^2 \left[\alpha - (\beta-\gamma)\,\theta_h\right] \Delta\theta}{(2\beta-\gamma)^3\,(2\beta+\gamma)} < 0,$$

where $\left.\frac{\partial \pi_{M_1-R_1}}{\partial \rho}\right|_{\nu=0} = 0$ since any $\rho \in [0,1]$ maximizes $M_1 - R_1$'s profits under full information (see Remark 1). For small values of ν, a lower ρ increases $M_1 - R_1$'s joint profits, and therefore full separation arises in equilibrium. This is because, when ν is low enough, the problem of asymmetric information is relatively mild and the benefits of softer competition driven by a reduction in ρ dominate the informational costs.

A comparison between panel (a) and panel (b) of Figure 2 indicates that a higher probability of the efficient retailer shifts the equilibrium ownership stake ρ^* in (20) upwards.[22] A more probable efficient retailer increases the expected informational rents, which exacerbates the upward price distortion for the inefficient retailer and inflates the informational costs. A higher level of integration is preferred since it mitigates these costs.

As Figure 2 illustrates, there exists a non-monotone relation between the degree of product differentiation γ and ρ^*, which implies that for intermediate values of the probability of the efficient retailer full separation can still occur in equilibrium for a range of product differentiation. When each supply chain acts as a monopolist ($\gamma = 0$), a pattern of full integration that removes any informational externality within the hierarchy is optimal. If γ increases, ρ^* declines over an initial range of γ. As competition intensifies, the benefits of softer competition become more attractive and this induces a reduction in ρ^*. However, above a certain threshold this pattern is reversed and a higher γ translates into a higher ρ^*. This is because, when competition is relatively fierce, the two supply hierarchies are reluctant to coordinate on high

[22] In Figure 2 the parameter values are $\beta = 1$, $\alpha = 32\,(1-\gamma)$, $\theta_h = \Delta\theta = 1.5$. Moreover, $\phi\,(\nu) = 1.2$ in panel (a), and $\phi\,(\nu) = 0.7$ in panel (b).

prices. At the extreme, when goods are close substitutes and competition tends to be perfect, prices converge to marginal costs and the equilibrium ownership stake approaches the outcome of full integration (if $\gamma \to \beta$, then $\alpha \to 0$ and $\rho^* \to 1$).

Note from (20) that a higher spread of the cost distribution $\Delta\theta \equiv \theta_h - \theta_l$ increases the equilibrium ownership stake ρ^*. A higher $\Delta\theta$ aggravates the problem of asymmetric information, which induces a higher ρ^* in order to mitigate the informational costs.

The result in Proposition 3 that the equilibrium ownership stake is lower than 1 holds whenever consumer demands are interdependent ($\gamma \neq 0$). Hence, partial vertical integration can emerge even with complementary goods ($\gamma < 0$). As (18) and (19) indicate, a higher price of the inefficient retailer due to a lower ownership stake than under full integration leads to a lower price for the complementary good of $M_2 - R_2$, since prices are now strategic substitutes. This stimulates the output of $M_1 - R_1$ and improves its profits.

7. Robustness

We now discuss some assumptions of the model to gain insights into the robustness of the results.

7.1. Type-contingent ownership stake

In our model the manufacturer's decision on the ownership stake is observable and takes place before the retailer learns its costs. In line with the literature on partial acquisitions, observability and commitment value of the ownership stake are relevant ingredients of our model. Now, suppose that the ownership stake is chosen after costs have materialized and it is incorporated into the vertical contract between the manufacturer and the retailer. In this case, M_1 secretly offers R_1 a contract of the form $\left\{\rho\left(\widehat{\theta}_1\right), t_1\left(\widehat{\theta}_1\right), p_1\left(\widehat{\theta}_1\right)\right\}$. It follows from Piccolo et al. (2014) that partial vertical integration can still emerge in this alternative scenario. This occurs when R_1's profits directly depend on the ownership stake ρ, and a higher ρ induces R_1 to internalize to a larger extent the hierarchy's joint profits. With positive cost correlation, R_1 anticipates that, after a report of high costs, M_1 conjectures that the competitor is more likely to be inefficient, which entails hierarchy's lower profits in the presence of strategic complementarity. Hence, M_1 is willing to compensate R_1 for this decline in the hierarchy's profits. A lower ownership stake designed for the inefficient retailer reduces the interest of the efficient retailer (that reports high costs) in the hierarchy's profits and mitigates its incentives to manipulate costs. Differently from our setting, a partial ownership stake does not arise from the benefits of softer competition but from a 'competing-contracts effect' à la Martimort (1996) and Gal-Or (1999).

7.2. Manufacturer's profit maximizing ownership stake

In line with the main literature, we derive the ownership stake of M_1 in R_1 from the joint profit maximization problem. We now consider the case in which M_1 chooses the ownership stake that maximizes its own profits in (2). The equilibrium value for the ownership stake ρ is the solution to the following program

$$\max_{\rho\in[0,1]} \nu \left\{ \left[p_{1l}^*(\rho) - \theta_l\right] E_{\theta_2}\left[q_1\left(p_{1l}^*(\rho), p_2^*(\rho)\right)|\theta_l\right] - (1-\rho)\,\pi_{R_{1l}}\left(p_{1h}^*(\rho)\right)\right\}$$
$$+ (1-\nu)\left[p_{1h}^*(\rho) - \theta_h\right] E_{\theta_2}\left[q_1\left(p_{1h}^*(\rho), p_2^*(\rho)\right)|\theta_h\right], \tag{21}$$

where $\pi_{R_{1l}}\left(p_{1h}^*(\rho)\right)$ is given by (7) evaluated in the competition stage equilibrium. As a comparison between (14) and (21) indicates, the strategic incentives to partially integrate are weaker than under joint profit maximization. A manufacturer that maximizes its own profits when choosing the ownership stake in its retailer internalizes not only the allocative costs of the double marginalization from asymmetric information but also the distributional costs arising from the inability to fully appropriate the retailer's rents. The informational costs of partial integration are higher than under joint profit maximization and full integration becomes more attractive. Partial vertical integration can still emerge in equilibrium as the manufacturer's profit maximizing outcome if the retailer's (expected) informational rents are not too large, which is typically the case when the spread of the cost distribution $\Delta\theta \equiv \theta_h - \theta_l$ is relatively small.

7.3. Two-part tariff

The contract that M_1 offers to R_1 directly specifies the retail price, which is known as resale price maintenance. Even though this type of vertical arrangements is sometimes viewed with skepticism by antitrust authorities, some countries (e.g., New Zealand) traditionally allow this practice if the beneficial effects can be shown to outweigh the detrimental effects. Remarkably, in the 2007 case 'Leegin Creative Leather Products, Inc., v. PSKS, Inc.' the US Supreme Court replaced the well-established doctrine of per se unlawfulness of resale price maintenance with a rule of reason which allows a firm to produce evidence that an individual resale price maintenance agreement is justified.[23]

Our qualitative results do not depend on the use of resale price maintenance. Suppose that this form of vertical contracting is not allowed, and M_1 secretly offers R_1 a two-part tariff $\{f_1, w_1\}$ that specifies a fixed franchise fee f_1 and a wholesale price w_1 for each unit of input that M_1 provides to R_1. Manufacturing costs are still normalized to zero. Using a standard assumption (e.g., Martimort and Piccolo 2007), R_1 converts M_1's input with a one-to-one technology into a final product supplied on the retail market. The interim expected profits of R_1 and M_1 are respectively given by

$$\pi_{R_1} = (p_1 - \theta_1 - w_1)\, E_{\theta_2}\left[q_1(p_1, p_2)|\theta_1\right] - f_1 \tag{22}$$

$$\pi_{M_1} = f_1 + w_1 E_{\theta_2}\left[q_1(p_1, p_2)|\theta_1\right] + \rho\left\{(p_1 - \theta_1 - w_1)\, E_{\theta_2}\left[q_1(p_1, p_2)|\theta_1\right] - f_1\right\}. \tag{23}$$

The game exhibits the same features as the baseline model, with the difference that M_1 cannot dictate the retail price to R_1. This implies that, after the offer $\{f_1, w_1\}$ from M_1, R_1 chooses the price that maximizes its own profits.

The following proposition summarizes the main results when a two-part tariff is adopted.

[23]For some empirical evidence on resale price maintenance in Europe, we refer to Bonnet and Dubois (2010).

Proposition 4 *Suppose that M_1 secretly offers R_1 a two-part tariff $\{f_1, w_1\}$. Then,*

(i) if M_1 is fully informed about R_1's costs $\theta_1 \in \{\theta_l, \theta_h\}$, the equilibrium wholesale price is $\widehat{w}_1 = 0$. The equilibrium ownership stake that M_1 holds in R_1 is any $\widehat{\rho}_{tp} \in [0, 1]$;

(ii) if R_1 is privately informed about its costs $\theta_1 \in \{\theta_l, \theta_h\}$, the equilibrium wholesale price is $w_1^* \in \{w_{1l}^*, w_{1h}^*\}$, where $w_{1l}^* = \widehat{w}_1$ and $w_{1h}^* \geq \widehat{w}_1$, with $w_{1h}^* = \widehat{w}_1$ if $\rho = 1$ and $\frac{\partial w_{1h}^*}{\partial \rho} < 0$. The equilibrium ownership stake that M_1 holds in R_1 is $\rho_{tp}^* < 1$ when consumer demands are interdependent. If consumer demands are independent, full vertical integration arises in equilibrium, i.e., $\rho_{tp}^* = 1$.

Proposition 4 shows that our main results are robust to the form of vertical contracting. Under full information within the supply hierarchy, a two-part tariff is equivalent to a contract specifying a retail price and a fixed fee, since either contractual form removes the double marginalization problem and therefore the ownership stake that the manufacturer acquires in the retailer is inconsequential. More relevantly, we find that, in the presence of asymmetric information, partial vertical integration can still emerge in equilibrium. A manufacturer using a two-part tariff inflates the wholesale price for the inefficient retailer above the full information level in order to curb the informational rents to the efficient retailer (e.g., Gal-Or 1991c). This occurs as long as the manufacturer does not fully own the retailer and therefore the informational rents are costly. Since a higher wholesale price translates into a higher retail price, partial vertical integration still constitutes a commitment device to relax competition.

The following corollary shows that the form of vertical contracting affects the equilibrium degree of vertical integration in a systematic manner.

Corollary 1 *It holds $\rho_{tp}^* \geq \rho^*$, where the equality follows if costs θ_1 and θ_2 are independent.*

A supply hierarchy prefers a higher level of integration under a two-part tariff than under resale price maintenance when retail costs are (positively) correlated. As the expressions for the informational rents (7) and (30) reveal, under either contractual form the efficient retailer envisages lower profits from a report of high costs in the presence of cost correlation, since it anticipates that the rival is more likely to be efficient and to set a relatively low price. The retailer's lower profits depend on the reduction in the expected quantity weighted by the price-cost markup. Cost correlation mitigates the retailer's incentive to manipulate costs, but it does so to a lower extent under a two-part tariff. This is because the upward distortion of the wholesale price for the inefficient retailer driven by asymmetric information leads to a lower price-cost markup and therefore alleviates the profit reduction that the efficient retailer expects from a report of high costs. In other terms, the double marginalization associated with a two-part tariff aggravates the manufacturer's incentive problem, which induces the acquisition of a higher ownership stake in order to mitigate the informational costs. Resale price maintenance can replicate the outcome of a two-part tariff at a lower cost in terms of informational rents, which makes the manufacturer better off. Hence, a two-part tariff will be adopted only when resale price maintenance arrangements are banned.

7.4. Cost correlation and ownership stake

The investigation of the impact of correlation between retail costs on the equilibrium ownership stake delivers results of some interest. Suppose that the retail costs θ_i of the supply hierarchy $M_i - R_i$, $i = 1, 2$, can be either θ_l or θ_h with ex ante probability $\nu \in (0, 1)$ and $1-\nu$, respectively. Following Piccolo and Pagnozzi (2013) and Piccolo et al. (2014), the vector of the retail costs (θ_1, θ_2) is drawn from a joint cumulative distribution function such that $\Pr(\theta_l, \theta_l) = \nu^2 + \mu$, $\Pr(\theta_l, \theta_h) = \Pr(\theta_h, \theta_l) = \nu(1-\nu) - \mu$, and $\Pr(\theta_h, \theta_h) = (1-\nu)^2 + \mu$. The parameter $\mu \in [0, \nu(1-\nu)]$ measures the degree of (positive) correlation between θ_1 and θ_2. Using Bayes' rule, posterior probabilities are $\Pr(\theta_l | \theta_l) = \nu + \frac{\mu}{\nu}$, $\Pr(\theta_l | \theta_h) = \nu - \frac{\mu}{1-\nu}$, $\Pr(\theta_h | \theta_l) = 1 - \nu - \frac{\mu}{\nu}$, and $\Pr(\theta_h | \theta_h) = 1 - \nu + \frac{\mu}{1-\nu}$.

The incentive constraints (5) and (6) for the efficient and inefficient retailer become

$$\begin{aligned}\pi_{R_{1l}} &= (p_{1l} - \theta_l)\left[\left(\nu + \frac{\mu}{\nu}\right) q_1(p_{1l}, p_{2l}) + \left(1 - \nu - \frac{\mu}{\nu}\right) q_1(p_{1l}, p_{2h})\right] - t_{1l} \\ &\geq (p_{1h} - \theta_l)\left[\left(\nu + \frac{\mu}{\nu}\right) q_1(p_{1h}, p_{2l}) + \left(1 - \nu - \frac{\mu}{\nu}\right) q_1(p_{1h}, p_{2h})\right] - t_{1h}\end{aligned}$$

$$\begin{aligned}\pi_{R_{1h}} &= (p_{1h} - \theta_h)\left[\left(\nu - \frac{\mu}{1-\nu}\right) q_1(p_{1h}, p_{2l}) + \left(1 - \nu + \frac{\mu}{1-\nu}\right) q_1(p_{1h}, p_{2h})\right] - t_{1h} \\ &\geq (p_{1l} - \theta_h)\left[\left(\nu - \frac{\mu}{1-\nu}\right) q_1(p_{1l}, p_{2l}) + \left(1 - \nu + \frac{\mu}{1-\nu}\right) q_1(p_{1l}, p_{2h})\right] - t_{1l}.\end{aligned}$$

Using the expression for $\pi_{R_{1h}}$ and the fact that $\pi_{R_{1h}} = 0$ in equilibrium, we can reformulate the binding incentive constraint for the efficient retailer as follows

$$\begin{aligned}\pi_{R_{1l}} &= \Delta\theta\left[\nu q_1(p_{1h}, p_{2l}) + (1-\nu) q_1(p_{1h}, p_{2h})\right] \\ &\quad - \frac{\mu}{\nu(1-\nu)}(p_{1h} - \theta_l - \nu\Delta\theta)\left[q_1(p_{1h}, p_{2h}) - q_1(p_{1h}, p_{2l})\right],\end{aligned} \tag{24}$$

which captures the retailer's informational rents. For the sake of tractability, we formally derive the impact of cost correlation μ on the equilibrium ownership stake when μ is relatively small and consumer demand takes the linear form in (15).

Proposition 5 *Suppose that the degree of correlation μ between costs θ_1 and θ_2 is small. If R_1 is privately informed about its costs $\theta_1 \in \{\theta_l, \theta_h\}$, the equilibrium ownership stake ρ^* that M_1 holds in R_1 increases (at an increasing rate) with μ.*

The numerical simulations illustrated in Figure 3 show that the result of Proposition 5 carries over for large values of μ.[24] As (24) indicates, a higher price for the inefficient retailer makes a report of high costs less attractive for the efficient retailer and it does so to a larger extent when costs are more closely correlated ($\frac{\partial \pi_{R_{1l}}}{\partial p_{1h}} < 0$ and $\frac{\partial^2 \pi_{R_{1l}}}{\partial p_{1h} \partial \mu} < 0$). In line with the discussion in Section 7.3, this is because the efficient retailer realizes that the rival is more likely to be efficient and to set a relatively low price, which reduces the retailer's rents from cost manipulation. As a consequence, a larger degree of cost correlation increases the manufacturer's benefits of a higher price for the inefficient retailer in terms of rent extraction. In other words,

[24]In Figure 3 the parameter values are $\nu = 0.5$, $\beta = 1$, $\gamma = 0.4$, $\alpha = 19.2$, $\theta_l = 0$. Moreover, $\theta_h = 1.5$ (the bottom line), $\theta_h = 2$ (the middle line), and $\theta_h = 2.5$ (the top line).

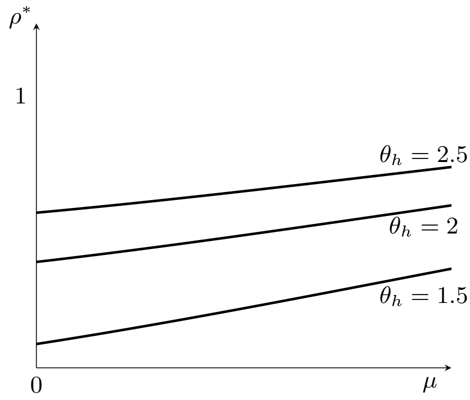


Figure 3: Cost correlation and ownership stake

for a given ownership stake, the manufacturer is more eager for an upward price distortion when cost correlation is higher. Since this increases the informational costs within the supply hierarchy, a higher ownership stake is preferred in equilibrium in order to mitigate these costs.

7.5. Competitor's vertical integration decision

Throughout the analysis we assume that manufacturer M_1 faces the fully integrated competitor $M_2 - R_2$ when deciding on the ownership stake ρ in its retailer R_1. This assumption is innocuous and the outcome of partial vertical integration carries over in alternative settings. It can be immediately seen from the proof of Proposition 2 that the result $\left.\frac{\partial \pi_{M_1-R_1}(\rho)}{\partial \rho}\right|_{\rho=1} < 0$ does not depend on the degree of vertical integration of $M_2 - R_2$. A similar result occurs if the decision on the level of ownership stake simultaneously takes place in the two supply hierarchies. This is because the benefits of softer competition arise regardless of what the rival does, and therefore each supply hierarchy has a unilateral incentive to depart from full integration.

8. Antitrust policy and empirical implications

In line with the theoretical literature, the empirical research on vertically related markets (exhaustively surveyed in Lafontaine and Slade 2007) has mainly focused on the binary choice between separation and integration. However, as documented in some relevant empirical works (e.g., Allen and Phillips 2000; Fee et al. 2006; Reiffen 1998), partial vertical acquisitions are a common phenomenon. The predictions of our model about the impact of partial vertical integration on retail prices lend themselves to empirically testable validations.

As discussed in Section 5.2, a higher degree of vertical integration in our model is welfare enhancing. A natural policy implication of this result is that any proposal of vertical acquisition should be approved by a myopic antitrust authority, since it improves welfare relative to full separation. However, a more sophisticated antitrust policy can achieve better outcomes. A forward-looking antitrust authority should block partial ownership agreements when it anticipates that firms will prefer full merger to separation. Alternatively, the antitrust authority should commit to only approving vertical acquisitions above a certain threshold that equates the marginal benefits of inducing a higher degree of vertical integration with the marginal costs of discouraging a vertical acquisition altogether.

We are aware that this kind of strategic commitment of the antitrust authority could be difficult to implement in practice. A more relevant implication of our results for vertical merger policy concerns the antitrust scrutiny of partial vertical divestitures. Our analysis suggests that the decision of a manufacturer to sell a fraction of the shares in its retailer can dampen competition. Remarkably, this is the case even when the acquirer is a silent investor or a firm operating in another market. Our conclusions complement the results of Foros et al. (2011) that show the anticompetitive effects of partial mergers relative to full mergers in horizontally related markets and recommend antitrust investigations of partial divestitures. By the same token, takeover regulations could be implemented, which favor full acquisitions over partial acquisitions.

The policy implications of our model are also in line with the results of Hunold et al. (2012), which show that, in a full information setting, passive ownership of downstream firms in their suppliers entails higher retail prices and is profitable with sufficiently intense competition. Greenlee and Raskovich (2006), however, find that a passive ownership interest of a downstream firm in an upstream monopoly is generally inconsequential and may harm consumers only in some circumstances, which limits the scope for antitrust intervention.

A well-known caveat of an antitrust policy recommendation in favor full integration is that it might induce anticompetitive input foreclosure. However, the empirical evidence suggests that lower retail prices tend to be associated with full integration (Lafontaine and Slade 2007), which is consistent with our results. Remarkably, Levy et al. (2016) show that under certain conditions partial integration is more likely to lead to input foreclosure than full integration. The predictions of our model provide further corroboration for the anticompetitive effects of partial vertical integration.

Our results indicate that partial vertical ownership emerges when it can induce an accommodating behavior of rivals, which is typically the case in markets where firms compete in prices with differentiated goods. This mode of competition can naturally arise in sectors with relationship-specific investments, such as vertically related markets. Hence, our analysis provides theoretical support for the empirical investigation of Ouimet (2013) that shows the preference for partial equity stakes over full integration in these sectors, where the number of patents is used as a proxy for relationship-specific investments. Conversely, we do not generally expect partial vertical ownership for strategic purposes when capacity constraints induce Cournot competition, since the partially integrated hierarchy's output reduction to curb the retailer's informational rents triggers a more aggressive behavior of rivals.

9. Concluding remarks

In this paper we investigate the strategic incentives for partial acquisitions in vertically related markets where two manufacturer-retailer pairs engage in differentiated good price competition and retailers are privately informed about their production costs. A partial ownership stake of a manufacturer in its retailer introduces a misalignment between the profit objectives of the two firms and entails an upward price distortion for the inefficient retailer in order to reduce the (costly) informational rents to the efficient retailer. This form of double marginalization from asymmetric information generates an information vertical effect that reduces the profitability of the supply hierarchy taken in isolation. In a competitive environment, the information vertical effect translates into an opposite competition horizontal effect. The partially integrated hierarchy's commitment to a higher price than under full integration induces the rival to increase its price as well. Therefore, partial vertical integration constitutes a strategic device to relax competition. The equilibrium degree of vertical integration trades off the benefits of softer competition against the informational costs.

Our analysis provides theoretical support for the empirical evidence on partial vertical integration and formulates antitrust policy recommendations for mergers and acquisitions in vertically related markets.

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Appendix

This Appendix collects the proofs.

Proof of Remark 1. Substituting t_1 with π_{R_1} from (1), M_1's problem of maximizing (2) becomes

$$\max_{p_1,\pi_{R_1}} \left(p_1-\theta_1\right) E_{\theta_2}\left[q_1\left(p_1,p_2\right)|\theta_1\right]-(1-\rho)\,\pi_{R_1} \quad s.t. \quad \pi_{R_1}\geq 0,$$

where the constraint ensures that R_1 is willing to participate in the contractual relationship with M_1. Since the maximand decreases with π_{R_1} for any $\rho\in[0,1]$, we find $\pi_{R_1}=0$ in equilibrium.[25] Taking the first-order condition for p_1 yields $E_{\theta_2}\left[q_1\left(p_1,p_2\right)|\theta_1\right]+\left(p_1-\theta_1\right)\frac{\partial E_{\theta_2}\left[q_1(p_1,p_2)|\theta_1\right]}{\partial p_1}=0$.

Using (3), M_2-R_2 solves

$$\max_{p_2} \left(p_2-\theta_2\right) E_{\theta_1}\left[q_2\left(p_1,p_2\right)|\theta_2\right],$$

which yields $E_{\theta_1}\left[q_2\left(p_1,p_2\right)|\theta_2\right]+\left(p_2-\theta_2\right)\frac{\partial E_{\theta_1}\left[q_2(p_1,p_2)|\theta_2\right]}{\partial p_2}=0$. Solving the system of the first-order conditions for the maximization problems of M_1 and M_2-R_2 gives the expression (4). Since the equilibrium prices do not depend on the ownership stake ρ, we have $\widehat{\rho}\in[0,1]$ in equilibrium. ■

[25]Indeed, π_{R_1} vanishes for $\rho=1$ and $\pi_{R_1}\geq 0$ can be supported in equilibrium. However, this does not affect the first-order condition for p_1.

Proof of Lemma 1 . The results in the lemma immediately follow from the first-order conditions for p_{1l} and p_{1h} in the maximization problem (8), and from the first-order condition for p_2 in the maximization problem (9). We now show that the incentive constraint (6) is satisfied in equilibrium. Combining terms in (6) yields

$$\begin{aligned}\pi_{R_{1h}} &= (p_{1h}-\theta_h)\,E_{\theta_2}\left[q_1(p_{1h},p_2)\,|\theta_h\right]-t_{1h}\\ &\geq (p_{1l}-\theta_h)\,E_{\theta_2}\left[q_1(p_{1l},p_2)\,|\theta_h\right]-t_{1l}\\ &= \pi_{R_{1l}}+p_{1l}\left\{E_{\theta_2}\left[q_1(p_{1l},p_2)\,|\theta_h\right]-E_{\theta_2}\left[q_1(p_{1l},p_2)\,|\theta_l\right]\right\}\\ &+\theta_l E_{\theta_2}\left[q_1(p_{1l},p_2)\,|\theta_l\right]-\theta_h E_{\theta_2}\left[q_1(p_{1l},p_2)\,|\theta_h\right].\end{aligned}$$

Using (7) evaluated in equilibrium and the fact that $\pi_{R_{1h}}=0$, we obtain after some manipulation

$$\begin{aligned}0 &\geq \Delta\theta\left\{E_{\theta_2}\left[q_1(p_{1h}^*,p_2^*)\,|\theta_h\right]-E_{\theta_2}\left[q_1(p_{1l}^*,p_2^*)\,|\theta_l\right]\right\}-(p_{1h}^*-\theta_l)\left\{E_{\theta_2}\left[q_1(p_{1h}^*,p_2^*)\,|\theta_h\right]\right.\\ &\left.-E_{\theta_2}\left[q_1(p_{1h}^*,p_2^*)\,|\theta_l\right]\right\}+(p_{1l}^*-\theta_h)\left\{E_{\theta_2}\left[q_1(p_{1l}^*,p_2^*)\,|\theta_h\right]-E_{\theta_2}\left[q_1(p_{1l}^*,p_2^*)\,|\theta_l\right]\right\}.\end{aligned}$$

Given the following first-order Taylor approximation

$$E_{\theta_2}\left[q_1(p_{1h}^*,p_2^*)\,|\theta_k\right]\approx E_{\theta_2}\left[q_1(p_{1l}^*,p_2^*)\,|\theta_k\right]+\frac{\partial E_{\theta_2}\left[q_1(p_{1l}^*,p_2^*)\,|\theta_k\right]}{\partial p_{1l}}(p_{1h}^*-p_{1l}^*),\ k=l,h,$$

we find

$$\begin{aligned}0 &\geq -(p_{1h}^*-p_{1l}^*)\,E_{\theta_2}\left[q_1(p_{1l}^*,p_2^*)\,|\theta_h\right]-(p_{1h}^*-\theta_h)(p_{1h}^*-p_{1l}^*)\frac{\partial E_{\theta_2}\left[q_1(p_{1l}^*,p_2^*)\,|\theta_h\right]}{\partial p_{1l}}\\ &+(p_{1h}^*-p_{1l}^*)\,E_{\theta_2}\left[q_1(p_{1l}^*,p_2^*)\,|\theta_l\right]+(p_{1h}^*-\theta_l)(p_{1h}^*-p_{1l}^*)\frac{\partial E_{\theta_2}\left[q_1(p_{1l}^*,p_2^*)\,|\theta_l\right]}{\partial p_{1l}}\\ &= -(p_{1h}^*-p_{1l}^*)\left\{E_{\theta_2}\left[q_1(p_{1l}^*,p_2^*)\,|\theta_h\right]-E_{\theta_2}\left[q_1(p_{1l}^*,p_2^*)\,|\theta_l\right]\right.\\ &\left.+(p_{1h}^*-\theta_h)\frac{\partial E_{\theta_2}\left[q_1(p_{1l}^*,p_2^*)\,|\theta_h\right]}{\partial p_{1l}}-(p_{1h}^*-\theta_l)\frac{\partial E_{\theta_2}\left[q_1(p_{1l}^*,p_2^*)\,|\theta_l\right]}{\partial p_{1l}}\right\}.\end{aligned}$$

Since $p_{1h}^*-p_{1l}^*>0$ (see (11) and (12)) and the expression in curly brackets is also positive (by Assumptions 1-2 and positively correlated, or independent, costs), the constraint (6) is satisfied in equilibrium. Finally, we check that the participation constraint $\pi_{R_{1l}}\geq 0$ is also fulfilled in equilibrium. Using the incentive constraint (7), sufficient (albeit not necessary) condition is that either the degree of cost correlation or the degree of substitutability is not too large. ■

Proof of Proposition 1 . The proof of the first sentence of the proposition immediately follows from a comparison between (4) and (11), (12), (13) for $\rho=1$. To prove the second sentence, denoting by $\frac{\partial\pi_{M_1}^e}{\partial p_{1l}}$, $\frac{\partial\pi_{M_1}^e}{\partial p_{1h}}$ and $\frac{\partial\pi_2}{\partial p_2}$ the left-hand side of (11), (12) and (13) respectively, the implicit function theorem yields

$$\begin{bmatrix}\frac{\partial p_{1l}^*}{\partial\rho}\\ \frac{\partial p_{1h}^*}{\partial\rho}\\ \frac{\partial p_2^*}{\partial\rho}\end{bmatrix}=-\begin{bmatrix}\frac{\partial^2\pi_{M_1}^e}{\partial p_{1l}^2} & \frac{\partial^2\pi_{M_1}^e}{\partial p_{1l}\partial p_{1h}} & \frac{\partial^2\pi_{M_1}^e}{\partial p_{1l}\partial p_2}\\ \frac{\partial^2\pi_{M_1}^e}{\partial p_{1h}\partial p_{1l}} & \frac{\partial^2\pi_{M_1}^e}{\partial p_{1h}^2} & \frac{\partial^2\pi_{M_1}^e}{\partial p_{1h}\partial p_2}\\ \frac{\partial^2\pi_2}{\partial p_2\partial p_{1l}} & \frac{\partial^2\pi_2}{\partial p_2\partial p_{1h}} & \frac{\partial^2\pi_2}{\partial p_2^2}\end{bmatrix}^{-1}\begin{bmatrix}\frac{\partial^2\pi_{M_1}^e}{\partial p_{1l}\partial\rho}\\ \frac{\partial^2\pi_{M_1}^e}{\partial p_{1h}\partial\rho}\\ \frac{\partial^2\pi_2}{\partial p_2\partial\rho}\end{bmatrix}.$$

It follows from Assumptions 2-4 and $\frac{\partial^2\pi_{M_1}^e}{\partial p_{1h}\partial p_{1l}}=\frac{\partial^2\pi_{M_1}^e}{\partial p_{1l}\partial p_{1h}}=0$ that the determinant of the Jacobian matrix is negative. Moreover, it can be immediately seen from (11), (12) and (13) that $\frac{\partial^2 \pi^e_{M_1}}{\partial p_{1l} \partial \rho} = \frac{\partial^2 \pi_2}{\partial p_2 \partial \rho} = 0$ and $\frac{\partial^2 \pi^e_{M_1}}{\partial p_{1h} \partial \rho} < 0$. Hence, standard computations show that

$$sign\left(\frac{\partial p^*_{1l}}{\partial \rho}\right) = sign\left(\frac{\partial^2 \pi^e_{M_1}}{\partial p_{1l} \partial p_2}\frac{\partial^2 \pi_2}{\partial p_2 \partial p_{1h}}\frac{\partial^2 \pi^e_{M_1}}{\partial p_{1h} \partial \rho}\right) \leq 0$$

$$sign\left(\frac{\partial p^*_{1h}}{\partial \rho}\right) = sign\left[\left(\frac{\partial^2 \pi^e_{M_1}}{\partial p^2_{1l}}\frac{\partial^2 \pi_2}{\partial p^2_2} - \frac{\partial^2 \pi^e_{M_1}}{\partial p_{1l} \partial p_2}\frac{\partial^2 \pi_2}{\partial p_2 \partial p_{1l}}\right)\frac{\partial^2 \pi^e_{M_1}}{\partial p_{1h} \partial \rho}\right] < 0$$

$$sign\left(\frac{\partial p^*_{2}}{\partial \rho}\right) = sign\left(-\frac{\partial^2 \pi^e_{M_1}}{\partial p^2_{1l}}\frac{\partial^2 \pi_2}{\partial p_2 \partial p_{1h}}\frac{\partial^2 \pi^e_{M_1}}{\partial p_{1h} \partial \rho}\right) \leq 0,$$

where the values of the signs follow from Assumptions 2-4, and the equalities hold when consumer demands are independent ($\frac{\partial^2 \pi^e_{M_1}}{\partial p_{1l} \partial p_2} = \frac{\partial^2 \pi_2}{\partial p_2 \partial p_{1h}} = 0$). ■

Proof of Proposition 2. Differentiating the objective function in (14) with respect to the ownership stake ρ yields

$$\nu\left\{\frac{\partial p^*_{1l}(\rho)}{\partial \rho}E_{\theta_2}\left[q_1\left(p^*_{1l}(\rho), p^*_2(\rho)\right)|\theta_l\right] + \left[p^*_{1l}(\rho) - \theta_l\right]\frac{\partial E_{\theta_2}\left[q_1\left(p^*_{1l}(\rho), p^*_2(\rho)\right)|\theta_l\right]}{\partial \rho}\right\}$$
$$+(1-\nu)\left\{\frac{\partial p^*_{1h}(\rho)}{\partial \rho}E_{\theta_2}\left[q_1\left(p^*_{1h}(\rho), p^*_2(\rho)\right)|\theta_h\right] + \left[p^*_{1h}(\rho) - \theta_h\right]\frac{\partial E_{\theta_2}\left[q_1\left(p^*_{1h}(\rho), p^*_2(\rho)\right)|\theta_h\right]}{\partial \rho}\right\}.$$

Applying the chain rule

$$\frac{\partial E_{\theta_2}\left[q_1\left(p^*_{1k}(\rho), p^*_2(\rho)\right)|\theta_k\right]}{\partial \rho} = \frac{\partial p^*_{1k}(\rho)}{\partial \rho}\frac{\partial E_{\theta_2}\left[q_1\left(p^*_{1k}(\rho), p^*_2(\rho)\right)|\theta_k\right]}{\partial p_{1k}}$$
$$+ E_{\theta_2}\left[\frac{\partial q_1\left(p^*_{1k}(\rho), p^*_2(\rho)\right)}{\partial p_2}\frac{\partial p^*_2(\rho)}{\partial \rho}|\theta_k\right], \quad k = l, h,$$

we obtain

$$\nu\left\{\frac{\partial p^*_{1l}(\rho)}{\partial \rho}\left[E_{\theta_2}\left[q_1\left(p^*_{1l}(\rho), p^*_2(\rho)\right)|\theta_l\right] + \left[p^*_{1l}(\rho) - \theta_l\right]\frac{\partial E_{\theta_2}\left[q_1\left(p^*_{1l}(\rho), p^*_2(\rho)\right)|\theta_l\right]}{\partial p_{1l}}\right]\right.$$
$$\left. + \left[p^*_{1l}(\rho) - \theta_l\right]E_{\theta_2}\left[\frac{\partial q_1\left(p^*_{1l}(\rho), p^*_2(\rho)\right)}{\partial p_2}\frac{\partial p^*_2(\rho)}{\partial \rho}|\theta_l\right]\right\} + (1-\nu)$$
$$\times\left\{\frac{\partial p^*_{1h}(\rho)}{\partial \rho}\left[E_{\theta_2}\left[q_1\left(p^*_{1h}(\rho), p^*_2(\rho)\right)|\theta_h\right] + \left[p^*_{1h}(\rho) - \theta_h\right]\frac{\partial E_{\theta_2}\left[q_1\left(p^*_{1h}(\rho), p^*_2(\rho)\right)|\theta_h\right]}{\partial p_{1h}}\right]\right.$$
$$\left. + \left[p^*_{1h}(\rho) - \theta_h\right]E_{\theta_2}\left[\frac{\partial q_1\left(p^*_{1h}(\rho), p^*_2(\rho)\right)}{\partial p_2}\frac{\partial p^*_2(\rho)}{\partial \rho}|\theta_h\right]\right\}.$$

Substituting (11) and (12) finally gives

$$\nu\left[p^*_{1l}(\rho) - \theta_l\right]E_{\theta_2}\left[\frac{\partial q_1\left(p^*_{1l}(\rho), p^*_2(\rho)\right)}{\partial p_2}\frac{\partial p^*_2(\rho)}{\partial \rho}|\theta_l\right] + (1-\nu)$$
$$\times\left\{\frac{\partial p^*_{1h}(\rho)}{\partial \rho}\phi(\nu)(1-\rho)\Omega\left(p^*_{1h}(\rho)\right) + \left[p^*_{1h}(\rho) - \theta_h\right]E_{\theta_2}\left[\frac{\partial q_1\left(p^*_{1h}(\rho), p^*_2(\rho)\right)}{\partial p_2}\frac{\partial p^*_2(\rho)}{\partial \rho}|\theta_h\right]\right\}.$$

If the second condition in Assumption 1 holds with inequality (interdependent demands), it follows from Proposition 1 (and $p^*_{1l} - \theta_l > 0$, $p^*_{1h} - \theta_h > 0$) that $\left.\frac{\partial \pi_{M_1 - R_1}(\rho)}{\partial \rho}\right|_{\rho=1} < 0$, which implies that the equilibrium ownership stake is $\rho^* < 1$. If the second condition in Assumption

1 holds with equality (independent demands), we have $\left.\frac{\partial \pi_{M_1-R_1}(\rho)}{\partial \rho}\right|_{\rho=1} = 0$, which implies that the equilibrium ownership stake is $\rho^* = 1$. ■

Proof of Proposition 3. Under full information, M_1's problem of maximizing (2) can be written as

$$\max_{p_{1k},\pi_{R_{1k}}} \left(p_{1k} - \theta_k\right)\left(\alpha - \beta p_{1k} + \gamma p_{2k}\right) - (1-\rho)\,\pi_{R_{1k}} \quad s.t. \quad \pi_{R_{1k}} \geq 0, \quad k = l, h,$$

where the constraint ensures that R_1 (with costs θ_l or θ_h) accepts the contractual offer of M_1. Since the maximand decreases with $\pi_{R_{1k}}$ for any $\rho \in [0,1]$, we have $\pi_{R_{1k}} = 0$ in equilibrium. Taking the first-order condition for p_{1k} yields $\alpha - 2\beta p_{1k} + \gamma p_{2k} + \beta\theta_k = 0$.

Using (3), $M_2 - R_2$ solves

$$\max_{p_{2k}} \left(p_{2k} - \theta_k\right)\left(\alpha - \beta p_{2k} + \gamma p_{1k}\right),\ k = l, h,$$

which yields $\alpha - 2\beta p_{2k} + \gamma p_{1k} + \beta\theta_k = 0$. Solving the system of the first-order conditions for the maximization problems of M_1 and $M_2 - R_2$ yields $\widehat{p}_{ik} = \frac{\alpha+\beta\theta_k}{2\beta-\gamma}$, $i = 1, 2$, $k = l, h$.

Now, we turn to the case of asymmetric information. Replacing (15) into (5) and (6), the incentive constraints write as

$$\begin{aligned}
\pi_{R_{1l}} &= \left(p_{1l} - \theta_l\right)\left(\alpha - \beta p_{1l} + \gamma p_{2l}\right) - t_{1l} \\
&\geq \left(p_{1h} - \theta_l\right)\left(\alpha - \beta p_{1h} + \gamma p_{2l}\right) - t_{1h} \\
&= \pi_{R_{1h}} + \gamma p_{1h}\left(p_{2l} - p_{2h}\right) + \theta_h\left(\alpha - \beta p_{1h} + \gamma p_{2h}\right) - \theta_l\left(\alpha - \beta p_{1h} + \gamma p_{2l}\right) \\
&= \pi_{R_{1h}} + \Delta\theta\left(\alpha - \beta p_{1h} + \gamma p_{2h}\right) - \gamma\left(p_{2h} - p_{2l}\right)\left(p_{1h} - \theta_l\right)
\end{aligned} \tag{25}$$

$$\begin{aligned}
\pi_{R_{1h}} &= \left(p_{1h} - \theta_h\right)\left(\alpha - \beta p_{1h} + \gamma p_{2h}\right) - t_{1h} \\
&\geq \left(p_{1l} - \theta_h\right)\left(\alpha - \beta p_{1l} + \gamma p_{2h}\right) - t_{1l} \\
&= \pi_{R_{1l}} + \gamma p_{1l}\left(p_{2h} - p_{2l}\right) - \theta_h\left(\alpha - \beta p_{1l} + \gamma p_{2h}\right) + \theta_l\left(\alpha - \beta p_{1l} + \gamma p_{2l}\right) \\
&= \pi_{R_{1l}} - \Delta\theta\left(\alpha - \beta p_{1l} + \gamma p_{2l}\right) + \gamma\left(p_{2h} - p_{2l}\right)\left(p_{1l} - \theta_h\right).
\end{aligned} \tag{26}$$

Since the constraint (25) is binding and $\pi_{R_{1h}} = 0$ in equilibrium, substituting (15) into (2) M_1's problem becomes

$$\begin{aligned}
&\max_{p_{1l},p_{1h}} \nu\left\{\left(p_{1l} - \theta_l\right)\left(\alpha - \beta p_{1l} + \gamma p_{2l}\right) - (1-\rho)\left[\Delta\theta\left(\alpha - \beta p_{1h} + \gamma p_{2h}\right) - \gamma\left(p_{2h} - p_{2l}\right)\right.\right. \\
&\quad \left.\left.\times\left(p_{1h} - \theta_l\right)\right]\right\} + (1-\nu)\left(p_{1h} - \theta_h\right)\left(\alpha - \beta p_{1h} + \gamma p_{2h}\right).
\end{aligned}$$

The first-order conditions for p_{1l} and p_{1h} are respectively $\alpha - 2\beta p_{1l} + \gamma p_{2l} + \beta\theta_l = 0$ and $\alpha - 2\beta p_{1h} + \gamma p_{2h} + \beta\theta_h + \phi(\nu)(1-\rho)\left[\beta\Delta\theta + \gamma\left(p_{2h} - p_{2l}\right)\right] = 0$.

Substituting (15) into (3), $M_2 - R_2$ solves

$$\max_{p_{2k}} \left(p_{2k} - \theta_k\right)\left(\alpha - \beta p_{2k} + \gamma p_{1k}\right), \quad k = l, h,$$

which yields $\alpha - 2\beta p_{2k} + \gamma p_{1k} + \beta\theta_k = 0$. The first-order conditions for the maximization problems of M_1 and $M_2 - R_2$ yield $p^*_{il} = \frac{\alpha+\beta\theta_l}{2\beta-\gamma}$, $i = 1, 2$, and the expressions in (16) and (17). To check that the incentive constraint (26) is satisfied in equilibrium, we substitute the binding constraint (25) into (26), which yields after some manipulation $0 \geq -(p_{1h}^* - p_{1l}^*)[\beta\Delta\theta + \gamma(p_{2h}^* - p_{2l}^*)]$. This condition is fulfilled since $p_{1h}^* - p_{1l}^* > 0$ and $p_{2h}^* - p_{2l}^* > 0$. Moreover, the binding constraint (25) implies that sufficient (albeit not necessary) condition for the participation constraint $\pi_{R_{1l}} \geq 0$ to be satisfied is that γ is not too high.

The optimal ownership stake is the solution to following program

$$\max_{\rho\in[0,1]} \nu\left(p_{1l}^* - \theta_l\right)\left(\alpha - \beta p_{1l}^* + \gamma p_{2l}^*\right) + (1-\nu)\left[p_{1h}^*(\rho) - \theta_h\right]\left[\alpha - \beta p_{1h}^*(\rho) + \gamma p_{2h}^*(\rho)\right].$$

Assuming for the time being an internal solution and using (16) and (17), the first-order condition for ρ can be written as

$$\begin{aligned}
&\left\{\frac{-\phi(\nu)\left[4\beta^3\Delta\theta - \gamma^2(\alpha+\beta\theta_h)\right]\left\{4\beta^2 - \gamma^2\left[1+\phi(\nu)(1-\rho)\right]\right\}}{(2\beta-\gamma)\left\{4\beta^2-\gamma^2\left[1+\phi(\nu)(1-\rho)\right]\right\}^2}\right.\\
&\left.-\gamma^2\phi(\nu)\frac{(\alpha+\beta\theta_h)\left(4\beta^2-\gamma^2\right)+\phi(\nu)(1-\rho)\left[4\beta^3\Delta\theta-\gamma^2(\alpha+\beta\theta_h)\right]}{(2\beta-\gamma)\left\{4\beta^2-\gamma^2\left[1+\phi(\nu)(1-\rho)\right]\right\}^2}\right\}\\
&\times\left\{\alpha - \frac{(\beta-\gamma)\left(4\beta^2-\gamma^2\right)(\alpha+\beta\theta_h)}{(2\beta-\gamma)\left\{4\beta^2-\gamma^2\left[1+\phi(\nu)(1-\rho)\right]\right\}} - \phi(\nu)(1-\rho)\right.\\
&\left.\times\frac{2\beta^2\left(2\beta^2-\gamma^2\right)\Delta\theta - \gamma^2(\alpha+\beta\theta_h)(\beta-\gamma)}{(2\beta-\gamma)\left\{4\beta^2-\gamma^2\left[1+\phi(\nu)(1-\rho)\right]\right\}}\right\}\\
&+\left\{\frac{(\alpha+\beta\theta_h)\left(4\beta^2-\gamma^2\right)+\phi(\nu)(1-\rho)\left[4\beta^3\Delta\theta-\gamma^2(\alpha+\beta\theta_h)\right]}{(2\beta-\gamma)\left\{4\beta^2-\gamma^2\left[1+\phi(\nu)(1-\rho)\right]\right\}} - \theta_h\right\}\\
&\times\left\{\phi(\nu)\frac{2\beta^2\left(2\beta^2-\gamma^2\right)\Delta\theta - \gamma^2(\alpha+\beta\theta_h)(\beta-\gamma)\left\{4\beta^2-\gamma^2\left[1+\phi(\nu)(1-\rho)\right]\right\}}{(2\beta-\gamma)\left\{4\beta^2-\gamma^2\left[1+\phi(\nu)(1-\rho)\right]\right\}^2}\right.\\
&+\gamma^2\phi(\nu)\left[\frac{(\beta-\gamma)\left(4\beta^2-\gamma^2\right)(\alpha+\beta\theta_h)}{(2\beta-\gamma)\left\{4\beta^2-\gamma^2\left[1+\phi(\nu)(1-\rho)\right]\right\}^2} + \phi(\nu)(1-\rho)\right.\\
&\left.\left.\times\frac{2\beta^2\left(2\beta^2-\gamma^2\right)\Delta\theta - \gamma^2(\alpha+\beta\theta_h)(\beta-\gamma)}{(2\beta-\gamma)\left\{4\beta^2-\gamma^2\left[1+\phi(\nu)(1-\rho)\right]\right\}^2}\right]\right\} = 0.
\end{aligned}$$

Combining terms implies

$$\begin{aligned}
&\phi(\nu)(1-\rho)\left(4\beta^2-\gamma^2\right)\left\{4\alpha\beta^3\gamma^2(2\beta-\gamma)\Delta\theta + \left[2\beta^2\left(2\beta^2-\gamma^2\right)\Delta\theta - \gamma^2(\alpha+\beta\theta_h)(\beta-\gamma)\right]\right.\\
&\times\left[4\beta^3\Delta\theta - \gamma^2(\alpha+\beta\theta_h)\right] + \gamma^2\left[2\beta^2\left(2\beta^2-\gamma^2\right)\Delta\theta - \gamma^2(\alpha+\beta\theta_h)(\beta-\gamma)\right](\alpha+\beta\theta_h)\\
&\left.+2\beta^2\left(2\beta^2-\gamma^2\right)\left[4\beta^3\Delta\theta-\gamma^2(\alpha+\beta\theta_h)\right]\Delta\theta + 2\beta^2\gamma^2(2\beta-\gamma)\left(2\beta^2-\gamma^2\right)\Delta\theta\theta_h\right\}\\
&+\left(4\beta^2-\gamma^2\right)^2\left[-4\alpha\beta^3(2\beta-\gamma)\Delta\theta + 4\beta^3(\beta-\gamma)(\alpha+\beta\theta_h)\Delta\theta\right.\\
&\left.+2\beta^2\left(2\beta^2-\gamma^2\right)(\alpha+\beta\theta_h)\Delta\theta - 2\beta^2(2\beta-\gamma)\left(2\beta^2-\gamma^2\right)\Delta\theta\theta_h\right] = 0,
\end{aligned}$$

which gives after further manipulation $\phi(\nu)(1-\rho)\left\{8\beta^3\left(2\beta^2-\gamma^2\right)\Delta\theta + \gamma^4\left[\alpha-(\beta-\gamma)\theta_h\right]\right\} - \gamma^2\left(4\beta^2-\gamma^2\right)\left[\alpha-(\beta-\gamma)\theta_h\right] = 0$. This yields the optimal ownership stake in (20). The remaining part of the proposition follows from straightforward computations. ■

Proof of Proposition 4. Proceeding backwards, R_1's problem of maximizing (22) with respect to p_1 gives the following first-order condition

$$E_{\theta_2}\left[q_1\left(p_1,p_2\right)|\theta_1\right]+\left(p_1-\theta_1-w_1\right)\frac{\partial E_{\theta_2}\left[q_1\left(p_1,p_2\right)|\theta_1\right]}{\partial p_1}=0. \tag{27}$$

Under full information within M_1-R_1, after substituting f_1 with π_{R_1} from (22) into (23), M_1 solves

$$\max_{w_1,\pi_{R_1}}\left[p_1\left(w_1\right)-\theta_1\right]E_{\theta_2}\left[q_1\left(p_1\left(w_1\right),p_2\right)|\theta_1\right]-\left(1-\rho\right)\pi_{R_1}\quad s.t.\ \ \pi_{R_1}\geq 0.$$

Since the maximand decreases with π_{R_1} for any $\rho\in[0,1]$, we find $\pi_{R_1}=0$ in equilibrium. The first-order condition for w_1 yields

$$\frac{\partial p_1\left(w_1\right)}{\partial w_1}\left\{E_{\theta_2}\left[q_1\left(p_1\left(w_1\right),p_2\right)|\theta_1\right]+\left[p_1\left(w_1\right)-\theta_1\right]\frac{\partial E_{\theta_2}\left[q_1\left(p_1\left(w_1\right),p_2\right)|\theta_1\right]}{\partial p_1}\right\}=0.$$

Using (27), this expression reduces to $w_1\frac{\partial p_1(w_1)}{\partial w_1}\frac{\partial E_{\theta_2}[q_1(p_1(w_1),p_2)|\theta_1]}{\partial p_1}=0$. It follows from $\frac{\partial p_1(w_1)}{\partial w_1}>0$ (see (27)) and $\frac{\partial E_{\theta_2}[q_1(p_1(w_1),p_2)|\theta_1]}{\partial p_1}<0$ (by Assumption 1) that we obtain $\widehat{w}_1=0$ in equilibrium. Since equilibrium prices do not depend on the ownership stake ρ, we have $\widehat{\rho}_{tp}\in[0,1]$ in equilibrium.

When R_1 is privately informed about its costs $\theta_1\in\{\theta_l,\theta_h\}$, M_1 offers R_1 a direct incentive compatible contract menu $\{(f_{1l},w_{1l}),(f_{1h},w_{1h})\}$ that satisfies the following incentive constraints

$$\begin{aligned}\pi_{R_{1l}}&=\left(p_{1l}-\theta_l-w_{1l}\right)E_{\theta_2}\left[q_1\left(p_{1l},p_2\right)|\theta_l\right]-f_{1l}\\&\geq\left(p_{1h}-\theta_l-w_{1h}\right)E_{\theta_2}\left[q_1\left(p_{1h},p_2\right)|\theta_l\right]-f_{1h}\end{aligned} \tag{28}$$

$$\begin{aligned}\pi_{R_{1h}}&=\left(p_{1h}-\theta_h-w_{1h}\right)E_{\theta_2}\left[q_1\left(p_{1h},p_2\right)|\theta_h\right]-f_{1h}\\&\geq\left(p_{1l}-\theta_h-w_{1l}\right)E_{\theta_2}\left[q_1\left(p_{1l},p_2\right)|\theta_h\right]-f_{1l}.\end{aligned} \tag{29}$$

Using the expression for $\pi_{R_{1h}}$ and the fact that $\pi_{R_{1h}}=0$ in equilibrium,[26] the binding constraint (28) can be rewritten as

$$\begin{aligned}\pi_{R_{1l}}&=p_{1h}\left\{E_{\theta_2}\left[q_1\left(p_{1h},p_2\right)|\theta_l\right]-E_{\theta_2}\left[q_1\left(p_{1h},p_2\right)|\theta_h\right]\right\}\\&\quad+\left(\theta_h+w_{1h}\right)E_{\theta_2}\left[q_1\left(p_{1h},p_2\right)|\theta_h\right]-\left(\theta_l+w_{1h}\right)E_{\theta_2}\left[q_1\left(p_{1h},p_2\right)|\theta_l\right]\\&=\Delta\theta E_{\theta_2}\left[q_1\left(p_{1h},p_2\right)|\theta_h\right]-\left(p_{1h}-\theta_l-w_{1h}\right)\\&\quad\times\left\{E_{\theta_2}\left[q_1\left(p_{1h},p_2\right)|\theta_h\right]-E_{\theta_2}\left[q_1\left(p_{1h},p_2\right)|\theta_l\right]\right\}.\end{aligned} \tag{30}$$

Hence, M_1's problem of maximizing its (expected) profits in (23) for a given ownership stake ρ

[26]Note from (28) and (29) that the efficient (inefficient) retailer that declares high (low) costs will set a price p_{1h} (p_{1l}), as defined by (27) for high (low) costs. Otherwise, M_1 would discover the retailer's cost misrepresentation and implement an adequate penalty. Moreover, in line with the proof of Lemma 1, the incentive constraint (29) for the inefficient retailer and the participation constraint $\pi_{R_{1l}}\geq 0$ for the efficient retailer can be checked ex post.

becomes

$$\max_{w_{1l},w_{1h}} \nu \left\{ \left[p_{1l}\left(w_{1l}\right) - \theta_l \right] E_{\theta_2}\left[q_1\left(p_{1l}\left(w_{1l}\right), p_2\right) | \theta_l \right] - (1-\rho)\, \pi_{R_{1l}}\left(p_{1h}\left(w_{1h}\right), w_{1h}\right) \right\}$$
$$+ (1-\nu) \left[p_{1h}\left(w_{1h}\right) - \theta_h \right] E_{\theta_2}\left[q_1\left(p_{1h}\left(w_{1h}\right), p_2\right) | \theta_h \right], \tag{31}$$

where $p_{1l}(w_{1l})$ and $p_{1h}(w_{1h})$ are given by (27). Taking the derivative of (30) with respect to w_{1h} yields

$$\begin{aligned}
\frac{\partial \pi_{R_{1l}}}{\partial w_{1h}} \equiv \Psi\left(p_{1h}\left(w_{1h}\right), w_{1h}\right) &= \Delta\theta \frac{\partial p_{1h}\left(w_{1h}\right)}{\partial w_{1h}} \frac{\partial E_{\theta_2}\left[q_1\left(p_{1h}\left(w_{1h}\right), p_2\right) | \theta_h \right]}{\partial p_{1h}} \\
&- \left[\frac{\partial p_{1h}\left(w_{1h}\right)}{\partial w_{1h}} - 1 \right] \left\{ E_{\theta_2}\left[q_1\left(p_{1h}\left(w_{1h}\right), p_2\right) | \theta_h \right] - E_{\theta_2}\left[q_1\left(p_{1h}\left(w_{1h}\right), p_2\right) | \theta_l \right] \right\} \\
&- \frac{\partial p_{1h}\left(w_{1h}\right)}{\partial w_{1h}} \left[p_{1h}\left(w_{1h}\right) - \theta_l - w_{1h} \right] \\
&\times \left\{ \frac{\partial E_{\theta_2}\left[q_1\left(p_{1h}\left(w_{1h}\right), p_2\right) | \theta_h \right]}{\partial p_{1h}} - \frac{\partial E_{\theta_2}\left[q_1\left(p_{1h}\left(w_{1h}\right), p_2\right) | \theta_l \right]}{\partial p_{1h}} \right\}.
\end{aligned} \tag{32}$$

Given Assumptions 1-2 and positively correlated (or independent) costs, sufficient (albeit not necessary) condition for $\Psi(.)$ to be negative is that the price-cost pass-through $\frac{\partial p_{1h}}{\partial w_{1h}}$ (which generally ranges between 0 and 1 from the combination of the first-order condition (27) and the associated second-order condition) is relatively large or costs are not highly correlated, which ensures that the positive expression in the first curly brackets is small enough.

Taking the first-order conditions for w_{1l} and w_{1h} in the maximization problem (31) yields after some manipulation

$$\frac{\partial p_{1l}\left(w_{1l}\right)}{\partial w_{1l}} \left\{ E_{\theta_2}\left[q_1\left(p_{1l}\left(w_{1l}\right), p_2\right) | \theta_l \right] + \left[p_{1l}\left(w_{1l}\right) - \theta_l \right] \frac{\partial E_{\theta_2}\left[q_1\left(p_{1l}\left(w_{1l}\right), p_2\right) | \theta_l \right]}{\partial p_{1l}} \right\} = 0 \tag{33}$$

$$\frac{\partial p_{1h}\left(w_{1h}\right)}{\partial w_{1h}} \left\{ E_{\theta_2}\left[q_1\left(p_{1h}\left(w_{1h}\right), p_2\right) | \theta_h \right] + \left[p_{1h}\left(w_{1h}\right) - \theta_h \right] \frac{\partial E_{\theta_2}\left[q_1\left(p_{1h}\left(w_{1h}\right), p_2\right) | \theta_h \right]}{\partial p_{1h}} \right\}$$
$$- \phi(\nu)(1-\rho)\, \Psi\left(p_{1h}\left(w_{1h}\right), w_{1h}\right) = 0. \tag{34}$$

Substituting (27) into (33) and (34) we find $w_{1l}^* = \widehat{w}_1 = 0$ and

$$w_{1h} \frac{\partial p_{1h}\left(w_{1h}\right)}{\partial w_{1h}} \frac{\partial E_{\theta_2}\left[q_1\left(p_{1h}\left(w_{1h}\right), p_2\right) | \theta_h \right]}{\partial p_{1h}} - \phi(\nu)(1-\rho)\, \Psi\left(p_{1h}\left(w_{1h}\right), w_{1h}\right) = 0,$$

which yields $w_{1h}^* = \widehat{w}_1 = 0$ for $\rho = 1$ and $w_{1h}^* > \widehat{w}_1$ for $\rho < 1$. Denoting by $\frac{\partial \pi_{R_{1l}}}{\partial p_{1l}}$, $\frac{\partial \pi_{R_{1h}}}{\partial p_{1h}}$, $\frac{\partial \pi^e_{M_1}}{\partial w_{1h}}$ and $\frac{\partial \pi_2}{\partial p_2}$ the left-hand side of (27) for p_{1k}, $k = l, h$, (34) and (13) respectively, the implicit function theorem yields

$$\begin{bmatrix} \frac{\partial p_{1l}^*}{\partial \rho} \\ \frac{\partial p_{1h}^*}{\partial \rho} \\ \frac{\partial w_{1h}^*}{\partial \rho} \\ \frac{\partial p_2^*}{\partial \rho} \end{bmatrix} = - \begin{bmatrix} \frac{\partial^2 \pi_{R_{1l}}}{\partial p_{1l}^2} & \frac{\partial^2 \pi_{R_{1l}}}{\partial p_{1l} \partial p_{1h}} & \frac{\partial^2 \pi_{R_{1l}}}{\partial p_{1l} \partial w_{1h}} & \frac{\partial^2 \pi_{R_{1l}}}{\partial p_{1l} \partial p_2} \\ \frac{\partial^2 \pi_{R_{1h}}}{\partial p_{1h} \partial p_{1l}} & \frac{\partial^2 \pi_{R_{1h}}}{\partial p_{1h}^2} & \frac{\partial^2 \pi_{R_{1h}}}{\partial p_{1h} \partial w_{1h}} & \frac{\partial^2 \pi_{R_{1h}}}{\partial p_{1h} \partial p_2} \\ \frac{\partial^2 \pi^e_{M_1}}{\partial w_{1h} \partial p_{1l}} & \frac{\partial^2 \pi^e_{M_1}}{\partial w_{1h} \partial p_{1h}} & \frac{\partial^2 \pi^e_{M_1}}{\partial w_{1h}^2} & \frac{\partial^2 \pi^e_{M_1}}{\partial w_{1h} \partial p_2} \\ \frac{\partial^2 \pi_2}{\partial p_2 \partial p_{1l}} & \frac{\partial^2 \pi_2}{\partial p_2 \partial p_{1h}} & \frac{\partial^2 \pi_2}{\partial p_2 \partial w_{1h}} & \frac{\partial^2 \pi_2}{\partial p_2^2} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial^2 \pi_{R_{1l}}}{\partial p_{1l} \partial \rho} \\ \frac{\partial^2 \pi_{R_{1h}}}{\partial p_{1h} \partial \rho} \\ \frac{\partial^2 \pi^e_{M_1}}{\partial w_{1h} \partial \rho} \\ \frac{\partial^2 \pi_2}{\partial p_2 \partial \rho} \end{bmatrix},$$

where p_{1k}^*, $k = l, h$, is the equilibrium price in (27) evaluated at w_{1k}^* and p_2^* is the equilibrium price arising from $M_2 - R_2$'s maximization problem (9). It follows from Assumptions 2-4 (where M_1's choice variables are w_{1l} and w_{1h}), the first-order condition in (27) and $\frac{\partial^2 \pi_{R_{1l}}}{\partial p_{1l}\partial p_{1h}} = \frac{\partial^2 \pi_{R_{1l}}}{\partial p_{1l}\partial w_{1h}} = \frac{\partial^2 \pi_{R_{1h}}}{\partial p_{1h}\partial p_{1l}} = \frac{\partial^2 \pi^e_{M_1}}{\partial w_{1h}\partial p_{1l}} = \frac{\partial^2 \pi^e_{M_1}}{\partial w_{1h}\partial p_{1h}} = \frac{\partial^2 \pi^e_{M_1}}{\partial w_{1h}\partial p_{2}} = \frac{\partial^2 \pi_{2}}{\partial p_{2}\partial w_{1h}} = 0$ that the determinant of the Jacobian matrix is positive. Moreover, it can be immediately seen from (13), (27) and (34) that $\frac{\partial^2 \pi_{R_{1l}}}{\partial p_{1l}\partial \rho} = \frac{\partial^2 \pi_{R_{1h}}}{\partial p_{1h}\partial \rho} = \frac{\partial^2 \pi_{2}}{\partial p_{2}\partial \rho} = 0$ and $\frac{\partial^2 \pi^e_{M_1}}{\partial w_{1h}\partial \rho} < 0$. Hence, standard computations show that

$$sign\left(\frac{\partial p_{1l}^*}{\partial \rho}\right) = sign\left(\frac{\partial^2 \pi_{R_{1l}}}{\partial p_{1l}\partial p_2}\frac{\partial^2 \pi_{R_{1h}}}{\partial p_{1h}\partial w_{1h}}\frac{\partial^2 \pi_{2}}{\partial p_{2}\partial p_{1h}}\frac{\partial^2 \pi^e_{M_1}}{\partial w_{1h}\partial \rho}\right) \leq 0$$

$$sign\left(\frac{\partial p_{1h}^*}{\partial \rho}\right) = sign\left[\left(\frac{\partial^2 \pi_{R_{1l}}}{\partial p_{1l}^2}\frac{\partial^2 \pi_{2}}{\partial p_{2}^2} - \frac{\partial^2 \pi_{R_{1l}}}{\partial p_{1l}\partial p_2}\frac{\partial^2 \pi_{2}}{\partial p_{2}\partial p_{1l}}\right)\frac{\partial^2 \pi_{R_{1h}}}{\partial p_{1h}\partial w_{1h}}\frac{\partial^2 \pi^e_{M_1}}{\partial w_{1h}\partial \rho}\right] < 0$$

$$sign\left(\frac{\partial w_{1h}^*}{\partial \rho}\right) = sign\left[\left(\frac{\partial^2 \pi_{R_{1l}}}{\partial p_{1l}\partial p_2}\frac{\partial^2 \pi_{R_{1h}}}{\partial p_{1h}^2}\frac{\partial^2 \pi_{2}}{\partial p_{2}\partial p_{1l}} + \frac{\partial^2 \pi_{R_{1l}}}{\partial p_{1l}^2}\frac{\partial^2 \pi_{R_{1h}}}{\partial p_{1h}\partial p_2}\frac{\partial^2 \pi_{2}}{\partial p_{2}\partial p_{1h}}\right.\right.$$
$$\left.\left. - \frac{\partial^2 \pi_{R_{1l}}}{\partial p_{1l}^2}\frac{\partial^2 \pi_{R_{1h}}}{\partial p_{1h}^2}\frac{\partial^2 \pi_{2}}{\partial p_{2}^2}\right)\frac{\partial^2 \pi^e_{M_1}}{\partial w_{1h}\partial \rho}\right] < 0$$

$$sign\left(\frac{\partial p_{2}^*}{\partial \rho}\right) = sign\left(-\frac{\partial^2 \pi_{R_{1l}}}{\partial p_{1l}^2}\frac{\partial^2 \pi_{R_{1h}}}{\partial p_{1h}\partial w_{1h}}\frac{\partial^2 \pi_{2}}{\partial p_{2}\partial p_{1h}}\frac{\partial^2 \pi^e_{M_1}}{\partial w_{1h}\partial \rho}\right) \leq 0, \tag{35}$$

where the values of the signs follow from Assumptions 2-4 (where M_1's choice variables are w_{1l} and w_{1h}), and the equalities hold when consumer demands are independent ($\frac{\partial^2 \pi_{R_{1l}}}{\partial p_{1l}\partial p_2} = \frac{\partial^2 \pi_{2}}{\partial p_{2}\partial p_{1h}} = 0$).

Differentiating (14) with respect to ρ and using (27), (33) and (34) yields after some manipulation

$$\nu\left[p_{1l}^*(\rho) - \theta_l\right]E_{\theta_2}\left[\frac{\partial q_1\left(p_{1l}^*(\rho), p_2^*(\rho)\right)}{\partial p_2}\frac{\partial p_2^*(\rho)}{\partial \rho}|\theta_l\right] + (1-\nu)\left\{\frac{\partial p_{1h}^*(\rho)}{\partial \rho}\phi(\nu)(1-\rho)\right.$$
$$\left.\times\frac{\Psi\left(p_{1h}^*(\rho), w_{1h}^*(\rho)\right)}{\frac{\partial p_{1h}^*(\rho)}{\partial w_{1h}}} + \left[p_{1h}^*(\rho) - \theta_h\right]E_{\theta_2}\left[\frac{\partial q_1\left(p_{1h}^*(\rho), p_2^*(\rho)\right)}{\partial p_2}\frac{\partial p_2^*(\rho)}{\partial \rho}|\theta_h\right]\right\}.$$

Proceeding along the same lines as in the proof of Proposition 2, if the condition in Assumption 1 holds with inequality (interdependent demands), it follows from (35) (and $p_{1l}^* - \theta_l > 0$, $p_{1h}^* - \theta_h > 0$) that $\left.\frac{\partial \pi_{M_1-R_1}(\rho)}{\partial \rho}\right|_{\rho=1} < 0$, which implies that the equilibrium ownership stake is $\rho_{tp}^* < 1$. If the condition in Assumption 1 holds with equality (independent demands), we have $\left.\frac{\partial \pi_{M_1-R_1}(\rho)}{\partial \rho}\right|_{\rho=1} = 0$, which implies that the equilibrium ownership stake is $\rho_{tp}^* = 1$. ■

Proof of Corollary 1. Suppose that the equilibrium under resale price maintenance is replicated with a two-part tariff. Substituting (12) into (34) yields

$$\phi(\nu)(1-\rho^*)\left[\frac{\partial p_{1h}^*}{\partial w_{1h}}\Omega\left(p_{1h}^*\right) - \Psi\left(p_{1h}^*, w_{1h}^*\right)\right].$$

As the inspection of (10) and (32) reveals, this expression is negative and vanishes when costs θ_1 and θ_2 are independent. It follows from Assumption 3 (where M_1's choice variables are w_{1l} and

w_{1h}) that a reduction in w_{1h} from the equilibrium under resale price maintenance is profitable unless costs are independent. Since we know from the proof of Proposition 4 that the optimal w_{1h} decreases with ρ, the equilibrium ownership stake under a two-part tariff is higher than the one under resale price maintenance, and they coincide with independent costs. ■

Proof of Proposition 5. With the demand function in (15), substituting (24) and $\pi_{R_{1h}} = 0$ into (8), M_1's maximization problem becomes

$$\max_{p_{1l}, p_{1h}} \nu \left\{ (p_{1l} - \theta_l) \left[\alpha - \beta p_{1l} + \gamma \left(\frac{\nu^2 + \mu}{\nu} p_{2l} + \frac{\nu (1-\nu) - \mu}{\nu} p_{2h} \right) \right] - (1-\rho) \pi_{R_{1l}} (p_{1h}) \right\}$$
$$+ (1-\nu) (p_{1h} - \theta_h) \left[\alpha - \beta p_{1h} + \gamma \left(\frac{\nu (1-\nu) - \mu}{1-\nu} p_{2l} + \frac{(1-\nu)^2 + \mu}{1-\nu} p_{2h} \right) \right],$$

where

$$\pi_{R_{1l}} = \Delta\theta \left\{ \alpha - \beta p_{1h} + \gamma \left[\nu p_{2l} + (1-\nu) p_{2h} \right] \right\} - \frac{\mu\gamma}{\nu (1-\nu)} (p_{2h} - p_{2l}) (p_{1h} - \theta_l - \nu\Delta\theta).$$

The first-order conditions for p_{1l} and p_{1h} are $\alpha - 2\beta p_{1l} + \gamma \left[\frac{\nu^2+\mu}{\nu} p_{2l} + \frac{\nu(1-\nu)-\mu}{\nu} p_{2h} \right] + \beta\theta_l = 0$ and $\alpha - 2\beta p_{1h} + \gamma \left[\frac{\nu(1-\nu)-\mu}{1-\nu} p_{2l} + \frac{(1-\nu)^2+\mu}{1-\nu} p_{2h} \right] + \beta\theta_h + \phi(\nu)(1-\rho) \left[\beta\Delta\theta + \frac{\mu\gamma}{\nu(1-\nu)} (p_{2h} - p_{2l}) \right] = 0$, respectively. Taking the first-order conditions for p_{2l} and p_{2h} of $M_2 - R_2$'s maximization program (9) yields $\alpha - 2\beta p_{2l} + \gamma \left[\frac{\nu^2+\mu}{\nu} p_{1l} + \frac{\nu(1-\nu)-\mu}{\nu} p_{1h} \right] + \beta\theta_l = 0$ and $\alpha - 2\beta p_{2h} + \gamma \left[\frac{\nu(1-\nu)-\mu}{1-\nu} p_{1l} + \frac{(1-\nu)^2+\mu}{1-\nu} p_{1h} \right] + \beta\theta_h = 0$, respectively. Plugging these conditions into M_1's maximization program (14), we can find analytically the equilibrium value $\rho^*(\mu)$. To establish the sign of the impact of μ on ρ^* when μ is small, we use a second-order Taylor approximation for $\rho^*(\mu)$ around $\mu = 0$. This yields $\rho^*(\mu)|_{\mu=\widetilde{\mu}>0} \approx \rho^*(\mu)|_{\mu=0} + \widetilde{\mu} \left. \frac{\partial\rho^*(\mu)}{\partial\mu} \right|_{\mu=0} + \frac{\widetilde{\mu}^2}{2} \left. \frac{\partial^2\rho^*(\mu)}{\partial\mu^2} \right|_{\mu=0}$, where

$$\rho^*(\mu)|_{\mu=0} = 1 - \frac{2\gamma^2 (2\beta+\gamma)(1-\nu) \left[\alpha - (\beta-\gamma)(\theta_h - \nu\Delta\theta) \right]}{(32\beta^4 - 16\beta^2\gamma^2 + \gamma^4) \Delta\theta}$$

$$\left. \frac{\partial\rho^*(\mu)}{\partial\mu} \right|_{\mu=0} = \frac{\gamma^3 (2\beta+\gamma) \left[\alpha - (\beta-\gamma)(\theta_h - \nu\Delta\theta) \right]}{\nu^2 \beta\Delta\theta (16\beta^4 - 8\beta^2\gamma^2 + \nu\gamma^4)}$$

$$\left. \frac{\partial^2\rho^*(\mu)}{\partial\mu^2} \right|_{\mu=0} = \frac{\gamma^2 (2\beta+\gamma)^2}{2\nu^3 (1-\nu) \beta^2 \Delta\theta^2 (16\beta^4 - 8\beta^2\gamma^2 + \nu\gamma^4)^2} \left\{ 8\nu\Delta\theta \left[\alpha\gamma^4 (\beta-\gamma) - \beta^2 (2\beta-\gamma)^2 \right. \right.$$
$$\times \left(2\beta^2 - \gamma^2\right) \theta_l + \left(8\beta^6 - 8\beta^5\gamma - 2\beta^4\gamma^2 + 4\beta^3\gamma^3 - 2\beta^2\gamma^4 + 2\beta\gamma^5 - \gamma^6\right) \theta_h \Big]$$
$$\left. + \nu^2\gamma^4\Delta\theta^2 \left(8\beta^2 - 12\beta\gamma + 5\gamma^2\right) + 4\gamma^4 \left[\alpha - (\beta-\gamma)\theta_h \right]^2 \right\}.$$

We have $\left. \frac{\partial\rho^*(\mu)}{\partial\mu} \right|_{\mu=0} > 0$ and $\left. \frac{\partial^2\rho^*(\mu)}{\partial\mu^2} \right|_{\mu=0} > 0$, where the inequalities follow from the assumptions on the parameters of the model. ■

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