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The OWA distance operator and its application in business failure

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Abstract

Purpose – The purpose of this paper is to develop a goodness index based on Hamming distance and ordered weighted averaging (OWA) distance, which is useful to make decisions. These alternative measures enrich the results of diagnostic fuzzy models and facilitate the experts' task in decision-making. An application to a set of firms to verify the results is also presented.

Design/methodology/approach – The paper follows the basis of OWA operators to design a methodology to reduce the map of causes of business failure into monitoring key areas.

Findings – The present paper introduces two alternative measures to test the proposal of grouping. In the empirical application, the superiority of the minimum T-norm over other decision rules is verified. The ordered weighted averaging distance (OWAD) goodness index predicts a better adjustment over the index built using OWA and Hamming distance measures.

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Practical implications – A useful mechanism to reduce the map of causes or diseases detected in key areas is added through this analysis. At the same time, these key areas can be disaggregated once some alert indicator is identified; this allows knowing the causes that require special attention. This application of OWA can encourage the development of suitable computer systems for monitoring the firm's problems, alerting regarding failures and easing decision-making.

Originality/value – A comparison of grouping causes into key areas through a goodness index based on Hamming distance and OWA distance is proposed. These contributions enrich the Vigier and Terceño (2008) model and could be applied to any model of fuzzy diagnosis to test the results.

Keywords Business failure, Cause-symptoms relations, Distance measures, Fuzzy relations, OWA distance, OWA operators

Paper type Research paper

1. Introduction

Predicting business failure has been an important research area in the literature. The traditional failure models compare and classify firms according to quantitative indicators to predict between healthy and unhealthy businesses. Most of these models try to improve estimation results without finding out the causes of failure[1]. The traditional methodologies have limitations when dealing with a diagnostic problem because they include elements of subjectivity and uncertainty. These considerations open up the possibility of using fuzzy logic tools and methods and methods, enabling working with qualitative variables and weak information and measuring the expert's knowledge. Since Zadeh (1965), there have been

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developments to model uncertainty and describe human behaviour and complex systems distinguished by incomplete information and multiple subjective variables. Fuzzy prediction models arise as an alternative in this context (Vigier and Terceño, 2008; Ng *et al.*, 2008; Maracine and Delcea, 2009; Quek *et al.*, 2009; Thapar *et al.*, 2009; Behbood *et al.*, 2010; Behbood and Lu, 2011; Scarlat and Delcea, 2011; Korol and Korodi, 2011; Delcea *et al.*, 2012; Kharola and Singh, 2014; Xu *et al.*, 2014; etc.) to overcome many restrictions of the traditional models (Balcaen and Ooghe, 2006; Kumar and Ravi, 2007; Bahrammirzaee, 2010; Sun *et al.*, 2014; Appiah *et al.*, 2015). Vigier and Terceño (2008) and Scarlat and Delcea (2011), among others, advance over other prediction models, understanding the process of business failure and the formal relationship between causes and symptoms. These models have the capacity to predict insolvency situations, diagnose problems and simulate (or complement) an analyst's task. The formalization of the expert's knowledge reduces discretion and the subjective results.

This research takes Vigier and Terceño (2008) as a reference work, which is based on the estimation of a financial–economic knowledge matrix (R) that arises from the operation between the incidence matrix of symptoms and the incidence matrix of causes. Also, it proposes the aggregation of matrices and the verification of trends that allows the result of an aggregate matrix with temporary validity and prediction capacity ($\mathfrak{R}$). This matrix is useful to detect the causes (or diseases) in firms. This model is chosen because it has a comprehensive analysis of causes and symptoms over the pioneers' models of Gil Aluja (1990) and Gil Lafuente (1996). The recent approaches of Maracine and Delcea (2009) and Scarlat and Delcea (2011), which took as reference the model developed by Vigier and Terceño (2008), derive the matrix of causes and symptoms based on the difference with the performance matrix. This difference introduces a greater degree of subjectivity in favour of the experts' points of view. In any case, none of these models completely define the symptoms and causes required to estimate the knowledge matrix. Also, they have an evaluation system of neither diagnosis nor prediction capacity. Therefore, based on Vigier and Terceño's (2008) model, a comparison of methods of grouping causes through a goodness index based on Hamming distance and ordered weighted averaging (OWA) distance is proposed. These contributions enrich the reference model, although they could be applied to any model of fuzzy business diagnosis.

2. Advantages and limitations of Vigier and Terceño's model

This section presents the advantages of Vigier and Terceño's (2008) model and some limitations. These limitations show possible contributions that can be made.

2.1 Advantages

2.1.1 The difference between causes and symptoms is recognized and formalized. The degree of incidence is determined. That is, to identify the problems, each firm should pay attention to the r_{ij} components in the R matrix that shows the level of incidence of the symptom S_i through the cause C_j.

2.1.2 Sample selection. Fuzzy logic tools reduce limitations of the properties and statistical representation of population. It is an essential condition that any of the companies used in the analysis were affected by any of the defined diseases without imposing a minimum number of firms. Also, relationships are estimated by combining the set of firms over time. This minimizes the risk of determining incidence levels between non-significant causes and symptoms (r_{ij}) that are the product of a temporary situation. A sample of similar size and sector is recommended to homogenize the accepted level of normality in the model's variables.

2.1.3 The problem of the independent variables. The cause–effect relationship linking the causes with the firms' diseases and the effects with economic–financial ratios is presented. The causes of failure may be found by the model, and moreover, if the ratios of a failure firm are available over time, it is possible to detect the reasons that led to this event. Fuzzy logic overcomes the statistical requirements of independent variables such as normality and multicollinearity problems. In this methodological conception, it is recommended to maximize the available information to estimate possible diseases and also explanatory and qualitative variables as independent variables in the modelling.

2.1.4 Misclassification errors. This model provides more information on the firm's situation regarding the binary models that only classify business in healthy or unhealthy firms. The incidence degree of each r_{ij} shows the disease grade of the firms, so measures can be taken to reverse or neutralize problems.

2.1.5 Static dimension of business failure. There is no temporal limitation when using fuzzy relations. If incidence-level estimations have temporal consistency, they may be considered as a continuous system, adding a dynamic perspective to the diagnostic process.

2.2 Limitations

2.2.1 Selection of symptoms. Regarding the symptoms, the model mentions that financial economic ratios that arise from the financial statements of companies need to be used without specifying which ones. In a partial study (Terceño *et al.*, 2012), the list of symptoms selected for the analysis is presented.

2.2.2 Detection of causes. The model only mentions the existence of internal and external causes of the firm, and identifies the difference and relationship between symptoms (or effects) and diseases (or causes). The causes are not explicitly defined in either a list or a handbook that should be considered by the analyst in the diagnosis. In the model mentioned above, there are endogenous and exogenous, as well as objective and subjective, causes that can be drawn from the theoretical models. In this regard, in Terceño *et al.* (2009, 2014) the integration of the fuzzy logic with the Balanced Scorecard is proposed to detect the causes of failure; in Scherger *et al.* (2015b), the method for measuring the causes is described; and in Scherger *et al.* (2014), a complete analysis of diseases is presented.

2.2.3 Lack of application. The model developed by Vigier and Terceño (2008) is a theoretical model that does not define specifically the set of causes and symptoms involved in the diagnosis. Furthermore, its simulation requires performance information from a large number of firms and it has not been empirically tested, although the internal consistency of the model has been demonstrated. Once causes and symptoms are defined, the empirical estimation is carried out. This application to a set of small and medium enterprises (SMEs) is presented by Scherger *et al.* (2015a).

2.2.4 Verification of prediction capacity. The model does not have any mechanism to test its capacity for diagnosis and prediction. There is no guide that evaluates the degree of adjustment of the predictions to the responses provided by experts.

In this work, a global proposal of treatment of causes through OWA operators that synthesize diseases of firms is presented. A comparison between two alternative measures to test and verify the model's capacity of diagnosis and prediction is also proposed.

3. OWA causes aggregation

When dealing with decision-making problems, it is necessary to aggregate the available information to make decisions. Therefore, given P', estimated by the inverse operation between Q and $\Re$, that shows the multiple causes (or diseases) that a firm can undergo, a reduction mechanism of the map of causes through OWA operators is introduced. In the empirical estimation presented by Scherger *et al.* (2015a), the R matrix with 2,952 r_{ij}

coefficients per year is estimated. That is a sum of 8,856 coefficients in the aggregated matrix $\Re$ that shows the true relationship between causes and symptoms. Therefore, a mechanism of grouping causes through OWA operators in key areas is introduced. These operators give a parameterized family of aggregation operators between the minimum and the maximum.

Yager (1993) introduces OWA operators. Then, the OWA operators have been extended and generalized by many authors to develop a more general framework. The heavy OWA (Yager, 2002), the generalized OWA – GOWA (Yager, 2004), the prioritized OWA (Yager, 2009), the linguistic-induced generalized aggregation distance operators – LOWA (Xu, 2006), the induced GOWA (Merigó and Gil-Lafuente, 2009), the uncertain probabilistic OWA – UPOWA (Merigó and Wei, 2011), the fuzzy probabilistic OWA – FPOWA (Merigó, 2011), the probabilistic OWA – POWA (Merigó, 2012), the induced generalized intuitionist fuzzy ordered weighted averaging – IGIFOWA (Xu and Wang, 2012), the ILPOWA using probabilistic information and induced aggregation operators (Merigó *et al.*, 2014a), among others. The OWAs are applied to reduce data dimensions without losing information to many problems such as investment decisions, diagnosis of health diseases, allocation of resources, selection of machines, human resource management and many other solving situations of decisions-making, statistics, health care, engineering, computing and business (Merigó and Gil-Lafuente, 2010; 2011; Zeng *et al.*, 2013; He *et al.*, 2015, etc.)

3.1 OWA operators

An OWA operator of dimension n is a mapping OWA: $R_n \rightarrow R$ that has an associated weighting vector W of dimension n such that the sum of the weights is 1 and $w_j \in [0, 1]$. Then, according to equation (1):

$$OWA(a_1, a_{2,\ldots,} a_n) = \sum\nolimits_{j=1}^{n} w_j b_j; \quad \text{where } b_j \text{ is the } j^{\text{th}} \text{ largest of the } a_i \tag{1}$$

The OWA operators satisfy the commutative, monotonic and idempotent property and the inequality: $Min\ (a_1, a_2, \ldots, a_n) \leq h\ (a_1, a_2, \ldots, a_n) \leq Max\ (a_1, a_2, \ldots, a_n)$; $\forall$ n-uplas $\in [0,1]^n$ (Yager, 1993; Beliakov, 2003; Xu and Da, 2003; Dubois and Prade, 2004; etc.).

3.2 The Hamming distance

The Hamming distance (Hamming, 1950) is a useful technique for calculating the differences between two sets (2). For two sets $A = \{a_1, a_2, \ldots, a_n\}$ and $B = \{b_1, b_2, \ldots, b_n\}$, then:

$$D_{HD} = \sum\nolimits_{j=1}^{n} \frac{1}{n} |a_i - b_i| \tag{2}$$

3.3 The OWA distance

The OWAD (or Hamming OWAD) operator is a measure proposed by Merigó and Gil-Lafuente (2010) that integrates the OWA operator with the Hamming distance. There are many extensions and applications to the decision-making problem in the literature (Merigó and Casanovas, 2010, 2011; Xu, 2011; Merigó, 2013; Zeng, 2013; Zeng *et al.*, 2013; Merigó *et al.*, 2014b; Su *et al.*, 2014; Xian and Sun, 2014; Zeng *et al.*, 2014; Blanco-Mesa *et al.*, 2015; Su *et al.*, 2015; etc.):

$$\text{This operator is defined as } OWAD = (\langle a_1, b_1\rangle, \langle a_2 b_2\rangle, \ldots \langle a_n, b_n\rangle) = \sum\nolimits_{j=1}^{n} \frac{1}{n} d_j \tag{3}$$

where d_j is the jth largest of the $|a_i - b_i|$

4. Aggregation and monitoring proposal

The dynamic perspective of failure introduced by Vigier and Terceño (2008) allows monitoring the evolution of firms and attacks the critical issues. Therefore, in monitoring terms, it is useful to concentrate the information in an aggregate number of key areas that represent the disaggregate firm's diseases (Terceño *et al.*, 2014). Although the model does not mention which factors are used, it provides analytical tools to formalize the relationship between causes and symptoms. The OWA operators are introduced in the original model to concentrate information and easily detect possible diseases in firms. Once a warning indicator in some area of the firm is detected, it is possible to disaggregate this key area into each of the causes or critical factors that generate problems, to evaluate and correct the situation.

4.1 The OWA and goodness index proposal

The OWA operators are used as an alternative to either grouping or aggregating the multiple causes of diseases and to synthetize the causes. This grouping methodology of causes from values of maximum, minimum and average r_{ij} is useful to detect the factors of greatest incidence and complement the task of the expert in monitoring. Therefore, OWA operators warrant the properties for the three fuzzy subsets.

Given $\Re = \{\Re_{ij}\}$, the $\Re^{Mean}$ matrix is obtained by applying the arithmetic mean to the causes within the monitoring area ($\Re^{Mean} = 1/\mathrm{m}\,\Sigma_{\mathrm{w}=1}^{\mathrm{m}} r_{ij}$). While, the matrices $\Re$ with maximum and minimum membership values ($\Re^{Max}$ and $\Re^{Min}$) are calculated through the maximum [$\Re^{Max} = Max\ (r_{ij})$] and the minimum [$\Re^{Min} = Min\ (r_{ij})$] selection of r_{ij} incidence levels within each group of causes. In Vigier *et al.* (2016) these three alternative methods of aggregation using OWA operators are introduced: minimum, maximum and average.

That is:

$$r_{iw}^{Mean} = (1/m)\,(r_{i1} + r_{i2} + \ldots + r_{iw})$$

$$r_{iw}^{Max} = Max\,(r_{i1}; r_{i2}; \ldots; r_{iw})$$

$$r_{iw}^{Min} = Min\,(r_{i1}; r_{i2}; \ldots; r_{iw})$$

This grouping allows the prediction of causes (Terceño and Vigier, 2011) $P' = Q\,\alpha\,\Re$; that is, $p'_{hj} = max\ [min\ (q_{hi}, r_{ij})]$ for the three possible levels of incidence of causes within each key area. As a result, the membership matrices of causes (or diseases) are estimated in the three levels of incidence [minimum (P'^{Min}), maximum (P'^{Max}) and average (P'^{Mean})]:

$$p'^{Min}_{iw} = \Lambda[(q_{ih}\,\alpha\,r^{min}{}_{h1}), (q_{ih}\,\alpha\,r^{min}{}_{h2}), \ldots, (q_{ih}\,\alpha\,r^{min}{}_{hw}),$$

$$p'^{Max}_{iw} = \Lambda[(\mathrm{q}_{ih}\,\alpha\,r^{\max}{}_{h1}), (\mathrm{q}_{ih}\,\alpha\,r^{\max}{}_{h2}), \ldots, (\mathrm{q}_{ih}\,\alpha\,r^{\max}{}_{hw}),$$

$$p'^{Mean}_{iw} = \Lambda[(q_{ih}\,\alpha\,r^{mean}{}_{h1}), (q_{ih}\,\alpha\,r^{mean}{}_{h2}), \ldots, (q_{ih}\,\alpha\,r^{mean}{}_{hw}),$$

Thus, the multiple causes detected in diagnosis or in prediction of firms' situations are grouped into key areas that facilitate the analyst's task by reducing the necessary information for the analysis. Once a warning indicator in some area of the firm is detected, it is possible to disaggregate this key area into each of the causes or factors that generate problems to evaluate and correct the situation. This option of monitoring through key areas (consistent with the disaggregated estimation of the model) allows a continuous and comprehensive tracking of the business areas through fuzzy logic advantages.

In this case, to verify the introduced methodology, a goodness index is suggested. As described in Section 1, Vigier and Terceño's (2008) model, like other fuzzy logic models, does not have a mechanism to verify its capacity of prediction.

To do so, the Hamming distance is adapted to check if the causes estimated by the model represent the true situation of the firm. Also, this index is useful to evaluate which of the three OWA aggregate methods is more efficient. The index represents the comparison between the original set of causes (P^*) aggregated into minimum [$P^{*Min} = Min\ (p_{hw})$], maximum [$P^{*Max} = Max\ (p_{hw})$] and average [$P^{*Mean} = 1/m \sum_{w=1}^{m} (p_{hw})$] incidence levels and the set of estimated causes (P'). For this, P' is aggregated in the same way as the original causes (P^*).

Therefore, the goodness index between P^* and P', that is, $\|P^* = P'\| = \|P^* \subseteq P'\| \wedge \|P' \subseteq P^*\|$ is represented by equation (4):

$$[P^* = P'] = 1 - 1/m \sum_{x \in X}^{n} |P^*_{hw} - P'_{hw}| \tag{4}$$

The main advantage of using distance measures in decision-making is that we can compare the alternatives of the problem with some ideal result, and therefore, select the rule with the closest result to the optimal choice.

Therefore:

$$[P^* = P']^{Min} = \left[1 - 1/m\left(|P^*_{h1} - P'_{h1}| + |P^*_{h2} - P'_{h2}| + \ldots + |P^*_{hk} - P'_{hw}|\right)\right]$$

$$[P^* = P']^{Max} = \left[1 - 1/m\left(|P^*_{h1} - P'_{h1}| + |P^*_{h2} - P'_{h2}| + \ldots + |P^*_{hw} - P'_{hw}|\right)\right]$$

$$[P^* = P']^{Mean} = \left[1 - 1/m\left(|P^*_{h1} - P'_{h1}| + |P^*_{h2} - P'_{h2}| + \ldots + |P^*_{hw} - P'_{hw}|\right)\right]$$

Where,

P^{*Min} = selects the minimum degree of incidence within the group of causes for each company;

P^{*Max} = chooses the maximum degree of incidence within the group of causes for each company; and

P^{*Mean} = shows the average of the causes within the group or key area of monitoring.

This test is useful to identify the best mechanism for aggregating causes using OWA operators (by maximum, minimum or average incidence values). It can also be used to estimate the degree of adjustment of the predictions to diseases that are present in the companies. In other words, it is helpful to prove this model's capacity (or any other) to predict insolvency situations and evaluate the three alternatives of synthesizing causes.

4.2 The OWA distance index

Another alternative to measure the fit of the proposal is using the operator proposed by Merigó and Gil-Lafuente (2010) that unifies the OWA operator with the Hamming distance. In this case, the OWA is adapted to show the adjustment of the disaggregate estimations to the answers given by the experts; and in the same operation, the three ways of aggregation are also tested. That means that equation (5) is valid to select the best rule of decision-making: maximum, minimum or average:

$$OWAD\,index = (\langle p^*_1, p'1\rangle, \langle p^*_2 p'2\rangle, \ldots \langle p^*_j, p'j\rangle) = \sum_{j=1}^{h} 1 - \frac{1}{h} \sum |p^*_j - p'_j| \tag{5}$$

This supposes the following steps:

- Given $\Re = \{\Re_{ij}\}$, the disaggregate diseases or causes are estimated through $P' = Q \alpha \Re$; that is, $P_{hj}' = Q_{hi} \alpha \Re_{ij}$.
- Then $[I(P^*_{hj}) - (p'_{hj}) I]$ is calculated for each of the causes, that is $\{|P^*_{h1} - p'_{h1}|; |P^*_{h2} - p'_{h2}|; [\ldots]; |P^*_{hj} - p'_{hj}|\}$.
- Within each key area, the minimum, the maximum and the average distance are selected, reducing the difference between P* and P′ in the three matrices of key areas for all the firms.
- Finally, the index based on OWAD is calculated. That is $[1 - 1/h\, (|P^*_{h1} - p'_{h1}| + |P^*_{h2} - p'_{h2}| + [\ldots] + |P^*_{hw} - p'_{hw}|)]$. This is done for the maximum, the minimum and the average matrix at the level of key area.

5. Empirical verification

Given $\Re$ ($\Re_{ij} = 41 \times 72$) with consistent and significant estimated r_{ij} coefficients (Scherger *et al.*, 2015a), the two alternative measures of cause grouping are applied.

5.1 The OWA and Hamming goodness index

This is carried out by applying the following steps:

(1) Proposed minimum, maximum and mean incidence aggregation operators are applied to P for obtaining the aggregated matrixes that reflect these three categories (P^{*Min}; P^{*Max}; and P^{*Mean}). Appendix 1 shows the three matrixes (P^*).

For example, taking as a reference the causes' membership matrix of Firm 1, the causes are grouped ($p_1, p_{2\,[\ldots]}, p_{10}\,[\ldots], p_{15}$) in a single cause ($P^*_1$) that reflects the problems related to entrepreneurial learning.

That is:

$$P^{*Mean}_{11} = (1/15)(0.25 + 1.00 + 0.50 + 0.20 + 0.20 + 0.50 + 0.86 + 0.33 + 0.29 + 0.43 + 0.29 + 0.36 + 1.00 + 0.20 + 0.43) = 0.46$$

$$P^{*Max}_{11} = Max\,(0.25; 1.00; 0.50; 0.20; 0.20; 0.50; 0.86; 0.33; 0.29; 0.43; 0.29; 0.36; 1.00; 0.20; 0.43) = 1.00$$

$$P^{*Min}_{11} = Min\,(0.25; 1.00; 0.50; 0.20; 0.20; 0.50; 0.86; 0.33; 0.29; 0.43; 0.29; 0.36; 1.00; 0.20; 0.43) = 1.00$$

(2) OWA operators are applied to $\Re$ to estimate the companies' diseases which are grouped into key areas [$\Re^{Min} = Min\,(r_{ij})]; \Re^{Max} = Max\,(r_{ij}); \Re^{Mean} = [1/m$

$$p'^{Max}_{11} = Max\,[(0.50), (0.50), \ldots, (0.60), \ldots, (0.57)] = 0.60$$

$\Sigma\,(r_{ij})]$. Through $P' = Q' \alpha \Re$; being; $p'_{hj} = max\,[min\,(q_{hi}, r_{ij})]$, three new matrixes of causes in minimum (P'^{Min}), maximum (P'^{Max}) and mean (P'^{Mean}) membership values are estimated (Appendix 2).

For example, the membership level of the cause $p'_{11}{}^{Max}$ for the aggregate is calculated through the operation of the first row of Q matrix with the first column of $\Re^{Max}$ matrix, that is $p'_{11'} = q(1 \times 41) \alpha \Re^{Max} (41x1) = p'_{11}\,(1 \times 1)$.

$$p'^{Max}_{11} = \Lambda\,[(0.60\,\alpha\,0.50), (0.67\,\alpha\,0.50), \ldots, (0.80\,\alpha\,0.60), \ldots, (0.80\,\alpha\,0.57)]$$

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Thus, P'^{Mean}_{11} and P'^{Min}_{11} are estimated by operating the first row of Q matrix with the first column of $\Re^{Max}$.

$$P'^{Mean}_{11} = \Lambda\,[(0.60\,\alpha\,0.29), (0.67\,\alpha\,0.28), \ldots, (0.93\,\alpha\,0.28), \ldots, (0.80\,\alpha\,0.29)]$$

$$p'^{Mean}_{11} = Max\,[(0.29), (0.28), \ldots, (0.28), \ldots, (0.29)] = 0.34$$

And,

$$p'^{Min}_{11} = \Lambda\,[(0.60\,\alpha\,0.14), (0.67\,\alpha\,0.14), \ldots, (0.13\,\alpha\,0.14), \ldots, (0.80\,\alpha\,0.14)]$$

$$p'^{Mean}_{11} = Max\,[(0.14), (0.14), \ldots, (0.14), \ldots, (0.14)] = 0.14$$

(3) The goodness index of the cause's grouping based on Hamming distance is estimated through the comparison of the two fuzzy subsets P^* and P'. Thus, the reduction of causes in monitoring key areas is verified and its degree of adjustment to the answers given by the experts is evaluated.
For example, for Firm 1:

$$\begin{aligned}[P^* = P']^{Min} = 1 - 1/14(&|0.20 - 0.14| + |0.12 - 0.13| + |0.57 - 0.56| \\ &+ |0.20 - 0.20| + |0.13 - 0.19| + |0.20 - 0.40| + |0.20 - 0.60| \\ &+ |0.13 - 0.13| + |0.20 - 0.26| + |0.43 - 0.43| + |0.20 - 0.20| \\ &+ |0.20 - 0.20| + |0.20 - 0.14| + |0.27 - 0.24|) = 0.94\end{aligned}$$

$$\begin{aligned}[P^* = P']^{Max} = 1 - 1/14(&|1.00 - 0.60| + |0.57 - 0.57| + |0.83 - 0.61| \\ &+ |0.67 - 0.67| + |1.00 - 0.93| + |1.00 - 0.62| + |1.00 - 0.80| \\ &+ |1.00 - 1.00| + |0.37 - 0.31| + |0.83 - 0.73| + |0.50 - 0.51| \\ &+ |1.00 - 0.93| + |1.00 - 0.67| + |0.86 - 0.80|) = 0.86\end{aligned}$$

$$\begin{aligned}[P^* = P']^{Mean} = 1 - 1/14(&|0.46 - 0.34| + |0.27 - 0.35| + |0.71 - 0.57| \\ &+ |0.46 - 0.47| + |0.43 - 0.47| + |0.70 - 0.51| + |0.60 - 0.70| \\ &+ |0.47 - 0.47| + |0.29 - 0.26| + |0.61 - 0.58| + |0.29 - 0.37| \\ &+ |0.72 - 0.61| + |0.64 - 0.35| + |0.55 - 0.49|) = 0.91\end{aligned}$$

5.2 The OWAD goodness index

The following steps are carried out:

(1) The difference in absolute value between the causes given by the experts (P^*) and the estimated causes (P') is calculated.
For example, for Firm 1:

$$\begin{aligned}\{|p^*_{11} - p'_{11}|; |p^*_{12} - p'_{12}|; \ldots; |p^*_{172} - p'_{172}|\} &= \{|0.25 - 0.25|; |1.0 - 0.50|; \\ &|0.5 - 0.36|; \ldots; |0.69 - 0.53|\} = \{0.0; 0.50; 0.14; \ldots; 0.16\}\end{aligned}$$

(2) The minimum, maximum and average distances are selected in each key area. This is equivalent to applying OWA and therefore the 72 causes are reduced in 14 key areas.

For example, taking as a reference the causes' membership matrix of Firm 1, the minimum, maximum and average distances are selected among the 15 causes of the key areas of entrepreneurial learning:

$$|P^* - P'|^{Min} = \{|p^*_{11} - p'_{11}|; |p^*_{12} - p'_{12}|; \ldots; |p^*_{115} - p'_{115}|\}$$
$$= Min\{0.0;0.50;0.14;\ldots;0.14\} = 0.0$$

$$|P^* - P'|^{Max} = \{|p^*_{11} - p'_{11}|; |p^*_{12} - p'_{12}|; \ldots; |p^*_{115} - p'_{115}|\}$$
$$= Max\{0.0;0.50;0.14;\ldots;0.14\} = 0.71$$

$$|P^* - P'|^{Mean} = \{|p^*_{11} - p'_{11}|; |p^*_{12} - p'_{11}|; \ldots; |p^*_{115} - p'_{115}|$$
$$= \{1/15(0.0 + 0.50 + 0.14 + \ldots + 0.14)\} = 0.18$$

(3) Finally, the index is calculated based on OWAD and the best rule of grouping is verified.

That is, for Firm 1:

$$OWAD\ goodness\ index = \left[1 - 1/14\left(|P^*_{11} - p'_{11}| + |P^*_{12} - p'_{12}| + \ldots + |P^*_{114} - p'_{114}|\right)\right]$$

$$[P^* = P']^{Min} = 1 - 1/14(0.0 + 0.0 + 0.0 + 0.0 + 0.0 + 0.20 + 0.0 + 0.0 + 0.06 + 0.0 + 0.0 + 0.0 + 0.0 + 0.0) = 0.98$$

$$[P^* = P']^{Max} = 1 - 1/14(0.71 + 0.24 + 0.22 + 0.0 + 0.73 + 0.40 + 0.47 + 0.40 + 0.07 + 0.10 + 0.29 + 0.20 + 0.71 + 0.29) = 0.66$$

$$[P^* = P']^{Mean} = 1 - 1/14(0.18 + 0.08 + 0.11 + 0.0 + 0.24 + 0.28 + 0.25 + 0.10 + 0.07 + 0.03 + 0.07 + 0.05 + 0.26 + 0.12) = 0.87$$

5.3 Comparative results

This analysis concludes that the best adjustment is obtained by the OWAD goodness index, with a degree of adjustment of 98 per cent following the minimum T- norm. With the OWA and the Hamming goodness index, the properties of the minimum T-norm are also validated over other decision rules (93 per cent). These results are consistent with
T1 the less adverse risk theories in terms of warnings and the analysis of results. Table I shows the comparative results obtained through the two alternative distance methods in the empirical estimation.

The performance of firms is also checked five years after diseases' estimation to verify the fit of the model's predictions. In most cases, the problems detected are confirmed in the empirical estimations; only one of the firms identified as moderately sick has few labour problems, and all of the healthier companies confirm their situation, showing no critical problems. These results confirm the utility of this methodology to diagnose diseases and identify causes of failure.

6. Conclusions

In this paper, we proposed the enrichment of the fuzzy models of business failure, particularly the Vigier and Terceño (2008) model, by introducing a mechanism of grouping causes through aggregation operators to facilitate the expert's task. Also, two alternatives of

Firm	Minimum (P'^{Min})	Maximum (P'^{Max})	Mean (P'^{Mean})
OWA and Hamming goodness index			
1	0.94	0.86	0.91
2	0.98	0.82	0.93
3	0.97	0.86	0.94
4	0.93	0.89	0.93
5	0.95	0.89	0.92
6	0.92	0.84	0.88
7	0.91	0.80	0.88
8	0.92	0.82	0.87
9	0.96	0.85	0.88
10	0.91	0.82	0.87
11	0.88	0.80	0.90
12	0.91	0.85	0.87
13	0.90	0.86	0.91
14	0.93	0.89	0.89
15	0.95	0.90	0.92
Mean	0.93	0.85	0.90
OWAD goodness index			
1	0.98	0.87	0.66
2	1.00	0.87	0.63
3	0.99	0.87	0.65
4	0.99	0.91	0.74
5	0.98	0.85	0.67
6	0.98	0.84	0.64
7	0.98	0.83	0.62
8	0.98	0.84	0.62
9	0.98	0.82	0.57
10	0.98	0.82	0.62
11	0.98	0.83	0.65
12	0.97	0.85	0.63
13	0.98	0.87	0.67
14	0.98	0.88	0.69
15	0.99	0.91	0.76
Mean	0.98	0.86	0.65

Table I.
Comparative results

goodness index, through Hamming distance and OWA distance, are introduced to test the model's functionality.

This approach allows synthesizing firms' diseases and supposes an advantage in predicting disease, as it involves the reduction of the general map of firms' causes into key areas. These causes are detected through the Balanced Scorecard and are grouped according to the OWA operators (minimum, maximum and average). Thus, once a warning indicator in some area of the firm is detected, it is possible to disaggregate this key area in the multiple factors that generate problems to evaluate and correct the situation. This methodology of grouping causes from values of maximum, minimum and average is useful to detect the greatest incidence of factors for comprehensive and continuous monitoring of the company.

Furthermore, the goodness measures are useful to evaluate the best way of grouping and to test the model's capacity to predict diseases. According to the empirical estimation – using $\mathfrak{R}$ for prediction – the superiority of the minimum T-norm is validated over other decision rules through both alternative measures. The OWAD goodness index predicts a better adjustment over the index built using OWA and Hamming distance.

An advance in the analysis of causes of business failure is proposed by synthesizing the importance of the causes in key areas through OWA operators. In addition, two alternative measures are introduced to select the best rule of aggregation; they are then applied and tested.

Note

1. Research of Argenti (1976, 1983), Gabás Trigo (1990) and Ooghe and De Prijcker (2008) partially analyse the causes of failure and their interrelationships. In other models like Keasey and Watson (1987), Flagg *et al.* (1991) and Platt *et al.* (1994) qualitative and external variables are introduced to improve the results.

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Appendix 1

Ta1-Ta3

	Learning and Growth				Business process			Customers perspective				Finance		
Areas	1	2	3	4	5	6	7	8	9	10	11	12	13	14
E1	1.00	0.57	0.83	0.67	1.00	1.00	1.00	1.00	0.37	0.83	0.50	1.00	1.00	0.86
E2	0.83	0.57	0.75	0.33	1.00	0.71	0.60	1.00	0.31	0.57	0.33	1.00	0.36	0.67
E3	0.73	0.53	0.79	0.83	0.73	0.71	0.80	1.00	0.40	0.71	0.75	1.00	0.83	0.71
E4	0.83	0.29	0.67	0.83	0.87	0.71	0.80	0.80	0.42	0.57	0.70	1.00	0.83	1.00
E5	0.71	0.57	0.83	0.50	0.73	0.64	0.80	0.79	0.37	0.73	0.63	1.00	0.67	0.72
E6	0.80	0.57	0.60	0.20	0.67	1.00	0.80	0.71	0.48	0.90	0.81	0.80	0.86	0.71
E7	0.80	0.57	0.60	0.20	0.63	1.00	0.80	0.71	0.48	0.90	0.81	0.80	0.86	0.80
E8	0.80	0.57	0.60	0.20	0.63	1.00	1.00	0.71	0.48	0.90	0.81	0.80	0.86	0.80
E9	1.00	0.63	0.80	0.67	1.00	0.71	1.00	1.00	0.40	1.00	0.75	0.80	1.00	1.00
E10	1.00	0.61	0.72	0.20	0.80	0.71	0.80	0.63	1.00	0.73	0.75	1.00	0.67	1.00
E11	1.00	0.57	1.50	0.83	0.93	0.79	0.80	0.80	0.47	0.86	0.58	0.81	0.83	1.00
E12	1.00	0.63	1.00	0.83	0.93	1.00	0.80	0.80	0.60	0.59	0.75	0.88	0.65	1.00
E13	1.00	0.63	0.80	1.00	0.80	0.71	0.80	1.00	0.43	0.83	0.75	1.00	0.83	0.69
E14	1.00	0.63	0.75	1.00	0.87	0.60	0.80	0.80	0.40	0.83	0.75	0.88	0.83	1.00
E15	0.80	0.57	0.80	0.67	0.80	0.80	0.80	0.80	0.48	0.83	0.61	1.00	0.65	1.00

Table AI.
Grouped membership matrix of causes (P^{*Max})

Areas	Learning and Growth 1	2	3	4	Business process 5	6	7	Customers perspective 8	9	10	11	Finance 12	13	14
E1	0.20	0.12	0.57	0.20	0.13	0.20	0.20	0.13	0.20	0.43	0.20	0.20	0.20	0.27
E2	0.14	0.13	0.57	0.17	0.07	0.40	0.60	0.13	0.20	0.39	0.20	0.20	0.14	0.27
E3	0.14	0.13	0.60	0.17	0.07	0.53	0.60	0.13	0.26	0.29	0.20	0.22	0.14	0.17
E4	0.14	0.13	0.14	0.20	0.13	0.40	0.40	0.25	0.40	0.43	0.20	0.20	0.14	0.27
E5	0.14	0.13	0.71	0.33	0.25	0.53	0.60	0.13	0.20	0.57	0.20	0.20	0.14	0.20
E6	0.14	0.13	0.56	0.17	0.25	0.47	0.80	0.20	0.40	0.43	0.13	0.11	0.55	0.27
E7	0.14	0.13	0.56	0.17	0.25	0.47	0.80	0.20	0.40	0.43	0.13	0.11	0.55	0.27
E8	0.14	0.13	0.56	0.17	0.25	0.47	0.80	0.20	0.40	0.43	0.13	0.11	0.55	0.27
E9	0.14	0.14	0.43	0.17	0.13	0.40	0.80	0.13	0.37	0.43	0.20	0.20	0.14	0.27
E10	0.14	0.38	0.57	0.17	0.25	0.20	0.60	0.25	0.42	0.57	0.38	0.33	0.14	0.27
E11	0.14	0.24	0.14	0.20	0.14	0.20	0.20	0.38	0.40	0.29	0.20	0.33	0.14	0.24
E12	0.20	0.57	0.71	0.20	0.14	0.40	0.60	0.13	0.52	0.57	0.20	0.20	0.14	0.27
E13	0.17	0.53	0.29	0.20	0.14	0.40	0.60	0.29	0.40	0.57	0.20	0.40	0.14	0.27
E14	0.20	0.53	0.57	0.40	0.25	0.40	0.60	0.13	0.32	0.43	0.20	0.20	0.14	0.27
E15	0.20	0.25	0.57	0.20	0.25	0.40	0.60	0.13	0.40	0.57	0.20	0.33	0.14	0.27

Table AII. Grouped membership matrix of causes (P^{*Min})

Areas	Learning and Growth 1	2	3	4	Business process 5	6	7	Customers perspective 8	9	10	11	Finance 12	13	14
E1	0.46	0.27	0.71	0.46	0.43	0.70	0.60	0.47	0.29	0.61	0.29	0.72	0.64	0.55
E2	0.29	0.40	0.67	0.23	0.46	0.50	0.60	0.50	0.25	0.46	0.26	0.49	0.26	0.47
E3	0.35	0.36	0.65	0.40	0.46	0.61	0.75	0.47	0.33	0.48	0.40	0.78	0.41	0.41
E4	0.45	0.19	0.52	0.51	0.46	0.54	0.65	0.54	0.41	0.48	0.38	0.59	0.48	0.58
E5	0.46	0.36	0.80	0.41	0.50	0.59	0.65	0.53	0.29	0.62	0.39	0.43	0.51	0.59
E6	0.39	0.39	0.57	0.18	0.45	0.66	0.80	0.45	0.44	0.63	0.35	0.41	0.71	0.52
E7	0.39	0.39	0.57	0.18	0.47	0.66	0.80	0.45	0.44	0.63	0.35	0.41	0.71	0.55
E8	0.39	0.39	0.57	0.18	0.47	0.66	0.85	0.45	0.44	0.63	0.35	0.41	0.71	0.55
E9	0.51	0.41	0.64	0.34	0.53	0.55	0.90	0.56	0.38	0.67	0.43	0.46	0.50	0.57
E10	0.42	0.52	0.65	0.18	0.51	0.53	0.75	0.51	0.71	0.62	0.61	0.55	0.41	0.59
E11	0.45	0.35	0.94	0.62	0.40	0.48	0.60	0.62	0.43	0.51	0.35	0.63	0.58	0.56
E12	0.59	0.59	0.85	0.62	0.43	0.70	0.70	0.57	0.56	0.58	0.43	0.69	0.42	0.56
E13	0.41	0.57	0.65	0.62	0.42	0.57	0.70	0.59	0.41	0.66	0.52	0.77	0.38	0.45
E14	0.48	0.57	0.67	0.80	0.52	0.53	0.75	0.47	0.36	0.61	0.43	0.61	0.49	0.57
E15	0.44	0.39	0.63	0.46	0.48	0.61	0.65	0.52	0.44	0.66	0.45	0.59	0.42	0.60

Table AIII. Grouped membership matrix of causes (P^{*Mean})

Appendix 2

Table AIV. Estimated membership matrix of diseases (P'^{Max})

Areas	Learning and Growth					Business process			Customers perspective			Finance		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
E1	0.57	0.57	0.61	0.67	0.87	0.80	0.80	0.93	0.31	0.83	0.51	1.00	0.83	0.67
E2	0.57	0.57	0.60	0.60	1.00	0.60	0.60	0.60	0.40	0.73	0.60	0.93	0.67	1.00
E3	0.67	0.43	0.61	0.80	1.00	0.60	0.80	0.80	0.27	0.59	0.63	0.80	0.55	1.00
E4	0.67	0.57	0.61	0.73	0.78	0.60	0.80	0.73	0.40	0.73	0.51	0.93	0.73	0.80
E5	0.83	0.57	0.60	0.67	0.63	0.60	0.80	0.80	0.40	0.73	0.63	1.00	0.67	0.92
E6	0.73	0.57	0.60	0.83	0.63	0.80	0.80	0.80	0.40	0.80	0.63	0.87	0.67	0.67
E7	0.57	0.57	0.60	0.67	0.63	0.80	0.80	1.00	0.40	0.83	0.63	1.00	0.83	0.73
E8	0.83	0.57	0.60	0.83	0.63	0.80	0.80	0.80	0.40	0.80	0.63	1.00	0.67	0.80
E9	0.67	0.57	0.61	1.00	1.00	0.60	0.80	0.87	0.33	0.83	0.63	1.00	0.83	1.00
E10	0.57	0.57	0.60	0.80	1.00	0.60	0.80	0.60	0.40	0.73	0.63	1.00	0.67	1.00
E11	0.67	0.57	0.61	0.67	0.89	0.67	0.67	0.67	0.40	0.67	0.51	0.80	0.67	0.89
E12	0.80	0.57	0.61	0.83	1.00	0.60	0.80	0.80	0.40	0.59	0.63	0.93	0.53	1.00
E13	0.67	0.57	0.61	0.93	1.00	0.60	0.80	0.80	0.40	0.83	0.63	0.93	0.83	1.00
E14	0.67	0.57	0.61	0.87	1.00	0.60	0.80	0.80	0.31	0.60	0.63	0.93	0.60	1.00
E15	0.83	0.57	0.60	0.67	0.80	0.80	0.80	0.80	0.31	0.83	0.51	1.00	0.50	1.00

Table AV. Estimated membership matrix of diseases (P'^{Min})

Areas	Learning and Growth				Business process			Customers perspective				Finance		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
E1	0.14	0.13	0.56	0.20	0.14	0.40	0.60	0.13	0.20	0.43	0.20	0.20	0.14	0.24
E2	0.14	0.13	0.56	0.20	0.14	0.40	0.60	0.13	0.26	0.43	0.20	0.20	0.14	0.24
E3	0.14	0.13	0.57	0.20	0.14	0.40	0.60	0.13	0.26	0.29	0.20	0.20	0.14	0.17
E4	0.14	0.13	0.57	0.20	0.14	0.40	0.60	0.13	0.26	0.43	0.20	0.20	0.14	0.24
E5	0.14	0.13	0.57	0.20	0.14	0.40	0.60	0.13	0.26	0.43	0.20	0.20	0.14	0.24
E6	0.14	0.13	0.56	0.20	0.14	0.40	0.60	0.13	0.26	0.43	0.20	0.20	0.14	0.24
E7	0.14	0.13	0.56	0.20	0.14	0.40	0.60	0.13	0.26	0.43	0.20	0.20	0.14	0.24
E8	0.14	0.13	0.56	0.20	0.14	0.40	0.60	0.13	0.26	0.43	0.20	0.20	0.14	0.24
E9	0.14	0.13	0.57	0.20	0.14	0.40	0.60	0.13	0.26	0.43	0.20	0.20	0.14	0.24
E10	0.14	0.13	0.56	0.20	0.14	0.40	0.60	0.13	0.26	0.43	0.20	0.20	0.14	0.24
E11	0.14	0.13	0.57	0.20	0.14	0.33	0.60	0.13	0.26	0.29	0.20	0.20	0.14	0.24
E12	0.14	0.13	0.56	0.20	0.14	0.40	0.60	0.13	0.26	0.29	0.20	0.20	0.14	0.24
E13	0.14	0.13	0.57	0.20	0.14	0.40	0.60	0.13	0.26	0.43	0.20	0.20	0.14	0.24
E14	0.14	0.13	0.57	0.20	0.14	0.40	0.60	0.13	0.20	0.33	0.20	0.20	0.14	0.24
E15	0.14	0.13	0.56	0.20	0.14	0.40	0.60	0.13	0.20	0.43	0.20	0.20	0.14	0.24

Areas	Learning and Growth 1	2	3	4	5	Business process 6	7	8	Customers perspective 9	10	11	Finance 12	13	14
E1	0.33	0.33	0.57	0.47	0.51	0.53	0.70	0.42	0.25	0.58	0.37	0.64	0.38	0.49
E2	0.32	0.33	0.57	0.51	0.47	0.51	0.60	0.42	0.33	0.58	0.35	0.65	0.38	0.49
E3	0.33	0.33	0.60	0.57	0.51	0.51	0.70	0.42	0.27	0.53	0.37	0.64	0.35	0.49
E4	0.32	0.33	0.60	0.57	0.47	0.53	0.65	0.42	0.33	0.58	0.33	0.61	0.38	0.49
E5	0.33	0.33	0.60	0.57	0.51	0.51	0.65	0.42	0.33	0.58	0.35	0.65	0.38	0.49
E6	0.33	0.33	0.57	0.51	0.41	0.53	0.70	0.42	0.33	0.58	0.35	0.65	0.38	0.49
E7	0.32	0.33	0.57	0.57	0.41	0.53	0.70	0.42	0.33	0.58	0.37	0.65	0.38	0.49
E8	0.33	0.33	0.57	0.57	0.40	0.53	0.70	0.42	0.33	0.58	0.37	0.65	0.35	0.49
E9	0.33	0.33	0.60	0.57	0.51	0.53	0.70	0.42	0.33	0.58	0.37	0.64	0.38	0.49
E10	0.32	0.33	0.57	0.53	0.47	0.51	0.70	0.42	0.33	0.58	0.37	0.61	0.38	0.49
E11	0.33	0.33	0.60	0.57	0.51	0.53	0.65	0.42	0.33	0.56	0.35	0.54	0.38	0.49
E12	0.33	0.33	0.57	0.57	0.51	0.53	0.70	0.42	0.33	0.53	0.37	0.64	0.36	0.49
E13	0.33	0.33	0.60	0.57	0.51	0.51	0.70	0.42	0.33	0.58	0.37	0.64	0.38	0.49
E14	0.33	0.33	0.60	0.57	0.51	0.51	0.70	0.42	0.25	0.56	0.37	0.64	0.36	0.49
E15	0.33	0.33	0.57	0.46	0.47	0.53	0.65	0.42	0.25	0.58	0.37	0.65	0.35	0.49

Table AVI. Estimated membership matrix of diseases (P'^{Mean})

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