UNIVERSITAT ROVIRA I VIRGILI

**RESOURCE ALLOCATION AS A CONFLICTING CLAIMS PROBLEM**

**Foroogh Salekpay**

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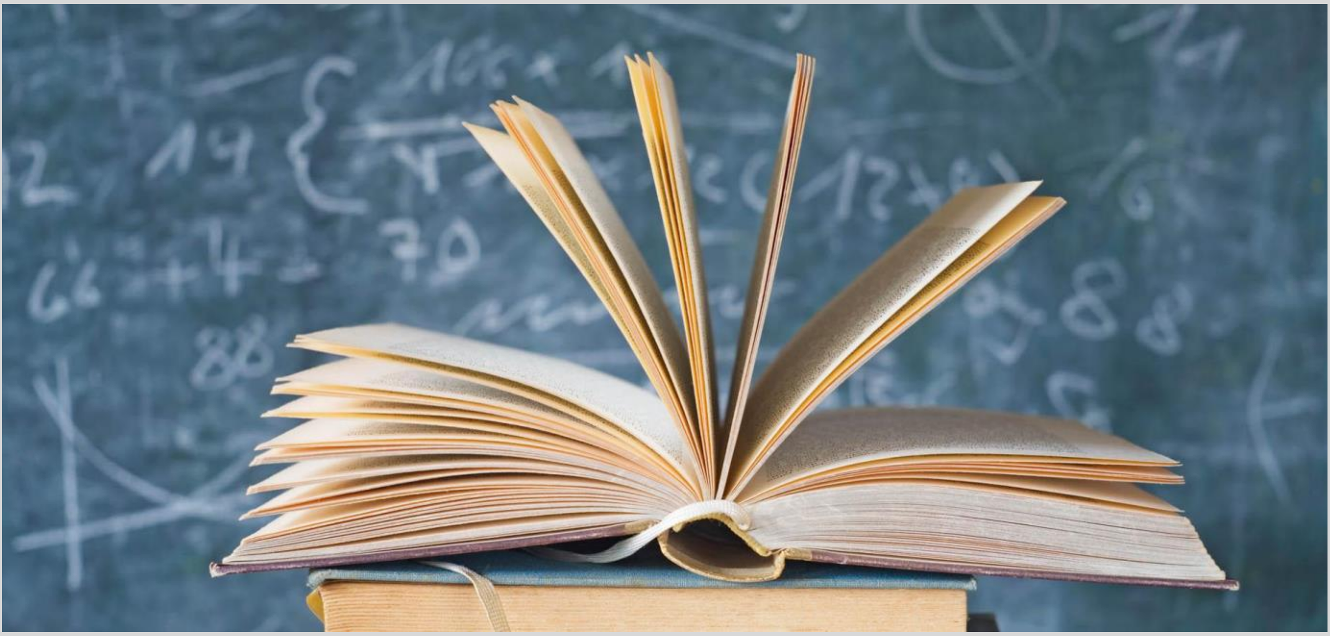

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UNIVERSITAT ROVIRA i VIRGILI

# RESOURCE ALLOCATION AS A CONFLICTING CLAIMS PROBLEM

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FAIG CONSTAR que aquest treball, titulat "**RESOURCE ALLOCATION AS A CONFLICTING CLAIMS PROBLEM**", que presenta **Foroogh Salekpay** per a l'obtenció del títol de Doctor, ha estat realitzat sota la meva direcció al Departament de **Economía** d'aquesta universitat.

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HAGO CONSTAR que el presente trabajo, titulado "**RESOURCE ALLOCATION AS A CONFLICTING CLAIMS PROBLEM**", que presenta **Foroogh Salekpay** para la obtención del título de Doctor, ha sido realizado bajo mi dirección en el Departamento de **Economía** de esta universidad.

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I STATE that the present study, entitled "**RESOURCE ALLOCATION AS A CONFLICTING CLAIMS PROBLEM**", presented by **Foroogh Salekpay** for the award of the degree of Doctor, has been carried out under my supervision at the Department of **Economics** of this university.

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Reus, 28/03/2023

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# DEDICATION

To my beloved family

## ACKNOWLEDGMENTS

With profound gratitude, I would like to express my appreciation to all those who have played a crucial role in my academic and personal growth during my doctoral studies.

First and foremost, I would like to express my sincere gratitude to my supervisor, Dr. José Manuel Giménez-Gómez, for his guidance, support, and encouragement throughout my Ph.D. journey. His expertise, insight, and feedback have been invaluable in helping me refine my research. His dedication to mentoring and his unwavering commitment to excellence has inspired me.

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In the end, I would like to take this opportunity to express my heartfelt thanks to my family. Their love, support, and encouragement have been unwavering and invaluable throughout

my academic journey. They have been my source of inspiration, and my motivation to keep pushing forward, even when the going got tough.

Sincerely,

Foroogh Salekpay

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# CHAPTER 1

# INTRODUCTION

The rationing problem is a fundamental challenge in resource allocation, where a scarce resource is divided among a group of individuals, each with their own preferences. The primary objective of each rationing situation is to design a fair and efficient mechanism. In doing so, various criteria, such as priorities, individual needs, cost-effectiveness, or market mechanisms, can ensure a degree of fairness and equality in resource allocation.

Priority-based resource allocation assigns resources based on defined priorities for the situation at hand. For example, during the Covid-19 vaccine allocation, governments prioritized individuals exposed to higher health risks, such as healthcare workers and the elderly. Needs-based resource allocation, on the other hand, allocates resources based on the needs of individuals. For instance, Birch et al. (1993) conduct a study on the allocation of the healthcare budget in Canada, considering the needs of different communities. The study employs the needs of individuals in each community as the allocation criterion and assigns more budget to communities with larger populations. However, the authors also take into account other factors such as the needs of different gender and age groups within each community. Indicators such as ethnicity, socioeconomic status (e.g., income, education, marital status), mortality rates, and disease complications can also be used to determine and assess the needs of society (Radinmanesh et al., 2021).

Cost-effective-based resource allocation is designed based on the principle of efficiency. This approach allocates limited resources to generate greater outcomes or benefits per unit cost. For instance, in the public sector, budgets for education, transportation, or other infrastructure are distributed based on expected social and economic returns. The goal is to maximize the overall welfare of the population receiving these budgets.

The market-based resource allocation distributes resources based on individuals' ability or willingness to pay. This approach is often used in the allocation of private goods, such as housing, where the allocation is based on the price of the goods and people's willingness and ability to pay.

Although the perception of fairness is subjective and context-dependent, the allocation criteria outlined above have been shown to result in a satisfactory level of allocation recipient contentment. This contentment is characterized by the absence of envy, which was introduced by Foley (1966) as a criterion for a fair distribution, known as the "envy-freeness". Moreover, when individuals are confident that the allocation mechanism treats all parties impartially and without prejudice, they are less likely to misrepresent their preferences or manipulate the allocation process. This characteristic is an important attribute of a fair allocation mechanism, referred to as "strategy-proofness" (Klaus et al., 2002).

In this context, the claims problem (O'Neill, 1982), also referred to as the bankruptcy problem, is a type of rationing problem in which the allocation of resources is based on individuals' demands or claims. The primary characteristic of the claims problem is that the available resources are insufficient to meet the total claims of all claimants. The objective is to identify a distribution method that maximizes the overall benefit of all claimants involved. The claims problem has been studied for years and has its roots in religious texts, as well as in ancient legal disputes. In the Talmud, for example, there are instances of claims problems in the form of arbitration by a third party, who resolves the dispute based on some allocation methods. Studies such as O'Neill (1982) and Aumann and Maschler (1985) aim to uncover these methods, formulate them, and generalize them for use in similar situations.

The claims problem has been addressed through various theoretical solutions, which can be broadly categorized into two methods: game theory and axiomatic methods. The game theory approach focuses on analyzing the interactions among claimants, whether cooperative or strategic and their impact on the resulting outcome. The axiomatic method, on the other hand, introduces a set of normative principles or axioms, whether ethical, legal, or social, and requires that the solutions (or rules) satisfy these principles.

The allocation of limited resources in disputes, such as bankruptcies where assets are insufficient to compensate creditors and shareholders, or competition for natural resources, like water, among farmers with varying needs and priorities, creates claims problems. Resolving such disputes requires a fair and efficient distribution of resources. To address this, various rules have been developed over the years in the context of rationing limited resources.

A rule is a function that maps a set of claims into a set of allocations. The main rules used in the literature are derived from ancient examples found in the Talmud. The Proportional rule, the primary rule that can be inferred from the ancient rationing examples, is a method of resource allocation among claimants based on their proportional involvement in a dispute. This means that resources are allocated based on each claimant's claim on the resource. According to Thomson (2019), this rule places all units of claims on the same footing, independently of who holds them, and results in an equal division of resources among the units. Proportionality of awards to claims is the end result. However, some results of resource allocation in ancient literature are different from what the Proportional rule can assign to parties.

A rule that derives from the Talmud is the Concede and divide rule (Aumann and Maschler, 1985), which is a rule applicable when only two claimants are involved. Under this rule, each claimant concedes to the other claimant the difference between her claim and the total endowment, if this difference is positive, or zero if it is negative. The rule then allocates to each claimant the amount conceded to them and divides the remaining resource equally between them.

Two other rules known as Constrained equal awards and Constrained equal losses (Maimonides, 1204) also referred to as Uniform gains and Uniform losses (Moulin, 2002), aim to equalize the award or loss that claimants receive subject to certain constraints or limitations. The Constrained equal awards rule distributes the resource equally among claimants, with the limitation that no claimant can receive more than their claim. This equal division is based on the absolute value of the claims, which differs from the Proportional rule that divides resources based on proportional involvement. The Constrained equal losses rule allocates losses equally, with the loss being the difference between the total of the claims and the available resource. If the result of the subtraction of the claimant's claims and allocated loss is negative, the rule allocates zero.

The Talmud rule (Aumann and Maschler, 1985) is a comprehensive ancient rule that extends the Concede and divides rule to cases involving more than two claimants. The behavior of this rule changes based on the magnitude of the resource. If the resource is less than or equal to half the aggregate claims, the rule corresponds to the Constrained equal awards. If the resource is greater than half of the aggregate claims, claimants receive half of what they required and incur an equal portion of the loss (i.e., the difference between the total remaining claims and the remaining resource).

The growing demand for finite resources like food, water, and energy highlights the importance of studying the claims problem approach. This approach can help resolve conflicts and address global challenges related to resource scarcity. Developing a fair and efficient allocation system for scarce resources can promote social justice, equality, and economic growth. For these reasons, the study of claims problems is a primary focus of this dissertation.

To this end, we dedicate Chapter 2 of this dissertation to focus on the application of the claims problem to reduce greenhouse gas emissions and address the threat of global warming. In response to this challenge, the European Union (EU), along with 169 other United Nations parties, signed an international treaty known as the Paris Agreement to resolve the global warming crisis. This chapter examines the conflict that EU member states face in attempting to meet the Paris Agreement target for the 2021-2030 period, which is a 55% reduction in greenhouse gas emissions by 2030 (compared to 1990 levels). To meet this target, EU member states must reduce their emissions to a level that is lower than their current emission levels.

To address this challenge, we propose a method based on the allocation of a limited emissions budget to each member state. We apply the rules of the claims problem to allocate the permitted emission budget and compare the outcomes of each rule using an axiomatic analysis to identify the most efficient and practical rule. We employ classical rules in conjunction with a novel rule known as $\alpha$-minimal (Giménez-Gómez and Peris, 2014). The $\alpha$-minimal rule combines the principles of the equal division of resources and the Proportional rule, offering a unique and effective approach to address claims problems in the allocation of limited resources.

We delve further into the application of the claims problem in Chapter 3, this time within the framework of the European regional development fund allocation. Our analysis leads us to introduce a new rule within the context of claims problems.

The rule we introduce is called CELmin, which combines the egalitarian allocation of resources with Constrained equal losses. Firstly, it guarantees a minimum allocation to all claimants regardless of the initial size of their claim. The remaining endowment is then allocated using the Constrained equal losses method. We propose to apply this rule to allocate the European regional development fund. This fund is distributed by the EU among member states to promote economic growth in less developed regions and achieve economic growth convergence across the region. CELmin addresses the limitations of other classical rules in the context of claims problems, making it a more effective method for distributing this financial aid.

Introducing CELmin provided us with the opportunity to shift our focus from studying individual rules to considering them as a family of rules. A family of rules is a group of rules that are organized based on shared features. In Chapter 4, we introduce a new family of rules called the CEL-family, which builds upon the intuition of the CELmin rule. The rules in the CEL-family combine two concepts of solidarity and resource allocation based on the claims. This family includes a range of rules that span opposite ends of the spectrum.

Some rules tend to neglect the smaller claims in an allocation, while others entitle these claims more than others. Constrained equal losses and Constrained equal awards are the extreme members of the CEL-family that represent these perspectives, respectively. The former completely neglects the smaller claims, while the latter fully honors them.

Finally, in Chapter 5, we present our general conclusions and discuss the contributions of the previous three chapters. Additionally, we propose avenues for future research in rationing problems and limited resource allocation.[1]

[1] I would like to state that this dissertation consists of three distinct papers. As you review the manuscript, you may notice that certain concepts and methods are repeated across the papers. I want to acknowledge and apologize for any redundancy that you may come across in the dissertation. I want to assure you that the repetition was intentional and necessary for the individual papers and was included to ensure clarity and consistency within each paper, allowing them to stand alone as coherent pieces of research.

# CHAPTER 2

# THE ALLOCATION OF GREENHOUSE GAS EMISSION IN THE EUROPEAN UNION THROUGH APPLYING THE CLAIMS PROBLEMS APPROACH

## 2.1 Introduction

The European Union (EU) has a significant impact on the context of global warming. This region is the fourth global greenhouse gas (GHG) emitter (UN, 2022). Nonetheless, the EU has always played the role of a leader to navigate activities for diminishing GHG emission (Parker and Karlsson, 2017).[2]

The most prominent role of the EU is its proposal to limit the global temperature increase up to 2ºC above the pre-industrial level in 1996 (Schleussner et al., 2016). This proposal has been the main target of all climate change protocols and agreements (Rayner and Jordan, 2013). After this, we can mention the Paris Agreement as a landmark in the EU's leading role to accelerate emission mitigation (Höhne et al., 2017). To achieve the targets of the Paris Agreement, the EU believes countries' efforts should be clear and quantifiable (Oberthür and Groen, 2017). Therefore, the EU itself set 3 objectives: 20% reduction in GHG emissions by 2020, 55% reduction by 2030 (compared with the 1990 level), and reaching net zero emissions by 2050. Member states have succeeded in starting a decreasing trend from 1990 onwards. This diminishing path has led to the target of 2020 being overachieved (32% emission reduction in 2020 compared with 1990 (Forster et al., 2021)).

[2] Chapter 2 of this dissertation is based on the following publication:
Salekpay, F. (2023). The Allocation of Greenhouse Gas Emission in European Union through Applying the Claims Problems Approach. Games, 14(1), 9.

Nevertheless, countries' emission projections for the period 2021 to 2030 indicate that this achievement is temporary. The member states' aggregate emission projections show that, in the best condition, their emission reduction is 41% less than 1990 (Forster et al., 2021). Thereby, the countries' national efforts are not sufficient in following the EU targets.

Several studies have tried to solve this problem. du Pont et al. (2017) and Pan et al. (2017) define different emission budget scenarios aligned with the Paris Agreement and allocate them among countries. The allocation each country receives can work as a criterion to limit their national emissions (Raupach et al., 2014). The studies allocate the emission budget based on some equity principles. For instance, countries with larger historical emissions and/or larger Gross domestic product (GDP) per capita should diminish more GHG emissions. Chen et al. (2022) consider the equity principle and compromise it with the efficiency principle. The efficiency principle focuses on the economic benefit and minimizing the cost of emission reduction. They define population size, economic development, and historical emissions as the indicators that are utilized for an equitable allocation. Regions with higher population size, GDP, and historical emissions should take more responsibility for GHG emissions reduction. To evaluate the efficiency, they use a model called Super-SBM. Capital, labor, and energy are the inputs of this model and the output is GDP and GHG emissions. The regions with higher efficiency are assigned higher emission allocations. Ju et al. (2021) discuss that the eventual allowable emission level that countries can emit is the difference between the target emission budget and countries' historical emission. They proposed to allocate this allowable budget based on equal per capita or equal per countries' current and future emissions. They also propose to enter the factor of historical emission into the aforementioned allocation methods.

However, the division of emission reduction responsibility is one method to abate the total GHG emission; another way is to sink and capture the cumulative GHG from the atmosphere. Fyson et al. (2020), Pozo et al. (2020) and Lee et al. (2021) focus on this issue rather than studying the emission reduction actions. They divide Carbon Dioxide Removal

(CDR) responsibilities fairly among countries. Based on their results, countries with larger GDPs and larger cumulative emissions per person will take more portion of CDR.

If we return to the solution of the GHG emission reduction through assigning a target emission budget to countries, Giménez-Gómez et al. (2016) provide a new approach to do that which is the application of the claims problem approach. Claims problem (O'Neill, 1982) distributes a limited resource in situations where the total needs of parties in a dispute are more than the available resource. Duro et al. (2020) study the distribution of the GHG emission budget between five main groups of countries by applying the claims problems approach. This method is used in a variety of resource allocation fields. Recently, Solís-Baltodano et al. (2022) implemented the claims problem to distribute the European regional development fund to different regions throughout the EU.

The case of GHG mitigation in the EU can evidently be defined as a claims problem. We estimate the total amount of GHG that member states must emit from 2021 to 2030 in the framework of the EU 2030 target and compare it with the member states' aggregate emission projections in this period. We observe that the aggregate projections exceed the desirable emission level and the claims problem approach is applicable. We apply several rules which are well-known in the claims problem literature and study their behavior. The allocation each member state receives by these rules tells how much they should abate their emission in the ten years to reach the target of 55% reduction by 2030 and obliges countries to adjust their emission projections to be more compatible with that target.

Here, the question rises, how countries should adjust the projection? We propose to look at this issue in a more dynamic way by applying the claims problem in each year. Now, countries can adjust their projections step by step according to the annual ceiling which is determined by the claims problem. The annual allocation is conducted as follows: in 2021, we allocate the permitted emission budget for this year to the projections of the year. As the aggregate projection is more than the emission budget, countries' projections can not be fully satisfied, and part of them will be lost. These losses are added to the projections for

2022 and then we divide the 2022 emission budget based on these revised projections and so on. This method is an opportunity for member states to adjust their annual projections by considering the losses and can accelerate the process of GHG mitigation. If member states define their annual projections without taking into account the losses, they will face large projections when the losses are added. We will see that some rules extremely penalize these countries by satisfying a slight portion of their projection, which means more loss for countries.

In the claims problem approach, countries are allowed to announce their demands and the claims problem allocates the emission budget on the basis of these demands. In addition, countries' final GHG emissions are limited to the amount that the claims problem assigns to them. Since in the claims problem no country can receive more than her claim (i.e. her need to emit), there is no chance for trading the emission allowances. Indeed, the claims problem establishes a strong limitation for rich countries to buy other countries' exceeded emission allowances.

This chapter is organized as follows: Section 2.2 defines the claims problem approach and the rules we apply to allocate the emission budget. This chapter also mentions the conditions and principles the rules should satisfy. Section 2.3 discusses the implementation of rules. In Section 2.4, the conclusions are provided.

## 2.2 Materials and Methods

Formally, we can define the claims problem as a set of agents $N = \{1, 2, ..., n\}$ and an amount $E \in \mathbb{R}_+$ the **endowment** that has to be allocated among them. Each agent has a **claim**, $c_i \in \mathbb{R}_+$ on it. Let $c \equiv (c_i)_{i \in N}$ be the claims vector.

Then, a **claims problem** (O'Neill, 1982) is a pair $(E, c)$ with $C = \sum_{i=1}^{n} c_i > E$.

Without loss of generality, we increasingly order the agents according to their claims, $c_1 \leq c_2 \leq \ldots \leq c_n$, and we denote by $\mathcal{B}$ the set of all claims problems.

We define the EU member states as the agents. To define the against' claims, we use countries' national projections of anthropogenic GHG emissions. The projections are the countries' estimations about their future GHG emissions in different sources and GHG removals for the period 2021 to 2030. The projections are prepared in two scenarios: 'with existing measures' (WEM) and 'with additional measures' (WAM).

In WEM scenario, projections reflect the effects of all adopted and implemented measures at the time the projections are prepared. These measures embrace all mitigation actions and instruments which are the yield of governments' official decisions. Measures are supported by assigning adequate financial and human resources and the process of implementation of these measures are guaranteed. In WAM scenario, projections consider all adopted and implemented measures and the measures are at the planning stage at the time the projections are prepared. Although these planned measures are under review when the projections are submitted, they have a realistic chance to be adopted and implemented in the future (European Parliament and Council of the European Union, 2018).

The member states are obliged to report their national measures and projections every two years to the Monitoring Mechanism Regulation (MMR). These reports are used to monitor the member states' national mitigation efforts and assess the capability of the current measures to serve the GHG emission mitigation (EEA, 2019). These measures are mainly implemented in industrial and agricultural sectors, energy supply (i.e., fuel extraction, distribution, and storage), and, energy consumption (i.e., consumption of fuels and electricity by households, services, industry, and agriculture). These measures appear in different forms such as economic incentives to reduce GHG, setting taxes on GHG emissions, building standard regulations, training programs, and research programs (EEA, 2019).

Afterward, we need to define the emission budget in line with the target of a 55% reduction by 2030. For this purpose, we assume a constant decreasing trend from the countries' last absolute emission to the desirable emission in 2030. According to the EU database

(Eurostat), the latest absolute emission released hitherto belongs to 2020 which is 3124.59 Megatonne (Mt). The desirable emission in 2030 is 2109.36 Mt which represents a 55% reduction compared with emissions in 1990. Let us show the emission in 2020 by $e^{2020}$ and desirable emission in 2030 by $e^{2030}$ and let us $d = \frac{e^{2020} - e^{2030}}{10}$ where 10 is the period in which the countries are diminishing the emission reduction (i.e, the number of years from 2021 to 2030). To achieve the constant decrease from 2021 and meet the desirable emission in 2030, the emission of EU in each year is the emission of the previous year minus $d$. Table 2.1 shows the total of this emission budget for 2021-2030 and the total of projections (in two scenarios) for these years.

| **Emission Budget** | **Projection (WEM)** | **Projection (WAM)** |
|---|---|---|
| 25662.12 | 33027.50 | 30927.98 |

Table 2.1: Total emission budget and projections for 2021- 2030, numbers are in megatonnes (Mt)

As Table 2.1 depicts the emission budget is not sufficient to satisfy the projections. There is a variety of rules in the claims problem that each proposes a particular way to divide the emission budget. A rule is a single-valued function $\varphi : \mathcal{B} \to \mathbb{R}^n_+$, such that $\varphi_i(E, c) \geq 0$. We use rules which were already implemented in the context of CO2 emission right by Giménez-Gómez et al. (2016) and Duro et al. (2020). These rules include Proportional, Constrained equal awards, Constrained equal losses, Talmud, Adjusted proportional, and $\alpha$-minimal.

The **Proportional (P)** (Thomson, 2003) divides the emission budget proportionally among countries according to their projections. In this rule for each $(E, c) \in \mathcal{B}$ and each $i \in N$, $P_i(E, c) \equiv \lambda c_i$, where $\lambda = E/\sum\limits_{i \in N} c_i$.

The **Constrained equal awards (CEA)** (Thomson, 2003) divides emission budget equally to all countries provided that non of them receive more than their projections. The process is as follows: If the average emission budget exceeds the projection of one country, the rule fully satisfies the country's projection, excludes this country from the allocation process, and continues to allocate the remaining emission budget equally to the rest of the countries.

For each $(E,c) \in \mathcal{B}$ and each $i \in N$, $CEA_i(E,c) \equiv \min\{c_i, \mu\}$, where $\mu$ is such that $\sum_{i\in N} \min\{c_i, \mu\} = E$. However, this rule neglects the differences between projections of countries.

The **Constrained equal losses (CEL)** (Thomson, 2003) proposes to divide the loss (difference between aggregate projections and emission budget) equally to all countries given that no country receives a negative amount. The allocation each country receives is the difference between its projection and the loss which is divided by CEL. If this difference is negative for a country, the country's allocation will be zero and she leaves the allocation process. CEL divides the loss equal to the remaining countries.

For each $(E,c) \in \mathcal{B}$ and each $i \in N$, $CEL_i(E,c) \equiv \max\{0, c_i - \mu\}$, where $\mu$ is such that $\sum_{i\in N} \max\{0, c_i - \mu\} = E$.

**Talmud (T)** (Aumann and Maschler, 1985) proposes a combination of CEA and CEL. This rule focuses on the half-sum of aggregate projections. If the emission budget is less than or equal to the half-sum of projections, CEA is applied. Countries receive the average emission budget or half of their projection (if the average emission budget is greater than half of the projection). If the emission budget is greater than half-sum of projections, the following process is conducted: countries receive half of their projections; The projections and emission budget are revised down by these initial allocations, and CEL is applied to these revised amounts.

For each $(E,c) \in \mathcal{B}$ and each $i \in N$, $T_i(E,c) \equiv CEA_i(E,c/2)$ if $E \leq c/2$; or $T_i(E,c) \equiv c_i/2 + CEL_i(E - c/2, c/2)$, otherwise.

The **Adjusted Proportional (AP)** (Curiel et al., 1987) has been introduced in two steps. In the first step, AP assigns to each country a minimal right ($m_i$). This minimal right is the remaining emission budget when the projections of the rest countries have been satisfied, with respect to the condition of $m_i(E, c) = \max\left\{0, E - \sum_{j \neq i} c_j\right\}$. In the second step, the projections are revised down by the minimal rights. Then the remaining emission budget is assigned proportionally among countries based on their revised projections.
For each $(E, c) \in \mathcal{B}$ and each $i \in N$, $AP_i(E, c) = m_i(E, c) + P(E - \sum_{i \in N} m_i(E, c), c - m(E, c))$

The **$\boldsymbol{\alpha}$-minimal** (**$\boldsymbol{\alpha}$-min**) (Giménez-Gómez and Peris, 2014) proposes to give each country a minimal amount equal to the lowest projection, in the case that the emission budget is enough. After revising down the projections by minimal amounts, the rule distributes the remaining emission budget proportionally among countries according to their revised projections. However, if the emission budget is not sufficient to give all countries the minimal amount, this rule recommends dividing the emission budget equally among countries.
For each $(E, c) \in \mathcal{B}$ and each $i \in N$, if $c_1 > E/n$ then $\alpha - min_i(E, c) = E/n$ and if $c_1 < E/n$ then $\alpha - min_i(E, c) = c_1 + P(E - nc_1, c - c_1)$.

All these rules must satisfy three basic requirements. First, the minimum amount countries receive by applying the rules is 0 (**non-negativity**), $\varphi_i(E, c) \geq 0$, for all $i \in N$. Second, countries cannot receive more than their projections (**claim-boundedness**), $\varphi_i(E, c) \leq c_i$, for all $i \in N$. Third, the whole emission budget should be divided among countries. (**efficiency**) $\sum_{i \in N} \varphi_i(E, c) = E$.

We also introduce some well-known properties in the context of resource distribution. These properties examine the characteristic of each division rule and assist us to select an optimal one.

**Equal treatment of equals** (Thomson, 2019) states that countries with the same projections should receive an equal amount of emission budget. For each $(E, c) \in \beta$ and $i, j \subseteq N$, if $c_i = c_j$ then $\varphi_i(E, c) = \varphi_j(E, c)$.

**Anonymity** (Thomson, 2019) says the allocation of emission budget exclusively depends on countries' projections. Other factors such as the identity of the countries cannot affect the emission allocation. For each $(E, c) \in \beta$ each $\pi \in \prod^N$ and each $i \in N$, $\varphi_{\pi(i)}(\mathrm{E},(\mathrm{c}_{\pi(i)})_{i\in N}) = \varphi(E, c)$, where $\prod^N$ is the permutations of $N$.

**Order preservation** (Aumann and Maschler, 1985) means the emission allocation assigned to countries with larger projections can not be smaller than the emission allocation of countries with lower projections. For each $(E, c) \in \beta$ and each $i, j \in N$ such that $c_i \geq c_j$, then $\varphi_i(E, c) \geq \varphi_j(E, c)$. Likewise, countries with larger projections bear an equal or larger amount of loss than countries with lower projections. $c_i - \varphi_i(E, c) \geq c_j - \varphi_j(E, c)$.

**Claims monotonicity** (Thomson, 2019) states if a country increases its projection, the allocation assigned to this country cannot be less than the initial amount. For each $(E, c) \in \beta$, $i \in N$ and each $c'_i > c_i$ we have $\varphi_i(E, c'_i, c_{-i}) \geq \varphi_i(E, c)$.

**Composition down** (Moulin, 2000) is a property for situations in which the emission budget is reduced after allocation due to re-evaluation of the emission budget or setting more strict emission reduction rules. For instance, the EU decides to increase the percentage of emission reduction by 2030 to more than 55%, while the emission budget of 55% reduction has been allocated before this announcement. In this case, we have two options: First, we cancel the initial emission budget allocation and reallocate the new amount of the emission budget. Second, we consider the initial emission allocation assigned to each country as their claims (rather than considering the projections) and divide the new emission budget by these new claims. Regardless of the option we select, both choices should produce equivalent outcomes. For each $(E, c) \in \mathcal{B}$, each $i \in N$, and each $0 \leq E' \leq E$, $\varphi_i(E', c) = \varphi_i(E', \varphi(E, c))$.

**Invariance under claims truncation** (Dagan and Volij, 1993) imposes an upper bound to countries projections. If a country's projection is greater than the emission budget, the exceeding part will be ignored. This property says that countries cannot request more than the available emission budget. For each $(E, c) \in \beta$ and each $i \in N$, $\varphi_i(E, c) = \varphi_i(E, (minc_i, E)_{i \in N})$.

Table 2.2 shows the rules and the properties which are satisfied by them. The Constrained equal awards (CEA) rule satisfies all these basic properties.

| **Properties/Rules** | **P** | **CEA** | **CEL** | **T** | **AP** | **$\alpha$-min** |
|---|---|---|---|---|---|---|
| Equal treatment of equals | Yes | Yes | Yes | Yes | Yes | Yes |
| Anonymity | Yes | Yes | Yes | Yes | Yes | Yes |
| Order preservation | Yes | Yes | Yes | Yes | Yes | Yes |
| Claims monotonicity | Yes | Yes | Yes | Yes | Yes | Yes |
| Composition down | Yes | Yes | Yes | No | No | Yes |
| Invariance under claims truncation | No | Yes | No | Yes | No | No |

Table 2.2: Properties and rules.

## 2.3 Results and Discussion

### 2.3.1 The allocation of total GHG emission

As we saw, the countries' total GHG emission from 2021 to 2030 is more than the estimated emission budget to achieve the target of 55% emission reduction in 2030 (see table 2.1). For solving this problem, we implement the aforementioned rules to allocate the total emission budget among countries according to their total projections for the period 2021-2030.

Table 2.3 and 2.4 show the results for WEM and WAM scenarios. In these tables, countries are ordered according to their projections from smaller to larger.

The CEA rule divides the emission budget equally among the member states. By analyzing the results depicted in the table 2.3 the first 21 countries receive the exact amount of their projections. Therefore, these countries are permitted to emit at the level they estimated (according to the requirement of claim-boundedness, countries cannot emit more than their projections, Therefore, if one rule assigns them an emission allocation more than their projections, that assignment is truncated to the countries' projections). In WAM, the first 22 countries are assigned equal to their projections (table 2.4). Hence, we can claim that CEA is an appropriate rule for countries with smaller projections. Since CEA assigns the emission budget equally to countries without taking in to account the magnitude of countries' projections. Thereby, countries with lower projections obtain more percentage of their projection.

The CEL divides the loss (i.e. aggregate projections minus current emission budget) equally among the member states. The CEL determines how much each country should decrease its emission. Each country's permitted emission is obtained by subtracting the allocated loss from the country's projections. In some cases (e.g Malta or Cyprus) this subtraction is negative. According to the requirement of non-negativity, the allocation of countries cannot be negative. Thereby, these countries are assigned zero emission permission. Indeed, CEL sacrifices countries with smaller projections and the reason lies in the equal division of loss without considering the size of the projections. T shows that this rule supports the countries with larger claims. Since the mission budget is greater than the half-sum of projections, CEL is applied. P, AP, and $\alpha$-min show moderate behaviors with respect to extremely smaller or larger projections, since they consider the size of the projections while they allocate the emission budget.

The main question is which of these rules should be selected to allocate the emission budget and meet the target of 2030. Countries tend to maximize the emission allocation

| **Country** | **Projection** | **P** | **CEA** | **CEL** | **T** | **AP** | **$\alpha$-min** |
|---|---|---|---|---|---|---|---|
| Malta | 30.11 | 23.40 | 30.11 | 0.00 | 15.05 | 23.28 | 30.11 |
| Cyprus | 79.16 | 61.51 | 79.16 | 0.00 | 39.58 | 61.21 | 67.95 |
| Sweden | 108.77 | 84.51 | 108.77 | 0.00 | 54.38 | 84.11 | 90.79 |
| Luxembourg | 115.98 | 90.12 | 115.98 | 0.00 | 57.99 | 89.69 | 96.35 |
| Slovenia | 120.76 | 93.83 | 120.76 | 0.00 | 60.38 | 93.38 | 100.03 |
| Estonia | 121.97 | 94.77 | 121.97 | 0.00 | 60.98 | 94.32 | 100.97 |
| Lithuania | 138.88 | 107.91 | 138.88 | 0.00 | 69.44 | 107.39 | 114.01 |
| Latvia | 140.36 | 109.06 | 140.36 | 0.00 | 70.18 | 108.54 | 115.15 |
| Croatia | 207.86 | 161.51 | 207.86 | 0.00 | 103.93 | 160.74 | 167.22 |
| Finland | 278.29 | 216.23 | 278.29 | 0.00 | 139.15 | 215.20 | 221.55 |
| Slovakia | 405.30 | 314.92 | 405.30 | 50.99 | 202.65 | 313.41 | 319.52 |
| Denmark | 428.22 | 332.72 | 428.22 | 73.91 | 214.11 | 331.14 | 337.20 |
| Bulgaria | 499.75 | 388.30 | 499.75 | 145.44 | 249.88 | 386.45 | 392.37 |
| Portugal | 499.77 | 388.32 | 499.77 | 145.46 | 249.88 | 386.47 | 392.39 |
| Hungary | 602.23 | 467.93 | 602.23 | 247.92 | 301.12 | 465.70 | 471.42 |
| Ireland | 685.67 | 532.76 | 685.67 | 331.36 | 342.83 | 530.22 | 535.79 |
| Austria | 734.51 | 570.71 | 734.51 | 380.20 | 367.25 | 567.99 | 573.46 |
| Romania | 798.82 | 620.68 | 798.82 | 444.51 | 399.41 | 617.72 | 623.07 |
| Greece | 801.66 | 622.88 | 801.66 | 447.35 | 400.83 | 619.91 | 625.26 |
| Czech | 1173.08 | 911.47 | 1173.08 | 818.77 | 677.29 | 907.13 | 911.76 |
| Belgium | 1260.61 | 979.48 | 1260.61 | 906.30 | 764.82 | 974.81 | 979.27 |
| Netherlands | 1764.67 | 1371.14 | 1764.67 | 1410.36 | 1268.88 | 1364.60 | 1368.09 |
| Spain | 2964.72 | 2303.57 | 2933.14 | 2610.41 | 2468.93 | 2292.58 | 2293.76 |
| Italy | 3641.23 | 2829.21 | 2933.14 | 3286.92 | 3145.44 | 2815.72 | 2815.60 |
| France | 3732.83 | 2900.38 | 2933.14 | 3378.52 | 3237.04 | 2886.55 | 2886.26 |
| Poland | 3787.25 | 2942.67 | 2933.14 | 3432.94 | 3291.46 | 2928.64 | 2928.24 |
| Germany | 7905.04 | 6142.16 | 2933.14 | 7550.73 | 7409.25 | 6235.22 | 6104.55 |

Table 2.3: The total GHG emission allocation 2021-2030 (WEM), numbers are in megatonnes (Mt)

they receive by accepting the rules giving them more allocation and denying the rules which seem not fair to them. Therefore, just by considering the amount each rule assigns to countries we cannot propose a specific division rule which can be accepted by all countries.

To offer an efficient division rule we need criteria to make rule selection independent from countries' tendencies. In doing so, we analyze division rules from equity and stability points of view. For this purpose, we consider two criteria: the Gini index and the Coefficient of variation.

| Country | Projection | P | CEA | CEL | T | AP | α-min |
|---|---|---|---|---|---|---|---|
| Malta | 30.11 | 24.98 | 30.11 | 0.00 | 15.05 | 24.51 | 30.11 |
| Cyprus | 75.29 | 62.47 | 75.29 | 0.00 | 37.65 | 61.28 | 67.39 |
| Luxembourg | 90.77 | 75.32 | 90.77 | 0.00 | 45.38 | 73.87 | 80.16 |
| Sweden | 108.77 | 90.25 | 108.77 | 0.00 | 54.38 | 88.52 | 95.02 |
| Lithuania | 119.70 | 99.32 | 119.70 | 0.00 | 59.85 | 97.42 | 104.03 |
| Estonia | 120.66 | 100.12 | 120.66 | 0.00 | 60.33 | 98.20 | 104.83 |
| Slovenia | 132.02 | 109.54 | 132.02 | 0.00 | 66.01 | 107.44 | 114.20 |
| Latvia | 132.76 | 110.16 | 132.76 | 0.00 | 66.38 | 108.05 | 114.81 |
| Croatia | 198.58 | 164.77 | 198.58 | 0.00 | 99.29 | 161.61 | 169.12 |
| Finland | 264.21 | 219.22 | 264.21 | 27.70 | 132.10 | 215.03 | 223.28 |
| Slovakia | 360.87 | 299.43 | 360.87 | 124.36 | 180.44 | 293.70 | 303.03 |
| Denmark | 428.22 | 355.31 | 428.22 | 191.71 | 214.11 | 348.51 | 358.61 |
| Portugal | 448.84 | 372.42 | 448.84 | 212.33 | 224.42 | 365.29 | 375.62 |
| Bulgaria | 483.58 | 401.24 | 483.58 | 247.07 | 241.79 | 393.56 | 404.29 |
| Hungary | 558.34 | 463.28 | 558.34 | 321.83 | 279.17 | 454.41 | 465.97 |
| Ireland | 627.44 | 520.61 | 627.44 | 390.93 | 336.65 | 510.64 | 522.99 |
| Greece | 672.19 | 557.74 | 672.19 | 435.68 | 381.40 | 547.06 | 559.92 |
| Austria | 699.54 | 580.43 | 699.54 | 463.03 | 408.75 | 569.32 | 582.48 |
| Romania | 757.57 | 628.58 | 757.57 | 521.06 | 466.78 | 616.55 | 630.37 |
| Czech | 1057.91 | 877.79 | 1057.91 | 821.40 | 767.12 | 860.98 | 878.19 |
| Belgium | 1151.02 | 955.04 | 1151.02 | 914.51 | 860.23 | 936.76 | 955.02 |
| Netherlands | 1769.12 | 1467.91 | 1769.12 | 1532.61 | 1478.33 | 1439.80 | 1465.04 |
| Spain | 2486.89 | 2063.47 | 2486.89 | 2250.38 | 2196.10 | 2023.96 | 2057.30 |
| Italy | 3159.62 | 2621.66 | 3159.62 | 2923.11 | 2868.83 | 2571.47 | 2612.40 |
| Poland | 3356.11 | 2784.69 | 3242.70 | 3119.60 | 3065.32 | 2731.38 | 2774.53 |
| France | 3732.83 | 3097.27 | 3242.70 | 3496.32 | 3442.04 | 3037.98 | 3085.38 |
| Germany | 7905.04 | 6559.11 | 3242.70 | 7668.53 | 7614.25 | 6924.81 | 6528.04 |

Table 2.4: The total GHG emission allocation 2021-2030 (WAM), numbers are in megatonnes (Mt)

**Gini index (Gi):** (Gini, 1921) Gini index is a statistic dispersion indicator that evaluates the degree of inequality in a resource allocation. Its value is in the interval of 0 and 1, 0 indicates perfect equality and 1 represents extreme inequality. Given an n-dimensional endowment, the Gini index is defined as follows:

$$Gi = \frac{1}{2n^2\mu}\sum_{i}\sum_{j<i}|r_i - r_{j<i}|$$

where vector $r$ is the assignment of the endowment to the individuals $i = \{1, \ldots, n\}$ and

$\mu = \frac{1}{N}\sum_i r_i.$

**Coefficient of variation (CV:)** For analyzing the stability of the allocation results, the countries' historical emissions should be considered. Countries decide about the fairness of a rule by comparing their historical emission with the emission allocation a particular rule assigns them. Countries would accept a division rule when they are ensured about the fairness of that rule (Lee, 2009). If the amount they receive by a rule is far from their historical emissions, countries would refuse that rule. For this purpose, we apply the CV to evaluate the weights of countries. CV calculates the dispersion of allocation around the mean. Formally, we can define it as:

$$CV = \frac{\delta}{\bar{P}I}$$

where $\delta$ is the standard deviation and $\bar{P}I$ is the mean of Power Index. The range of CV is a value between 0 and $\sqrt{N-1}$ for a finite sample of $N$ (Abdi, 2010) that $\sqrt{N-1}$ shows the complete instability.

Tables 2.5 and 2.6 show the values of the Gini index and Coefficient of variation for the different division rules. As we can see CEA, P, and $\alpha$-min are more equitable and stable division rules, they have lower Gini index and Coefficient of variation compared with other division rules.

| **Criterion** | **P** | **CEA** | **CEL** | **T** | **AP** | **$\alpha$-min** |
|---|---|---|---|---|---|---|
| Gi | 0.63 | 0.55 | 0.75 | 0.70 | 0.63 | 0.63 |
| CV | 1.45 | 1.10 | 1.82 | 1.74 | 1.47 | 1.44 |

Table 2.5: Gini index (Gi) and Coefficient of variation (CV) for total GHG emission allocation (WEM scenario)

| textbfCriterion | **P** | **CEA** | **CEL** | **T** | **AP** | **$\alpha$-min** |
|---|---|---|---|---|---|---|
| Gi | 0.64 | 0.57 | 0.74 | 0.71 | 0.64 | 0.63 |
| CV | 1.50 | 1.17 | 1.79 | 1.76 | 1.55 | 1.50 |

Table 2.6: Gini index (Gi) and Coefficient of variation (CV) for total GHG emission allocation (WAM scenario)

### 2.3.2 Annual allocation

By comparing the annual emission budget and annual projections, we can clearly observe that projections are surplus to the emission budget in both WEM and WAM scenarios (table 2.7).

| **Year** | **Emission Budget** | **Projection (WEM)** | **Projection (WAM)** |
|---|---|---|---|
| 2021 | 3023064.45 | 3393431.76 | 3326137.63 |
| 2022 | 2921541.73 | 3408314.62 | 3305192.39 |
| 2023 | 2820019.00 | 3408667.03 | 3269079.93 |
| 2024 | 2718496.27 | 3377677.61 | 3200327.53 |
| 2025 | 2616973.55 | 3356436.31 | 3143477.10 |
| 2026 | 2515450.82 | 3310391.92 | 3077607.66 |
| 2027 | 2413928.10 | 3268123.97 | 3012592.09 |
| 2028 | 2312405.37 | 3217965.41 | 2938835.50 |
| 2029 | 2210882.64 | 3173982.99 | 2872139.24 |
| 2030 | 2109359.92 | 3112504.27 | 2782595.63 |

Table 2.7: The annual emission budget and projections for 2021- 2030, numbers are in megatonnes (Mt)

In annual allocation, we limit our study to WEM scenario which shows fewer efforts to diminish the GHG emission. As we mentioned, from 2022, we revise the claims of each country by adding the loss of the previous year to the projection of the current year. More precisely, we allocate the annual emission of the year 2021 to the countries based on their projections for this year, by implementing the aforementioned rules. As the emission budget is not sufficient to cover most of the countries' projections, a part of the countries' projections in 2021 remains unsatisfied. We add this unsatisfied part to the projection of countries in 2022. Then, we allocate the 2022 emission budget to these revised projections and so on. Table 2.8 represents the sum of the annual allocation which are dedicated by rules from 2021 to 2030.

We narrow our interpretation to the behavior of two extreme rules, CEA and CEL. The former serves countries with lower projections and the latter supports countries with higher projections. In 2021, all countries except Germany (with the largest projection) are fully honored, by applying CEA. The allocation of Germany is around 480 Mt which decrease to 372 Mt in 2022. The reason is a significant growth in Germany's 2022 projection due to adding the loss of 2021 to that. From 2022 onward, countries such as France, Poland, and Italy are also punished by CEA. While, despite the growth in projections of countries such as Malta, CEA increases their allocation. This confirms our previous results which show that CEA supports countries with smaller projections. In 2021, CEL imposes an equal amount of loss (15.02 Mt) to all countries which cause zero allocation to some countries with smaller claims such as Malta and Cyprus. From 2021 onward, the number of countries that receives zero allocation by applying CEL increases. On one hand, the annual emission is decreasing with a constant slope and on the other hand, most of the countries increase their projections. Therefore, the loss which is the difference between the aggregate projections and the emission budget increases.

Table 2.9 indicates the average measures of the Gini index and Coefficient of variation in this method. The result shows: CEA $<$ P, AP, $\alpha$-min $<$ T $<$ CEL

It is noteworthy to mention that in annual allocation, the rules satisfy all basic requirements and properties except Order preservation. As we can see in table 2.8 the projection of Lithuania is less than Latvia, while the allocation she receives is greater than Latvia (except CEA).

## 2.4 Conclusions

In this study, all the proposed rules are evaluated from different aspects. First, their capability to satisfy a set of basic properties. We saw that Constrained equal awards (CEA) is the rule which satisfies all the properties (table 2.2). The second aspect is fairness in allocation. To evaluate the degree of fairness of each rule, we used two indicators, the Gini

| **Country** | **Projection** | **P** | **CEA** | **CEL** | **T** | **AP** | **$\alpha$-min** |
|---|---|---|---|---|---|---|---|
| Malta | 30.11 | 22.85 | 30.11 | 0.00 | 26.95 | 22.86 | 30.11 |
| Cyprus | 79.16 | 60.99 | 79.16 | 0.00 | 71.31 | 61.03 | 67.95 |
| Sweden | 108.77 | 87.48 | 108.77 | 0.00 | 99.73 | 87.51 | 94.32 |
| Luxembourg | 115.98 | 88.97 | 115.98 | 0.00 | 104.26 | 89.02 | 95.70 |
| Slovenia | 120.76 | 92.10 | 120.76 | 0.00 | 108.46 | 92.15 | 98.76 |
| Estonia | 121.97 | 93.68 | 121.97 | 0.00 | 109.77 | 93.73 | 100.37 |
| Lithuania | 138.88 | 108.63 | 138.88 | 0.23 | 125.91 | 108.69 | 115.25 |
| Latvia | 140.36 | 104.62 | 140.36 | 0.00 | 124.72 | 104.69 | 111.15 |
| Croatia | 207.86 | 159.70 | 207.86 | 5.32 | 187.03 | 159.79 | 165.87 |
| Finland | 278.29 | 217.62 | 278.29 | 26.23 | 252.29 | 217.73 | 223.42 |
| Slovakia | 405.30 | 311.47 | 405.30 | 74.48 | 364.66 | 311.63 | 316.40 |
| Denmark | 428.22 | 336.49 | 428.22 | 101.32 | 389.14 | 336.65 | 341.40 |
| Bulgaria | 499.75 | 390.00 | 499.75 | 149.90 | 452.86 | 390.20 | 394.43 |
| Portugal | 499.77 | 403.29 | 499.77 | 168.57 | 459.74 | 403.47 | 407.92 |
| Hungary | 602.23 | 466.18 | 602.23 | 239.45 | 543.40 | 466.42 | 469.96 |
| Ireland | 685.67 | 529.00 | 685.67 | 322.90 | 617.76 | 529.28 | 532.23 |
| Austria | 734.51 | 567.38 | 734.51 | 371.73 | 662.40 | 567.68 | 570.31 |
| Romania | 798.82 | 620.65 | 798.82 | 436.03 | 722.28 | 620.98 | 623.25 |
| Greece | 801.66 | 621.47 | 801.66 | 438.88 | 724.09 | 621.79 | 624.03 |
| Czech | 1173.08 | 921.42 | 1173.08 | 810.30 | 1065.12 | 921.87 | 921.99 |
| Belgium | 1260.61 | 963.89 | 1260.61 | 897.83 | 1131.79 | 964.42 | 963.44 |
| Netherlands | 1764.67 | 1374.69 | 1764.67 | 1401.89 | 1598.57 | 1375.38 | 1371.37 |
| Spain | 2964.72 | 2289.44 | 2739.55 | 2601.94 | 2473.20 | 2290.63 | 2278.63 |
| Italy | 3641.23 | 2822.37 | 2939.66 | 3278.45 | 2792.57 | 2823.82 | 2807.41 |
| France | 3732.83 | 2886.75 | 2961.27 | 3370.04 | 2829.77 | 2888.24 | 2871.20 |
| Poland | 3787.25 | 2902.18 | 2956.14 | 3424.48 | 2816.08 | 2903.73 | 2885.97 |
| Germany | 7905.04 | 6218.81 | 3069.07 | 7542.26 | 4808.34 | 6208.75 | 6179.30 |

Table 2.8: The aggregate annual GHG emission allocation 2021–2030 (WEM), numbers are in megatonnes (Mt)

| **Criterion** | **P** | **CEA** | **CEL** | **T** | **AP** | **$\alpha$-min** |
|---|---|---|---|---|---|---|
| Gi | 0.63 | 0.55 | 0.75 | 0.58 | 0.63 | 0.63 |
| CV | 1.46 | 1.10 | 1.83 | 1.27 | 1.46 | 1.45 |

Table 2.9: Gini index (Gi) and Coefficient of variation (CV) of the annual GHG emission allocation

index, and the Coefficient of variation. The results also confirm that CEA shows fair behavior. Another reason that makes CEA the final solution of this study is the rule's protective behavior to countries with smaller projections. This rule imposes the emission reduction pressure on the countries with larger projections. The countries with higher projections are mainly industrialized and well-developed countries (e.g. Germany). These countries are the main target of GHG emission reduction. Since, on one hand, they are the larger GHG emitter, and on the other hand, they have the technological and financial ability to reduce GHG emissions. CEA is the rule that reduces these countries' emission allocation in the favor of less developed countries.

# CHAPTER 3

# HOW TO DISTRIBUTE THE EUROPEAN REGIONAL DEVELOPMENT FUNDS THROUGH A COMBINATION OF EGALITARIAN ALLOCATION: THE CELMIN

## 3.1 Introduction

The European regional development fund (ERDF) is part of the European structural and investment fund, which has been designed under the European Union (EU) cohesion policy. With the aim of reducing inequalities in the level of development among regions throughout the EU and compensating the backwardness of less developed regions, the ERDF is invested to support small and medium-sized enterprises, improve the health system, develop the digital infrastructure, enforce non-polluting transportation and reduce greenhouse gas emissions to achieve the target of being carbon neutral by 2050.[3]

The European Commission together with member states are responsible for allocating the ERDF budget to regions. To allocate the ERDF, each member state is classified into three regions according to their Gross domestic product (GDP) per capita: less developed, transition, and more developed regions; and the ERDF is distributed to cover the need of the regions according to the so-called Berlin method. However, we can consider the ERDF budget as the limited endowment to be allocated, and the funds that the regions need to develop some projects (mainly in infrastructures: airports, universities, hospitals, etc.) that they could not afford individually, can be defined as claims. The available ERDF budget

[3]Chapter 3 of this dissertation is based on the following paper currently under review:
Salekpay, F., Vilella, C., Giménez-Gómez, J. M. How to distribute the ERDF funds through a combination of egalitarian allocations: the CELmin

is not enough to satisfy all the claims that the regions have on it, thus, we have a claims problem (O'Neill, 1982).

Within this context, the current approach complements Fragnelli and Kiryluk-Dryjska (2019) and Solís-Baltodano et al. (2022), by defining a new way of distributing the ERDF. As Fragnelli and Kiryluk-Dryjska (2019) mention "this approach has the great advantage that solutions may be obtained with a fast computation." Particularly, Solís-Baltodano et al. (2022) identify the agents (the EU NUTS level 2 regions) and the endowment (the ERDF budget to be allocated) and use four claims solutions: the Proportional rule, the Constrained equal awards rule, the Constrained equal losses rule, and the $\alpha$-minimal galitarian rule. Among the analyzed rules, the one that performs best (promoting convergence) is the one that proposes the most unequal (per capita) distribution of the ERDF budget: the Constrained equal losses rule.

It is noteworthy that there are other related economic and social problems where the claims problem approach is implemented: in the education sector Pulido et al. (2002) use this approach to obtain an efficient allocation of university funds; in the fishing sector, it is a useful tool for seeking possible solutions to address fish shortages, by proposing fishing quotas among a number of agents within an established perimeter (Iñarra and Prellezo, 2008; Iñarra and Skonhoft, 2008; Kampas, 2015); Kiryluk-Dryjska (2014, 2018) propose a formal framework for rural development budget allocation by using fair division techniques; or, in the negotiations on CO2 emissions, a relevant issue nowadays, Giménez-Gómez et al. (2016) and Duro et al. (2020) propose an appealing distribution by analyzing this situation as a conflicting claims problem.

To solve claims problems, we have several division rules that propose a unique way to divide the endowment among agents. As aforementioned, Solís-Baltodano et al. (2022) study the allocation of ERDF as a claims problem by investigating different division rules. The authors show that the Constrained equal losses (CEL) rule, an egalitarian rule that divides the difference between the aggregate claims and the endowment (the part that cannot be honored, i.e., the losses) equally among the agents is the best proposal to achieve the EU convergence target. The CEL gives priority to agents with larger claims per capita, that is, the less developed regions.

Nonetheless, the CEL assigns no funds to some regions with smaller claims; Hence, it is usually not applied in real situations (no region will accept to receive zero amount)[4]. Therefore, it seems clear that, in any real situation, smaller claimants should be protected.

Following Giménez-Gómez and Peris (2014), we propose to guarantee a minimal amount for all the claimants, and, then, divide the remainder by applying the CEL rule. This minimal amount is based on the sustainability and preeminence concepts (Herrero and Villar, 1998, Herrero and Villar, 2002). On the one hand, if region $i$'s claim is as small as when we truncate the claims of other regions to agent $i$'s claim, then we do not have a claims problem anymore, as this claim is sustainable. Sustainability states that these types of claims should be completely honored. On the other hand, we should take into account the excess of claims, i.e., the losses. Preeminence, which is the dual concept of sustainability, establishes that if a claim is removed from the problem, and we still have a claims problem, then this so-called residual claim should not be satisfied. Hence, preeminence, which is satisfied by the CEL rule, gives priority to larger claims, the claims that can change the situation of the claims problem.

Our proposed solution, called CELmin rule, keeps a balance between sustainability and preeminence. The CELmin is a compromise between the egalitarian distribution of the endowment and the CEL rule. The rule proposes that: if the smallest claim is sustainable, CELmin assigns a minimal guarantee equal to the smallest claim to all agents (or the egalitarian distribution of the endowment) and revises down the claims and the endowment to implement CEL and distributes the remaining. If not, the rule proposes an equal division of the endowment.

The rest of the chapter is organized as follows: In Section 3.2 we formally present the

[4]See Solís-Baltodano et al. (2022), where regions with smaller claims receive nothing from the ERDF, or Duro et al. (2020), where small countries receive no CO2 emission permission.

notion of claims problems and some of the main rules in the literature. In Section 3.3 we define our new solution, the CELmin. In Section 3.4 we make an axiomatic analysis of the proposed rule. In Section 3.5 we apply the previous analysis to the ERDF problem and Section 3.6 analyses and compares the proposed allocations from the convergence point of view. Some final comments in Section 3.7 conclude the paper.

## 3.2 Preliminaries: claims problems

The agents are defined as a set of $N = \{1, 2, ..., n\}$. Each agent is identified by her *claim*, $c_i$, $i \in N$, on the *endowment* $E \in R_+$. A **claims problem** occurs when the endowment is not sufficient to cover all the claims, which means $\sum_{i=1}^{n} c_i > E$. Without loss of generality, we order agents according to their claims: $c_1 \leq c_2 \leq \cdots \leq c_n$. The pair $(E, c)$ represents the claims problem and $\mathcal{B}$ is the set of all claims problems. A *claim rule* (**solution**) is a single value function $\varphi : \mathcal{B} \to \mathbb{R}^n_+$ such that, for each $i \in N$, $0 \leq \varphi_i(E, c) \leq c_i$, (**non-negativity** and **claim-boundedness**) and $\sum_{i=1}^{n} \varphi_i(E, c) = E$, (**efficiency**).

Next, we define three well-known classic rules of claims problems (see Thomson (2003) and Thomson (2019)).

The **Proportional (P)** divides the endowment proportionally according to the agents' claims. For each $(E, c) \in \mathcal{B}$ and each $i \in N$, $P_i(E, c) = \lambda c_i$, where $\lambda = \frac{E}{\sum_{i \in N} c_i}$.

The **Constrained equal awards (CEA)** assigns the endowment equally by imposing a constraint on the allocation, such that no agent receives more than her claim. For each $(E, c) \in \mathcal{B}$ and each $i \in N$, $CEA_i(E, c) \equiv \min\{c_i, \mu\}$, where $\mu$ is chosen so that $\sum_{i \in N} \min\{c_i, \mu\} = E$.

Note that the CEA rule is based on the **Equal awards division** $(EA)$ . This method assigns the endowment equally among all members, i.e., for each $(E, c) \in \mathcal{B}$ and each $i \in N$, $EA_i(E, c) = \frac{E}{n}$. However, it is easy to see that in some situations with equal distribution, an agent may receive more than her claim, violating the claim-boundedness condition of a claim rule.

The **Constrained equal losses (CEL)** allocates the loss which is the difference between aggregate claims and the endowment. This measure is divided equally, such that no agent receives a negative amount. For each $(E, c) \in \mathcal{B}$ and each $i \in N$, $CEL_i(E, c) \equiv \max\{0, c_i - \mu\}$, where $\mu$ is such that $\sum_{i \in N} \max\{0, c_i - \mu\} = E$.

In addition, we mention **$\alpha$-minimal ($\alpha$-min)** rule (Giménez-Gómez and Peris (2014)), which is a compromise of the Equal awards division and the Proportional.

The **$\alpha$-min** guarantees a minimal right equal to the smallest claim to all agents, and if the endowment is sufficient it distributes the remaining endowment proportionally to the agents' revised claims. If the endowment is not enough, it is divided equally. For each $(E, c) \in \mathcal{B}$ and each $i \in N$, if $c_1 \geq \frac{E}{n}$ then $\alpha_{min_i}(E, c) = \frac{E}{n}$ and if $c_1 < \frac{E}{n}$ then $\alpha_{min_i}(E, c) = c_1 + P_i(E - nc_1, c - c^1)$, where $\mathbf{c}^1 = (c_1, \ldots, c_1)_{1 \times n}$

### 3.3 The CELmin solution

By combining CEL and EA we define a solution that, if possible, assigns the minimum positive claim $c_1$ to all agents and distributes the remaining endowment $E' = E - nc_1$ by implementing the CEL rule among the agents with respect to the remaining claims $c'_i = c_i - c_1$. If the endowment is not enough to assign the $c_1$ to all agents, then we give an equal division of the endowment to the agents.

**Definition 1** *For each $(E, c) \in \mathcal{B}$ with $c_i > 0$ and each $i \in N$,*

$$CELmin(E, c) = \begin{cases} (E/n)\mathbf{1} & \text{if } c_1 \geq E/n \\ \mathbf{c}^1 + CEL(E - nc_1, c - \mathbf{c}^1) & \text{otherwise} \end{cases}$$

*where $\mathbf{c}^1 = (c_1, \ldots, c_1)_{1xn}$ and $\mathbf{1} = (1, \ldots, 1)_{1xn}$*

*In the event of some claims being equal to zero, $c_1 = c_2 = \ldots c_k = 0$, $c_j > 0$, for each*

*$j > k$, we extend the solution in a consistent manner:*

$$CELmin(E, c) = (\boldsymbol{0}, CELmin(E, \bar{c}))$$

*where $\boldsymbol{0} = (0, \ldots, 0)_{1xk}$ and $\bar{c} = (c_{k+1}, \ldots, c_n)$.*

The following example shows how the rule proceeds.

**Example 1** *Consider $(E, c) = (2000; (500, 2000, 2400)$. $CELmin(E, c) = (500, 500, 500)$ $+CEL(500, (0, 1500, 1900) = (500, 500, 500)$ $+(0, 50, 450) = (500, 550, 950)$. Notice that with the CEL we have $CEL(E, c) = (0, 800, 1200)$, agent one receives nothing and with the CELmin everyone receives at least a minimal amount of 500.*

*Compared with the Proportional rule, which allocates $P(E, c) = (204.08, 816.33, 979.59)$, in this example, larger claimants receive larger amounts by applying CELmin (in section 6 we see that this is not always the case). Although CELmin and $\alpha$-min assign equal minimal rights, the allocation of $\alpha$-min$(E, c) = (500, 750.59, 779.41)$ shows CELmin protects larger claimants more than $\alpha$-min. CEA is the rule which allocated the smallest share to larger claimants, $CEA(E, c) = (500, 750, 750)$.*

Similarly, Hougaard et al. (2013a) and Alcalde and Peris (2022) provide insights also about the combination of equal sharing of losses and awards. Specifically, Hougaard et al. (2013a) ensure each claimant an endogenous minimal amount that depends on the claims and the endowment, called the baseline, $b$. For each $(b, E, c)$, let $t_i(b, c) = \min(b_i, c_i)$ for each $i \in N$ and $t(b, c) = \{t_i(b, c)\}_{i \in N}$ denotes the truncated baseline-claim vector and $T = \sum_{i \in N} t_i(b, c)$. In this context, the authors define a family of rules, $S^b$, through a composition operator (Hougaard et al. (2012) and Hougaard et al. (2013b)) as $S^b(b, E, c) = S(E, t(b, c))$, if $E \leq T(b, c)$, or $S^b(b, E, c) = t(b, c) + S(E - T(b, c), c - t(b, c))$, if $E \geq T(b, c)$.

It is noteworthy that if we define the baseline as the smallest claimant and take CEL as the starting rule then, the CELmin is retrieved,

$$CELmin(E, c) = CEL^b(b, E, c),$$

with $b_i = c_1$ for each $i$.

### 3.4 Axiomatic analysis

In this section, we analyze the CELmin rule from an axiomatic point of view and we compare it with the CEL, which is the rule that is most related to it. At the end of this section, there is a table that summarizes the axiomatic comparison of the axioms satisfied by both rules. Next, we propose some properties considered by the literature as a minimum requirement and some additional principles [5], with the aim of studying whether the CELmin satisfies them or not [6].

**Equal treatment of equals** considers that agents with equal claims must receive equal allocations. For each $(E, c) \in \mathcal{B}$, and each $i, j \in N$, such that $c_i = c_j$, then $\varphi_i(E, c) = \varphi_j(E, c)$.

**Anonymity** requires invariance under permutations of agents; the names of the agents should not matter. Denoting by $\Pi^N$ the class of bijections from $N$ into itself, the requirement is the following:

For each $(E, c) \in \mathcal{B}$, such that $\pi \in \Pi^N$, and each $i \in N$, then $\varphi_{\pi(i)}(E, c') = \varphi_i(E, c)$, where $c' \equiv (c_{\pi(j)})_{j \in N}$.

**Order preservation** considers that the order of the claims must be respected. If agent $i$'s claim is at least as large as agent $j$'s claim, the awards and losses allocated to agent $i$ must be at least as much as the ones allocated to agent $j$. For each $(E, c) \in \mathcal{B}$, and each $i, j \in N$, such that $c_i \geq c_j$, then $\varphi_i(E, c) \geq \varphi_j(E, c)$, and $c_i - \varphi_i(E, c) \geq c_j - \varphi_j(E, c)$.

[5] See Rose et al. (1998) for a better understanding and implications of the equity principles.

[6] For technical details about all properties we refer to Thomson (2019).

**Resource monotonicity** (Curiel et al., 1987) indicates that if the endowment increases, all agents should receive at least the amount of the endowment that was allocated to them before the increase. For each $(E, c) \in \mathcal{B}$ and each $E' \in \mathbb{R}_+$ such that $C > E' > E$, then $\varphi_i(E', c) \geq \varphi_i(E, c)$, for each $i \in N$.

**Super-modularity** (Dagan et al., 1997) requires that if the endowment increases, given two agents, the one with the greater claim should receive a greater portion of the increment than the other. For each $(E, c) \in \mathcal{B}$, all $E' \in \mathbb{R}_+$ and each $i, j \in N$ such that $C > E' > E$ and $c_i \geq c_j$, then $\varphi_i(E', c) - \varphi_i(E, c) \geq \varphi_j(E', c) - \varphi_j(E, c)$.

**Order preservation under claims variations** (Thomson, 2019) demands that if the claim of one agent decreases, given two other agents, the one with the greater claim receives more than the other. For each $k \in N$, each pair $(E, c)$ and $(E, c') \in \mathcal{B}$, with $c' = (c'_k, c_{-k})$ and $c'_k < c_k$ and each pair $i$ and $j \in N \setminus k$ with $c_i \leq c_j$, $\varphi_i(E, c') - \varphi_i(E, c) \leq \varphi_j(E, c') - \varphi_j(E, c)$[7].

**Composition down** requires that, if after distributing the endowment, the endowment decreases, two options are available: first, cancel the initial allocation and apply the rule for the revised endowment. Second, consider the agents' initial awards as their claims and apply the rule to allocate the revised endowment in this situation. Both ways should lead to the same award vector. For each $(E, c) \in B$, each $i \in N$, and each $0 \leq E' \leq E$, $\varphi_i(E', c) = \varphi_i(E', \varphi(E, c))$.

**Limited consistency** states that adding an agent with a zero claim does not affect the award of other agents [8]. For each $(E, c) \in \mathcal{B}$ and each $i \in N$, $\varphi_i(E, c) = \varphi_i(E, (0, c_1, ..., c_n))$.

In the next proposition, we show that the CELmin solution satisfies all the axioms mentioned above.

[7]Notice that $(c'_k, c_{-k})$ is the claims vector obtained from $c$ by replacing $c_k$ by $c'_k$.
[8]Clearly if $(E, (c_1, \ldots, c_n))$ is a claims problem with n agents, then $(E, (0, c_1, \ldots, c_n))$ is a claims problem with n+1 agents.

**Proposition 1** *The CELmin solution satisfies Equal treatment of equals, Anonymity, Order preservation, Resource monotonicity, Super modularity, Order preservation under claims variation, Composition down, and Limited consistency.*

**Proof.** For each $(E, c) \in \mathcal{B}$ , if $c_1 \geq \frac{E}{n}$, then CELmin equals EA rule. Otherwise, each agent receives $CELmin(E, c) = c_1 + CEL(E - nc_1, c - c^1)$, then since the CEL satisfies the properties it is straightforward that the CELmin satisfies Equal treatment of equals, Anonymity, and Order preservation.

Regarding Resource monotonicity, the only case to be considered is when is when $c_1 \geq \frac{E}{n}$ and $c_1 < \frac{E'}{n}$, then $CELmin(E, c) = \frac{E}{n}$ and $CELmin(E', c) = c_1 + CEL(E' - nc_1, c - c^1)$ which shows that the axiom is satisfied. Similarly, we can prove that the rule satisfies Super modularity.

To prove that the CELmin satisfies Composition down, we confine to the following condition: If $c_1 < \frac{E'}{n} \leq \frac{E}{n}$, then $CELmin(E, c) = c_1 + CEL(E - nc_1, c - c^1)$ and $CELmin(E', c) = c_1 + CEL(E' - nc_1, c - c^1)$. Based on the definition of Composition down, we have $c_1 + CEL(E' - nc_1, c - c^1) = c_1 + CEL(E' - nc_1, +CEL(E - nc_1, c - c^1))$.

To show that Order preservation under claims variations is fulfilled by the CELmin, we must distinguish three cases:

1. If $c_1 \geq c'_1 \geq \frac{E}{n}$, then, $CELmin(E, c) = CELmin(E, c') = \frac{E}{n}$.

2. If $c_1 \geq \frac{E}{n} > c'_1$, then, $CELmin(E, c) = (\frac{E}{n})$ and $CELmin(E, c') = c_1 + CEL(E - nc'_1, c - c^{1\prime})$.

   According to the definition of Order preservation under claims variations and the fact that the CEL satisfies the property we have: $c_1 + CEL_i(E - nc'_1, c - c^{1\prime}) - \frac{E}{n} \leq c_1 + CEL_j(E - nc'_1, c - c^{1\prime}) - \frac{E}{n}$.

3. If $c_1 < \frac{E}{n}$, then $CELmin(E, c) = c_1 + CEL(E - nc_1, c - c^1)$.

   (a) If $k = 1$, $c' = (c'_1, c_{-1}) = (c'_1, c_2, c_3, \dots)$, $c'_1 < c_1$. Then,

$c_1' + CEL_i(E - nc_1', c' - c^{1\prime}) - c_1 - CEL_i(E - nc_1, c - c^1) \leq c_1' + CEL_j(E - nc_1', c' - c^{1\prime}) - c_1 - CEL_j(E - nc_1, c - c^1)$.

(b) If $k \neq 1$, $c_1 = c_1' < \frac{E}{n}$ and $c' = (c_k', c_{-k}) = (c_1, \dots, c_k', \dots)$. Then, $c_1 + CEL_i(E - nc_1, c' - c^1) - c_1 - CEL_i(E - nc_1, c - c^1) \leq c_1 + CEL_j(E - nc_1, c' - c^1) - c_1 - CEL_j(E - nc_1, c - c^1)$, again since the CEL satisfies the property, it is satisfied.

Finally, note that Limited consistency follows directly from the definition of CELmin.

■

Although CELmin and CEL have shown similar behavior in axiomatic analysis so far, Composition up is satisfied by CEL but not fulfilled by CELmin. On the other hand, CELmin fulfills Min lower bound, and CEL does not.

**Composition up** demonstrates the opposite situation of Composition down in which after distributing the endowment, re-evaluation shows the endowment to increase. Again, two options are available: First, cancel the initial distribution and apply the rule for revised endowment. Second, the claims of agents are revised down by their initial gains. The rule divides the increment part of the endowment to revised claims. The result of both options should coincide. For each $(E, c) \in \mathcal{B}$, and each $E' > E$, $\varphi(E', c) = \varphi(E, c) + \varphi(E' - E, c - \varphi(E, c))$.

**Min lower bound** (Dominguez and Thomson, 2006) proposes that each agent should receive at least $1/n$ of the smallest claim truncated by the endowment. For each $(E, c) \in \mathcal{B}$, each $i \in N$, and each $E' > E$, $\varphi(E, c)_i \geq \frac{1}{n} \min\{\min\{c_j\}_{j \in N}, E\}$

It is straightforward to check that CELmin fulfills the Min lower bound since, by the definition of the rule, each agent receives either an equal division of the endowment or the smallest claim. The following example shows that the CELmin does not satisfy Composition up.

**Example 2** *Consider* $(E, c) = (30, (10, 20, 30))$*. Then,* $CELmin(E, c) = (10, 10, 10)$*. If the endowment increases to* $E' = 50$*, according to the definition of Composition up, the below equation should be obtained:*

$$CELmin(50, (10, 20, 30)) = CELmin(30, (10, 20, 30)) + CELmin(20(0, 10, 20)).$$

*But,* $CELmin(E', c) = (10, 15, 25)$ *and* $CELmin(E, c) + CELmin(20, (0, 10, 20)) = (10, 20, 20)$ *which does not coincide, therefore Composition Up is not satisfied.*

Next, we define some axioms that the CELmin does not satisfy and are not met by CEL either. To show that we refer to *Example 1*.

**Invariance under claims truncation** requires that the part of the claim of agent $i$ that exceeds the endowment should be ignored. Indeed, agent $i$ cannot ask for more than the available resource. For each $(E, c) \in B$, $\varphi(E, c) = \varphi(E, \min\{c_i, E\})$.

**Self-duality** requires the solution to recommend the same allocation when dividing gains and losses, where losses are defined as the difference between the sum of the claims and the estate. For each $(E, c) \in B$, and each $i \in N$, $\varphi_i(E, c) = c_i\varphi_i(L, c)$.

**Midpoint property** ensures to each agent half of her claim when the estate is equal to half of the aggregate claim. For each $(E, c) \in B$, and each $i \in N$, if $E = C/2$, then $\varphi_i(E, c) = c_i/2$.

**Reasonable lower bounds on awards** ensures that each individual receives at least the minimum of her claim and the endowment divided by the number of individuals.For each $(E, c) \in B$, and each $i \in N$, if $E = C/2$, then $\varphi_i(E, c) \geq \frac{\min\{c_i,E\}}{n}$.

| **Principles / Rules** | **CELmin** | **CEL** |
|---|---|---|
| Equal treatment of equals | Yes | Yes |
| Anonymity | Yes | Yes |
| Order preservation | Yes | Yes |
| Resource monotonicity | Yes | Yes |
| Super modularity | Yes | Yes |
| Order preservation under claims variation | Yes | Yes |
| Composition down | Yes | Yes |
| Limited consistency | Yes | Yes |
| Min lower bound | Yes | No |
| Composition up | No | Yes |
| Invariance under claims truncation | No | No |
| Self-duality | No | No |
| Midpoint property | No | No |
| Reasonable lower bounds on awards | No | No |

Table 3.1: The table shows the axioms satisfied by the rules. The two columns correspond to the CELmin and the CEL rules, and each row shows the proposed axioms.

### 3.5 Distribution of the European regional development fund (ERDF)

The ERDF budget that the European Council and Parliament determined for the 2014-2020 programming period is approximately 182,150 million euros, which corresponds to almost 44% of the total budget. This budget is intended for the second level of the EU nomenclature of territorial units for statistics (NUTS2) which involves regions with populations between 800,000 and 3,000.000 inhabitants. According to this division, the regional eligibility for the ERDF is calculated by taking into account the regional GDP per capita. Regions in NUTS level 2 are split and classified into three different categories according to their GDP per capita measured in purchasing power standards, as follows:

**More developed regions (R1):** with GDP per capita above 100% of the average GDP per capita of the EU-27 .

**Transition regions (R2):** with GDP per capita between 75% and 100% of the average GDP per capita of the EU-27.

**Less developed regions (R3):** with GDP per capita less than 75% of the average GDP per capita of the EU-27.

According to this classification, there were 265 regions in NUTS 2 for the 2014-2020 programming period. This number declines to 47 if the regions of the same category are considered together (Solís-Baltodano et al., 2022). From the claims problems perspective, these 47 regions form the claimants who have a claim on the ERDF budget. We use the same claims that Solís-Baltodano et al. (2022) offer in their study. In their method, each agent claims a fixed amount which is equal for all regions, the allocation per inhabitant obtained for the region with the highest GDP per inhabitant (it can be interpreted as a minimal allocation), plus an amount that depends on the gap between the specific region GDP per capita and the highest GDP per capita. The attribute of this method is that the less developed regions claim more than the others. The claims of the regions are depicted in table 4.2. Moreover, the table illustrates a comparison between the regional allocation of our proposed rule and the rules that have been already studied. The absolute allocation of ERDF to each country is provided in table 3.2.

The CEA rule distributes the fund as equally as possible to all regions without taking into account the measure of their demands. In contrast, the CEL rule imposes equal losses to all regions. Therefore, it helps regions with larger claims, which are regions in R3 to obtain more ERDF. On the other hand, CEL causes some more developed regions (R1) to receive nothing. Notice that, the rest of the studied rules, P, $\alpha$-min, and CELmin stand somewhere between CEA and CEL. In particular, as table 3.3 illustrates, the total ERDF that CELmin allocates to R3 regions is equal to the allocation of the CEL with a slight difference. Nonetheless, CELmin supports some regions in R1 that are ignored by the CEL (e.g. Czech R1). This is the main objective of proposing the CELmin; By implementing the CELmin, every region receives the minimal right that the rule guarantees for all regions.

| Country | Current | P | CEA | CEL | α-min | CELmin |
|---|---|---|---|---|---|---|
| Austria | 536.26 (0.29%) | 2,990.30 (1.64%) | 3,605.90 (1.98%) | 1,664.43 (0.91%) | 3,023.98 (1.66%) | 1,656.43 (0.91%) |
| Belgium | 953.01 (0.52%) | 4,153.60 (2.28%) | 4,658.91 (2.56%) | 3,047.36 (1.67%) | 4,181.13 (2.30%) | 3,037.02 (1.67%) |
| Bulgaria | 3,567.67 (1.96%) | 3,714.38 (2.04%) | 2,881.54 (1.58%) | 5,426.21 (2.98%) | 3,668.28 (2.01%) | 5,419.82 (2.98%) |
| Croatia | 4,321.50 (2.37)% | 2,130.59 (1.17%) | 1,678.03 (0.92%) | 3,059.63 (1.68%) | 2,105.53 (1.16%) | 3,055.90 (1.68%) |
| Cyprus | 299.90 (0.16%) | 373.82 (0.21%) | 353.24 (0.19%) | 413.15 (0.23%) | 372.66 (0.20%) | 412.37 (0.23%) |
| Czech | 11,940.69 (6.56%) | 4,556.74 (2.50%) | 4,336.62 (2.38%) | 5,352.62 (2.94%) | 4,544.31 (2.49%) | 5,386.12 (2.96%) |
| Denmark | 206.62 (0.11%) | 1,968.98 (1.08%) | 2,362.93 (1.30%) | 1,119.93 (0.61%) | 1,990.53 (1.09%) | 1,114.69 (0.61%) |
| Estonia | 1,856.56 (1.02%) | 595.11 (0.33%) | 539.17 (0.30%) | 706.46 (0.39%) | 591.99 (0.33%) | 705.27 (0.39%) |
| Finland | 486.64 (0.27%) | 2,092.38 (1.15%) | 2,253.37 (1.24%) | 1,731.82 (0.95%) | 2,101.10 (1.15%) | 1,726.82 (0.95%) |
| France | 7,978.14 (4.38%) | 26,643.93 (14.6%) | 27,395.44 (15.0%) | 24,782.70 (13.6%) | 26,683.47 (14.7%) | 24,721.92 (13.57%) |
| Germany | 10,773.84 (5.91%) | 29,153.27 (16.0%) | 33,839.48 (18.6%) | 18,992.84 (10.4%) | 29,409.23 (16.2%) | 18,917.76 (10.39%) |
| Greece | 8,622.33 (4.73%) | 5,203.71 (2.86%) | 4,390.21 (2.41%) | 6,859.19 (3.77%) | 5,158.57 (2.83%) | 6,849.45 (3.76%) |
| Hungary | 10,756.78 (5.91%) | 4,675.35 (2.57%) | 3,996.69 (2.19%) | 6,052.90 (3.32%) | 4,637.67 (2.55%) | 6,044.03 (3.32%) |
| Ireland | 410.78 (0.23%) | 902.04 (0.50%) | 1,974.31 (1.08%) | 0.00 (0.00%) | 961.01 (0.53%) | 156.50 (0.09%) |
| Italy | 21,507.18 (11.8%) | 25,193.38 (13.8%) | 24,721.44 (13.6%) | 25,919.76 (14.2%) | 25,165.68 (13.8%) | 25,864.92 (14.20%) |
| Latvia | 2,401.25 (1.32%) | 933.67 (0.51%) | 790.63 (0.43%) | 1,224.56 (0.67%) | 925.74 (0.51%) | 1,222.81 (0.67%) |
| Lithuania | 3,501.41 (1.92%) | 1,276.29 (0.70%) | 1,148.07 (0.63%) | 1,532.40 (0.84%) | 1,269.15 (0.70%) | 1,529.85 (0.84%) |
| Luxembourg | 19.50 (0.01%) | 6.31 (0.00%) | 19.50 (0.01%) | 0.00 (0.00%) | 19.50 (0.01%) | 19.50 (0.01%) |
| Malta | 384.35 (0.21%) | 195.76 (0.11%) | 194.43 (0.11%) | 196.49 (0.11%) | 195.67 (0.11%) | 196.06 (0.11%) |
| Netherlands | 510.28 (0.28%) | 5,750.65 (3.16%) | 7,022.38 (3.86%) | 3,016.17 (1.66%) | 5,820.26 (3.20%) | 3,000.59 (1.65%) |
| Poland | 40,213.87 (22.1%) | 18,186.09 (9.98%) | 15,552.10 (8.52%) | 23,654.01 (13.0%) | 18,038.20 (9.90%) | 23,720.33 (13.02%) |
| Portugal | 10,661.23 (5.85%) | 4,778.73 (2.62%) | 4,206.22 (2.31%) | 5,932.01 (3.26%) | 4,746.89 (2.61%) | 5,922.68 (3.25%) |
| Romania | 10,726.08 (5.89%) | 9,586.80 (5.26%) | 7,983.86 (4.38%) | 12,855.80 (7.06%) | 9,497.91 (5.21%) | 12,896.23 (7.08%) |
| Slovakia | 7,291.46 (4.00%) | 2,569.88 (1.41%) | 2,224.75 (1.22%) | 3,367.28 (1.85%) | 2,550.70 (1.40%) | 3,384.02 (1.86%) |
| Slovenia | 1,416.69 (0.78%) | 906.04 (0.50%) | 844.79 (0.46%) | 1,025.26 (0.56%) | 902.61 (0.50%) | 1,023.38 (0.56%) |
| Spain | 20,079.13 (11.0%) | 20,013.10 (11.0%) | 19,070.57 (10.5%) | 21,783.55 (12.0%) | 19,959.85 (11.0%) | 21,741.24 (11.94%) |
| Sweden | 727.83 (0.40%) | 3,600.08 (1.98%) | 4,136.42 (2.27%) | 2,432.44 (1.34%) | 3,629.36 (1.99%) | 2,425.27 (1.33%) |

Table 3.2: The current allocations of ERDF and proposed allocation of different rules to each country (numbers are in million euros). The percentage of the funds allocated to each country is shown in the brackets.

| Country | Region | Claim | Current | P | CEA | CEL | α-min | CELmin |
|---|---|---|---|---|---|---|---|---|
| Austria | R1 | 1,038 | 57.40 | 335.79 | 408.73 | 178.90 | 339.78 | 178.00 |
| Austria | R2 | 1,332 | 160.60 | 430.93 | 408.73 | 473.05 | 429.67 | 472.15 |
| Belgium | R1 | 1,009 | 36.30 | 326.20 | 408.73 | 149.25 | 330.72 | 148.35 |
| Belgium | R2 | 1,425 | 203.10 | 460.89 | 408.73 | 565.70 | 457.99 | 564.80 |
| Bulgaria | R3 | 1,629 | 506.00 | 526.86 | 408.73 | 769.67 | 520.32 | 768.76 |
| Croatia | R3 | 1,605 | 1,052.60 | 518.96 | 408.73 | 745.25 | 512.86 | 744.35 |
| Cyprus | R1 | 1,337 | 347.00 | 432.54 | 408.73 | 478.05 | 431.20 | 477.15 |
| Czech | R1 | 565 | 244.80 | 182.70 | 408.73 | 0.00 | 195.13 | 32.40 |
| Czech | R3 | 1,434 | 1,247.80 | 463.77 | 408.73 | 574.59 | 460.70 | 573.68 |
| Denmark | R1 | 1,004 | 33.30 | 324.65 | 408.73 | 144.45 | 329.26 | 143.55 |
| Denmark | R2 | 1,345 | 50.30 | 434.97 | 408.73 | 485.55 | 433.49 | 484.65 |
| Estonia | R2 | 1,395 | 1,407.40 | 451.14 | 408.73 | 535.55 | 448.77 | 534.65 |
| Finland | R1 | 1,173 | 88.20 | 379.53 | 408.73 | 314.13 | 381.11 | 313.22 |
| France | R1 | 1,142 | 67.60 | 369.35 | 408.73 | 282.68 | 371.50 | 281.77 |
| France | R2 | 1,418 | 145.30 | 458.69 | 408.73 | 558.90 | 455.91 | 558.00 |
| France | R3 | 1,530 | 1,003.50 | 494.80 | 408.73 | 670.55 | 490.03 | 669.65 |
| Germany | R1 | 1,039 | 61.40 | 335.99 | 408.73 | 179.51 | 339.97 | 178.61 |
| Germany | R2 | 1,351 | 491.40 | 436.89 | 408.73 | 491.50 | 435.31 | 490.59 |
| Greece | R1 | 1,327 | 419.00 | 429.03 | 408.73 | 467.20 | 427.89 | 466.29 |
| Greece | R2 | 1,573 | 795.60 | 508.58 | 408.73 | 713.16 | 503.05 | 712.26 |
| Greece | R3 | 1,622 | 1,181.90 | 524.53 | 408.73 | 762.48 | 518.12 | 761.57 |
| Hungary | R1 | 1,202 | 85.60 | 388.88 | 408.73 | 343.05 | 389.95 | 342.15 |
| Hungary | R3 | 1,601 | 1,551.50 | 517.85 | 408.73 | 741.83 | 511.81 | 740.92 |
| Ireland | R1 | 577 | 85.00 | 186.74 | 408.73 | 0.00 | 198.95 | 32.40 |
| Italy | R1 | 1,158 | 91.20 | 374.43 | 408.73 | 298.36 | 376.29 | 297.46 |
| Italy | R2 | 1,440 | 277.00 | 465.85 | 408.73 | 581.04 | 462.67 | 580.13 |
| Italy | R3 | 1,556 | 973.90 | 503.39 | 408.73 | 697.12 | 498.15 | 696.21 |

Table 3.3: The current allocations of ERDF and proposed allocation of different rules to each region, per capita (numbers are in million euros).

| **Country** | **Region** | **Claim** | **Current** | **P** | **CEA** | **CEL** | **$\alpha$-min** | **CELmin** |
|---|---|---|---|---|---|---|---|---|
| Latvia | R3 | 1,492 | 1,241.40 | 482.67 | 408.73 | 633.05 | 478.57 | 632.15 |
| Lithuania | R3 | 1,405 | 1,246.50 | 454.37 | 408.73 | 545.55 | 451.83 | 544.65 |
| Luxembourg | R1 | 32.4 | 32.40 | 10.48 | 32.40 | 0 | 32.40 | 32.40 |
| Malta | R2 | 1,272 | 808.00 | 411.52 | 408.73 | 413.05 | 411.34 | 412.15 |
| Netherlands | R1 | 1,035 | 29.70 | 335.79 | 408.73 | 175.55 | 338.76 | 174.65 |
| Poland | R1 | 840 | 880.50 | 271.64 | 408.73 | 0.00 | 279.17 | 32.40 |
| Poland | R3 | 1,536 | 1074.40 | 496.81 | 408.73 | 676.76 | 491.93 | 675.86 |
| Portugal | R1 | 1,276 | 274.80 | 412.61 | 408.73 | 416.41 | 412.36 | 415.50 |
| Portugal | R2 | 1,370 | 514.70 | 443.05 | 408.73 | 510.55 | 441.13 | 509.65 |
| Portugal | R3 | 1,513 | 1,417.40 | 489.37 | 408.73 | 653.77 | 484.90 | 652.86 |
| Romania | R1 | 867 | 268.40 | 280.54 | 408.73 | 8.05 | 287.57 | 32.40 |
| Romania | R3 | 1,604 | 586.60 | 518.88 | 408.73 | 745.00 | 512.78 | 744.09 |
| Slovakia | R1 | 707 | 402.60 | 228.79 | 408.73 | 0.00 | 238.68 | 32.40 |
| Slovakia | R3 | 1,562 | 1,466.80 | 505.18 | 408.73 | 702.65 | 499.84 | 701.74 |
| Slovenia | R1 | 1,225 | 467.40 | 396.16 | 408.73 | 365.55 | 396.82 | 364.65 |
| Slovenia | R3 | 1,472 | 928.60 | 476.21 | 408.73 | 613.05 | 472.46 | 612.15 |
| Spain | R1 | 1.248 | 253.60 | 403.76 | 408.73 | 389.04 | 404.00 | 388.14 |
| Spain | R2 | 1,485 | 743.10 | 480.1 | 408.73 | 625.21 | 476.17 | 624.30 |
| Spain | R3 | 1,512 | 1,473.00 | 489.14 | 408.73 | 653.05 | 484.90 | 652.15 |
| Sweden | R1 | 1,100 | 71.90 | 355.77 | 408.73 | 240.55 | 358.62 | 239.65 |

Table 3.3: The current allocations of ERDF and proposed allocation of different rules to each region, per capita (numbers are in million euros).

## 3.6 Convergence

It is noteworthy to re-emphasize that the objective of the EU for allocating the ERDF is to elevate the growth rate of less developed regions to achieve convergence in the EU territory. Supporting the less developed regions requires detecting the division rules that distribute the ERDF in a way that is more favorable for larger claimants.

Lorenz dominance is an appropriate criterion that explores how the rules treat smaller claimants relative to larger claimants. A Lorenz dominant rule is an equitable rule which is favorable for smaller claimants.

Let $\mathbb{R}^n_+$ be the set of positive n-dimensional vectors $x = (x_1, x_2, \ldots, x_n)$ ordered from small to large, i.e., $0 < x_1 \leq x_2 \leq \ldots \leq x_n$. Let $x$ and $y$ be in $\mathbb{R}^n_+$. We say that $x$ Lorenz dominates $y$, $x \succ_L y$, if for each $k = 1, 2, \ldots, n-1$: $x_1 + x_2 + \cdots + x_k \geq y_1 + y_2 + \ldots + y_k$ and $x_1 + x_2 + \ldots + x_n = y_1 + y_2 + \ldots + y_n$. If $x$ Lorenz dominates $y$ and $x \neq y$, then at least one of these $n-1$ inequalities is a strict inequality. The following definition extends Lorenz dominance to claims problem situations.

**Definition 2** *Given two solutions $\varphi$ and $\psi$ it is said that $\varphi$ Lorenz dominates $\psi$, $\varphi \succ_L \psi$, if for any claims problem $(E, c)$ the vector $\varphi(E, c)$ Lorenz dominates $\psi(E, c)$. Indeed, it states that $\varphi(E, c)$ is more equitable and more supportive of smaller claims.*

Bosmans and Lauwers (2011) proved that the $CEA$ is the most equitable rule, since this rule Lorenz dominates all other rules. Their comparison shows that the $CEL$ is the most inequitable rule: $CEA \succ_L \alpha_{\min} \succ_L P \succ_L CEL$.

The following result shows the Lorenz relationships between our solution and the main ones.

**Proposition 2**

*(a) $CEA \succ_L \alpha_{\min} \succ_L CELmin \succ_L CEL$.*

*(b) There is no Lorenz dominance relation between CELmin and P.*

**Proof.** Part (a) is easily obtained by definition and previous results. By definition, the CELmin rule has an egalitarian part that makes the rule more equitable than the CEL.

Part (b) is directly obtained from the case analysis. If $c_1$ is unsustainable, CELmin corresponds to CEA. Therefore, the rule Lorenz dominates P. If $c_1$ is sustainable, the result of table 4.2 shows that P Lorenz dominates CELmin. ■

Note that CEA Lorenz dominates all the rules since it distributes the endowment as equally as possible. However, in our context, where the goal is convergence, those regions with larger claims should receive larger awards. So, the best solution to ensure convergence is just the opposite one, the Lorenz dominated rule, CEL.

Another method that allows us to examine whether the rules promote convergence is to study the effect of the rules' allocations $x \geq 0$ on the regions' GDP per capita. It is expected that after allocating the ERDF to regions, their GDP per capita increases to a new measure.

Solís-Baltodano et al. (2022) introduce *divergence ratio* as a criterion that evaluates which division rule can achieve convergence in the EU faster than the others. They assume that the regions' new GDP per capita ($\widehat{GDP^h}$) equals their initial GDP per capita ($GDP^h$) plus the allocation amount ($x$). The divergence ratio is as follows:

$$d_{(\alpha,\beta)} = 1 - \frac{GDP^h_\alpha}{GDP^h_\beta}$$

where $GDP^h_\alpha$ is the GDP per capita of the less developed region $\alpha$ and $GDP^h_\beta$ is the GDP per capita of the most developed region $\beta$. The divergence ratio is always greater than 0 and the amount close to 0 reflects convergence.

To compute the divergence ratio of the rules, we consider the GDP per capita of the least developed region (Bulgaria R3) and the GDP per capita of the most developed region (Luxembourg R1). Then, the initial divergence ratio before any allocation is 0.8054. To calculate the divergence ratio after the ERDF allocation, we add the assignment of each rule to the GDP per capita of the aforementioned regions and compute the ratio for each rule. The divergence ratio of CEA is 0.8003. It is expected that this rule has the largest ratio. Since this rule distributes the budget in the most egalitarian manner possible, maintaining the existing differences before the budget was allocated. On the contrary, the CEL rule provides the less egalitarian distribution of the fund. The ratio for CEL is 0.7957. Therefore, CEL rules may be most appropriate to achieve the convergence goal of the ERDF. However, this rule does not allocate the fund to some regions, which makes it difficult to implement in real life. The ratio for the $\alpha$-min and P are 0.7989 and 0.7987 respectively. The divergence ratio for the CELmin is about 0.7957 which is almost equal to the CEL's ratio. It is significant to consider that, since all the rules satisfy order preservation, they increase the convergence in the EU, since they allocate more assignments to poor regions. However, the results of the divergence ratio confirm that CELmin is the most appropriate rule to meet the convergence target.

### 3.7 Conclusions

We conclude this study by highlighting the findings. The aim of the European regional development fund (ERDF) allocation is to help less developed regions in the European Union (EU) to achieve a welfare level like that experienced by developed regions. Therefore, it can be inferred that the fair allocation of ERDF lies in the unequal division of the fund and assigning more to less developed regions.

The claims problem approach contributes to allocate this fund as unequally as possible. The pillar of the claims problem approach is the regions' demand for money needed to boost their development level. These demands are estimated in such a way that the well-developed regions require smaller funds (Solís-Baltodano et al., 2022). First, the four division rules are applied in this study and the results prove that they are not able to serve the objective of the ERDF allocation. The Constrained equal awards (CEA) assigns an equal portion of the ERDF (408.073 million euros) to all regions. The only exception is Luxembourg R1 which receives her claim due to the claims-boundedness assumption and the sustainability properties. In addition, the result of the Lorenz dominance illustrates that CEA is the most equitable rule between the rules we apply. Thus, CEA is not an appropriate solution for the ERDF allocation. The Proportional (P) rule considers the claims of the regions in the fund allocation process. Therefore, the portion it assigns to less developed regions is larger than what the more developed regions receive. The $\alpha$-minimal ($\alpha$-min) has two phases, egalitarian allocation and proportional allocation, which makes it more equitable than Proportional and not appropriate for the ERDF allocation. The Constrained equal losses (CEL) completely neglects the well-developed regions (e.g. Czech R1 or Ireland R1) and supports the less developed ones. Although the degree of inequality of CEL (which is more than P) suggests it would be the best choice for the ERDF fund, zero allocation to developed regions is a noticeable obstacle to use this rule in a real situation.

To adjust this problem and simultaneously support larger claims, we propose CELmin, which guarantees a fixed allocation right to all agents by reaching a compromise between Equal awards (EA) allocation and CEL. The results in table 3.3 confirm that CELmin supports less developed regions, compared with CEA and $\alpha$-min. Moreover, the Lorenz dominance of CELmin is less than these rules. To compare CELmin with P, we compute the divergence ratio for them. The results depict that CELmin with a ratio equal to 0.7957 is able to reach convergence in the EU much faster than P (with a ratio equal to 0.7987).

# CHAPTER 4

# THE CEL-FAMILY OF RULES FOR THE CLAIMS PROBLEMS

## 4.1 Introduction

If a firm goes bankrupt, how should its liquidation be divided among stakeholders? How can fixed taxes be assigned to individuals with different incomes? These kinds of situations can be addressed in the so-called claims problems. In general, a claims problem is defined as a situation in which an infinitely dividable endowment is allocated to a group of agents while the agents' aggregate claim exceeds the endowment (to deepen their understandings, readers are referred to O'Neill (1982), Aumann and Maschler (1985) and a comprehensive survey was conducted by Thomson (2019)).[9]

Solving these problems requires the proposal of some allocation methods, called rules, which should be based on some appealing or commonly accepted principles, called properties. Within the literature, we can find a large number of rules, and properties, trying to combine fairness principles, with equity, solidarity, effort, and rights. For instance, the Babylonian Talmud already proposed different ways of allocating the resources, such as, proportionally to the claims, or an equal division (Aumann and Maschler, 1985). Maimonides (1204) proposed an equal distribution of the endowment, but depending on if we are distributing awards (Constrained equal awards rule), or the part of the resources that are not obtained, the average loss (Constrained equal losses rule). Indeed, these two rules may be considered as two extreme and opposite ways of allocating resources. In this sense, Alcalde and Peris

[9]Chapter 4 of this dissertation is based on the following paper currently under review:
Salekpay, F., Vilella, C., Giménez-Gómez, J. M. The CEL-family of rules for the claims problems.

(2022) propose a convex combination of these rules to analyze the whole spectrum of possible allocation between them.

The idea of joining all the possible allocations satisfying some properties into a family is not so new. For instance, Young (1987) introduces the parametric family, which includes the Proportional, the Constrained equal awards (Maimonides, 1204), the Constrained equal losses (Maimonides, 1204), and the Talmud (Aumann and Maschler, 1985) rules. Thomson (2003) defines the ICI family which includes the Minimal overlap rule (O'Neill, 1982) and, Moreno-Ternero and Villar (2006) propose the TAL-family, which is an extension of the Talmud rule.

On the other hand, it must be noted that some claims rules may not be considered admissible depending on the context (Alcalde and Peris, 2022). Indeed, the Constrained equal losses rule focuses on loss allocation (i.e. the subtraction of aggregate claims from the endowment) and it allocates the average loss to agents. It is noteworthy that the main feature of this rule is its extreme behavior with the so-called 'residual claims'. The residual claims are so small that their elimination from the allocation process does not change the situation of the claims problem. The Independence of residual claims is an axiom states that residual claims should not be allocated anything (Herrero and Villar, 2002). So, some agents may receive zero awards, and it does not seem like a fair rule in an award-sharing situation. However, in tax sharing, if the claims of firms represent their incomes, the Independence of residual claims means that the firms with small incomes are exempted from paying tax.

In order to guarantee that each agent has access to a minimum level of resources, various lower bounds have been proposed in the literature (Thomson, 2019), with minimal rights being a prominent one. The concept of minimal rights was introduced by Tijs et al. (1981) in the context of cooperative games, Moulin (2000) for rationing problems, and Herrero and Villar (2002) for claims problems, and it states that a claimant should receive at least what is left when all the other claimants are completely satisfied in their claims.

The current paper follows this vein considering two key concepts: solidarity and contribution (Giménez-Gómez and Peris, 2014; Giménez-Gómez and Osorio, 2015). Particularly, we combine the concept of sustainability (Herrero and Villar, 2002) with the amount required by the claimants. In doing so, our starting point is the CELmin rule (Salekpay et al., 2023). CELmin is established to keep a balance between the Independence of residual claim and its dual property, Sustainability. Agent $i$'s claim is sustainable if it is so small that when we replace the claims of other agents with agent $i$'s claim, there is no claims problem. The Sustainability property states that such claims should be fully satisfied. If the endowment is sufficient, CELmin guarantees a minimal right equal to the smallest claim ($c_1$) to all agents and then, applies the Constrained equal losses to divide the remaining endowment. If the endowment is not sufficient, all agents receive the endowment average. Note that this way of allocating the resources may fully honor the smallest claimant, without considering the rest of the claims. However, in certain contexts, this approach may not be suitable or permissible.

For instance, Solís-Baltodano et al. (2022) study the allocation of European regional development fund to the European Union member states, which is a financial aid that aims to boost the economies of the poor regions in the European Union. In their study, each region's claim indicates the degree to which they need money to improve their economy. The results depict that Constrained equal losses supports poor regions more than other rules, so the convergence among regions increases. However, some regions with smaller claims (which represent the more developed regions) receive no resources. To address the problem of zero allocation to more developed regions while continuing to support less developed regions, Salekpay et al. (2023) proposes the use of the CELmin rule, but it recommends fully honoring regions with the smallest claims, i.e, the richest regions.

In this paper, we introduce a family of rules called CEL-family which is the extension of CELmin rule. In CEL-family, we study all the possible combinations of the egalitarian division of the resources and the Constrained equal losses rule. For any given value of $\theta \in [0, 1]$, the rules in CEL-family assign the endowment such that each agent receives at least $\theta$ average of the endowment (constrained on the smallest claim to ensure no agent receives more than she claims). The rules of this family serve the concept of unequal allocation and allow us to select either the egalitarian division of resources or of losses, depending on the context; So, it includes both the Constrained equal losses rule if $\theta = 0$ and the Constrained equal awards if $\theta = 1$.

Once we define the family of rules, we provide an axiomatic analysis of the main properties for the sake of comparison between the main rules. In doing so, we consider basic properties, besides some solidarity and invariance principles. Finally, we apply the CEL-family to the European regional development fund problem, studying the combination of the convergence and solidarity principles. As the data depicts, if our main goal is to ensure a faster convergence among regions, the Constrained equal losses should be implemented ($\theta = 0$). However, since each region should receive a strictly positive amount of resources, the larger the $\theta$, the larger the minimum amount received by the richest regions.

The rest of the chapter is organized as follows. Section 4.2 provides the definition of the claims problems and the rules. Sections 4.3 and 4.4 define the CEL-family and introduce the axiomatic analysis, respectively. Section 4.5 applies this family to the distribution of the European regional development fund. Finally, Section 4.6 provides some final remarks.

## 4.2 Preliminary definitions

Consider a set of agents $N = \{1, 2, ..., n\}$ and an amount $E \in \mathbb{R}_+$ of an infinitely divisible resource, the **endowment**, that has to be allocated among them. Each agent has a **claim**, $c_i \in \mathbb{R}_+$ on it. Let $c \equiv (c_i)_{i \in N}$ be the claims vector. A **claims problem** (O'Neill, 1982) is a pair $(E, c)$ with $C = \sum_{i \in N} c_i \geq E > 0$ and $\mathcal{B}$ is the set of all claims problems. Without loss of generality, we assume that agents are ordered according to their claims so that $c_1 \leq c_2 \leq \ldots \leq c_n$.

A **rule** is a single-valued function $\varphi : \mathcal{B} \to \mathbb{R}^n_+$ such that, $0 \leq \varphi_i(E, c) \leq c_i$ for all $i \in N$ (**non-negativity** and **claims-boundedness**) and $\sum_{i \in N} \varphi_i(E, c) = E$ (**efficiency**).

We provide the definitions of rules used throughout the paper. Note that we also mention the rules included in the CEL-family: the Equal awards division, the Constrained equal awards, the Constrained equal losses and the CELmin, which is a combination of the previous ones.

The **Proportional (P)** rule divides the endowment proportionally according to the agents' claims. For each $(E, c) \in \mathcal{B}$ and each $i \in N$, $P_i(E, c) = \lambda c_i$, where $\lambda = \dfrac{E}{\sum\limits_{i \in N} c_i}$.

The **Constrained equal awards (CEA)** rule assigns the endowment equally such that no agent receives more than her claim. For each $(E, c) \in \mathcal{B}$ and each $i \in N$, $CEA_i(E, c) \equiv \min\{c_i, \mu\}$, where $\mu$ is chosen so that $\sum\limits_{i \in N} \min\{c_i, \mu\} = E$.

Note that the CEA solution is based on the **Equal Awards division** $(EA)$. This method assigns the endowment equally among all members, i.e., for each $(E, c) \in \mathcal{B}$ and each $i \in N$, $EA_i(E, c) = \frac{E}{n}$. However, it is easy to see that in some situations with the equal distribution an agent may receive more than her claim, violating the claim-boundedness condition of a claim rule.

The **Constrained equal losses (CEL)** rule allocates the difference between aggregate claims and the endowment (i.e. losses) equally to each agent, such that no agent receives a negative amount. For each $(E, c) \in \mathcal{B}$ and each $i \in N$, $CEL_i(E, c) \equiv \max\{0, c_i - \mu\}$, where $\mu$ is such that $\sum\limits_{i \in N} \max\{0, c_i - \mu\} = E$.

The **$\alpha$-minimal($\alpha$-min)** rule is a compromise of the EA and P. If the endowment is sufficient, the rule guarantees a minimal right equal to the smallest claim to all agents and distributes the remaining endowment proportionally to the agents' revised claims. If the endowment is not enough, then it is divided equally among the agents. For each $(E, c) \in \mathcal{B}$ and each $i \in N$, if $c_1 \geq \frac{E}{n}$ then $\alpha - min_i(E, c) = \frac{E}{n}$ and if $c_1 < \frac{E}{n}$ then $\alpha - min_i(E, c) = c_1 + P(E - nc_1, c - c_1)$.

The **CELmin** rule is a compromise between the EA and the CEL. It assigns a minimal positive right equal to the smallest claim $c_1$ to each agent and, distributes the remaining endowment $E' = E - nc_1$ by applying CEL with respect to agents' remaining claims $c_i' = c_i - c_1$. If the endowment is not enough to assign the minimal right to all agents, the rule allocates an equal division of the endowment to all agents. For each $(E, c) \in \mathcal{B}$ with $c_i > 0$ and each $i \in N$,

$$CELmin(E, c) = \begin{cases} (E/n)\mathbf{1} & \text{if } c_1 \geq E/n \\ \mathbf{c}^1 + CEL(E - nc_1, c - \mathbf{c}^1) & \text{otherwise} \end{cases}$$

where $\mathbf{c}^1 = (c_1, \ldots, c_1)_{1\times n}$ and $\mathbf{1} = (1, \ldots, 1)_{1\times n}$.

In the event of some claims being equal to zero, $c_1 = c_2 = \ldots c_k = 0$, $c_j > 0$, for each $j > k$, we extend the solution in a consistent manner:

$$CELmin(E, c) = (\mathbf{0}, CELmin(E, \bar{c}))$$

where $\mathbf{0} = (0, \ldots, 0)_{1\times k}$ and $\bar{c} = (c_{k+1}, \ldots, c_n)$.

The next example shows how the previous rules work.

**Example 1** *Let* $(E, c) = (600, (100, 200, 300, 400))$.

*The* $EA(E, c) = (150, 150, 150, 150)$, *notice that the first agent receives more than her claim.*

*The* $CEA(E, c) = (100, 166.66, 166.66, 166.66)$, *here the first agent is fully compensated and the remaining endowment is divided equally among the rest of the agents.*

*The* $CEL(E, c) = (0, 100, 200, 300)$, *divides the losses (*$L = 1000 - 600$*) equally among the agents such that each agent receives the subtraction of her claim and the equal portion of the losses.*

*The* $CELmin(E, c) = (100, 100, 150, 250)$, *allocates a minimal right to all agents equal the* $c_1 = 100$, *then the claims of the rest of the agents are revised down by the minimal right,* *and the CEL is applied to allocate the remaining endowment.*

## 4.3 The CEL-family rules

The main reason why the CELmin rule is introduced by Salekpay et al. (2023) is to adjust the situations in which CEL allocates nothing to smaller claims. The allocation of CELmin depends on the magnitude of average endowment $E/n$ with respect to agent one's claim $c_1$. If the average endowment is less than agent one's claim, all the agents receive an identical allocation equal to the average endowment and the rule corresponds to CEA. If the average endowment is greater than agent one's claim, the agents receive at least agent one's claim. In other words, what CELmin does when the $c_1 < \frac{E}{n}$ is to fully satisfy $c_1$ in an award allocation.

As mentioned by Salekpay et al. (2023), this rule is designed to show the highest degree of inequality in a resource allocation, while a non-zero allocation is guaranteed to all agents. Although the rule assigns the main part of the endowment to larger claims, it also protects smaller claims by assigning a minimal right to them. However, what is observable in CELmin allocation is that the smallest claim $c_1$ never enters the process of loss allocation which is performed by applying CEL.

Here we present a family of rules called **CEL-family** that generalizes the idea of the CELmin rule. The strength of the CEL-family lies in the different levels of inequality that a rule can impose on the agents. Despite the fact that $c_1$ is less than or greater than $\frac{E}{n}$, the rules in the CEL-family allocate a minimal right smaller than the agents' claims and enter them into the second phase of the allocation which is loss allocation through the application of CEL. The rules in the CEL-family are differentiated with respect to the factor $\frac{\theta E}{n}$. For any value of $\theta \in [0, 1]$, if $\frac{\theta E}{n}$ is less than or equal to $c_1$, the agents are allocated at least $\frac{\theta E}{n}$ and, if $\frac{\theta E}{n}$ is greater than $c_1$, no agent receives less than $c_1$. Formally, the following holds.

The **CEL-family** $(CEL^\theta)$ is defined as all rules such that for some $\theta \in [0, 1]$, for all

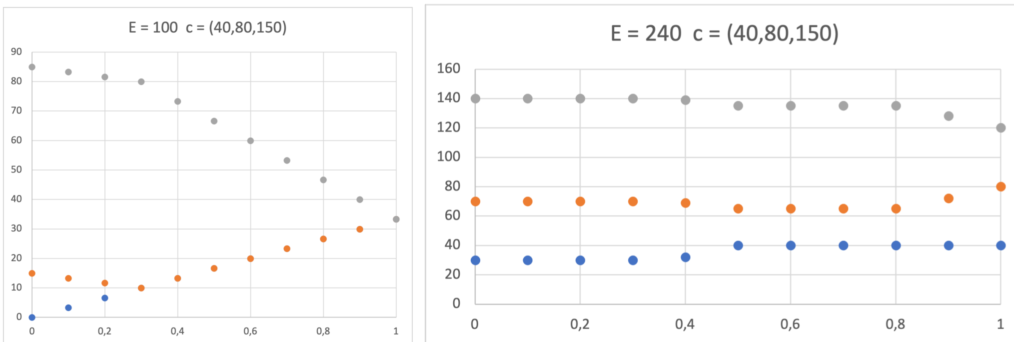


Figure 4.1: CEL-family in the three-claimant case. The blue, orange, and grey dots lines depict the allocation (vertical axis) of the endowment for different values of $\theta$ (horizontal axis) for claimants one, two, and three, respectively. Specifically, the left-hand side shows a situation where CEA is retrieved when $\theta = 1$. On the right-hand side, the endowment increase does not allow it, since the smallest claimant is totally honored.

$(E, c) \in \mathcal{B}$, and all $i \in N$,

$$CEL^{\theta}(E,c) = \begin{cases} (\frac{\theta E}{n})\mathbf{1} + CEL(E - \theta E, c - (\frac{\theta E}{n})\mathbf{1}) & \text{if } \frac{\theta E}{n} \leq c_1 \\ \mathbf{c}^1 + CEL(E - nc_1, c - \mathbf{c}^1) & \text{otherwise} \end{cases}$$

Equivalently we can say,

$CEL^{\theta}(E,c) = min^1 + CEL(E - MIN, c - min^1)$, where $MIN \equiv n \min\{\frac{\theta E}{n}, c_1\}$, $min \equiv \min\{\frac{\theta E}{n}, c_1\}$, and $min^1 = (\min\{\frac{\theta E}{n}, c_1\}, \ldots, \min\{\frac{\theta E}{n}, c_1\})_{1\times n}$.

The $CEL^{\theta}$ rule initially assigns a minimal right equal to $min$ to all agents. Then, after revising down the endowment and the claims by the minimal rights, applies an equal division of their losses to the revised claims. When $\theta = 0$ and $\theta = 1$ the rule $CEL^{\theta}$ corresponds to the CEL and the CELmin rules, respectively. See figure 4.1 for further details.

**Example 2** *For the sake of comprehension, let us use Example 1 and* $\theta = 0.25$. *First, the CEL-family assigns a minimal right equal to* $\min\{\frac{\theta E}{n}, c_1\} = 37.5$. *Then, the endowment and the claims are revised down by the minimal right. After that, the revised endowment is divided by applying CEL to the revised claims. Therefore, the final allocation is* $CEL^{\theta}(E,c) = (37.5, 87.5, 187.5, 287.5)$.

## 4.4 Axiomatic analysis of the CEL-family

In this section, we study several properties that are commonly used in the bankruptcy literature and investigate if the rules in the CEL-family satisfy them. We analyze some basic properties (Equal treatment of equals, Order preservation, Homogeneity, Claim monotonicity, Resource monotonicity, Composition down, and Anonymity) which are satisfied by all the rules in the family and we study some more specific properties ( Population monotonicity, Composition from minimal rights, Composition up, Continuity, Consistency, Independence of residuals claims, Exclusion, and Sustainability) that are only satisfied by the extreme members of the family. For the sake of comparison within the literature, we have studied the properties following the organization established by Moreno-Ternero and Villar (2006).

### 4.4.1 Basic properties

**Homogeneity** states that if the claims and the endowment are multiplied by the same positive number, then so should all awards. For each $(E, c) \in \mathcal{B}$ and each $\lambda > 0$, $\varphi(\lambda E, \lambda c) = \lambda\varphi(E, c)$.

**Equal treatment of equals** states that all agents with equal claims should receive an equal allocation. For each $(E, c) \in \mathcal{B}$, and each $i, j \in N$, such that $c_i = c_j$, then $\varphi_i(E, c) = \varphi_j(E, c)$.

**Anonymity** means that the identification of the agents should be ignored in the process of allocation. The only factor recognized as the allocation base is the agents' claims. For each $(E, c) \in \mathcal{B}$, such that $\pi \in \Pi^N$, and each $i \in N$, then $\varphi_{\pi(i)}(E, c') = \varphi_i(E, c)$, where $c' \equiv (c_{\pi(j)})_{j \in N}$.

**Order preservation** (Aumann and Maschler, 1985) states that if the agent $i$'s claim is smaller than agent $j$'s claim, the agent $i$ cannot receives an allocation greater than agent $j$'s allocation. For each $(E, c) \in \mathcal{B}$, and each $i, j \in N$, such that $c_i \geq c_j$, then $\varphi_i(E, c) \geq \varphi_j(E, c)$, and $c_i - \varphi_i(E, c) \geq c_j - \varphi_j(E, c)$.

**Consistency** (O'Neill, 1982; Thomson, 2003) states that if some agents leave the problem, the remaining agents should not be affected. This property makes the rule invariant from partitions and mergers of claimants. For each $(E, c) \in \mathcal{B}$, and each $N' \subseteq N$, if $x = \varphi(E, c)$, then $x_{N'} = \varphi(\sum_{N'} x_i, c_{N'})$.

**Proposition 1** *All the rules in the CEL-family satisfy Homogeneity, Equal treatment of equals, Anonymity, and Order preservation.*

**Proof.** Let $(E, c) \in \mathcal{B}$ and $\lambda > 0$. To prove Homogeneity we have two cases.

Case 1, if $\frac{\theta\lambda E}{n} \leq \lambda c_1$. Since $CEL$ satisfies Homogeneity we have:[10]

$$\begin{aligned} CEL^{\theta}(\lambda E, \lambda c) &= (\frac{\theta\lambda E}{n}, \dots \frac{\theta\lambda E}{n}) + CEL(\lambda E - \theta\lambda E, (\lambda c_1 - \frac{\theta\lambda E}{n}, \dots, \lambda c_n - \frac{\theta\lambda E}{n})) = \\ &= \lambda(\frac{\theta E}{n}, \dots, \frac{\theta E}{n}) + CEL(\lambda E - \theta\lambda E, (\lambda c_1 - \frac{\theta\lambda E}{n}, \dots, \lambda c_n - \frac{\theta\lambda E}{n}) = \\ &= \lambda(\frac{\theta E}{n}, \dots, \frac{\theta E}{n}) + \lambda(CEL(E - \theta E, (c_1 - \frac{\theta E}{n}, \dots, c_n - \frac{\theta E}{n}))) = \\ &= \lambda CEL^{\theta}(E, c). \end{aligned}$$

Case 2, if $\frac{\theta\lambda E}{n} > \lambda c_1$. Again by Homogeneity of the CEL, we have

$$\begin{aligned} CEL^{\theta}(\lambda E, \lambda c) &= (\lambda c_1, \dots, \lambda c_1) + CEL(\lambda E - n\lambda c_1, (0, \lambda c_2 - \lambda c_1, \dots, \lambda c_n - \lambda c_1)) = \\ &= \lambda(c_1, \dots, c_1) + \lambda(CEL(E - nc_1, (0, c_2 - c_1, \dots, c_n - c_1)) = \\ &= \lambda CEL^{\theta}(E, c). \end{aligned}$$

Next, we prove Equal treatment of equals. Let $i, j \in N$ such that $c_i = c_j$. We must distinguish two cases.

[10] For the sake of simplicity, hereinafter, we remove the expression "$1 \times n$" from all the vectors of the type $(a, ..., a)$.

Case 1, if $c_1 > \frac{\theta E}{n}$ and $\frac{\theta E}{n} < c_1 \leq c_2 \leq \cdots \leq c_i = c_j \leq \cdots \leq c_n$. Then we have $CEL^\theta(E,c) = (\frac{\theta E}{n}, \ldots, \frac{\theta E}{n}) + CEL(E - \theta E, (c_1 - \frac{\theta E}{n}, \ldots, c_n - \frac{\theta E}{n}))$. Therefore for $i, j$ we have $CEL_i^\theta(E,c) = \frac{\theta E}{n} + CEL_i(E - \theta E, (c_1 - \frac{\theta E}{n}, \ldots, c_n - \frac{\theta E}{n}))$ and $CEL_j^\theta(E,c) = \frac{\theta E}{n} + CEL_j(E - \theta E, (c_1 - \frac{\theta E}{n}, \ldots, c_n - \frac{\theta E}{n}))$. Since CEL satisfies Equal treatment of equals, it is proved that $CEL_i^\theta(E,c) = CEL_j^\theta(E,c)$.

Case 2, if $c_1 \leq \frac{\theta E}{n}$, $CEL^\theta(E,c) = (c_1, \ldots, c_1) + CEL(E - nc_1, c - c^1)$. Then $CEL_i^\theta(E,c) = c_1 + CEL_i(E - nc_1, c - c^1)$ and $CEL_j^\theta(E,c) = c_1 + CEL_j(E - nc_1, c - c^1)$. Since CEL satisfies Equal treatment of equals, $CEL^\theta$ satisfies this property.

To prove Anonymity, let $\pi \in \Pi^N$, and $c' \equiv (c_{\pi(j)})_{j \in N}$. We have two cases.

Case 1, if $c_1 \geq \frac{\theta E}{n}$, then

$$CEL^\theta(E,c) = (\frac{\theta E}{n}, \ldots, \frac{\theta E}{n}) + CEL(E - \theta E, (c_1 - \frac{\theta E}{n}, \ldots, c_n - \frac{\theta E}{n})).$$

Therefore

$$CEL_\pi^\theta(E,c') = (\frac{\theta E}{n}, \ldots, \frac{\theta E}{n}) + CEL_\pi(E - \theta E, (c'_1 - \frac{\theta E}{n}, \ldots, c'_n - \frac{\theta E}{n})).$$

By Anonymity of CEL, we have that for all $j \in N$, $CEL_{\pi(j)}^\theta(E,c') = CEL_j^\theta(E,c)$.

Case 2, if $c_1 < \frac{\theta E}{n}$, then $CEL^\theta(E,c) = (c_1, \ldots, c_1) + CEL(E - nc_1, c - c^1)$. Therefore,

$$CEL_\pi^\theta(E,c') = (c_1, \ldots, c_1) + CEL_\pi(E - nc_1, c' - c^1).$$

Again by Anonymity of CEL, we have that for all $j \in N$, $CEL_{\pi(j)}^\theta(E,c') = CEL_j^\theta(E,c)$.

Finally, we prove Order preservation. Let $i, j \in N$ such that $c_i \leq c_j$. Since CEL satisfies Order preservation we have,

$$CEL_i^{\theta}(E,c) = \min\{\frac{\theta E}{n}, c_1\} + CEL_i(E - MIN, c - min^1) \leq$$
$$\leq \min\{\frac{\theta E}{n}, c_1\} + CEL_j(E - MIN, c - min^1) = CEL_j^{\theta}(E,c).$$

Again by Order preservation satisfied by CEL,

$$c_i - CEL_i^{\theta}(E,c) = c_i - \min\{\frac{\theta E}{n}, c_1\} - CEL_i(E - MIN, c - min^1) \leq$$
$$\leq c_j - \min\{\frac{\theta E}{n}, c_1\} - CEL_j(E - MIN, c - min^1) =$$
$$= c_j - CEL_j^{\theta}(E,c)$$

■

**Proposition 2** *The only rule in the CEL-family that satisfies Consistency is CEL.*

**Proof.** The rule $CEL^0(E,c) = CEL(E,c)$ satisfies consistency (Thomson, 2003). Let us see that there is no other rule within the CEL-family for which this happens. Note that for any $\theta \in (0,1]$ whenever $c_n - c_{n-1} \geq E - MIN$, Consistency does not hold. ■

### 4.4.2 Solidarity properties

In this section, we study monotonicity properties that have been considered as solidarity axioms in the literature (Moreno-Ternero and Roemer, 2006).

**Resource monotonicity** (Curiel et al., 1987; Young, 1987) requires that if the endowment increases, the agents are allocated at least the amounts they receive initially. For each $(E,c) \in \mathcal{B}$ and each $E' \in \mathbb{R}_+$ such that $C > E' > E$, then $\varphi_i(E',c) \geq \varphi_i(E,c)$, for each $i \in N$.

**Claim monotonicity** (Thomson, 2003) states that if the agents' claims increase, they should receive at least as much as they did initially. For each $(E,c) \in \mathcal{B}$, each $i \in N$ and each $c_i' > c_i$, $\varphi_i(E,(c_i', c_{-i})) \geq \varphi_i(E,c)$.

**Population monotonicity** (Thomson, 1983) means that if the number of agents increases while the amount of the endowment remains unchanged, the allocation of each agent is at most as much as the initial amount. For each $\{N, N'\} \in \mathcal{N}$ such that $N \subset N'$, and each $(E, c) \in \mathcal{B}$, $\varphi_N(E, c) \leq \varphi(E, c_{N'})$.

**Proposition 3** *Let $CEL^\theta$, $\theta \in [0, 1]$, be a rule in the CEL-family. The following statements hold:*

*(i) All the rules in the CEL-family satisfy Claim monotonicity and Resource monotonicity.*

*(ii) The only rules in the CEL-family that satisfy Population monotonicity are $CEL$ and the particular case when $CEL^1 = CEA$.*

**Proof.** (i) To prove Claim monotonicity we must consider two cases.

Case 1. If $\frac{\theta E}{n} \leq c_1$, then,

$$CEL^\theta(E, c) = (\frac{\theta E}{n}, \ldots, \frac{\theta E}{n}) + CEL\big(E - \theta E, (c_1 - \frac{\theta E}{n}, \ldots, c_n - \frac{\theta E}{n})\big).$$

If we increase the claim of claimant $i \in N$ from $c_i$ to $c'_i$ then,

$$CEL^\theta(E, (c'_i, c_{-i})) = (\frac{\theta E}{n}, \ldots, \frac{\theta E}{n}) + CEL\big(E - \theta E, (c_1 - \frac{\theta E}{n}, \ldots, c'_i - \frac{\theta E}{n}, \ldots, c_n - \frac{\theta E}{n})\big)$$

As the $CEL$ satisfies Claim monotonicity, we have $CEL^\theta_i(E, c) \leq CEL^\theta_i(E, (c'_i, c_{-i}))$.

Case 2. If $\frac{\theta E}{n} > c_1$

Case 2.1. Let $\frac{\theta E}{n} > c_1$ and $c'_i > c_i$ for $i \neq 1$. Then,

$$CEL^\theta(E, c) = (c_1, \ldots, c_1) + CEL\big(E - nc_1, (0, c_2 - c_1, \ldots, c_n - c_1)\big).$$

If we increase the claim of the claimant $i$ from $c_i$ to $c'_i$ then,

$$CEL^{\theta}(E,(c_i',c_{-i})) = (c_1,\dots,c_1) + CEL\big(E - nc_1,(0,c_2-c_1,\dots,c_i'-c_1\dots,c_n-c_1)\big)$$

Again, since $CEL$ satisfies Claim monotonicity, we have

$$CEL_i^{\theta}(E,c) \leq CEL_i^{\theta}(E,(c_i',c_{-i})).$$

Case 2.2. Let $i=1$ and $c_1 < c_1' < \frac{\theta E}{n}$. Then,

$$CEL_1^{\theta}(E,c) = c_1 + CEL_1\big(E - nc_1,(0,c_2-c_1,\dots,c_n-c_1)\big)$$

and

$$CEL_1^{\theta}(E,(c_1',c_2,\dots,c_n)) = c_1' + CEL_1\big(E - nc_1',(0,c_2-c_1',\dots,c_n-c_1')\big).$$

Therefore,

$$CEL_1^{\theta}(E,c) = c_1 \leq CEL_1^{\theta}(E,(c_1',c_2,\dots,c_n)) = c_1'.$$

Case 2.3. Let $i=1$ and $c_1 < \frac{\theta E}{n} < c_1'$. Then,

$$CEL_1^{\theta}(E,c) = c_1 + CEL_1\big(E - nc_1,(0,c_2-c_1,\dots,c_n-c_1)\big)$$

and

$$CEL_1^{\theta}(E,(c_1',c_2,\dots,c_n)) = \frac{\theta E}{n} + CEL_1\big(E - \theta E,(c_1'-\frac{\theta E}{n},c_2-\frac{\theta E}{n},\dots,c_n-\frac{\theta E}{n})\big).$$

Therefore,

$$CEL_1^\theta(E,c) = c_1 \leq CEL_1^\theta(E,(c_1',c_2,\ldots,c_n)) =$$
$$= \frac{\theta E}{n} + CEL_1(E-\theta E,(c_1' - \frac{\theta E}{n}, c_2 - \frac{\theta E}{n},\ldots,c_n - \frac{\theta E}{n})).$$

Next, we prove Resource monotonicity. Let $C \geq E^* > E$. We have three cases.

Case 1. If $c_1 \leq \frac{\theta E}{n} \leq \frac{\theta E^*}{n}$, since CEL satisfies Resource monotonicity we have,

$$CEL^\theta(E,c) = (c_1,\ldots,c_1) + CEL(E-nc_1,(0,c_2-c_1,\ldots,c_n-c_1)) \leq$$
$$\leq (c_1,\ldots,c_1) + CEL(E^*-nc_1,(0,c_2-c_1,\ldots,c_n-c_1)) = CEL^\theta(E^*,c).$$

Case 2. If $\frac{\theta E}{n} \leq \frac{\theta E^*}{n} \leq c_1$. Since $E - \theta E < E^* - \theta E^*$ and CEL satisfies Resource monotonicity we have,

$$\begin{aligned}
CEL^\theta(E,c) &= (\frac{\theta E}{n},\ldots,\frac{\theta E}{n}) + CEL(E-\theta E,(c_1-\frac{\theta E}{n},\ldots,c_n-\frac{\theta E}{n})) \leq \\
&\leq (\frac{\theta E^*}{n},\ldots,\frac{\theta E^*}{n}) + CEL(E-\theta E,(c_1-\frac{\theta E}{n},\ldots,c_n-\frac{\theta E}{n})) \leq \\
&\leq (\frac{\theta E^*}{n},\ldots,\frac{\theta E^*}{n}) + CEL(E^*-\theta E^*,(c_1-\frac{\theta E}{n},\ldots,c_n-\frac{\theta E}{n})) \leq \\
&\leq (\frac{\theta E^*}{n},\ldots,\frac{\theta E^*}{n}) + CEL(E^*-\theta E^*,(c_1-\frac{\theta E^*}{n},\ldots,c_n-\frac{\theta E^*}{n})) = CEL^\theta(E^*,c).
\end{aligned}$$

Case 3. If $\frac{\theta E}{n} \leq c_1 \leq \frac{\theta E^*}{n}$. Since CEL satisfies Resource monotonicity and Claim monotonicity, the following inequalities hold.

$$CEL^{\theta}(E,c) = (\frac{\theta E}{n}, \ldots, \frac{\theta E}{n}) + CEL(E - \theta E, (c_1 - \frac{\theta E}{n}, \ldots, c_n - \frac{\theta E}{n})) \leq$$
$$\leq (c_1, \ldots, c_1) + CEL(E - \theta E, (c_1 - \frac{\theta E}{n}, \ldots, c_n - \frac{\theta E}{n})) \leq$$
$$\leq (c_1, \ldots, c_1) + CEL(E^* - nc_1, (c_1 - \frac{\theta E}{n}, \ldots, c_n - \frac{\theta E}{n})) =$$
$$= (c_1, \ldots, c_1) + CEL(E^* - nc_1, (0, c_2 - c_1, \ldots, c_n - c_1)) = CEL^{\theta}(E^*, c).$$

(ii) The CEL rule and, in the particular case that $c_1 \geq \frac{E}{n}$, $CEL^1$= CEA satisfy Population monotonicity (Thomson, 2003).

Finally, note that for any $\theta \in (0,1)$ whenever $c_1 \leq \frac{\theta E}{n}$, and for any $i \neq 1 \in N$, $c_i \geq \frac{\theta \sum_{i=2}^{n} CEL_i^{\theta}(E,c)}{n-1}$, Population monotonicity does not hold.[11] ■

### 4.4.3 Composition properties

In this subsection, we consider situations in which after allocating the endowment for a problem by applying a rule, the amount of the endowment changes. The following two properties propose two ways to handle this situation.

**Composition down** (Moulin, 2000) considers a situation in which we distribute the endowment by applying a rule. Then, we re-evaluate the endowment and find that the value of the endowment is less than what we had evaluated initially. To deal with this situation, we have two options. First, we cancel the initial distribution and re-allocate the revised endowment by implementing the division rule. Second, we consider the initial allocation as agents' claims and divide the revised endowment into these new claims. Composition down requires that the result of both options should be equal. For each $(E,c) \in \mathcal{B}$, and each $E' < E$, $\varphi(E',c) = \varphi(E', \varphi(E,c))$.

**Composition up** (Young, 1988) illustrates the opposite situation of composition down.

[11] For instance, consider $(E,c) = (30,(5,20,30))$, $CEL^1(E,c) = (5,7.5,17.5)$. By considering, $N' = \{2,3\}$, $CEL^1(E,c) = (15,15)$, since $c_2 \leq (CEL^1_2(E,c) + CEL^1_3(E,c))/2$.

This property deals with a situation in which after dividing the endowment by applying the division rules, the re-evaluation of the endowment shows that it is worth more than its initial value. To deal with this situation we have two options. First, cancel the initial allocation and re-allocate the revised endowment. The second option is to give the agents their initial allocation, revise their claims down by these allocations and divide the incremental amount of the endowment to agents based on their revised claims. What the agents receive is the sum of the initial allocation and incremental allocation. Composition up states that the allocation of the two options is equal. For each $(E, c) \in \mathcal{B}$, and each $E' > E$, $\varphi(E', c) = \varphi(E, c) + \varphi(E' - E, c - \varphi(E, c))$.

**Proposition 4** *Let $CEL^\theta$, $\theta \in [0, 1]$, be a rule in the CEL-family. The following statements hold:*

*(i) All the rules in the CEL-family satisfy Composition down.*

*(ii) The only rules in the CEL-family that satisfy Composition up are $CEL$ and the particular case when $CEL^1 = CEA$.*

**Proof.** (i) To prove Composition down we must consider three cases.

Case 1. If $c_1 \geq \frac{\theta E}{n} \geq \frac{\theta E'}{n}$,

$$CEL^\theta(E', c) = (\frac{\theta E'}{n}, \ldots, \frac{\theta E'}{n}) + CEL(E' - \theta E', (c_1 - \frac{\theta E'}{n}, \ldots, c_n - \frac{\theta E'}{n}) =$$
$$= (\frac{\theta E'}{n}, \ldots, \frac{\theta E'}{n}) + CEL(E' - \theta E', (c_1 - \frac{\theta E'}{n}, \ldots, c_n - \frac{\theta E'}{n}) + (\frac{\theta E}{n}, \ldots, \frac{\theta E}{n}) - (\frac{\theta E}{n}, \ldots, \frac{\theta E}{n})) =$$

Since the CEL satisfies Composition down and $E' - \theta E' < E - \theta E$,

$$= (\frac{\theta E'}{n},\ldots,\frac{\theta E'}{n}) + CEL\big(E'-\theta E', CEL(E-\theta E,(c_1-\frac{\theta E}{n},\ldots,c_n-\frac{\theta E}{n}))+ + (\frac{\theta E}{n},\ldots,\frac{\theta E}{n}) - (\frac{\theta E'}{n},\ldots,\frac{\theta E'}{n})\big) =$$

$$= CEL^{\theta}\big(E',(\frac{\theta E}{n},\ldots,\frac{\theta E}{n}) + CEL(E-\theta E,(c_1-\frac{\theta E}{n},\ldots,c_n-\frac{\theta E}{n}))\big) =$$

$$= CEL^{\theta}(E', CEL^{\theta}(E,c)).$$

Case 2. If $\frac{\theta E}{n} \geq c_1 \geq \frac{\theta E'}{n}$, by Composition down of the CEL and $E'-\theta E' < E-\theta E \leq E-nc_1$ the following is satisfied,

$$CEL^{\theta}(E',c) = (\frac{\theta E'}{n},\ldots,\frac{\theta E'}{n}) + CEL(E'-\theta E',(c_1-\frac{\theta E'}{n}+c_1-c_1,\ldots,c_n-\frac{\theta E'}{n}+c_1-c_1)) =$$

$$= (\frac{\theta E'}{n},\ldots,\frac{\theta E'}{n}) + CEL(E'-\theta E',(c_1,\ldots,c_1)+CEL(E-nc_1,(0,c_2-c_1,\ldots,c_n-c_1)) - (\frac{\theta E'}{n},\ldots,\frac{\theta E'}{n})) = (\frac{\theta E'}{n},\ldots,\frac{\theta E'}{n}) + CEL(E'-\theta E',(CEL^{\theta}(E,c)-(\frac{\theta E'}{n},\ldots,\frac{\theta E'}{n})) =$$

$$= CEL^{\theta}(E', CEL^{\theta}(E,c)).$$

Case 3. If $c_1 \leq \frac{\theta E'}{n} \leq \frac{\theta E}{n}$, by Composition down of the CEL and $E'-nc_1 < E-nc_1$ the following is satisfied,

$$CEL^{\theta}(E',c) = (c_1,\ldots,c_1) + CEL(E'-nc_1, c-c^1) =$$

$$= (c_1,\ldots,c_1) + CEL(E'-nc_1, c^1 + CEL(E-nc_1,c-c^1)) - c^1 =$$

$$= (c_1,\ldots,c_1) + CEL(E'-nc_1, CEL^{\theta}(E,c)-c^1)) =$$

$$= CEL^{\theta}(E', CEL^{\theta}(E,c)).$$

(ii) It is proven by Moulin (2000) that the two rules in the CEL-family which are CEL and the particular case when $c_1 \geq \frac{\theta E}{n}$, so, $CEL^1$= CEA satisfy Composition up.

Finally, note that for any $\theta \in (0,1)$ whenever $CEL_{n-1}(E-MIN,\, c-min) \neq 0$, then,

$c_n - CEL_n(E - MIN, c - min) = c_{n-1} - CEL_{n-1}(E - MIN,\ c - min)$. So, $E' - E$ is equally distributed between these two claimants. It is straightforwardly obtained that this allocation does not coincide with the direct distribution of $E'$, where the distance between $c_n$ and $c_{n-1}$ remains. ■

### 4.4.4 Independence and related properties

In this section, we study some properties related to the structure of the claims problems, considering changes in the claims or in the endowment.

**Composition from minimal rights** (Dagan and Volij, 1993) guarantees that each agent receives a minimum amount of resources. This minimum amount is determined by the portion of the total resources to be allocated that is assigned to the agent after all other claims have been fully honored, provided that this amount is non-negative. For each $(E, c) \in B$, $\varphi(E, c) = m(E, c) + \varphi(E - M(E, c), c - m(E, c))$, where $m(E, c) = (m_i(E, c))_{i \in N} = (\max\{0, E - \sum_{j \in N - \{i\}} c_j\})_{i \in N}$ and $M(E, c) = \sum_{i \in N} m_i(E, c)$.

**Claims truncation invariance** (Curiel et al., 1987; Dagan and Volij, 1993) states that agents cannot demand more than the available endowment. If an agent's claim is larger than the endowment, the part of the claim that exceeds the endowment is ignored. For each $(E, c) \in B$, $\varphi(E, c) = \varphi(E, \min\{c_i, E\})$.

**Self-duality** (Aumann and Maschler, 1985) looks at the problem from two opposite perspectives: from the obtained awards and from the part of the claim that is not honored. So, it implies that the problem of dividing 'what is available' or 'what is missing' should result in the same awards: for each $(E, c) \in \mathcal{B}$ and each $i \in N$, $\varphi_i(E, c) = c_i - \varphi_i(C - E, c)$.

**Proposition 5** *The following statements hold:*

*(i) The only rule in the CEL-family that satisfies Composition from minimal rights is* $CEL$.

*(ii) The only rule in the CEL-family that satisfies Claims truncation invariance is the particular case when $CEL^1 = CEA$.*

*(iii) CEL-family rules do not satisfy Self-duality.*

Note that this statement is straightforwardly obtained by the definitions of the rules. On the one hand, from Thomson (2003), it is clear enough that CEA and CEL satisfy Claims truncation invariance and Composition from minimal rights, respectively. Moreover, none of them satisfies Self-duality (Aumann and Maschler, 1985).

On the other hand, note that if $\theta \in (0, 1)$, then each claimant will receive a strictly positive allocation of the endowment, violating minimal rights. Furthermore, by definition, CEL recommendation depends on the distance between claimants. Then, if this distance varies, the recommendation also varies, so the CEL-family does not satisfy Claims truncation invariance either. Finally, by definition of the CEL-family, a part of the endowment is distributed by CEL, so it does not satisfy Self-duality.

### 4.4.5 Protective properties

In this section, we study a group of properties that put restrictions on the behavior of the rule when the claims are very different. They give some criteria to protect the agents with extreme claims, big or small. These properties were studied in the literature by Herrero and Villar (2001), Herrero and Villar (2002), and Yeh (2006). The properties are Sustainability, Exclusion, and Independence of residual claims.

**Sustainability** implied that agents with comparatively smaller claims are entitled to receive their entire claims. To do so, the larger claims are replaced with the smaller ones and we have enough to compensate everyone. For all $(E, c) \in B$, and each $i \in N$, if $\sum_{j \in N} \min\{c_i, c_j\} \leq E$ then $\varphi_i(E, c) = c_i$.

**Exclusion** is a property underlying the idea of Sustainability. The idea is that all agents with a claim at most as large as equal division should be fully compensated. If a claim is at most as large as equal division, it is clearly sustainable. Therefore in the general case, Sustainability is stronger than Exclusion (Herrero and Villar, 2001), and for the two claimant cases the two properties are equivalent. For all $(E, c) \in B$, and each $i \in N$, if $c_i \leq \frac{C-E}{n}$ then $\varphi_i(E, c) = 0$.

**Independence of residual claims** shifts the attention from gains to losses, thereby emphasizing on agents with larger claims. In this scenario, larger claims are accorded priority, and smaller ones are not entitled to any compensation. A claim is identified as residual (Herrero and Villar, 2001) when the overall surplus claim in relation to the agent possessing the claim surpasses the amount to be divided. The property says that if an agent's claim is residual, she should receive nothing. For all $(E, c) \in B$, if $E \leq \sum_{j \in N} \max\{0, c_j - c_i\}$ then $\varphi_i(E, c) = 0$. Independence of residual claims is the dual property of Sustainability. Therefore Independence of residual claims implies Exclusion.

**Proposition 6** *The following statements hold:*

*(i) The only rule in the CEL-family that satisfies Sustainability is the particular case when* $CEL^1 = CEA$.

*(ii) The only rule in the CEL-family that satisfies Exclusion and Independence of residual claims is the* $CEL$.

**Proof.** (i) By (Herrero and Villar, 2001) CEA satisfies Sustainability, i.e., if $c_1 \geq \frac{\theta E}{n}$ and $\theta = 1$, then $CEL^\theta$ also does. If $\theta \neq 1$ it is straightforward to see that it does not. Consider the following example, $(E, c) = (30, (5, 10, 50))$, $CEL_2^\theta(E, c) < c_2$, violating Sustainability.

(ii) By Herrero and Villar (2001), CEL satisfies the Independence of residual claims and Exclusion, i.e., if $\theta = 0$, then $CEL^\theta$ also does. If *theta* $\neq 0$ it is straightforward to see that it does not, since $CEL_i^\theta(E, c) > 0$, for any $(E, c) \in \mathcal{B}$. ■

### 4.4.6 Summary of properties and families

To conclude the axiomatic analysis of the CEL-family, we study its relationship with two prominent families of rules: the parametric rules (Young, 1987) and the ICI rules (Thomson, 2003).

As defined by Young (1987) a rule is parametric if the $i$th agent's award is a function that depends only on $c_i$ and a parameter $\lambda$, which is related to the size of the amount to be divided. Formally,

A **rule** $\varphi$ **is parametric** if there exists a function $f : [a, b] \times \mathbb{R}_+ \to \mathbb{R}_+$, where $[a, b] \subset \mathbb{R} \cup \{\pm\infty\}$, continuous and weakly monotonic in its first argument, such that:

- $\varphi_i(E, c) = f(\lambda, c_i)$, for each $(E, c) \in \mathcal{B}$, and for some $\lambda \in [a, b]$;
- $f(a, x) = 0$, for each $x \in \mathbb{R}_+$; and
- $f(b, x) = x$, for each $x \in \mathbb{R}_+$.

Young (1987) shows that a rule is parametric if and only if it satisfies Equal treatment of equals, Continuity, and Consistency. Therefore, the Proportional rule, the Constrained equal awards (Maimonides, 1204), the Constrained equal losses (Maimonides, 1204), and the Talmud (Aumann and Maschler, 1985) rules are parametric rules. However, as a consequence of Proposition 2, the CEL-family of rules are not parametric rules.

The ICI family (Thomson, 2003) contains the Constrained equal awards (Maimonides, 1204), the Constrained equal losses (Maimonides, 1204), the Talmud (Aumann and Maschler, 1985), and Minimal overlap (O'Neill, 1982) rules.

ICI rules exhibit the evolution of each claimant's award as a function of the endowment: it is increasing first, constant next, and finally increasing again. Formally, let $\mathcal{G}^N$ be the family of lists $G \equiv \{E_k, F_k\}_{k=1}^{n-1}$, where $n = |N|$, of real-valued functions of the claims vector, satisfying for each $c \in \mathbb{R}^N_+$, the following relations:

$$\frac{E_1(c)}{n} + \frac{C - F_1(c)}{n} = c_1$$

$$c_1 + \frac{E_2(c) - E_1(c)}{n-1} + \frac{F_1(c) - F_2(c)}{n-1} = c_2$$

$$\vdots$$

$$c_{k-1} + \frac{E_k(c) - E_{k-1}(c)}{n-k+1} + \frac{F_{k-1}(c) - F_k(c)}{n-k+1} = c_k$$

$$\vdots$$

$$c_{n-1} + \frac{-E_{n-1}(c)}{1} + \frac{F_{n-1}(c)}{1} = c_n$$

The ICI rule relative to $G \equiv \mathcal{G}^N$, is defined as follows. For $k = 1$ each $c \in \mathbb{R}^N_+$, the awards vector is given as the following function of the endowment $E$, as it varies from 0 to $C$. As $E$ increases from 0 to $E_1(c)$, equal division prevails; as it increases from $E_1(c)$ to $E_2(c)$, claimant 1's award remains constant, and equal division of each new unit prevails among the other claimants. As $E$ increases from $E_2(c)$ to $E_3(c)$, awards of claimants 1 and 2 remain constant, and equal division of each new unit prevails among the other claimants, and so on. This process goes on until $E$ reaches $E_{n-1}(c)$. The next units go to claimant $n$ until $E$ reaches $F_{n-1}(c)$, at which point an equal division of each new unit prevails among claimants $n$ and $n-1$. This goes on until $E$ reaches $F_{n-2}(c)$, at which point equal division of each new unit prevails among claimants $n$ through $n-2$. The process continues until $E$ reaches $F_1(c)$, at which point claimant 1 re-enters the scene and equal division of each new unit prevails among all claimants.

Thomson (2008) shows that a rule belongs to the ICI-family if and only if it is consistent, among other properties. Therefore, as a consequence of Proposition 2, the CEL-family of rules are not parametric rules.

In the following table, we summarize all the results from the previous sections. Here we have the main properties and which are the rules in the CEL-family that satisfy each one of them.

| **Properties** | **Rules that satisfy the properties** |
|---|---|
| Equal treatment of equals | $CEL^\theta$ for all $\theta \in [0,1]$ |
| Anonymity | $CEL^\theta$ for all $\theta \in [0,1]$ |
| Order preservation | $CEL^\theta$ for all $\theta \in [0,1]$ |
| Homogeneity | $CEL^\theta$ for all $\theta \in [0,1]$ |
| Claim monotonicity | $CEL^\theta$ for all $\theta \in [0,1]$ |
| Resource monotonicity | $CEL^\theta$ for all $\theta \in [0,1]$ |
| Composition down | $CEL^\theta$ for all $\theta \in [0,1]$ |
| Population monotonicity | $CEL^0$, the particular case when $CEL^1 = CEA$ |
| Composition up | $CEL^0$, the particular case when $CEL^1 = CEA$ |
| Claims truncation invariance | The particular case when $CEL^1 = CEA$ |
| Independence of residual claims | $CEL^0$ |
| Self-duality | Not satisfied |
| Sustainability | The particular case when $CEL^1 = CEA$ |
| Exclusion | $CEL^0$ |
| Consistency | $CEL^0$ |
| Composition from minimal rights | $CEL^0$ |

Table 4.1: Summary of the properties that are satisfied by the CEL-family

## 4.5 ERDF allocation

The European regional development fund (ERDF) is an aid program established by the European Union (EU) to support its member states. The fund's primary objective is to reduce the economic development gap between regions within the EU. In other words, it aims to address the backwardness of less developed countries and push them towards improving their development level.

To distribute this fund, member states are categorized into three groups of regions based on their Gross domestic product (GDP) per capita. The less developed regions (R3) have a GDP per capita less than 75% of the average GDP per capita of EU-27, transition regions (R2) have a GDP per capita between 75% and 100% of the average GDP per capita of EU-27, and more developed regions (R1) have a GDP per capita above 100% of the average GDP per capita of EU-27. This classification results in the definition of 47 regions within the EU.

Solís-Baltodano et al. (2022) studied the allocation of the ERDF and proposed that this allocation can be viewed as a claims problem situation. Their research focused on the 2014-2020 period when the budget was around 182.150 million euros. They defined the claims of the regions in a way that reflects the amount of money each region needs. As a result, it can be deduced that less developed regions demand more money and have larger claims.

The EU aims for development convergence across the territory, which means that less developed regions should be supported more than well-developed regions and receive more ERDF to boost their development level. The implementation of different claims problem rules in the research by Solís-Baltodano et al. (2022) resulted that CEL can serve convergence in the EU. CEL is a rule that sacrifices smaller claimants (in this case, more developed regions) and allocates the main part of the endowment to smaller claimants. Although their proposal seems logical, their results show that CEL allocates nothing to some of the smallest regions and completely neglects them.

Later, in Salekpay et al. (2023), the authors discuss the results of the previously mentioned study and highlight a shortcoming of CEL in allocating zero amounts to regions with relatively smaller claims. They argue that such zero allocation is not practical in the real world and propose a new allocation method called CELmin. This rule not only supports less developed regions and achieves development convergence, but also guarantees a minimal allocation to all regions.

In accordance with the definition of CELmin provided in Section 2, this rule grants a minimal right to all regions, equivalent to the smallest claim belonging to Luxembourg (R1) (Salekpay et al., 2023). Upon comparing this result with the findings of Solís-Baltodano et al. (2022), which assign zero allocation to Luxembourg (R1), we observe that both CEL and CELmin exhibit extreme behavior towards this highly developed region. While the former rule completely honors its claim, the latter entirely ignores it. Although zero allocation may not be practical in the real world, fully honoring a wealthy region can also impede the objective of convergence in the EU. In this context, the CEL-family of rules may offer the most optimal solution since we can adjust the amount of the minimal right by modifying the parameter $\theta$. As CEL is more unequal than CELmin, reducing $\theta$ would move the CEL-family rule closer to CEL and, consequently increase the degree of inequality, making it more likely to achieve the goal of convergence. The CEL-family rule provides greater allocation to less developed regions and a small amount (but not zero) to more developed regions.

We analyzed different values of $\theta$ and found that for values greater than 0.08, the allocation result corresponds to CELmin, as the minimal right is $c_1$ for this interval. However, when $\theta$ is reduced to the range of (0, 0.08), the minimal right becomes $\frac{\theta E}{n}$. Decreasing $\theta$ towards zero would result in a smaller minimal right and a more inequitable distribution. Table 4.2 presents the allocation of the CEL-family rule when $\theta$ is set to 0.001. By comparing these results with other rules, particularly the three extreme cases of the CEL-family (CEL, CELmin, and CEA), we can draw more informed conclusions.

| Country | Region | Claim | Current | P | CEA | CEL | $\alpha$-min | CELmin | $CEL^{0.001}$ |
|---|---|---|---|---|---|---|---|---|---|
| Austria | R1 | 1,038 | 57.40 | 335.79 | 408.73 | 178.90 | 339.78 | 178.00 | 178.81 |
| Austria | R2 | 1,332 | 160.60 | 430.93 | 408.73 | 473.05 | 429.67 | 472.15 | 472.95 |
| Belgium | R1 | 1,009 | 36.30 | 326.20 | 408.73 | 149.25 | 330.72 | 148.35 | 149.15 |
| Belgium | R2 | 1,425 | 203.10 | 460.89 | 408.73 | 565.70 | 457.99 | 564.80 | 565.61 |
| Bulgaria | R3 | 1,629 | 506.00 | 526.86 | 408.73 | 769.67 | 520.32 | 768.76 | 769.57 |
| Croatia | R3 | 1,605 | 1,052.60 | 518.96 | 408.73 | 745.25 | 512.86 | 744.35 | 745.15 |
| Cyprus | R1 | 1,337 | 347.00 | 432.54 | 408.73 | 478.05 | 431.20 | 477.15 | 477.95 |
| Czech | R1 | 565 | 244.80 | 182.70 | 408.73 | 0.00 | 195.13 | 32.40 | 4.08 |
| Czech | R3 | 1,434 | 1,247.80 | 463.77 | 408.73 | 574.59 | 460.70 | 573.68 | 574.49 |
| Denmark | R1 | 1,004 | 33.30 | 324.65 | 408.73 | 144.45 | 329.26 | 143.55 | 144.35 |
| Denmark | R2 | 1,345 | 50.30 | 434.97 | 408.73 | 485.55 | 433.49 | 484.65 | 485.45 |
| Estonia | R2 | 1,395 | 1,407.40 | 451.14 | 408.73 | 535.55 | 448.77 | 534.65 | 535.45 |
| Finland | R1 | 1,173 | 88.20 | 379.53 | 408.73 | 314.13 | 381.11 | 313.22 | 314.03 |
| France | R1 | 1,142 | 67.60 | 369.35 | 408.73 | 282.68 | 371.50 | 281.77 | 282.58 |
| France | R2 | 1,418 | 145.3 | 458.69 | 408.73 | 558.90 | 455.91 | 558.00 | 558.81 |
| France | R3 | 1,530 | 1,003.50 | 494.80 | 408.73 | 670.55 | 490.03 | 669.65 | 670.45 |
| Germany | R1 | 1,039 | 61.40 | 335.99 | 408.73 | 179.51 | 339.97 | 178.61 | 179.42 |
| Germany | R2 | 1,351 | 491.40 | 436.89 | 408.73 | 491.50 | 435.31 | 490.59 | 491.40 |
| Greece | R1 | 1,327 | 419.00 | 429.03 | 408.73 | 467.20 | 427.89 | 466.29 | 467.10 |
| Greece | R2 | 1,573 | 795.60 | 508.58 | 408.73 | 713.16 | 503.05 | 712.26 | 713.07 |
| Greece | R3 | 1,622 | 1,181.90 | 524.53 | 408.73 | 762.48 | 518.12 | 761.57 | 762.38 |
| Hungary | R1 | 1,202 | 85.60 | 388.88 | 408.73 | 343.05 | 389.95 | 342.15 | 342.95 |
| Hungary | R3 | 1,601 | 1,551.50 | 517.85 | 408.73 | 741.83 | 511.81 | 740.92 | 741.73 |
| Ireland | R1 | 577 | 85.00 | 186.74 | 408.73 | 0.00 | 198.95 | 32.40 | 4.08 |

Table 4.2: The current allocation of ERDF, and the rules proposed allocation, per capita (numbers are in million euros).

| Country | Region | Claim | Current | P | CEA | CEL | $\alpha$-min | CELmin | $CEL^{0.001}$ |
|---|---|---|---|---|---|---|---|---|---|
| Italy | R1 | 1,158 | 91.20 | 374.43 | 408.73 | 298.36 | 376.29 | 297.46 | 298.27 |
| Italy | R2 | 1,440 | 277.00 | 465.85 | 408.73 | 581.04 | 462.67 | 580.13 | 580.94 |
| Italy | R3 | 1,556 | 973.90 | 503.39 | 408.73 | 697.12 | 498.15 | 696.21 | 697.02 |
| Latvia | R3 | 1,492 | 1,241.40 | 482.67 | 408.73 | 633.05 | 478.57 | 632.15 | 632.95 |
| Lithuania | R3 | 1,405 | 1,246.50 | 454.37 | 408.73 | 545.55 | 451.83 | 544.65 | 545.45 |
| Luxembourg | R1 | 32.4 | 32.40 | 10.48 | 32.40 | 0.00 | 32.40 | 32.40 | 4.08 |
| Malta | R2 | 1,272 | 808.00 | 411.52 | 408.73 | 413.05 | 411.34 | 412.15 | 412.95 |
| Netherlands | R1 | 1,035 | 29.70 | 335.79 | 408.73 | 175.55 | 338.76 | 174.65 | 175.45 |
| Poland | R1 | 840 | 880.50 | 271.64 | 408.73 | 0.00 | 279.17 | 32.40 | 4.08 |
| Poland | R3 | 1,536 | 1,074.40 | 496.81 | 408.73 | 676.76 | 491.93 | 675.86 | 676.67 |
| Portugal | R1 | 1,276 | 274.80 | 412.61 | 408.73 | 416.41 | 412.36 | 415.50 | 416.31 |
| Portugal | R2 | 1,370 | 514.70 | 443.05 | 408.73 | 510.55 | 441.13 | 509.65 | 510.45 |
| Portugal | R3 | 1,513 | 1,417.40 | 489.37 | 408.73 | 653.77 | 484.90 | 652.86 | 653.67 |
| Romania | R1 | 867 | 268.40 | 280.54 | 408.73 | 8.05 | 287.57 | 32.40 | 7.95 |
| Romania | R3 | 1,604 | 586.60 | 518.88 | 408.73 | 745.00 | 512.78 | 744.09 | 744.90 |
| Slovakia | R1 | 707 | 402.60 | 228.79 | 408.73 | 0.00 | 238.68 | 32.40 | 4.08 |
| Slovakia | R3 | 1,562 | 1,466.80 | 505.18 | 408.73 | 702.65 | 499.84 | 701.74 | 702.55 |
| Slovenia | R1 | 1,225 | 467.40 | 396.16 | 408.73 | 365.55 | 396.82 | 364.65 | 365.45 |
| Slovenia | R3 | 1,472 | 928.60 | 476.21 | 408.73 | 613.05 | 472.46 | 612.15 | 612.95 |
| Spain | R1 | 1,248 | 253.60 | 403.76 | 408.73 | 389.04 | 404.00 | 388.14 | 388.95 |
| Spain | R2 | 1,485 | 743.10 | 480.10 | 408.73 | 625.21 | 476.17 | 624.30 | 625.11 |
| Spain | R3 | 1,512 | 1,473.00 | 489.14 | 408.73 | 653.05 | 484.90 | 652.15 | 652.95 |
| Sweden | R1 | 1,100 | 71.90 | 355.77 | 408.73 | 240.55 | 358.62 | 239.65 | 240.45 |

Table 4.2: The current allocation of ERDF, and the rules proposed allocation, per capita (numbers are in million euros).

## 4.6 Final remarks

The paper presents a family of rules called CEL-family which is an extension of the CELmin rule. The CEL-family gathers all the possible combinations of the egalitarian division and the Constrained equal losses rule for the resources to be distributed. The rules of this family serve the concept of unequal allocation and allow us to select either the egalitarian division of resources or of losses, depending on the context. So, if $\theta = 0$, it would be the Constrained equal losses, and if $\theta = 1$, it would be the CELmin and in some particular cases the Constrained equal awards rule.

We analyze the main properties for the sake of comparison between the main rules and we apply the CEL-family to the European regional development fund problem, by combining the convergence and the solidarity principles. As the data depicts, the Constrained equal losses should be implemented to ensure a faster convergence between regions, and the larger the $\theta$, the larger the minimum amount received by the richest regions. To conclude, and as a possible extension of this work, we note that it would be interesting to find a characterization of the rules within the CEL-family.

# CHAPTER 5

# GENERAL CONCLUSION AND FUTURE STUDY

## 5.1 General conclusion

Our research has made significant contributions to the literature on claims problems in two important ways. Firstly, we applied well-established division rules to different contexts of claims problems and analyzed their performance to identify the fairest and most efficient rule in each case. Secondly, we extended the scope of claims problems by proposing a novel division rule and exploring a new family of rules that can adjust the degree of equal or unequal allocation to suit the specific needs of different allocation problems.

We began our investigation in Chapter 2 by studying the application of the claims problems in the allocation of the European Union (EU) emission budget and identifying a way to achieve the target of the Paris Agreement by 2030, which is a 55% reduction in greenhouse emissions compared to 1990 levels. We applied a set of different rules in the context of the claims problems to distribute the EU emission budget among member states. We then evaluated these rules comprehensively to find the most compatible rule, using both axiomatic analysis and fairness metrics. We identified a set of axioms and investigated whether the rules satisfy them. Additionally, we employed two fairness metrics, the Gini index and the coefficient of variation, to compare the degree of equity among the rules. We showed that the Constrained equal awards (CEA) rule satisfies all the axioms and exhibits a lower Gini index and Coefficient of variation, indicating a higher degree of equality compared to the other rules.

In Chapter 3, our investigation continued by exploring the application of the claims problem in the allocation of the European regional development fund (ERDF), which aims to support less developed regions in the EU and promote territorial convergence. Achieving this convergence requires an unequal allocation of the ERDF in favor of the less developed regions.

A previous study on ERDF allocation using claims problem rules (Solís-Baltodano et al., 2022) identified the Constrained equal losses (CEL) rule as the best choice. However, this rule may allocate zero shares to some significantly smaller claimants which are more developed regions in our study, rendering it unusable in real-world applications. To address this problem, we introduce a new division rule called CELmin that solves the issue of zero allocation. CELmin represents a compromise between the egalitarian allocation of resources and the CEL rule, ensuring that all regions receive a minimum allocation of resources before distributing the remainder using the CEL rule.

Through axiomatic analysis and data implementation, our study has demonstrated that CELmin is the best choice for ERDF allocation. It promotes economic convergence in the EU by allocating more funds to less developed regions while guaranteeing a positive allocation for more developed regions.

In Chapter 4, we proposed a novel family of rules called the CEL-family, which is a generalization of the CELmin rule. The CEL-family includes all possible combinations of the egalitarian division rule and the CEL rule, offering a wide range of options for adjusting the degree of equal or unequal allocation. The CEL rule, which allocates the endowment in the most unequal way, and the CELmin rule, which shows the highest degree of inequality in endowment allocation while guaranteeing a non-zero allocation to all claimants, represent the extreme rules of the family.

To evaluate the CEL-family, we conducted a comprehensive axiomatic analysis, considering a large set of axioms to observe their satisfaction. Our analysis showed that the CEL-family satisfies several fundamental and significant axioms, including Equal treatment of equals, Order preservation, Claims and Endowment monotonicity, Composition down, and Homogeneity. In cases where all the rules in the family failed to satisfy a specific axiom, we narrowed our analysis to determine if a specific rule in the family could satisfy the said axiom. We also implemented the rules in the CEL-family to allocate the ERDF and observed that the rules which behave more similarly to CEL tend to generate higher levels of economic convergence within the EU. Our results indicate that the CEL-family is a flexible and practical approach to allocation problems, allowing decision makers to adjust the degree of equal or unequal allocation to meet their specific needs.

## 5.2 Future study

One way to address the challenge of limiting global temperature increase to 2 degrees Celsius or less, as required by the Paris Agreement, is to allocate the greenhouse gas emission budget among emitter countries. As demonstrated in Chapter 2 of this dissertation, a method to accomplish this is by rationing the emission budget using the claims problem approach. It was found that different rules in the context of the claims problem allocate the emission budget in various ways. To select a rule that allocates the emission budget in a preferred way, it is necessary to ensure that the rules satisfy a group of axioms that evaluate their fairness and efficiency. Additionally, the countries' perception of fairness is taken into account by considering their historical emissions. Furthermore, other equity criteria are included to provide additional evaluation of each rule from an equity perspective.

All of these steps were undertaken to find a fair rule for allocating greenhouse gas emissions. The question remains whether finding a fair rule is sufficient to determine the best method for allocating greenhouse gas emissions. It is our belief that the claims problem approach overlooks the potential impact of the division rules on the social and economic factors of countries. Consequently, we propose a new method to allocate the emission budget by considering these social and economic factors.

One of the crucial factors in assessing the level of welfare in a society is the Human

Development Index (HDI). The HDI is a composite measure that takes into account a country's achievements in health, education, and standard of living, providing a comprehensive framework to evaluate a country's developmental status (Anand and Sen, 1994). The HDI comprises six major components that promote human development, including empowerment, sustainability, cooperation, security, productivity, and equity (Javaid et al., 2018). In addition, some scholars argue that lifestyle and the environment also play a critical role in human development (Badulescu et al., 2019).

Previous studies have primarily focused on the impact of greenhouse gas emissions on economic growth while neglecting the crucial link between these emissions and human welfare. The HDI offers a more comprehensive measure of a country's overall development and well-being, beyond just economic indicators such as GDP. A country's level of development can significantly affect its ability to reduce emissions and its vulnerability to the impacts of climate change. For instance, countries with a lower HDI may have fewer resources to invest in clean energy technologies, making it more challenging for them to reduce emissions and protect their citizens. Such countries require greater assistance in developing their clean energy infrastructure. Allocating greenhouse gas emission budgets based on HDI could help ensure that these countries are not disproportionately impacted by emission reduction policies, while also promoting global equity and fairness.

Developed countries, which generally have higher HDI, have historically contributed more to global greenhouse gas emissions than developing countries. Allocating greenhouse gas emission budgets based on HDI could help rectify this historical imbalance by giving developing countries more space to increase their emissions as they continue to develop, while developed countries would be required to reduce their emissions more quickly. Thus, considering HDI in greenhouse gas emission allocation can provide valuable insights to policymakers.

To this end, we propose an alternative method that departs from the claims problems approach. Our proposed method is a mathematical optimization model that seeks to find an optimal assignment of greenhouse gas emissions to each member state. The model is as follows:

$$\max_{x,p_1,p_2} \sum_{i\in N} x_i$$

$$\text{s.t.}$$

$$x_i \leq c_i, \forall i \in N$$

$$p_1 + p_2 = 1$$

$$x_i \leq \frac{p_1 * E * pop_i}{pop_E} + (p_2 * E * \frac{\alpha_i}{\sum_{i\in N} \alpha_i})$$

$$\sum_{i\in N} x_i \leq E \; x_i \geq 0, \forall i \in N, p_1 \geq 0, p_2 \geq 0$$

where, $N = \{1, 2, \ldots, n\}$ is the set of countries, $c_i$, $i \in N$ is country $i$'s per capita claims or demand for greenhouse gas emissions, $E$ is the EU greenhouse gas emission budget, $\alpha_i$ is a decreasing function of country $i$'s HDI (e.i. $\frac{1}{HDI}$), and $x_i$ the allocation of the greenhouse gas emissions that country $i$ receives.

The model has two phases of budget distribution. $p_1$, represents the percentage of $E$ that will be equally distributed among countries; $p_2$, represents the percentage of $E$ that will be proportionally distributed among countries. The objective function of the optimization problem is to maximize the sum of each country's share of greenhouse gas emissions per capita. The model is subject to four constraints:

The share of greenhouse gas emissions per capita for each country cannot exceed her claim; the sum of the percentages allocated to the two distribution phases ($p_1$ and $p_2$) must add up to 1; the share of greenhouse gas emissions per capita for each country cannot exceed the sum of its proportional share and its equal share of the budget, as determined by its HDI and the total HDI of all countries; the total sum of greenhouse gas emissions per capita shares cannot exceed the EU emission budget. The optimization problem also includes non-negativity constraints for the variables $x_i, p_1$, and $p_2$.

This is the direction of our future research. Our next step involves using HDI as a factor for allocating the emission budget. We will demonstrate that an increase in emissions can lead to an increase in HDI and vice versa. To achieve this, we will define the various components of HDI and develop a regression model to analyze their relationship to emissions. We will then apply the optimal model we identified to allocate the emission budget.

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