A model of a team contest, with an application to incentives under list proportional representation*

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Abstract

We develop a model of a team contest for multiple, indivisible prizes. Team members exert costly effort to improve their team's success. We analyze two intrateam allocation rules. Under a list rule, prizes are allocated according to a predetermined list. Under an egalitarian rule, prizes are allocated according to a fair lottery. We show that which allocation rule maximizes team success depends on the degree of complementarity between members' efforts and the convexity of the individual cost of effort function. We then apply the model to the context of elections under proportional representation with both open and closed lists. We derive conditions under which closed lists generate stronger incentives than open lists. Our results offer a rationale for the lack of evidence on the negative incentive effects of closed lists.

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Keywords: Elections; Open lists; Closed lists; Proportional Representation; Contests; Multiple prizes.

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1 Introduction

We develop a model of a team contest for multiple, indivisible prizes. Such contests are observed in many different settings. The political arena offers the most prominent example: legislative elections under proportional representation (PR) with pre-defined party candidate lists, as in Spain or Sweden. In such elections, political parties propose to voters a pre-defined list of politicians who compete as a team to win as many legislative seats as possible. Each seat is a prize that can be won by any politician on a party list. Under closed-list proportional representation, like in Spain or Israel, a party's electoral list determines the order in which the seats a party has won are allocated to the different politicians on the list. Under open lists, like in Finland or Brazil, the set of candidates who are attributed the seats their party won depends on the number of personal, preference votes these candidates received. The system to allocate these seats among candidates on the list is *de facto* a way to provide incentives to candidates. We wish to pin down the conditions under which closed lists generate stronger incentives than open ones.

For another example, consider promotion contests between departments that make up an organization when the firm's ultimate success depends on each department's output, which itself depends on the efforts of its members. The firm thus cares about its employees' effort provision not per se, but because these effort choices determine the outputs of the firm's departments. Departments that generate higher output typically need to see their workforce grow. Then, the firm's hiring and promotion policy favors departments which generate higher outputs. An important and open challenge for the management of the firm is how to design the rules governing the allocation of promotions *within* each department, so as to offer a menu of individual incentives that maximize the output of each department. We wish to pin down the conditions under which an intradepartmental promotion allocation rule generates stronger incentives than another one.

We develop a model of a contest among $T \geq 2$ teams of n identical players for n identical and indivisible prizes, such as legislative seats or promotions. Team members exert individually costly effort to help their team win as many prizes as possible among the n available ones.[1] The allocation of prizes among teams depends on their aggregate output, which we model as a CES function of the efforts exerted by individual team members. The number of prizes won by a team follows a binomial distribution with parameters n, the total number of available prizes, and p, the ratio of

[1]As the number of prizes is equal to the number of individuals within a team, the inter-team aspect of the contest is necessary to (match the characteristics of the examples above and to) avoid the trivial outcome in which all team members are certain of receiving one of the identical prizes and thus exert no effort.

that team's output to the sum of all team outputs. Our framework is thus an extension of the Tullock (1980) contest success function (CSF) to the case of many prizes.

We consider two types of intrateam prize allocation rules.[2] We first analyze a list rule. Under this rule, the n team members are ranked on a list before they exert effort. The prizes a team wins are then attributed to list members in the order of this ranking. The member in position j on the list receives a prize if and only if the team wins at least j prizes. The ranking is independent of effort provision by the different members. The composition of the list is thus not directly related to effort but is instead based on seniority, age, gender, geographical origin or some other member characteristic. Team members, who are ex-ante identical from any payoff-relevant perspective, face unequal treatment under this rule.

We then analyze an egalitarian rule. Under this rule, all team members have the same probability of being allocated one of the prizes the team won, regardless of their individual effort contribution. The egalitarian rule is equivalent to a fair lottery among all members of the team. As with the list rule, effort provision plays no direct role in the allocation of prizes under the egalitarian rule. Yet, contrary to the list rule, the egalitarian rule treats all team members equally.

Our focus on these two rules is not arbitrary. In the online appendix, we consider all rules that allocate prizes within a team independently of individual effort. When we impose a weak monotonicity condition, we show that these two turn out to be the only two rules that maximize team output.

Turning to our results, we first derive equilibrium efforts under the two rules. We show that the equilibrium distribution of efforts is bell-shaped under the list rule while under the egalitarian rule, all team members exert the same effort. This difference underpins the main result of the paper: the list rule generates higher team output than the egalitarian rule whenever the degree of complementarity between efforts within teams is low and/or the cost of effort function is not too convex.[3] A simple intuition for this result is that, when efforts are substitutable, having just a few team members exert high effort is superior to having all team members exert some effort. Thus, even though the list rule is associated to unequal treatment, it may still be superior to an equal treatment rule.

We then use our model to analyze incentives under open- and closed-list PR. This application is the second contribution of the paper. The model is flexible enough to take into account that 1)

[2]We do not use the contest literature's terminology – sharing rules – as prizes are indivisible in our model .

[3]We obtain the same result when we analyze monotonic team output-maximizing rules in the online appendix.

more than two parties compete in the election; 2) some parties may enjoy an ideological advantage; 3) parties compete as teams of candidates; 4) voters may or may not vote for individual politicians; 5) voters may or may not be well informed about the choices of individual candidates.

Closed-list PR corresponds exactly to the list rule. To model open-list PR, we extend the model to take into account preference votes. When lists are open, preference votes determine the winners of the seats a party won. Candidates thus exert effort not only to increase the number of seats won by their party, but also to attract preference votes. To model the mapping from preference votes to seat allocation, we use the contest model of Clark and Riis (1996). As voters need information about individual effort to cast preference votes, we parametrize the sensitivity of the outcome of the intraparty contest to individual effort with a noise parameter. A noisy contest corresponds to the case of poorly informed voters. In the limit case, in which voters have no information about candidates' efforts and thus randomize their preference vote over the set of candidates of their preferred party, open-list PR is equivalent to the egalitarian rule.

Our main finding is that closed lists generate stronger incentives than open lists when voters are poorly informed about the candidates' choices, when party electoral outputs exhibit low complementarity, and when the individual cost of effort is not too convex. We then interpret this finding in terms of the level at which the election takes place – in second tier elections voters are often poorly informed about candidates – and in terms of the type of issue that is at the forefront of the election – routine elections on routine issues can be viewed as elections in which complementarity and the convexity of the cost function are low; the opposite holds for elections on novel, complex issues. We believe our findings provide an important theoretical rationale for the lack of evidence that politicians' incentives to perform under open and closed lists differ significantly.

2 Related Literature

Our paper contributes to the literature on team contests and to the one on contests for multiple prizes.[4] We offer a bridge between these two strands of the literature via the introduction of a novel interteam CSF, the binomial Tullock, and the analysis of two intrateam prize allocation rules that are directly applicable to important realworld settings, the list and egalitarian rules.

The analysis of team contests has focused on situations where several teams compete in order to win a single prize. The teams' sharing rules determine how the (private part of the) prize is

[4] The literature on contests is too vast to be reviewed here. See Konrad (2009) and Vojnovic (2015).

allocated to the winning team's members. This rule affects rent dissipation and the group size paradox. Contributions include Nitzan (1991), Lee (1995), Esteban and Ray (2001), Baik and Lee (2001), Ueda (2002), Nitzan and Ueda (2011), Baik and Lee (2012) and Balart, Flamand and Troumpounis (2016).[5] These papers use the Tullock CSF. Other contributions use all-pay auctions; see for instance Fu, Lu and Pan (2015), Barbieri, Malueg and Topolyan (2014), Barbieri and Malueg (2016) and Eliaz and Wu (2018). We extend this literature to the case of multiple, indivisible prizes by developing a novel interteam CSF, which we label the binomial Tullock. The intrateam allocation of prizes is governed by what we label the list and egalitarian rules.

Moldovanu and Sela (2001) consider a contest among individuals in which the contest designer can decide on both the number and value of the prizes on offer. As their contest is among individuals, the issue of the intrateam prize allocation rule is absent, by construction, whereas it is the focus of our analysis. Their analysis also focuses mostly on the convexity of the cost of effort function.

By comparing the incentive effects of the egalitarian and list rules, we also contribute to the literature on incentives in teams, and in particular to the literature that links incentives and non-equal treatment of ex-ante identical team members. Winter (2004) analyses the optimality of asymmetric rewards to improve incentives when team production follows an "O-Ring" technology. Bose, Pal and Sappington (2010), show that sequencing effort decisions is superior to having a simultaneous game when there are complementarities in team production. Closer to us is Ray, Baland and Dagnelie (2007). They also use a CES function to model team production. They find that unequal sharing rules are efficient when efforts are substitutes.

We contribute to the recent research on electoral incentives. In particular we build on Caillaud and Tirole (2002) and Castanheira, Crutzen and Sahuguet (2010): they model candidates who exert costly effort to improve the electoral platforms of their party. These models study first-past-the post elections while we offer a novel and tractable model of elections under PR with closed and open lists, thanks to the binomial Tullock CSF. To model closed-list PR, we use the list rule. To model open lists under PR, we use Clark and Riis (1996) to model the intrateam competition for preference votes, augmented by a noise term that captures how well informed voters are about each politicians' effort. Clark and Riis (1996) is the key reference to model competition for multiple prizes between individuals when each individual can win at most one prize.[6] In Clark and Riis, the number and values of the prizes to be distributed are given, because there is a single team

[5] For a recent survey on sharing rules in collective rent seeking, see Flamand and Troumpounis (2015).

[6] Fu and Lu (2012) showed that Clark and Riis (1996)'s mechanism possesses the desirable axiomatic properties.

in the game. In our game, the values of the prizes are endogenous in that they depend on the probability that the party wins a given number of seats. We show that closed lists can provide better incentives than open lists. This contrasts with the received wisdom in both economics and political science about the incentive effects of PR, which is quite negative, especially when lists are closed; see Persson, Tabellini and Trebbi (2003), for example.

A few other recent papers compare the various types of PR systems (closed, open, flexible), but they all focus on selection; see for example Galasso and Naniccini (2015) and Buisseret and Prato (2019). Buisseret and Prato (2019) propose an accountability model to study how the degree of flexibility of the list impacts the candidates' incentives to favor their party's goals over their desire to 'cultivate a personal vote', to use the terminology of Carey and Shugart (1995). Buisseret et al. (2019) propose a model in which candidates differ in quality to examine how parties rank these candidates on their party lists, and test their model's predictions with Swedish data. Their empirical estimates of a candidate's winning probability as a function of their rank on the party list are isomorphic to the ones the Binomial Tullock generates. Hangartner, Nelson and Tukiainen (2019) analyze list type choices (and their consequences) in Colombia. Finally, Crutzen and Sahuguet (2019) compares closed-list PR and plurality rule and also offers some cautionary tales about the negative views regarding closed-list PR with respect to PR.

3 A model of a team contest

3.1 Individual efforts and team outputs

$T \geq 2$ teams compete in a contest for n indivisible prizes, $n \geq 3$ and odd.[7] Each team is composed of n team members; each team member can win at most one prize. Each prize has value V. Member i in team j exerts effort $e_{ij} \geq 0$ at cost e_{ij}^{β}/β with $\beta > 1$. All team members are equally productive, and team j's output is denoted by E_j. We assume that the production function aggregating individual efforts exhibits constant elasticity of substitution:

$$E_j = \left(\sum_{i=1}^{n} (e_{ij})^{1-\sigma} \right)^{\frac{1}{1-\sigma}}, \text{ with } \sigma \in [0, 1). \tag{1}$$

When $\sigma = 0$, individual efforts are perfect substitutes and a team's output is the simple sum of efforts. The complementarity of efforts increases with σ.

[7]Imposing that n is odd is simply to ensure that there is a unique effort maximizing candidate under the list rule, which makes the description of the forces at play simpler.

3.2 Allocation of prizes among teams

The allocation of prizes among teams depends on their output and follows a Binomial-Tullock distribution. This distribution generalizes the Tullock contest (1980) to multiple prizes. As in a Tullock contest, the probability that team j wins a particular prize is given by $p_j = \frac{E_j^r}{\sum_{t=1}^{T} E_t^r}$. Prizes are then awarded to team j using independent draws from a Bernoulli distribution with parameter p_j. The probability that team j wins $k \leq n$ prizes follows a binomial distribution and is given by:

$$P_j(k) = C_k^n \left(p_j\right)^k \left(1 - p_j\right)^{n-k}, \tag{2}$$

where $C_k^n = \frac{n!}{(n-k)!k!}$ denotes the binomial coefficient.

Parameter $r \in [0, 1]$ is a return to scale parameter. Values of r lower than 1 make the allocation of prizes among teams more noisy and less responsive to teams' outputs. Lower values of r also makes the objective functions of team members more concave; r thus plays an important role to ensure equilibrium existence.

3.3 Allocation of prizes within teams

We consider two allocation mechanisms. First, we consider a **list rule**: n team members are ordered on a list that determines the allocation of seats won by the team. The individual in m^{th} position on the list wins a seat if their team wins at least m seats. List member in m^{th} position on the list of team j chooses e_{mj} to maximize:

$$V \sum_{k=m}^{n} P_j(k) - \frac{\left(e_{mj}\right)^\beta}{\beta}. \tag{3}$$

Notice that the summation goes from m to n and not from 1 to n, as the list member in the m^{th} position only gets a prize when their team wins at least m prizes.

We then consider an **egalitarian rule**: prizes won by the team are distributed by a random lottery after the contest. The probability of getting one of the $k \leq n$ prizes is thus independent of effort decision and is the same for all team members, k/n. Team member i in team j chooses their level of effort to maximize:

$$V \sum_{k=1}^{n} P_j(k) \frac{k}{n} - \frac{\left(e_{ij}\right)^\beta}{\beta}.$$

Note that under both rules team members' effort does not directly increase their chance of winning a prize.

4 Equilibrium efforts

We first characterize individual effort and team output under the list rule. The proof of this proposition and of all the results that follow are in the appendix.

Proposition 1 *Under the list rule, in a team-symmetric pure strategy Nash equilibrium, team output E_L^* is equal to:*

$$E_L^* = (rV)^{\frac{1}{\beta}} \left(\sum_{k=1}^{n} \left(kC_k^n \left(\frac{1}{T}\right)^k \left(\frac{T-1}{T}\right)^{n-k+1} \right)^{\frac{1-\sigma}{\beta+\sigma-1}} \right)^{\frac{\beta+\sigma-1}{\beta(1-\sigma)}}. \tag{4}$$

Individual effort e_m^* of list member in m^{th} position is equal to:

$$e_m^* = \left(\frac{\left(mC_m^n \left(\frac{1}{T}\right)^m \left(\frac{T-1}{T}\right)^{n-m+1} \right)^{\frac{\beta}{\beta+\sigma-1}}}{\sum_{k=1}^{n} \left(kC_k^n \left(\frac{1}{T}\right)^k \left(\frac{T-1}{T}\right)^{n-k+1} \right)^{\frac{1-\sigma}{\beta+\sigma-1}}} rV \right)^{1/\beta}. \tag{5}$$

To characterize equilibrium efforts, we first take the first order condition of the maximization problem for each team member. As the objective function is not always globally concave, we derive sufficient conditions for equilibrium existence.

The probability density of the binomial distribution function drives the marginal benefit of members' efforts and thus incentives. As this distribution is unimodal and bell-shaped with a maximum at $\frac{n+1}{T}$, the highest effort comes from the member in position $\frac{n+1}{T}$. With two teams, the effort distribution is also symmetric around $(n+1)/2$. Figure 1 below illustrates this for $n = 31$.

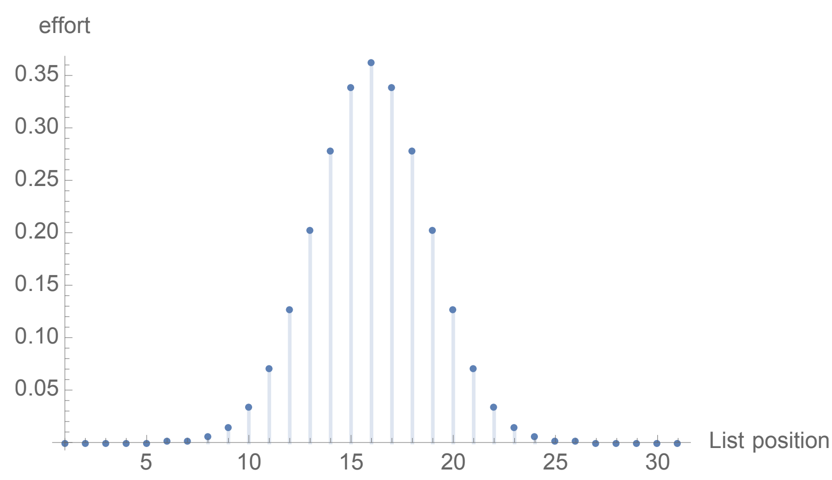

Figure 1: Effort as a function of the position on the list

We now turn to the equilibrium under the egalitarian allocation rule; we have:

Proposition 2 *Under the egalitarian rule, in a team-symmetric pure strategy Nash equilibrium, team output E_E^* is equal to:*

$$E_E^* = n^{\frac{1}{(1-\sigma)}} \left(r \frac{V}{n} \frac{T-1}{T^2} \right)^{1/\beta}. \tag{6}$$

Individual effort is equal to:

$$e^* = \left(r \frac{V}{n} \frac{T-1}{T^2} \right)^{1/\beta}. \tag{7}$$

Under the egalitarian rule, the equilibrium is symmetric across and within teams: all team members face the same maximization problem and exert the same level of effort. When $\sigma = 0$ and $\beta = 2$, the contest under the egalitarian rule is equivalent to a contest over one fully divisible prize of value nV with an egalitarian intrateam sharing rule, as in Nitzan (1991).

4.1 Comparison of efforts

Under the list rule, as shown in Figure 1, effort is a bell-shaped and right-skewed function of the position on the list, with the highest effort exerted by the member in position $(n+1)/T$. Under the egalitarian rule, all team members exert the same effort. As team output is a CES function of individual efforts, which system leads to higher team output depends on the degree of complementarity of the CES function and on the convexity of the cost function. Comparing team outputs under the two rules yields our main result:

Theorem 3 *The list rule leads to higher team output than the egalitarian rule, $E_L^* \geq E_E^*$, if and only if $\beta \leq 2 - 2\sigma$.*

The intuition behind this result is as follows. Incentives are uniform under the egalitarian rule and bell-shaped under the list rule. Suppose that the cost of effort function is close to linear (β is close to 1). When individual efforts are substitutes (σ is close to 0), team output is close to the sum of individual efforts. In this case, inducing differences in efforts can be optimal. When efforts are complements ($\sigma > 1/2$), inducing differences in individual efforts is inefficient as a team output depends relatively more on the lowest efforts. The role of convexity of the cost function is similar. Suppose that efforts are substitutes ($\sigma = 0$). When the cost function is very convex ($\beta > 2$), its marginal cost is also convex so that asymmetric incentives are bad. Indeed, starting from equal marginal benefits of effort, increasing the marginal benefit of one individual and decreasing the benefit of another has a positive effect provided that the marginal cost increases more slowly for the individual with stronger incentives than for the one with weaker incentives. However, when the marginal cost of effort is convex, this is simply not possible. The noise parameter r does not influence the comparison of efforts in Theorem 3 as r impacts how prizes are allocated among teams irrespective of how teams allocate prizes to their members.

4.2 Group Size Paradox

Our paper also contributes to the literature on the group size paradox (GSP); see Olson (1965) and Esteban and Ray (2001). The GSP states that, as groups grow larger, they become less effective beacuse incentives to free-ride increase and individual benefits linked to group success decrease.

Our model sheds light on optimal team size when prizes are multiple and indivisible. We assumed above that team size is exogenous and equal to the number of prizes available. We now extend the model to allow for teams composed of more members than the total number of available prizes. Under a list rule, a larger team has no benefit as members beyond position n on the list have no chance of getting a prize and thus exert no effort. Under the egalitarian rule, increasing the number of members can have an impact on team output. We have:

Proposition 4 *An increase in the number of team members does not impact team output under the list rule. Under the egalitarian rule, increasing the number of members leads to higher team output if and only if* $\beta \geq 2 - \sigma$.

In our model, the GSP is linked not only to the group technology (in terms of complementarity and cost of effort) but also to the intrateam prize allocation rule. When $\beta > 2 - \sigma$, the paradox does not hold and teams should (rely on the egalitarian rule and) be as large as possible. This last result is in line with Esteban and Ray (2001) who show that the GSP disappears when the cost of effort becomes very convex. When $\beta \leq 2 - 2\sigma$, teams should use the list rule and increasing team size has no impact on team output.

5 Elections under List Proportional Representation

In this section, we use the model above to analyze candidate incentives under PR. PR is the world's most frequently used electoral rule. As of 2017, more than 90 out of the 147 democracies rely on

PR for their legislative elections (Cruz, Keefer and Scartascini, 2018). Under PR, each party's seat share in parliament is proportional to its vote share. Also, PR systems use multi-member constituencies. Political parties thus offer lists of candidates to the electorate. Of the democracies relying on PR, roughly 25 use open or flexible lists; the rest relies more heavily on closed lists. With closed lists, parties propose an ordered list of candidates to voters. As the ballot structure allows the electorate to vote only for a party list, voters cannot express their preferences for individual candidates. The legislative seats a party won are allocated to its candidates following the order on the list. Closed lists are used for example in Argentina, Costa Rica, Guatemala, Israel, Spain and Turkey. With open lists, the ballot structure allows voters to cast a preference vote for an individual candidate on one of the party lists. The total number of preference votes received by all candidates on a party's list determines the party's vote share and its number of seats in parliament. Contrary to what happens with closed lists, the intraparty allocation of legislative seats to candidates is determined by the number of preference votes each candidate received, as seats go to the candidates having received the highest number of such votes. Open lists are used for example in Austria, Brazil, Finland, Greece, Indonesia and Japan. Under PR, electoral success requires cooperation among list members but, with open lists, there is also competition between party members.

Persson, Tabellini and Trebbi (2003, p. 961) offer a crisp summary of the issues at stake: "Politicians' incentives are [...] diluted by two effects. First, a free-rider problem arises among politicians on the same list. Under proportional representation, the number of seats depends on the votes collected by the whole list, rather than the votes for each individual candidate. Second, [if] the list is closed and voters cannot choose their preferred candidate, an individual's chance of re-election depends on his rank on the list, not his individual performance".

Before diving into the analysis, we discuss why our model is an appropriate model of incentives under PR. First, the binomial Tullock contest success function (CSF) generates a simple but realistic distribution of party seats. The figure below displays the probability that a politician wins a seat as a function of their rank on the party list under closed-list PR in the symmetric equilibrium for $n = 15$ and T equal to 2 and 5. The inverted S-shape is strikingly in line with its empirical counterpart estimated by Buisseret et al. (2019) in their analysis of party nomination strategies in Sweden (see their Figure 1).

Second, the binomial Tullock CSF is flexible and allows for various extensions. For example, we can account for a party to be ideologically advantaged by modifying the success ratios to

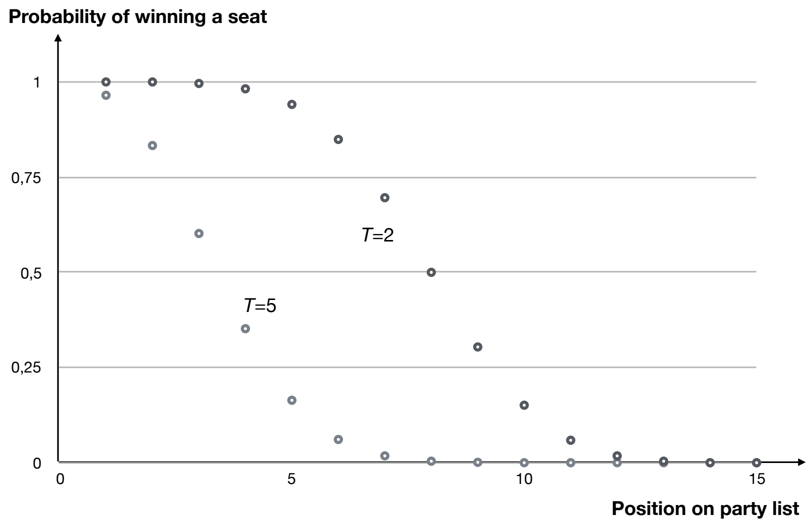

Figure 2: A candidate's probability of being elected as a function of their list rank

$p_j = \frac{\lambda_j E_j^r}{\sum_{k=1}^T \lambda_k E_k^r}$. A party with a high λ has an ideological advantage over a party with a lower λ. As this vector of λ's impacts the first order condition of the candidates maximization problem similarly across all allocation rules, our comparison between rules would still obtain.[8]

5.1 Modelling closed-list and open-list PR

The list rule of section 3 applies directly to model closed-list PR. To model preference votes under open lists, we use Clark and Riis (1996)'s model of a contest among n players for $m \leq n$ prizes.

The probability that i ends up among the m candidates with the highest number of preference votes is given by:

$$Q_i(m) = q_1 + \sum_{j=2}^{m} q_j \left(\prod_{s=1}^{j-1} (1 - q_s) \right) \tag{8}$$

where q_j is the probability that i ends up with the j^{th} largest amount of preference votes once the $j-1$ candidates with the highest number of preference votes have been identified. This probability is given by a standard Tullock CSF based on individual effort choices among the $n-j+1$ remaining candidates:

$$q_j = \frac{e_i^\rho}{e_i^\rho + \sum_{k \neq i} e_k^\rho}, \#k = n - j + 1 \tag{9}$$

[8] The argument used to prove Theorem 3 and Proposition 6 below, applies directly to a biased contest as $\frac{\partial p_j}{\partial e_{mj}} = r\frac{p_j(1-p_j)}{E_j}$.

with $\rho \in [0,1]$.

This contest can be interpreted as the result of a sequential process. Each contestant exerts effort once. The winner of the first prize, the candidate who obtains the highest number of preference votes, is decided via the Tullock CSF based on the efforts of all the n contestants. The winner and their effort are then erased and the candidate with the second largest number of preference votes is chosen via the Tullock CSF based on the efforts of the remaining $n-1$ contestants. This process continues until all candidates have been ranked in terms of preference votes.

We use ρ to parametrize the noise in preference voting, that is, the extent to which voters observe the candidates' efforts. Candidate i in party j chooses e_{ij} to maximize:

$$V \sum_{m=1}^{n} P_j(m) Q_i(m) - \frac{(e_{ij})^\beta}{\beta}, \tag{10}$$

where $Q_i(m) = q_1 + \sum_{j=2}^{m} q_j \left(\prod_{s=1}^{j-1} (1-q_s) \right)$; $q_j = \frac{e_i^\rho}{e_i^\rho + \sum_{k \neq i} e_k^\rho}$, $\#k = n-j+1$; $P_j(m) = C_m^n p_j^m (1-p_j)^{n-m}$; and $p_j = \frac{E_j^r}{\sum_{k=1}^{T} E_k^r}$.

If $\rho = 0$, the intraparty allocation becomes random as voters do not have any information about the effort of individual candidates. Voters cannot cast an informed vote and thus simply randomize across all candidates of the party they wish to support. As all party candidates are treated equally, they end up with the same probability of winning one of the k legislative seats their party won, $Q_i(k) = k/n, \forall i$. Thus, open-list PR coincides with the egalitarian rule of section 3 when $\rho = 0$. We have:

Proposition 5 *When lists are open, in a team-symmetric Nash equilibrium, a party's electoral output E^*_{OL} is equal to:*

$$E^*_{OL} = n^{\frac{1}{1-\sigma}} \left\{ \frac{T-1}{nT^2} rV + \rho V \sum_{m=1}^{n} C_m^n \left(\frac{1}{T}\right)^m \left(\frac{T-1}{T}\right)^{n-m} \left(1 - \frac{m}{n}\right) \sum_{j=1}^{m} \frac{1}{n-j+1} \right\}^{\frac{1}{\beta}} \tag{11}$$

Individual effort e^*_{OL} is equal to:

$$e^*_{OL} = \left\{ \frac{T-1}{nT^2} rV + \rho V \sum_{m=1}^{n} C_m^n \left(\frac{1}{T}\right)^m \left(\frac{T-1}{T}\right)^{n-m} \left(1 - \frac{m}{n}\right) \sum_{j=1}^{m} \frac{1}{n-j+1} \right\}^{\frac{1}{\beta}} \tag{12}$$

The first term in between parentheses corresponds to the incentives to work for the party, while the second term corresponds to the incentives to get preferences votes. As ρ goes to zero, the second term vanishes and effort converges to its value under the egalitarian rule, namely $\left(\frac{(T-1)}{nT^2} rV\right)^{1/\beta}$.

5.2 Comparing effort under closed and open list PR

The two ballot structures differ in two main dimensions. The first difference comes from preference votes. With open lists, individual effort influences both the party's electoral success and the candidate's intraparty ranking (in terms of preference votes). Competing for preference votes within one's party is a source of incentives that does not exist with closed lists. The second difference comes from the heterogeneity of incentives that we analyzed in Section 3.

The received wisdom in the political economy and political science literature typically views open lists as superior to closed lists. The literature argues that preference votes under open lists generate extra incentives and candidates at the top and bottom of a closed list face little incentives to exert effort. However, we show that, under closed lists, incentives are strong for politicians close to the *marginal* position of the list – this is the list position that corresponds to the number of seats a party is expected to win in the symmetric equilibrium of the game – a fact that the literature seems to have overlooked.

The interplay of the two differences between ballot structures implies the following:

Proposition 6 *Closed lists lead to higher party output than open lists PR,* $E^*_{CL} \geq E^*_{OL}$ *when* $\beta \leq 2 - 2\sigma$ *and* ρ *is small enough.*

Open lists lead to higher party output than closed lists, $E^*_{OL} \geq E^*_{CL}$*, when* $\beta \geq 2 - 2\sigma$ *or when* $\beta \leq 2 - 2\sigma$ *and* ρ *is large enough.*

The two types of ballot yield the same party's output, $E^*_{OL} = E^*_{CL}$*, when* $\beta = 2 - 2\sigma$ *and* $\rho = 0$*.*

To interpret the findings of Proposition 6 in the context of elections, we can associate standard and routine electoral issues with low effort complementarity (low σ) and low convexity of the cost of effort function (low β). Indeed, when the election is about 'conventional' issues such as defence or the minimum wage, say, candidates can simply stand behind their party's official position on such conventional issues. To the contrary, when electoral issues are novel and complex, such as how to deal with ethical questions that could be generated by the further development of genetical engineering or AI, say, parties need to use a portfolio of diverse skills and competencies. Candidates' efforts become complementary, as team work and brainstorming becomes highly relevant across the different skill sets and areas of specialization of candidates, and the cost of the required efforts should increase faster, as candidates find themselves in largely uncharted territory.

When voters are poorly informed ($\rho = 0$), the comparison between the two ballot structures follows the same condition as in Theorem 3. From Eq (11), we see that an improvement in voter information – an increase in ρ – leads to an increase in effort. When $\beta \leq 2 - 2\sigma$, open lists lead to higher electoral output than closed lists, and an increase in ρ reinforces the effect. When $\beta \geq 2 - 2\sigma$, closed lists lead to higher effort when $\rho = 0$. An increase in ρ can however reverse the ranking of effort as E^*_{OL} is increasing in ρ while E^*_{CL} does not change with ρ. This finding suggests that open-list PR is particularly efficient when voters are well-informed about individual candidates. Nevertheless, we should also stress that this benefit of open lists requires that candidates's individual efforts are used to contribute to the party's success and not to build a personal group of supporters to ensure the candidate gets elected via preference votes. To say it another way, in our model, we impose that there is congruence between personal success and party success. Yet, strategies to build a personal basin of support to the detriment of one's party fortunes are observed in many open-list systems; see for example Myerson (1993), Ames (1995a,b and 2002) and Zittel (2015).

Two real world determinants of the value of ρ are the size of electoral districts and the type and level of the election. Carey and Hix (2011 p. 385) argue that voters in "[...] a lower magnitude multimember district say, with magnitude of two to six should have a relatively clear preference ordering over the candidates or lists [...]. By contrast, in a high magnitude multimember district say, with magnitude above 10 [...] voters are unlikely to have clear preference ranking over all the options [...]." Thus, our theory offers one rationale for the use of closed lists instead of open ones in democracies in which district magnitude is large.[9] Similarly, there is evidence that voters in second-tier elections are not well informed. For example, Hobolt and Wittrock (2011, p. 39) conclude their study about voting behavior in elections for the European Parliament by stating that "voters are likely to base their EP vote choices on sincere preferences relating to the dominant dimension of contestation in *national* politics"(emphasis added).

5.3 Choice of electoral system

The design of electoral systems is at the center of many public debates. Many countries, like France, are thinking about moving towards PR but participants in the debate find it difficult to agree on the type of PR to adopt. Proposition 6 has direct implications in terms of constitutional design. A benevolent constitution would choose the electoral system that maximizes electoral outputs. If voters are well informed about candidates' efforts, then open lists appear superior to closed ones.

[9] An analysis of the determinants of district magnitude across advanced democracies and the associated theory of the optimal size of electoral districts is beyond the scope of this article, but is certainly an interesting avenue for future research.

If voters are poorly informed about politicians' effort, open-list PR loses its edge. In that case, the convexity of the cost of effort and the complementarity of candidates' efforts should guide the choice of electoral system.

The argument follows our assumption that effort exerted by candidates is beneficial to society. To justify this choice, remark that in most elections, there is a gap of several months between the moment parties announce their list and the day of the election. Such a gap gives candidates time to improve their party's electoral platform, in which case effort provision is beneficial to society at large. Nevertheless, we could also interpret effort as advertising resources spent during the campaign to convince voters. As campaign spending does not enhance society's welfare, a good constitution would aim at minimizing this type of effort. In this case, all the results still apply but their implications need to be reversed.

Yet, in practice, the party leadership also has its say in the way candidates are selected. A striking example is Colombia. Since the 2003 reform, parties can choose the type of ballot to use. They can opt for closed or open lists, and the choice can vary across districts; see Shugart, Moreno and Fajardo (2006) and Hangartner, Ruiz and Tukiainen (2019) for more on this case. Less extreme examples can be found in many countries in which parties adopt strategies and practices as a strategic reaction to the electoral rule. For instance, Italy before the Mani Pulite scandal of 1992 was officially using open-list PR. However, in practice, many parties were using methods resembling a closed-list system. Katz (1985) argues that the Communist party was closely controlling the list of candidates and would give instructions to their partisans about preference votes. The final outcome would look like the one under a closed list, as the party decided the ranking of candidates and voters implemented it. The opposite situation also happens in countries using closed-list PR. Some parties organize primary elections to decide the names and positions of list candidates. Even if the set of voters in the primary and in the general election is not exactly the same, organizing a primary clearly shifts the electoral system towards an open-list one. For instance, in recent Israeli elections, several parties, including Likud, Labor, the Jewish Home, and Meretz had systems in which the leadership and most candidates on their lists were first elected in primary elections. Similarly, some small Turkish parties use primary elections to set up their list of candidates.

To study such strategies, we add a stage to the model, in which the party leadership chooses between the egalitarian rule and a closed list.[10] In the second stage, after observing the choice of all

[10] All results can be adapted to account for voters being informed about individual candidates' choices ($\rho > 0$), at the cost of additional algebra.

parties, candidates choose how much effort to exert. We show that, in equilibrium, the condition driving the parties' choice of the allocation rule is the same as in Theorem 3.

Proposition 7 *In the subgame perfect equilibrium of the two-stage game, parties choose a closed list if and only if* $\beta < 2 - 2\sigma$ *and choose the egalitarian rule if and only if* $\beta > 2 - 2\sigma$.

6 Conclusion

We studied a multi-prize contest between teams of individual players when prizes are fully indivisible and each team member can win at most one prize. We considered the list rule – in which the prizes a team won are allocated to members in the order of a pre-defined list – and the egalitarian rule – in which all team members have the same probability of receiving one of the prizes their team won. We showed that the degree of complementarity between individual efforts in the team production function and the degree of convexity of the cost of effort function determine which allocation rule maximizes team output. We also showed that only the egalitarian rule may give rise to the group size paradox.

We then applied our model to study incentives under list PR. The list rule can be applied directly to elections under closed-list PR. The egalitarian rule is the limit case of open-list PR when voters are not informed about the choices of individual politicians. Comparing closed- and open-list PR, our main finding runs parallel to that in our contest model: closed lists can be superior to open ones in terms of incentives to exert effort. This contrasts quite markedly with the unconditional negative views closed-list PR suffers from in the existing literature.

The team contest model is tractable and is amenable to many extensions and applications. For instance, Crutzen and Sahuguet (2019) adapt the present set-up to compare politicians' effort in elections under plurality rule and closed-list PR when political parties play an active role in the selection of their candidates. They find that the received wisdom according to which that plurality rule provides incentives more efficiently than PR may need qualification.

The main limitation of our contest model is its symmetry: team members have the same effort productivity (or cost of effort) and are competing for prizes of identical value. Crutzen et al. (2019) allows for heterogeneous productivity and allow heterogeneity in the value of the prizes. The paper then studies how teams should constitute their lists and which position the most productive individuals should occupy on the list.

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Appendix: proofs

Proof of Proposition 1

We start by proving two useful lemmas.

Lemma 1: $\frac{dE_j}{de_{ij}} = \left(\frac{E_j}{e_{ij}}\right)^{\sigma}$

Proof:

$$\frac{dE_j}{de_{ij}} = \frac{1}{1-\sigma}\left(1-\sigma\right)\left(e_{ij}\right)^{-\sigma}\left(\sum_{i=1}^{n}\left(e_{ij}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}-1} = \left(e_{ij}\right)^{-\sigma}\left(\sum_{i=1}^{n}\left(e_{ij}\right)^{1-\sigma}\right)^{\frac{\sigma}{1-\sigma}} = \left(\frac{E_j}{e_{ij}}\right)^{\sigma}$$

■

Lemma 2

$\sum_{k=m}^{n} C_k^n \left(kp^{k-1}\left(1-p\right)^{n-k} - \left(n-k\right)\left(1-p\right)^{n-k-1}p^k\right) = mC_m^n p^{m-1}\left(1-p\right)^{n-m}$

Proof:

Consider the second term within the summation sign: $\left(n-k\right)C_k^n\left(1-p\right)^{n-k-1}p^k$. Using the identity $\left(n-k\right)C_k^n = \left(k+1\right)C_{k+1}^n$ we can write it as $\left(n-k\right)C_k^n\left(1-p\right)^{n-k-1}p^k = \left(k+1\right)C_{k+1}^n\left(1-p\right)^{n-k-1}p^k$, which corresponds to the first term within the summation sign for the index $k+1$. These two terms cancel out, leaving only the first and last term of the sum. The first term is $mC_m^n p^{m-1}\left(1-p\right)^{n-m}$. The last term is equal to zero. ■

The individual mth position on the list maximizes:

$$\sum_{k=m}^{n} C_k^n \left(\frac{E_j^r}{\sum_{t=1}^{T} E_t^r}\right)^k \left(1-\frac{E_j^r}{\sum_{t=1}^{T} E_t^r}\right)^{n-k} V - \frac{e_{mj}^{\beta}}{\beta}$$

Denoting $p_j = \frac{E_j^r}{\sum_{t=1}^{T} E_t^r}$, the first-order condition is:

$$V\sum_{k=m}^{n}\frac{dE_j}{de_{mj}}\frac{dp_j}{de_{mj}}C_k^n\left(k\left(p_j\right)^{k-1}\left(1-p_j\right)^{n-k} - \left(n-k\right)\left(p_j\right)^k\left(1-p_j\right)^{n-k-1}\right) - \left(e_{mj}\right)^{\beta-1} = 0$$

As

$$\frac{dp_j}{de_{mj}} = r\frac{E_j^{r-1}\sum_{t=1}^{T}E_t^r - E_j^{r-1}E_j^r}{\left(\sum_{t=1}^{T}E_t^r\right)^2} = \frac{p_j\left(1-p_j\right)}{E_j}$$

we get:

$$rV\sum_{k=m}^{n}\frac{dE_j}{de_{mj}}\frac{p_j\left(1-p_j\right)}{E_j}C_k^n\left(k\left(p_j\right)^{k-1}\left(1-p_j\right)^{n-k} - \left(n-k\right)\left(p_j\right)^k\left(1-p_j\right)^{n-k-1}\right) - \left(e_{mj}\right)^{\beta-1} = 0.$$

At a symmetric equilibrium, using lemmas 1 and 2, the first-order condition simplifies to:

$$\left(\frac{E}{e_m}\right)^{\sigma} \frac{m}{E} C_m^n \left(\frac{1}{T}\right)^m \left(\frac{1-T}{T}\right)^{n-m+1} rV - e_m^{\beta-1} = 0.$$

Thus

$$\begin{aligned}
(e_m)^{\beta+\sigma-1} &= E^{\sigma-1} m C_m^n \left(\frac{1}{T}\right)^m \left(\frac{1-T}{T}\right)^{n-m+1} rV, \\
e_m &= \left(E^{\sigma-1} m C_m^n \left(\frac{1}{T}\right)^m \left(\frac{1-T}{T}\right)^{n-m+1} rV\right)^{\frac{1}{\beta+\sigma-1}}.
\end{aligned}$$

Team output is thus:

$$\begin{aligned}
E &= \left(\sum_{k=1}^{n} \left(E^{\sigma-1} k C_k^n \left(\frac{1}{T}\right)^k \left(\frac{1-T}{T}\right)^{n-k+1} rV\right)^{\frac{1-\sigma}{\beta+\sigma-1}}\right)^{\frac{1}{1-\sigma}}, \\
E^{\frac{\beta}{\beta+\sigma-1}} &= \left(\sum_{k=1}^{n} \left(k C_k^n \left(\frac{1}{T}\right)^k \left(\frac{1-T}{T}\right)^{n-k+1} rV\right)^{\frac{1-\sigma}{\beta+\sigma-1}}\right)^{\frac{1}{1-\sigma}}, \\
E &= (rV)^{\frac{1}{\beta}} \left(\sum_{k=1}^{n} \left(k C_k^n \left(\frac{1}{T}\right)^k \left(\frac{1-T}{T}\right)^{n-k+1}\right)^{\frac{1-\sigma}{\beta+\sigma-1}}\right)^{\frac{\beta+\sigma-1}{\beta(1-\sigma)}}.
\end{aligned}$$

And individual effort of team member m is:

$$\begin{aligned}
e_m^* &= \left(E_{CL}^{*\sigma-1} m C_m^n \left(\frac{1}{T}\right)^m \left(\frac{1-T}{T}\right)^{n-m+1} rV\right)^{\frac{1}{\beta+\sigma-1}}, \\
&= \left(\frac{rV}{\sum_{k=1}^{n} \left(k C_k^n \left(\frac{1}{T}\right)^k \left(\frac{1-T}{T}\right)^{n-k+1}\right)^{\frac{1-\sigma}{\beta+\sigma-1}}}\right)^{1/\beta} \left(m C_m^n \left(\frac{1}{T}\right)^m \left(\frac{1-T}{T}\right)^{n-m+1}\right)^{\frac{1}{\beta+\sigma-1}}.
\end{aligned}$$

We now look at second-order conditions. We start with some relabeling: team member m maximizes:

$$G_{mj}(p_j(e_{mj}))V - c(e_{mj}),$$

where $G_{mj}=\sum_{k=m}^{n}\binom{n}{k}p_j^k\left(1-p_j\right)^{n-k}$ denotes the probability that member m gets a prize. The first-order condition is:

$$G'_{mj}\cdot p'\left(e_{mj}\right)V-\left(e_{mj}\right)^{\beta-1}=0,$$

while the second-order condition is:

$$G''_{mj}\cdot\left(p'\left(e_{mj}\right)\right)^2V+G'_{mj}\cdot p''\left(e_{mj}\right)V-\left(\beta-1\right)\left(e_{mj}\right)^{\beta-2}.$$

If G_{mj} is globally concave, then the second-order condition is satisfied, as $p''\left(\cdot\right)\leq 0$. An issue can arise when G_{mj} can be locally convex. For instance, $G_{nj}=p_j^n$ is globally convex.

From the FOC, we have $G'_{mj}\cdot p'\left(e_{mj}\right)V=e_{mj}^{\beta-1}$ or $G'_{mj}V=\frac{\left(e_{mj}\right)^{\beta-1}}{p'\left(e_{mj}\right)}$

We also have:

$$\begin{aligned}
p'(e) &= r\frac{p\left(1-p\right)}{E}\left(\frac{E}{e}\right)^{\sigma}\\
p''\left(e\right) &= r\left(\left(1-2p\right)\frac{E^{\sigma-1}}{e^{\sigma}}p'(e)+\left(1-p\right)p\left(\left(\sigma-1\right)E^{\sigma-2}\left(\frac{E}{e}\right)^{\sigma}e^{-\sigma}+E^{\sigma-1}\left(-\sigma e^{-\sigma-1}\right)\right)\right)\\
&= r\left(\left(1-2p\right)r\frac{p\left(1-p\right)}{E}\left(\frac{E}{e}\right)^{\sigma}\frac{E^{\sigma-1}}{e^{\sigma}}+\left(1-p\right)p\left(\left(\sigma-1\right)E^{\sigma-2}\left(\frac{E}{e}\right)^{\sigma}e^{-\sigma}+E^{\sigma-1}\left(-\sigma e^{-\sigma-1}\right)\right)\right)\\
&= rpE^{\sigma-2}e^{-2\sigma}\left(p-1\right)\left((2p-1)rE^{\sigma}+\left(1-\sigma\right)E^{\sigma}+\sigma Ee^{2\sigma}e^{-\sigma-1}\right).
\end{aligned}$$

Thus:

$$\begin{aligned}
\frac{p''\left(e\right)}{p'(e)} &= \frac{pE^{\sigma-2}e^{-2\sigma}\left(p-1\right)\left((2p-1)rE^{\sigma}+\left(1-\sigma\right)E^{\sigma}+\sigma Ee^{2\sigma}e^{-\sigma-1}\right)}{\frac{p(1-p)}{E}\left(\frac{E}{e}\right)^{\sigma}}\\
&= -\left(\frac{E}{e}\right)^{\sigma}\frac{1}{E}\left((2p-1)r+\left(1-\sigma\right)+\sigma\left(\frac{e}{E}\right)^{\sigma-1}\right).
\end{aligned}$$

Manipulate the second-order conditions as follows:

$$\begin{aligned}
& G''_{mj}\left(p'\left(e_{mj}\right)\right)^2V+G'_{mj}p''\left(e_{mj}\right)V-\left(\beta-1\right)\left(e_{mj}\right)^{\beta-2}\\
= & VG''_{mj}\left(r\frac{p_j\left(1-p_j\right)}{E_j}\left(\frac{E_j}{e_{mj}}\right)^{\sigma}\right)^2+p''\left(e_{mj}\right)\left(e_{mj}\right)^{\beta-1}/p'\left(e_{mj}\right)-\left(\beta-1\right)\left(e_{mj}\right)^{\beta-2}\\
= & VG''_{mj}\left(r\tfrac{p_j(1-p_j)}{E_j}\left(\tfrac{E_j}{e_{mj}}\right)^{\sigma}\right)^2-\left(e_{mj}\right)^{\beta-1}\left(\tfrac{E_j}{e_{mj}}\right)^{\sigma}\tfrac{1}{E_j}\left(\left(2p_j-1)r+1-\sigma+\sigma\left(\tfrac{e_{mj}}{E_j}\right)^{\sigma-1}\right)+\left(\beta-1\right)\tfrac{\left(e_{mj}\right)^{\sigma-1}}{E_j^{\sigma-1}}\right)\\
= & \tfrac{1}{E_j}\left(\tfrac{E_j}{e_{mj}}\right)^{\sigma}\left(r^2VG''_{mj}\left(p_j\left(1-p_j\right)\right)^2\tfrac{1}{E_j}\left(\tfrac{E_j}{e_{mj}}\right)^{\sigma}-\left(e_{mj}\right)^{\beta-1}\left((2p_j-1)r+\left(1-\sigma\right)+\left(\sigma+\beta-1\right)\left(\frac{e_{mj}}{E_j}\right)^{\sigma-1}\right)\right)
\end{aligned}$$

The second-order conditions are thus satisfied for a choice of effort that solves the first-order conditions if:

$$r^2 V G''_{mj} \left(p_j \left(1-p_j\right)\right)^2 \frac{1}{E_j} \left(\frac{E_j}{e_{mj}}\right)^{\sigma} - \left(e_{mj}\right)^{\beta-1} \left((2p_j-1)r + (1-\sigma) + (\sigma+\beta-1)\left(\frac{e_{mj}}{E_j}\right)^{\sigma-1}\right) \leq 0$$

We have, from the first order condition and our derivations above:

$$rV \frac{1}{E_j} \left(\frac{E_j}{e_{mj}}\right)^{\sigma} = \frac{\left(e_{mj}\right)^{\beta-1}}{mC^n_m p^m_j \left(1-p_j\right)^{n-m+1}}$$

and

$$\left(\frac{e_{mj}}{E_j}\right)^{\sigma-1} = \frac{\sum_{k=1}^{n} \left(kC^n_k p^k_j \left(1-p_j\right)^{n-k+1}\right)^{(1-\sigma)/(\sigma+\beta-1)}}{\left(mC^n_m p^m_j \left(1-p_j\right)^{n-m+1}\right)^{(1-\sigma)/(\sigma+\beta-1)}}$$

and

$$\begin{aligned} G'_m &= \sum_{k=m}^{n} \binom{n}{k} p^{k-1} \left(1-p\right)^{n-k-1} \left(k-np\right) = mp^{m-1} \binom{n}{m} \left(1-p\right)^{n-m} \\ G''_m &= m \binom{n}{m} \left(p^{m-1}\left(\left(m-n\right)\left(1-p\right)^{n-m-1}\right) + \left(p^{m-2}\left(m-1\right)\right)\left(1-p\right)^{n-m}\right) \end{aligned}$$

As

$$\frac{p^{m+1}\left(\left(m-n\right)\left(1-p\right)^{n-m+1} + p^m \left(m-1\right)\right)\left(1-p\right)^{n-m+2}}{p^m \left(1-p\right)^{n-m+1}} = m + p - np - 1,$$

the second-order conditions depend on the sign of:

$$r\left(m+p-np-1\right) - \left((2p_j-1)r + (1-\sigma) + (\sigma+\beta-1)\left(\frac{\sum_{k=1}^{n}\left(kC^n_k p^k (1-p)^{n-k+1}\right)^{(1-\sigma)/(\sigma+\beta-1)}}{\left(mC^n_m p^m (1-p)^{n-m+1}\right)^{(1-\sigma)/(\sigma+\beta-1)}}\right)\right).$$

The first term is positive for individuals whose position in the list is above the expected number of prizes won by the team. So for individuals high on the list, the second-order conditions are satisfied. The second term is negative.

We see that as r becomes smaller the first term goes to zero while the second term remains negative. Thus a small value of r is sufficient for the second-order condition to be satisfied and for an equilibrium in pure strategies to exist. For larger values of r, the condition is not necessarily satisfied, though.

We also have

$$Lim_{\beta\to\infty}(\sigma+\beta-1)\left(\frac{\left(\sum_{k=1}^{n}\left(kC_k^n p^k(1-p)^{n-k+1}\right)^{(1-\sigma)/(\sigma+\beta-1)}\right)}{\left(mC_m^n p^m(1-p)^{n-m+1}\right)^{(1-\sigma)/(\sigma+\beta-1)}}\right) = \infty.$$

$$Lim_{\sigma\to 1}(\sigma+\beta-1)\left(\frac{\left(\sum_{k=1}^{n}\left(kC_k^n p^k(1-p)^{n-k+1}\right)^{(1-\sigma)/(\sigma+\beta-1)}\right)}{\left(mC_m^n p^m(1-p)^{n-m+1}\right)^{(1-\sigma)/(\sigma+\beta-1)}}\right) = n\beta$$

So the SOCs are satisfied when β is large enough or when σ is close to 1.

In the main text, parameters r, σ and β are assumed to satisfy the second order conditions. ■

Proof of Proposition 2

As $\sum_{k=1}^{n} P_j(k)k = np_j^k$, the maximization problem can be rewritten as:

$$Vp_j^k - \frac{(e_{ij})^\beta}{\beta}.$$

The first-order conditions are:

$$rV\left(\frac{E_j}{e_{ij}}\right)^\sigma \frac{p_j(1-p_j)}{E_j} - (e_{ij})^{\beta-1} = 0$$

As $p''(e) < 0$, the second-order conditions are satisfied. At a symmetric equilibrium we get:

$$rV\frac{T-1}{T^2}(E_j)^{\sigma-1} = (e_{ij})^{\sigma+\beta-1},$$

which yields

$$e_{ij} = \left(r\frac{V}{n}\frac{T-1}{T^2}\right)^{1/\beta}.$$

■

Proof of Theorem 3

Team output is higher under the egalitarian rule when:

$$n^{\frac{\beta+\sigma-1}{\beta(1-\sigma)}} > \left(\sum_{k=1}^{n}\left(kC_k^n\left(\frac{1}{T}\right)^{k-1}\left(\frac{1-T}{T}\right)^{n-k}\right)^{\frac{1-\sigma}{\beta+\sigma-1}}\right)^{\frac{\beta+\sigma-1}{\beta(1-\sigma)}},$$

which simplifies to:

$$n > \sum_{k=1}^{n}\left(kC_k^n\left(\frac{1}{T}\right)^{k-1}\left(\frac{1-T}{T}\right)^{n-k}\right)^{\frac{1-\sigma}{\beta+\sigma-1}}.$$

As

$$\sum_{k=1}^{n} kC_k^n \left(\frac{1}{T}\right)^{k-1} \left(\frac{1-T}{T}\right)^{n-k} = n,$$

we can use Jensen's inequality to complete the proof.

As the function $g(x) = x^{\frac{1-\sigma}{\beta+\sigma-1}}$ is concave when $\beta \leq 2 - 2\sigma$, we conclude that so effort is higher under the egalitarian rule when $\beta \geq 2 - 2\sigma$.■

Proof of Proposition 4

Under the egalitarian allocation rule, with $l > n$ politicians, candidates maximize:

$$\sum_{k=1}^{n} P_j(k)\frac{k}{l}V - \frac{e_{ij}^{\beta}}{\beta} = \frac{E_j^r}{\sum_{t=1}^{T} E_t^r} V \frac{n}{l} - \frac{e_{ij}^{\beta}}{\beta} \tag{13}$$

Taking the first-order condition and then imposing symmetry, we get:

$$r\frac{T-1}{T^2 E} V n^{\frac{\sigma}{1-\sigma}} \frac{n}{l} - e^{\beta-1} = 0.$$

and total output is:

$$\begin{aligned} E &= \left\{ \sum_{k=1}^{l} \left[\left[r\frac{T-1}{T^2 E} V n^{\frac{\sigma}{1-\sigma}} \frac{n}{l} \right]^{\frac{1}{\beta-1}} \right]^{1-\sigma} \right\}^{\frac{1}{1-\sigma}} \\ &= l^{\frac{1}{1-\sigma}} \left(r\frac{T-1}{T^2 E}\frac{n}{l} V \right)^{\frac{1}{\beta-1}} n^{\frac{\sigma}{(1-\sigma)(\beta-1)}} \\ &= l^{\frac{\beta+\sigma-2}{\beta(1-\sigma)}} \left(r\frac{(T-1)V}{T^2} \right)^{\frac{1}{\beta}} n^{\frac{\sigma+1}{\beta(1-\sigma)}}. \end{aligned}$$

Thus, whether total output increases or decreases with l depends on the sign of $\beta + \sigma - 2$. ■

Proof of Proposition 5

The first order condition to the problem of candidate i in party j:

$$\left(\sum_{m=1}^{n} \frac{\partial P_j(l)}{\partial e_{ij}} Q_i(m) + \sum_{m=1}^{n} P_j(m) \frac{\partial Q_i(m)}{\partial e_{ij}} \right) V = e_i^{\beta-1}.$$

As n candidates are competing, the equilibrium probability of being offered slot m on the list is $Q^*(m) = \frac{m}{n}$ and $P_j^*(m) = C_m^n \left(\frac{1}{T}\right)^m \left(\frac{T-1}{T}\right)^{n-m}$. Then, in the symmetric equilibrium and dropping subscripts we get:

$$\sum_{m=1}^{n} \frac{\partial P_L(l)}{\partial e} Q_i(m) = n^{\frac{\sigma}{1-\sigma}} \sum_{m=1}^{n} \frac{T-1}{T^2 E} r C_m^n \left(m \left(\frac{1}{T}\right)^{m-1} \left(\frac{T-1}{T}\right)^{n-m} - (n-m) \left(\frac{1}{T}\right)^m \left(\frac{T-1}{T}\right)^{n-m-1} \right) \frac{m}{n}$$

Using the moment generating function for the binomial distribution, we have:

$$\sum_{m=1}^{n} C_m^n \left(\frac{1}{T}\right)^m \left(\frac{T-1}{T}\right)^{n-m} m = \frac{n}{T},$$

and

$$\sum_{m=1}^{n} C_m^n \left(\frac{1}{T}\right)^m \left(\frac{T-1}{T}\right)^{n-m} m^2 = n(n-1)\left(\frac{1}{T}\right)^2 + \frac{n}{T}.$$

We get:

$$\sum_{m=1}^{n} \frac{\partial P_L(l)}{\partial e} Q_i(m) = n^{\frac{\sigma}{1-\sigma}} \frac{r(T-1)}{T^2 E},$$

Finally:

$$\frac{dQ_i(m)}{de_{iL}} = \frac{\rho}{e^*}\left(1 - \frac{m}{n}\right) \sum_{j=1}^{m} \frac{1}{n-j+1}.$$

In the symmetric equilibrium, $E^* = n^{\frac{1}{1-\sigma}} e^*$. Thus we get:

$$e^* = \left\{ \frac{T-1}{nT^2} rV + \rho V \sum_{m=1}^{n} C_m^n \left(\frac{1}{T}\right)^m \left(\frac{T-1}{T}\right)^{n-m} \left(1 - \frac{m}{n}\right) \sum_{j=1}^{m} \frac{1}{n-j+1} \right\}^{\frac{1}{\beta}},$$

and

$$E^* = n^{\frac{1}{1-\sigma}} \left\{ \frac{T-1}{nT^2} rV + \rho V \sum_{m=1}^{n} C_m^n \left(\frac{1}{T}\right)^m \left(\frac{T-1}{T}\right)^{n-m} \left(1 - \frac{m}{n}\right) \sum_{j=1}^{m} \frac{1}{n-j+1} \right\}^{\frac{1}{\beta}}.$$

■

Proof or Proposition 6

Set ρ to 0. Then the proof of Theorem 3 applies directly. To complete the proof, simply remark that team output under open list is increasing in ρ. ■

Proof of Proposition 7

Given the allocation rules chosen by other parties, a party chooses the allocation rule that maximizes its output. We compare party j's electoral output under both allocation rules, given the other parties' allocation rules. Let $p_j = \frac{E_j^r}{\sum_{t=1}^{T} E_t^r}$. Manipulating the first order condition reported in the proof of Proposition 2, party j's output under the egalitarian rule is:

$$E_j^E = \left(rp_j \left(1 - p_j\right) V\right)^{\frac{1}{\beta}} n^{\frac{\beta+\sigma-1}{\beta(1-\sigma)}},$$

Exploiting Proposition 1 and the lemmas in its proof, party j's output under the list rule is:

$$E_j^L = \left(\sum_{k=1}^{n} \left(kC_k^n p_j^{k-1} \left(1 - p_j\right)^{n-k}\right)^{\frac{1-\sigma}{\beta+\sigma-1}}\right)^{\frac{\beta+\sigma-1}{\beta(1-\sigma)}} \left(rp_j \left(1 - p_j\right) V\right)^{\frac{1}{\beta}}.$$

We now divide the two outputs, and use Jensen's inequality to prove th result. Parties have a dominant strategy to choose the egalitarian rule if and only if $\beta > 2 - 2\sigma$. If $\beta = 2 - 2\sigma$, both rules yield the same payoff.

Appendix B (online appendix, not for publication)

Effort-independent, team output maximizing rules

We show that the list and egalitarian rule are the two effort-independent rules that maximize team's output under a weak monotonicity constraint. To simplify notation, we focus on a contest between two teams, let efforts be perfect substitutes, $\sigma = 0$ and assume second order conditions are satisfied.

An allocation rule can be represented as a $n * (n+1)$ matrix of weights $[\lambda_{ik}]$. Entry λ_{ik} corresponds to the probability that team member i gets a seat when their team has won k prizes. Feasibility constraints require that $0 \leq \lambda_{ik} \leq 1$ and $\sum_{i=1}^{n} \lambda_{ik} = k$. Under a monotonic rule, the probability that a team member wins a prize needs to be (weakly) increasing in the number of prizes won by their team, that is $\lambda_{ik} \leq \lambda_{ik+1}$ for any i and k.

The egalitarian allocation rule can be represented as a matrix in which each column has equal entries $\lambda_{ik} = k/n$. The list rule can be represented as a matrix with $\lambda_{ik} = 0$ if $i > k$ and $\lambda_{ik} = 1$ if $i \leq k$.

We then have:

Proposition 8 *Among monotonic rules, the egalitarian rule maximizes team output when $\beta > 2$; the list rule maximizes team output when $\beta < 2$; both rules maximize team output when $\beta = 2$.*

The intuition behind the result is simple. The intrateam prize allocation rule determines individual incentives and thus effort choices. When $\beta > 2$, it is efficient to equalize incentives across members within a team, while when $\beta < 2$, it is efficient to make incentives as strong as possible for some members.

Member i of team j maximizes $V \sum_{k=1}^{n} \lambda_{ik} P_j(k) - \frac{e_{ij}^{\beta}}{\beta}$.

The first order condition is:

$$e_i = \max \left\{ \left(V \sum_{k=1}^{n} \tfrac{dE}{de_{ij}} \tfrac{E_i}{(E_1+E_2)^2} \lambda_{ik} C_k^n \left(k p_j^{k-1} (1-p_j)^{n-k} - (n-k) p_j^k (1-p_j)^{n-k-1} \right) \right)^{\frac{1}{\beta-1}}, 0 \right\}.$$

At a symmetric Nash equilibrium, we get:

$$e_i = \max \left\{ \left(V \sum_{k=1}^{n} \frac{dE}{de_i} \frac{1}{4E} \lambda_{ik} C_k^n \left(\frac{1}{2} \right)^{n-1} (2k-n) \right)^{\frac{1}{\beta-1}}, 0 \right\}.$$

Simplifying and forgetting for now the non-negativity constraint on effort, we get:

$$E = \left(\sum_i \left(\frac{V}{2^{n+1}} \sum_{k=1}^{n} \lambda_{ik} C_k^n (2k - n) \right)^{\frac{1}{\beta-1}} \right)^{\frac{\beta-1}{\beta}}$$

Denoting $\Delta_{ik} = \lambda_{i(n-k)} - \lambda_{ik}$ and exploiting the fact that $\lambda_{i0} = 0$, we can rewrite team's output as:

$$E = \left(\sum_i \left(\frac{V}{2^{n+1}} \sum_{k=0}^{\lfloor n/2 \rfloor} \Delta_{ik} C_k^n (n - 2k) \right)^{\frac{1}{\beta-1}} \right)^{\frac{\beta-1}{\beta}}.$$

The constraints take two forms. First, there is the budget constraint: $0 \leq \sum_i \Delta_{ik} \leq 1$. Second, individual probabilities must be between 0 and 1, implying that: $-1 \leq \Delta_{ik} \leq 1$. Thus, a team's output is maximized when $\sum_i \left[\sum_{k=0}^{\lfloor n/2 \rfloor} \Delta_{ik} C_k^n (n - 2k) \right]^{\frac{1}{\beta-1}}$ is maximized. There are two cases to consider depending on the value of β.

Case 1: $\beta \geq 2$ (concave objective function)

When $\beta \geq 2$, we have that $\frac{1}{\beta-1} \leq 1$ and a team's output is the sum over team members of a concave function of their individual efforts.

The team's objective is given by

$$E^{\frac{\beta}{\beta-1}} = \sum_i \left(\frac{V}{2^{n+1}} \sum_{k=0}^{\lfloor n/2 \rfloor} \max \left(\Delta_{ik} C_k^n (n - 2k), 0 \right) \right)^{\frac{1}{\beta-1}},$$

and the budget constraint implies $\sum_{i=1}^{n} \Delta_{ik} = n - k$.

Note first that the monotonicity constraint imposes that $\Delta_{ik} \geq 0$ for any i and k. The first order conditions to this maximization problem (forgetting for now about the non-negativity constraints) yield that the partial derivatives $\frac{\partial E^{\frac{\beta}{\beta-1}}}{\partial \Delta_{ik}}$ and $\frac{\partial E^{\frac{\beta}{\beta-1}}}{\partial \Delta j}$ are equal to μ, the Lagrange multiplier associated with the budget constraint, for any i and j. To meet these conditions, the coefficients Δ_{ik} must be equal across team members. The optimal Δ_{ik} that solve the unconstrained problem are also such that $0 \leq \Delta_{ik} \leq 1$. The solution of the unconstrained problem is thus also a solution to the optimization problem with the constraint. Thus, the egalitarian rule is the optimal rule when $\beta \geq 2$.

Case 2: $\beta \leq 2$ (convex objective function)

Assume that in column k there exists λ_{ik} with $0 < \lambda_{ik} < 1$. By construction, this means that there exists λ_{jk} with $j \neq i$, such that $0 < \lambda_{jk} < 1$. Now, suppose without loss of generality that $\sum_{k=1}^{n} \lambda_{ik} C_k^n (2k - n)$ is greater than $\sum_{k=1}^{n} \lambda_{jk} C_k^n (2k - n)$. Given that the objective function is convex in $\sum_{k=1}^{n} \lambda_{mk} C_k^n (2k-n)$, increasing the larger of the two terms above and decreasing the smaller one increases the sum. The only way the allocation rule cannot be improved upon is when it is deterministic with all the entries λ_{mk} being either 0 or 1.

Applying the monotonicity requirement implies that the list rule is the optimal rule. Indeed, for $k = 1$, a deterministic rule gives the prize to one team member. To be monotonic, the rule also needs to allocate a prize to that member when more prizes are won, when $k > 1$. This means that this member has the top spot on the list. The reasoning is similar for all following prizes and thus confirms that the optimal monotonic rule is the list rule. ■

Contractible efforts

We now extend the analysis of contractible efforts. Teams allocate prizes according to the efforts of its members. Effort is determined by each member's participation constraint. Under the egalitarian rule, the contract between the team and each member specifies that, provided the member exerts at least e^*, their chance of getting one of the m prizes won by their team is equal to m over the number of members who honored their team contract. In equilibrium, this chance is thus m/n. Any deviation below e^* makes that probability go to zero. As before, the egalitarian rule treats all members equally, and when efforts are contractible these are pinned down by the participation constraint $e^\beta/\beta = V/T$, which yields $e = (\beta V/T)^{1/\beta}$.

Under the list rule, the contract still assigns to each member a rank in the list, but now also specifies a minimal effort level associated with each rank. If a member exerts the specified effort (or more), they get a prize if their team wins a number of prizes equal to at least their rank. If they exert less effort or if the team wins fewer prizes than their rank, they get no prize. Because the participation constraint pins down the level of effort, and the probability of winning a prize is increasing in one's ranking, all else equal, we have that effort levels are decreasing in list rank, whereas the distribution of efforts is hump-shaped when efforts are not contractible. Indeed, we get $e_m = \left(\sum_{k=m}^{n} C_k^n \left(\frac{1}{T}\right)^k \left(\frac{T-1}{T}\right)^{n-k} \beta V\right)^{1/\beta}$, which increases in m. We then have:[11]

Proposition 9 *When efforts are contractible, the egalitarian allocation rule leads to higher team output than the list rule for all values of parameters β and σ.*

[11] For the contract to be credible, the team must have a list composed of at least $n+1$ members.The team uses these additional members as a credible threat to enforce the contract with its first n members by committing to offering these extra members a prize in case the number of prizes it wins exceeds the size of the subgroups of n members who honored their contract.

Proof. In a symmetric equilibrium, when both teams use the egalitarian rule, team members have a probability $1/T$ of winning a prize. The participation constraint $e_i^\beta/\beta = V/T$ pins down individual effort, which is given by:

$$e_i = (\beta V/T)^{1/\beta}.$$

This leads to a team's output being equal to

$$E = \left(\sum_{i=1}^{n} e_i^{1-\sigma}\right)^{\frac{1}{1-\sigma}} = n^{\frac{1}{1-\sigma}} \left(\frac{\beta V}{T}\right)^{1/\beta}.$$

Turning to the list rule, individual effort of the member in mth position is given by the participation constraint:

$$\begin{aligned} (e_m)^\beta/\beta &= \sum_{k=m}^{n} C_k^n \left(\frac{1}{T}\right)^k \left(\frac{T-1}{T}\right)^{n-k} V, \\ e_m &= \left(\sum_{k=m}^{n} C_k^n \left(\frac{1}{T}\right)^k \left(\frac{T-1}{T}\right)^{n-k} \beta V\right)^{1/\beta}. \end{aligned}$$

Team output is equal to:

$$\begin{aligned} E &= \left(\sum_{m=1}^{n} e_m^{1-\sigma}\right)^{\frac{1}{1-\sigma}} = \left(\sum_{m=1}^{n} \left(\sum_{k=m}^{n} C_k^n \left(\frac{1}{T}\right)^k \left(\frac{T-1}{T}\right)^{n-k} \beta V\right)^{\frac{1-\sigma}{\beta}}\right)^{\frac{1}{1-\sigma}} \\ &= \left(\sum_{m=1}^{n} \left(\sum_{k=m}^{n} C_k^n \left(\frac{1}{T}\right)^{k-1} \left(\frac{T-1}{T}\right)^{n-k}\right)^{\frac{1-\sigma}{\beta}}\right)^{\frac{1}{1-\sigma}} \left(\frac{\beta V}{T}\right)^{1/\beta}. \end{aligned}$$

■