

Light effects of the rose windows in Mallorca cathedral

Abstract

This research shows the light effects and geometric alignments created by the sunlight passing through the stained glasses of the Eastern rose window in Mallorca cathedral and projecting on the inner side of the cathedral's main façade. Besides, we show other novel effects which occur in coincidence with certain religious festivities.

Introduction

It is well known, particularly in Mallorca, that every year on the same dates, and almost at the same time, the morning sunlight passes through the giant Eastern rose window and projects on the inner side of the main façade right underneath the Western rose window, thus forming the celebrated "Eight Effect" or "Festival of Light" (Daniel R. and Josep Lluís Pol 2010). This light effect occurs every 2nd of February (Candlemas Day) and every 11th of November (Saint Martin of Tours). These dates are 40 days and 43 days off Christmas, respectively, and the positions of both projections differ slightly. Also, in the days prior to the winter solstice, the sunlight passing through the Eastern rose projects almost exactly on the Western rose window, thus creating another light effect which can be seen from outside the cathedral.

This paper shows the light effects and geometric alignments created by the sunlight passing through the stained glasses of the Eastern rose window in Mallorca cathedral and projecting on the inner side of the cathedral's main façade. As well as providing more accurate information about these already known light effects, this paper makes use of laser scanning techniques and astronomical and geographical concepts in order to graphically display other novel effects which occur in coincidence with certain religious festivities throughout the year.

The Research

In order to calculate and graphically represent our results, it was necessary to create a digital model of the cathedral. Using a RTC360 Leica scanner, we carried out a laser scanner survey with 1250 stations and determined a cloud made up by 21.000.000 points covering the cathedral close and the space under consideration (Figure 1). In order to have a better graphic control of results and incorporate the numerical calculations into the architectural drawings, some parts of the building were outlined with CAD on the basis of the abovementioned survey. After this graphical analysis, the following parameters were established: Distance between the Eastern rose window \mathcal{R}_L ; Height to the center \mathcal{C}_L of the Eastern rose window \mathcal{R}_L and height to the center \mathcal{C}_P of the Western rose window \mathcal{R}_P with respect to the central nave's floor level; Distance between \mathcal{O}_L , which is the orthogonal projection of \mathcal{C}_L on the wall σ , and the vertical axis containing \mathcal{C}_P (Figure 2 and 3); and, Amplitude of the angle formed between the inner side of the Western façade and the Equatorial plane in the direction of rotation North-East.

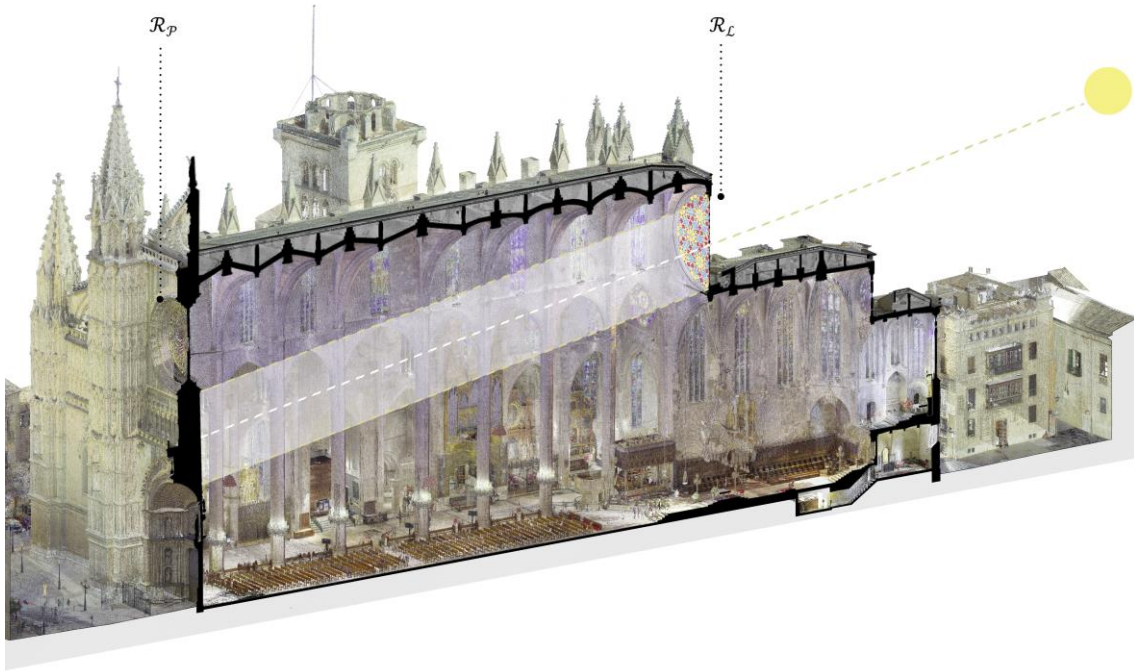


Figure 1. Image of the resulting cloud point.

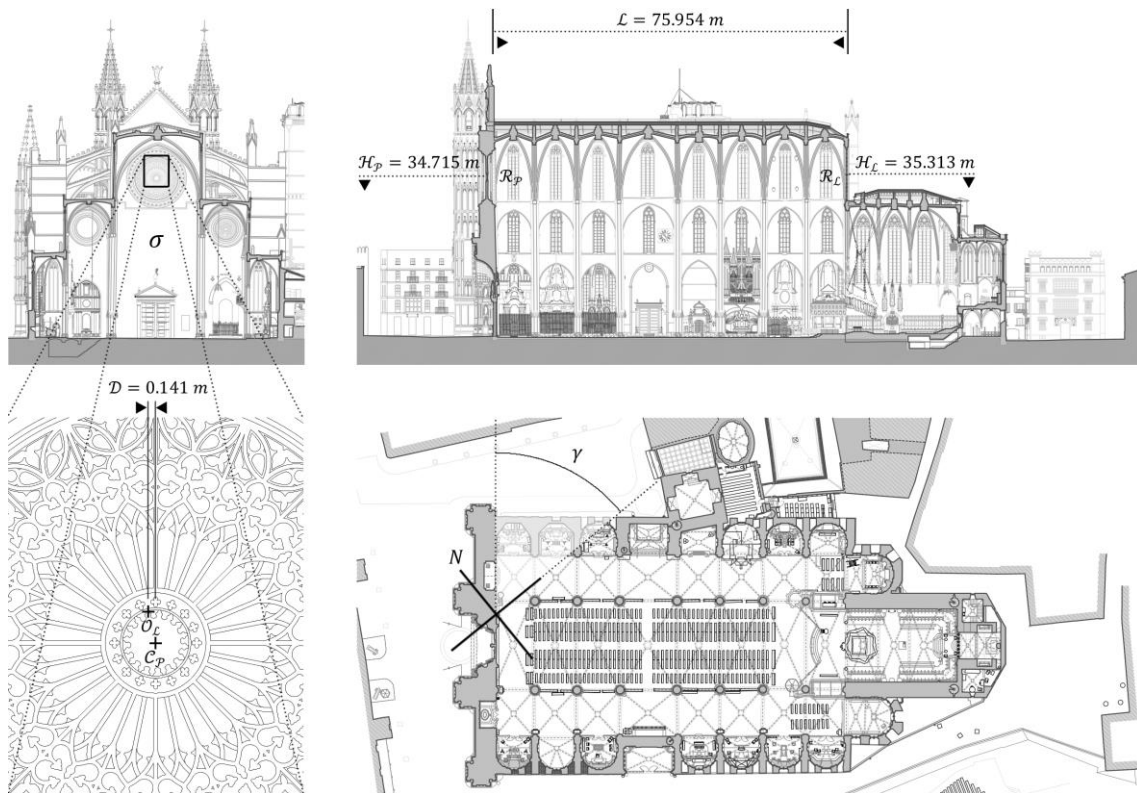


Figure 2. Parameters considered in our research, with $\gamma = 57.163^\circ$.

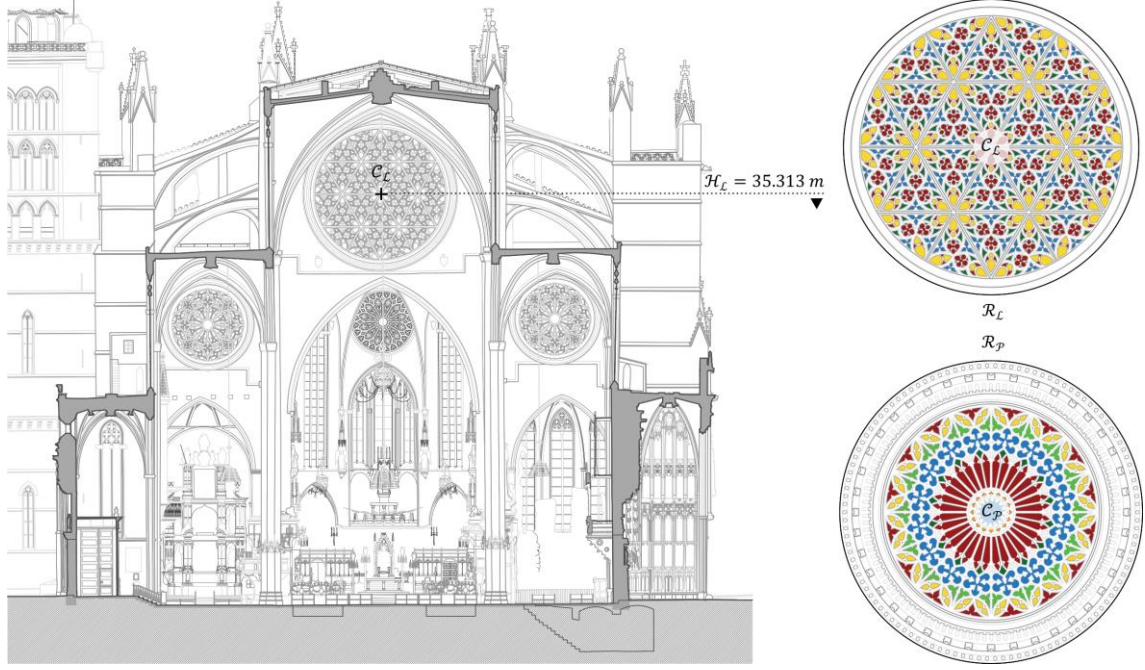


Figure 3. Position of C_L on the Eastern façade.

The graphical results of this paper (Figure 4, 5 and 6), are based on solving some astronomical and geometric problems. Assuming that the center θ_E of the Earth is the center of the universe and also the center of a celestial sphere where the stars are, we need to: 1) determine the position θ_S of the center of the Sun in the celestial sphere (apparent right ascension α_{app} and apparent declination δ_{app}) at every given instant JD , 2) determine the time equation E^m at the instant JD , and 3) determine the intersection $p_L = \sigma \cap r_S$ of plane σ with the straight line r_S passing through C_L and having the direction of vector $\overrightarrow{\theta_E \theta_S}$. We use the well-known algorithms mentioned in (Martín 1990) and (Meeus 1991). The initial variable in these three problems is a calendar date. Thus, we start from an initial variable date: year a , month $m \in \mathbb{Z} \cap (1,12)$, day of the month d , hour h , minute m , second s , in Greenwich Civil Time, i.e., $GMT + 12^h$. Please note that in Mallorca, due to the delays and advancements imposed by the Spanish Government, we must consider the Greenwich Civil Time plus 1 hour in Autumn-Winter or plus 2 hours in Spring-Summer. Next, the date is converted to Julian Date JD . Readers may turn to (Martín 1990) and (Meeus 1991) for definitions and calculations of: CE Gregorian year (from 14th October 1582 onwards), CE and BCE Julian year (until 4th October 1582), and Julian Day $JD(\#)$ according to the time measurement system proposed by Joseph Scaliger.

Now that we know JD , we can calculate δ_{app} , which is the apparent declination of the Sun in the celestial sphere. We will use the algorithms in (Meeus 1991) with an error of $0.01''$ sexagesimal seconds. Even though we do not need such level of accuracy for our purposes, we follow the steps and calculate the following astronomical variables: T (the time in Julian centuries of 36525 ephemeris days), L_0 (geometric mean longitude of the Sun referred to the mean equinox of date), M (the mean anomaly of the Sun), e (the eccentricity of the Earth's orbit), C (the equation of the Center of the Sun), \odot (the true geometric longitude of the Sun), λ (the apparent longitude of the Sun referred to the true equinox of date T), ε_0 (the mean obliquity of the ecliptic), L' (the mean longitude of the Moon), $\Delta\varepsilon$ (the nutation in obliquity of the ecliptic), ε (the true obliquity of the ecliptic) and δ_{app} (the apparent declination of the Sun).

Next, given the date JD , we proceed to calculate the time equation $E^m = 4E$. Again, we use the algorithms provided in (Meeus 1991) to calculate the following astronomical variables: $\Delta\psi$ (the nutation in longitude of the ecliptic), α_{app} (the apparent right ascension of the Sun taking into account the aberration and the nutation) and E (the time equation).

Having calculated δ_{app} and E^m based on JD , the required parameters are collected previously to carry out the solar projection of \mathcal{C}_L on the plane σ and determine the point p_L . Cartesian orthonormal reference system $\{\theta; \vec{e}_1, \vec{e}_2, \vec{e}_3\}$ is used, such that a) if points in space have coordinates (x,y,z) , then the equation of plane σ is $x=0$, b) \vec{e}_3 has direction towards the zenith, c) \vec{e}_1 has direction towards \mathcal{C}_L , d) \vec{e}_2 is such that the base $\vec{e}_1, \vec{e}_2, \vec{e}_3$ is clockwise, and e) $(0,0,0)$ are the coordinates of \mathcal{O}_L , which is the orthogonal projection of \mathcal{C}_L on σ . Using the algorithms described in (Martín 1990) and (Meeus 1991), we determined the coordinates of $p_L = (0, y_1, z_1)$ the projection of \mathcal{C}_L . The variation of this projection p_L over time results in paths $\mathcal{T}_{i/j}$ on the plane σ .

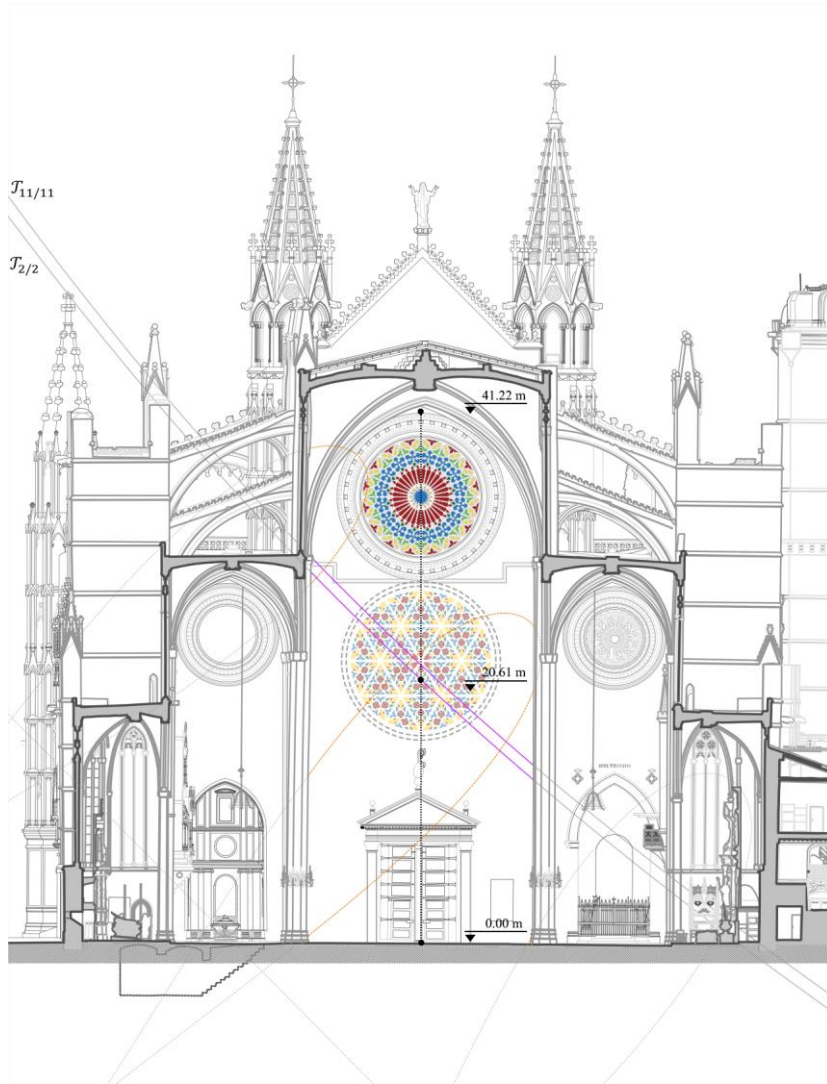


Figure 4. Graphical representation of $\mathcal{T}_{11/11}$ (St. Martin of Tours) and $\mathcal{T}_{2/2}$ (Candlemas) on the plane σ .

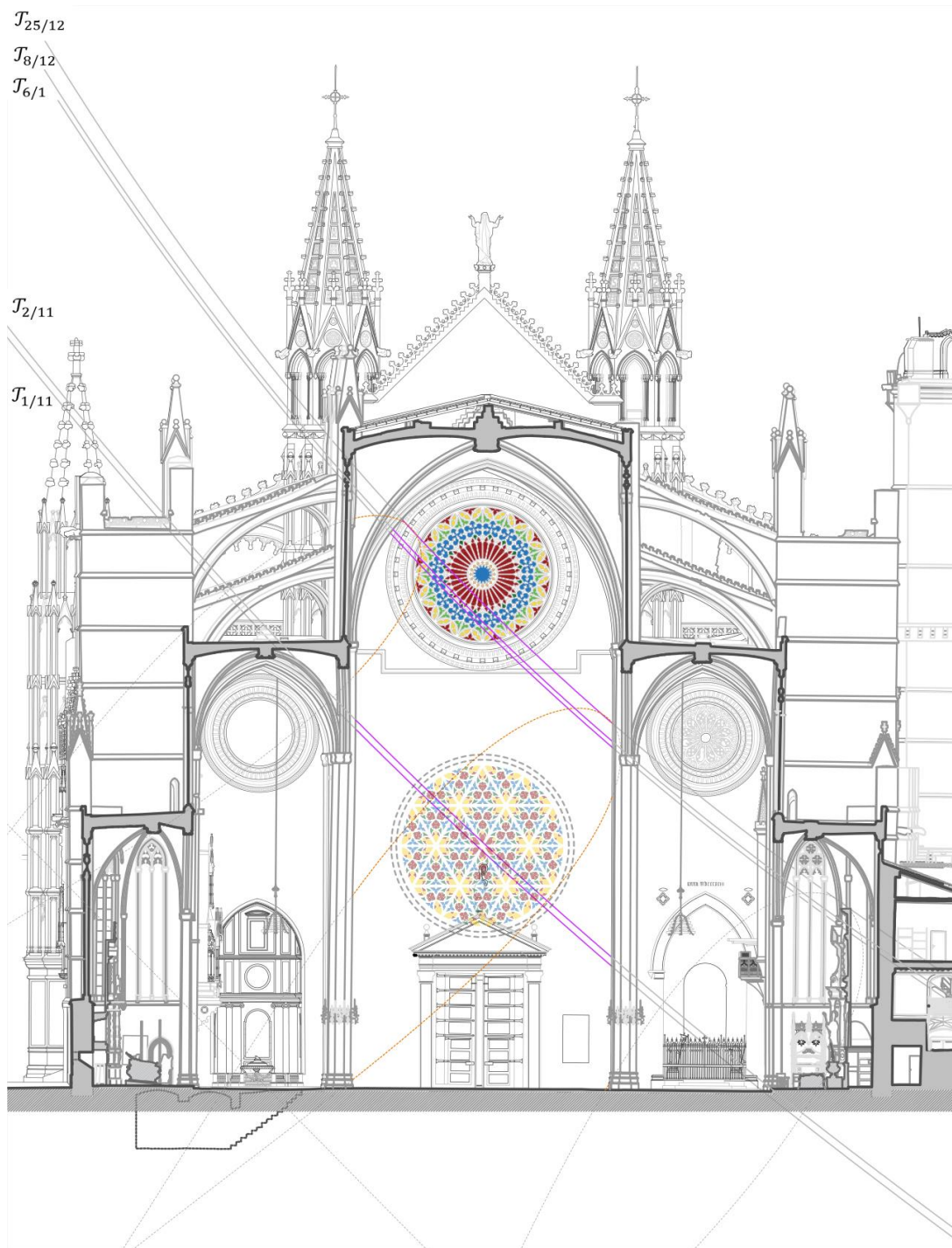


Figure 5. Graphical representation of $T_{25/12}$ (Christmas), $T_{8/12}$ (Immaculate Conception), $T_{6/1}$ (Epiphany), $T_{2/11}$ (All Souls Day) and $T_{1/11}$ (All Saints' Day) on the plane σ .

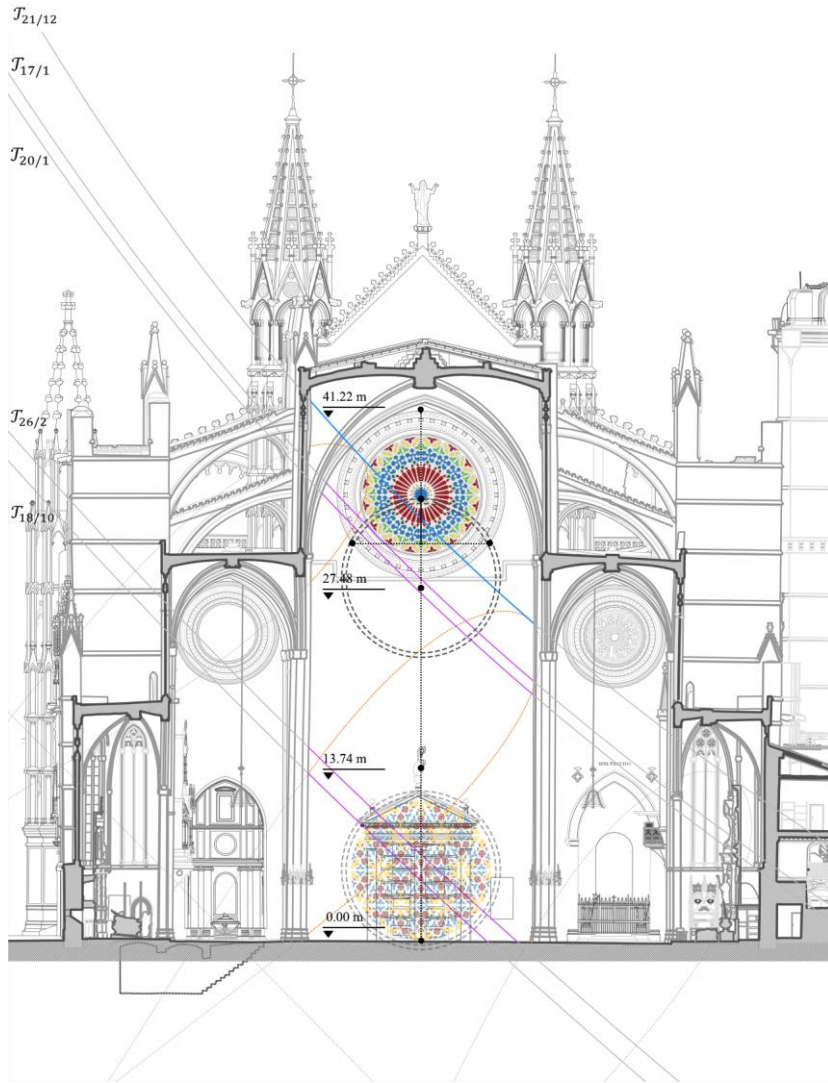


Figure 6. Graphical representation of $J_{21/12}$ (Winter Solstice), $J_{17/1}$ (St. Anthony, local party), $J_{20/1}$ (St. Sebastian, local party), $J_{26/2}$ (St. Alexander) and $J_{18/10}$ (St. Luke) on the plane σ .

Conclusion

On the 11th of November (St. Martin of Tours) at 8.33 am \mathcal{C}_L is projected on the vertical straight line passing through the center of \mathcal{R}_P ; the distance between p_L and \mathcal{C}_P is 13.05 meters. Similarly, on the 2nd of February (Candlemas Day) at 9.03 am \mathcal{C}_L is projected on the vertical straight line passing through the center of \mathcal{R}_P ; the distance between p_L and \mathcal{C}_P is 14.19 meters. Thus, the "Eight Effect" is more clearly visible on St. Martin's Day. All of this is shown in the Figure 4

On 2nd February at 9.03 am the projection of \mathcal{C}_L on the plane σ is exactly at half the distance between the cathedral's floor level and the height of the topmost point in the edge of the rose window \mathcal{R}_P (Figure 4). This middle point lies at a height of 20.61 meters.

The paths $J_{21/12}$, $J_{25/12}$, $J_{8/12}$ and $J_{6/1}$ cross the stained glasses of the Western rose window between 8.00 am and 8.30 am. Due to this, the sun rays passing through \mathcal{R}_L are projected onto \mathcal{R}_P , illuminating it from the inside of the cathedral. This effect is strongest on Winter Solstice Day at 8:17 am, when the distance between \mathcal{C}_P and the path $J_{21/12}$ is the shortest. All of this is shown in Figure 5 and 6.

In addition to the above effects, we present other sunlight effects which are not commonly known:

- On the 1st and 2nd of November at 8.45 am \mathcal{C}_L is projected onto the ornamental element located on top of the frieze which crowns the main entrance porch to the cathedral. The mean distance between both projections p_L and this element is 0.89 meters (Figure 5).
- On 17th January and on 20th January at 9.03 am the projection of \mathcal{C}_L on the plane σ is positioned exactly at two thirds of the distance between the cathedral's floor level and the height of the topmost point in the edge of the rose window \mathcal{R}_P (Figure 6). This position lies at a height of 27.48 meters.
- As shown in the Figure 6, on the 18th of October at 10.03 am and on the 26th of February at 9.31 am the rose window \mathcal{R}_L is projected exactly onto the access door to the cathedral. On 18th October, this projection is even tangent to the horizontal plane of the cathedral.

In summary, this paper makes use of laser scanning techniques and astronomical and geometric concepts in order to graphically display the light effects and alignments (both known and unknown) which occur when the sunlight passes through the stained glasses of the Eastern rose window in Mallorca cathedral and projects this rose window on the inner side of the cathedral's main façade.

References

- Daniel R. and Josep Lluís Pol. 2010. Els efectes de la llum solar a la seu de Mallorca. *Nova Època* 3(1): 37–47.
- Martín, F. 1990. *Astronomía*. Madrid: Parainfo.
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