Abstract

This paper presents a Systematic Review of the Literature (SRL) on Value-at-Risk (VaR), and more specifically on the models that have been applied to estimate VaR, to achieve two purposes: to gather the most used models in the literature and to verify whether their popularity has changed since the 2008 financial crisis. The SRL is based on SCOPUS in the period from 1996 to 2017. The results show that ARCH/GARCH models and Extreme Value Theory, together with Monte Carlo Simulation, Historical Simulation and Variance and Covariance, are the most used models. Since the crisis, the ARCH/GARCH models have clearly been the most popular, while no significant differences were found in the percentage of articles on the other models. This study can be considered the first systematic review of the literature on Value-at-Risk models because no previous work of a similar nature has been carried out on this topic. It provides a rich background for researchers and professionals interested in the topic, contributing with detailed information about the papers published and classified by models, including authors, citation count, journals and year published, among others.

Keywords Value-at-Risk; Systematic Review of the Literature; Financial Risk; Models

Value-at-Risk Models: A Systematic Review of the Literature

1. Introduction

Measuring different types of risk became a necessity after the issuance of Basel I, published in 1988 by the Basel Committee on Banking Supervision (BCBS). The main goal of this agreement was to ensure that financial institutions maintain a capital that is sufficient to cover any unexpected losses and meet financial obligations.

The Value-at-Risk (VaR) of a portfolio is the maximum amount of loss that an investor may face over a certain time horizon with a given probability. Financial institutions could use their own models for calculating VaR as per the amendment to Basel I issued by the BCBS (1996), which explains the evolution of the academic literature on the topic, with proposals of new models for its estimation.

VaR is calculated according to three basic methodologies:

1. The Historical Simulation approach (non-parametric method), consisting in ordering historical returns and then taking the assumption that history repeats itself.
2. The Variance – Covariance approach (parametric method), taking the assumption that stock returns are normally distributed.
3. The Monte Carlo Simulation approach (semi-parametric method), consisting in developing a model for predicting future stock returns and running multiple trials through this model.

In this paper, the models developed on estimating VaR between 1996 and 2017 are reviewed, determining the most used ones and analysing the evolution and trend of growth in the number of articles published on each model.

This paper is the first of its kind in the domain of VaR estimation and it will be of enormous help to both academics who are interested in developing new models, and to financial institutions, who depend on in-house VaR models developed by their analytics teams.

The paper is structured as follows. Section 2 presents the Systematic Review of the Literature (SRL). Section 3 contains the search criteria in the selected database. Section 4 sets out the search results of the review obtained. Section 5 is a summary of the entire SRL carried out. Last, the conclusions are drawn in Section 6.

2. Systematic Review of the Literature

This paper follows the SRL that emerged in the 1990s, first developed in the field of medicine (Dickersin et al., 1994; Evans, 2001; Mulrow et al., 1997). This methodology was later introduced to other fields.

One of the main differences between the literature review and the SRL is that the former is more affected by selection bias since authors may include in their research only the material that supports their own personal opinion. The SRL, on the other hand, achieves an unbiased and balanced summary of the literature (Nightingale, 2009) by identifying all research addressing a specific question. Ordinary literature reviews, furthermore, mainly include the easily identified studies that are published in high impact journals (Nightingale, 2009).

What really determines the quality of a review is the extent to which it incorporates scientific review methods to minimize the error and bias, which is the major factor that distinguishes narrative reviews from an SRL (Cook, Mulrow, et al., 1997). As per Rosenthal (1991) and Cook, Greengold, et al. (1997) a major difference between the two is that the former incorporates critical analysis of the obtained results and does not generate subjective opinions.

As per Tranfield et al. (2003), the SRL has three stages: (1) planning, (2) execution, and (3) reporting and dissemination.

In the planning stage, the researcher must define the importance of and need for the review, along with its general and specific objectives, also determining the research criteria and the database used. The execution stage involves searching in the chosen database for the articles related to the subject of study, according to the terms and search question(s). In the third stage, the search results are analysed, and a summary is conducted to answer the research question(s) and objectives.

3. SRL: Planning Stage

What are the most popular VaR models? This is an indirect question on the accuracy of the VaR models used prior to the global financial crisis of 2008, and which were not accurate enough to prevent the financial crash. To answer this question, the Scopus database and the period 1996 to 2017 are used.

The search was based on using the “title, abstract, keyword” to find documents on VaR, using the terms “value-at-risk”, “value at risk” and “VaR”. Using “VaR”, the result was a huge number of documents (105,000 as of November 21, 2018), many of which were not relevant to the topic and were removed from the study. Thereafter, the search was limited to the following topics: “Economics, Econometrics and Finance”, “Business, Management and Accounting” and “Decision Sciences”. The document type was limited to Articles, in English, and the source type was limited to Journals.

Based on the results, the VaR models with more than 100 articles and more than 1,000 citations were analysed. The aim of this analysis is to show when each of these models was first introduced into the estimation of VaR, with a focus on their evolution trend and the growth in the number of articles published on them.

4. SRL Execution Stage: Search in Scopus

The first publication on VaR, as per the field selections, appeared in 1996 and was classified in the topic "Economics, Econometrics and Finance". Consequently, this paper marks the start date of the research, with 2017 chosen as the end date. After applying all the above-mentioned filters, the result was 1,505 articles, a figure that was reduced to 1,502 after removing duplicates.

Table 1. Distribution of filtered articles on VaR per year

Year	Number of Articles	Cumulative Number of Articles	Year	Number of Articles	Cumulative Number of Articles
1996	1	1	2007	71	360
1997	10	11	2008	71	431
1998	6	17	2009	86	517
1999	16	33	2010	103	620
2000	21	54	2011	109	729
2001	30	84	2012	116	845
2002	37	121	2013	118	963
2003	27	148	2014	134	1,097
2004	30	178	2015	136	1,233
2005	42	220	2016	116	1,349
2006	69	289	2017	153	1,502
Total Number of Articles				**1,502**	

As shown in Table 1, the number of publications per year follows an increasing trend, from 1 in 1996 to an average of 22 between 1997 and 2004; 56 between 2005 and 2006; 71 between 2007 and 2008; and 119 thereafter. The tremendous increase in the average number of publications per year between the periods 1997 to 2004 and 2004 to 2006 can be explained by the issuance of Basel II in 2004. The increase from the latter period to the years 2007 – 2008 was due to the global financial crisis, which prompted practitioners and academics to start working on developing better models for estimating VaR.

The next step consisted in searching for the models of VaR in the "Abstract, Title or Keywords", this search producing articles that actually focus on the models and not just mention them. After data cleaning, the analysis focused on the models, the outcome of which is shown in Table 2 as a list of models in descending order according to the number of articles. The total number of models of VaR was 27 and those with more than 100 articles on them and having a citation count above 1,000 were analysed.

Some risk measures, alternatives to VaR, were excluded from the study since they did not precisely model it. Among these models, can be mentioned the Conditional VaR, the Conditional Autoregressive VaR, the Liquidity Adjusted VaR, the Modified VaR, the Stressed VaR, the Entropic VaR, and the Multidimensional VaR. The reason for their exclusion was that they are not VaR models as such, but measures that apply existing models.

Table 2. Distribution of articles per model

Model	Sub Model	Number of Articles	Total	Citation Count
ARCH/GARCH Family	GARCH	307	335	4,825

	ARCH	32		
	EGARCH	24		
	FIGARCH	22		
	FIAPARCH	12		
	APARCH	11		
	Hyperbolic GARCH	4		
	AVGARCHM model	1		
Extreme Value Theory	Multivariate Extreme Value at risk	1	178	3,213
	Extreme Value-at-risk	3		
	Extreme Value Theory	178		
Monte Carlo Simulation	Monte Carlo Simulation Method	160	160	2,302
	Quasi Monte Carlo	4		
Historical Simulation	Historical Simulation Method	154	142	2,279
	Filtered Historical Simulation	19		
	Derivative-based VaR/Diversified VaR	1		
Variance Covariance Method	Variance-Covariance Method	134	134	1,733
Copulas	Asymptotic Value-at-Risk	1	108	1,008
	Copulas	108		
Moving Average Models	Moving Average	7	59	499
	ARMA	26		
	Exponentially Weighted Moving Average	20		
	ARFIMA	5		
	ARIMA	4		
	Equally Weighted Moving Average	4		
Quantile Regression	Quantile Regression	58	58	732
Generalized Pareto Distribution	Generalized Pareto Distribution	62	62	882
RiskMetrics	RiskMetrics	44	47	643
	Component Value at Risk	3		
Fourier	Fourier	16	21	89
	Entropic Value-at-Risk	6		

G-and-H Distribution	Generalized Hyperbolic Distribution	17	17	172
	Tukey's G-and-H	2		
Cornish-Fisher Value-at-Risk	Cornish-Fisher	13	13	116
Heterogeneous Autoregressive Model	Heterogeneous Autoregressive	12	12	78
Wavelet Value-at-Risk	Wavelet Value-at-Risk	12	12	234
Levy Model	Levy Model	8	8	41
Quadratic Value at Risk	Quadratic Value at Risk	1	8	168
	Delta-Gamma	8		
Multivariate Value-at-Risk	Multivariate Value at Risk	7	7	67
CreditRisk+	CreditRisk+	7	7	528
CreditMetrics	CreditMetrics	6	6	575
Fuzzy Models	Fuzzy Models	6	6	105
Granularity Adjustment	Granularity Adjustment	6	6	34
Black-Litterman Model	Black-Litterman	4	4	31
KMV	KMV	4	4	551
Artificial Intelligence	Artificial Intelligence	3	3	11
Expectile	Expectile	2	2	60
Threshold stochastic volatility model	Threshold stochastic volatility model	2	2	16
Total before removing duplicates		**1,551**	**1,421**	**20,992**
Total after removing duplicates		**818**	**818** [1]	**11,058**
Excluded Models		**391**	**391**	
Others		**293**	**293**	
Total Number of Articles		**1,502**	**1,502**	

Table 2 shows that the ARCH/GARCH family of models is the most widely used in the estimation of VaR, with an increasing trend especially after 2008. The remaining 684 articles not included in Table 2 are grouped in two categories, "Excluded Models" and "Others". These articles are mostly related to "alternative models to VaR" or are slightly related to VaR modelling.

Regarding the development of VaR models, Table 3 gives the percentage of articles on the models published in the same year. Those with more than 100 published articles and more than 1,000 citations are ARCH/GARCH Models, Extreme Value Theory, the Monte Carlo Simulation Method, the Historical

[1] It is worth noting that the number of articles decrease from 1,502 to 818 after focusing our analysis on "Models on VaR" and excluding "alternative measures to VaR".

Simulation Method, the Variance Covariance Method and Copulas. The number of articles corresponding to these six models constitutes almost 75% of the total number of articles.

Table 3. Evolution of VaR models by percentage of total articles per year

Year	ARCH/GARCH Models	% of Total (ARCH/GARC	Extreme Value Theory (EVT)	% of Total (EVT)	Monte Carlo Simulation	% of Total (MCS)	Historical Simulation	% of Total (HS)	Variance Covariance (V-	% of Total (V-C)	Copulas	% of Total (Copulas)	Total Number of Articles Per Year
1996	0	**0%**	0	**0%**	0	**0%**	0	**0%**	0	**0%**	0	**0%**	1
1997	1	**10%**	0	**0%**	0	**0%**	0	**0%**	0	**0%**	0	**0%**	10
1998	2	**33%**	0	**0%**	0	**0%**	0	**0%**	0	**0%**	0	**0%**	6
1999	2	**13%**	1	**6%**	1	**6%**	1	**6%**	2	**13%**	0	**0%**	16
2000	4	**19%**	4	**19%**	7	**33%**	3	**14%**	6	**29%**	0	**0%**	21 [2]
2001	3	**10%**	3	**10%**	0	**0%**	3	**10%**	3	**10%**	1	**3%**	30
2002	7	**19%**	2	**5%**	3	**8%**	1	**3%**	2	**5%**	0	**0%**	37
2003	3	**11%**	5	**19%**	2	**7%**	1	**4%**	1	**4%**	1	**4%**	27
2004	8	**27%**	3	**10%**	6	**20%**	4	**13%**	3	**10%**	2	**7%**	30
2005	9	**21%**	7	**17%**	4	**10%**	7	**17%**	5	**12%**	1	**2%**	42
2006	11	**16%**	10	**14%**	6	**9%**	10	**14%**	7	**10%**	2	**3%**	69
2007	10	**14%**	7	**10%**	7	**10%**	5	**7%**	6	**8%**	1	**1%**	71
2008	19	**27%**	9	**13%**	7	**10%**	9	**13%**	2	**3%**	1	**1%**	71
2009	19	**22%**	7	**8%**	12	**14%**	10	**12%**	9	**10%**	9	**10%**	86
2010	24	**23%**	12	**12%**	10	**10%**	14	**14%**	13	**13%**	7	**7%**	103
2011	29	**27%**	10	**9%**	9	**8%**	8	**7%**	7	**6%**	11	**10%**	109
2012	21	**18%**	14	**12%**	10	**9%**	9	**8%**	10	**9%**	5	**4%**	116
2013	36	**31%**	12	**10%**	9	**8%**	10	**8%**	7	**6%**	14	**12%**	118
2014	33	**25%**	23	**17%**	21	**16%**	15	**11%**	13	**10%**	13	**10%**	134
2015	29	**21%**	15	**11%**	19	**14%**	12	**9%**	15	**11%**	14	**10%**	136
2016	26	**22%**	14	**12%**	14	**12%**	7	**6%**	8	**7%**	10	**9%**	116
2017	39	**25%**	20	**13%**	13	**8%**	12	**8%**	15	**10%**	16	**10%**	153
Total	**335**	**22%**	**178**	**12%**	**160**	**11%**	**141**	**9%**	**134**	**9%**	**108**	**7%**	**1,502**

To add clarity to the analysis, the time span of the research is divided into two intervals, the first prior to the crisis, 1996 to 2007, and the second during and after the crisis, 2008 to 2017. For the ARCH/GARCH models, the average percentage of articles published in the first period was 16%, increasing to 24% in the second period. The Extreme Value Theory percentage of articles also underwent an increase from 9% to 12%. The Variance-Covariance method did not vary in terms of the average percentage of articles

[2] The sum of articles in a year might exceed the corresponding total number of articles in the same year. This is due to the fact that some articles are classified to belong to more than one model.

published, with 8% in both periods. There was a slight increase of 2% for the Monte Carlo Simulation, from 9% to 11%. Among the three traditional methods, the Historical Simulation method was the one with the highest percentage of growth, from 7% in the first interval to 10% in the second. The number of articles published on Copulas increased enormously between the periods, from 2% before the crisis to 8% after it. In the following sub-sections, the models mentioned above are analysed.

4.1. ARCH/GARCH Models

The Generalized Autoregressive Conditional Heteroskedastic (GARCH) model was introduced by Bollerslev (1986) as a generalization of the ARCH model from Engle (1982). The main reason for the development of the GARCH model was to facilitate the computation of the polynomial lag operator when it presented a high order. It was mostly used in VaR estimation, where the volatility of returns is a central issue. The ARCH/GARCH models that have so far been used in the estimation of VaR are the original ARCH, GARCH, the Exponential GARCH (EGARCH), the Fractionally Integrated Asymmetric Power ARCH (FIAPARCH), the Fractionally Integrated GARCH (FIGARCH), the Asymmetric Power ARCH (APARCH), the Hyperbolic GARCH (HYGARCH) and the Absolute Value GARCH in the mean (AVGARCHM).

The GARCH (p, q) process, where "p" stands for the order of the polynomial, referring to the autoregressive term, and "q" is the order of the polynomial, referring to the Moving Average term, improves ARCH (q) because it allows for lagged conditional variances, unlike the ARCH (q) process where the conditional variance is specified as a linear function of past sample variances only (Bollerslev, 1986). GARCH models deal with volatility, specifically with Heteroskedasticity, which arises when the variances of error terms in a given data are not equal, which is the real case rather than the assumption of normality of distribution of returns.

The use of GARCH models in the estimation of VaR started in 1997 (Table 4), with the first article published in the "Journal of Derivatives". The average number of articles published before the release of Basel II was 3 per year between 1997 and 2003. In 2004, the number of articles increased to 8, then to an average of 10 articles per year between 2005 and 2007. In 2008, the number of articles almost doubled and kept increasing thereafter with an average of 28 articles between 2009 and 2017, showing the general increasing trend of the evolution of work on VaR estimation using GARCH models. Table 5 shows the details of articles on ARCH/GARCH models with the highest citation count.

Table 4. Distribution of articles on VaR estimation using ARCH/GARCH per year

Year	Articles	Year	Articles
1997	1	2008	19
1998	2	2009	19
1999	2	2010	24
2000	4	2011	29
2001	3	2012	21
2002	7	2013	36
2003	3	2014	33
2004	8	2015	29
2005	9	2016	26
2006	11	2017	39
2007	10		

Total	**335**

Table 5. Articles on ARCH/GARCH models with the highest citation count

Title	Author(s)	Year	I. Journal	Citation Count
Estimation of tail-related risk measures for heteroscedastic financial time series: An extreme value approach	McNeil, Frey.	2000	Journal of Empirical Finance	583
Value-at-risk prediction: A comparison of alternative strategies	Kuester, *et al*.	2006	Journal of Financial Econometrics	224
Modelling daily Value-at-Risk using realized volatility and ARCH type models	Giot, Laurent.	2004	Journal of Empirical Finance	172
An econometric analysis of emission allowance prices	Paolella, Taschini.	2008	Journal of Banking and Finance	132
Returns synchronization and daily correlation dynamics between international stock markets	Martens, Poon.	2001	Journal of Banking and Finance	113
Value at risk when daily changes in market variables are not normally distributed	Hull, White.	1998	Journal of Derivatives	111
Systemic risk measurement: Multivariate GARCH estimation of CoVaR	Girardi, Tolga Ergün.	2013	Journal of Banking and Finance	91
Using extreme value theory to measure value-at-risk for daily electricity spot prices	Fong Chan, Gray.	2006	International Journal of Forecasting	75
On a threshold heteroscedastic model	Chen, So.	2006	International Journal of Forecasting	74
Value-at-Risk: A multivariate switching regime approach	Billio, Pelizzon.	2000	Journal of Empirical Finance	72

4.2. Extreme Value Theory

According to Ho et al. (2000) , the estimation of VaR using EVT focuses on modelling the tail of the return distribution rather than obtaining the tail as an outcome of modelling the entire density function, which is a primary objective of other approaches for estimating VaR such as the RiskMetrics, Historical Simulation and Monte Carlo Simulation methods. Notably, there are a large number of works that combine the EVT and the GARCH models, including Byström (2004), Huang et al.(2011), McNeil and Frey (2000), Yi et al. (2014) among others. This combination has proved to yield more precise results compared to other VaR estimation methods.

The first article issued on the estimation of VaR using EVT was published in 1999 in the "Journal of International Money and Finance" by Pownall and Koedijk. As Table 6 shows, between 1999 and 2004, the average number of articles published on this topic was 3 per year, increasing to an average of 8 between 2005 and 2008 after the issuance of Basel II. The years 2009 and 2010 saw an average of 10 articles published per year, with a rapid increase thereafter to an average of 15 articles per year between 2009 and 2017, attributable the issuance of Basel III in 2010. Table 7 provides details of the articles on EVT with the highest citation count.

Table 6. Distribution of articles on VaR estimation by EVT per year

Year	Articles	Year	Articles
1999	1	2009	7
2000	4	2010	12
2001	3	2011	10
2002	2	2012	14
2003	5	2013	12
2004	3	2014	23
2005	7	2015	15
2006	10	2016	14
2007	7	2017	20
2008	9		
Total		**178**	

Table 7. Articles on EVT with the highest citation count

Title	Author(s)	Year	Journal	Citation Count
Estimation of tail-related risk measures for heteroscedastic financial time series: An extreme value approach	McNeil, Frey.	2000	Journal of Empirical Finance	583
Value-at-risk prediction: A comparison of alternative strategies	Kuester, *et al.*	2006	Journal of Financial Econometrics	224
From value at risk to stress testing: The extreme value approach	Longin.	2000	Journal of Banking and Finance	210
Extreme value theory and Value-at-Risk: Relative performance in emerging markets	Gençay, Selçuk.	2004	International Journal of Forecasting	170
Value-at-risk versus expected shortfall: A practical perspective	Yamai, Yoshiba.	2005	Journal of Banking and Finance	110
An Extreme Value Approach to Estimating Volatility and Value at Risk	Bali.	2003	Journal of Business	97
Using extreme value theory to measure value-at-risk for daily electricity spot prices	Fong Chan, Gray.	2006	International Journal of Forecasting	79
Procyclical leverage and value-at-risk	Adrian, Shin.	2014	Review of Financial Studies	75

4.3. Monte Carlo Simulation Method

The Monte Carlo Simulation (MCS) is one of the three basic methodologies for estimating VaR. The main difference between this model and Historical Simulation is that in the first a statistical distribution is assumed to approximate the changes in the market factors. The hypothetical changes in market factors are then generated based on this assumed distribution, along with a pseudo-random number generator. The hypothetical profit and loss portfolios in the current portfolio and the distribution of possible profit or loss

are constructed based on these hypothetical changes. This distribution is then used to determine the VaR of the portfolio (Linsmeier & Pearson, 2000).

According to Linsmeier and Pearson (2000), one of the major drawbacks of the MCS in estimating VaR is the flexibility in choosing the statistical distribution for market factors. However, despite this, MCS proved its usefulness as a numerical method for defining market models due to its adaptability to different market conditions. Furthermore, although MCS can be an expensive method, there are certain methods for obtaining a fast MCS, for instance using importance sampling, like in Hsieh et al. (2014).

The first article on this subject was published in 1999 in the "Journal of Portfolio Management" by Papageorgiou and Paskov. This figure fluctuated between 1999 and 2008, with an average of 5 per year, increasing enormously from 7 in 2008 to 12 in 2009. From 2009 onwards the number of articles also fluctuated, but with a larger number of articles, illustrating that the MCS method was increasingly involved in VaR estimation after the financial crisis of 2008, with an average number of 13 articles per year (see Table 8). Table 9 contains details of the articles with the highest citation count on the MCS method.

Table 8. Distribution of articles per year on VaR estimation using the Monte Carlo Simulation method

Year	Articles	Year	Articles
1999	1	2009	12
2000	7	2010	10
2002	3	2011	9
2003	2	2012	10
2004	6	2013	9
2005	4	2014	21
2006	6	2015	19
2007	7	2016	14
2008	7	2017	13
Total		**160**	

Table 9. Articles on the Monte Carlo Simulation Method with the highest citation count

Title	Author(s)	Year	Journal	Citation Count
Estimation of tail-related risk measures for heteroscedastic financial time series: An extreme value approach	McNeil., Frey.	2000	Journal of Empirical Finance	583
Value at Risk	Linsmeier, Pearson.	2000	Financial Analysts Journal	116
Evaluating value-at-risk models with desk-level data	Berkowitz, *et al.*	2011	Management Science	107
Variance reduction techniques for estimating value-at-risk	Glasserman, *et al.*	2000	Management Science	93
On a threshold heteroscedastic model	Chen, So.	2006	International Journal of Forecasting	74

4.4. Historical Simulation Method

Historical Simulation (HS) consists in constructing the distribution of the potential future data based on historical data and then estimating the quantile of the distribution to obtain the VaR (Linsmeier & Pearson, 2000). The major drawback of this method lies in the assumption that history repeats itself, and this was the main reason why the tremendous losses of the crisis of 2008 were not accurately predicted.

Evidence on the low level of accuracy of the HS method in estimating VaR is given in Miletic and Miletic (2015), which investigates the performance of the VaR models in the middle of the financial crisis of 2008 in selected Central and Eastern European emerging capital markets. After the back-testing analysis, it was concluded that GARCH-type models outperform HS and RiskMetrics models in measuring VaR (Miletic & Miletic, 2015).

One of the main advantages of the HS method is that it includes no assumptions on the shape of the distribution of the risk factor affecting the value of the portfolio, which is what makes the HS perform somewhat better than other models that assume the normality of this distribution (Pritsker, 2006). However, the main pitfall of the HS is the assumption that the risk factors, and consequently the historically simulated returns, are independently and identically distributed (i.i.d.) over time (Pritsker, 2006).

On applying the filters established for this study, the first article on the estimation of VaR using HS was published in the "Journal of Derivatives" in 1999 by Taylor J. After 1999, the number of articles published on this topic fluctuated, with an average of 3 articles per year between 1999 and 2005. In 2006, for the first time the number of articles on this topic reached 10, which can be attributed to the release of Basel II. Thereafter, between 2006 and 2008 (inclusive), the average number of articles was 8, after which the amount of research on this topic increased, reaching its peaks in 2010 (15 articles) and 2014 (16 articles), due to the issuance of Basel III. However, it was noticed that there was an overall fluctuation in the number of articles during the period after the financial crisis, with an overall average of 12 per year, i.e., almost triple the average number of articles published between 1999 and 2005. (See Table 10). Table 11 contains the details of the articles on the HS method with the highest citation count.

Table 10. Distribution of articles per year on VaR estimation using the Historical Simulation method

Year	Articles	Year	Articles
1999	1	2009	10
2000	3	2010	14
2001	3	2011	8
2002	1	2012	9
2003	1	2013	10
2004	4	2014	15
2005	7	2015	12
2006	10	2016	7
2007	5	2017	12
2008	9		
Total		**140**	

Table 11. Articles on the Historical Simulation method with the highest citation count

Title	Author(s)	Year	Journal	Citation Count
Value-at-risk prediction: A comparison of alternative strategies	Kuester, *et al*.	2006	Journal of Financial Econometrics	229
Extreme value theory and Value-at-Risk: Relative performance in emerging markets	Gençay, Selçuk.	2004	International Journal of Forecasting	171
The level and quality of Value-at-Risk disclosure by commercial banks	Pérignon, Smith.	2010	Journal of Banking and Finance	129
Value at Risk	Linsmeier, Pearson.	2000	Financial Analysts Journal	116
Using extreme value theory to measure value-at-risk for daily electricity spot prices	Fong Chan, Gray.	2006	International Journal of Forecasting	80
The hidden dangers of historical simulation	Pritsker.	2006	Journal of Banking and Finance	75

4.5. Variance Covariance Method

The Variance-Covariance (V-C) approach for estimating VaR, also known as the delta-normal approach, the normal linear or Linear VaR, is a widely used technique for VaR estimation. In their paper "Value at Risk", Linsmeier and Pearson (2000) state that "the delta-normal approach is based on the assumption that the underlying market factors have a multivariate normal distribution". The distribution of the mark-to-market profits and losses is then determined to also be normal, after which mathematical properties of the normal distribution are used to determine the quantile of the distribution (the VaR). The major drawback of this approach is the assumption of normality where, in the real world, the distribution of actual data is far from the normal distribution.

Again, according to Gençay & Selçuk (2004)**,** the V-C method sometimes underestimates the risk due to its assumptions, and another major drawback is that it was not found appropriate for asymmetric distributions. As per Table 12, the first two articles on the estimation of VaR using the V-C method were published in 1999 in the "European Financial Management" and "Journal of Derivatives". The number of articles published thereafter fluctuated throughout the period 1999 to 2008, with an average of 4, reaching a maximum of 7 articles in 2006, attributable to the issuance of Basel II. From 2 in 2008, this figure rose to 9 in 2009, attributable to the financial crisis of 2008. From 2009 onwards, the number of articles continued to fluctuate, but with a larger number, the minimum and maximum being 7 and 15 in 2015 and 2017, respectively, and a global average of 11 articles per year between 2009 and 2017. Table 13 contains the details of the articles published on the V-C method with the highest citation count.

Table 12. Distribution of articles per year on VaR estimation using the Variance-Covariance method

Year	Articles	Year	Articles
1999	2	2009	9
2000	6	2010	13
2001	3	2011	7
2002	2	2012	10

2003	1	2013	7
2004	3	2014	13
2005	5	2015	15
2006	7	2016	8
2007	6	2017	15
2008	2		
Total	**134**		

Table 13. Articles on the Variance Covariance method with the highest citation count

Title	Author(s)	Year	Journal	Citation Count
Extreme value theory and Value-at-Risk: Relative performance in emerging markets	Gençay, Selçuk.	2004	International Journal of Forecasting	162
Value at Risk	Linsmeier, Pearson.	2000	Financial Analysts Journal	110
Using extreme value theory to measure value-at-risk for daily electricity spot prices	Fong Chan, Gray.	2006	International Journal of Forecasting	79

4.6. Copulas

This research found that the use of Copulas in estimating VaR started in 2001 with Cherubini and Luciano (Cherubini & Luciano, 2001). These authors evaluated the tail probabilities and market risk trade-offs at a given confidence level using copula functions, while dropping assumption of joint normality of returns. Copulas were introduced into the field of finance to calculate the correlations between the returns of assets that are not normally distributed. According to these authors, although copulas are a useful tool to represent any joint distribution function when computed at the marginal distribution values, contrary to the traditional joint distributions they separate the effect of the marginal distribution from the effect of association or dependence between returns. More specifications for copula models with time-varying dependence structures were brought together in Manner and Reznikova (2012) which compared different copula models and their applicability.

The estimation of VaR using copulas can be considered a recent technique, as can be seen in Table 14. The first article was published in “Economic Notes” by Cherubini U. and Luciano E. in 2001. Until 2008, the number of articles published on this topic fluctuated between 1 and 2, with an average of 2 per year. However, in 2008, the number of articles witnessed a rapid increase to 9, after which the number remained considerably high compared to the years prior to the crisis, averaging 11 in the period 2009 to 2017, and reaching a peak in 2017 with 16. This huge difference in the average number of articles between the period before the crisis (2 per year) and after (11 per year) could be linked to the efficiency of this method in the estimation of VaR compared to other methods. The details of the articles published on Copulas in the estimation of VaR with the highest citation count are shown in Table 15.

Table 14. Distribution of articles per year on VaR estimation using Copulas

Year	Articles	Year	Articles
2001	1	2010	7
2003	1	2011	11
2004	2	2012	5
2005	1	2013	14
2006	2	2014	13
2007	1	2015	14
2008	1	2016	10
2009	9	2017	16
Total		**108**	

Table 15. Articles on Copulas with the highest citation count

Title	Author(s)	Year	Journal	Citation Count
Model uncertainty and VaR aggregation	Embrechts, *et al.*	2013	Journal of Banking and Finance	88

5. Conclusion

This paper analyses the different models that researchers have used to calculate Value-at-Risk, focusing on the evolution of the number of papers on each model and examining their application before and after the financial crisis. Some models have obviously been adopted more than others due to their advantages in terms of performance, implementation and application. However, the six models that have been used most represent 75% of the total number of papers on VaR modelling.

Considering the period from 1996 to 2017, the ARCH/GARCH models, used to model volatility and the issuance of Heteroskedasticity, are the most widely used in the estimation of risk, with 335 published articles. The GARCH models have been proven to successfully predict conditional variance, even in its simplest form. The highest increase in the evolution of the number of articles on models used to estimate VaR was for the ARCH/GARCH models, rising from 16% in the period 1996 to 2007 to 24% in the period 2008 to 2017, highlighting its growing importance in this field.

The Extreme Value Theory is another widely used model in the estimation of VAR, and in the greatest number of works it is coupled with GARCH models. It has proven its superiority over other models in terms of accuracy and precision, including both RiskMetrics, when applied in the Asian market during periods of financial turmoil (Pownall & Koedijk, 1999), and the Historical Simulation method (Ho et al., 2000). The average percentage of articles published on the estimation of VaR using EVT increased from 9% in the period 1996 to 2007 to 12% in the period 2008 to 2017.

The Monte Carlo Simulation is a traditional semi-parametric approach for estimating VaR, gaining in popularity because of its advantages compared to other modelling methodologies, despite being a considerably expensive, time-consuming method. It became more popular after the crisis of 2008, which may be sound proof of its accuracy. The Monte Carlo simulation witnessed a slight growth in the number

of articles published, although the average increased by just 2%, reaching 9% of the total number of articles in the period 2008 to 2017.

The Historical Simulation method is another traditional approach for estimating VaR. Its major drawback is the assumption that history repeats itself, while one of its advantages is that unlike other methodologies it does not make any assumptions regarding the shape of the distribution of risk factors, since empirically these distributions tend to be fat-tailed while most models take the assumption of normality, which is far from reality. In addition, the number of articles published on Historical Simulation increased from an average of 7% in the period 1996 to 2007 to 10% in the period 2008 to 2017.

The Variance-Covariance method is the third basic methodology used for estimating VaR. It is a widely used technique and a parametric approach, which assumes that the underlying market factors have a multivariate normal distribution, this assumption being one of this method's disadvantages. The average percentage of the number of articles published on the Variance-Covariance method remained stable throughout the two periods, at 8%.

Copulas are widely used in the estimation of VaR because they aim to evaluate the tail probabilities and market risk trade-offs at a given confidence level. Contrary to some model assumptions, they are mainly used to calculate the correlations between the returns of assets that are not normally distributed, and this fact in turn makes the use of correlation coefficients insufficient. The first article published on this topic was in 2001, since when the number of articles increased from an average 6% in the first period to an average of 8% in the second.

Last, regarding the current trend, the ARCH/GARCH models and Copulas, followed by the EVT, are taking the lead in the estimation of VaR over the major three traditional methods, the Variance Covariance method, Monte Carlo Simulation and Historical Simulation. The latter methods also witnessed a growth after the outbreak of the financial crisis, but it was minor compared to the now leading methods and models.

Declarations of Interest

"The authors report no conflicts of interest. The authors alone are responsible for the content and writing of the paper."

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