



Leaders, factions and the determinants of electoral success[☆]

Benoît S.Y. Crutzen^{a,*}, Sabine Flamand^b

^a Erasmus Universiteit Rotterdam, Tinbergen Institute, Netherlands

^b Universitat Rovira i Virgili, ECO-SOS, Spain

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ABSTRACT

We model the internal game between the leader and the factions of a party, to study the effect of party leadership on the determinants of electoral success. Factions are of interest or of principle. The probability of winning the election is increasing in how close the party is to the median voter, the leader's charisma, party coherence and the factions' contributions. The leader offers both types of factions their favourite rewards in exchange for contributions. We show that party coherence and factions' contributions are not necessarily increasing in the leader's charisma and ideological proximity to factions. Further, we show that factions of interest constraint the party's electoral strategy less than factions of principle. In particular, factions of interest always contribute more, are less of an obstacle towards achieving party coherence, and offer the party more freedom in its choice of the ideological location and charisma of the party leader.

1. Introduction

Electoral success and policymaking are the overarching priority of any mainstream party. To meet this goal, parties follow multi-faceted strategies. To deepen our understanding of the forces that shape these strategies, we need to analyse not only the characteristics of the electoral arena parties are in, but also the inner workings of parties themselves. It is on the inner workings of parties that this paper focuses on.

Our premise is that parties are aware that voters value ideological proximity (Downs, 1957; Enelow and Hinich, 1982), contributions by factions and activists (Aldrich, 1983; Schofield, 2003), party coherence behind the party leader and manifesto (Greene and Haber, 2015) and charismatic leaders (Schofield, 2003; Bittner, 2011; Cross and Pilet, 2015).¹ These four characteristics of a party are essential in determining its electoral success: an ideologically appealing leader and platform can be quickly undermined if the party does not defend and acts coherently with its leader and platform (Snyder and Ting, 2002; Dewan and Myatt, 2007, 2008; Andeweg and Thomassen, 2010; Ceron, 2012; Vivyan and Wagner, 2012; Marx and Schumacher, 2013; Butler and Powell, 2014; Greene and Haber, 2014, 2015).

Tony Blair was an asset for New Labour in the 1997 election precisely because he managed to convince his party to unite behind him and the platform he was defending was particularly appealing to the British electorate of the time (Dewan and Myatt, 2007). Theresa May's lack of charisma and poor leadership was one of the key reasons for the Tories' poor performance in 2017. One of the key tasks of the successor to Boris Johnson will be to restore party coherence behind the party leader and manifesto. The French

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* Corresponding author.

E-mail addresses: crutzen@ese.eur.nl (B.S.Y. Crutzen), sabine.flamand@urv.cat (S. Flamand).

¹ Given that our theory rests on the ideological internal heterogeneity of parties, our theory speaks to mainstream parties with a relatively broad appeal, rather than niche and extreme parties, which are typically, though not always, ideologically much narrower and homogeneous.

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Socialist party in the June 2017 Presidential election offers another vivid example of the importance of parties meeting the four conditions above if they wish to perform in elections. The then leader of the Socialists, Benoît Hamon, notwithstanding the fact that a primary had given him the leadership of the party, found himself competing intensely and vocally during the whole electoral contest with Manuel Valls, the head of the most centrist faction in that party. Such public display of divisions inside socialists severely reduced Hamon's electoral appeal.

In the USA, think of Obama's 2008 campaign. By generating enthusiasm, goodwill and hope under his "Yes we Can" campaign slogan, Obama's charisma led the democrats to follow coherently Obama's main campaign message. To the contrary, Hillary Clinton did not manage to tame down and control the dissenting voices within her party, especially those coming from the left wing ideological side of the party and from Bernie Sanders in particular. This generated a lot of incoherence between the actions and messages of the Clinton campaign and those of other prominent democrats. Voters were also sceptical about her and her electoral platform, as she admits herself in her account of her presidential defeat (Clinton, 2017).

In line with the above examples, we wish to understand better how the charisma and ideological position of a party leader influence the decisions of other key party players—especially party factions. We then build on our findings to analyse how the type and ideological position of both the party leader and party factions impact the determinants of electoral success we referred to above. We give factions centre stage because, very often, a party's prominent figures are the top members of a faction. Factions are thus key players within parties (Bettcher, 2005; Persico et al., 2011; Ceron, 2012, 2019; Dewan and Squintani, 2016).

We develop a formal theory of parties as complex, multi-agent organizations (Laver and Benoit, 2003; Laver and Schofield, 1998). The game we analyse is that of a mainstream party – such as the US Republicans and Democrats or the U.K.'s Labour and Conservatives – which has to choose at its annual conference a new leader, knowing that this leader then needs to strike an agreement with the different party factions to ensure that both party work and the general election strategy are carried out and supported as well as possible. We model the relationship between factions and the leader as a Tullock contest game in which the leader offers factions some rewards in exchange for their contributions (Tullock, 1980). To the best of our knowledge, our model is the first to analyse the effects of leadership charisma and ideology on factional choices and party coherence, and thus electoral success.

We consider two types of factions.² They can be either of principle or of interest, to follow the terminology of Bettcher (2005). Factions of principle are ideological in nature and wish to influence the party manifesto, to reap 'collective benefits' from the exchange with the leader. Examples of such factions were evident in U.K.'s Labour in the eighties or the Australian Labour Party (Brand, 1989; McAllister, 1991) and in France's Socialist party before their dismal defeat in 2017, an example we return to later on. They were very visible in U.K.'s conservative party during Brexit. Factions of interest wish to obtain party funds or other 'selective benefits', allowing them to continue developing and sustaining their activities, as was the case for Chicago's Daley machine or Italy's *Democrazia Cristiana*.

There is a key difference between these two types of factions that we build upon. A faction of interest does not care *directly* about which factions get the funds they did not manage to secure for themselves, as the allocation of these other funds does not impact its activities *directly*. To the contrary, a faction of principle cares *directly* about the total distribution of influence across all factions, as the latter determines the overall electoral message the party sends to voters, and thus can interfere *directly* with the faction's own plans. The decision of a faction of principle generates an externality on the other factions of principle, something that does not happen with factions of interest.

We also consider two types of leader: strong and charismatic or weak and dull. Our definition of charisma is that of *authority charisma*: an individual is authority charismatic when its charisma's first purpose is to have followers abide by their leadership.³ Well known examples of authority charismatic leaders, on top of those mentioned earlier, include Baroness Thatcher and General De Gaulle. We follow the literature on valence politics (see for example Enelow and Hinich, 1982; Ansolabehere and Snyder, 2000; or Schofield, 2006 for theoretical accounts and Bittner, 2011 for empirical evidence): given that charisma is an important facet of a politician's valence (but, obviously, not the only one), voters value charismatic leaders more than non-charismatic ones. This is the bright side of having a charismatic leader. Yet, authority charismatic leaders can bias the exchange game they play with factions in favour of those they prefer, as they can use their authority and electoral appeal as a lever to give their views prominence (Dewan and Myatt, 2008; Schumacher et al., 2013; Landa and Tyson, 2017; Bittner, 2011). This is the dark side of authority charisma. Thus, the contest among factions is fair and unbiased under a weak leader. To the contrary, a charismatic and strong leader biases the game in favour of those factions that are closest ideologically to them. These factions end up receiving a fraction of the rewards that is larger than what their contribution would command.

Empirical research confirms that the bargaining game between the leader and factions of a party is central to the determination of that party's platform. For example, Ceron (2012, 2019) provides strong support for the idea that "factions negotiate over party positions according to the bargaining power of each subgroup". His key finding is that factions affect the party manifesto proportionally to their strength.⁴ Yet, Ceron also finds that this proportionality is weakened when leaders have a degree of autonomy from the parliamentary party and thus can bias the distribution of factional influence to their liking. We add that strong and

² Many preceding contributions also group factions into two main types; see for example Janda (1980), Panebianco (1988), Bettcher (2005), König (2006), Persico et al. (2011) and Dewan and Squintani (2016).

³ Weber (1968, chapter 6), is among the first to discuss this concept. He remarks that this type of charisma can serve the organization – the party, here – only if the leader aligns the effects of their charisma with the objectives of the organization. As in our theory the party selects the leader to maximize its chances of winning the forthcoming election, such a need is automatically satisfied in our case. For a recent popular report on charisma and authority charisma, see BBC (2017).

⁴ Lo et al. (2014) offer similar evidence.

charismatic leaders, because of the support they benefit among voters and other followers, should find it easier than weak leaders to be granted more autonomy and influence inside their party. This justifies and rationalizes our modelling strategy for the contest between the leader and factions.

We again follow the literature on valence politics also to model the extent to which the party appears united behind its leader via both the factions' contributions and their influence or activities (Miller and Schofield, 2003; Schofield, 2003, 2006; Schofield and Sened, 2005). A party achieves coherence only when the distribution of the contributions (and rewards) of the different factions is monotonically decreasing in the ideological distance between the leader and the different factions. Indeed, if the faction that is endowed with most resources is the one that is ideologically most distant from the current leader, the message the party is sending to voters is very unclear, not to say outright contradictory. Such a situation can only be electorally costly for that party. As we argued above, a vivid example of such a suboptimal situation is offered by the French Socialist party in the June 2017 Presidential election. Another important example is the 2016 Presidential election fight within the Democratic party between Hillary Clinton and Bernie Sanders.

That diverse and possibly contradictory influences on the party manifesto can reduce its coherence is self-evident. That the activities of factions of interest also impact the party's image less so. But they do, as they have the scope and ideological colour of the faction carrying them out – think of the trading of votes for jobs under Daley's Chicago machine or the public jobs the different factions within Italy's Christian Democratic party *Democrazia Cristiana* were fighting over (Bettcher, 2005; Persico et al., 2011).

Tuning to our results, we find that charisma generates a trade-off between faction contributions and party coherence. For any given ideological position of the leader and factions, weak leaders are always associated to higher faction contributions than charismatic ones. Yet, charisma increases (weakly) intraparty coherence, and the more so if factions are of interest. We then derive sufficient conditions for the party to select a charismatic leader (whose ideology is as close as possible to that of the median voter). We then show that parties with factions of interest should find it less costly to track any ideological shifts in the electorate, all else equal.

Another directly related consequence of our findings concerns the ability of parties to adapt to changes in the distribution of factions within the ideological support of a party. After a leadership change and/or an electoral defeat, the composition and distribution of factions can change quite dramatically, as documented for example by Bettcher (2005), Persico et al. (2011) and Dewan and Squintani (2016). Our findings suggest that parties face less constraints in adapting to such factional changes when factions are of interest.

Turning to the characteristics of elections, these have grown more personalized, with party leaders gaining importance; see for example McAllister (2007), Cross and Pilet (2015) and Wauters et al. (2016). Our theory suggests that parties are more likely to favour and sustain electoral systems that are more personalized if their factions are of interest. These electoral systems increase the importance of having strong and charismatic leaders, and having such leaders is less costly with factions of interest. In fact, selecting a charismatic leader allows parties with factions of interest to take full advantage of the personalization of the electoral contest and to foster party coherence, even though there is a cost in terms of factional contributions.

Finally, the growing personalization of campaigns also appears to have gone hand in hand with changes in the media environment (think about not only television, but also the internet and social media) and with the professionalization of political campaigns; see for example Lilleker (2005), Farrell (2006), Strömbäck and Kioussis (2014) and Steffan and Venema (2019). By their very nature, factions of interest are in a better and more natural position to reap the benefits of such personalization and professionalization of campaign. Some, if not many campaign professionals are likely to originate in these factions, for example because by their very nature these factions are likely to count such professionals in their ranks and they also do not want their party's resources to go to external companies or consultants. This gives factions of interest an edge over factions of principle.

The rest of the paper is structured as follows. The next section reviews the related literature. Section three presents the model. Section four solves for the optimal behaviour of factions. The following section focuses on how our findings impact the determinants of electoral success. The last section discusses our findings and concludes. All proofs are relegated to the Appendix.

2. Related literature

The main goal of our analysis is to study how a party leader's type and ideological position jointly impact the willingness of different types of factions to contribute to their party's work and electoral strategy and thereby the determinants of its electoral success.

We build on three strands of the existing literature. First, we borrow from the literature on valence politics and the personalization of politics the idea that strong and charismatic leaders bring about an electoral bonus, all else equal. Important contributions are Schofield (2003), Bittner (2011) and Cross and Pilet (2015). For example, Schofield (2003) builds a formal model that adds to the literature on spatial electoral competition by letting the party leader be of high or low valence, and Bittner (2011) shows empirically that the charisma of leaders – as opposed to their competence, for example – is the most important factor determining success in elections. Recently, Wauters et al. (2016) also show that, at least in the Belgian flexible proportional system, the personalization of politics seems to be increasingly limited to party leaders, in line with our working assumption that having a strong and charismatic leader is an electoral advantage, all else equal. We add to this literature by suggesting two additional consequences of having a strong and charismatic leader that may be important for the electoral score of a party: factional contributions are lower under a strong leader, but the degree of internal coherence can be higher with such leaders.

Second, we also take seriously the idea of the literature on valence politics that a party's main activists and factions are key to determine the party's electoral fortunes. See for example Miller and Schofield (2003), Schofield (2006) and the literature that

builds on these articles, such as Schofield and Sened (2005), Karp et al. (2007), Schofield et al. (2011) and Kernell (2015).⁵ Schofield (2003) states that “[b]y contributing time and money to the candidate, activists allow the candidate to advertise his (or her) policies and persuade otherwise uncommitted voters that the candidate is a suitable choice”. Our contribution here is to allow factions to be of one of the two main types – namely of interest or of principle, following Bettcher (2005) and Persico et al. (2011) – to study the willingness of different types of factions to contribute under different types of leaders.

Third, our formal theory contributes to the literature on the electoral consequences of party coherence by suggesting one mechanism through which the interactions between the position and type of the leader and that of the factions map into the degree of party coherence that can be achieved. Caillaud and Tirole (1999, 2002) were among the first to propose a formal model in which disagreement among factions can reduce the electoral appeal of the party, but their theory offers no explicit role for the party leader and his views. Other formal theories which focus on the effects of disagreements among factions are Hortala-Vallve and Mueller (2015) and Mutlu-Eren (2015). In these contributions it is the effect of party splits that takes centre stage. Dewan and Myatt (2007, 2008) are contributions which focus on the role and effects of the characteristics and choices of a party leader, but in their theory factions play no explicit role. Our results also resonate with those of Dewan and Squintani (2016), even though our model is very different from theirs. Invernizzi and Prato (2022) also develop a formal theory of how a party leader can incentivize (two) factions to contribute to the party’s electoral efforts. They do not distinguish explicitly between factions of interests and of principle and they have only one type of leader. They focus on the relationship between the characteristics of the electoral system (the extent to which the election is a winner-take-all one) and the optimal shape of the function governing the allocation of power across factions. They find, like us, that if the median voter cares sufficiently strongly about ideology and a party’s manifesto, then parties are often better off not distorting power sharing between factions, which in the context of our model requires the leader to be weak.⁶ Another important strategy factions may follow but that we do not consider in our work (because of the nature of the leader-faction interaction in our model) is intraparty sabotage. Invernizzi (2023) studies a model of elections between two parties in which factions can either campaign for the party or sabotage the other factions within their party. Parties redistribute electoral spoils to factions based on a noisy signal of the choices of each faction. She shows factions trade off the benefits of sabotage with its costs in terms of electoral success. This implies that intra- and interparty competition are substitutes. And, as the electoral stakes increase, factions invest fewer resources in sabotage and devote more resources to promote the party. Yet, Invernizzi (2023) does not study how the incentives to use sabotage vary with the type of faction. This question is thus an interesting avenue for further research.

Empirical exercises in this strand of the literature include Andeweg and Thomassen (2010), Ceron (2012), Vivyan and Wagner (2012), Marx and Schumacher (2013), Butler and Powell (2014) and Greene and Haber (2014, 2015). Greene and Haber (2015) offer empirical backing to our contribution as they provide evidence that party incoherence is viewed negatively by voters, all else equal.

Finally, we contribute to the advancement of our understanding of which party characteristics make parties more or less reactive to changes in the environment they are embedded in Schumacher et al. (2013) and more or less favourable to electoral systems that give more prominence to their key players, especially their leader (Cross and Pilet, 2015; Wauters et al., 2016)

3. The model

We consider a game between a party leader and factions in a setting where voters favour a party: (1) that is close to them ideologically (Downs, 1957); (2) whose leader is charismatic (Bittner, 2011); (3) that appears and acts coherently with the ideological position of its leader (Greene and Haber, 2015); and (4) in which factions contribute to party work (Schofield, 2003). Thus, when choosing its leader, the party takes into account not only the direct electoral consequences of the leader’s ideological position (which also determines the party’s electoral position) and type, but also their indirect consequences on party coherence and factional contributions.

Leaders are characterized by an ideology and a degree of authority charisma. Authority charisma is defined as someone’s capacity to have followers or subordinates obey their command.⁷ The leader’s ideology is $L \in [0, 1]$, which is also the ideological location of the party manifesto. The leader’s authority charisma S can be weak ($S = 0$) or strong ($S = 1$). Whenever the leader is charismatic, the party receives an electoral boost. The leader wishes to give prominence to the views that are in line with theirs.⁸ An authority charismatic leader is in a position to do this, whereas a leader with weak charisma is not. In other words, authority charisma allows the leader to favour the factions with an ideology that is close to theirs.

Party factions are of two types. Factions of principle are populated by politicians motivated by ideology, while factions of interest are populated by politicians motivated by the desire to obtain personal benefits (in our case, party funds). While factions of principle would like to craft the party manifesto in line with their ideology, factions of interest would like to get party funds to carry out their factional activities. There are three equally-sized factions of each type, labelled A , B and C .⁹ Each faction is characterized by

⁵ Aldrich (1983) is one of the very first formal models that includes activists, yet in that article activists do not influence the platform of the party.

⁶ Casas and Kselman (2022) analyse how organizational centralization relates to the mobilization effort of parties’ representatives.

⁷ See Weber (1968, chapter 6) and BBC (2017).

⁸ See for example Schelling (1978) and Benhabib et al. (2010) on the pervasiveness of such homophily in very diverse environments.

⁹ Allowing factions to differ in size/power would generate the results that Ceron (2012) uncovered empirically, namely that more powerful factions would benefit more from the exchange with the leader. We abstract from this effect for the sake of simplicity only.

an ideological position on the ideological spectrum: A has its ideological bliss point in 0; C has it in 1 and that of B is somewhere in $[0, 1]$.¹⁰

To incentivize factions to contribute to the party’s strategy and goals, the leader sets up two exchanges with factions, one per faction type.¹¹ In the exchange with factions of interest, the leader offers party funds worth V in exchange for the factions’ contributions. These funds can then be used by the different factions to finance their own activities. In the exchange with factions of principle, the reward offered to factions in return for their contributions is influence on a share I of the party manifesto. I is the share of that part of the party manifesto all players involved expect their party to be able to implement after the election (provided all players behave according to the rules of the game and in accordance with the incentives provided to them). For the sake of simplicity and ease of exposition, we take V and I as exogenously given.¹² We also assume the leader’s type does not influence V and I .¹³ Yet, in Appendix B we extend our game to endogenize V and I to show that our results survive qualitatively such an extension.

Let c_i be faction i ’s contribution to the party. In the spirit of Tullock (1980), the mapping governing the exchange is given by the following standard imperfectly discriminating contest success function:

$$Sh_i = \frac{b_i c_i}{\sum_{j=A,B,C} b_j c_j} \tag{1}$$

where Sh_i is faction i ’s share of party funds or party manifesto influence and where the different b_i ’s are the weights the leader uses in evaluating each faction’s contribution to party work. All weights are between zero and one. The closer to one is b_i , the more the leader favours faction i . The leader’s capacity to bias the exchange with factions is a function of the leader’s charisma. If the leader is weak, the exchange with factions is unbiased and $b_i = 1$ for all $i = A, B, C$. If the leader is strong, the exchange is biased. For the sake of analytical simplicity, we assume the biases the leader uses are given by one minus the Euclidean distance between the leader’s ideological position L and that of any faction i . Let $L \in [0, 1/2]$ (the biases are defined symmetrically for $L \in (1/2, 1)$). Given that $A = 0$ and $C = 1$, these biases are given by:

- If $B \geq L$:

$$\begin{aligned} b_A &= 1 - (L - A) = L \\ b_B &= 1 - (B - L) = 1 - B + L \\ b_C &= 1 - (C - L) = L \end{aligned}$$

- If $B < L$:

$$\begin{aligned} b_A &= L \\ b_B &= 1 - (L - B) = 1 - L + B \\ b_C &= 1 - L \end{aligned}$$

Without some form of compensation, factions do not contribute to their party beyond a certain level. This is why the party leader enters in an exchange with the party factions. While there may be diverse reasons why factions may need to be compensated to follow their leader’s desires and choices, we capture this aspect of the problem by assuming that factions bear a strictly positive cost for their contributions – for the sake of simplicity only, we actually set this cost equal to the contribution itself – and thus normalize to zero the minimal contribution factions are willing to offer for free (such a minimal contribution can also be thought of as the contribution factions offer given the baseline expectation of how well the party will fare in the election if no further help and contribution is offered). Thus, in the exchange with factions of interest, faction $i = A, B, C$ solves the following problem:

$$\max_{c_i} \frac{b_i c_i}{\sum_{j=A,B,C} b_j c_j} V - c_i, \tag{2}$$

where c_i is faction i ’s contribution cost.

Contrary to factions of interest, factions of principle, as they care about the overall ideological stance of the party, also care directly about how the other factions of principle influence the party manifesto. And, logically, any faction of principle would rather see other factions who are ideologically close to them influence the manifesto. Thus, for ideological factions, the objective function takes into account the influence of other factions, and the weight attached to each of the other factions’ influence is decreasing in the (Euclidean) ideological distance between them and these other factions. In other words, there are externalities in their objective function. Faction $i = A, B, C$ solves the following problem:

$$\max_{c_i} \frac{b_i c_i}{\sum_m b_m c_m} I + (1 - d_{ij}) \frac{b_j c_j}{\sum_m b_m c_m} I + (1 - d_{ik}) \frac{b_k c_k}{\sum_m b_m c_m} I - c_i \tag{3}$$

¹⁰ The reason why factions of interest have an ideological colour too is simply that the actions they undertake are obviously in line with their views.

¹¹ We split the exchange between the leader and the factions into two different contests for the sake of expositional ease. Indeed, as the goals of the two types of factions differ and so do the rewards available, the choices of one set of factions does not influence directly the choices of the other set of factions. Thus there is not too much loss of generality in splitting the exchange the way we do.

¹² In what follows we implicitly restrict V and I to both be large enough so as to ensure that the equilibria we characterize are well defined and unique.

¹³ Assuming that V and I are smaller under a weak leader would reinforce all the results that follow.

where $d_{AC} = C - A = 1$, $d_{AB} = B - A = B$, $d_{BC} = C - B = 1 - B$.

Given the above structure, intuition suggests that the type and position of both the leader and the factions influences the distribution of equilibrium factional contributions and rewards. This joint distribution is what determines the party's degree of coherence behind its leader. For either factional type, we say that *strong* party coherence is achieved if the mapping from the distance between the position of the leader and factions to their contributions and rewards is *strictly decreasing* in this distance: the more distant a faction is, the less it contributes and the less funds/influence it gets. Indeed, with such a decreasing mapping, the message the party and the leader send to voters is most coherent. If the mapping from distance to contributions and/or funds/influence is *weakly decreasing* in distance, we have *weak* party coherence. If the mapping from distance to contributions and/or funds/influence is *not monotonically decreasing* in distance, we say that party coherence is *absent*.¹⁴

The party wants to maximize its electoral success, and thus the median voter's utility, which is given by¹⁵:

$$U = \alpha S + \beta C + \gamma \sum_j c_j + \delta(1 - d) \tag{4}$$

where $S = \{0, 1\}$ is the leader's charisma, and takes value 0 (resp. 1) when the leader is weak (resp. authority charismatic); $C = \{0, 1/2, 1\}$ is the degree of party coherence, and takes value 0, 1/2, or 1 when party coherence is absent, weak, or strong, respectively; $\sum_j c_j$ is the sum of factional contributions to party work (be them of interest or principle), and d is the ideological distance between the median voter's ideological bliss point and that of the leader.

Given that we focus on elections in which voters value ideological proximity, contributions by factions and activists, party coherence behind the party leader and manifesto and charismatic leaders, parameters α, β, γ and δ are all restricted to be positive. Besides that, we do not take a stand on which aspect of the election should be given centre stage, that is, on which parameter(s) should take on a higher value. In Section 5 we analyse how the decisions of the party and its factions impact its electoral success.

The timing of the game is as follows:

1. The party selects the type and ideological location of its leader;
2. The three factions of each type contribute;
3. The three factions receive their reward which, together with the contributions, define the degree of party coherence achieved;
4. The median voter observes, for each party, the location and type of the leader, the degree of party coherence, the sum of factional contributions and then casts their ballot for the party they prefer.

We use backward induction to solve for the factions' decisions and the party's optimal choices.

4. Equilibrium factional behaviour

As steps 3 and 4 in the timing above are non strategic, our analysis starts at $t = 2$ with the factions' problem.

4.1. Factions of interest

Suppose first that the leader is weak and thus the exchange procedure for the allocation of party funds is unbiased. All factions compete for the funds on an equal footing: $\forall i, b_i = 1$. In such case, the share of the funds going to faction i is exactly proportional to their contribution to the party, c_i :

$$Sh_i = \frac{c_i}{\sum_j c_j} \tag{5}$$

As a result, equilibrium behaviour is symmetric, as each faction contributes equally and receives the same share of party funds:

Result 1. *Suppose the leader is weak so that the exchange with factions of interest is unbiased. Then, for all $i = A, B, C$, we have:*

- $c_i^* = \frac{2V}{9}$;
- $Sh_i^* = \frac{1}{3}$.

The exchange between a weak leader and factions of interest reduces to a standard [Tullock \(1980\)](#) contest with identical players, in which the equilibrium contribution is increasing in the value of the available reward and decreasing in the number of participants in the contest.

What if the leader is authority charismatic? With such a leader, the exchange procedure is biased towards those factions that are closer to the leader, and equilibrium behaviour across factions is more involved. First of all, when one of the factions is too far from both the leader and the other two factions, this isolated faction decides not to contribute. The intuition for why this isolated

¹⁴ Using a continuous measure such as $\sum_j (1 - d_{Lj})Sh_j^*$ yields the same results as the mapping from distance to contributions and funds/influence for the factions that participate in the exchange turns out to be of three types only: (1) strictly decreasing in distance, (2) independent of distance; or (3) neither strictly decreasing in nor independent of distance.

¹⁵ We could add a further term in the equation, to capture the effect of factional activities beyond their contributions to party work on their party's popularity. As our model does not focus on how such activities vary and impact the faction's popularity across elections, this term would take the form of a constant ϵ in Eq. (4) and would not affect party and factional incentives at the margin.

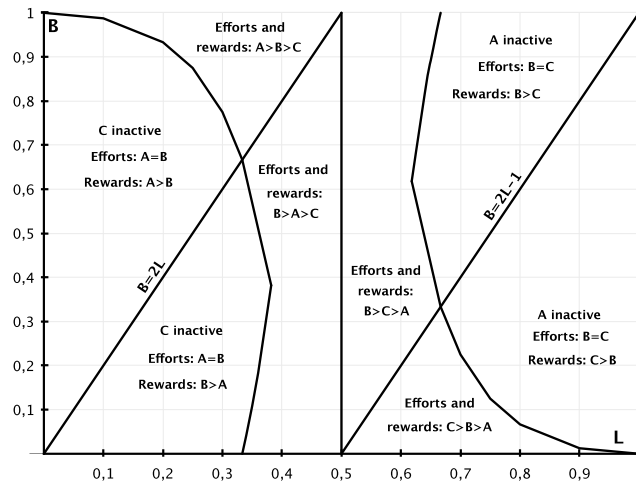


Fig. 1. Contributions and fund shares of factions of interest.

player decides to drop out of the exchange is quite simply that they feel that the procedure is so biased against them that there is no point in participating in it. Further, whenever one faction does not participate in the exchange with the leader, the two active factions behave symmetrically, even though they are subject to different biases in the exchange procedure.¹⁶ This implies that the active faction which obtains the largest share of the party funds is the one whose bliss ideological point is closest to that of the leader. We thus have:

Result 2. Suppose the leader is authority charismatic. In any equilibrium in which only factions of interest *i* and *j* participate in the exchange with the leader, these two factions are the ones that are closest to the leader, and:

- $c_i^* = c_j^* = \frac{b_i b_j}{(b_i + b_j)^2} V$;
- Total contributions are maximized and equal to $V/2$ when *L* is equidistant to the two active factions, that is, when $L = B/2$ (resp. $L = \frac{1+B}{2}$);
- $Sh_i^* > Sh_j^*$ if and only if $d_{Li} < d_{Lj}$.

What if the positions of the three factions and the leader are such that all three factions actively participate in the exchange with the leader? Then we have the following equilibrium contributions and party fund shares¹⁷:

Result 3. Suppose the leader is authority charismatic. When all three factions of interest participate in equilibrium, we have:

1. If *B* is closer to *L* than *A* (resp. *C*), then:
 - (a) *B* contributes most, followed by *A* and then *C* (resp. *C* then *A*);
 - (b) *B* also receives the largest share of party funds, followed again by *A* and then *C* (resp. *C* then *A*).
2. If *B* is further away from *L* than *A* (resp. *C*), then:
 - (a) *A* (resp. *C*) contributes most, followed by *B* and then *C* (resp. *A*);
 - (b) *A* (resp. *C*) also receives the largest share of party funds, followed again by *B* and then *C* (resp. *A*).

Fig. 1 is a stylized representation of all the equilibria of the exchange between an authority charismatic leader and factions of interest, as a function of the ideological position of *B* and *L*. The figure also reports the ranking of factional contributions and fund shares.

Let us focus on the left-hand half of the figure, as for the right-hand side of the figure the reasoning is symmetric. An intuition for the shapes and locations of the areas where the faction furthest away from *L* is inactive is as follows. Imagine that *L* is close to *A*, in 0.2, say, and so is *B*, in 0.3 say. Then *C* has a very low probability of receiving any funds because of his very low bias and the large biases that both *A* and *B* enjoy. As any contribution is costly to *C*, *C* is better off dropping out of the exchange. Now move *B* closer to *C* and further away from *L*, keeping all else equal. Then the share of funds *C* can hope to obtain increases because

¹⁶ The fact that the contest between the two factions generates an equilibrium in which each faction chooses the same contribution (but their shares of the party funds are a function of their ideological distance to the leader) is a standard side product of contest models à la Tullock (1980).

¹⁷ In the proof of this result in the Appendix, we pin down precisely all parameter configurations leading to all three factions being active.

the leader favours B less and C (and A) relatively more (the bias of B decreases hence the expected share of funds of C increases because of the reduction in the denominator of Sh_C). Hence, as B gets closer to C , there can be a position of B beyond which C starts to participate in the exchange.

Let us now turn to the intuition for why the set in which only two factions are active has a missile-like shape. Focus on the area in the figure with L around 0.35. Start from a point where all three factions are active, say at B in 0.9. Then, as B gets closer to L , B 's expected share of party funds increases, to reach a pinnacle for $B = L$. This must depress (A 's and) C 's incentives to participate in the exchange (or more generally the incentives of the faction that is furthest from L), so that C is inactive. Finally, B 's bias starts to decrease as B moves beyond 0.35, so that C has an incentive to participate.

4.2. Factions of principle

How do factions of principle behave when their leader is weak and thus the exchange is unbiased? As with factions of interest, all factions are then on an equal footing. However, these factions care directly about how the other factions influence the manifesto, so we should expect the behaviour of factions not to be symmetric even when the exchange is unbiased. Indeed, when the leader is weak and thus the exchange is unbiased, we have:

Result 4. *Suppose the leader is weak so that the exchange with factions of principle is unbiased. Then, in equilibrium we have:*

- $c_A^* = c_C^* = \frac{L}{4}$ and $c_B^* = 0$;
- $Sh_A^* = Sh_C^* = \frac{1}{2}$ and $Sh_B^* = 0$.

With factions of principle and when the leader is weak, only factions A and C actively participate in the exchange: B does not find it worthwhile to participate, the two active factions behave symmetrically and are offered the same influence on the manifesto.¹⁸

The behaviour of the different factions of principle within the French Socialist party at the last presidential and legislative elections of 2017 are very much in line with this prediction: Benoit Hamon, the socialist leader, chose a markedly leftwing platform but turned out to be a very weak and electorally unappealing candidate. As predicted by our model, the faction of Manuel Valls, which was the one furthest away from that of the Socialist leader, was also the most active and vociferous in the election. The findings above also rationalize the struggles between Hillary Clinton and Bernie Sanders in the 2016 US Presidential election, who on many policy issues and ideas were quite clearly at opposite ends of the ideological spectrum within the Democratic party.

Suppose now that the leader is authority charismatic so that the exchange is biased towards those factions which are ideologically closer to the leader. As was the case for factions of interest, whenever one faction does not participate in party work in equilibrium, the two active factions behave symmetrically even though they are subject to different biases in the exchange procedure. Hence, the faction which ends up being most influential over the party manifesto is the one whose bliss ideological point is closest to the one of the leader, thereby contributing to party coherence. We have:

Result 5. *Suppose the leader is authority charismatic. In any equilibrium in which only two factions of principle i and j participate in the exchange with the leader, these two factions are the ones that are closest to the leader, and:*

- $c_i^* = c_j^* = \frac{b_i b_j d_{ij}}{(b_i + b_j)^2} I$;
- Total contributions are maximized and equal to $\frac{BI}{2}$ (resp. $\frac{(B-1)I}{2}$) when L is equidistant to the two active factions, that is when $L = B/2$ (resp. $L = \frac{1+B}{2}$);
- $Sh_i^* > Sh_j^*$ if and only if $d_{Li} < d_{Lj}$;

What does participation look like when all three factions of principle are active? We have:

Result 6. *Suppose the leader is authority charismatic. When all three factions of principle contribute, which can happen only if B is closer to L than A (resp. C), we have:*

1. If A (resp. C) and B are on opposite ideological sides of L , then:
 - (a) A (resp. C) contributes most, followed by B and then C (resp. A);
 - (b) B has the largest influence on the manifesto, followed by A (or C) and then C (or A).
2. If A (resp. C) and B are on the same ideological side of L , then:
 - (a) C (or A) contributes most, followed by B and then A (resp. C);
 - (b) B has the largest influence on the party manifesto, followed by A or C .

Fig. 2 displays the set of equilibria with factions of principle.

¹⁸ This result is reminiscent of similar findings in the economics corporate governance literature. See for example, Osborne et al. (2000) and Flamand and Troumpounis (2014), who provide two different but related formal models in which moderate individuals – like B in our model – tend to not participate in costly meetings when participants are *principled* instead of *interested*, to use our factional terminology.

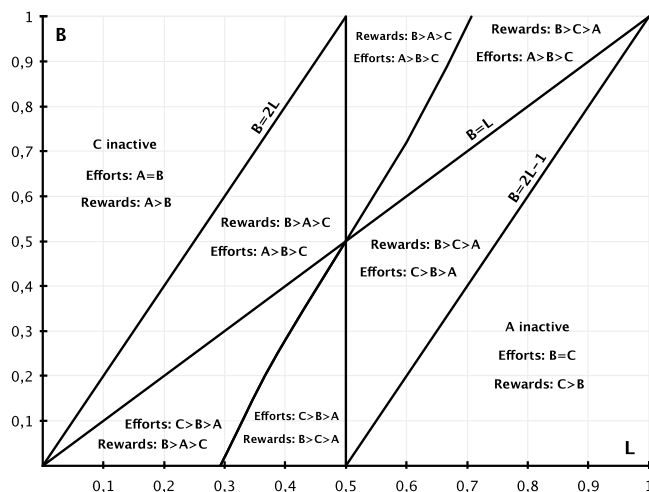


Fig. 2. Contributions and influence shares of factions of principle.

Consider the left-hand side of the figure, where $L \in [0, 1/2]$ (the right-hand side of the figure is symmetric). An intuition for the shapes and locations of the areas where C , the faction furthest away from L , is inactive is as follows. Imagine that L is close to A , say at 0.2. If B is very close to C , say at 0.9, it means that C has high incentives to free-ride on B , as their ideology is very close, and A and B have low incentives to free-ride on each other, hence their contributions are rather large. Thus, it is not worthwhile for C to contribute to the party cause. As B moves closer to A , C has lower incentives to free-ride on B , whereas A and B have higher incentives to free-ride on each other, meaning that their contribution decreases. The fact that the bias of B increases reinforces this process, as it allows B to reduce its contribution while keeping the same expected influence. After some point (namely when $B < 2L$), C finds it worthwhile to devote some time and energy to the party cause. This mechanism goes on as B moves even closer to A until reaching the position of L , at which point the three factions contribute equally. Finally, when $B < L$, and thus A and B are very close to each other, it is C that contributes the most: it has to compensate for the large biases in favour of A and B , whose ideology it dislikes a lot.

Further, notice that faction C is inactive in quite different situations than before. Indeed, with factions of interest, C does not contribute when L is too far from them, unless B is quite close to C so that C does not feel isolated ideologically. To the contrary, with factions of principle, C does not contribute if (L is too far from C and) B is quite close to C , as then C can free-ride on B 's contribution to see the manifesto being influenced along lines that C does not dislike too much.

5. Leader selection and electoral success

Having pinned down how factions behave as a function of the position and type of the party leader, we now turn to the main questions of the paper: what trade-offs does a party face when it sets its electoral strategy and chooses its leader position and type to maximise its electoral success? What leader type and position will it choose? We first pin down how the charisma of the leader impacts factional contributions and intraparty coherence and then build on these findings to discuss the optimal leader selection strategy of the party. Examining how total contributions change with the type and position of the leader across the two types of factions, we have:

Proposition 1. *Given any position of B and L , a weak leader always generates higher total contributions than a strong leader. Also, for any position of B and L and any leader type, factional contributions as a share of the available rewards (V or I) are always higher with factions of interest than with factions of principle.*

Fig. 3 illustrates this last finding for three different positions of B under a charismatic leader.

Turning to party coherence, we have:

Proposition 2. *If the leader is weak, party coherence is weak with factions of interest and absent with factions of principle.*

If the leader is charismatic:

- *If two factions are active (irrespective of their type):*
 - *party coherence is strong if $L = B/2$ (resp. $L = \frac{1+B}{2}$); and*
 - *it is weak otherwise.*
- *If all factions of interest are active, party coherence is strong;*

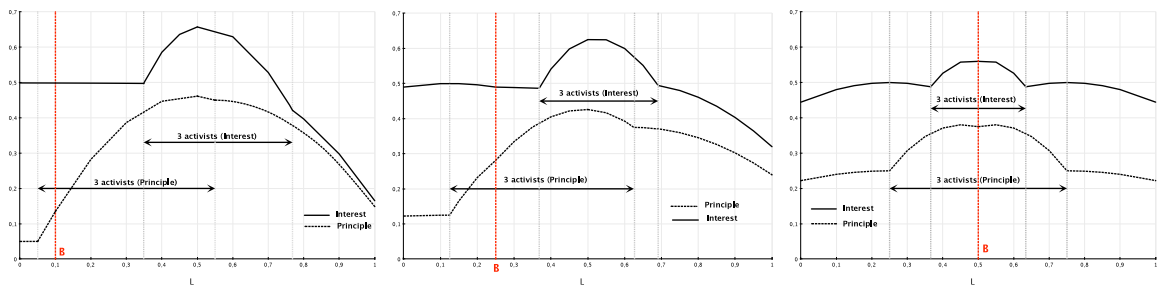


Fig. 3. Total contributions under a charismatic leader.

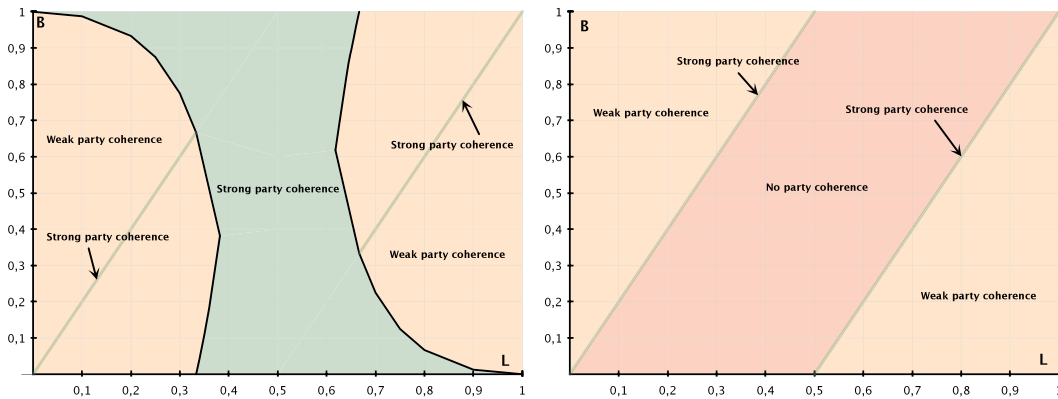


Fig. 4. Party coherence with factions of interest (left panel) and principle (right panel). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

• If all factions of principle are active, party coherence is absent

With factions of interest, when the leader is weak, party coherence is weak, but this is because all three factions contribute the same and receive the same share of resources. In many respects, this is a situation that many real world parties could welcome. Even though no premium is given to factions that are ideologically closer to the leader, all factions in the party participate in the electoral effort. Thus, a weak leader can achieve full party mobilization when factions are of interest.

With factions of principle, when the leader is weak, only the two factions that are furthest ideologically from the leader, A and C, actively participate in the exchange: B does not find it worthwhile to participate. Thus, with a weak leader and factions of principle, party coherence is absent. The message to the electorate that comes out of the party manifesto is likely to be unclear and unappealing to voters, as the views that are contained in there are those of the two most distant factions and thus are somewhat incoherent, if not altogether incompatible.

Having a charismatic leader improves weakly intraparty coherence irrespective of the type of faction, but factions of interest allow for a larger intraparty coherence premium when the leader is charismatic. Indeed, with factions of principle, strong intraparty coherence can only be achieved in the knife edge situations in which $L = B/2$ or $L = \frac{1+B}{2}$, whereas with factions of interest strong intraparty coherence can be achieved in the previous two knife edge situations but also for a non-empty (and quite large) set of ideological positions of the leader, as Fig. 4 demonstrates. This figure is an exact graphical representation of the second part of Proposition 2. The colours regarding the level of party coherence mirror the ones of a standard traffic light: in the green areas, party coherence is strong, while it is weak (resp. absent) in the orange (resp. red) areas. Notice that party coherence is strong along the lines corresponding to $B = 2L$ and $B = 2L - 1$ for both faction types.

Given the above, when will a party select a charismatic leader? When will it shy away from doing so? Remember that the median voter's utility is given by:

$$U = \alpha S + \beta C + \gamma \sum_j c_j(L) + \delta(1 - d)$$

where $S = \{0, 1\}$ is the leader's charisma, and takes value 0 (resp. 1) when the leader is weak (resp. authority charismatic); $C = \{0, 1/2, 1\}$ is the degree of party coherence, and takes value 0, 1/2, or 1 when party coherence is absent, weak, or strong,

respectively; $\sum_j c_j(L)$ is the sum of factional contributions to party work (be them of interest or principle), and d is the ideological distance between the median voter's ideological bliss point and the leader.

From Propositions 1 and 2 above, we have that the parties' problem is most interesting to analyse when weights α, β, γ and δ in the median voter's utility function generate an important trade off for parties via the effect of the leader type and position on intraparty coherence and factional contribution. We focus on the case most analysed in the literature, namely the one in which the median voter cares most about the leader being ideologically as close as possible to them: $\delta > \alpha, \beta, \gamma$.¹⁹ This case is interesting because the trade off in terms of contributions, intraparty coherence, the leader's ideology and charisma is stark. Suppose without loss of generality – as we demonstrate in the proof of Proposition 3 – that the party can actually set the ideological position of the leader to be equal to that of the median voter. For what values of α, β and γ does the party select a charismatic leader whose ideology is equal to that of the median voter? The following proposition offers sufficient conditions for this to happen.

Proposition 3. *Suppose that the median voter cares most about the leader being ideologically close to them: $\delta > \alpha, \beta, \gamma$. Given the ideological position of faction B and that of the median voter, a sufficient condition for the party to select a charismatic leader whose ideology is equal to the of the median voter is:*

- $\alpha \geq \gamma V \left[\frac{2}{3} - \sum_j c_j(L) \right]$ if factions are of interest; and
- $\alpha \geq \gamma I \left[\frac{1}{2} - \sum_j c_j(L) \right]$ if factions are of principle.

Given that we cannot compute in closed form the position of the charismatic leader that maximizes total factional contributions – because total contributions are not a well-behaved function of the charismatic leader's position – Proposition 3 pins down a sufficient conditions only. The two conditions in Proposition 3 inform us that the median voter must also value sufficiently more having a charismatic leader over maximizing total factional contributions.²⁰ Parameter β is not constrained by our sufficient conditions because these conditions are derived under the assumption that intraparty coherence with a charismatic leader is not higher than that with a weak leader (see the proof of Proposition 3 for details). Of course, Proposition 2 informs us that this is likely to be often violated, because intraparty coherence is often higher when the leader is charismatic. Hence, parties are likely to select a charismatic leader even when the sufficient conditions of Proposition 3 are not satisfied. Somewhat intuitively, a reduction in the stakes of the game for factions, a reduction in V or I , also makes it more likely that the party selects a charismatic leader, as a reduction in V or I reduces the electoral costs of selecting such a leader. The shape of the median voter's utility function and the electoral stakes thus generates a ranking of incentives for parties.

Setting $V = I$, we can go one step further and compare the net electoral effect of each type of leader for parties with factions of interest and for parties with factions of principle when it is optimal for parties to select the same type of leader:

Proposition 4. *Suppose $V = I$. If the optimal leader type for the party with factions of interests is the same as that of the party with factions of principle, then the party with factions of interest always generates more utility for the median voter than the party with factions of principle.*

Our findings generate three interesting predictions. First, they predict that with a charismatic leader, parties with factions of interest find it less costly than parties with factions of principle to track closely the median voter's position (if the leader is weak, tracking the median voter's position involves no costs). Indeed, with a charismatic leader, intraparty coherence and factional contributions are higher with factions of interest (Propositions 1 and 2). Our results thus complement the findings of Schumacher et al. (2013) on the determinants of the reactivity of parties to changes in the characteristics of the electorate. These authors contrast parties that are dominated by the party elite on the one hand and by party activists on the other. Our theory adds that the type of factions that are present within parties also matters for how reactive parties can afford to be. Thus, one testable implication of our theory is that parties with factions of interest should track more closely any ideological shifts in the electorate, all else equal.

A second consequence of our findings concerns the ability of parties to adapt to changes in the distribution of factions within the ideological support of a party—this corresponds to a change in the position of B in our model. It is well documented that, after a leadership change and/or an electoral defeat, the composition and distribution of factions can change quite dramatically; see for example Bettcher (2005), Persico et al. (2011) and Dewan and Squintani (2016). Our findings suggest that parties face less constraints in adapting to such factional changes when factions are of interest.

The third prediction is about the electoral success of parties as the characteristics of elections evolve. Elections are now more personalized, with party leaders gaining importance; see for example McAllister (2007), Cross and Pilet (2015) and Wauters et al. (2016). Our theory suggests that parties are more likely to favour and sustain electoral systems that are more personalized if their factions are of interest. These electoral systems increase the importance of having strong and charismatic leaders, and having such leaders is less costly with factions of interest. In fact, selecting a charismatic leader allows parties with factions of interest to take full advantage of the personalization of the electoral contest and to foster party coherence, even though there is a cost in terms of factional contributions. Yet, if the increased personalization of politics and the growing importance of the party leader go hand in

¹⁹ For example, the case in which the median voter's utility function weights is such that all the median voter cares about is either to maximize total factional contributions or to have a charismatic leader irrespective of the leader's ideological position – this requires α or γ to be very much greater than all the other parameters – yields the immediate result that it is a dominant strategy for both parties to select a non-charismatic or a charismatic leader, respectively.

²⁰ These conditions are not satisfied for all values of α and γ as both $\frac{2}{3} - \sum_j c_j(L)$ and $\frac{1}{2} - \sum_j c_j(L)$ are guaranteed to be positive, by Proposition 1.

hand with a decrease in the importance of factional contributions, such a trade off may be losing in salience. This last claim is in itself an interesting avenue for further empirical research. Some evidence exists that advances in ICT, which arguably allow for the increased use of microtargeting, is further personalizing electoral campaigns and is making small donor contributions an increasingly important facet of campaigns – at least in systems that allow for such contributions –, the flip side of which could be a decrease in the importance of factional contributions. See for example Bouton et al. (2022). The growing personalization of campaigns also appears to have gone hand in hand with changes in the media environment (think about not only television, but also the internet and social media) and with the professionalization of political campaigns; see for example Lilleker (2005), Farrell (2006), Strömbäck and Kiouisis (2014) and Steffan and Venema (2019). By their very nature, factions of interest are in a better and more natural position to reap the benefits of such personalization and professionalization of campaign. Some, if not many campaign professionals are likely to originate in these factions. And these factions do not want their party's resources to go to external companies or consultants. This gives factions of interest an edge over factions of principle.

6. Conclusion

We developed and analysed a model of the intraparty exchange game the party leader plays with the party factions, to understand the conditions that are most conducive to electoral success. We analysed how this success is impacted by the position and strength of the leader and the position and type of the factions.

Our results imply that an uncharismatic leader is conducive to weak party coherence at best, irrespective of the type of factions. Thus having such a leader is a double penalty for the party, as uncharismatic leaders are less valued than charismatic ones, and they do not generate party coherence, whereas authority charismatic leaders can. This is especially true when the party's main factions are of principle, as the dramatic counter-performance of the French socialist party at the last Presidential and legislative elections in France in 2017 shows quite vividly. The only advantage a weak leader brings about is in terms of total factional contributions to party work, which are maximized under a weak leader. We also proved that when the leader is authority charismatic, party coherence is more easily achieved with factions of interest than with factions of principle. Finally, we showed that parties can more easily adapt to changes in the ideology of the median voter and in the distribution of factions when these are of interest. The general message that emerges from our analysis is therefore that factions of interest constrain less the party in its choices than factions of principle.

While we substantiated the findings of our model with important examples from recent electoral races in several countries such as France, the UK and the USA, we believe that our findings could be taken to the data in a more systematic manner, to further our understanding of the mapping from the pre-electoral choices of parties and their leaders to their post-election fates.

Where do we go from here? Whenever we observe a leader change within parties, especially after an electoral defeat, we also observe a reshuffling of personalities in the party's most important positions. This is probably a natural consequence of the new leader wanting to have at least some of their *garde rapprochée* in the key party positions, so as to be able to deploy their electoral and political strategy. Analysing the consequences of such intraparty senior member reshufflings is a very interesting avenue for further research.²¹

Declaration of competing interest

We declare that we have no conflicting interest to declare as pertains our resubmission of manuscript, Leaders, Factions and the Determinants of Electoral Success.

Data availability

No data was used for the research described in the article.

Appendix A. Proofs

Generalities

All the proofs that follow are for $L \in [0, 1/2]$ and $B \in [0, 1]$. The proofs for $L \in [1/2, 1]$ are symmetric, one only needs to interchange C and A .

Remark that three factions offering no contribution to party work cannot be an equilibrium. Indeed, in that case a profitable deviation is for any faction to offer an infinitesimally small contribution and receive the whole of V or I . Following, for example, Corchón (2007) in his proof of Propositions 3.1 (for symmetric equilibria) and 4.1 (for asymmetric equilibria), we also restrict V and I to both be large enough so that the maximization problem each active faction faces is well defined and the equilibria we characterize exist and are unique.

²¹ We thank an anonymous referee for suggesting this idea.

Proofs of results and propositions

Proof of Result 1. Faction $i = A, B, C$ chooses its contribution to party work by maximizing

$$\frac{c_i}{\sum_j c_j} V - c_i$$

The first order condition yields the best response

$$c_i = \sqrt{(c_j + c_k)V} - c_j - c_k$$

while the second order condition is satisfied in this candidate equilibrium:

$$-\frac{2(c_j + c_k)V}{(c_i + c_j + c_k)^3} < 0$$

and thus the unique symmetric equilibrium is such that:

$$c_i^* = \frac{2V}{9} \text{ and } Sh_i^* = \frac{1}{3} \text{ for all } i = A, B, C$$

Proof of Result 2. As we show in the proof of Result 3 below, the two factions that are active are always the ones that are closest to the leader. There, we also identify the parameter spaces for which only two or all three factions are active in equilibrium.

When only two factions are active, the game reduces to a contest with two players. Faction i chooses its contribution to party work by maximizing

$$\frac{b_i c_i}{b_i c_i + b_j c_j} V - c_i$$

The first order condition yields the best response

$$c_i = \frac{1}{b_i} \sqrt{b_i b_j c_j V} - b_j c_j$$

while the second order condition is satisfied:

$$-\frac{2b_i^2 b_j c_j V}{(b_i c_i + b_j c_j)^3} < 0$$

and thus the unique symmetric equilibrium is such that:

$$c_i^* = c_j^* = \frac{b_i b_j V}{(b_i + b_j)^2}$$

To prove that contributions are maximal for $L = B/2$, we need to distinguish between the case $B < L$ and the case $B > L$. For $B < L$, $c_i = \frac{(1-L)(1-L+B)V}{(2-2L+B)^2}$. It is then straightforward to check that $\frac{\partial c_i}{\partial L} < 0$ for all $B < L$. Thus contributions decrease as L moves away from B . For $L < B$, $c_i = \frac{(1-L)(1-B+L)V}{(2-B)^2}$. Then $\frac{\partial c_i}{\partial L} = \frac{B-2L}{(2-B)^2} V$ which is negative for $L > B/2$, positive for $L < B/2$ and equal to zero for $L = B/2$. There is thus a global maximum at $L = B/2$.

Finally, the shares of party funds are given by

$$Sh_i^* = \frac{b_i}{b_i + b_j} \text{ and } Sh_j^* = \frac{b_j}{b_i + b_j}$$

For $b_i = 1 - d_{Li}$ and $b_j = 1 - d_{Lj}$, we have that $Sh_i^* > Sh_j^*$ if and only if $d_{Li} < d_{Lj}$.

Proof of Result 3. Faction $i = A, B, C$ chooses its contribution to party work by maximizing

$$\frac{b_i c_i}{\sum_j b_j c_j} V - c_i$$

The first order condition yields the best response

$$c_i = \frac{1}{b_i} \left[\sqrt{b_i(b_j c_j + b_k c_k)V} - b_j c_j - b_k c_k \right]$$

while the second order condition is satisfied:

$$-\frac{2b_i^2(b_j c_j + b_k c_k)V}{(b_i c_i + b_j c_j + b_k c_k)^3} < 0$$

and thus the unique equilibrium is such that:

$$c_i^* = \frac{2b_j b_k [b_i(b_j + b_k) - b_j b_k] V}{[b_j b_k + b_i(b_j + b_k)]^2}$$

$$SH_i^* = \frac{b_i(b_j + b_k) - b_j b_k}{b_i(b_j + b_k) + b_j b_k}$$

where $b_i = 1 - d_{Li}$.

For what parameter values does the above problem include an inactive faction and thus we are back to [Result 2](#)? If $B < L$, we have the following equilibrium contributions:

$$c_A^* = \frac{2(1 + B - L)L [(1 - L)^2 + B(1 - 2L)] V}{(1 + B - L^2)^2}$$

$$c_B^* = \frac{2 [B + (1 - L)^2] (1 - L)LV}{(1 + B - L^2)^2}$$

$$c_C^* = \frac{2(1 + B - L)(1 - L) [(4 - 3L)L - 1 - B(1 - 2L)] V}{(1 + B - L^2)^2}$$

We have that $c_A^* > 0$ and $c_B^* > 0$, while $c_C^* > 0$ if and only if $B < \frac{3L^2 - 4L + 1}{2L - 1}$. Observe that the latter condition is never satisfied for $L \leq 1/3$ – implying that faction C does not contribute –, while it is always satisfied for $L \geq \frac{1}{2}(3 - \sqrt{5})$.

If $B > L$, we have the following equilibrium contributions:

$$c_A^* = \frac{2(1 - B + L)L [1 - 3L^2 - B(1 - 2L)] V}{[B - 1 - (2 - L)L]^2}$$

$$c_B^* = \frac{2(1 - L)L(1 - B + L^2)V}{[B - 1 - (2 - L)L]^2}$$

$$c_C^* = \frac{2(1 - B + L)(L - 1)[1 - L(2 + L) - B(1 - 2L)] V}{[B - 1 - (2 - L)L]^2}$$

We have that $c_A^* > 0$ and $c_B^* > 0$, while $c_C^* > 0$ if and only if $B > \frac{L^2 + 2L - 1}{2L - 1}$. Observe that the latter condition is always satisfied for $L \geq \frac{1}{2}(3 - \sqrt{5})$.

Consider the parameter configurations in which all three factions are contributing to party work:

1. $L \leq \frac{1}{3}$ and $B > \frac{L^2 + 2L - 1}{2L - 1}$
2. $\frac{1}{3} < L < \frac{1}{2}(3 - \sqrt{5})$ and $B < \frac{3L^2 - 4L + 1}{2L - 1}$ or $B > \frac{L^2 + 2L - 1}{2L - 1}$
3. $\frac{1}{2}(3 - \sqrt{5}) < L \leq \frac{1}{2}$

If $B < L$, we have

$$c_A^* - c_B^* = \frac{2BL [1 + B(1 - 2L) + L(3L - 4)] V}{(1 + B - L^2)^2} < 0$$

Indeed, let $K = 1 + B(1 - 2L) + L(3L - 4)$. We have that $\frac{\partial K}{\partial B} = 1 - 2L > 0$. For $B = \frac{3L^2 - 4L + 1}{2L - 1}$, we have that $K = 0$. For $B = 0$, we have that $K = (1 - L)(1 - 3L)$, which is negative for all $L > 1/3$. It follows that $K < 0$ for all $B < \frac{3L^2 - 4L + 1}{2L - 1}$.

$$c_A^* - c_C^* = \frac{2 [B + (1 - L)^2] (1 + B - L)(1 - 2L)V}{(1 + B - L^2)^2} > 0$$

$$c_B^* - c_C^* = \frac{2(1 + B - 2L)(1 - L) [(1 - L)^2 + B(1 - 2L)] V}{(1 + B - L^2)^2} > 0$$

It is direct that the above two expressions are positive for $B < L \leq 1/2$.

If $B > L$, we have

$$c_A^* - c_B^* = \frac{2(2L - B)L[1 - L(2 + L) - B(1 - 2L)]V}{[B - 1 - (2 - L)L]^2} > 0 \text{ if and only if } B > 2L$$

Indeed, let $K = 1 - L(2 + L) - B(1 - 2L)$. We have that $\frac{\partial K}{\partial B} = 2L - 1 < 0$. For $B = \frac{L^2 + 2L - 1}{2L - 1}$, we have that $K = 0$. For $B = 1$, we have that $K = -L^2 < 0$. Therefore, $K < 0$ for all $B > \frac{L^2 + 2L - 1}{2L - 1}$, and thus $c_A^* - c_B^* > 0$ if and only if $B > 2L$.

$$c_A^* - c_C^* = \frac{2(1 - B + L)(1 - 2L)(1 - B + L^2)}{[B - 1 - (2 - L)L]^2} > 0$$

It is direct that the above expression is positive for $L \leq 1/2$ and $L < B < 1$.

$$c_B^* - c_C^* = \frac{2(1 - B)(1 - L) [1 - B(1 - 2L) - 3L^2] V}{[B - 1 - (2 - L)L]^2} > 0$$

Indeed, let $K = 1 - B(1 - 2L) - 3L^2$. We have that $\frac{\partial K}{\partial B} = 2L - 1 < 0$. For $B = \frac{L^2 + 2L - 1}{2L - 1}$, we have that $K = 2L(1 - L) > 0$. For $B = 1$, we have that $K = L(2 - 3L) > 0$. Therefore, $K > 0$ for all $B > \frac{L^2 + 2L - 1}{2L - 1}$.

Therefore, we finally obtain that:

- $c_A^* > c_B^* > c_C^*$ if $B > 2L$
- $c_B^* > c_A^* > c_C^*$ if $B < 2L$

We know that for faction $i = A, B, C$, the equilibrium share of party funds is given by

$$Sh_i^* = \frac{b_i(b_j + b_k) - b_j b_k}{b_i(b_j + b_k) + b_j b_k}$$

We have that $Sh_i^* - Sh_j^* = \frac{2(b_i - b_j)b_k}{b_j b_k + b_i(b_j + b_k)}$, from which it follows that $Sh_i^* > Sh_j^*$ if and only if $b_i > b_j$. Therefore, as the biases are linear in the distance:

- $Sh_A^* > Sh_B^* > Sh_C^*$ if $B > 2L$
- $Sh_B^* > Sh_A^* > Sh_C^*$ if $B < 2L$

Proof of Result 4. Faction $i = A, B, C$ chooses its contribution by maximizing

$$\frac{c_i}{\sum_m c_m} I + \frac{c_j}{\sum_m c_m} I(1 - d_{ij}) + \frac{c_k}{\sum_m c_m} I(1 - d_{ik}) - c_i$$

The first order condition yields the best response

$$c_i = \sqrt{(d_{ij}c_j + d_{ik}c_k)I} - c_j - c_k$$

while the second order condition is satisfied:

$$-\frac{2(c_i d_{ij} + c_k d_{ik})I}{(c_i + c_j + c_k)^3} < 0$$

and thus the unique equilibrium is such that:

$$c_i^* = \frac{2d_{ij}d_{ik}(d_{ij} + d_{ik} - d_{jk})d_{jk}^2 I}{\left[d_{ij}^2 + (d_{ik} - d_{jk})^2 - 2d_{ij}(d_{ik} + d_{jk}) \right]^2}$$

Given that $d_{AB} = B$, $d_{BC} = 1 - B$ and $d_{AC} = 1$, this yields:

$$c_A^* = c_C^* = \frac{I}{4} \text{ and } c_B^* = 0$$

and the equilibrium individual influences on the party manifesto are given by

$$Sh_A^* = Sh_C^* = \frac{1}{2} \text{ and } Sh_B^* = 0$$

Proof of Result 5. Whenever one faction does not contribute to party work, the game reduces to a contest with two players. As we show in the proof of the next result, this can only happen if $L \leq B/2$. Faction $i = A, B, C$ chooses its contribution by maximizing

$$\frac{b_i c_i}{b_i c_i + b_j c_j} I + \frac{b_j c_j}{b_i c_i + b_j c_j} I(1 - d_{ij}) - c_i$$

The first order condition yields the best response

$$c_i = \frac{1}{b_i} \sqrt{b_i b_j d_{ij} c_j I} - b_j c_j$$

while the second order condition is satisfied:

$$-\frac{2b_i^2 b_j c_j d_{ij} I}{(b_i c_i + b_j c_j)^3} < 0$$

and thus the unique equilibrium is such that:

$$c_i^* = c_j^* = \frac{b_i b_j d_{ij} I}{(b_i + b_j)^2}$$

Given that, as we show in the proof of the next result, only two factions contribute if and only if $L \leq B/2$, replacing b_i and b_j by their values as a function of L and B yields immediately that c_i is strictly increasing in L and therefore contributions are maximized for $L = B/2$.

Finally, the individual influences on the party manifesto are given by

$$Sh_i^* = \frac{b_i}{b_i + b_j} \text{ and } Sh_j^* = \frac{b_j}{b_i + b_j}$$

As $b_i = 1 - d_{Li}$ and $b_j = 1 - d_{Lj}$, we have that $Sh_i^* > Sh_j^*$ if and only if $d_{Li} < d_{Lj}$.

Proof of Result 6. Faction $i = A, B, C$ chooses its contribution by maximizing

$$\frac{b_i c_i}{\sum_m b_m c_m} I + \frac{b_j c_j}{\sum_m b_m c_m} I(1 - d_{ij}) + \frac{b_k w_k}{\sum_m b_m c_m} I(1 - d_{ik}) - c_i$$

The first order condition yields the best response

$$c_i = \frac{1}{b_i} \left[\sqrt{b_i(d_{ij} b_j c_j + d_{ik} b_k c_k)} I - b_j c_j - b_k c_k \right]$$

while the second order condition is satisfied:

$$-\frac{2b_i^2(b_j c_j d_{ij} + b_k c_k d_{ik}) I}{(b_i c_i + b_j c_j + b_k c_k)^3} < 0$$

and thus the unique equilibrium is such that:

$$c_i^* = \frac{2b_j b_k d_{ij} d_{ik} d_{jk}^2 (b_i b_j d_{ij} + b_i b_k d_{ik} - b_j b_k d_{jk}) I}{\left[b_i(d_{ij} - d_{ik})(b_j d_{ij} - b_k d_{ik}) - (b_j(b_i + b_k)d_{ij} + b_k(b_i + b_j)d_{ik})d_{jk} + b_j b_k d_{jk}^2 \right]^2}$$

$$SH_i^* = \frac{d_{jk} [b_j b_k d_{jk} - b_i(b_j d_{ij} + b_k d_{ik})]}{b_i(d_{ij} - d_{ik})(b_j d_{ij} - b_k d_{ik}) - [b_j(b_i + b_k)d_{ij} + b_k(b_i + b_j)d_{ik}] d_{jk} + b_j b_k d_{jk}^2}$$

If $B < L$, we have the following equilibrium contributions:

$$c_A^* = \frac{(1 + B - 2L)L I}{2(1 + B - L)}$$

$$c_B^* = \frac{(1 - L)L [1 + B(1 - 2L) - 2(1 - L)L] I}{2(1 - B)(1 + B - L)^2}$$

$$c_C^* = \frac{(1 - L)[2(1 - L)L - B(1 + B - 2L)] I}{2(1 - B)(1 + B - 2L)}$$

and thus $c_i^* > 0$ for all $i = A, B, C$.

If $B > L$, we have the following equilibrium contributions:

$$c_A^* = \frac{L [2L(B - L) - (1 - B)B] I}{2B(1 - B + L)}$$

$$c_B^* = \frac{(1 - L)[B - 2L(B - L)] I}{2B(1 - B + L)^2}$$

$$c_C^* = \frac{(2L - B)(1 - L) I}{2(1 - B + L)}$$

and thus $c_A^* > 0$ and $c_B^* > 0$, while $c_C^* > 0$ if and only if $B < 2L$.

Consider the parameter configurations such that all three factions are active, hence $B < 2L$. If $B < L$ we have:

$$c_A^* - c_B^* = \frac{(B - L)[2L(1 + B - L) - B(1 + B)] I}{2(1 - B)(1 + B - L)^2} < 0$$

Indeed, let $K = 2L(1 + B - L) - B(1 + B)$. We have that $\frac{\partial K}{\partial B} = 2(L - B) - 1 < 0$. For $B = 0$, we have that $K = 2L(1 - L) > 0$. For $B = L$, we have that $K = L(1 - L) > 0$. Therefore, $K > 0$ for all $B < L$.

$$c_A^* - c_C^* = \frac{(L - B)[2L(1 + B - L) - (1 + B)] I}{2(1 - B)(1 + B - L)} < 0$$

Indeed, let $K = 2L(1 + B - L) - (1 + B)$. We have that $\frac{\partial K}{\partial B} = 2L - 1 < 0$. For $B = 0$, we have that $K = 2L(1 - L) - 1 < 0$. For $B = L$, we have that $K = L - 1 < 0$. Therefore, $K < 0$ for all $B < L$.

$$c_B^* - c_C^* = \frac{(1 + B)(1 + B - 2L)(L - B)(1 - L) I}{2(B - 1)(1 + B - L)^2} < 0$$

It is direct that the above expression is negative for $B < L \leq 1/2$.

If $B > L$ we have:

$$c_A^* - c_B^* = \frac{(2 - B)(2L - B)(B - L)L I}{2B(1 - B + L)^2} > 0$$

$$c_A^* - c_C^* = \frac{(B - L)[B(1 - 2L) + 2L^2] I}{2B(1 + L - B)} > 0$$

$$c_B^* - c_C^* = \frac{(B - L)(1 - L)[B(1 - B) + 2L(B - L)] I}{2B(1 - B + L)^2} > 0$$

It is direct that the three above expressions are positive for $L < B < 2L \leq 1$. And thus:

1. If $B < L$, we have $c_C^* > c_B^* > c_A^*$
2. If $B > L$, we have $c_A^* > c_B^* > c_C^*$

Further, if $B < L$, the equilibrium influences on the party manifesto are given by

$$SH_A^* = \frac{1 + B - 2L}{2(1 + B - L)}$$

$$SH_B^* = \frac{(B + 1) - 2L(1 + B - L)}{2(1 - B)(1 + B - L)}$$

$$SH_C^* = \frac{2L(1 + B - L) - B(1 + B)}{2(1 - B)(1 + B - L)}$$

and thus we have

$$SH_A^* - SH_B^* = \frac{2L(2B - L) - B(B + 1)}{2(1 - B)(1 + B - L)} < 0$$

Indeed, let $K = 2L(2B - L) - B(B + 1)$. We have that $\frac{\partial K}{\partial L} = 4(B - L) < 0$. For $L = B$, we have that $K = B(B - 1) < 0$. For $L = 1/2$, we have that $K = B(1 - B) - 1/2 < 0$. Therefore, $K < 0$ for all $B < L \leq 1/2$.

$$SH_A^* - SH_C^* = \frac{2L(L - 2) + 1 + B}{2(1 - B)(1 - L)} > 0 \text{ if and only if } B > 4L - 2L^2 - 1$$

Indeed, let $K = 2L(L - 2) + 1 + B$. We have that $\frac{\partial K}{\partial B} = 1 > 0$. For $B = 4L - 2L^2 - 1$, we have that $K = 0$. For $B = L$, we have that $K = 1 + L - 2L(2 - L) > 0$. Therefore, $K > 0$ for all $B > 4L - 2L^2 - 1$.

$$SH_B^* - SH_C^* = \frac{(1 + B - 2L)^2}{2(1 - B)(1 + B - L)} > 0$$

It is direct that the above expression is positive for $B < L \leq 1/2$.

Finally, if $B > L$, the equilibrium influences on the party manifesto are given by

$$SH_A^* = \frac{2L(B - L) + B(1 - B)}{2B(1 - B + L)}$$

$$SH_B^* = \frac{B - 2L(B - L)}{2B(1 - B + L)}$$

$$SH_C^* = \frac{2L - B}{2(1 - B + L)}$$

and thus we have

$$SH_A^* - SH_B^* = -\frac{(B - 2L)^2}{2B(1 - B + L)} < 0$$

It is direct that the above expression is negative for $L < B < 2L \leq 1$.

$$SH_A^* - SH_C^* = \frac{B - 2L^2}{2B(1 - B + L)} > 0$$

It is direct that the above expression is positive for $L < B < 2L \leq 1$.

$$SH_B^* - SH_C^* = \frac{B(1 + B) - 2L(2B - L)}{2B(1 - B + L)} > 0$$

Indeed, let $K = B(1 + B) - 2L(2B - L)$. We have that $\frac{\partial K}{\partial L} = 4(L - B) < 0$. For $L = 0$, we have that $K = B(B + 1) > 0$. For $L = B$, we have that $K = B(1 - B) > 0$. Therefore, $K > 0$ for all $L < B$.

Therefore:

- If $B < 4L - 2L^2 - 1$, we have $SH_B^* > SH_C^* > SH_A^*$
- If $B > 4L - 2L^2 - 1$, we have $SH_B^* > SH_A^* > SH_C^*$

Proof of Proposition 1. First we will show that total contributions are always higher under a weak leader than under a charismatic one. We start with **factions of interest**. The fact that total contributions are higher when the contest is not biased is a direct application of standard results in the theory of Tullock contests; see for example [Drugov and Ryvkin \(2017\)](#) and the references therein.

Things are less straightforward when factions are of principle, as this scenario is understudied in the theory of contests. If the leader is **weak**, only factions of interest A and C are active and total effort is given by $E_W^* = \frac{I}{2}$.

If the leader is strong and all three factions are active, we have two cases to consider. First, if $B < L$, total effort is given by

$$E_S^* = \frac{B^3 + B^2(2 - 4L) + 4L(L - 1)^3 + B[1 - 4L(L - 2)(L - 1)]}{2(B - 1)(1 + B - L)^2} I.$$

The above total effort turns out to be strictly smaller than $\frac{I}{2}$. Indeed, we have that

$$E_S^* - \frac{I}{2} = \frac{B^2(1 - 2L) + (1 - 3L + 2L^2)^2 + B(2 - 8L + 11L^2 - 4L^3)}{2(B - 1)(1 + B - L)^2} I$$

and the three terms in the sum in the numerator are positive for $L \leq 1/2$ and $B \in [0, 1]$ whereas the denominator is negative, given that $B \leq 1/2$. Hence $E_S^* \leq I/2$.

Second, if $L \leq B < 2L$, we have

$$E_S^* = \frac{B^3 - 4L^4 - B^2(1 + 4L) + 4BL(1 + L^2)}{2B(1 - B + L)^2} I,$$

which is again smaller than $\frac{I}{2}$. Indeed, $E_S^* - I/2$ is negative if

$$\begin{aligned} 2B^3 - 8L^4 - 2B^2(1 + 4L) + 8BL(1 + L^2) - 2B(1 - B + L)^2 &< 0 \\ \iff 4BL [2L^2 - B] - 8L^4 + B [2L(2 - L) - 2(1 - B)] &< 0 \end{aligned}$$

which is always true. Indeed, $2L^2 - B \leq 0$ for any $L \leq 1/2$ and $L < B < 2L$ as $2L^2 \leq L$ for $L \leq 1/2$; also, even though the term $B [2L(2 - L) - 2(1 - B)]$ is increasing in B , even for $B = 2L$, $8L^2 [2L^2 - 2L] - 8L^4 + 2L [2L(2 - L) - 2(1 - 2L)] \leq 0$.

Finally, we know from [Result 5](#) that when faction C is inactive, total contributions are at most equal to $BI/2$. Given that $B \in [0, 1]$, this concludes the proof of the first statement of [Proposition 1](#).

We now turn to the comparison of total contributions across factions. That is, we want to show that factions of interest always contribute more than factions of principle. For any positions of B and L such that the number of active factions of interest is equal to the number of active factions of principle, we can rely on the first order condition to the factions' problem; see the proofs of [Results 2, 3, 5](#) and [6](#). It is then immediate to see that factions of interest contribute more (as a fraction of the available rewards) given that the (marginal) cost of effort is the same for both types of factions, namely c_i , whereas the (marginal) benefit of contributing is strictly greater for factions of interest.

To prove that total contributions are still higher (as a fraction of the available rewards) with factions of interest for values of B and L such that only two of them contribute but three factions of principle are active (this happens if $B < 2L$ and B inside the missile-shaped area), we proceed as follows. We know that total contributions (as a fraction of V) of the two factions of interest are equal to $2 \frac{(1-L)(1-L+B)}{(2-2L+B)^2}$ if $B < L$ and to $2 \frac{(1-L)(1-B+L)}{(2-B)^2}$ if $B > L$. For factions of principle, total contributions (as a fraction of I) are given by $\frac{B^3+B^2(2-4L)+4L(L-1)^3+B[1-4L(L-2)(L-1)]}{2(B-1)(1+B-L)^2}$ if $B < L$, and by $\frac{B^3-4L^4-B^2(1+4L)+4BL(1+L^2)}{2B(1-B+L)^2}$ if $B > L$. We thus need to show that:

$$2 \frac{(1-L)(1-L+B)}{(2-2L+B)^2} > \frac{B^3 + B^2(2 - 4L) + 4L(L - 1)^3 + B [1 - 4L(L - 2)(L - 1)]}{2(B - 1)(1 + B - L)^2}$$

if $B < L$, and

$$2 \frac{(1-L)(1-B+L)}{(2-B)^2} > \frac{B^3 - 4L^4 - B^2(1 + 4L) + 4BL(1 + L^2)}{2B(1 - B + L)^2}$$

if $B > L$.

Re-writing the first inequality, we need to show that

$$1 > \frac{(1-L+B/2)^2 (B^3 + B^2(2 - 4L) + 4L(L - 1)^3 + B [1 - 4L(L - 2)(L - 1)])}{(1-L)(B-1)(1+B-L)^3}$$

which is always true as $\frac{(1-L+B/2)^2}{(1-L)(1+B-L)} \in [1, 2]$ for $L \leq \frac{1}{2}(3 - \sqrt{5})$ and $B < L$ and we know from above that $\frac{B^3+B^2(2-4L)+4L(L-1)^3+B[1-4L(L-2)(L-1)]}{2(B-1)(1+B-L)^2} < 1/2$.

For $B > L$, we need to show that

$$2 \frac{(1-L)(1-B+L)}{(2-B)^2} > \frac{B^3 - 4L^4 - B^2(1 + 4L) + 4BL(1 + L^2)}{2B(1 - B + L)^2}$$

Simple but tedious algebra proves that the above is true for both $B = L$ and $B = 2L$ (or the relevant upper bound on B when L approaches $\frac{1}{2}(3 - \sqrt{5})$). The LHS is strictly increasing in B as long as $B < 2L$, which always holds. Additional tedious algebra confirms that the RHS is strictly decreasing in B . The above inequality thus holds for any admissible values of B and L .

Finally, we need to prove that total contributions are still higher (as a fraction of the available rewards) with factions of interest for values of B and L such that all three of them contribute but only two factions of principle are active (this happens if $B > 2L$ and B outside the missile-shaped area). We need to show that:

$$\frac{2B [1 + L(4L^2 - 4 + L)] + L [4 + L(2 - L(4 + 5L))] - 1 - B^2(1 - 2L)^2}{[B - 1 - (2 - L)L]^2} > \frac{B(1 - B + L)(1 - L)}{(2 - B)^2}$$

Given the subset of the parameter space we are focusing on, the denominator of the RHS is larger than that of the LHS of the inequality. A comparison of the numerators thus suffices to prove that the above inequality always holds. Starting with the terms in the numerator on the LHS, $B^2(1 - 2L)^2$ is always smaller than B^2 for $L \leq 1/2$. Also, $L(4L^2 - 4 + L)$ is negative but never smaller than -1 , hence $2B [1 + L(4L^2 - 4 + L)] - 1 - B^2(1 - 2L)^2 > 2B - 1 - B^2$, which is greater than the numerator of the RHS. As $L [4 + L(2 - L(4 + 5L))]$ is strictly positive for any $L \leq 1/2$, this concludes the proof.

Proof of Proposition 2. Direct from the ranking of factional efforts and rewards (Results 1 to 6).

Proof of Proposition 3. Remember that the median voter’s utility is given by:

$$U = \alpha S + \beta C + \gamma \sum_j c_j + \delta(1 - d)$$

where $S = \{0, 1\}$ is the leader’s charisma, and takes value 0 (resp. 1) when the leader is weak (resp. authority charismatic); $C = \{0, 1/2, 1\}$ is the degree of party coherence, and takes value 0, 1/2, or 1 when party coherence is absent, weak, or strong, respectively; $\sum_j c_j$ is the sum of factional contributions to party work (be them of interest or principle), and d is the ideological distance between the median voter’s ideological bliss point and the leader.

Suppose without loss of generality that the median voter’s ideology is included in the set of ideologies the party can choose from for its leader. Then the party sets its leader’s ideology to be equal to that of the median voter. This has no cost in terms of total factional contributions if the leader is weak, but does have such a cost if the leader is charismatic.

Consider a party with factions of interest. We first pin down the utility a party with a weak leader can guarantee the median voter. Intraparty coherence is weak (see Proposition 2), factional contributions are equal to $\frac{2V}{3}$, the ideological payoff is equal to δ but the party does not enjoy the bonus from having a charismatic leader. The median voter’s utility is thus

$$U(\text{weak}) = \frac{\beta}{2} + \gamma \frac{2V}{3} + \delta.$$

Suppose now the party selects a charismatic leader. This leader biases the factional contest, which depresses total contributions with respect to the case of a weak leader (see Proposition 1) but yields the electoral bonus from charisma and generates at least weak intraparty coherence. The lowest intraparty coherence it can achieve then is weak coherence. Suppose this is what the party achieves. Then the utility of the median voter is equal to:

$$U(\text{charismatic}) = \alpha + \frac{\beta}{2} + \gamma \sum_j c_j(L) V + \delta,$$

where $\sum_j c_j(L)$ is total factional contributions given the charismatic leader’s position, L .

A sufficient condition for the party to choose a leader (whose ideological position is equal to that of the median voter and) who is charismatic is:

$$\alpha + \frac{\beta}{2} + \gamma \sum_j c_j(L) V + \delta \geq \frac{\beta}{2} + \gamma \frac{2V}{3} + \delta \iff \alpha \geq \gamma V \left[\frac{2}{3} - \sum_j c_j(L) \right].$$

Consider now a party with factions of principle. Following the same steps as above, when the leader is weak and has the same ideological position as the median voter, this voter’s utility is equal to:

$$U(\text{weak}) = \gamma \frac{I}{2} + \delta.$$

With a charismatic leader (whose ideological position is equal to that of the median voter), the party may not achieve any intraparty coherence. Suppose this is the case. Then the median voter’s utility is equal to:

$$U(\text{charismatic}) = \alpha + \gamma \sum_j c_j(L) I + \delta,$$

and a sufficient condition for the party choose a leader whose ideological position is equal to that of the median voter and who is charismatic is:

$$\alpha + \gamma \sum_j c_j(L) I + \delta \geq \gamma \frac{I}{2} + \delta \iff \alpha \geq \gamma I \left[\frac{1}{2} - \sum_j c_j(L) \right]. \quad \square$$

Proof of Proposition 4. From Proposition 3, we know that with a weak leader the party with factions of interest yields the median voter utility equal to

$$\frac{\beta}{2} + \gamma \frac{2V}{3} + \delta.$$

whereas the party with factions of principle yields the median voter

$$\gamma \frac{I}{2} + \delta < \frac{\beta}{2} + \gamma \frac{2V}{3} + \delta$$

Given that $V = I$, $\gamma \frac{I}{2} + \delta < \frac{\beta}{2} + \gamma \frac{2V}{3} + \delta$.

The same ranking across party types holds when it is optimal to have a charismatic leader because of our findings in Propositions 1 and 2.

Appendix B. Endogenous V and I

We now extend the game to let V and I be endogenous to the electoral outcome, be conditional on the party winning the election. We modify the game as follows. Let the probability that the party wins the election be linear in the utility of the median voter:

$$P(\text{win}) = a + \frac{1-a}{U} U = a + b(a)U$$

where $a \in (0, 1)$ and \bar{U} is the maximal utility the median voter can get from the party. A high value of a implies the party is ex-ante preferred by the median voter. More importantly for what follows, $P(\text{win})$ is increasing in each faction's contribution.

Once the party has chosen the type and location of its leader, factions decide how much to contribute knowing the contest they are playing in and knowing $P(\text{win})$ above. Faction i thus maximizes:

$$P(\text{win}) \frac{b_i c_i}{\sum_j b_j c_j} V - c_i$$

if it is a faction of interest, or

$$P(\text{win}) \left\{ \frac{b_i c_i}{\sum_m b_m c_m} I + (1 - d_{ij}) \frac{b_j c_j}{\sum_m b_m c_m} I + (1 - d_{ik}) \frac{b_k c_k}{\sum_m b_m c_m} I \right\} - c_i$$

if it is a faction of principle.

Each faction now faces a more complex problem. Yet, given that $P(\text{win})$ is increasing in c_i , its optimal decisions is qualitatively similar to the one derived in the main body of our article. Indeed, the benefit side of the first order condition now comprises two terms, one that includes term $\frac{\partial P(\text{win})}{\partial c_i} > 0$ and the one we have in the main body of the article multiplied by $P(\text{win})$. Thus solving for the equilibrium of this extended game delivers results that are qualitatively similar to those we have in the main body of the article.

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