

Passive Damping Stabilization for Constant-Power Loaded Boost Converter

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Abstract—This paper presents a methodology to stabilize an open-loop boost power converter feeding a constant power load. The stabilization is ensured by means of three different passive damping filters, an RC parallel branch placed at the output port, an RL series branch placed at the input, and an RL parallel branch placed also in the input port. The presented methodology allows the design of the system under a pre-defined dynamic behavior and the established size, weight, and cost limitations. Analytical models are obtained for each topology and a theoretical study is carried in order to determine the optimal resistance that ensures the damping of the peak of the output impedance of the converter with each stabilizer. The loci of the poles of the system in the complex plane is studied in terms of the stabilizer parameter values to estimate the dynamics of the complete system. Finally, numerical examples and PSIM simulations are discussed for the RC damping topology to validate the presented concepts.

Keywords—stability, constant power load, passive damping, optimal damping resistance

I. INTRODUCTION

Damping filters are used in several applications in the power electronics field. LC input filters are placed between the source and the converter that, from the point of view of the filter, acts as a load (Fig. 1 (a)). These filters are used to protect the system from electromagnetic interferences, to avoid unstable operation in the duty cycle controlled converters or, in some applications, even to avoid the degradation of the control system. In [1]–[3], RC parallel, RL series and RL parallel branches are applied as damping filters to ensure the proper functioning of the system by damping the filter peak output impedance at the resonance frequency.

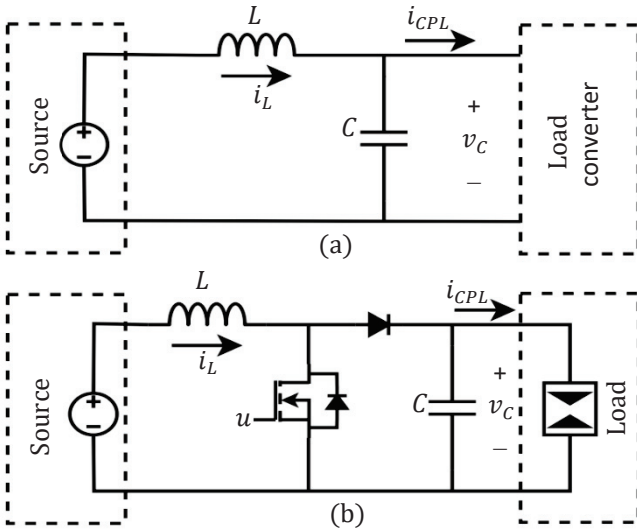


Fig. 1. Circuit diagram of the system. (a) LC input filter (b) boost converter loaded with a constant power load.

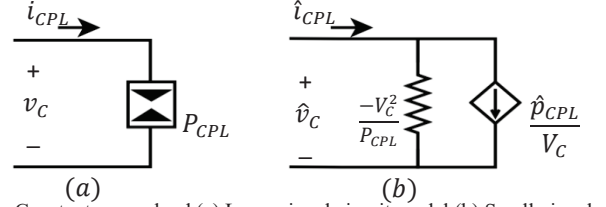


Fig. 2. Constant power load (a) Large signal circuit model (b) Small-signal linearized circuit model.

In addition, in the cascade connection of several power converters, strict voltage regulation results in a constant power load (CPL) behavior for the upstream elements in the power chain. In [4] it was shown that this CPL can lead to an unstable operating regime due to the negative incremental resistance that introduces into the system. Considering that the current of the CPL is $i_{CPL} = P_{CPL}/v_C$, where P_{CPL} is the power of the load and v_C is the voltage across it, we can write:

$$\Delta i_{CPL} = \frac{d\left(\frac{P_{CPL}}{v_C}\right)}{dv_C} \Delta v_C + \frac{d\left(\frac{P_{CPL}}{v_C}\right)}{dP_{CPL}} \Delta P_{CPL} \quad (1)$$

Hence, the linearized model of the CPL (Fig. 2 (b)) is given by (2) where we can identify the negative incremental resistance and a current source that can be considered as an external perturbation.

$$\hat{i}_{CPL} = -\frac{P_{CPL}}{V_C^2} \hat{v}_C + \frac{\hat{p}_{CPL}}{V_C} \quad (2)$$

Reference [5] addresses this issue in the same way as mentioned above however, adding the analysis of the stability requirements that appear due to the presence of the CPL.

On the other hand, in certain applications, an increase in voltage with respect to the source, in the ports where constant power is demanded, may be crucial. That is why the stage that interconnects these two points in Fig. 1 (b) is a boost converter. Usual approaches to stabilize this type of systems require closed-loop operation with an appropriate control in the boost converter [6], a modification of the load control characteristics or alternatively adequate techniques for either passive or active damping modification in open-loop operation. In this article we focus on passive damping stabilization implemented with the three topologies presented in Fig. 3 and considering that the load converter behaves as a CPL. Finally, the steady state power losses caused by the proposed dampers are in the range of milliwatts. Realize that such losses only depend on the ripple of the state variables and the damping resistor value, as the steady state average current through that resistor is null. The dynamic losses will depend on the application.

The rest of this paper is structured in such a way that the study of stability limitations of each damping filter is discussed in Section II. The design methodology to implement the circuit with the desired dynamic requirements is carried out in Section III.

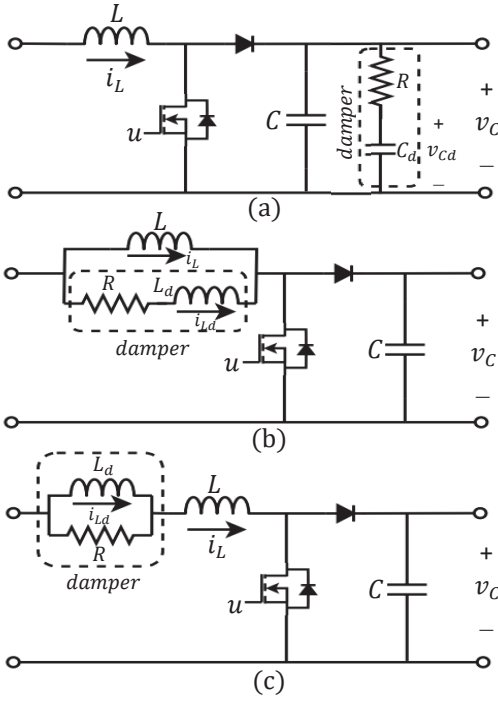


Fig. 3. Circuit diagram of the three passive damping filters integrated into the boost converter topology. (a) RC parallel damping (b) RL parallel damping (c) RL series damping.

The study of the optimal value of the damping resistance for the reduction of the filter output impedance peaking is performed in Section IV. To validate the theoretical results, in Section V numerical simulations are performed for the RC parallel damping topology. Finally, Section VI presents the main conclusions of the work.

II. STABILITY CONDITIONS

It must be ensured that the system operates within the stability conditions. For this, it is necessary to find the transfer functions that define the small-signal dynamic behavior of the converter and to study what conditions they impose.

The small-signal averaged behavior of the boost converter with CPL can be modeled according to Fig. 4 assuming that the duty cycle is constant and therefore $\hat{d} = 0$. The small-signal power at the output is also considered constant $\hat{p}_{CPL} = 0$.

To obtain the dynamic model of the converter in the frequency domain, the Laplace transform is used and the equations that describe the electrical circuit are solved. The characteristic equation for the converter when a damping filter is added can be expressed as follows

$$s^3 + a_2 s^2 + a_1 s + a_0 = 0 \quad (3)$$

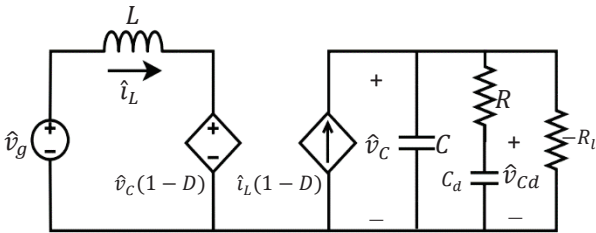


Fig. 4. Small-signal averaged model of a "boost" with RC parallel damping filter in the output, where D is the steady-state duty cycle.

TABLE I. TERMS OF THE CHARACTERISTIC POLYNOMIAL FOR THE THREE DAMPERS

	a_2	a_1	a_0
RC parallel damping	$\frac{C + C_d}{CC_d R} - \frac{P_{CPL}}{V_C^2 C}$	$\frac{D'^2}{CL} - \frac{P_{CPL}}{V_C^2 CC_d R}$	$\frac{D'^2}{RLCC_d}$
RL parallel damping	$\frac{R}{L_d} - \frac{P_{CPL}}{V_C^2 C}$	$\frac{D'^2(L_d + L)}{CL_d L} - \frac{P_{CPL} R}{V_C^2 CL_d}$	$\frac{RD'^2}{L_d LC}$
RL series damping	$\frac{R(L_d + L)}{L_d L} - \frac{P_{CPL}}{V_C^2 C}$	$\frac{D'^2}{LC} - \frac{RP_{CPL}(L_d + L)}{L_d LV_C^2 C}$	$\frac{RD'^2}{L_d LC}$

Considering the three different topologies shown in Fig. 3, the terms a_0 , a_1 and a_2 depicted in TABLE I are obtained for the different damping filters used.

Let the incremental resistance introduced by the CPL be replaced by R_l

$$R_l = \frac{V_C^2}{P_{CPL}} = \frac{V_C}{I_{CPL}} \quad (4)$$

Let also n be defined as follows

$$n = \frac{C_d}{C} = \frac{L_d}{L} \quad (5)$$

By applying the Routh-Hurwitz criterion to the characteristic polynomial in (3), the stability region of the system can be determined in the parametric space thus resulting in the stability conditions shown in TABLE II for the different damping filters used. Namely, the conditions $a_0 > 0$, $a_2 > 0$ and $a_2 a_1 - a_0 > 0$ must hold.

It can be observed that the inequality $a_0 > 0$ is satisfied for all the proposed topologies regardless of the value of the stabilization resistor R . Based on the condition $a_2 > 0$ it can be predicted that the power V_C^2/R consumed by the RC damping filter resistance must be larger than a lower limit P_{low} given by the following equation:

$$P_{low} = \frac{n}{1+n} P_{CPL} \quad (6)$$

The graphical representation of the parametric region in the $n - R$ parameter plane that ensures the stability of the system, for the three considered topologies, is presented in Fig. 5. For this example, the set of parameters values used are $V_g = 200$ V, $V_o = 400$ V, $C = 10$ μ F, $L = 500$ μ H and $P_{CPL} = 1000$ W.

TABLE II. STABILITY CONDITIONS FOR THE THREE DAMPING FILTERS

	$a_2 > 0$	$a_2 a_1 - a_0 > 0$
RC parallel damping	$R < \frac{R_l(1+n)}{n}$	$\frac{1}{C} \left(\frac{(1+n)}{nR} - \frac{1}{R_l} \right) \left(nCR - \frac{L}{R_l D'^2} \right) > 1$
RL parallel damping	$R > \frac{Ln}{CR_l}$	$L \left(\frac{R}{nL} - \frac{1}{R_o C} \right) \left(\frac{n+1}{R} - \frac{1}{R_l D'^2} \right) > 1$
RL series damping	$R > \frac{Ln}{CR_l(1+n)}$	$L \left(\frac{R(1+n)}{nL} - \frac{1}{R_o C} \right) \left(\frac{n}{R} - \frac{(n+1)}{R_l D'^2} \right) > 1$

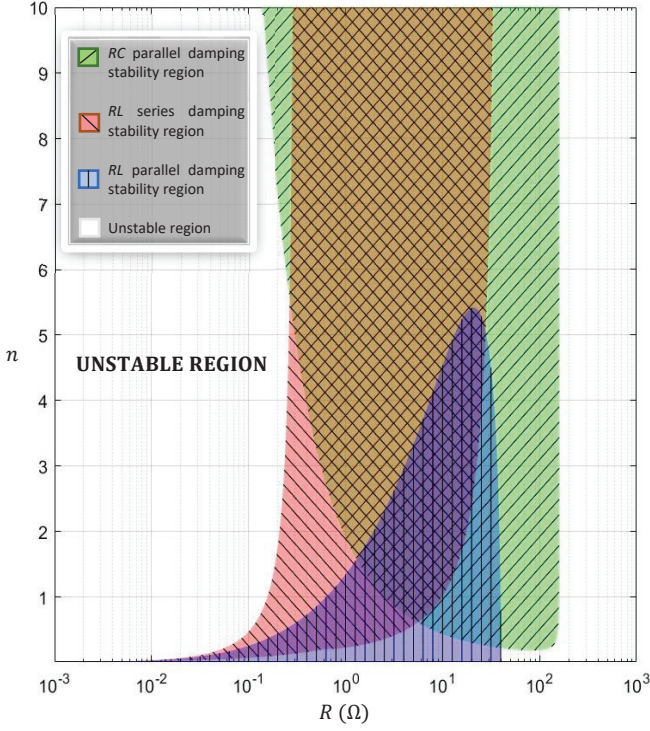


Fig. 5. Representation of the parametric stability region in the $n - R$ plane for the different damping filters.

The previous figure shows the limitations of each topology. In particular, the RC parallel topology has the highest upper limit in resistance value, and it is also noted that this limit saturates at the value of 160Ω as n increases. Regarding the RL series damping filter, it can be noted that it allows lower values of parameters R and n , even though, at this range, the system presents a marginally stable response. Finally, for the RL parallel damping filter, it is observed that its maximum value of n is around 5.5, which means that the stabilizer inductor can only be this 5.5 higher than the plant inductor. As will be shown later, this topology allows values of n lower than 1 and in fact, for a fixed value of R , the lower the value of n is, the more damped the response will be.

III. FILTER DESIGN

In the previous section, it was shown how, there is still two unknown parameters in the design process. Since there is no criterion to select the n value other than limitations of the size, weight and cost of the prototype, a methodology is proposed in order to determine the value of the capacitor C_d (alternatively L_d) and the damping resistor R . The aforementioned methodology considers the maximum damping that each topology can achieve as a representative parameter of the dynamic behavior of the system.

For example, considering the characteristic polynomial of the boost converter with CPL and the parallel RC damper, equation (7) is obtained.

$$s^3 + s^2 \left(\frac{1+n}{nCR} - \frac{1}{R_1C} \right) + s \left(\frac{(1-D)^2}{CL} - \frac{1}{RC^2nR_1} \right) + \frac{(1-D)^2}{RLC^2n} = 0 \quad (7)$$

It is possible to arrange the previous expression to determine those terms with common factor $1/R$. Next, the whole polynomial is divided by the terms that do not depend on $1/R$.

$$1 + \frac{1}{R} \left[\frac{s^2 \left(\frac{1}{nC} + \frac{1}{C} \right) - s \left(\frac{PCPL}{V_C^2 C^2 n} \right) + \frac{(1-D)^2}{C_1 LC}}{s^3 - s^2 \left(\frac{PCPL}{V_C^2 C} \right) + s \left(\frac{(1-D)^2}{CL} \right)} \right] = 0 \quad (8)$$

or using $G = 1/R$, the following equivalent design-oriented characteristic equation is obtained

$$1 + G \cdot T(s) = 0 \quad (9)$$

where $T(s)$ is given by the following expression

$$T(s) = \frac{s^2 \left(\frac{1}{nC} + \frac{1}{C} \right) - s \left(\frac{PCPL}{V_C^2 C^2 n} \right) + \frac{(1-D)^2}{C_1 LC}}{s^3 - s^2 \left(\frac{PCPL}{V_C^2 C} \right) + s \left(\frac{(1-D)^2}{CL} \right)} \quad (10)$$

The design-oriented expression shown above can be interpreted as the characteristic polynomial of a negative feedback control system with proportional control (9) where

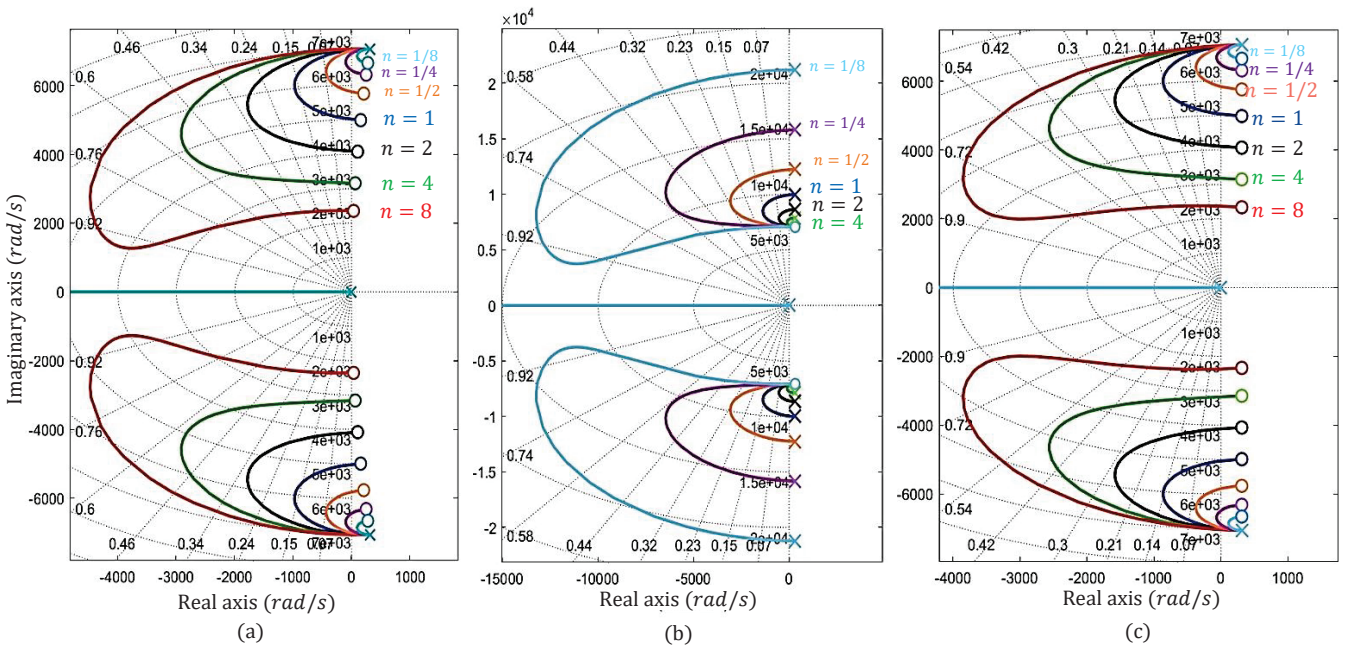


Fig. 6. Root loci in function of R and for different values of n . (a) RC parallel damping (b) RL parallel damping (b) RL series damping.

the proportional ‘gain’ is given by the conductance $G = 1/R$ of the damping filter. With this methodology, the *root-locus* of the system can be represented in order to estimate the position of the poles as a function of the stabilization conductance/ resistance. This process is repeated for a wide range of values of n in Fig. 6.

This representation allows the validation of the stability analysis realized in the previous section since, as can be observed, the poles of the system are positioned in the right half-plane for certain values of damping resistance R . It can be proved that the limits are the same as in the stability analysis which validates the methodology presented.

IV. OPTIMAL RESISTANCE

Often the constant power demand behavior observed in the load is imposed by the converter connected to the output port of the system. To not disturb the normal operation of the mentioned converter and to avoid the appearance of instabilities, it must be ensured that:

$$\|Z_o\| \ll \|Z_{in}\| \quad (11)$$

The previous inequality limits the maximum peak allowed in the output impedance of the boost converter with the damping filter and constitutes a design constraint. In fact, it is common that, in the undamped variant, the output impedance of the converter can tend to infinity at the resonance frequency.

In this section, the optimization of the damping filter is addressed with the purpose of minimizing the peak in the output impedance of the plant through the selection of the resistive element of the damping filter.

The objective is to minimize the dependence of R in the expression of the output impedance $Z_o(R, f)$. This dependence can be defined with Middlebrook's extra element theorem (12).

$$Z_o(R, f) = Z_o(\infty, f) \left(\frac{1 + \frac{Z_N(s)}{R}}{1 + \frac{Z_D(s)}{R}} \right) = Z_o(0, f) \left(\frac{1 + \frac{R}{Z_N(s)}}{1 + \frac{R}{Z_D(s)}} \right) \quad (12)$$

In the previous expression $Z_o(\infty, f)$ and $Z_o(0, f)$ represents the output impedance when R tends to infinity and zero, respectively. $Z_N(s)$ and $Z_D(s)$ are the impedances seen by R when the filter output voltage is zero, and when the filter is unloaded, respectively. In [3] is demonstrated that there exists a frequency f_m at which equation (12) is independent of R . At this frequency the modulus of the output impedance when R tends to zero and infinity are equal.

$$\|Z_o(\infty, f_m)\| = \|Z_o(0, f_m)\| \quad (13)$$

The problem to find the optimal resistance is solved in two steps. Initially equation (13) is solved to find the frequency f_m . With this result the resistance $R = R_{opt}$ that causes the output impedance to have a minimum at the obtained frequency can be found. Meaning that the derivative of the output impedance as a function of the frequency should be equal to zero at the frequency f_m and for the optimal resistance R_{opt} . The results for each topology of dampers are presented in the following points. First, let us define f_0 and R_0 as follows

$$f_0 = \frac{1}{2\pi\sqrt{CL}} \quad (14)$$

$$R_0 = \sqrt{\frac{L}{C}} \quad (15)$$

A. RC Parallel damping

$$f_m = f_0(1 - D)\sqrt{\frac{2}{(2+n)}} \quad (16)$$

$$R_{opt} = \frac{R_0}{n(1-D)}\sqrt{\frac{(2+n)(3n+4)}{2(4+n)}} \quad (17)$$

B. RL Parallel damping

$$f_m = f_0(1 - D)\sqrt{\frac{(2n+1)}{2n}} \quad (18)$$

$$R_{opt} = R_0(1 - D)\sqrt{\frac{n(2n+1)(4n+3)}{2(4n+1)}} \quad (19)$$

C. RL Series damping

$$f_m = f_0(1 - D)\sqrt{\frac{(n+2)}{2(n+1)}} \quad (20)$$

$$R_{opt} = R_0 \frac{n(1-D)}{n+1} \sqrt{\frac{(2+n)(3n+4)}{2(4+n)(1+n)}} \quad (21)$$

An equivalent study is carried out in [5]. In that paper, instead of stabilizing a boost converter, it is an *LC* filter that interconnects the source and the constant power load. Since this paper focuses on the same dampers, it is illustrative to compare the expressions obtained. For this purpose, the results of the *LC* filter are called with the following nomenclature:

- the optimum resistance is R'_{opt}
- the frequency f_m is f'_m .

In the three cases the frequencies are related as follows:

$$f_m = (1 - D)f'_m \quad (22)$$

For the *RL* damping filter, the optimal resistance is:

$$R_{opt} = (1 - D)R'_{opt} \quad (23)$$

Finally, considering the *RC* parallel damping branch, the optimal resistance is:

$$R_{opt} = \frac{1}{(1-D)}R'_{opt} \quad (24)$$

V. NUMERICAL EXAMPLE

Consider the circuit shown in Fig. 1 with the *RC* parallel damping filter in the output. For this example, the same parameters values are used as before. An overshoot *PO*(%) lower than 45% in the dynamic response of the output voltage is added as a design constraint. Firstly, to fulfill the overshoot requirement, the damping should be:

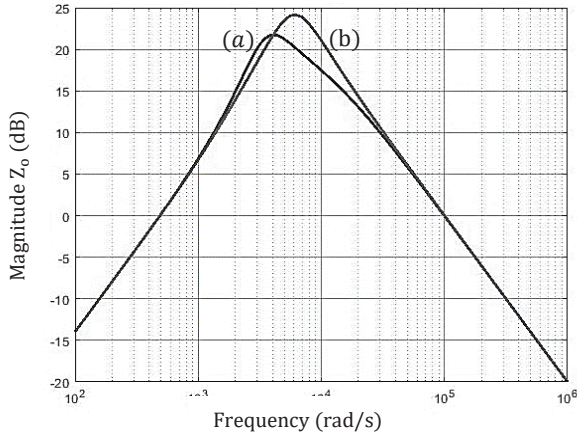


Fig. 7. Representation of the output impedance for two different values of R . (a) $R = R_{opt}$. (b) $R = 15 \Omega$

$$\zeta \geq \frac{\left| \ln \left(\frac{PO(\%)}{100} \right) \right|}{\sqrt{\ln \left(\frac{PO(\%)}{100} \right)^2 + \pi^2}} = 0.25 \quad (25)$$

For this reason, the capacitor C_d must be at least 2 times greater than C . In this example $n = 4$ is chosen to ensure the fulfillment of the requirements. The optimal value of the damping resistance for this configuration is:

$$R_{opt} = \frac{R_o}{n(1-D)} \sqrt{\frac{(2+n)(3n+4)}{2(4+n)}} = \frac{\sqrt{\frac{5 \times 10^{-4}}{10^{-5}}}}{4 \times (1-0.5)} \sqrt{\frac{(2+4) \times (3 \times 4 + 4)}{2(4+4)}} = 8.66 \Omega$$

In Fig. 6 (a) it can be seen that there exists an equivalent solution for the same damping ratio at $R = 15 \Omega$. In this case, the natural frequency of the poles is greater, and the system response will be faster, but the damping resistance is not optimal and the peak in the output impedance of the system needs to be further studied.

As can be seen in Fig. 7 the difference between the bode plots of the output impedance with the optimal resistance and with the resistance that allows the same damping is not significative. Therefore, both systems will be considered. Considering that the system operates in open loop, the behavior of the system will be represented by its input-to-output transfer function.

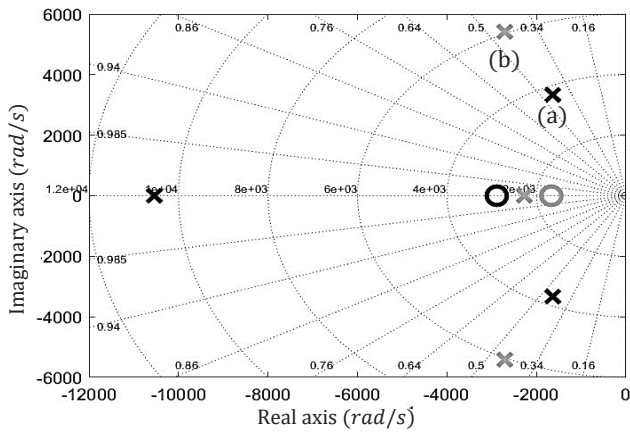


Fig. 8. Representation of the poles and zeros of the system for two different values of R . (a) $R = R_{opt}$. (b) $R = 15 \Omega$.

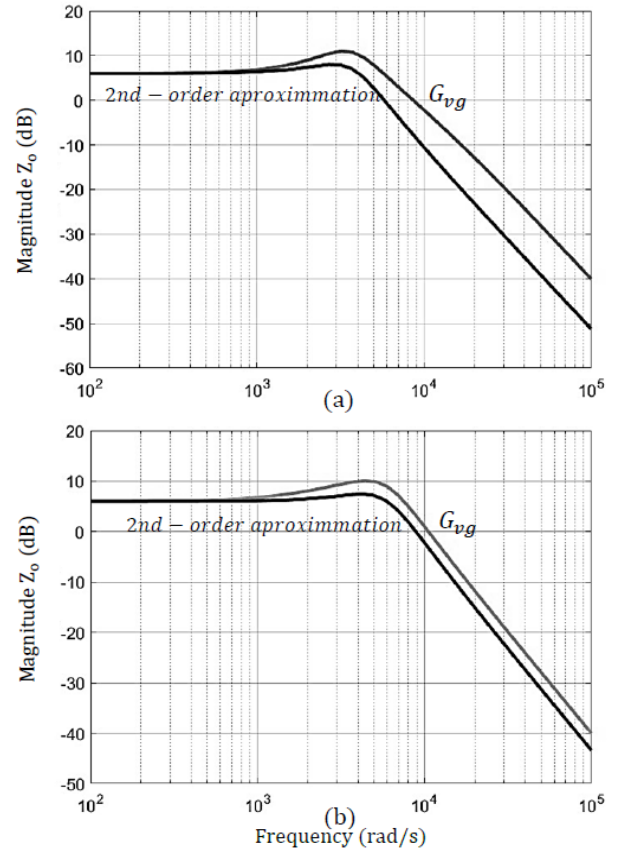


Fig. 9. Bode plots of the two proposed solutions and its second order approximation. (a) $R = R_{opt}$. (b) $R = 15 \Omega$

$$s = -\frac{1}{C_d R} \quad (26)$$

This transfer function presents a real negative zero (26) which is also dependent on R .

This zero will allow to determine if there is a pole dominance, and therefore dynamically imposes the second order behavior of the two complex conjugate poles. There may also exist the situation of non-dominance of any of the poles and consequently the behavior of the system cannot be studied in a simple way. The first order behavior due to the real pole dominance is rather improbable because of the cancellation of this pole with the real zero when this pole gets close of the imaginary axis.

In Fig. 8 the distribution of poles and zeros is represented for the two proposed dampers. In the case that the resistance value used is the optimal one, there is no dominance in the dynamics although the real pole is significantly far from the complex conjugate poles, the real zero is close. On the contrary, in the other case, there is a cancellation between the pole and the real zero and as a result, the dynamics are close to the second order behavior imposed by the two complex conjugate poles. This can also be seen in Fig. 9, in which the difference between the original bode plot of the input-to-output voltage transfer function and its second order approximation are represented.

The time-domain system response is represented in Fig. 10 which was obtained by numerical simulation performed on PSIM software using both the circuit-level switched and averaged models.

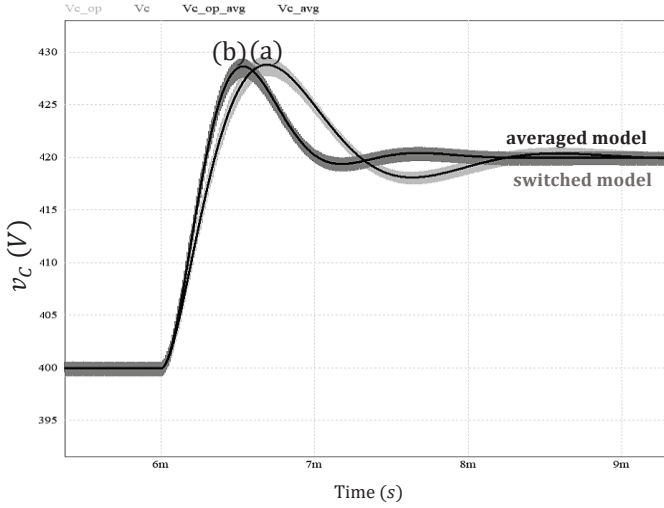


Fig. 10. Time-domain response of the boost converter showing the output voltage due to a step change of 10 V in the input voltage for two different values of R . (a) $R = R_{opt}$. (b) $R = 15 \Omega$.

It can be seen that for the two values of the used parameter R the system is stable and reaches the desired steady state. Calculations are presented below to validate that the damping ratio is lower than 45% and therefore, that the system requirements have been satisfied by the design.

$$PO_1(\%) = 100 \cdot \frac{v_{C, peak} - v_{C, ss}}{\Delta v_{C, ss}} = 100 \cdot \frac{428.8 - 420}{20} = 44.15\% \quad (27)$$

$$PO_2(\%) = 100 \cdot \frac{v_{C, peak} - v_{C, ss}}{\Delta v_{C, ss}} = 100 \cdot \frac{428.7 - 420}{20} = 43.36\% \quad (28)$$

The obtained overshoot in both cases is smaller than the limitation imposed. It is observed that despite having overdimensioned the damping, the effect of the real zero, modifies the dynamic response making the damping lower than the expected from the second order approximation.

Knowing this limitation in the process, and supposing that more restrictive requirements are imposed, the presented process can be repeated starting from a higher damping value (by selecting the damping filter resistance) or increasing the size of the damping filter capacitor C_d and repeating the previous steps.

VI. CONCLUSIONS

Three damping filters have been presented to stabilize an open-loop boost power converter feeding a constant power load. A canonical third-order polynomial characteristic equation has been obtained for the converter when three different damping filters are used. Based on the obtained characteristic polynomial, a methodology for designing the filters has been also explained. Although the damping cannot be directly estimated by a second order approximation of the system, it has been verified that there exist two cases with equivalent damping. One of them will present a faster response than the other. In addition, it has been shown that obtaining the optimal value of the damping resistance, according to the criterion of mitigating the peak in the output impedance, can lead to an undesired dynamic response. In fact, it is possible to reach a compromise in order to ensure that the output impedance resonance is mitigated and to obtain the desired dynamic response. Furthermore, the representation of the poles of the system when the resistance has its optimum value (Fig. 11), shows how it is not possible to reach certain values of damping at this operating point.

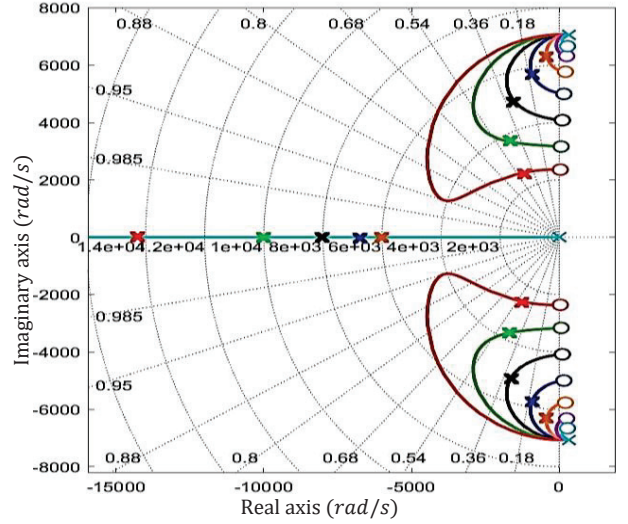


Fig. 11. Representation of the position of the poles when the resistance R has its optimal value.

To sum up, the presented methodology allows the sizing of the damping capacitor through an iterative process.

It is possible to intuitively discard in advance those designs that in their geometric location of the roots do not reach at any point the desired damping value. Next, a compromise between size, cost, weight, and dynamic behavior must be reached. Once the capacitor is selected, it is necessary to find a resistor whose value allows the damping of the output impedance peak and a desired dynamic response. Finally, since the selection of this resistance is done using the second order system approximation, if the obtained response in simulation is not satisfactory, the process should be restarted and a response with greater damping must be chosen, by increasing the capacitor size or changing the value of R .

Further analysis needs to be carried in the other topologies and for different sets of parameters.

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