Pressure recovery model for gas-liquid two-phase flow across sudden expansions

Abderraouf Arabi[1]✉, Ghassan H. Abdul-Majeed[2], Houari B. Boukrouche[1] and Zakaria Larbi[3]

1. SONATRACH, Direction Centrale Recherche et Développement, Avenue 1er Novembre, 35000, Boumerdes, Algeria

2. University of Baghdad, Baghdad, Iraq

3. University of Sciences and Technology Houari Boumediene USTHB, Physics' Faculty Laboratory of Theoretical and Applied Fluid Mechanics, LMFTA, BP 32 El Alia 16111, Bab Ezzouar, Algiers, Algeria

✉ Arabi.abderraouf@sonatrach.dz

Abstract

The presence of a sudden expansion generates a variation of the static pressure commonly called Pressure Recovery (*PR*). In this paper, we made firstly an extensive literature survey to list existing gas-liquid two-phase flow pressure recovery models and to collect an experimental database. Thus, a total of 305 data was collected from 6 recent works and 18 predictive models was identified. An analysis of the different existing models was carried out firstly. Then, the predictive capability of nine existing models was assessed using the collected database. It was reported that none of the models can predict the experimental results for a large range of experimental conditions. This finding highlighted the necessity to propose a new model. The proposed predictive model was developed using the two-phase multiplier and mass quality. These two parameters were correlated using 157 data points from the collected database, while the other data was used to validate it. It was found that the proposed model gives better predictions compared to existing ones in the range of conditions and parameters of the experimental database used in this analysis.

Keywords

Gas-liquid two-phase flow, sudden expansion, literature review, pressure recovery, two-phase multiplier.

Nomenclature

A	Area [m^2]
$AARE$	Absolute Average Relative Error
Bo	Bond number
C_h	Chishom factor
D	Pipe diameter [m]
fe	Total friction factor
Fr	Froude number
g	Gravitational acceleration [m/s^2]
G	Mass flux [kg/m^{2}s]
K	Wadle's pressure recovery coefficient
L	Length
La	Laplace constant
P	Pressure [Pa]
PR	Pressure recovery [Pa]
q	Empirical coefficient
r	Correction factor
Re	Reynolds number
R^2	Coefficient of determination
S	Slip ratio
V	Velocity [m/s]
We	Weber number
x	Mass quality
X	Lockhart-Martinelli parameter
z	Axial direction [m]
Z	Normalized pressure recovery

Greek symbols

α	Void fraction
α_E	Mean volumetric liquid entrainment,
β	Volumetric quality
Γ_e	Base pressure coefficient
θ	Pipe inclination [°]
θ_σ	Function of the area ratio
μ	Viscosity [Pa.s]
ρ	Density [kg/m^3]
ρ'	Fictitious mixture density [kg/m^3]
ρ''	Fictitious mixture density [kg/m^3]
ρ_{eff}	Mean effective density [kg/m^3]
σ	Area ratio
σ^*	Surface tension [N/m]
τ	Shear stress [N/m^2]
Φ	Characteristic function
Φ^2	Two-phase multiplier
Ω_1	Correction factors
Ω_2	Correction factors
Ω_3	Correction factors

Subscripts

cal	Calculated
d	Developing region
fD	Fully developed region
G	Gas
H	Homogenous
I	Irreversible
L	Liquid

M	Mixture
mea	Measured
O	Only
R	Reversible
s	Single-phase
S	Superficial
TP	Two-phase
0	At the sudden expansion section
1	Upstream the sudden expansion
2	Downstream the sudden expansion

1 Introduction

Gas-liquid two-phase flow in macro pipes are widely encountered in the industry as oil and gas production and transportation (Arabi et al., 2022, Boutaghane et al., 2023), nuclear power plants (Yeo, 2019; Hibiki, 2019), wastewater industry (Hundshagen et al., 2021) or in chemical and biochemical applications (Wang et al., 2021, Levitsky et al., 2022). These kinds of flows are also found in mini pipes such as those used in high-power electronic devices, heat exchangers and air-conditioning systems (Hazuku et al., 2020)

Due to the complexity of industrial installations, many parts of fluid transport systems are constituted by singularities, or fittings. These geometrical 'anomalies' lead to a change of the flow behavior and static pressure. In industrial fluid transport networks, the pressure losses generated by the singularities can be more important than the regular pressure drop generated by the straight pipes (Azzopardi and Hills, 2003). Thus, the engineers need to predict the pressure change correctly to design and optimize the industrial device's performances (Abdul-Majeed and Al-Mashat, 2000; Ghajar, 2020). The sudden expansion or sudden enlargement, where an upstream pipe is connected to a larger diameter downstream pipe, is a common singularity. Indeed, this singularity can be found in the hydrocarbon production wells (Ramirez-Jaramillo et al., 2010), petroleum surface installations (Arabi et al., 2021b) or heat exchangers (Aloui and Souhar, 1996a; Lewis and Wang, 2018). The sudden expansions are characterized by the aspect ratio (σ), given by the following equation:

$$\sigma = \frac{A_1}{A_2} = \left(\frac{D_1}{D_2}\right)^2 \qquad (1)$$

where A and D are the cross-section area, and pipe diameter, respectively. The subscripts 1 and 2 refer to the upstream and downstream pipes, respectively.

Compared to straight pipes, few studies have investigated the influence of sudden expansions on gas-liquid two-phase flows. Some of these works was devoted to the study of the fitting's influence on the flow pattern (Yang et al., 2001; Kondo et al., 2002; Ahmed et al., 2007, 2008; Arabi et al., 2018, 2021b; Zhang and Goharzadeh, 2019; Roman et al., 2020) or on the hydrodynamic flow parameters (Aloui and Souhar, 1996a; Arabi et al., 2018, 2021b; Liang et al., 2022). Some authors were interested in the behavior of a specific flow regime through the sudden expansions. For example, the bubbly flow was investigated by Aloui et al. (1996a, 1996b), Lobanov and Pakhomov (2017), Lobanov et al. (2019), Kim and Park (2019), Celis et al. (2021) and Bogatko et al. (2021). On the other hand, Arabi et al. (2018, 2021b) and Zhang and Goharzadeh (2019) dealt with the intermittent flow. However, as mentioned by Arabi et al. (2021b), most studies was focused on the estimation of the pressure change caused by the passage of the gas-liquid two-phase flow through this fitting. This pressure change is called pressure recovery (*PR*). In these studies, the authors carried out experimental measurements of *PR* and proposed predictive models. Most of these predictive models was not validated beyond their original data set. Wang et al. (2010) evaluated the applicability of eight *PR* models. Their study was carried out by comparing the predictions given by eight existing models with an experimental database composed of 282 points collected from the literature. The authors pointed out that most available models can predict correctly the pressure recovery only for their database, and the utilization of the models outside their database is not recommended.

Since the work of Wang et al. (2010), additional databases have been generated and other models have also been developed. Thus, it is important to carry out an assessment study by incorporating an up-to-date experimental database. This aim constitutes the motivation of this work. To achieve this objective, an extensive literature review was first carried out in order to analyze the existing models. This literature survey allowed to collect 305 data from 6 recent sources. The comparison between the predictions of the pressure recovery given by the existing models with the collected experimental data showed that none of them can predict accurately this parameter in the whole range of experimental conditions. Based on the state-of-the-art analysis of existing models, the separated approach was used for the development of a new predictive model. The predictive capacity of the developed pressure recovery model was compared with the existing ones and validated using the collected database.

2 Review of existing models

2.1 Definition of the pressure recovery through sudden expansions and the phenomena associated

Before discussing the existing *PR* models, it is important to explain as a first step how the pressure recovery is calculated. The passage of the two-phase flow, as a single-phase flow, through the sudden expansion causes its deceleration in the area just downstream called the recovery region; this deceleration causes an increase of the static pressure. When it reaches its maximum, the static pressure begins to decrease, joining the pressure gradient downstream of the singularity: the flow re-establishes. The pressure recovery is calculated by extrapolating the linear lines representing the gradient of the pressure upstream and downstream of the singularity at the location of the sudden expansion, as shown in Fig.1. The difference between the intersections of these two lines at the location of the sudden expansion (position $z = 0$) gives the pressure recovery.

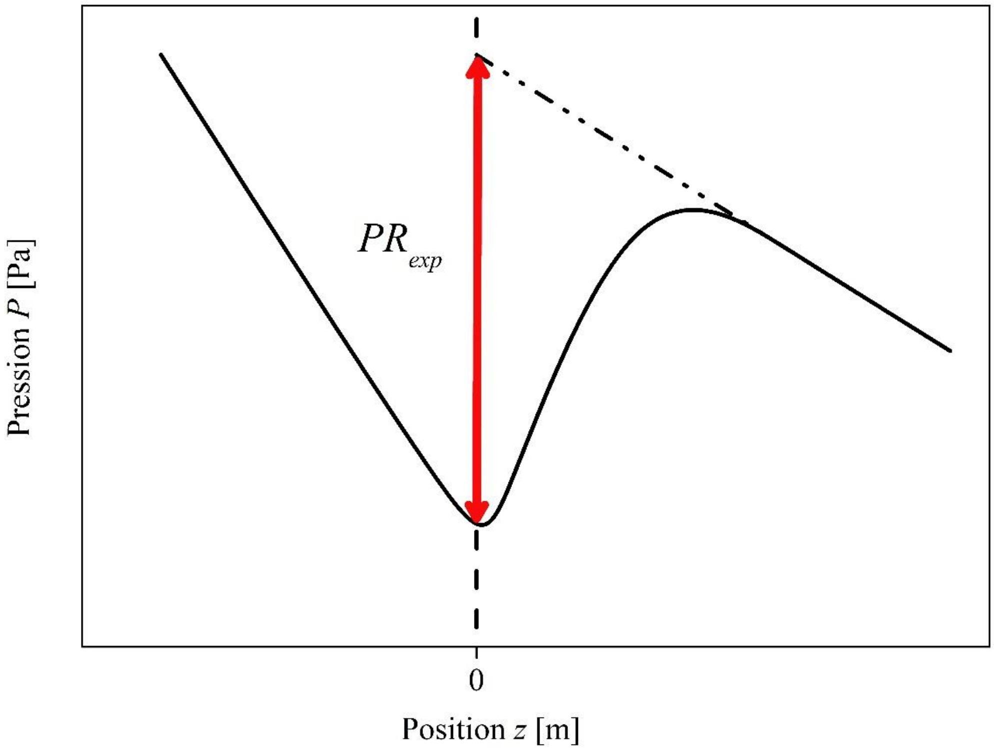

Fig. 1 Variation of the static pressure before and after the sudden expansion.

Due to the difference in physical properties between the gas and the liquid (i.e., density and viscosity), the two phases don't decelerate in the same way downstream of the singularity. Thus, the downstream region close to the singularity exhibits recirculation zones (Salhi, 2010, Roul and Dash, 2011). The latter lead to a redistribution of the void fraction (Ahmed, 2007; Roman et al., 2020; Liang et al., 2022) and a modification of the flow regime or sub-regime downstream the singularity (Attou et al., 2000; Kondo et al., 2002; Arabi et al., 2018, 2021b; Zhang and Goharzadeh, 2019). The hydrodynamics parameters (Aloui and Souhar, 1996b; Ahmed et al., 2008; Rosero et al., 2021) as well as the heat transfer characteristics (Lobanov et al., 2019; Chinak et al., 2019), in the case of non-isothermal flows, are also disturbed.

2.2 Existing pressure recovery models

In this section, we have started with the presentation of existing *PR* models in the case of single-phase flow in order to better understand the two-phases models.

Solving the momentum equation inside the control volume between the upstream and downstream (after the flow recovery) pipes gives Eq. 2, which is known as the Borda-Carnot equation.

$$PR = \frac{\sigma(1-\sigma)G_1^2}{\rho} \tag{2}$$

where G_1 is the total upstream mass flux and ρ the fluid density.

The mechanical energy balance equation leads to the following equation:

$$PR = \frac{\frac{1}{2}(1-\sigma^2)G_1^2}{\rho} \tag{3}$$

Regarding the two-phase flow, a variety of models have been developed through the years. These models are summarized in Table 1. The analytical models are obtained by applying the homogenous or heterogeneous model to the single-phase conservation equations (momentum or mechanical energy). The model of Romie (1958) was obtained by applying the heterogeneous model to the single-phase momentum conservation equation. The model of Delhaye (1981) is based on the assumption that the void fraction, upstream (α_1) and downstream (α_2) the singularity, remains constant. Lottes (1958), in his model, assumed that the dynamic pressure recovery occurs only on the liquid phase. For Chisholm and Sutherland (1969), the *PR* is the product of the liquid pressure recovery (PR_L) and the two-phase multiplier (ϕ_L^2).

Ahmed et al. (2007) developed an analytical expression by adding two terms to the pressure recovery model of Romie (1958). The first is the pressure difference between the upstream flow and the downstream face of the expansion, whereas the second one is due to the shear stress in the developing region downstream of the enlargement. The exponent *r*, used in the model of Attou and

Bolle (1997) is a correction factor that depends on the nature of the working fluids. The authors recommended to use r = -1.4 for air-water mixture and r = 1 for a water-steam mixture.

Wadle (1989) proposed a semi-empirical model which was not developed from the theoretical momentum or mechanical energy conservation equations. Thus, this model is based on the assumption that the pressure recovery is proportional to the dynamic pressure difference caused by the deceleration of the two phases due to the sudden expansion. This model recourses to an empirical coefficient K. Based on his experimental data, Wadle (1989) suggested to use different values of K for air-water and steam-water systems. Owen et al. (1992) found that their experimental results were well correlated with Wadle's model using K = 0.22. The difference between this value and those suggested by Wadle (1989) can be explained by the geometry used as well as the conditions of each experiment. Using experimental results obtained with sudden expansions of the area ratio of 0.26 and 0.39, Chen et al. (2007) modified the empirical coefficient of Wadle's model by correlating it with the aspect ratio.

It also exists some purely empirical models. For example, Ahmed et al. (2003) developed their empirical model using their data obtained with air-oil mixture. Wang et al. (2010) compared the predictions of several models (Homogenous models, Richardson, 1958; Chisholm and Sutherland, 1969; Wadle, 1989; Schmidt and Friedel, 1996; Attou and Bolle,1997; Abdelall et al., 2005) with published experimental data. They found that none of the evaluated models can predict accurately all the databases. Then, Wang et al. (2010) proposed a new model by introducing a coefficient to the homogeneous model. A total of 282 experimental data collected from the literature were used to build this empirical coefficient. This coefficient combines four dimensionless parameters, namely: Bond number (Bo), Weber number (We), Froude number (Fr), and liquid only Reynolds number (Re_{LO}). Kourakos (2011) proposed an empirical model to predict the pressure recovery based on its data obtained with two aspect ratios (0.328 and 0.646).

It also exists phenomenological (mechanistic) models that were not reviewed in the present paper. In fact, these kinds of models were built based on the complex phenomena specific for each flow regime (Hewitt, 1983). As a consequence, the mechanistic models recourse to assumptions and input parameters specific for each flow pattern, which limit their use to only one flow regime. As examples of the *PR* mechanistic models, we can cite the models of Anupriya and Jayanti (2014, 2018) developed for annular flow, and that of Attou et al. (1997) specific for bubbly flow.

Table 1 Summary of existing models to predict *PR*

Authors and year	Model	
Homogeneous model applied to the momentum balance	$PR = \sigma(1-\sigma)G_1^2\left[\frac{x}{\rho_G}+\frac{(1-x)}{\rho_L}\right]$	(4)
Homogeneous model applied to the mechanical energy balance	$PR = \frac{1}{2}(1-\sigma^2)G_1^2\left[\frac{x}{\rho_G}+\frac{(1-x)}{\rho_L}\right]$	(5)
Romie (1958)	$PR = \sigma(1-\sigma)G_1^2\left[\left(\frac{x^2}{\rho_G\alpha_1}+\frac{(1-x)^2}{\rho_L(1-\alpha_1)}\right)-\sigma\left(\frac{x^2}{\rho_G\alpha_2}+\frac{(1-x)^2}{\rho_L(1-\alpha_2)}\right)\right]$	(6)
Delhaye (1981)	$PR = \frac{\sigma(1-\sigma)G_1^2}{\rho_L}\left[\frac{(\rho_L/\rho_G)x^2}{\alpha}+\frac{(1-x)^2}{(1-\alpha)}\right]$	(7)
Lottes (1961)	$PR = \frac{\sigma(1-\sigma)G_1^2}{\rho_L(1-\alpha)^2}$	(8)
Chisholm and Sutherland (1969)	$PR = PR_L\phi_L^2$	(9)
	$PR_L = \frac{\sigma(1-\sigma)(1-x)^2G_1^2}{\rho_L}$	(10)
	$\phi_L^2 = 1+\frac{C_h}{X}+\frac{1}{X^2}$	(11)
	$X = \left(\frac{1-x}{x}\right)\sqrt{\frac{\rho_G}{\rho_L}}$	(12)
	$C_h = \left[1-\frac{1}{2}\sqrt{\frac{\rho_L-\rho_G}{\rho_L}}\right]\left[\sqrt{\frac{\rho_L}{\rho_G}}+\sqrt{\frac{\rho_G}{\rho_L}}\right]$	(13)

Ahmed et al. [26]	$PR = \sigma(1-\sigma)G_1^2\left[\left(\frac{x^2}{\rho_G\alpha_1}+\frac{(1-x)^2}{\rho_L(1-\alpha_1)}\right)-\sigma\left(\frac{x^2}{\rho_G\alpha_2}+\frac{(1-x)^2}{\rho_L(1-\alpha_2)}\right)\right] - (P_1-P_0)(1-\sigma)-\frac{4}{D_2}\left(\int_0^{L_d}\tau_d(z)dz-\tau_{fD}L_d\right)$	(14)
Collier and Thome [60]	$PR = \frac{(1-\sigma^2)G_1^2}{2\left(\frac{x}{\rho_G}+\frac{1-x}{\rho_L}\right)}\left[\frac{x^3}{\alpha^2{\rho_G}^2}+\frac{(1-x)^3}{(1-\alpha)^2{\rho_L}^2}\right]$	(15)
Richardson (1958)	$PR = \frac{1}{2}(1-\sigma^2)G_1^2\left[\frac{\sigma(1-x)^2}{\rho_L(1-\alpha)}\right]$	(16)
Wadle (1989)	$PR = \frac{1}{2}(1-\sigma^2)G_1^2K\left[\frac{x^2}{\rho_G}+\frac{(1-x)^2}{\rho_L}\right]$	(17)
	K = 0.83 for air-water K = 0.667 for steam-water	(18)
Owen et al. (1992)	K = 0.22	(19)
Chen et al. (2007)	$K = \frac{1}{1.551-7.64\sigma^2}$	(20)
Schmidt and (1996)	$PR = \frac{G_1^2\left[\frac{\sigma}{\rho_{eff}}-\frac{\sigma^2}{\rho_{eff}}-f_e\rho_{eff}\left(\frac{x}{\rho_G\alpha}-\frac{(1-x)}{\rho_L(1-\alpha)}\right)\left(1-\sqrt{\sigma}\right)^2\right]}{1-\Gamma_e(1-\sigma_1)}$	(21)
	$\frac{1}{\rho_{eff}} = \frac{x^2}{\rho_G\alpha}+\frac{(1-x)^2}{\rho_L(1-\alpha)}+\rho_L(1-\alpha)\left(\frac{\alpha_E}{1-\alpha_E}\right)\times\left[\frac{x}{\rho_G\alpha}-\frac{1-x}{\rho_L(1-\alpha)}\right]^2$	(22)
	$\alpha_E = \frac{1}{S}\left[1-\frac{1-x}{1-x(1-0.05We_{G01}^{0.27}Re_{SL1}^{0.05})}\right]$	(23)
	$S = \frac{x}{1-x}\frac{(1-\alpha)}{\alpha}\frac{\rho_L}{\rho_G}$	(24)
	$We_{GO1} = G_1^2x^2\frac{D_1}{\rho_G\sigma^*}\frac{(\rho_L-\rho_G)}{\rho_G}$	(25)
	$Re_{SL1} = \frac{G_1(1-x)D_1}{\mu_L}$	(26)
	$\Gamma_e = 1-\sigma^{0.25}$	(27)
	$f_e = 4.9\times10^{-3}x^2(1-x)^2\left(\frac{\mu_L}{\mu_G}\right)^{0.7}$	(28)
	$\alpha = 1-\frac{2(1-x)^2}{1-2x+\sqrt{1+4x(1-x)\left(\frac{\rho_L}{\rho_G}-1\right)}}$	(29)

Attou and Bolle (1997)	$PR = \sigma(1-\sigma)\theta_\sigma^r G^2 \Phi + \frac{(1-\theta_\sigma^r)\sigma(1-\sigma)G^2}{\rho_l}$	(30)
	$\Phi = \frac{\dot{x}^2}{\alpha\rho_g} + \frac{(1-x)^2}{(1-\alpha)\rho_l}$	(31)
	$\theta_\sigma = \frac{3}{1+\sigma^{0.5}+\sigma}$	(32)
Abdelall et al. (2005)	$PR = (P_2 - P_1)_R + (P_2 - P_1)_I$	(33)
	$PR_R = \frac{G_1^2}{2\rho''^2}(1-\sigma)$	(34)
	$PR_I = \frac{G_1^2}{2\rho_L}\left[\frac{2\rho_L\sigma(\sigma-1)}{\rho'} - \frac{\rho_H\rho_L(1-\sigma^2)}{\rho''^2}\right]$	(35)
	$\rho' = \frac{1}{\left[\frac{(1-x^2)}{\rho_L(1-\alpha)} + \frac{x^2}{\rho_G\alpha}\right]}$	(36)
	$\rho'' = \frac{1}{\sqrt{\frac{(1-x)^3}{\rho_L^2(1-\alpha)^2} + \frac{x^3}{\rho_G^2\alpha^2}}}$	(37)
Ahmed et al. (2003)	$PR = \frac{1}{2} Z \rho_H V_{M1}^2$	(38)
	$Z = \left(2.795 \times 10^{-5} + \frac{4 \times 10^{-7}}{x^2}\right)^{1/2}$	(39)
	$\rho_{TP} = x\rho_G + (1-x)\rho_L$	(40)
	$V_{M1} = V_{SL1} + V_{SG1}$	(41)
Wang et al. (2010)	$PR = (1 - \Omega_1 + \Omega_2)(1+\Omega_3)\sigma(1-\sigma)G^2\left[\frac{x}{\rho_G} + \frac{(1-x)}{\rho_L}\right]$	(42)
	$\Omega_1 = \left(\frac{WeBo}{Re_{LO}}\right)^2 \left(\frac{1-x}{x}\right)^{0.3} \frac{1}{Fr^{0.8}}$	(43)
	$\Omega_2 = 0.2\left(\frac{\mu_G}{\mu_L}\right)^{0.4}$	(44)
	$\Omega_3 = 0.4\left(\frac{x}{1-x}\right)^{0.3} + 0.3e^{\frac{1.6}{Re_{LO}^{0.1}}} - 0.4\left(\frac{\rho_G}{\rho_L}\right)^{-0.2}$	(45)
	$Bo = \frac{(\rho_L - \rho_G)}{\sigma^*}$	(46)
	$We = \frac{G_1^2 D_1}{\sigma^* \rho_H}$	(47)

	$$Fr = \frac{G_1^2}{\rho_H^2 g D_1}$$	(48)
	$$Re_{LO1} = \frac{G_1 D_1}{\mu_L}$$	(49)
Kourakos (2009)	$$PR = \frac{1}{2}\rho_L(V_{M1}^2 - V_{M2}^2) - \frac{1}{2}\rho_L Re_{SL1}^2(-0.772\sigma - 6.210^{-7} Re_{SL1} + 0.0825\beta + 0.706)$$	(50)
	$$\beta = \frac{V_{SG1}}{V_{SL1} + V_{SG1}}$$	(51)
	$$Re_{SL1} = \frac{G_1(1-x)D_1}{\mu_L}$$	(52)

2.3 Analysis of existing models

After presenting the existing pressure recovery models, it seems important to deeply analyze them. A primary analysis of the models, described above, highlights their great variety, either in the approach used for their developement or in the choice of input parameters (summarized in Table 2). As it appears in the Table, all the models require the use of flux mass (G_1) and mass quality (x), except the model of Lottes (1961) which is independent of x. The model of Ahmed et al. (2003) is the only one that does not recourse to the aspect ratio.

The mechanistic models are the most rigorous, as they consider the real phenomena occurring in each flow regime (Hewitt, 1983). However, the problem in predicting the flow regimes downstream the singularity, when the flow pattern maps are not valid (Kondo et al., 2002; Arabi et al., 2018; Zhang and Goharzadeh, 2019), adds further complexity to pressure recovery modeling using the phenomenological approach. In addition, these models are also known by their complexity due to their dependance of several parameters. This last statement is also valid for the models of Schmidt and Friedel (1996) and Attou and Bolle (1997).

Table 2 Independent parameters of the existing two-phase PR models

Model	Parameters											
	Upstream mass flux (G_1)	Mass quality (x)	Volume flow rate ratio (β)	Aspect ratio (σ)	Upstream pipe diameter (D_1)	Liquid density (ρ_L)	Gas density (ρ_G)	Liquid dynamic viscosity (μ_L)	Gas dynamic viscosity (μ_G)	Surface tension (σ^*)	Upstream void fraction (α_1)	Downstream void fraction (α_2)
Homogeneous model applied to the momentum balance	+	+	-	+	-	+	+	-	-	-	-	-
Homogeneous model applied to the mechanical energy balance	+	+	-	+	-	+	+	-	-	-	-	-
Romie (1958)	+	+	-	+	-	+	+	-	-	-	+	+
Delhaye (1981)	+	+	-	+	-	+	+	-	-	-	+	-
Lottes (1961)	+	-	-	+	-	+	-	-	-	-	+	-
Chisholm and Sutherland (1969)	+	+	-	+	-	+	+	-	-	-	-	-
Ahmed et al. (2007)	+	+	-	+	-	+	+	-	-	-	+	+
Collier and Thome (1994)	+	+	-	+	-	+	+	-	-	-	+	-
Richardson (1958)	+	+	-	+	-	+	-	-	-	-	+	-
Wadle (1989)	+	+	-	+	-	+	+	-	-	-	-	-
Owen et al. (1993)	+	+	-	+	-	+	+	-	-	-	-	-
Chen et al. (2007)	+	+	-	+	-	+	+	-	-	-	-	-
Schmidt and Friedel (1996)	+	+	-	+	+	+	+	+	+	+	+	-
Attou and Bolle (1997)	+	+	-	+	-	+	+	-	-	-	+	-
Abdelall et al. (2005)	+	+	-	+	-	+	+	-	-	-	+	-
Ahmed et al. (2003)	+	+	-	-	-	+	+	-	-	-	-	-
Wang et al. (2010)	+	+	-	+	+	+	+	+	+	+	-	-
Kourakos (2011)	+	+	+	+	+	+	+	+	-	-	-	-

Based on their own results, Ahmed et al. (2007) found that the *PR* is affected by the pressure difference between the upstream flow and the downstream face of the expansion, as well as the wall shear stress in the recovery region. However, these two parameters are difficult to estimate, which make this model unsuitable in practice. Furthermore, similar to the models of Romie (1958) and Collier and Thome (1994), this model depends on the value of the void fraction in the downstream pipe, which is also difficult to predict. Indeed, the gas decelerates more rapidly than liquid after crossing the expansion. Therefore, the slip factor and the void fraction increase close to recovery region (Ahmed, 2005). After the flow recovery, the downstream void fraction's value may decrease or increase compared to the upstream value (Ahmed, 2005; Liang et al., 2022). The relationship between the void fractions on either side of the sudden expansion remains not well correlated. Besides, there is no consensus on the parameter controlling the change of the void fraction, as reported by Ahmed (2005) and Arabi (2019).

The estimation of the upstream void fraction is also complicated. The literature is full of models to predict the void fraction in straight pipes. The prediction level of each model depends on the fluids' physical properties, pipe geometry, and flow regime (Marquez-Torres et al., 2020). Therefore, the models developed for the straight pipes are not necessarily valid in pipes fitted with a singularity (Zitouni et al., 2021). Schmidt and Friedel (1996) suggested to use Huq and Loth's correlation (1992) (Eq. 29) to estimate the void fraction in their pressure recovery model, which is based on their experimental results. Doubts remain on the confidence level of this model for conditions different from those of Schmidt and Friedel's experiments.

To demonstrate the consistency of the model based on the momentum balance on the mechanical energy conservation, the evolution of the pressure recovery, in the case of the single-phase, was plotted as a function of the diameter ratio $\sigma^{0.5}$ in Fig. 2. The dimensionless normalized singular pressure recovery (K_s), given by Eq. 53, was used instead of the pressure recovery. As shown, for $\sigma = 0$, when the downstream cross-section is too large, and for $\sigma = 1$, when the cross-section does not change, $K_s = 0$, which agrees with the physics of the phenomenon. The representation of single-phase flow experimental results collected from various studies (Delhaye, 1981; Suleman, 1990; Aloui and Souhar, 1994; Ahmed, 2005) shows that the momentum equation predicts the majority of the experimental data with a confidence level of ± 20 %.

$$K_s = \frac{PR}{\frac{G_1^2}{2\rho}} \tag{53}$$

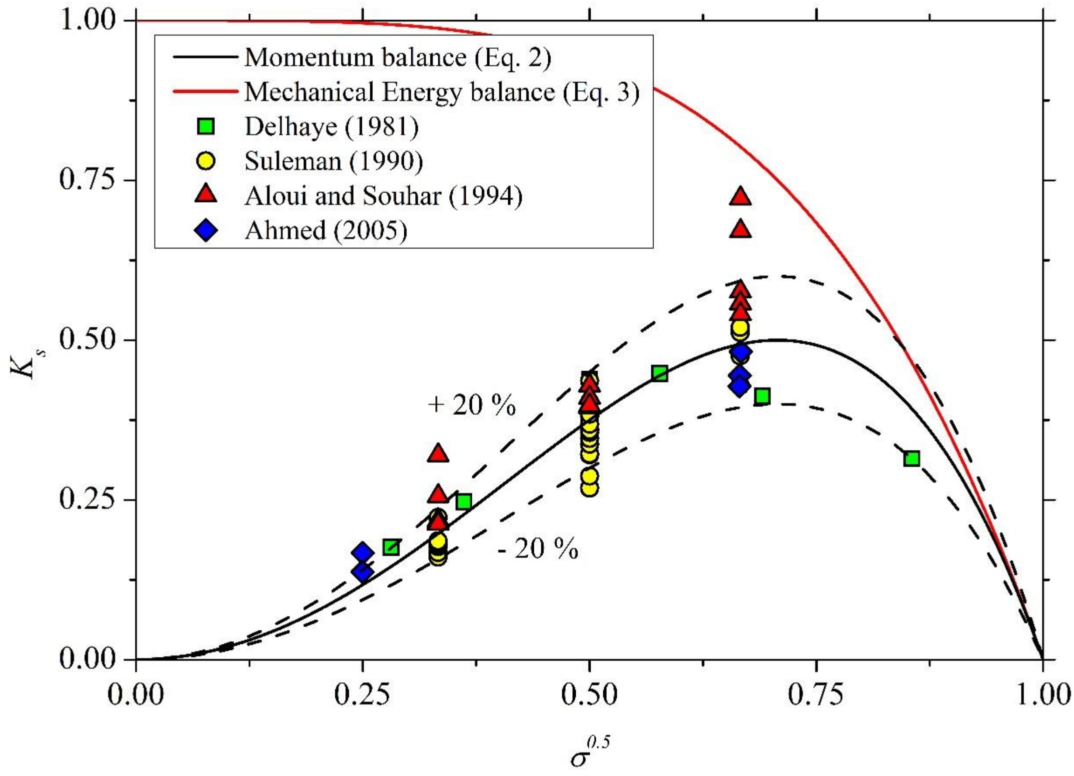

Fig. 2 Normalized liquid single phase *PR* as a function of $\sigma^{0.5}$.

Regarding the homogeneous models, the fact that they neglect the slippage between the two phases leads to an overestimation of the *PR* predictions, as noted by Abdelall et al. (2005) and Wang et al. (2010). However, the homogeneous model has the advantage of being practical and easy to apply. This aspect led Wang et al. (2010) to propose a correction of this model by incorporating an empirical coefficient. Generally, the empirical models' predictions are valid only within the range of experimental conditions close to those of data used for their development. This observation is also valid with the purely empirical models of Ahmed et al. (2003) and Kourakos (2011). It is important to note that these three models of Ahmed et al. (2003), Wang et al. (2010) and Kourakos (2011) were not validated with independent data. In addition, the model of Wang et al. (2010) has also the disadvantage of being applicable only for cases where $(1 - \Omega_1 + \Omega_2)(1 + \Omega_3) > 0$.

The modeling approach of Wadle (1989) is interesting since it is based on the Bernoulli principle. The weakness of this approach lies in difficulty to estimate the coefficient K. Indeed, according to Wadle (1989), it depends on the nature of the fluids. Chen et al. (2007) correlated it with the aspect ratio. The use of this model remains limited, since it is not valid in the case when $\sigma \geq 0.451$.

The model of Chisholm and Sutherland (1969), based on the separated flow model, recourses to the two-phase multiplier and the liquid pressure recovery. The separated flow approach remains relatively reliable especially considering its popularity for pressure drop modeling. The latter is notably used for the estimation of frictional pressure drop in the straight pipes (Muzychka and Awad, 2010; Sassi et al., 2020, Arabi et al., 2021a), as well as for the estimation of the pressure drop generated by the presence of different kinds of singularity, such as the bend (Azzi et al., 2000; Azzi and Friedel, 2005; Hayashi et al., 2020), the orifice (Zeghloul et al., 2017), the multi-hole orifice (Zeghloul et al., 2018), the gate and ball valves (Zeghloul et al., 2020), the pump valve (Ma et al., 2020), the Venturi (Messilem et al., 2020) or the static mixer (Hosni et al., 2023). The liquid pressure recovery of the model of Chisholm and Sutherland (1969) is calculated using the Borda-Carnot formula based on the momentum equation. Attou and Bolle (1997) reported that the model of Chisholm and Sutherland (1969) tends to underestimate the results with a mean relative error of -11% for steam-water mixture. The opposite was observed for air-water mixture. Wadle (1989) noted that the model of Chisholm and Sutherland (1969) overestimates their experimental results obtained using steam-water and air-water mixtures, with a mean relative error of 2.1% and 46.4%, respectively.

3 Existing database for pressure recovery

In order to perform an assessment study about the performance of the existing pressure recovery models, we have collected a total of 305 data from six recent papers. Table 3 summarizes the ranges of flow parameters covered by the collected experimental database. Some published data, like those of Ahmed (2005), were not included due to the lack of some critical information regarding the experimental conditions as well as the fluid properties (i.e, density and viscosity). The collected database covers a wide range of aspect ratio values ($0.0568 \leqslant \sigma \leqslant 0.646$). All experiments were conducted using an air-water mixture at atmospheric pressure, except those of Schmidt and Friedel (1996), which have been carried out at pressure equals to 5 bars. This experimental dataset covers horizontal (H) and vertical (V) configurations.

The measured data are plotted in Fig.3.a in terms of the geometric parameters σ and D_I. The hatched line represents the calculated value of Laplace constant (La) (Eq. 54). According to Triplett et al. (1999) and Cheng et al. (2008), the Laplace constant represents the critical value of pipe diameter to distinguish between mini and macro pipes.

$$La = \sqrt{\frac{\sigma^*}{g(\rho_L - \rho_G)}} \tag{54}$$

where σ^* and g are the surface tension and gravity, respectively.

As shown in Fig. 3.a, only the experiments of Abdellal et al. (2005) and Chalfi and Gaisshian (2008), which were carried out on the same experimental device, concerned mini pipes. Compared to the latter, the data of Abdellal et al. (2005) covered larger values of upstream flow mass, as it appears in Fig. 3.b.

Table 3 Summary of the experimental conditions of the database used in the present study.

Authors	Working fluids	Pipe inclination (θ) [°]	Working pressure [bar]	Upstream mass flux range (G_1) [kg/m^2.s]	Mass quality (x)	Upstream pipe diameter (D_1) [mm]	Area ratio (σ)	*PR* range [Pa]	Data points
Schmidt and Friedel (1996)	Air-water	0 and 90	5	1,000.00 - 1,0000.00	0.0022 – 0.9996	17.20, 19.00	0.0568, 0.0937, 0.1150	224.72 - 35,777.60	157
Abdellal et al. (2005)	Air-water	0	1	3,074.36 – 4,211.65	0.0019 – 0.0129	0.84	0.2760	7,26.00 – 4,619.00	14
Chalfi and Gausshian (2008)	Air-water	0	1	506.71 – 665.45	0.0315 – 0.1828	0.84	0.2760	534.39 – 3,391.30	24
Kourakos (2011)	Air-water	0	1	1,792.04 – 5,243.98	2.454×10^{-5} – 0.00038	41.00, 62.70	0.4280, 0.6460	310.15 – 10,446.67	53
Salhi et al. (2011)	Air-water	0	1	15.52 – 188.57	0.0042 – 0.1176	40.00	0.4440	5.43 – 42.33	14
Anupriya and Jayanti (2014)*	Air-water	90	1	85.40 – 357.80	0.187 – 0.796	25.00, 40.00	0.2500, 0.4440	298.00 - 3,520.00	43

***The data of Anupriya and Jayanti (2014) were collected from Anupriya and Jayanti (2018).**

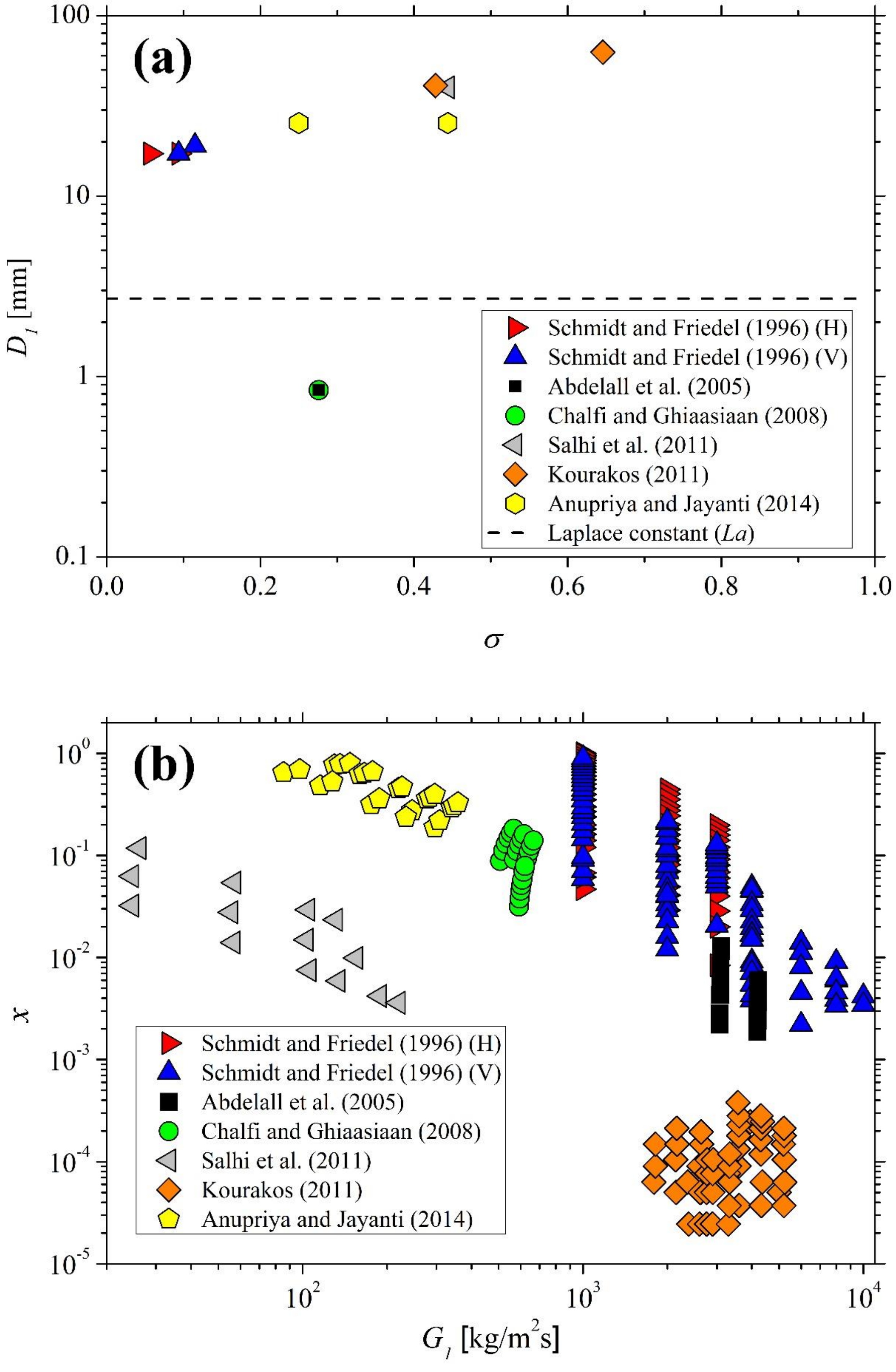

Fig. 3 Representation of the database in terms of (a): aspect ratio (σ) and upstream pipe diameter (D_1); (b): upstream mass flux (G_1) and mass quality (x).

4 Results and discussion

4.1 Comparison of existing pressure recovery models with collected data

The results of the confrontation between the experimental collected database and the predictions of the existing pressure recovery models are presented and discussed in this section. Based on the observation reported in section 2.3 about the difficulty to estimate the void fraction, we have chosen to study the performances of the only nine models which are independent from this parameter. These models are the homogeneous models (based on the momentum and energy balance equations), the models of Chisholm and Sutherland (1969), Wang et al. (2010), Ahmed et al. (2003), Kourakos (2011), and the models based on the Wadle's approach (Wadle, 1989; Owen et al., 1993; Chen et al., 2007). The comparison evaluation is based on the absolute average relative error (*AARE*) given by Eq. 55. The calculated values of the *AARE*, expressed in %, are listed in Table 4.

$$AARE = \frac{1}{N}\sum_{I=1}^{N}\left|\frac{PR_{cal} - PR_{exp}}{PR_{exp}}\right| \times 100 \qquad (55)$$

with PR_{cal} and PR_{mea} refer to calculated and experimental *PR*, respectively.

Table 4 reveals that the best predictions are obtained by the models of Chisholm and Sutherland (1969) and Wadle (1989) with *AARE* equal to 57.94% and 58.47%, respectively. Moreover, it appears clearly that the predictions given by each model are different for each data sources. Important differences exist also among the predictions of different models for each data source. The model of Chisholm and Sutherland (1969) is found to give the best predictions of the results obtained by Salhi et al. (2011) and Anupriya and Jayanti (2014). The data of Chalfi and Ghiassian (2008) are best predicted by the model of Wadle (1989) with *AARE* = 34.49 %. The model of Wang et al. (2010) gives good results only for the data used in their development (i.e., Schmidt and Friedel (1996), Abdelall et al. (2005) and Chalfi and Ghiassian (2008)). The same observation was reported for the correlation of Kourakos (2011). The model of Ahmed et al. (2003) gives acceptable predictions (*AARE* < 50 %) only for the data of Schmidt and Friedel (1996).

Table 4 Performance evaluation of the selected existing models against the experimental database. The results are given in %.

	Homogeneous model applied to the momentum balance	Homogeneous model applied to the mechanical energy balance	Chisholm and Sutherland (1969)	Wadle (1989)	Owen et al. (1993)	Chen et al. (2007)	Ahmed et al. (2003)	Wang et al. (2010)	Kourakos (2011)
All Data (305 data)	80.89	394.02	**57.94***	58.47	68.13	372.92[1]	482.37	91.54[2]	37947.70
Schmidt and Friedel (1996) (H) (75 data)	25.96	494.88	46.03	90.87	59.58	71.92	44.03	**24.27***	30110.99
Schmidt and Friedel (1996) (V) (82 data)	36.96	355.85	51.74	43.22	73.17	41.86	42.29	**35.58***	10263.45
Abdellal et al. (1996) (14 data)	449.18	1169.49	236.52	137.14	45.00	194.85	840.21	**22.10***	6700
Chalfi and Ghiassian (2008) (24 data)	269.15	753.33	106.26	**34.49***	78.05	39.23	495.48	40.46	76206.74
Kourakos (2011) (53 data)	19.62	48.72	21.46	22.6	70.14	870.92[3]	1853.54	-[4]	**14.95***
Salhi et al. (2011) (14 data)	97.73	191.85	**52.20***	75.24	93.44	564.67	119.86	534.52	11777.05
Anupriya and Jayanti (2014) (43 data)	105.5	329.29	**52.24***	57.54	64.75	1328.83	390.31	222.52	148504.36

*** Bold numbers mean the best predictions for each database.**

[1]Only 285 data were considered due to the conditions required for the application of the model of Chen et al. (2007) ($\sigma > 0.451$).

[2]Only 252 data were considered due to the conditions required for the application of the model of Wang et al. (2010) ($(1 - \Omega_1 + \Omega_2)(1 + \Omega_3) > 0$).

[3]Only 33 data were considered due to the conditions required for the application of the model of Chen et al. (2007) ($\sigma > 0.451$).

[4]None data were considered due to the conditions required for the application of the model of Wang et al. (2010) ($(1 - \Omega_1 + \Omega_2)(1 + \Omega_3) > 0$).

The cross plots of the predicted vs measured *PR* values, given by the models of Chisholm and Sutherland (1969) and Wadle (1989), are presented in Figs. 4.a and 4.b, respectively. The solid line represents the perfect case with the relative error equals to 0. The dashed lines represent the relative error of ± 50%. This threshold value is the same as used by Ahmed et al. (2007) and Wang et al. (2010). It appears from Fig. 4.a that the majority of the data of Schmidt and Friedel (1996), Kourakos (2011), Salhi et al. (2011) and Anupriya and Jayanti (2014) are fallen within ± 50% range. On the other hand, the model of Chisholm and Sutherland (1969) overestimates the results obtained in mini pipes (Abdellal et al., 2005; Chalfi and Ghiassian, 2008).

In contrast to this model, the model of Wadle (1989) gives two different trends for the data obtained in mini pipes (Fig. 4.b). Indeed, this model overestimates the results of Abdellal et al. (2005) and underestimates those of Chalfi and Ghiassian (2008). As a reminder, both authors have used the same experimental setup and the same pipe diameters. This result is an example to demonstrate that that the prediction of each model depends to the flow conditions.

The inexistence of an ‘universal’ model which can predict the data for a large range of experimental conditions encouraged us to propose a new model to predict the pressure recovery.

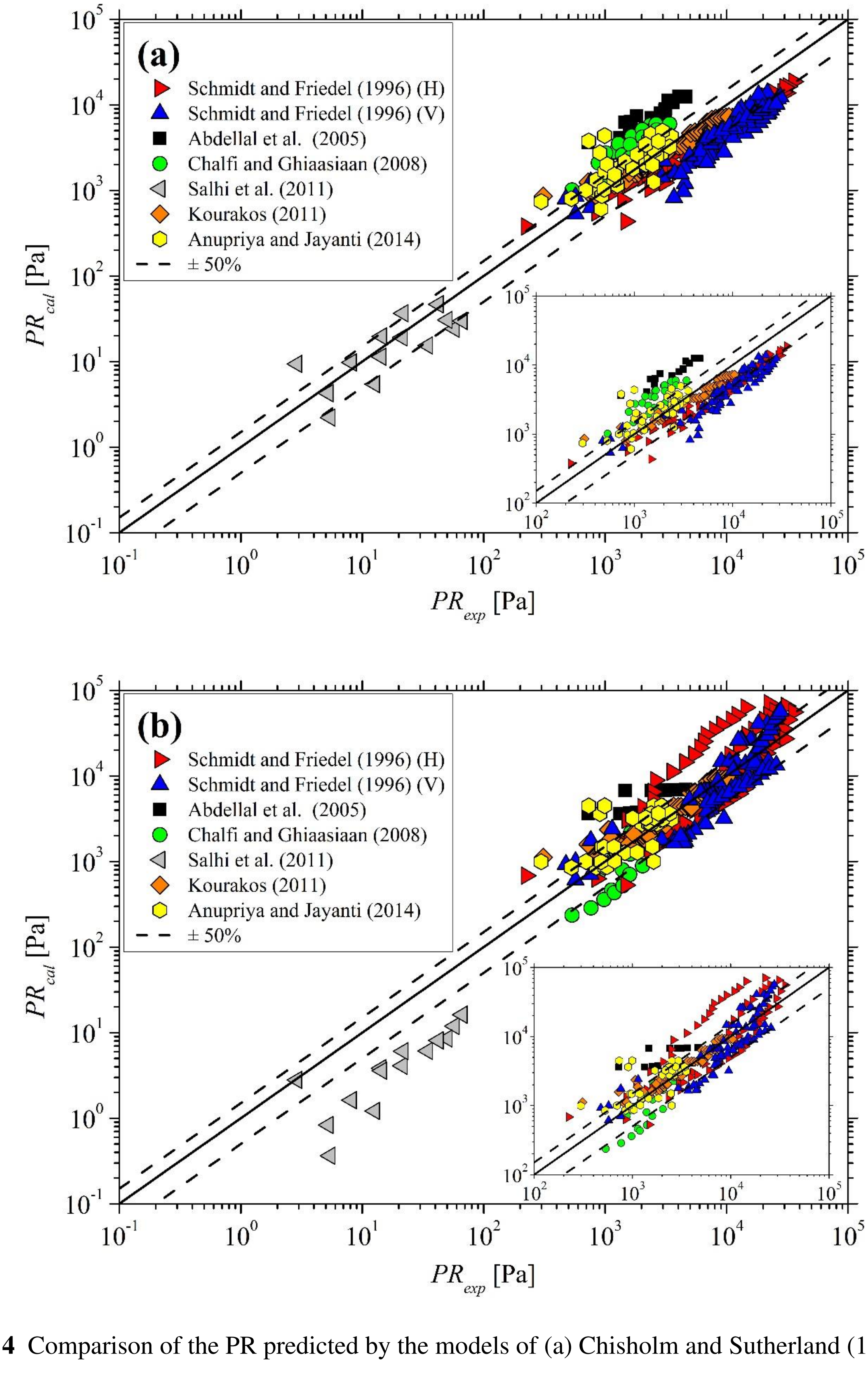

Fig. 4 Comparison of the PR predicted by the models of (a) Chisholm and Sutherland (1969); (b) Wadle (1989) with the experimental database.

4.2 Development of new pressure recovery model

To develop a model for estimating pressure recovery, we have chosen to use the separated model approach. Indeed, as explained in the section 2.3, this method is very popular for modeling the pressure drop in straight pipes or those generated by the presence of fittings. For the sudden expansion, this approach has only been used by Chisholm and Sutherland (1969). The fact that this model was found to give the best prediction of the collected database, as shown in section 4.1, can only confirm this choice. Using the latter, the *PR* is the product of the liquid pressure recovery (PR_L) and the two-phase multiplier (ϕ_L^2) (Eq. 9). The liquid pressure recovery is calculated using the Borda-Carnot formula based on the momentum equation (Eq. 10), which is more consistent than the one based on the energy conservation equation, as explained in section 2.3. The expression of the two-phase multiplier ϕ_L^2 (Eq. 56) is obtained from Eq. 9. In Fig. 5, we plotted all the data points of the database using ϕ_L^2 in function of the mass quality (x). The mass quality was chosen because it considers the densities of the two phases, unlike the volumetric quality.

$$\phi_L^2 = \frac{PR_{exp}}{PR_L} = \frac{\rho_L PR_{exp}}{\sigma(1-\sigma)(1-x)^2 G_1^2} \tag{56}$$

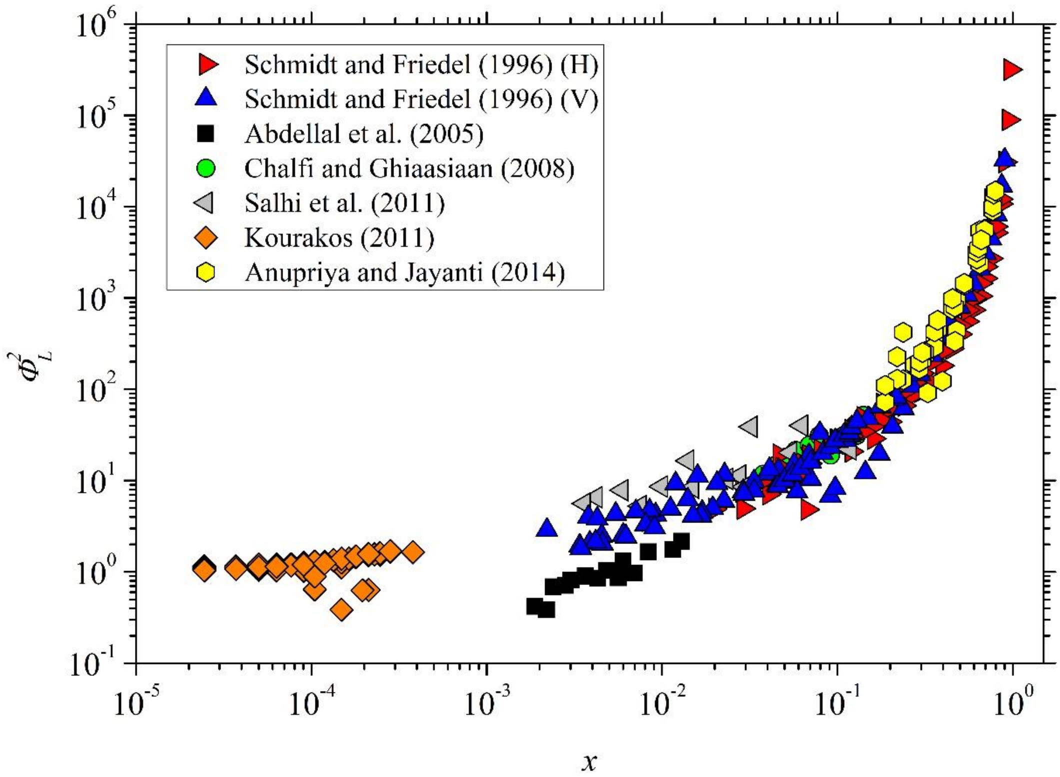

Fig. 5 Representation of the experimental database using ϕ_L^2 as a function of *x*.

As can be seen in Fig. 5, all the data follow the same trend of increasing ϕ_L^2 with increasing of *x*. An exponential function can be used to fit the data. We decided to consider only the data of Schmidt and Friedel (1996) for developing the new expression for the estimation of ϕ_L^2. Indeed, as shown in Figs. 3.a and 3.b, this database includes a wide range of diameters, aspect ratios, mass flow rates and mass qualities. These data were represented, in Fig. 6, using $ln\phi_L^2$ versus *x*. The obtained results can be satisfactorily correlated ($R^2 = 0.9779$) with the following equation:

$$ln\phi_L^2 = \frac{127x}{1 + 29.45x - 20.48x^2} \quad (57)$$

By substituting the expression of ϕ_L^2 extracted from Eq. 57 into Eq. 9, the proposed model of *PR* reads as follows:

$$PR = \frac{\sigma(1-\sigma)(1-x)^2 G_1^2}{\rho_L} e^{\left(\frac{127x}{1+29.45x-20.48x^2}\right)} \tag{58}$$

Comparison of Eq. 58 with the model of Chisholm and Sutherland (1969) (Eqs. 10, 11, 12 and 13) demonstrates the simplicity of the proposed model, which correlates ϕ_L^2 only with *x*.

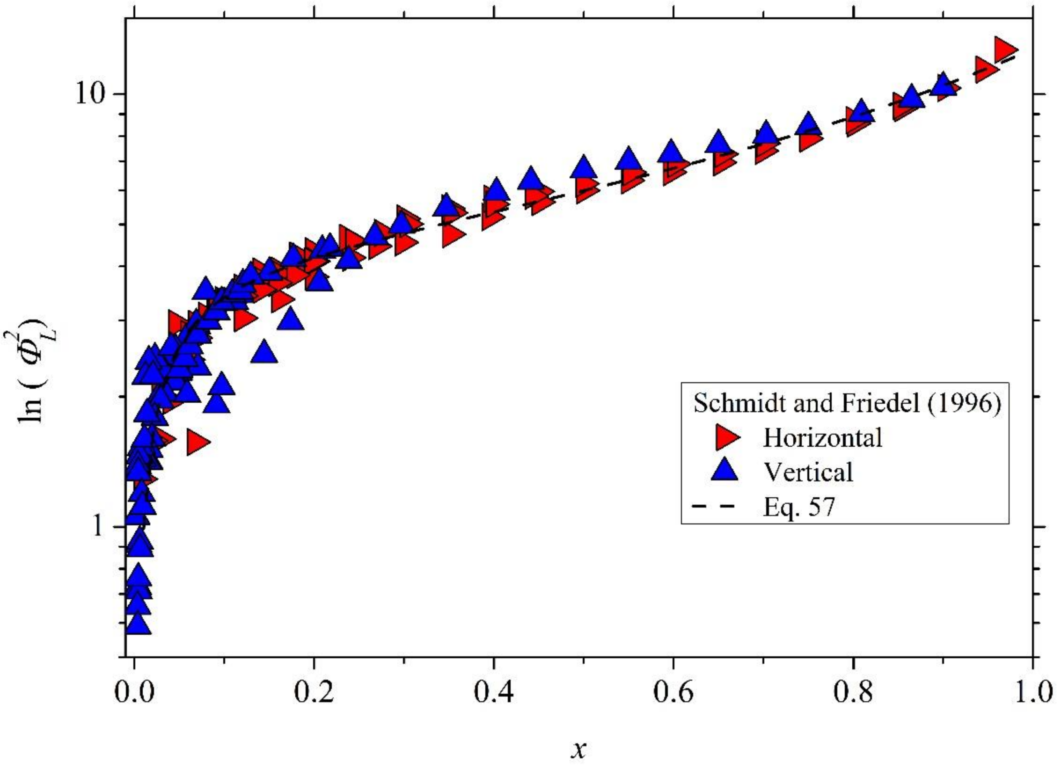

Fig. 6 Representation of the data of Schmidt and Friedel (1996) using ln ϕ_L^2 as a function of *x*.

4.3 Assessment of predictive capability of the proposed pressure recovery model

The performance of the proposed model was also evaluated. The calculated values of *AARE* are summarized in Table 5. With *AARE* = 36.73 %, the proposed model gives the best performance of the experimental database compared to the other models (see for instance, Table 4). The fact that the proposed model gives good prediction with data that was not used for its development confirms its validity. Indeed, this model gives good predictions for a majority of data source except those of Abdellal et al. (2005). For these data, only the model of Owen et al. (1993) and Wang et al. (2010)

gives better predictions. It appears also that the new model gives acceptable results (*AARE* < 50%) for a great majority of data points, as shown in Fig. 7.

Table 5 Performance evaluation of the proposed model against the experimental database. The results are given in %.

	Developed model
All Data (305 data)	**36.73***
Schmidt and Friedel (1996) (H) (75 data)	**25.60***
Schmidt and Friedel (1996) (V) (82 data)	**38.68***
Abdellal et al. (2005) (14 data)	97.86
Chalfi and Ghiassian (2008) (24 data)	**16.84***
Kourakos (2011) (53 data)	**22.86***
Salhi et al. (2011) (14 data)	57.14
Anupriya and Jayanti (2014) (43 data)	54.08

***Bold number means the predictions with *AARD* < 50%.**

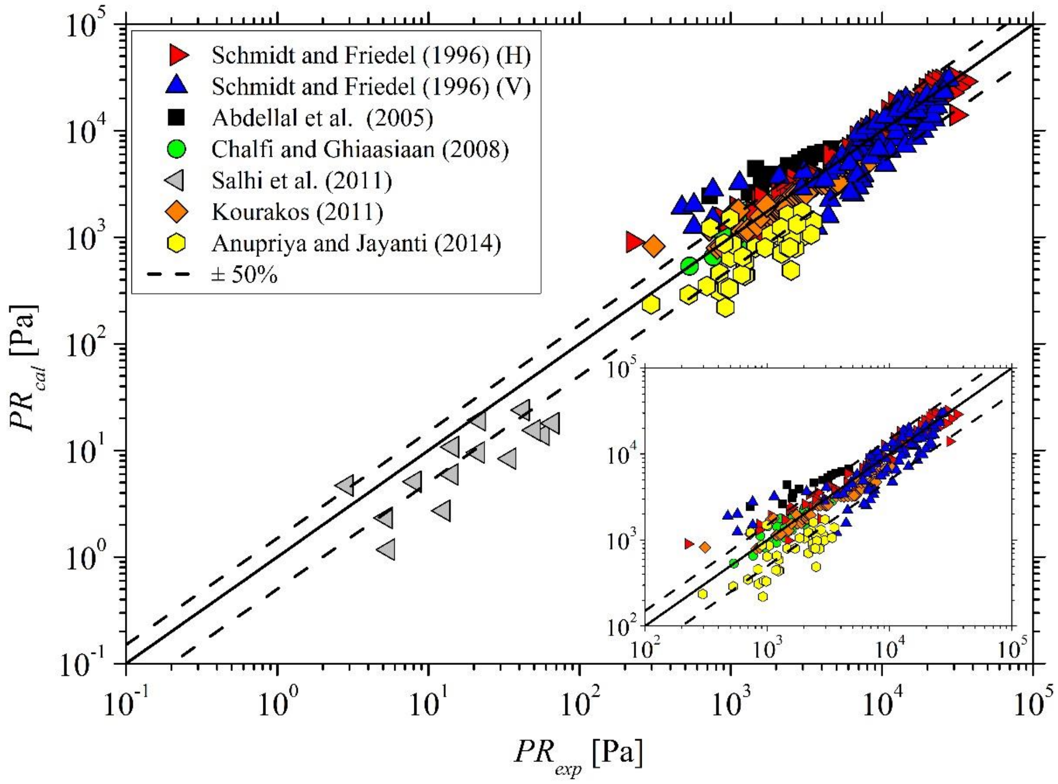

Fig. 7 Comparison of the PR predicted by the proposed model with the experimental database.

To highlight further the best performance of the proposed model comparatively to the models of Chisholm and Sutherland (1969) and Wadle (1989), we have represented the ratio of calculated to experimental *PR* given by the three models as a function of x in Fig. 8. We can see qualitatively that the proposed model is the model that give the larger number of points within the range of relative error of ± 50%.

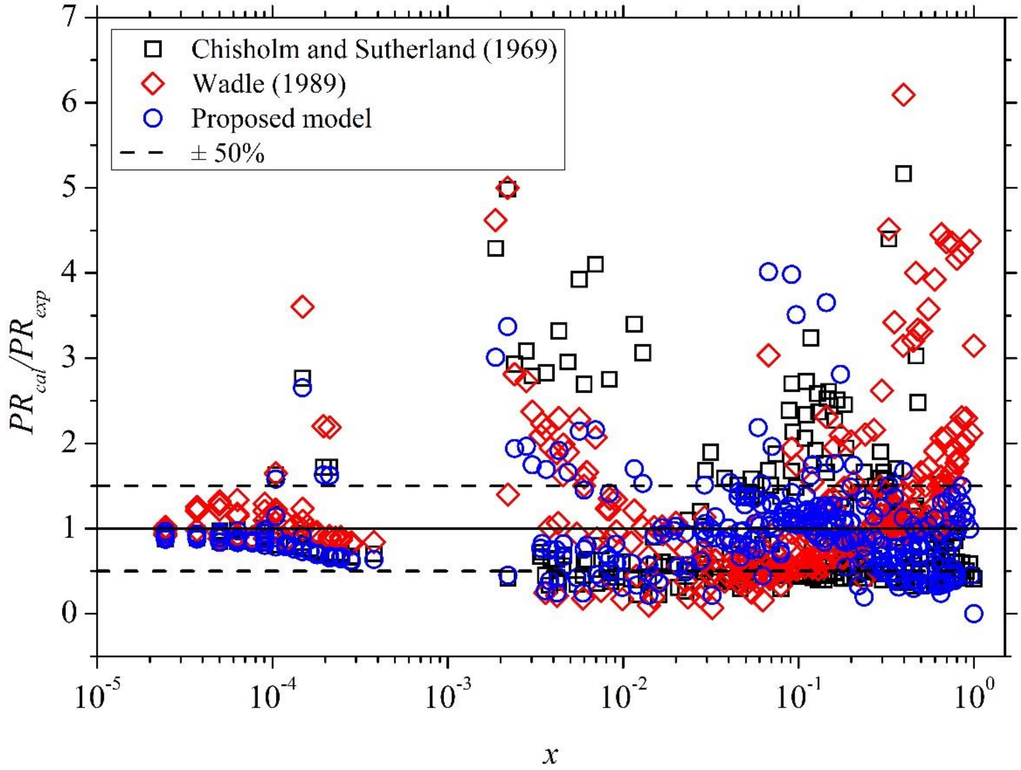

Fig. 8 Comparison of the predictive capability of the developed correlation with those of Chisholm and Sutherland (1969) and Wadle (1989).

The performances of the three models have also been evaluated quantitatively using the absolute average absolute error (*AAAE*), given by Eq. 59, as well as the percentage of data points predicted within the considered confidence range of ±50%. The results of both parameters are summarized in Table 6. The best results of both parameters obtained with the developed models confirm its superiority to the models of Chisholm and Sutherland (1969) and Wadle (1989)..

$$AAAE = \frac{1}{N}\sum_{I=1}^{N}\left|PR_{cal} - PR_{exp}\right| \times 100 \tag{59}$$

Table 6 Performance evaluation of the three best models against the experimental database using *AAAE* and % of points laying between ±50%.

	Developed model	Chisholm and Sutherland (1969)	Wadle (1989)
Rate of points laying between ±50% (%)	**74.75***	53.44	63.93
AAAE [Pa]	**170,666.37***	349,853.00	391,278.04

*** Bold numbers mean the best results.**

5 Conclusions

In this paper, a review of existing models to predict the pressure recovery generated by the passage of gas-liquid two-phase flow through the sudden expansion was firstly presented. The various existing models can be classified into three categories: mechanistic, analytical, and empirical. The mechanistic models, built based on the flow regimes, have the disadvantage of being limited only for a specific regime. The majority of the theoretical models have the disadvantage of being dependent to the void fraction values, which remains difficult to estimate. On the other hand, the models based on momentum conservation are more consistent with the involved physical phenomena compared to those built from mechanical energy conservation. The models based on Bernoulli's principle have the advantage to be simple, even though there is no consensus about the value of the empirical coefficient *K*. All empirical models have the inconvenience of being not validated with independent data.

The results given by nine existing models to predict the pressure recovery were compared with the collected database. It was found that the prediction accuracy of each model was not consistent on all data sources. Significant differences exist among the predictions of different models for each data source. Among the examined models, it was reported that the best predictions are obtained with the models of Chisholm and Sutherland (1969) and Wadle (1989).

A new model, based on the use of the two-phase multiplier based on the liquid phase and mass quality, was proposed to model the pressure recovery. The experimental results of Schmidt and Friedel (1996) were used to correlate these two parameters. The proposed model was validated with data that was not used in its development. The calculated *AARE, AAAE* as well the number of the data predicted within the ranges of ± 50% have demonstrated the reliability of the proposed model in the range of conditions and parameters of the experimental database used in this study.

In view of the encouraging results obtained with the present predictive model using air-water data, it would be interesting to compare its predictions with other data to extend its range of validity. Especially since the literature is not full of data on pressure recovery generated by this type of fitting. Therefore, further experimental investigations and simulations obtained with CFD would be needed to generate additional database.

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Declaration of competing interest

The authors have no competing interests to declare that are relevant to the content of this article.

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Figures

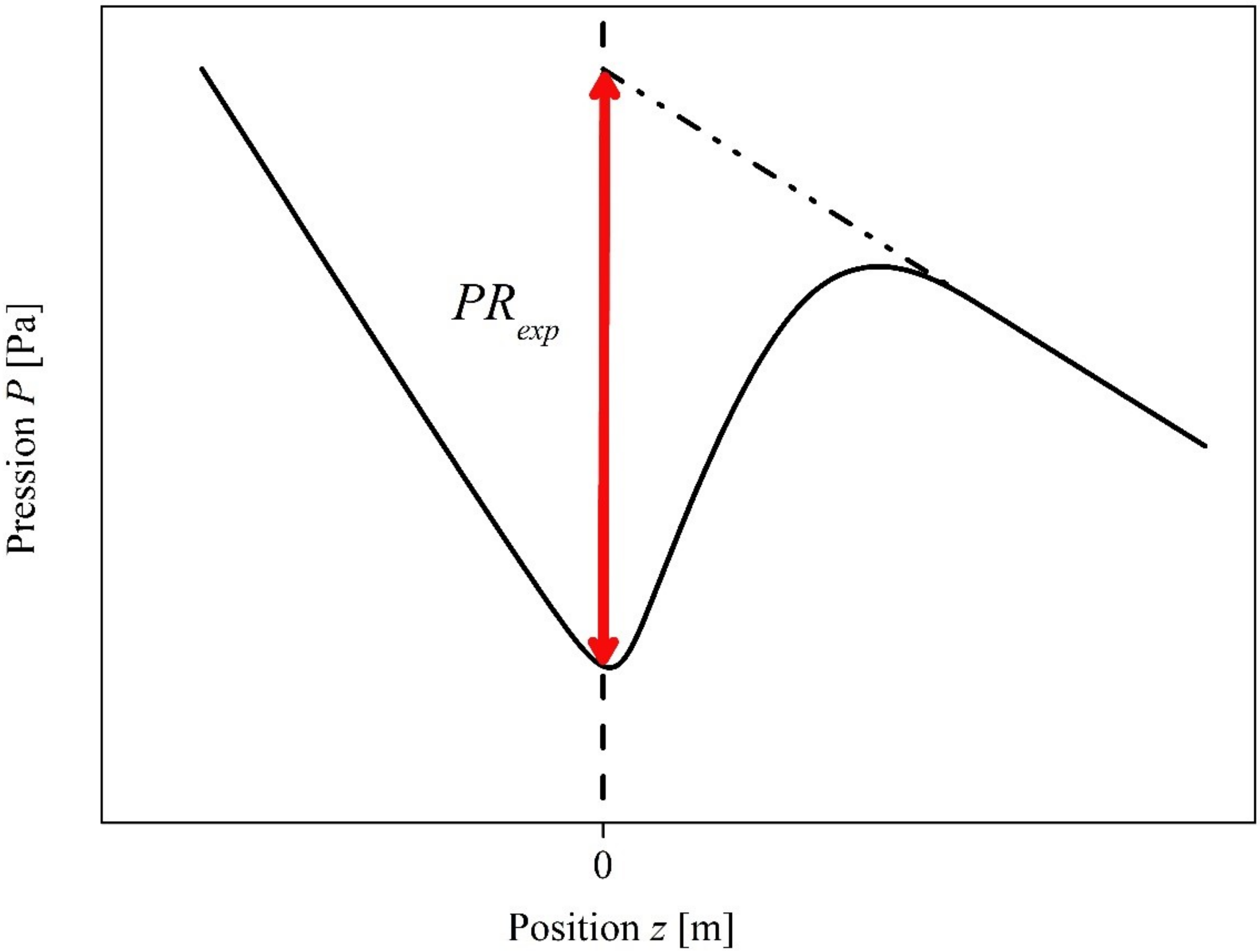

Figure 1

Variation of the static pressure before and after the sudden expansion.

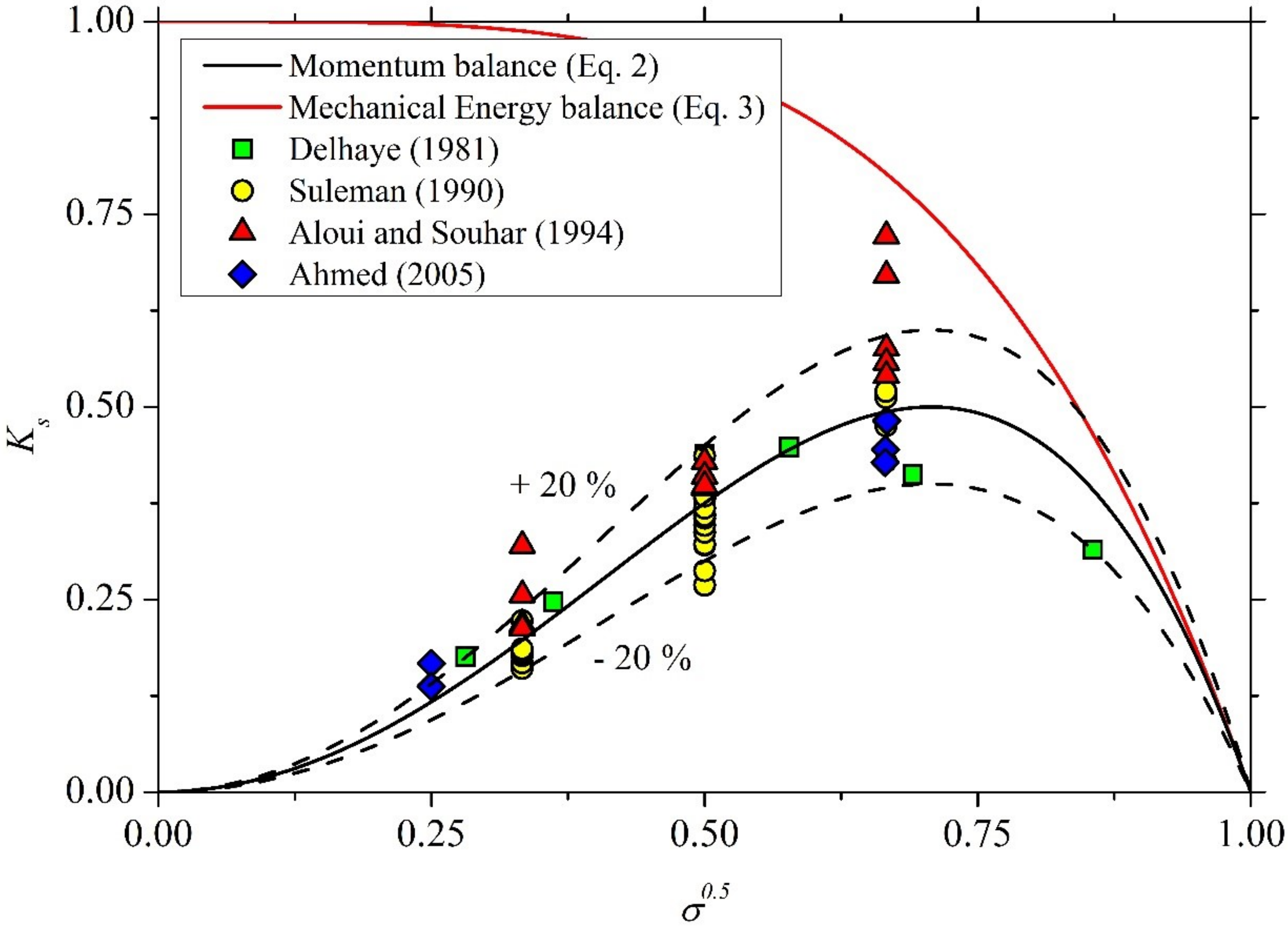

Figure 2

Normalized liquid single phase *PR* as a function of $\sigma^{0.5}$.

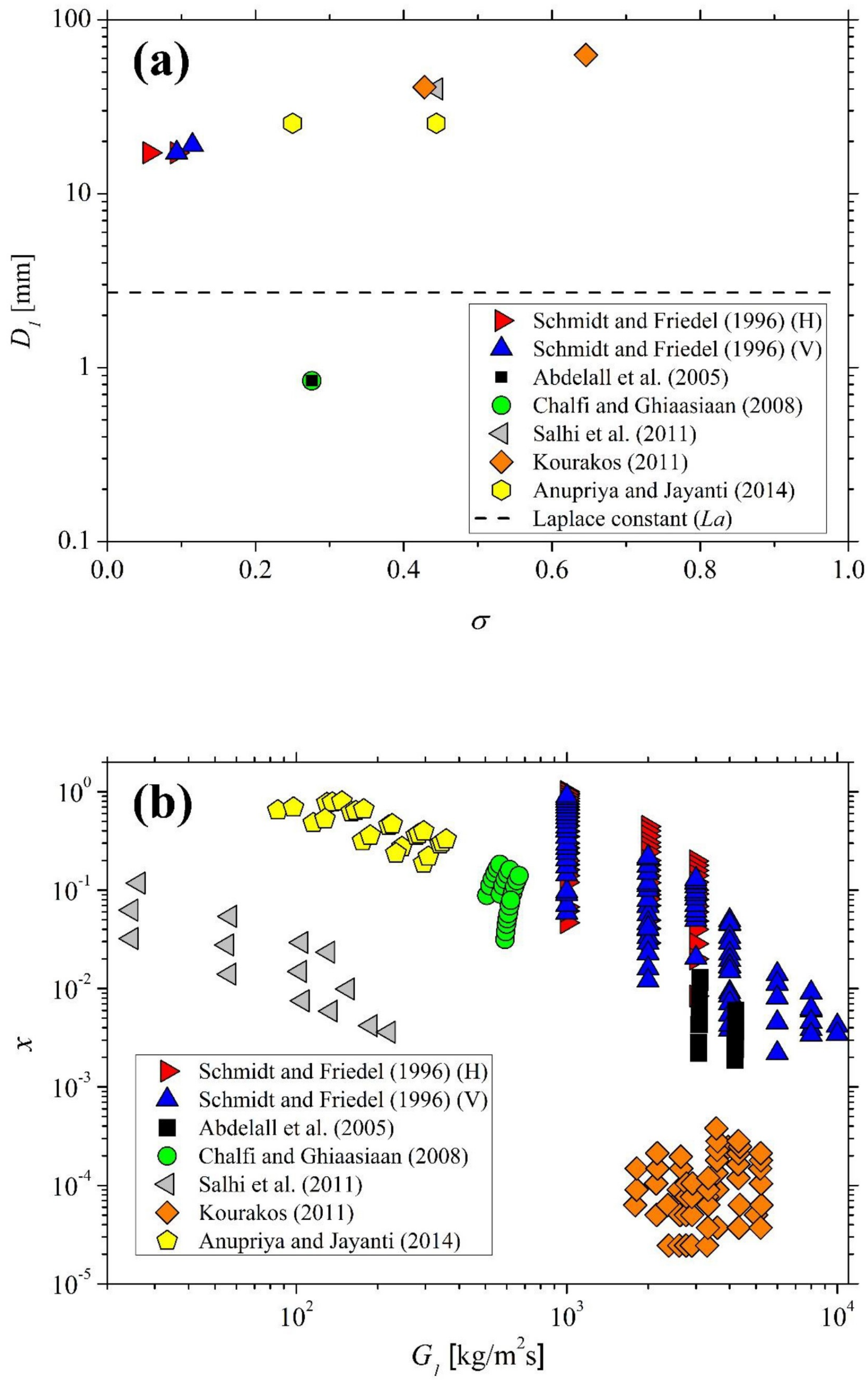

Figure 3

Representation of the database in terms of (a): aspect ratio (σ) and upstream pipe diameter (D_1); (b): upstream mass flux (G_1) and mass quality (x).

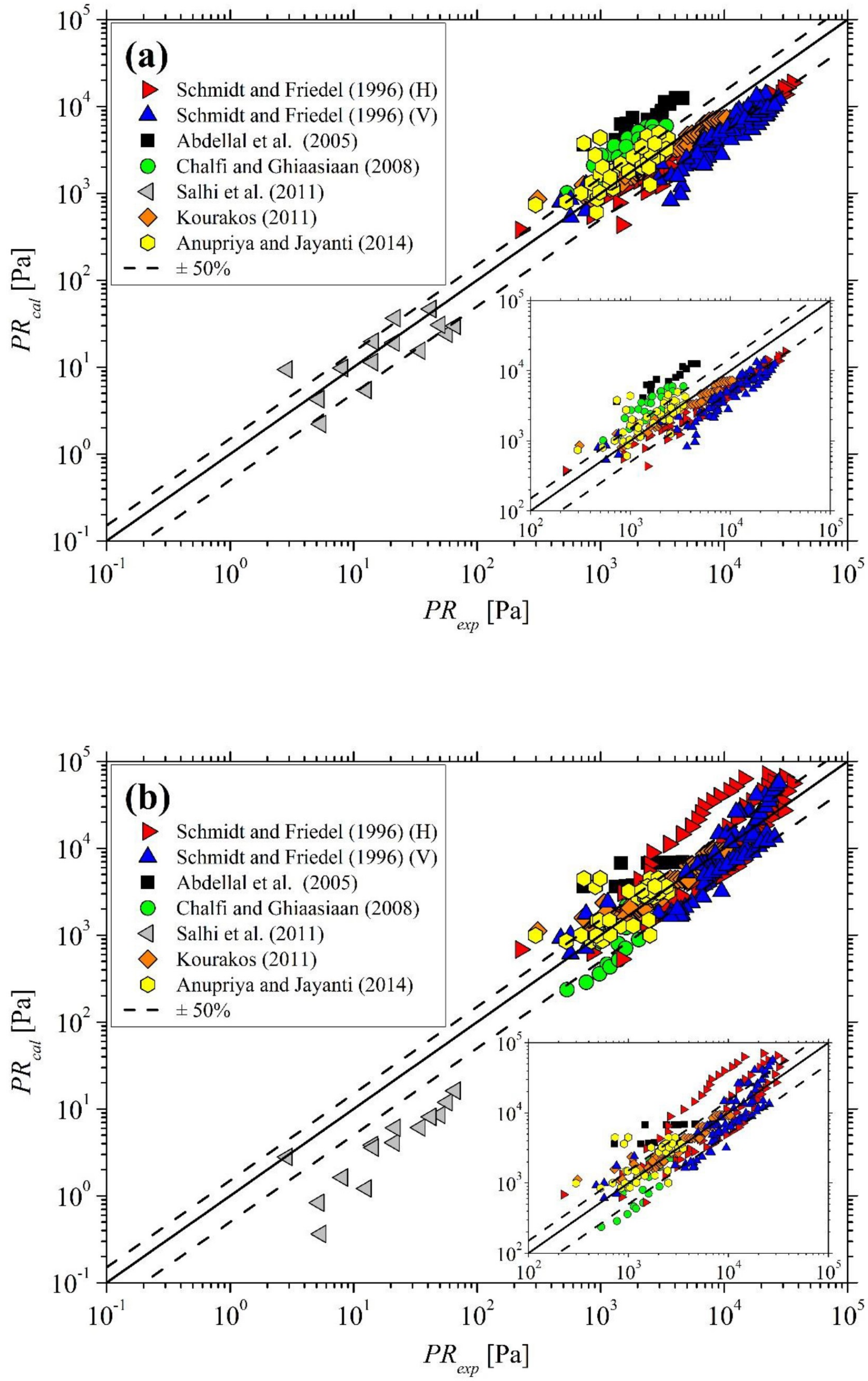

Figure 4

Comparison of the PR predicted by the models of (a) Chisholm and Sutherland (1969); (b) Wadle (1989) with the experimental database.

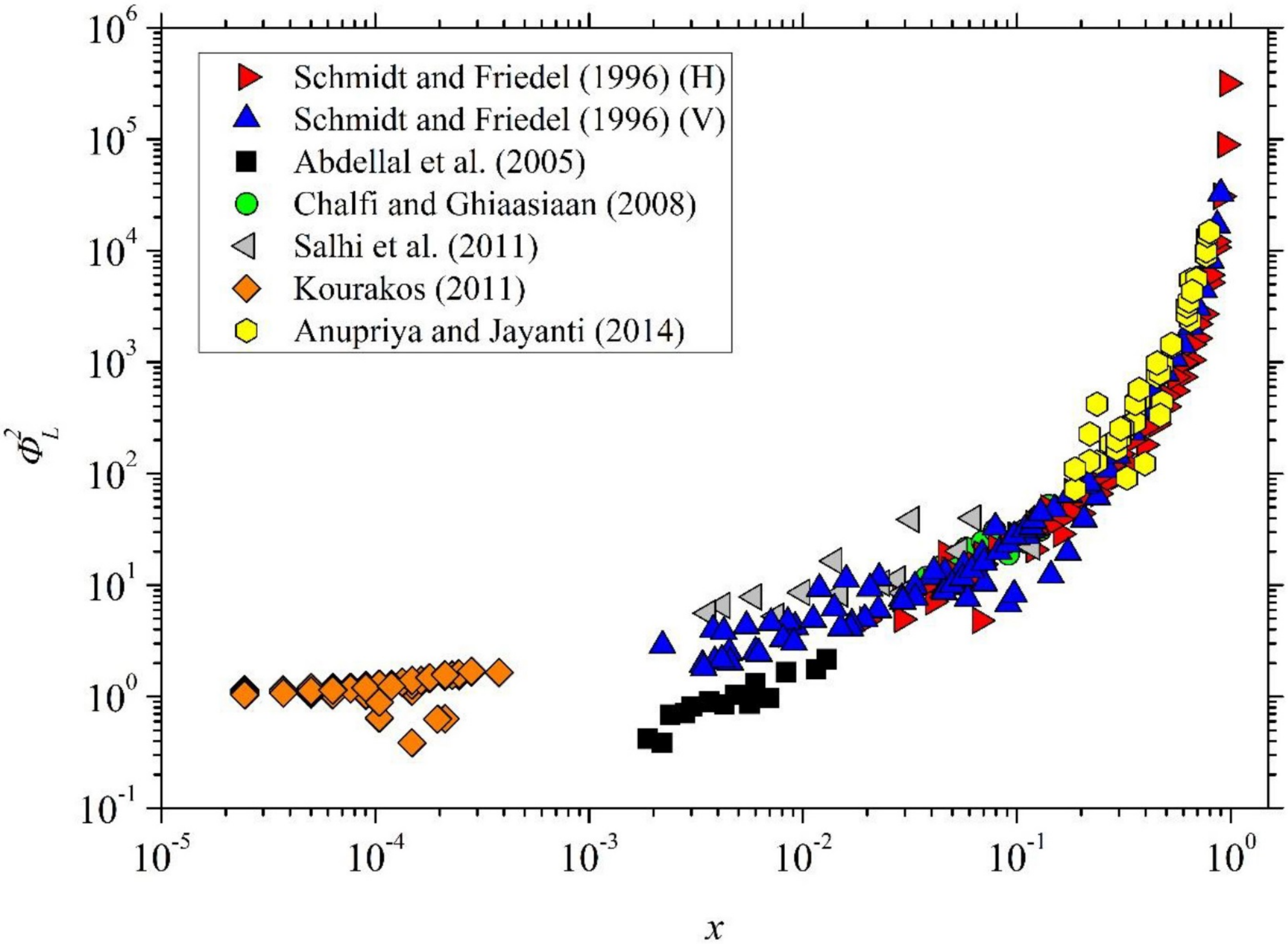

Fig. 5 Representation of the experimental database using ϕ_L^2 as a function of x.

Figure 5

See image above for figure legend.

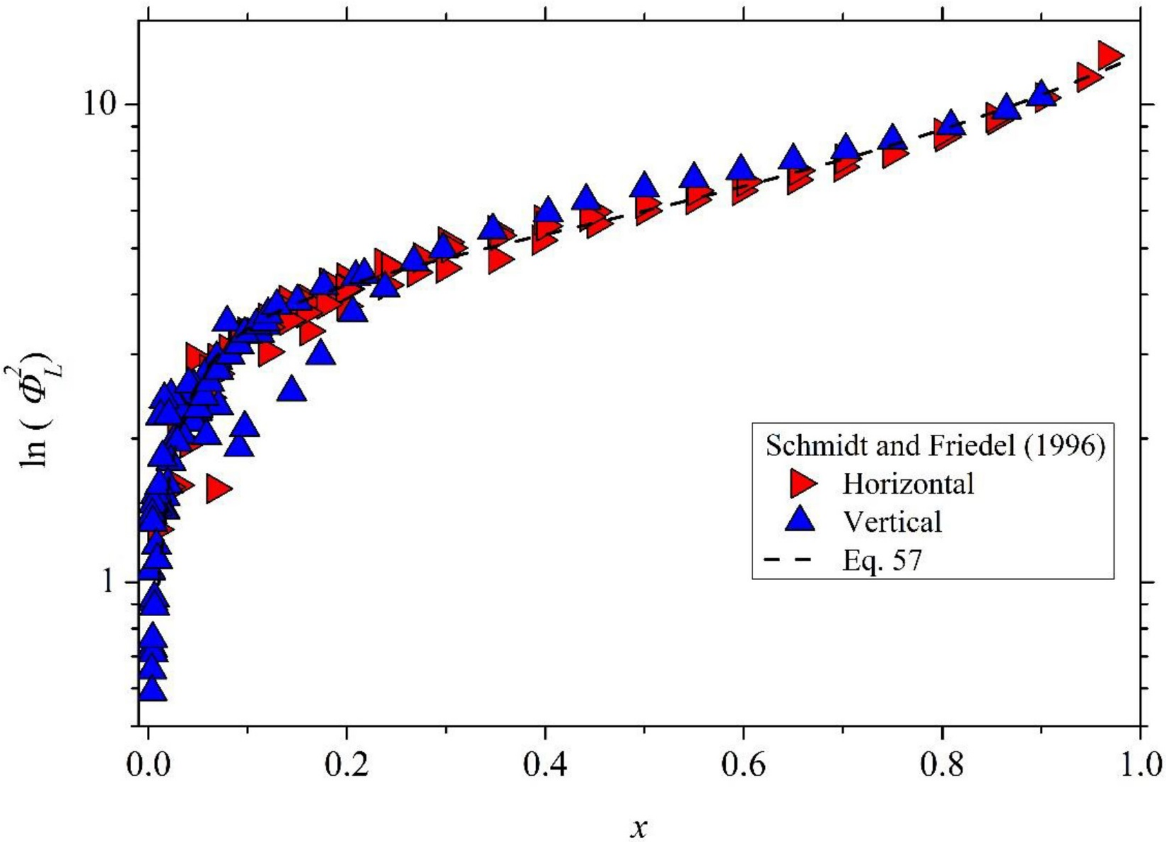

Fig. 6 Representation of the data of Schmidt and Friedel (1996) using ln ϕ_L^2 as a function of *x*.

Figure 6

See image above for figure legend.

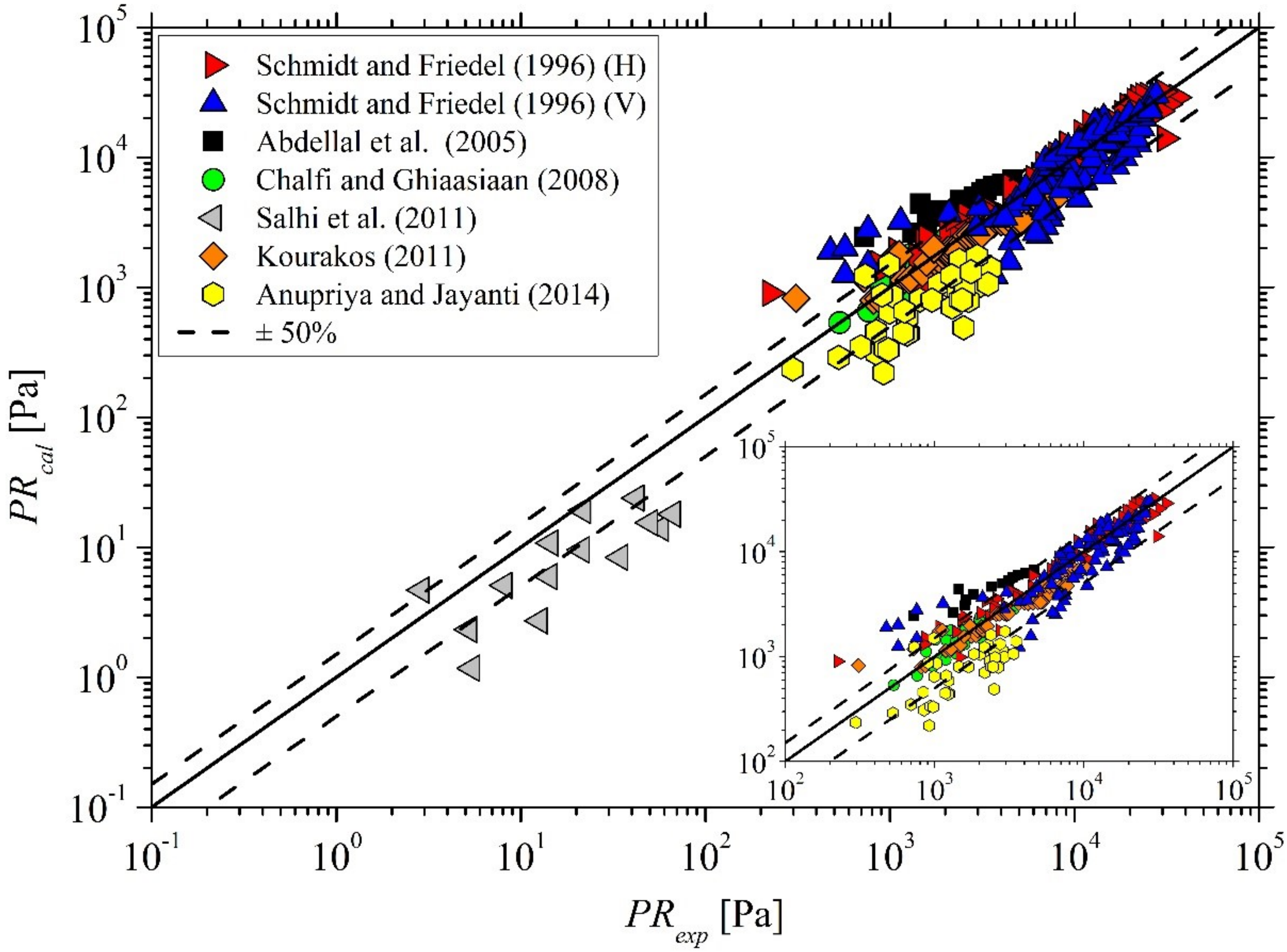

Figure 7

Comparison of the PR predicted by the proposed model with the experimental database.

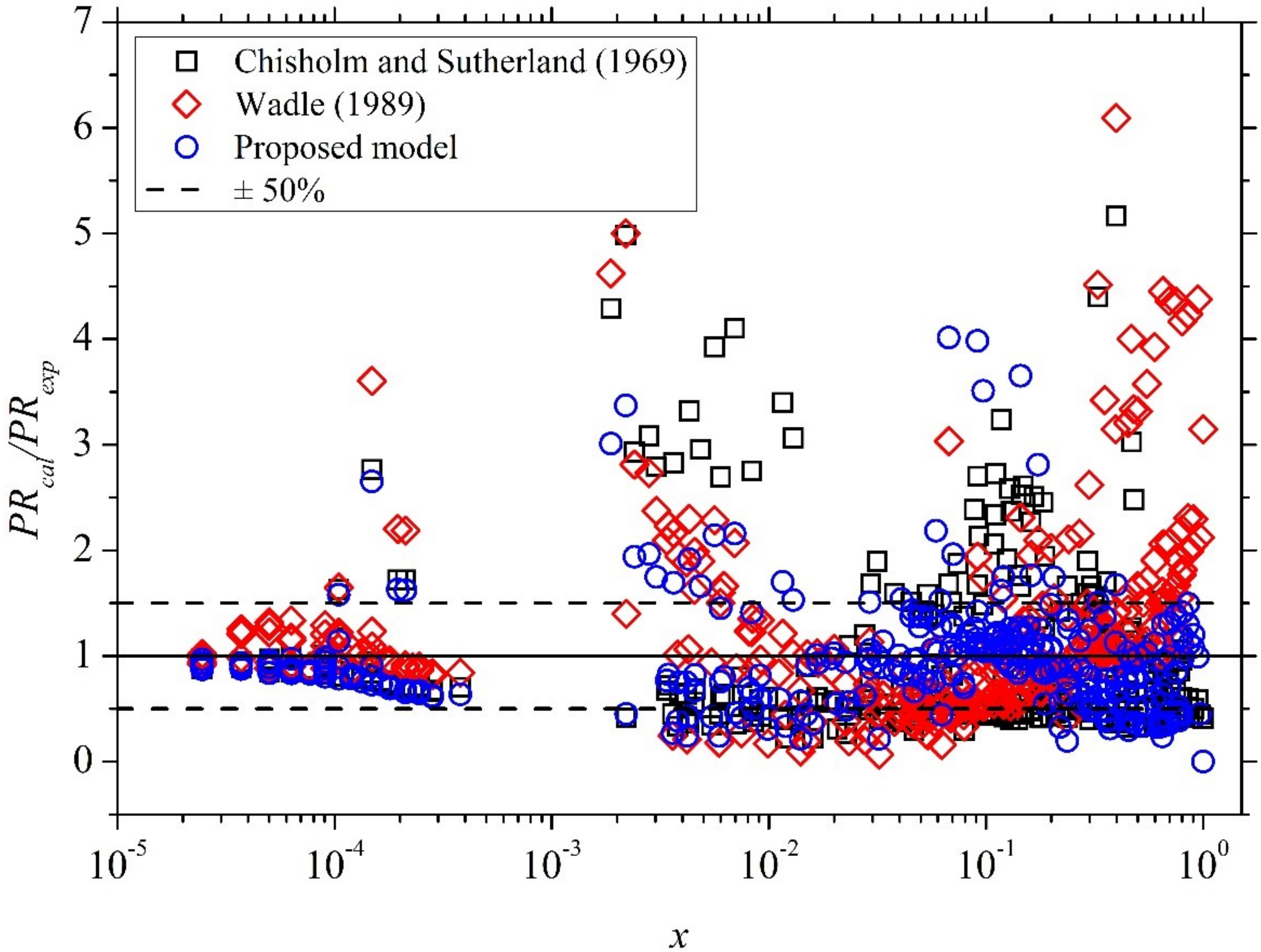

Figure 8

Comparison of the predictive capability of the developed correlation with those of Chisholm and Sutherland (1969) and Wadle (1989).