

Global Dynamical Analysis of a Boost Converter with a Constant Power Load and Actively Damped by a Series Loss-Free Resistor

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Abstract Two-stage cascaded DC-DC converters appear in many power electronics engineering applications. In this cascaded scheme, the first stage is called the source or upstream converter while the second one is called the load or downstream converter. If the downstream converter is controlled to maintain a tightly regulated output voltage on the load, it behaves as a constant power load (CPL) for the source converter. The overall dynamics of this arrangement tends to be unstable, so it can be stabilized by means of a suitable feedback, or instead in open loop operation, by using an additional resistor, which can be a loss free resistor (LFR), thus avoiding efficiency losses. This work deals with the dynamical analysis of an open loop DC-DC boost converter with a stabilizing resistor in series with the inductor and a constant power load by means of an averaged model. Fold and Hopf bifurcations explain existence and stability conditions of the two possible equilibrium points. Moreover, an appropriate design requires not only that the equilibrium point must be stable, but also that its basin of attraction must be large enough to include normal initial conditions. Then, we make an extended analysis taking into account the global dynamics of the system, mainly related to the saddle stable manifold or to an unstable large limit cycle, whenever it exists.

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1 Introduction

Two-stage cascaded DC-DC converters are widely used in various applications such as electric vehicles and ships, microgrids, more electric power aircrafts and more electric ships among others [1, 2, 3, 4, 5]. In these systems, the first stage is called the source or the upstream converter and the second stage is the load or the downstream converter. When the second stage is controlled to maintain a tightly regulated output voltage on the load, it behaves as a constant power load (CPL) for the source converter [3, 4, 5]. An electrical machine in an electric vehicle with tight velocity control and under constant torque operation will absorb constant power from the three phase DC-AC inverter hence resulting in a CPL to the upstream DC-DC converter [4]. With a constant power load, the output voltage decreases as the load current increases to maintain the constant power demand. In such a scenario, the load converter exhibits a negative incremental impedance to the source converter leading to a high risk of instability [1, 2, 4, 6, 7]. Since the overall dynamics of the converter with CPL tends to be unstable, it can be stabilized by means of a suitable feedback.

The stabilization and control of DC-DC converters with CPLs have been extensively studied, and various control strategies have been proposed to achieve stable performance [6, 7, 8, 9, 10, 11]. Instead of using feedback, to make stable a DC-DC converter with CPL in open loop operation, damping filters can be used [12]. Another way to stabilize these systems is by aggregating parasitic resistances to some key elements such as inductors and switching elements such as transistors and diodes [5]. Because this strategy would imply a reduction in the efficiency of the converter, a convenient alternative solution would be the use of a loss free resistor (LFR). Namely, an LFR is a two-port element with resistive behavior at its input port and transfers the absorbed power at the input port to its output port without losses. This LFR can be synthesized using a power converters by imposing the proportionality between the voltage and current at its input port [13, 14].

This work deals with the analysis of the dynamics of an open loop DC-DC boost converter with a stabilizing resistor in series with the inductor and a CPL. To simplify the analysis and to provide algebraic explicit mathematical results, an averaged model of the circuit is used with a reduced three dimensional parameter space. This model has two possible equilibrium points, a node and a saddle, whose existence and stability are linked to codimension one smooth fold and Hopf bifurcations, both emerging from a co-dimension-two Bogdanov-Takens bifurcation. Actually, the node equilibrium, which can be a focus or not, is the approximation of the desired limit cycle of the real switched system. Then, we get parameter constraint for a correct operation from this analysis. Moreover, an appropriate design requires not only a stable limit cycle with small amplitude, but also that its basin of attraction being wide enough to include normal initial conditions. Then, we make an extended analysis taking into account the global dynamics of the system, mainly related to the saddle stable manifold or to an unstable large limit cycle (ULLC) whenever it exists. The boundaries for existence in the parameter space of this unstable large amplitude cycle are those corresponding to Hopf bifurcation and a global homoclinic bifurcation, which also emerges from the Bogdanov-Takens bifurcation.

An averaged model of a bidirectional boost DC-DC converter with constant power load (CPL) is introduced in Section 2 making use of normalized variables and parameters. In Section 3, different limit sets and related bifurcations are justified. The accessibility of the desired equilibrium point under reasonable initial conditions is extensively studied in Section 4. Finally, in the last section the main conclusions are summarized.

2 Circuit: diagram, expressions and normalization

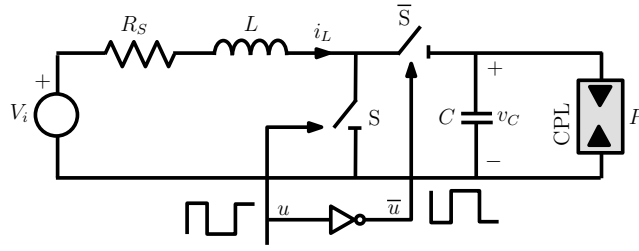


Fig. 1 Circuit schematic diagram of a bidirectional boost converter loaded by a CPL.

A simplified scheme of the boost converter is shown in Fig. 1. Let τ be the physical time and v_C and i_L the capacitor voltage and the inductor current, which are the state variables of the system. Physical parameters are C and L , that is the capacitance and inductance of corresponding elements, R_S is the resistance of the LFR, P is the constant power load and V_i is the main voltage source. The following differential equations are obtained by applying standard Kirchhoff laws to the circuit shown in Fig.1.

$$\begin{aligned} C \frac{dv_C}{d\tau} &= -\frac{P}{v_C} + (1-u)i_L \\ L \frac{di_L}{d\tau} &= -(1-u)v_C - R_S i_L + V_i \end{aligned} \quad (1)$$

The two switches S and $\bar{S}$ are activated in a complementary way, that is $\bar{u} = 1 - u$. Then if $u = 1$, S is ON (closed) and $\bar{S}$ is OFF (open). The opposite action is done when $u = 0$. Concerning the driving signal applied to the switches, T_S and d are its period and duty ratio (fraction for which $u = 1$), respectively. From this set of variables and parameters, the following normalized variables are defined,

$$t = \omega_0 \tau, \quad x = \frac{v_C}{V_i}, \quad y = \frac{i_L Z_0}{V_i}, \quad (2)$$

where t, x, y are the normalized time and state variables and

$$\omega_0 = \sqrt{\frac{1}{LC}}, \quad Z_0 = \sqrt{\frac{L}{C}}.$$

Besides, normalized period, power and equivalent resistance parameters are

$$T = \omega_0 T_S, \quad p = \frac{P Z_0}{V_i^2}, \quad r = \frac{R_S}{Z_0}. \quad (3)$$

Finally, the averaged vector field, that is the normalized differential averaged equations for state variables (x, y) with parameters (p, r, d) can be summarized as

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}) \quad \text{with} \quad \begin{cases} F_x(x, y) = \dot{x} = -\frac{p}{x} + (1-d)y \\ F_y(x, y) = \dot{y} = -(1-d)x - ry + 1 \end{cases}, \quad (4)$$

where $\mathbf{x} = (x, y)^\top$ is the state vector. Notice that the binary variable $u \in \{0, 1\}$ in the switched model has been substituted by the duty ratio d , which is valid in the open interval $(0, 1)$. In the averaged approach, a continuous time and vector field are assumed for the evolution of the system, such that the state variables are in fact their averaged values within a switching cycle, thus replacing the exact switched model by the simplified averaged one. Note that the small amplitude desired limit cycle corresponding to the switched model becomes an equilibrium point for the averaged model. In Fig. 2, several limit cycles are represented in the state plane with parameters indicated in the own figure. Note that cycles grow as T increases around the corresponding averaged value represented by a red point. It is also worth to emphasize that relevant properties of the limit cycles corresponding to the switched model such as stability suffer minimal variations regarding those of the equilibrium points of the averaged approach analyzed below.

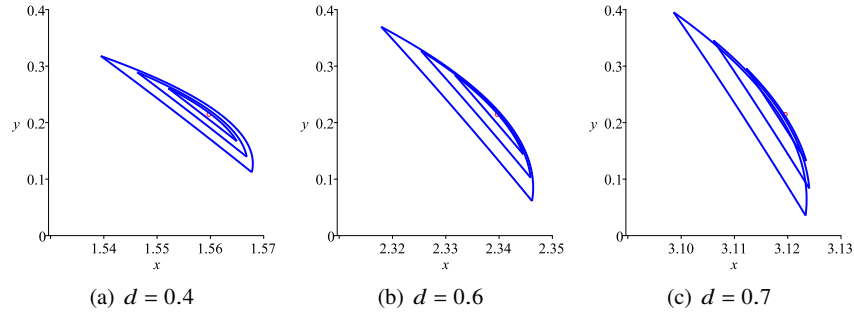


Fig. 2 Limit cycles (periodic orbits) with parameters $r = 0.3$, $p = 0.2$, $T \in \{0.25, 0.4, 0.55\}$ and d in the caption. Red points correspond to equilibrium in the averaging approach.

3 Stability and bifurcation analysis

3.1 Equilibrium points and associated bifurcations

Two equilibrium points exist for the averaged system (4), whenever $4pr < 1$. These will be called here $\mathbf{x}_N = (x_N, y_N)^\top$ and $\mathbf{x}_S = (x_S, y_S)^\top$ and their expressions are straightforward from $\mathbf{F}(\mathbf{x}) = \mathbf{0}$.

$$\mathbf{x}_N = \begin{pmatrix} \frac{1 + \sqrt{1 - 4rp}}{2(1 - d)} \\ \frac{1 - \sqrt{1 - 4rp}}{2r} \end{pmatrix}, \quad \mathbf{x}_S = \begin{pmatrix} \frac{1 - \sqrt{1 - 4rp}}{2(1 - d)} \\ \frac{1 + \sqrt{1 - 4rp}}{2r} \end{pmatrix}. \quad (5)$$

The eigenvalues pairs λ_N and λ_S corresponding to $\mathbf{x}_N$ and $\mathbf{x}_S$ respectively, are obtained by means of the Jacobian matrix of the vector field $\mathbf{F}(\mathbf{x})$, that is the equivalent linear system of (4) around equilibrium (5)

$$\lambda_N = \frac{p}{2x_N^2} - \frac{r}{2} \pm \sqrt{W_N} \quad \text{where} \quad W_N = \left(\frac{p}{2x_N^2} + \frac{r}{2} \right)^2 - (1 - d)^2, \quad (6)$$

$$\lambda_S = \frac{p}{2x_S^2} - \frac{r}{2} \pm \sqrt{W_S} \quad \text{where} \quad W_S = \left(\frac{p}{2x_S^2} + \frac{r}{2} \right)^2 - (1 - d)^2. \quad (7)$$

Note that under the critical parameter condition $4rp = 1$, the two equilibrium collapse at the common fold point $\mathbf{x}_F$ with eigenvalues λ_{F1} and λ_{F2} ,

$$\mathbf{x}_F = \left(\frac{1}{2(1 - d)}, \frac{1}{2r} \right)^\top, \quad \lambda_{F1} = 0, \quad \lambda_{F2} = \frac{(1 - d)^2}{r} - r = \delta \left(2 + \frac{\delta}{r} \right), \quad (8)$$

where $\delta = 1 - r - d$. These results can be summarized in the following Remark.

Remark 1 In system (4), for the critical parameter condition $p = p_F = (4r)^{-1}$, the two equilibrium collapse, being null one of the common eigenvalues thus resulting in a smooth fold, also called saddle-node, codimension one bifurcation.

Note that for this condition, the nonzero eigenvalue $\lambda_{F2} > 0$ has the same sign than δ . In the boundary, $\delta = 0$, we have

Remark 2 In system (4), for the two critical parameter conditions $p = p_F = (4r)^{-1}$ and $d = 1 - r$, in addition to the equilibrium extinction, their two common eigenvalues become null, thus resulting a smooth codimension two Bogdanov-Takens bifurcation with two zero eigenvalues.

Moreover, a codimension three parameter point defined as $\mathcal{P}_1 \equiv \{r = 1, p = 1/4, d = 1\}$, such that the Bogdanov-Takens bifurcation is located at the extreme of

the duty ratio domain ($d = 1$), is associated to a relevant change in the bifurcation pattern. As a consequence the following remark results

Remark 3 If $r > 1$, the Hopf bifurcation cannot be given. This bifurcation only exists if $r < 1$, thus starting at the codimension two Bogdanov-Takens, according to Remark 2.

In order to check the evolution of the eigenvalues λ_N and λ_S for the two equilibrium $\mathbf{x}_N$ and $\mathbf{x}_S$, with parameters $r > 0, 0 < p < p_F = (4r)^{-1}$ and $d \in (0, 1)$, we evaluate W_N and W_S in (6) and (7) respectively, taking into account x_N and x_S in (5). Noticing that either W_N or W_S are expressed as a difference of two squared terms, both of them positive, they can only permute its sign when the difference of that two terms, called here Σ_N or Σ_S vanishes. These magnitudes are

$$\Sigma_N = \frac{(1-d)^2}{\sigma_N} + r - 2(1-d), \text{ where } \sigma_N = \frac{1 - 2rp + \sqrt{1-4rp}}{2p} \quad (9)$$

$$\Sigma_S = \frac{(1-d)^2}{\sigma_S} + r - 2(1-d), \text{ where } \sigma_S = \frac{1 - 2rp - \sqrt{1-4rp}}{2p} \quad (10)$$

We check that $\Sigma_S > 0$ in our parameter domain, so $\mathbf{x}_S$ is always a saddle.

However, concerning $\mathbf{x}_N$, $\Sigma_N = 0$ if $d = d_W = 1 - \sigma_N \left(1 \pm \sqrt{1 - r/\sigma_N}\right)$. Besides, $\Sigma_N = 0$ if $d = 1 - r/2$ with $p = 0$ and also, $\Sigma_N = 0$ if $d = 0$ with $p = p_0 = (2-r)(1+2r-r^2)^{-2}$. Therefore, if $r < 1$, the positive branch is valid in the range $d_W \in (0, 1-r)$ corresponding to p from p_0 to p_F and the negative branch is valid in the range $d_W \in (1-r, 1-r/2)$ corresponding to p from p_F to 0. If $1 < r < 2$, the positive branch for d_W is not valid and the negative one is given for $d_W \in (0, 1-r/2)$ associated to p from p_0 to 0. If $r > 2$, no branch applies for d_W , so the node is always non spiral and stable.

Crossing the positive d_W branch implies a transition for the equilibrium node $\mathbf{x}_N$ between spiral or focus and non spiral behavior. The positive branch stands for unstable node and the negative one for the stable node.

The next and relevant question concerns stability. The spiral node $\mathbf{x}_N$, with an associated frequency $\sqrt{-W_N}$, becomes unstable when the real part of λ_N vanishes, which occurs at the critical value

$$d_H = 1 - \sqrt{r\sigma_N} = 1 - \frac{1 + \sqrt{1-4rp}}{2} \sqrt{\frac{r}{p}}, \quad (11)$$

where σ_N is defined in (9). Actually, this is a smooth subcritical Hopf bifurcation, at which an ULLC is born. More details about the existence and significance of this large limit cycle, which coexists with the stable spiral equilibrium $\mathbf{x}_N$, will be given below.

In Fig. 3, the continuous black and red lines stands for the smooth fold (emergence of the saddle and stable node pairs) and the subcritical Hopf (stability boundary of the spiral node) bifurcations respectively. The continuous gray line is associated to

the saddle and unstable node pair appearance. The sparse dotted line corresponds to the transition between spiral and non spiral behavior of the node.

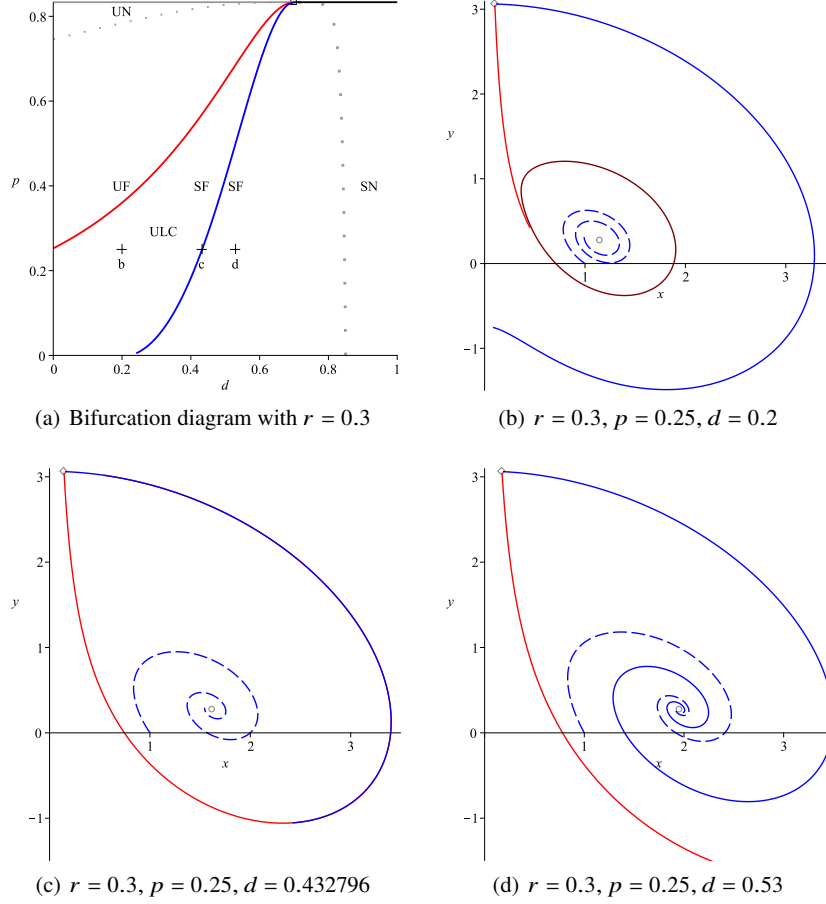


Fig. 3 (a) Diagram using fixed parameter $r = 0.3$ in the complementary (d, p) parameter plane, including bifurcation lines: fold (black), Hopf (red), homoclinic (blue), and also the spiral to non spiral transition (dotted gray line). The possible equilibrium, SF or UF and SN or UN (stable or unstable focus and stable or unstable node) are indicated in their existence regions. Besides, the ULLC exists in the parameter region between Hopf and homoclinic bifurcation. Notice also that the saddle equilibrium exists for $p < (4r)^{-1}$ and the codimension two bifurcation is given at $p = (4r)^{-1}$, $d = 1 - r$ (squared point), from which the bifurcating lines emerge. Panels (b-d) stand for dynamics in the state plane with parameters in the caption (see cross dots in panel (a) standing for these values), including relevant invariant sets: the stable spiral node and the saddle equilibrium (circle and diamond points), the stable and unstable saddle manifolds (red and blue lines), and the orbit from $\mathbf{x}_1 = (1, 0)^T$ state (dashed blue line). Case (b) includes also the ULLC (maroon closed line) and case (c) close to the homoclinic bifurcation for which the stable and unstable saddle manifolds merge together.

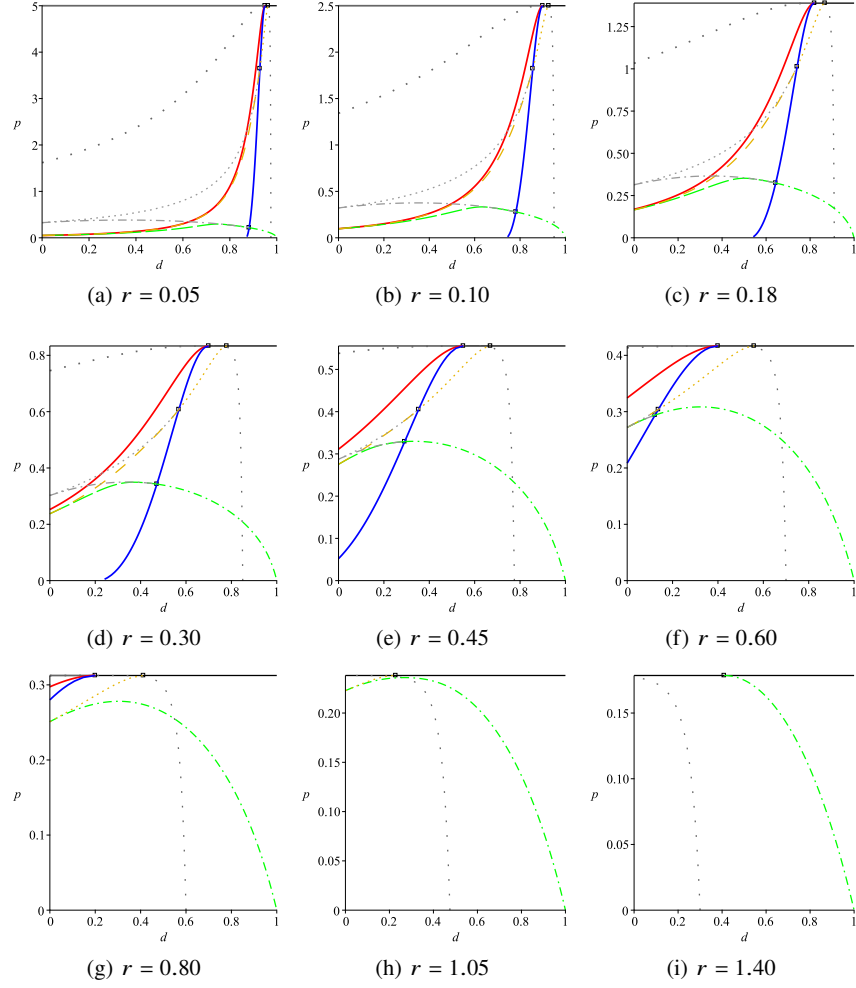


Fig. 4 Diagrams in (d, p) parameter plane with parameter r fixed given in the caption.

3.2 Homoclinic bifurcation and existence of the ULLC

In order to delimit the parameter subset for which the ULLC exists, we must pay attention to the homoclinic bifurcation emerging from the smooth Bogdanov-Takens bifurcation of the equilibrium examined above. This is a global bifurcation, for which the stable saddle manifold (SSM) is superimposed to the unstable saddle manifold (USM) of the same saddle equilibrium point. From a numerical continuation procedure implemented on Maple platform, we have determined the set of points for

this bifurcation, fixing one parameter and solving in the complementary plane. In Fig. 3(a), made using fixed parameter $r = 0.3$, the continuous blue line stands for the homoclinic bifurcation set in the complementary parameter plane. Figure 3(c) is a state diagram with a parameter selection near the homoclinic bifurcation. Note that in the parameter region between the Hopf (red) and homoclinic (blue) lines, the node is a spiral equilibrium (focus), and the unstable saddle manifold cannot end at that point as it avoids crossing the stable saddle manifold (SSM). Thus forces the existence of an extra invariant set in the form of an ULLC splitting the state space in two regions. Orbits starting inside the ULLC flow to the stable focus (SF) equilibrium, see Fig. 3(b). Conversely, with parameters such in Fig. 3(d), the SSM itself is the separatrix of the desired orbits flowing to the SF or the undesired ones collapsing as approaching to $x = 0$.

Taking into account, according to Remark 3 that the codimension two Bogdanov-Takens bifurcation only exists for parameter $r < 1$, with values $p = (4r)^{-1}$, $d = 1 - r$, the homoclinic bifurcation can only be given with the same constraint $r < 1$. In such a case, the basin of attraction of the stable focus is an ULLC in the parameter region between Hopf and homoclinic bifurcations. Outside this parameter region if $r < 1$, or in any case if $r > 1$, the stable saddle manifold defines the attraction basin of the stable (spiral or not) node, whenever it exists.

4 Parameter sets associated to the boundary of the basin of attraction of the stable equilibrium

This section deals with the feasibility of reaching the equilibrium point whenever it exists and it is stable, thus concerning the boundary of its basin of attraction. We will concentrate on two conditions. As a stricter one, the inclusion in that basin of the state $\mathbf{x}_1 = (1, 0)^\top$ as a likely starting point of the dynamics, is considered. As a second more relaxed condition, we impose that at least one point of the abscissa, that is $\mathbf{x} = (x, 0)^\top$, belongs to the basin. In order to find critical parameter values for which these conditions apply, two excluding cases are possible. One refers to the ULLC, and the stable saddle manifold (SSM) is involved in the second case. As it is well known these two invariant sets can be boundaries of an attraction basin. If the ULLC exists, this set will determine the critical condition, otherwise the SSM will do. Let us define some codimension three bifurcation points concerning the basin of attraction of the stable equilibrium.

A first (defined above) codimension three bifurcation point corresponds to the parameter set $\mathcal{P}_1 \equiv \{r = r_1 = 1, p = 1/4, d = 1\}$. For this set, the Bogdanov-Takens bifurcation is done at one extreme of the parameter d domain. Therefore, Hopf and homoclinic bifurcation cannot exist if $r > r_1$.

A second codimension three point is obtained numerically by imposing the following constraints: homoclinic bifurcation with tangency at $\mathbf{x}_1$ at the extreme value $d = 0$. Approximately, this is $\mathcal{P}_2 \equiv \{r = r_2 \simeq 0.695540, p \simeq 0.262139, d = 0\}$. If $r > r_2$, the state $\mathbf{x}_1$ cannot be attained by the ULLC, then the critical boundary

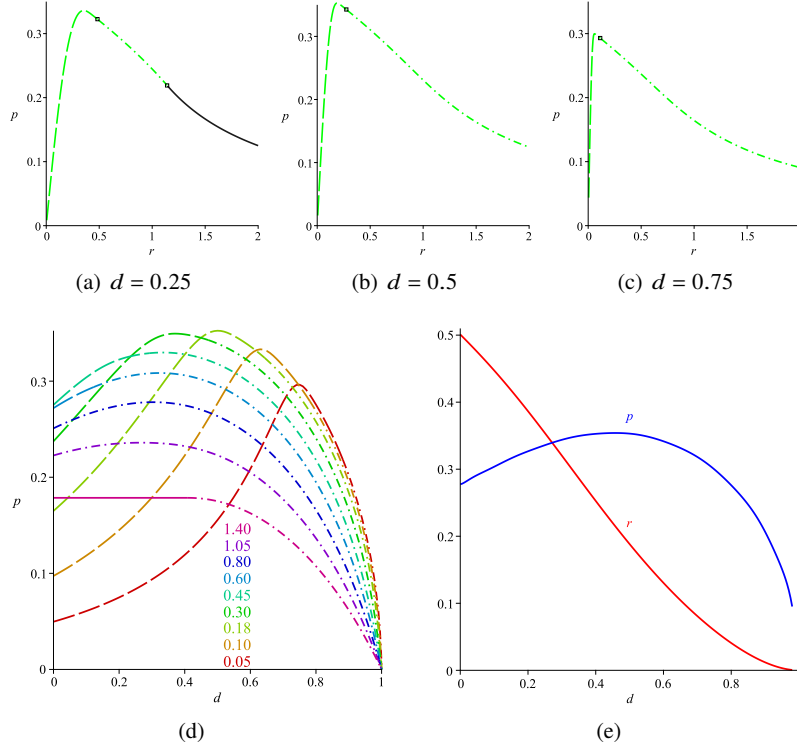


Fig. 5 Diagrams related to basin of attraction boundary at $\mathbf{x}_1 = (1, 0)^\top$. Panels (a-c) are for $p(r)$ relation with fixed d shown in the caption. Dashed, dot-dashed and black lines refer to ULLC, SSM and no bound cases respectively. In panel (d), $p(d)$ is represented for several values of parameter r from 0.05 to 1.4 (see legend). In panel (e), $r(d)$ parameter relation is such that gives a maximum value of p that optimizes the boundary and in the same diagram $p(d)$ indicates that maximum value.

condition at $\mathbf{x}_1$ is limited exclusively by the SSM. If $r < r_2$, the homoclinic bifurcation with tangency at $\mathbf{x}_1$ is given at a codimension two point with $d = d_C > 0$. If $d < d_C$, the ULLC determines the critical boundary of the basin at $\mathbf{x}_1$. Otherwise if $d > d_C$, the SSM must be used instead. Summarizing, if the ULLC exists, this set will determine the boundary of the basin of attraction, otherwise only the SSM define this boundary.

4.1 Boundary of the basin of attraction includes state $\mathbf{x}_1$

We will first determine the parameter regions for which orbits starting at the state $\mathbf{x}_1 = (1, 0)^\top$ flows to the stable equilibrium. In order to determine the frontier of this region, two scenarios are possible. If $r < r_2 \simeq 0.695540$ (parameter r at $\mathcal{P}_2$) and

inside the parameter region between the Hopf and the homoclinic bifurcation, the frontier line should be determined by forcing the ULLC passing through $\mathbf{x}_1$. This case is represented in Fig. 4 with long dashed green lines in several panels for fixed values of parameter r . Otherwise, the frontier line, if exists, is determined by forcing the SSM passing through $\mathbf{x}_1$. Dot-dash green lines, also in Fig. 4, correspond to this second occurrence.

In order to give a more global view of the parameter region with appropriate starting conditions, some alternative representations are given in Fig. 5. In panels (a-c) of this figure, the critical $p(r)$ condition such that $\mathbf{x}_1$ belongs to the boundary is plotted for some fixed values of parameter d . Herein, dashed and dot dashed green lines stand for ULLC and SSM cases respectively and the solid black line represents the fold existence limit, because in these case, the state $\mathbf{x}_1$ is always inside the basin of attraction. The connection between those cases is represented by squared points. In panel (d), the boundary (green lines in Fig. 4) are plotted all together. From these diagrams, we can appreciate that certain parameter combinations are more suitable in order to optimize the basin of attraction of equilibrium, thus avoiding undesired dynamics if $\mathbf{x}_1$ is assumed as the starting state. In the panel (e), we represent the optimal value of r (red line) in front of the duty ratio d , thus providing a maximum available value of parameter p with safe conditions. The blue line in this diagram indicated this maximum value of p . Note that this critical situation corresponds to the ULLC as the basin of attraction including the state $\mathbf{x}_1$.

4.2 Boundary of the basin of attraction tangent to x -axis

As a second more relaxed condition, we impose that at least one point of the abscissa, that is $\mathbf{x} = (x, 0)^\top$ belongs to the basin. Actually, $\mathbf{x}$ is tangent to the abscissa if it is an isolated point. If that condition were not satisfied, dynamics will not flow to the stable equilibrium point with a zero initial inductor current condition for any value of the initial capacitor voltage. As in the above cases for boundary at $\mathbf{x}_1$, this condition should be applied either to the ULLC or to the SSM. Orange lines in Fig. 4 stand for these more relaxed cases, dashed for ULLC and dotted for SSM cases.

5 Overview

The dynamics of a DC-DC boost open loop controlled converter, with a CPL and an actively damped loss free resistor, has been analyzed by means of nonlinear tools in a averaged model, that is a smooth approximation of the exact switched model. Invariant sets such an equilibrium, saddle manifolds and unstable large limit cycles are found, and some related transitions, as fold, Hopf and homoclinic bifurcations are elucidated. Parameter optimization has been studied focusing on the accessibility of the desired periodic orbit (equilibrium in the averaging approach), by considering the

inclusion or not of reasonable starting condition in the equilibrium basin of attraction. Future research will deal with changes due to the discontinuous conduction mode, if a diode acts as complementary switch.

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References

1. Belkhatay M., Cooley R., Witulski A.: Large signal stability criteria for distributed systems with constant power loads. In Proceedings of the 26th Annual IEEE Power Electronics Specialists Conference **2**, 1333–1338 (1995)
2. Emadi A., Fahimi B., Ehsani M.: On the concept of negative impedance instability in the more electric aircraft power systems with constant power loads.: SAE Transactions **108**, 689–699. (1999)
3. Choi B., Cho, B.H., Hong S.S.: Dynamics and control of DC-to-DC converters driving other converters downstream. IEEE Transactions on Circuits and Systems I: Fundamental Applications **46** 1240–1248 (1999)
4. Emadi A., Khaligh A., Rivetta C.H., Williamson G.A.: Constant power loads and negative impedance instability in automotive systems: definition, modeling, stability, and control of power electronic converters and motor drives. IEEE Transactions on Vehicular Technology **55**, 1112–1125 (2006)
5. Khaligh A.: Realization of Parasitics in Stability of DC–DC Converters Loaded by Constant Power Loads in Advanced Multiconverter Automotive Systems. IEEE Transactions on Industrial Electronics **55**, 2295–2305 (2008)
6. Benadero L., Cristiano R., Pagano D., Ponce E.: Nonlinear analysis of interconnected power converters: A case study. IEEE Journal on Emerging and Selected Topics in Circuits **5**, 326–335 (2015)
7. Cristiano R., Pagano D., Benadero L., Ponce E.: Bifurcation analysis of a DC–DC bidirectional power converter operating with constant power loads. International Journal of Bifurcation and Chaos **26** (2016)
8. Kwasinski A, Krein P.T.: Stabilization of constant power loads in Dc-Dc converters using passivity-based control. INTELEC 07 - 29th International Telecommunications Energy Conference, 867–874 (2007)
9. Onwuchekwa C.N., Kwasinski A.: Analysis of boundary control for boost and buck-boost converters in distributed power architectures with constant-power loads. Twenty-Sixth Annual IEEE Applied Power Electronics Conference and Exposition (APEC), 1816–1823 (2011)
10. Kwasinski A., Onwuchekwa C.N.: Dynamic Behavior and Stabilization of DC Microgrids With Instantaneous Constant-Power Loads. IEEE Transactions on Power Electronics **26**, 822–834 (2011)
11. Martinez-Treviño B.A., Aroudi A., Cid-Pastor A., Martinez-Salamero L.: Nonlinear Control for Output Voltage Regulation of a Boost Converter With a Constant Power Load. IEEE Transactions on Power Electronics **34**, 10381–10385 (2019)
12. Erickson R.W., Maksimovic D., *Input Filter Design. Fundamentals of Power Electronics*. (Springer International Publishing, 2020), 675–723
13. Cid-Pastor A., Martinez-Salamero L., El Aroudi A., Giral R., Calvente J., Leyva R.: Synthesis of loss-free resistors based on sliding-mode control and its applications in power processing. Control Engineering Practice **21**, 689–699 (2013)
14. Haroun R., El Aroudi A., Cid-Pastor A., Vidal-Idiarte E., Valderrama-Blavi H., Martinez-Salamero L.: Modelling and Control of Modular DC-Nanogrids Based on Loss-Free Resistors. IEEE Access **8**, 33305–33317 (2020)