

Central bank communication: Inflation target transparency with fiscal policy

Montserrat Ferré¹  | Òscar Macaya² | Carolina Manzano¹

¹Universitat Rovira i Virgili and ECO-SOS, Reus, Spain

²Centre d'Estudis Superiors de l'Aviació (CESDA), Reus, Spain

Correspondence

Montserrat Ferré, Universitat Rovira i Virgili and ECO-SOS, Av. de la Universitat 1, Reus 43204, Spain.
Email: montserrat.ferre@urv.cat

Funding information

Agència de Gestió d'Ajuts Universitaris i de Recerca, Grant/Award Number: SGR2021-00729; Ministerio de Ciencia, Innovación y Universidades, Grant/Award Number: PID2022-137382NB-I00; Universitat Rovira i Virgili, Grant/Award Number: 2022PFR-URV-50

Abstract

This article examines the impact of opacity in announcing the central bank's inflation target on macroeconomic volatility and welfare, considering the interaction between an independent central bank and a fiscal authority. Our findings suggest that a key factor influencing the effects of central bank transparency is the relative importance placed by the central bank, compared to the government, on output stabilization versus inflation stabilization. Specifically, when output stabilization is relatively more important for the central bank than for the government, full opacity may benefit the central bank, but not necessarily society as a whole.

KEYWORDS

central bank transparency, inflation target, macroeconomic volatility, welfare

JEL CLASSIFICATION

E52, E58, E62, E63, H30

1 | INTRODUCTION

During the 1990s, the move toward greater independence of central banks brought important changes to the communication of information on their policy objectives. While, previously, central banks would provide relatively little information or make imprecise announcements (Stein, 1989), independent central banks started to be more explicit and, thus, more transparent. An example of this increased dissemination of information was the introduction of publicly announced numerical inflation targets. New Zealand was the first country to adopt a target in 1990, but many countries followed in the subsequent years: Canada in 1991, the UK in 1992, Sweden and Finland in 1993... A numerical goal for monetary policy is considered essential for institutional or political transparency (Eijffinger & Geraats, 2006; Kuttner & Posen, 2000). It is thought that a more transparent framework for inflation targeting greatly contributes to the credibility of central banks (Bordo & Siklos, 2017) and to their ability to bring inflation down (Dincer & Eichengreen, 2014).

The reduction of inflation rates that followed in many countries seemed to provide support for greater independence and transparency. However, the low inflation rates observed in the decade preceding the 2020s, together with the zero lower bound, brought into question the capacity of central banks to *raise* inflation. Did the low inflation rates contribute to a decline in inflation expectations, risking a vicious circle between low inflation, low inflation expectations and low

Abbreviation: Fed, US Federal Reserve.

This is an open access article under the terms of the [Creative Commons Attribution-NonCommercial-NoDerivs](https://creativecommons.org/licenses/by-nc-nd/4.0/) License, which permits use and distribution in any medium, provided the original work is properly cited, the use is non-commercial and no modifications or adaptations are made.

© 2024 The Author(s). Contemporary Economic Policy published by Wiley Periodicals LLC on behalf of Western Economic Association International.

economic activity? To fight this possibility, in August 2020, the US Federal Reserve (Fed) formally committed to a flexible average inflation target: it would be more inclined to allow inflation to run higher than the standard 2% target before hiking interest rates. This new released target, hinting toward a more tolerant stance on inflation, appears somehow vaguely defined: how much higher than 2% will the Fed let inflation go? Will it generate uncertainty, as the public tries to guess the new central bank objectives? If that were to be the case, it would lead to what Cukierman and Meltzer (1986) define as imperfect transparency.

This paper examines the effect of imperfect transparency, understood as the presence of asymmetric information regarding the central bank's inflation target between the monetary policymaker and other economic agents. Unlike the existing literature on central bank transparency, which mostly focuses on models with a single policy, monetary policy, this article considers a framework with both fiscal and monetary policies. This departure is motivated by the fact that the degree of central bank transparency could influence government actions, thereby potentially influencing the behavior of the central bank itself. Consequently, the impact of central bank transparency on macroeconomic volatility and welfare could be altered by the interaction between fiscal and monetary policies.

To enhance our understanding of the impact of including fiscal policy, we first examine the effects of transparency in a model with only monetary policy. In this benchmark model, there is asymmetric information about the central bank's inflation target: while the central bank perfectly knows this target, private agents receive a signal about it, which they use to rationally form private-sector inflation expectations. Achieving transparency relies on effective communication from the central bank. A clearer communication strategy regarding the inflation target results in a more precise signal, thereby promoting increased transparency. It is shown that increased transparency leads to higher inflation variability, lower output variability, but does not affect their average levels. Then, we propose a Stackelberg game between fiscal and monetary authorities, with the government acting as the leader and the central bank as the follower. We show that the economic consequences of central bank transparency in such an environment critically depend on which policymaker, the central bank or the government, is more concerned about output stabilization relative to inflation stabilization. In this setup, an increase in the value of the signal about the central bank's inflation target raises private agents' expectations of inflation, while reducing the tax rate set by the government. In scenarios where the government is more concerned than the central bank about output stability relative to inflation stability (i.e., the central bank is sufficiently conservative), the government's reaction to the signal is more moderate than that of private agents, and consequently, the results derived in the benchmark model hold. However, when the government is less concerned than the central bank about output stability relative to inflation stability (i.e., the central bank is sufficiently populist), the government's reaction to the signal is stronger than that of private agents, leading to results opposite to those derived in the framework without fiscal policy. Finally, we also obtain that full transparency is preferred by the central bank if it is sufficiently conservative, while complete opacity is preferred if it is sufficiently populist.

This study is related to the literature focused on the political transparency of central banks. Geraats (2002) provides an excellent review of earlier studies in this area and highlights that the impact of asymmetric information regarding the central bank's preferences varies depending on the source of uncertainty. Building upon this insight, it is worth specifying that this study contributes to the existing literature investigating the transparency of central bank targets. Geraats (2002) argues that when the central bank's inflation target is stochastic for the private sector, increased transparency reduces the variability of inflation and output, though it does not significantly affect their average levels.¹ Nevertheless, given that many central banks have a quantitative inflation target, whereas opacity prevails for output (gap) targets (Geraats, 2006), most theoretical studies have considered the effects of uncertainty on the output (or employment) objectives (see, for instance, Demertzis & Hughes-Hallett, 2007; Faust & Svensson, 2001, 2002, Westelius, 2009, among others).

It is worth noting that, as Hahn (2009) highlights, the literature on preference transparency often relies on variance-reduction frameworks. Consequently, alterations in transparency levels not only influence the extent of information asymmetries but also impact the distribution of preferences, thereby altering the central bank's loss function. An alternative approach to modeling transparency is to consider a framework where private agents receive signals about the parameters of the central bank's preferences, which are unknown to them. Under this approach, Geraats (2007) studies opacity on both the inflation and the output targets in a framework where the assumption of perfect common knowledge about the degree of central bank transparency does not hold. The paper argues that, while actual transparency can be beneficial by reducing communication noise, perceived transparency can have drawbacks by making markets overly sensitive to potentially noisy information. Additionally, this paper suggests that central banks may benefit from maintaining misperceptions regarding transparency, particularly concerning information about the output target and supply shocks.

Other papers that use the alternative approach of modeling transparency through signals are Hahn (2009) and Hielscher (2012). Hahn (2009) shows that in a model with information asymmetries regarding the central bank's employment target and the central bank's relative weight of the conflicting goals for employment and inflation, society prefers full transparency over full opacity if it places significant value on employment stabilization. Conversely, society prefers full opacity to full transparency if it is very concerned about inflation stabilization. Hielscher (2012) proposes a two-stage model of monetary policy with asymmetric information regarding both the central bank's inflation target and output target. This paper shows potential conflicts of interest between society and the central bank regarding transparency. While the central bank prefers to reveal its inflation target while concealing its output target, society would prefer either transparency or opacity for both targets. In this paper we adopt the new approach to modeling transparency, similar to the recent papers mentioned. Unlike these works, which focus solely on monetary policy, we examine a model with both monetary and fiscal policies.

The remainder of the paper is organized as follows. Section 2 analyses the effects of transparency in a model with only monetary policy. Section 3 presents a model incorporating both monetary and fiscal policies and explores the influence of transparency on the stability of inflation, output, and welfare. Section 4 concludes. Proofs and extensions are provided in the Appendices A–C.

2 | THE BENCHMARK MODEL

This section presents a basic framework based on Geraats' model (2007), with only one policy, monetary policy.² Output is given by

$$x = \pi - \pi^e, \quad (1)$$

where π and π^e are the actual and expected inflation rates, respectively.³ Expectations (i.e., wage contracts) are set before the policymaker chooses π .

In order to reflect the possible uncertainty generated about the new inflation target recently announced by the Fed, the central bank's inflation target, denoted by π_{CB} , is allowed to be stochastic for the private sector, with $\pi_{CB} \sim N(\bar{\pi}_{CB}, \sigma_{\pi_{CB}}^2)$.⁴ We suppose that this target is known perfectly by the central bank, while the private sector observes a risky signal of this target, denoted by s , where $s = \pi_{CB} + \nu$, where ν is a white noise, that is, $\nu \sim N(0, \sigma_{\nu}^2)$ and $cov(\nu, \pi_{CB}) = 0$. Moreover, it is assumed that the distributions of these random variables are common knowledge. Therefore, once the private sector observes the signal s , it expects the central bank's inflation target to be as follows:

$$\mathbb{E}[\pi_{CB}|s] = \mathcal{A}s + (1 - \mathcal{A})\bar{\pi}_{CB}, \quad (2)$$

where

$$\mathcal{A} = \frac{\sigma_{\pi_{CB}}^2}{\sigma_{\pi_{CB}}^2 + \sigma_{\nu}^2}. \quad (3)$$

Note that \mathcal{A} is a measure of the accuracy of the signal s and ranges from 0 to 1. When the signal s is completely accurate (i.e., there is no noise, $\nu = 0$, and therefore, $s = \pi_{CB}$ and $\sigma_{\nu}^2 = 0$), then there is perfect transparency about the central bank's inflation target ($\mathcal{A} = 1$) and Equation (2) becomes $\mathbb{E}[\pi_{CB}|s] = s = \pi_{CB}$. By contrast, when the signal s is completely inaccurate (i.e., $\sigma_{\nu}^2 \rightarrow \infty$), then there is perfect opaqueness about this target ($\mathcal{A} \rightarrow 0$) and Equation (2) implies $\mathbb{E}[\pi_{CB}|s] \rightarrow \bar{\pi}_{CB}$.

The timing is as follows: firstly, the central bank's inflation target, π_{CB} , is realized, but observed only by the central bank. Subsequently, the private sector observes the signal s , which is used to rationally form the private sector inflation expectations, π^e . Finally, the central bank chooses π .

The loss function of the central bank will be:

$$L_{CB} = \frac{1}{2} \left((\pi - \pi_{CB})^2 + \delta_{CB} x^2 \right),$$

with $\delta_{CB} \geq 0$, indicating that the central bank cares about stabilizing inflation and output around the targets π_{CB} and 0, respectively.⁵ Note that a smaller value of δ_{CB} suggests a higher degree of conservatism exhibited by the central bank.

It is shown in Appendix A that the optimal inflation rate and the corresponding output are given by

$$\pi = \frac{\pi_{CB}}{1 + \delta_{CB}} + \frac{\delta_{CB}}{1 + \delta_{CB}}\pi^e = \frac{\pi_{CB}}{1 + \delta_{CB}} + \frac{\delta_{CB}}{1 + \delta_{CB}}\mathbb{E}[\pi_{CB}|s] \text{ and} \tag{4}$$

$$x = \frac{\pi_{CB} - \mathbb{E}[\pi_{CB}|s]}{1 + \delta_{CB}}. \tag{5}$$

Using Equation (2), note that the last expressions show that inflation and output are affected by the degree of accuracy of the signal s . Moreover, from Equations (4) and (5), we can derive the following results:

Proposition 1. *In the benchmark model without fiscal policy, greater transparency (higher A) increases the variability of inflation and reduces the volatility of output, but does not affect their average levels.*

According to the neoclassical Phillips curve, output differs from its natural level whenever inflation expectations diverge from inflation. Transparency makes inflation expectations more accurate and thus reduces output fluctuations. To understand the effect of transparency on the variance of inflation, one has to note that Equation (4) shows that the central bank's optimal choice of inflation is an increasing function of both the inflation target and private-sector inflation expectations. Under transparency, inflation expectations covary more strongly with the inflation target. This results in larger fluctuations of inflation.

3 | MONETARY-FISCAL POLICY INTERACTION

In this section, we will extend the previous framework to develop a model with two policies (monetary and fiscal policy). In this setup, there are three players: the private sector, which sets inflation expectations; a fiscal authority (the government), responsible for setting (distortionary) taxes; and the central bank, responsible for directly setting the inflation rate through monetary policy. Moreover, in this new context output is given by

$$x = \pi - \pi^e - \tau, \tag{6}$$

where π and π^e are the actual and expected inflation rates, as in Equation (1), while τ represents the tax rate levied on output.⁶

The government's budget constraint is:

$$g = \tau + \pi,$$

where g denotes the ratio of public expenditures over output. Note that public spending will be financed by a distortionary tax (controlled by the fiscal authority) and/or by money creation (controlled by the authority responsible for monetary policy).

Fiscal policy will be under the control of the government and has the following loss function:

$$L_G = \frac{1}{2}((\pi - \bar{\pi}_G)^2 + \delta_G x^2 + \gamma_G (g - \bar{g})^2),$$

with δ_G and γ_G indicating the weight given by the government on output and government spending stabilization relative to inflation stabilization, respectively ($\delta_G, \gamma_G > 0$). Thus, the government wishes to minimize the deviations of inflation, output and public spending from some targets, $\bar{\pi}_G$, 0 and \bar{g} , respectively. We allow for $\delta_G \neq \delta_{CB}$; if $\delta_G > \delta_{CB}$, then the government prioritizes output over inflation to a greater extent compared to the central bank. In addition, when γ_G is low enough, $\delta_G > \delta_{CB}$ would also indicate that the central bank is more conservative than the government.

As in the benchmark model, we assume that the central bank knows perfectly its inflation target. However, both the private sector and the government now observe the same risky signal of this target (s).

The timing of events is as follows: firstly, the central bank's inflation target, π_{CB} , is realized, but observed only by the central bank. Subsequently, the private sector and the government observe the same risky signal of this target, and the private sector uses this signal to rationally form its inflation expectations. Then, fiscal-monetary interactions involve a Stackelberg game, where the government takes the leading role and the central bank acts as the follower. Therefore, the fiscal authority chooses the tax rate taking into account the response of the central bank to its action. This type of setting has been used by Dixit and Lambertini (2003), Acocella et al. (2007) and Oros and Zimmer (2020), among others, as monetary policy can be adjusted more quickly than fiscal policy.

3.1 | Equilibrium

This sequential game is solved by backward induction. This requires starting with the player that acts last, which in our setup is the central bank. The central bank minimizes its loss function with π^e and τ given. The first-order condition with respect to π implies that

$$\pi = \frac{\pi_{CB}}{1 + \delta_{CB}} + \frac{\delta_{CB}}{1 + \delta_{CB}}(\pi^e + \tau). \quad (7)$$

Hence, when fiscal policy is included, the optimal inflation rate is equal to the optimal level in the benchmark model given in Equation (4) plus an additional term $\left(\frac{\delta_{CB}}{1 + \delta_{CB}}\tau\right)$, which increases with τ whenever $\delta_{CB} \neq 0$. Thus, when the central bank is concerned with output stabilization, an increase in taxes raises the incentives for the central bank to inflate in order to offset the negative effect of the tax increase on output (see Equation 6).

The fiscal authority's optimization problem is as follows:

$$\min_{\tau} \mathbb{E}[L_G|s] = \mathbb{E}\left[\frac{1}{2}\left((\pi - \bar{\pi}_G)^2 + \delta_G x^2 + \gamma_G(g - \bar{g})^2\right) \middle| s\right]$$

$$\text{s.t. } x = \pi - \pi^e - \tau$$

$$g = \tau + \pi$$

$$\pi = \frac{\pi_{CB}}{1 + \delta_{CB}} + \frac{\delta_{CB}}{1 + \delta_{CB}}(\pi^e + \tau).$$

We solve this optimization problem by applying the substitution method. Specifically, we substitute the previous expressions for output, public spending and inflation in the (conditional) expected loss function of the government. This results in an objective function with only one variable, τ . Then, the first-order condition (in the optimization problem with only one variable) implies that

$$(\mathbb{E}[\pi|s] - \bar{\pi}_G) \frac{\delta_{CB}}{1 + \delta_{CB}} + \delta_G(\mathbb{E}[\pi|s] - \pi^e - \tau) \left(\frac{-1}{1 + \delta_{CB}}\right) + \gamma_G(\mathbb{E}[\pi|s] + \tau - \bar{g}) \left(\frac{1 + 2\delta_{CB}}{1 + \delta_{CB}}\right) = 0.$$

Using the rational expectations hypothesis ($\mathbb{E}[\pi|s] = \pi^e$), we have

$$(\pi^e - \bar{\pi}_G) \frac{\delta_{CB}}{1 + \delta_{CB}} + (-\tau) \left(\frac{-\delta_G}{1 + \delta_{CB}}\right) + \gamma_G(\pi^e + \tau - \bar{g}) \left(\frac{1 + 2\delta_{CB}}{1 + \delta_{CB}}\right) = 0, \quad (8)$$

and therefore,

$$\tau = -\frac{\delta_{CB} + \gamma_G(1 + 2\delta_{CB})}{\delta_G + \gamma_G(1 + 2\delta_{CB})}\pi^e + \frac{\delta_{CB}}{\delta_G + \gamma_G(1 + 2\delta_{CB})}\bar{\pi}_G + \frac{\gamma_G(1 + 2\delta_{CB})}{\delta_G + \gamma_G(1 + 2\delta_{CB})}\bar{g}. \tag{9}$$

In this expression, we can see that an increase in π^e triggers a response of the government, reducing the tax rate. Moreover, this reduction in the tax rate is smaller (larger) than the increase in π^e if and only if $\delta_{CB} < \delta_G$ ($\delta_{CB} > \delta_G$). To understand intuitively this result, we rewrite the first-order condition given in Equation (8) as follows:

$$(\pi^e - \bar{\pi}_G)\frac{\delta_{CB}}{1 + \delta_{CB}} + (-\tau)\left(\frac{-\delta_G}{1 + \delta_{CB}}\right) = \gamma_G(\bar{g} - \pi^e - \tau)\left(1 + \frac{\delta_{CB}}{1 + \delta_{CB}}\right).$$

The left-hand side of this expression can be interpreted as the marginal cost, measured in terms of higher inflation (in expected terms) and lower output,⁷ resulting from an increase in taxes. Meanwhile, the right-hand side represents the marginal benefit, measured in terms of higher tax revenue, derived from an increase in τ .

According to this expression, a decrease in π^e exerts two effects on the government's decision. On the one hand, it reduces the marginal cost of an increase in τ , encouraging the government to raise taxes. On the other hand, it increases the marginal benefit of an increase in τ , leading the government to raise taxes. Therefore, we conclude that a decrease in π^e causes the fiscal authority to increase the tax rate. In addition, when the government and the central bank are equally concerned about output stabilization relatively to inflation stabilization ($\delta_G = \delta_{CB}$), the previous expression becomes

$$(\pi^e - \bar{\pi}_G + \tau)\frac{\delta_{CB}}{1 + \delta_{CB}} = \gamma_G(\bar{g} - \pi^e - \tau)\left(1 + \frac{\delta_{CB}}{1 + \delta_{CB}}\right),$$

which shows that a decrease in π^e leads to an increase in the optimal value of τ of the same magnitude. However, if the fiscal authority is relatively more concerned about output than the central bank ($\delta_G > \delta_{CB}$), then the increase in the tax rate is lower than the decrease in π^e .

By taking (conditional) expectations in Equation (7) and solving for π^e , it follows that $\pi^e = \mathbb{E}[\pi_{CB}|s] + \delta_{CB}\tau$. Solving the system of two equations, formed by the previous equation and Equation (9), and two unknowns (i.e., π^e and τ), we have

$$\pi^e = \frac{\gamma_G + \delta_G + 2\gamma_G\delta_{CB}}{\Delta}\mathbb{E}[\pi_{CB}|s] + \frac{\delta_{CB}^2\bar{\pi}_G + \delta_{CB}\gamma_G(2\delta_{CB} + 1)\bar{g}}{\Delta} \text{ and} \tag{10}$$

$$\tau = -\frac{\gamma_G + \delta_{CB} + 2\gamma_G\delta_{CB}}{\Delta}\mathbb{E}[\pi_{CB}|s] + \bar{g} + \frac{\delta_{CB}\bar{\pi}_G - (\delta_{CB}^2 + \delta_G + \gamma_G\delta_{CB}(2\delta_{CB} + 1))\bar{g}}{\Delta}, \tag{11}$$

where

$$\Delta = \delta_{CB}^2 + \delta_G + \gamma_G(\delta_{CB} + 1)(2\delta_{CB} + 1). \tag{12}$$

Finally, substituting Equations (10) and (11) into (7), we obtain

$$\pi = \frac{1}{1 + \delta_{CB}}\pi_{CB} + \frac{\delta_{CB}(\delta_G - \delta_{CB})}{(1 + \delta_{CB})\Delta}\mathbb{E}[\pi_{CB}|s] + \frac{\delta_{CB}^2\bar{\pi}_G + \delta_{CB}\gamma_G(2\delta_{CB} + 1)\bar{g}}{\Delta}. \tag{13}$$

Next, we make some remarks regarding Equations (11) and (13). Note that taxation is increasing in the public spending target (\bar{g}) and in the government inflation target ($\bar{\pi}_G$). Inflation depends positively on the realized target of the central bank (π_{CB}), on the government inflation target ($\bar{\pi}_G$) and on the government spending target (\bar{g}).

Expressions (11) and (13) also show that the conditional expectations that private agents and the government have on the central bank's inflation target ($\mathbb{E}[\pi_{CB}|s]$) affect taxation and inflation. If the government expects the central bank

to have a high target for inflation, taxes will be set lower—see Equation (11). Intuitively, if the government and the private sector expect the central bank to have a high target for inflation, then the private sector's inflation expectations are higher (π^e), resulting in a reduction of the tax rate—see Equation (9).

Furthermore, the higher $\mathbb{E}[\pi_{CB}|s]$, the higher the central bank will set inflation whenever $\delta_G > \delta_{CB}$, as shown in Equation (13). To intuitively understand this result, notice that Equations (7), (10) and (11) show that $\mathbb{E}[\pi_{CB}|s]$ affects the central bank's inflation decisions through the private sector's inflation expectations and the government's taxes. However, a change in $\mathbb{E}[\pi_{CB}|s]$ impacts π^e and τ in opposite ways, as can be seen in Equations (10) and (11). The higher the value of $\mathbb{E}[\pi_{CB}|s]$, the higher the private agents' expected inflation will be, and the lower the taxes set by the government. Additionally, as previously stated, when $\delta_G = \delta_{CB}$, the reduction in the tax rate is exactly the same as the increase in the inflation expectations, and these effects cancel out. Hence, the inflation set by the central bank does not depend on $\mathbb{E}[\pi_{CB}|s]$. Conversely, when $\delta_G > \delta_{CB}$, private agents react more to a change in $\mathbb{E}[\pi_{CB}|s]$ than the government. Consequently, an increase in $\mathbb{E}[\pi_{CB}|s]$ results in higher inflation set by the central bank.

In relation to output, substituting Equations (10), (11) and (13) in (6), we have

$$x = \frac{1}{1 + \delta_{CB}} \pi_{CB} - \frac{\delta_G - \delta_{CB}}{(1 + \delta_{CB})\Delta} \mathbb{E}[\pi_{CB}|s] - \frac{\delta_{CB}}{\Delta} \bar{\pi}_G - \frac{\gamma_G(1 + 2\delta_{CB})}{\Delta} \bar{g}. \quad (14)$$

Thus, the combination of Equations (2) and (13) along with the last expression indicates that inflation and output are affected by the degree of accuracy of the signal s except for when $\delta_G = \delta_{CB}$; however, their unconditional expected values (i.e., $\mathbb{E}[\pi]$ and $\mathbb{E}[x]$) are not affected, in line with Geraats (2007). By contrast, we obtain a significantly different result for the variance of these variables, as discussed in the following paragraphs.⁸

Proposition 2. *When the interaction of monetary policy and fiscal policy is present, greater transparency (higher \mathcal{A}) increases the variability of inflation and reduces the volatility of output whenever $\delta_G > \delta_{CB}$. The opposite results hold when $\delta_G < \delta_{CB}$. Furthermore, in any case, a change in the degree of transparency does not affect the average levels of inflation and output.*

According to Proposition 2, the relationship between the central bank's degree of transparency and the volatility of both inflation and output will depend on the relationship between δ_G and δ_{CB} , in other words, on whether the central bank is more or less concerned than the government about the stabilization of output relative to inflation. This proposition shows that when $\delta_G > \delta_{CB}$, the least stable inflation and the most stable output are obtained when the central bank is perfectly transparent. By contrast, when $\delta_G < \delta_{CB}$, the least stable inflation and the most stable output occur when the central bank is fully opaque.

To grasp the relationship between inflation volatility and signal precision, note that Equation (13) indicates that when $\delta_G = \delta_{CB}$, the inflation set by the central bank is not affected by $\mathbb{E}[\pi_{CB}|s]$. Recall that in this scenario, a change in the signal s leads to adjustments in both the tax rate and inflation expectations, with equal magnitude but opposite signs, resulting in the cancellation of these effects when considering the inflation rate chosen by the central bank. Hence, in this case, the volatility of inflation remains independent of the accuracy of the signal. In contrast, when $\delta_G > \delta_{CB}$, the inflation set by the central bank increases with $\mathbb{E}[\pi_{CB}|s]$. Thus, greater transparency leads to a stronger co-movement of the first two terms in the expression of π given in Equation (13), resulting in higher inflation volatility.

To understand the relationship between the volatility of output and the accuracy of the signal, let us consider the formula derived from the first-order condition of the optimization problem of the central bank

$$x = \frac{1}{\delta_{CB}} (\pi_{CB} - \pi). \quad (15)$$

As it can be seen, the volatility of output will be related to the difference between the central bank's inflation target and realized inflation. Let us consider first the relationship between transparency and the deviation of inflation from the central bank's inflation target. Note that an increase in this target has two effects on the behavior of the central bank. Firstly, considering the inflation objective, the rise in this target increases the incentives to inflate. Secondly, regarding the output objective, a change in π_{CB} influences the incentives for inflation because it affects the expectations of private agents and the tax rate determined by the government. If $\delta_G > \delta_{CB}$, a change in π_{CB} results in an adjustment in $\pi^e + \tau$ in the same direction as π_{CB} , thereby increasing the incentives to inflate. Therefore, in this case, both effects

align, leading to the conclusion that inflation increases with a change in π_{CB} . Increased transparency magnifies the latter effect, thus intensifying the co-movement between π_{CB} and π and resulting in a decrease in output volatility.

The results presented in Proposition 2 diverge from those obtained in the benchmark model. In that case, inflation was the most stable when the central bank was the least transparent about its inflation target, due to the greatest stability of π^e . By contrast, in the present setup, the government also observes the signal, which affects the tax rate τ . Whenever τ reacts less to the signal than π^e , the aforementioned result derived in the benchmark model holds. Otherwise, the opposite result occurs. A similar analysis can be applied to the relationship between transparency and output volatility.

The Fed has expressed views in recent years that could be interpreted as reflecting a higher δ_{CB} or a higher output target.⁹ While the level of the central bank's output target does not change the results of our analysis, an increase in δ_{CB} does. Then, we end this subsection by examining how an increase in δ_{CB} affects the sensitivity of inflation and output volatility to central bank transparency. These comparative statics results are illustrated in Figures 1 and 2, and summarized in Corollary 3.

According to Proposition 2, Figure 1 illustrates different patterns regarding the relationship between inflation volatility and the accuracy of the signal, depending on whether $\delta_{CB} \leq \delta_G$ or $\delta_{CB} > \delta_G$. For the first region (i.e., $\delta_{CB} \leq \delta_G$), inflation volatility increases with the accuracy of the signal, except when $\delta_{CB} = 0$ (represented by the blue solid line) and $\delta_{CB} = \delta_G = 1$ (represented by the black solid line), where inflation volatility remains unaffected by the level of central bank transparency. It is noteworthy that in the case of $\delta_{CB} = 0$, the central bank sets inflation equal to its target,

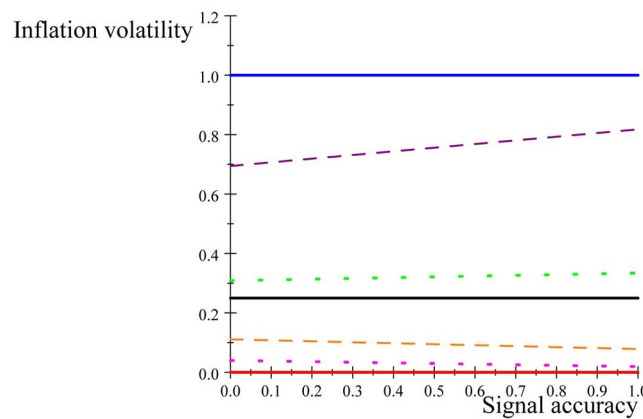


FIGURE 1 Relationship between inflation volatility and accuracy of the signal for different values of δ_{CB} : $\delta_{CB} = 0$ (blue solid line), $\delta_{CB} = 0.2$ (purple dashed line), $\delta_{CB} = 0.8$ (green dotted line), $\delta_{CB} = 1$ (black solid line), $\delta_{CB} = 2$ (orange dashed line), and $\delta_{CB} = 4$ (magenta dotted line), and $\delta_{CB} \rightarrow \infty$ (red solid line). Other parameter values: $\delta_G = 1$, $\gamma_G = 0.5$, and $\sigma_{\pi_{CB}}^2 = 1$.

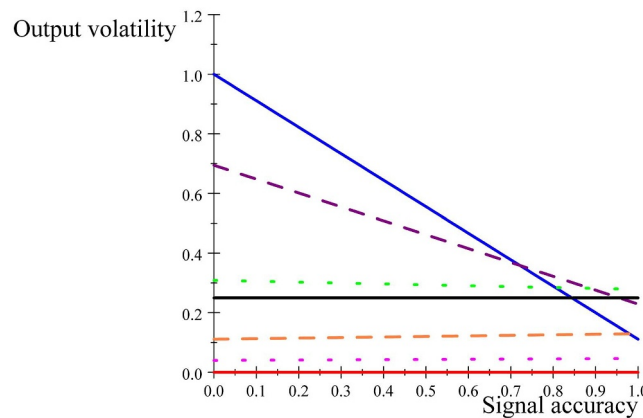


FIGURE 2 Relationship between output volatility and accuracy of the signal for different values of δ_{CB} : $\delta_{CB} = 0$ (blue solid line), $\delta_{CB} = 0.2$ (purple dashed line), $\delta_{CB} = 0.8$ (green dotted line), $\delta_{CB} = 1$ (black solid line), $\delta_{CB} = 2$ (orange dashed line), and $\delta_{CB} = 4$ (magenta dotted line), and $\delta_{CB} \rightarrow \infty$ (red solid line). Other parameter values: $\delta_G = 1$, $\gamma_G = 0.5$, and $\sigma_{\pi_{CB}}^2 = 1$.

resulting in an inflation unaffected by the accuracy of the signal. Similarly, in the case of $\delta_{CB} = \delta_G$, it should be recalled that the reactions of government and private agents to the signal counterbalance each other, leaving the inflation rate set by the central bank unaffected by the signal. Moreover, Figure 1 allows us to understand how the sensitivity of the inflation volatility to the accuracy of the signal is affected by a change in δ_{CB} . For low values of δ_{CB} in the first region (i.e., $\delta_{CB} \leq \delta_G$), an increase in δ_{CB} leads to inflation volatility being more sensitive to the accuracy of the signal. Observe that for $\delta_{CB} = 0$ the blue solid line is horizontal and for $\delta_{CB} = 0.2$ the purple dashed line is increasing. However, the opposite result holds for high values of δ_{CB} in the first region (i.e., $\delta_{CB} \leq \delta_G$). Observe that at $\delta_{CB} = 0.8$ the green dotted line is increasing and for $\delta_{CB} = \delta_G = 1$ the black solid line is flat. Thus, in this part of the first region an increase in δ_{CB} leads to inflation volatility being less sensitive to the signal's accuracy. Concerning the second region (i.e., $\delta_{CB} > \delta_G$), there is a negative relationship between inflation volatility and the accuracy of the signal. For low values of δ_{CB} within this region, an increase in δ_{CB} leads to greater sensitivity of inflation volatility to the signal's accuracy. Conversely, for high values of δ_{CB} within this region, an increase in δ_{CB} diminishes this sensitivity. As δ_{CB} approaches infinity, inflation volatility becomes independent of the level of central bank transparency (red solid line).

Analogously to Figures 1 and 2 also illustrates different patterns regarding the relationship between output volatility and the accuracy of the signal, depending on whether $\delta_{CB} \leq \delta_G$ or $\delta_{CB} > \delta_G$. For the first region (i.e., $\delta_{CB} \leq \delta_G$), a negative relationship exists between output volatility and the accuracy of the signal, except when $\delta_{CB} = \delta_G = 1$. In this case, output volatility remains unaffected by the level of central bank transparency, as the reactions of government and private agents to the signal counterbalance each other, resulting in both inflation rate and output being independent of the signal. Consequently, the line representing output volatility (the black solid line) is flat. Additionally, the lines tend to flatten as δ_{CB} increases, indicating a reduced sensitivity of output volatility to the accuracy of the signal. Concerning the second region (i.e., $\delta_{CB} > \delta_G$), there is a positive relationship between output volatility and the accuracy of the signal. For low values of δ_{CB} within this region, an increase in δ_{CB} leads to an increase in the sensitivity of output volatility to the signal's accuracy. However, as δ_{CB} increases to higher values within this range, this sensitivity diminishes. As δ_{CB} approaches infinity, output volatility becomes unaffected by the level of central bank transparency (red solid line).

These results are formalized in the following corollary:

Corollary 3. *The effect of an increase in δ_{CB} on the sensitivity of inflation volatility and output volatility to the central bank transparency level is ambiguous. Specifically, the following results hold:*

- a) *There exist two values of δ_{CB} , denoted by $\underline{\delta}$ and $\bar{\delta}$, with $\underline{\delta} < \delta_G < \bar{\delta}$, such that $\left| \frac{\partial \text{var}[\pi]}{\partial A} \right|$ increases with δ_{CB} whenever $0 < \delta_{CB} < \underline{\delta}$ or $\delta_G < \delta_{CB} < \bar{\delta}$, while $\left| \frac{\partial \text{var}[\pi]}{\partial A} \right|$ decreases with δ_{CB} whenever $\underline{\delta} < \delta_{CB} < \delta_G$ or $\delta_{CB} > \bar{\delta}$.*
- b) *There exists a value of δ_{CB} , denoted by $\tilde{\delta}$, with $\delta_G < \tilde{\delta}$, such that $\left| \frac{\partial \text{var}[x]}{\partial A} \right|$ decreases with δ_{CB} whenever $0 < \delta_{CB} < \delta_G$ or $\delta_{CB} > \tilde{\delta}$, while $\left| \frac{\partial \text{var}[x]}{\partial A} \right|$ increases with δ_{CB} whenever $\delta_G < \delta_{CB} < \tilde{\delta}$.*

3.2 | The optimal degree of transparency

We are interested in studying what degree of transparency makes the central bank and the government better off, in order to determine if a conflict of interest may arise between these policymakers. We initially analyze the incentives of the central bank for revealing information about its inflation target. To that end, we compute the expected loss for the monetary authority in equilibrium. Note that using Equation (15), we get

$$\mathbb{E}[L_{CB}] = \frac{1}{2} \delta_{CB} (1 + \delta_{CB}) \mathbb{E}[x^2],$$

where $\mathbb{E}[x^2] = (\mathbb{E}[x])^2 + \text{var}[x]$.

Taking into account the previous expression and Proposition 2, which states that $\mathbb{E}[x]$ is independent of the degree of transparency but affects output volatility, the central bank favors the transparency level that minimizes output volatility. Therefore, Proposition 2 enables us to derive the following result:

Proposition 4. *When the interaction of monetary policy and fiscal policy is present, a more accurate message (higher \mathcal{A}) decreases the expected loss of the central bank whenever $\delta_G > \delta_{CB}$. The opposite results hold when $\delta_G < \delta_{CB}$.*

Proposition 4 indicates that the optimal degree of transparency of the central bank depends on which policymaker is relatively more concerned about output stabilization. If the government prioritizes stabilizing output to a greater extent, then maximum transparency would be optimal for the central bank. Otherwise, that is, if the central bank is more concerned about output than the government, full opacity about the inflation target is preferred by the central bank.

After determining the optimal degree of transparency from the central bank's viewpoint, let us examine how the central bank transparency affects government's welfare. Unsurprisingly, the next proposition shows that the government always benefits from complete transparency.

Proposition 5. *When the interaction of monetary policy and fiscal policy is present, a more accurate message (higher \mathcal{A}) decreases the expected loss of the government.*

It is interesting to consider the normative question of whether transparency is socially desirable. In this regard, it should be mentioned that many articles assume that the government's preferences coincide with society's preferences, as an elected government seeks to align its preferences closely with those of society in order to secure re-election. In such cases, Proposition 5 suggests that society is better off with complete transparency.

In instances where the government's preferences diverge from those of society, our results are ambiguous and depend on society's preferences regarding inflation, output, and government spending. For example, suppose the government (relatively) prioritizes output stabilization more than the central bank ($\delta_G > \delta_{CB}$), a scenario typical of a very conservative central bank. If society's primary concern is output stabilization, Proposition 2 suggests that society would benefit from the central bank announcing its inflation target in a fully transparent manner. Proposition 4 also indicates that the central bank would benefit from this transparency. Conversely, if society's main concern is inflation stabilization, Proposition 2 shows that opacity would improve society's welfare. In such cases, a conflict of interest arises between society and the central bank.

4 | CONCLUSION

Following recent announcements by the Fed regarding the flexibilization of its inflation target, this article explores how altering the level of transparency surrounding the central bank's inflation objective affects macroeconomic volatility and welfare within a framework where an independent central bank interacts with a fiscal authority.

Our results suggest that a key factor influencing the effects of central bank transparency is the relative importance placed by the central bank, in comparison to the government, on output stabilization versus inflation stabilization. When the central bank assigns less weight to output stabilization relative to inflation stabilization than the government does, a more transparent inflation target will result in lower output volatility, but higher inflation volatility.

We also find that when the central bank is relatively less concerned about output stabilization than the government, the former prefers maximum transparency of its inflation target. Conversely, when output stabilization is relatively more important to the central bank than to the government, the central bank is better off with full opacity. In any case, the government always prefers full transparency. Additionally, if the government aligns with society's preferences, then transparency is also socially optimal. Otherwise, that is, when the government does not represent society's preferences, full opacity of the central bank's inflation target could be beneficial for society.

The analysis presented here could be extended in several ways. For instance, introducing other forms of uncertainty into the model could help assess the robustness of the findings outlined in this article.

ACKNOWLEDGMENTS

We thank two anonymous referees for providing valuable comments and suggestions to the manuscript. The authors acknowledge financial support from the Government of Spain (AEI/FEDER, UE PID2022-137382NB-I00), the Government of Catalonia (SGR2021-00729) and the Universitat Rovira i Virgili (2022PFR-URV-50). Any errors are our own responsibility.

DATA AVAILABILITY STATEMENT

The data that support the finding of this study are available from the corresponding author by reasonable request.

ORCID

Montserrat Ferré  <https://orcid.org/0000-0003-4959-608X>

ENDNOTES

- ¹ By contrast, uncertainty about the central bank's stabilization parameters affects average levels of inflation and output. Specifically, Schaling and Nolan (1998) show that expected inflation rises with uncertainty about inflation stabilization preferences. Similarly, Eijffinger et al. (2000, 2003) find that this uncertainty adversely affects inflation bias and variability, but improves output stabilization, suggesting potential societal benefits. Sorensen (1991) and Grüner (2002) demonstrate that this uncertainty may be socially desirable when the private sector acts strategically. Other papers examine interactions between monetary and fiscal policies with uncertainty about the central bank's stabilization parameters. Ciccarone et al. (2007), and Hefeker and Zimmer (2011) find that this uncertainty could have a fiscal disciplining effect in terms of reducing the public expenditures and taxes, leading to lower inflation and higher output gap. It could also reduce the macroeconomic volatility if the initial degree of opacity is sufficiently high. Oros and Zimmer (2015) show that in a monetary union with uncertain central bank preferences, private agents expect the central bank to act more conservatively, lowering inflation and improving macroeconomic outcomes through better communication. Our paper differs from these articles in the source of uncertainty and the way of modeling transparency.
- ² This benchmark model could also include a fixed tax rate to make it more comparable with the full model presented in Section 3. Nevertheless, this change would not affect the results obtained in this section.
- ³ The inclusion of a supply shock does not alter the results of this paper, as shown in Appendix B. This is the reason why we have omitted it in our analysis.
- ⁴ Uncertainty could arise because of unclear communication by the central bank, be it misstatements, bad timing or miscalibration of messages to different audiences (Filardo & Guinigundo, 2008). It could also be due to, as stated by Blinder (2009), different messages being recounted by different members of the monetary policy committee. Further, Haldane and McMahon (2018) explore how in the presence of low financial and macroeconomic literacy, the public might misunderstand the central bank communication.
- ⁵ Without any loss of generality, we normalize the target level of output to zero.
- ⁶ As in Alesina and Tabellini (1987) and Debelle and Fischer (1994), this expression is obtained assuming that there is a competitive private sector in this economy. There is a continuum of firms that are price takers in both the output and in the labor market and aim to maximize their profits net of taxes. Distortionary taxes are levied on production and are the only tax available to the government. Workers set the nominal wage to achieve a real wage target (which is normalized to zero) in a competitive labor market populated by uncoordinated small agents. Since the wage is chosen at the beginning of the period, private agents need to set their inflation expectations.
- ⁷ In this interpretation, we assume that $\bar{\pi}_G$ is small enough. Otherwise, we would rewrite this first term on the right-hand side of this equality, representing the corresponding term a marginal benefit of an increase in τ .
- ⁸ In Appendix C, we show that the results derived in Proposition 2 also hold in an extension of our model, where the output target of the central bank is uncertain.
- ⁹ We would like to thank a referee for this suggestion.

REFERENCES

- Acocella, N., Di Bartolomeo, G. & Tirelli, P. (2007) Monetary conservatism and fiscal coordination in a monetary union. *Economics Letters*, 94(1), 56–63. Available from: <https://doi.org/10.1016/j.econlet.2006.08.002>
- Alesina, A. & Tabellini, G. (1987) Rules and discretion with noncoordinated monetary and fiscal policies. *Economic Inquiry*, 25(4), 619–630. Available from: <https://doi.org/10.1111/j.1465-7295.1987.tb00764.x>
- Blinder, A. (2009) Talking about monetary policy: the virtues (and vices?) of central bank communication. BIS Working Papers no 274.
- Bordo, M. & Siklos, P. (2017) Central bank credibility before and after the crisis. *Open Economies Review*, 28(1), 19–45. Available from: <https://doi.org/10.1007/s11079-016-9411-2>
- Ciccarone, G., Marchetti, E. & Di Bartolomeo, G. (2007) Unions, fiscal policy and central bank transparency. *The Manchester School*, 75(5), 617–633. Available from: <https://doi.org/10.1111/j.1467-9957.2007.01033.x>
- Cukierman, A. & Meltzer, A. (1986) A theory of ambiguity, credibility and independence under discretion and asymmetric information. *Econometrica*, 54(5), 1099–1128. Available from: <https://doi.org/10.2307/1912324>
- Debelle, G. & Fischer, S. (1994) How independent should central banks be? In Fuhrer, J. (Ed.) *Goals, Guidelines and Constraints facing Monetary Policymakers*. Boston: Federal Reserve Bank of Boston Conference Series, pp. 195–221.
- Demertzis, M. & Hughes-Hallett, A. (2007) Central bank transparency in theory and practice. *Journal of Macroeconomics*, 29(4), 760–789. Available from: <https://doi.org/10.1016/j.jmacro.2005.06.002>

- Dincer, N.N. & Eichengreen, B. (2014) Central bank transparency and independence: updates and new measures. *International Journal of Central Banking*, 10(1), 189–259.
- Dixit, A. & Lambertini, L. (2003) Interactions of commitment and discretion in monetary and fiscal policies. *The American Economic Review*, 93(5), 1522–1542. Available from: <https://doi.org/10.1257/000282803322655428>
- Eijffinger, S.C.W. & Geraats, P.M. (2006) How transparent are central banks? *European Journal of Political Economy*, 22(1), 1–21. Available from: <https://doi.org/10.1016/j.ejpoleco.2005.09.013>
- Eijffinger, S.C.W., Hoerberichts, M. & Schaling, E. (2000) Why money talks and wealth whispers: monetary uncertainty and mystique. *Journal of Money, Credit, and Banking*, 32(2), 218–235. Available from: <https://doi.org/10.2307/2601240>
- Eijffinger, S.C.W., Hoerberichts, M. & Schaling, E. (2003) Why money talks and wealth whispers: monetary uncertainty and mystique: A reply. *Journal of Money, Credit, and Banking*, 35(1), 137–139.
- Faust, J. & Svensson, L.E.O. (2001) Transparency and credibility: monetary policy with unobservable goals. *International Economic Review*, 42(2), 369–397. Available from: <https://doi.org/10.1111/1468-2354.00114>
- Faust, J. & Svensson, L.E.O. (2002) The equilibrium degree of transparency and control in monetary policy. *Journal of Money, Credit, and Banking*, 34(2), 520–539. Available from: <https://doi.org/10.1353/mcb.2002.0037>
- Filardo, A. & Guinigundo, D. (2008) Transparency and communication in monetary policy: a survey of Asian central banks. In: *BSP-BIS High-Level Conference on Transparency and Communication in Monetary Policy*, 1 vol. Manila.
- Geraats, P.M. (2002) Central bank transparency. *The Economic Journal*, 112(483), 532–565. Available from: <https://doi.org/10.1111/1468-0297.00082>
- Geraats, P.M. (2006) Transparency of monetary policy: theory and practice. *CESifo Economic Studies*, 52(1), 111–152. Available from: <https://doi.org/10.1093/cesifo/ifj004>
- Geraats, P.M. (2007) The mystique of central bank speak. *International Journal of Central Banking*, 3(1), 37–80.
- Grüner, H.P. (2002) How much should central banks talk?: A new argument. *Economics Letters*, 77(2), 195–198. Available from: [https://doi.org/10.1016/s0165-1765\(02\)00125-8](https://doi.org/10.1016/s0165-1765(02)00125-8)
- Hahn, V. (2009) Transparency of central bank preferences. *German Economic Review*, 10(1), 32–49. Available from: <https://doi.org/10.1111/j.1468-0475.2008.00440.x>
- Haldane, A. & McMahon, M. (2018) Central bank communications and the general public. *AEA Papers and Proceedings*, 108, 578–583. Available from: <https://doi.org/10.1257/pandp.20181082>
- Hefeker, C. & Zimmer, B. (2011) The optimal choice of central bank independence and conservatism under uncertainty. *Journal of Macroeconomics*, 33(4), 595–606. Available from: <https://doi.org/10.1016/j.jmacro.2011.09.005>
- Hielscher, K. (2012) Monetary policy delegation and transparency of policy targets: a positive analysis. *German Economic Review*, 13(1), 21–40. Available from: <https://doi.org/10.1111/j.1468-0475.2011.00537.x>
- Kuttner, K.N. & Posen, A.S. (2000) Inflation, monetary transparency, and G3 exchange rate volatility. Available at SSRN: <https://ssrn.com/abstract=239300> [Accessed 23 August 2024].
- Oros, C. & Zimmer, B. (2015) Uncertainty and fiscal policy in a monetary union: why does monetary policy transmission matter? *Economic Modelling*, 50, 85–93. Available from: <https://doi.org/10.1016/j.econmod.2015.06.006>
- Oros, C. & Zimmer, B. (2020) Budget uncertainty in a monetary union. *European Journal of Political Economy*, 63, 101884. Available from: <https://doi.org/10.1016/j.ejpoleco.2020.101884>
- Schaling, E. & Nolan, C. (1998) Monetary policy uncertainty and inflation: the role of central bank accountability. *De Economist*, 146(4), 585–602. Available from: <https://doi.org/10.1023/a:1003494413926>
- Sorensen, J.R. (1991) Political uncertainty and macroeconomic performance. *Economics Letters*, 37(4), 377–381. Available from: [https://doi.org/10.1016/0165-1765\(91\)90074-u](https://doi.org/10.1016/0165-1765(91)90074-u)
- Stein, J. (1989) Cheap talk and the Fed: a theory of imprecise policy announcements. *The American Economic Review*, 79(1), 32–42.
- Westelius, N.J. (2009) Imperfect transparency and shifts in the central bank's output gap target. *Journal of Economic Dynamics and Control*, 33(4), 985–996. Available from: <https://doi.org/10.1016/j.jedc.2008.11.002>

SUPPORTING INFORMATION

Additional supporting information can be found online in the Supporting Information section at the end of this article.

How to cite this article: Ferré, M., Macaya, Ò. & Manzano, C. (2024) Central bank communication: inflation target transparency with fiscal policy. *Contemporary Economic Policy*, 42(4), 642–659. Available from: <https://doi.org/10.1111/coep.12667>

APPENDIX A

Derivations of Equations (4) and (5). Substituting the expression for output given in Equation (1) into the loss function of the central bank and taking the first-order condition, one obtains $\pi - \pi_{CB} + \delta_{CB}(\pi - \pi^e) = 0$, which implies

$$\pi = \frac{\pi_{CB}}{1 + \delta_{CB}} + \frac{\delta_{CB}}{1 + \delta_{CB}} \pi^e. \quad (\text{A1})$$

Taking (conditional) expectations and solving for π^e , it follows that

$$\pi^e = \mathbb{E}[\pi_{CB}|s]. \quad (\text{A2})$$

Substituting (A2) into (A1), Equation (4) is derived. Finally, using Equations (4) and (A2) in (1), we obtain Equation (5). \square

Proof of Proposition 1. Taking expectations in Equations (4) and (5), we have $\mathbb{E}[\pi] = \bar{\pi}_{CB}$ and $\mathbb{E}[x] = 0$. Therefore, these expected values are not affected by the degree of central bank transparency. In addition, it can be shown that the variability of inflation and output are given by $\text{var}[\pi] = \frac{\mathcal{A}\delta_{CB}^2 + 2\mathcal{A}\delta_{CB} + 1}{(1 + \delta_{CB})^2} \sigma_{\pi_{CB}}^2$ and $\text{var}[x] = \frac{1 - \mathcal{A}}{(1 + \delta_{CB})^2} \sigma_{\pi_{CB}}^2$, where Equation (3) is used to substitute for σ_v^2 . These expressions allow us to conclude that greater transparency (higher \mathcal{A}) increases the variability of inflation and reduces the volatility of output. \square

Proof of Proposition 2. After some algebra, using $\sigma_v^2 = (\frac{1}{\mathcal{A}} - 1) \sigma_{\pi_{CB}}^2$, we get

$$\text{var}[\pi] = \frac{\delta_{CB}(\delta_G - \delta_{CB})(2\Delta + \delta_G\delta_{CB} - \delta_{CB}^2)\mathcal{A} + \Delta^2}{\Delta^2(1 + \delta_{CB})^2} \sigma_{\pi_{CB}}^2 \quad \text{and}$$

$$\text{var}[x] = \frac{(\Delta^2 - \mathcal{A}(\delta_G - \delta_{CB})(2\Delta - \delta_G + \delta_{CB}))}{\Delta^2(1 + \delta_{CB})^2} \sigma_{\pi_{CB}}^2.$$

Using the expression of Δ given in Equation (12), it follows that

$$\frac{\partial \text{var}[\pi]}{\partial \mathcal{A}} = \frac{\sigma_{\pi_{CB}}^2 \delta_{CB}(\delta_G - \delta_{CB})(2\delta_G + 2\gamma_G + (6\gamma_G + \delta_G)\delta_{CB} + (4\gamma_G + 1)\delta_{CB}^2)}{(1 + \delta_{CB})^2(\Delta)^2} \quad \text{and} \quad (\text{A3})$$

$$\frac{\partial \text{var}[x]}{\partial \mathcal{A}} = -\frac{\sigma_{\pi_{CB}}^2 (\delta_G - \delta_{CB})(\delta_G + 2\gamma_G + (6\gamma_G + 1)\delta_{CB} + 2(2\gamma_G + 1)\delta_{CB}^2)}{(1 + \delta_{CB})^2(\Delta)^2}. \quad (\text{A4})$$

Therefore, $\frac{\partial \text{var}[\pi]}{\partial \mathcal{A}} > 0$ if and only if $\delta_G > \delta_{CB}$, and $\frac{\partial \text{var}[x]}{\partial \mathcal{A}} > 0$ if and only if $\delta_G < \delta_{CB}$. Finally, by taking expectations in Equations (13) and (14), it follows that $\mathbb{E}[\pi]$ and $\mathbb{E}[x]$ are not affected by the degree of transparency of the central bank. \square

Proof of Corollary 3. a) Differentiating Equation (A3) with respect to δ_{CB} , we have

$$\frac{\partial}{\partial \delta_{CB}} \left(\frac{\partial \text{var}[\pi]}{\partial \mathcal{A}} \right) = \sigma_{\pi_{CB}}^2 \frac{2P(\delta_{CB})}{(1 + \delta_{CB})^2 (\delta_{CB}^2 + \delta_G + \gamma_G(\delta_{CB} + 1)(2\delta_{CB} + 1))^2},$$

where $P(\delta) = p_6\delta^6 + p_5\delta^5 + p_4\delta^4 + p_3\delta^3 + p_2\delta^2 + p_1\delta + p_0$, with

$$\begin{aligned}
 p_6 &= (2\gamma_G + 1)(4\gamma_G + 1), \quad p_5 = 3\gamma_G(3 - 2\delta_G)(2\gamma_G + 1), \\
 p_4 &= (-2(2\gamma_G + 1)\delta_G^2 - \delta_G(10\gamma_G + 34\gamma_G^2 - 3) + \gamma_G(7\gamma_G + 3)), \\
 p_3 &= -((13\gamma_G + 6)\delta_G^2 + 4\delta_G\gamma_G(8\gamma_G + 3) + 10\gamma_G^2), \\
 p_2 &= -3(3\gamma_G + \delta_G(3\gamma_G + 1))(\gamma_G + \delta_G), \\
 p_1 &= 2(-\gamma_G + \delta_G(\gamma_G - 1))(\gamma_G + \delta_G), \quad \text{and } p_0 = \delta_G(\gamma_G + \delta_G)^2.
 \end{aligned}$$

Next, we show that $P(\delta)$ has two positive roots. In order to prove that, we distinguish two cases: (1) $3 \geq 2\delta_G$, and (2) $3 < 2\delta_G$.

Case 1. If $3 \geq 2\delta_G$, independently of the signs of coefficients p_4 and p_1 , $P(\delta)$ has two changes in the sign of its coefficients.

Case 2. If $3 < 2\delta_G$, then the coefficients of p_5 and p_4 are negative. Consequently, independently of the sign of the coefficient of p_1 , we can state that $P(\delta)$ has two changes in the sign of its coefficients.

Then, applying the Descartes' rule, we conclude that the maximum number of positive roots of $P(\delta)$ is two. In addition, since $P(0) > 0$, $P(\delta_G) < 0$ and $\lim_{\delta \rightarrow \infty} P(\delta) = \infty$, we can conclude that $P(\delta)$ has two positive roots. Hence, taking into account the previous expression for $\frac{\partial}{\partial \delta_{CB}} \left(\frac{\partial \text{var}[\pi]}{\partial \mathcal{A}} \right)$, it follows that there exist two values of δ_{CB} , denoted by $\underline{\delta}$ and $\bar{\delta}$, with $\underline{\delta} < \delta_G < \bar{\delta}$, such that $\frac{\partial}{\partial \delta_{CB}} \left(\frac{\partial \text{var}[\pi]}{\partial \mathcal{A}} \right) > 0$ whenever $\delta_{CB} < \underline{\delta}$, $\frac{\partial}{\partial \delta_{CB}} \left(\frac{\partial \text{var}[\pi]}{\partial \mathcal{A}} \right) < 0$ whenever $\underline{\delta} < \delta_{CB} < \bar{\delta}$, and $\frac{\partial}{\partial \delta_{CB}} \left(\frac{\partial \text{var}[\pi]}{\partial \mathcal{A}} \right) > 0$ whenever $\delta_{CB} > \bar{\delta}$. By taking absolute values, the results stated in Corollary 3.a are satisfied.

b) Differentiating Equation (A4) with respect to δ_{CB} , we get

$$\frac{\partial}{\partial \delta_{CB}} \left(\frac{\partial \text{var}[x]}{\partial \mathcal{A}} \right) = \frac{2Q(\delta_{CB})}{(1 + \delta_{CB})^3 (\delta_{CB}^2 + \delta_G + \gamma_G(\delta_{CB} + 1)(2\delta_{CB} + 1))^3} \sigma_{\pi_{CB}}^2,$$

where $Q(\delta) = q_5\delta^5 + q_4\delta^4 + q_3\delta^3 + q_2\delta^2 + q_1\delta + q_0$, with

$$\begin{aligned}
 q_5 &= -3(2\gamma_G + 1)^2, \quad q_4 = (2\gamma_G + 1)(4(2\gamma_G + 1)\delta_G - (17\gamma_G + 3)), \\
 q_3 &= (2\gamma_G + 1)(25\gamma_G + 3)\delta_G - (4\gamma_G + 1)(8\gamma_G + 1), \\
 q_2 &= 3((2\gamma_G + 1)\delta_G^2 + (19\gamma_G^2 + 7\gamma_G + 1)\delta_G - 3\gamma_G^2), \\
 q_1 &= 9\gamma_G\delta_G^2 + (28\gamma_G^2 + 7\gamma_G + 1)\delta_G + \gamma_G(2\gamma_G + 1), \quad \text{and} \\
 q_0 &= \delta_G^3 + 3\gamma_G\delta_G^2 + \gamma_G(5\gamma_G + 1)\delta_G + \gamma_G^2.
 \end{aligned}$$

Next, we show that $Q(\delta)$ has a unique positive root. In order to prove that, we distinguish two cases: (1) $\delta_G \geq \frac{17\gamma_G+3}{4(2\gamma_G+1)}$, and (2) $\delta_G < \frac{17\gamma_G+3}{4(2\gamma_G+1)}$.

Case 1. If $\delta_G \geq \frac{17\gamma_G+3}{4(2\gamma_G+1)}$, it can be shown that all the coefficient of $Q(\delta)$, except the coefficient q_5 , are negative. Therefore, we can state that $Q(\delta)$ has one change in the sign of its coefficients.

Case 2. If $\delta_G < \frac{17\gamma_G+3}{4(2\gamma_G+1)}$ (and therefore, $q_4 < 0$), then we distinguish two subcases: (2.1) $\delta_G \geq \frac{(4\gamma_G+1)(8\gamma_G+1)}{(2\gamma_G+1)(25\gamma_G+3)}$, and (2.2) $\delta_G < \frac{(4\gamma_G+1)(8\gamma_G+1)}{(2\gamma_G+1)(25\gamma_G+3)}$.

Case 2.1. If $\delta_G \geq \frac{(4\gamma_G+1)(8\gamma_G+1)}{(2\gamma_G+1)(25\gamma_G+3)}$ (and therefore, $q_3 \geq 0$), then it can be shown that q_2 is also positive, and hence, that $Q(\delta)$ has one change in the sign of its coefficients.

Case 2.2. If $\delta_G < \frac{(4\gamma_G+1)(8\gamma_G+1)}{(2\gamma_G+1)(25\gamma_G+3)}$ (and therefore, $q_3 < 0$), then, independent of the sign of the coefficient q_2 , one can see that $Q(\delta)$ has one change in the sign of its coefficients.

Then, applying the Descartes' rule, we conclude that the maximum number of positive roots of $Q(\delta)$ is one. In addition, since $Q(0) > 0$ and $\lim_{\delta \rightarrow \infty} Q(\delta) = -\infty$, we conclude that $Q(\delta)$ has a unique root, denoted by $\tilde{\delta}$. Moreover, as $Q(\delta_G) > 0$, we conclude that $\delta_G < \tilde{\delta}$. Hence, taking into account the previous expression for $\frac{\partial}{\partial \delta_{CB}} \left(\frac{\partial \text{var}[x]}{\partial \mathcal{A}} \right)$, it follows that $\frac{\partial}{\partial \delta_{CB}} \left(\frac{\partial \text{var}[x]}{\partial \mathcal{A}} \right) > 0$ whenever $\delta_{CB} < \tilde{\delta}$, and $\frac{\partial}{\partial \delta_{CB}} \left(\frac{\partial \text{var}[x]}{\partial \mathcal{A}} \right) < 0$ whenever $\delta_{CB} > \tilde{\delta}$. By taking absolute values, the results stated in Corollary 3.b are satisfied. \square

Proof of Proposition 4. Using (15) and after some algebra, it follows that the expected loss for the monetary authority in equilibrium satisfies

$$\mathbb{E}[L_{CB}] = \frac{1}{2} \delta_{CB} (1 + \delta_{CB}) ((\mathbb{E}[x])^2 + \text{var}[x]).$$

In addition, Proposition 2 shows that $\mathbb{E}[x]$ is independent of the degree of transparency of the central bank, while $\text{var}[x]$ decreases with the degree of transparency of the central bank whenever $\delta_G > \delta_{CB}$. This leads us to conclude that the expected loss of the central bank decreases with its degree of transparency if and only if $\delta_G > \delta_{CB}$. \square

Proof of Proposition 5. The unconditional expected loss of the government can be written as

$$\mathbb{E}[L_G] = \frac{1}{2} [(\mathbb{E}[\pi - \bar{\pi}_G])^2 + \delta_G (\mathbb{E}[x])^2 + \gamma_G (\mathbb{E}[g - \bar{g}])^2 + \text{var}[\pi] + \delta_G \text{var}[x] + \gamma_G \text{var}[g]].$$

Direct computations yield that $\mathbb{E}[\pi - \bar{\pi}_G]$, $\mathbb{E}[x]$ and $\mathbb{E}[g - \bar{g}]$ are not affected by \mathcal{A} . In addition, after some algebra, using $\sigma_v^2 = \left(\frac{1}{\mathcal{A}} - 1\right) \sigma_{\pi_{CB}}^2$, we get

$$\text{var}[g] = \frac{((1 + \delta_{CB})^2 (\delta_G - \delta_{CB})^2 - \Delta^2) \mathcal{A} + \Delta^2}{\Delta^2 (1 + \delta_{CB})^2} \sigma_{\pi_{CB}}^2.$$

Taking into account Equations (A3) and (A4), the previous expression and the expression of Δ given in Equation (12), we have

$$\frac{\partial \mathbb{E}[L_G]}{\partial \mathcal{A}} = \frac{-\gamma_G (\delta_{CB}^2 + \delta_G + \gamma_G (1 + \delta_{CB}) (1 + 2\delta_{CB}))^2 - (\delta_G - \delta_{CB})^2 (\delta_{CB}^2 + \delta_G + \gamma_G (1 + \delta_{CB}) (1 + 3\delta_{CB}))}{2\Delta^2 (1 + \delta_{CB})^2} \sigma_{\pi_{CB}}^2.$$

Therefore, $\frac{\partial \mathbb{E}[L_G]}{\partial \mathcal{A}} < 0$, which implies that an increase in \mathcal{A} makes the government better off. \square

APPENDIX B

We will show here that the main results of the paper are not affected by the inclusion of a supply shock. We will adopt the same framework as before, but now output is given by

$$x = \pi - \pi^e - \tau + \varepsilon,$$

where π and π^e are the actual and expected inflation rates, τ represents the tax rate levied on output and ε is a productivity shock, with $\varepsilon \sim N(0, \sigma_\varepsilon^2)$ and independent of the other random variables of the model.

The timing of events is as follows: firstly, the central bank's inflation target, π_{CB} , is realized, but observed only by the central bank. Subsequently, the private sector and the government observe the same risky signal of this target (s), and the private sector uses this signal to rationally form its inflation expectations. Afterward, the productivity shock ε occurs. Then, fiscal-monetary interactions involve a Stackelberg game, where the fiscal authority takes the leading role and the monetary authority acts as the follower.

We apply backward induction and, using similar computations as in Section 3, obtain

$$\pi = \pi^{NSS} - \frac{\delta_{CB}\gamma_G(1 + 2\delta_{CB})}{(1 + \delta_{CB})(\delta_G + \gamma_G(1 + 2\delta_{CB}))} \varepsilon, \tag{B1}$$

with

$$\pi^{NSS} = \frac{\pi_{CB}}{1 + \delta_{CB}} + \frac{\delta_{CB}(\delta_G - \delta_{CB})}{(1 + \delta_{CB})\Delta} \mathbb{E}[\pi_{CB}|s] + \frac{\delta_{CB}(\delta_{CB}\bar{\pi}_G + \gamma_G(2\delta_{CB} + 1)\bar{g})}{\Delta}.$$

The first term in Equation (B1), denoted by π^{NSS} , is the optimal inflation in the case that there are no supply shocks. Direct computations yield

$$\mathbb{E}[\pi] = \mathbb{E}[\pi^{NSS}] \text{ and } \text{var}[\pi] = \text{var}[\pi^{NSS}] + \left(\frac{\delta_{CB}\gamma_G(1 + 2\delta_{CB})}{(1 + \delta_{CB})(\delta_G + \gamma_G(1 + 2\delta_{CB}))} \right)^2 \sigma_\varepsilon^2.$$

Concerning output, the first-order condition of the optimization problem of the central bank implies $x = \frac{1}{\delta_{CB}}(\pi_{CB} - \pi)$. Using Equation (B1), it follows that

$$x = x^{NSS} + \frac{\gamma_G(1 + 2\delta_{CB})}{(1 + \delta_{CB})(\delta_G + \gamma_G(1 + 2\delta_{CB}))} \varepsilon, \tag{B2}$$

with $x^{NSS} = \frac{1}{\delta_{CB}}(\pi_{CB} - \pi^{NSS})$ and x^{NSS} is the optimal level of output in the case where there are no supply shocks. Moreover, $\mathbb{E}[x] = \mathbb{E}[x^{NSS}]$ and

$$\text{var}[x] = \text{var}[x^{NSS}] + \left(\frac{\gamma_G(1 + 2\delta_{CB})}{(1 + \delta_{CB})(\delta_G + \gamma_G(1 + 2\delta_{CB}))} \right)^2 \sigma_\varepsilon^2.$$

Note that the inclusion of a supply shock increases the volatility of macroeconomic variables without affecting their expected values. Consequently, we can deduce that changes in transparency levels do not alter the average levels of inflation and output within this extended framework. Furthermore, since the terms involving ε in Equations (B1) and (B2) are not affected by the accuracy of the signal s , the impact of central bank transparency on inflation and output variability is the same as that in the model without supply shocks.

APPENDIX C

We extend the previous model assuming that the output target of the central bank is uncertain. Suppose that the loss functions of policymakers are given by:

$$L_{CB} = \frac{1}{2}((\pi - \pi_{CB})^2 + \delta_{CB}(x - x_{CB})^2) \text{ and}$$

$$L_G = \frac{1}{2}((\pi - \bar{\pi}_G)^2 + \delta_G(x - \bar{x}_G)^2 + \gamma_G(g - \bar{g})^2).$$

In this extension, the central bank's targets are allowed to be both stochastic, with $\pi_{CB} \sim N(\bar{\pi}_{CB}, \sigma_{\pi_{CB}}^2)$, $x_{CB} \sim N(\bar{x}_{CB}, \sigma_{x_{CB}}^2)$ and $\text{cov}(\pi_{CB}, x_{CB}) = 0$. The central bank's targets, π_{CB} and x_{CB} , are only known perfectly by the

central bank, while both the private sector and the government observe risky signals of these targets, denoted by s_1 , where $s_1 = \pi_{CB} + \nu_1$ and $s_2 = x_{CB} + \nu_2$, where ν_i , $i = 1, 2$, is a white noise, that is, $\nu_i \sim N(0, \sigma_{\nu_i}^2)$ and they are uncorrelated between them and with the central bank's targets. Note that

$$\mathbb{E}[\pi_{CB}|s_1] = \mathcal{A}_1 s_1 + (1 - \mathcal{A}_1)\bar{\pi}_{CB} \quad \text{and} \quad \mathbb{E}[x_{CB}|s_2] = \mathcal{A}_2 s_2 + (1 - \mathcal{A}_2)\bar{x}_{CB},$$

where $\mathcal{A}_1 = \frac{\sigma_{\pi_{CB}}^2}{\sigma_{\pi_{CB}}^2 + \sigma_{\nu_1}^2}$ and $\mathcal{A}_2 = \frac{\sigma_{x_{CB}}^2}{\sigma_{x_{CB}}^2 + \sigma_{\nu_2}^2}$, which are measures of accuracy of the corresponding signals. Observe that $0 \leq \mathcal{A}_i \leq 1$, $i = 1, 2$.

The timing of events will be as follows. Firstly, the central bank's targets π_{CB} and x_{CB} are realized but only observed by the central bank. Subsequently, the private sector and the government observe the signals s_1 and s_2 , which are used to rationally form the private sector inflation expectations, π^e . Subsequently, the government and the central bank will choose their policies sequentially, with the government acting first.

We apply backward induction and, using similar computations as in Section 3, we obtain

$$\begin{aligned} \pi = & \frac{\pi_{CB} + \delta_{CB}x_{CB}}{1 + \delta_{CB}} \\ & + \frac{\delta_{CB}(\delta_G - \delta_{CB})}{(1 + \delta_{CB})\Delta} (\mathbb{E}[\pi_{CB}|s_1] + \delta_{CB}\mathbb{E}[x_{CB}|s_2]) \\ & + \frac{\delta_{CB}(\delta_{CB}\bar{\pi}_G - \delta_G\bar{x}_G + \gamma_G(1 + 2\delta_{CB})\bar{g})}{\Delta} \end{aligned} \quad (C1)$$

In relation to output, the first-order condition of the optimization problem of the central bank implies $x = x_{CB} + \frac{\pi_{CB} - \pi}{\delta_{CB}}$. Using Equation (C1) in the expression of the output, we have that

$$\begin{aligned} x = & \frac{\pi_{CB} + \delta_{CB}x_{CB}}{1 + \delta_{CB}} - \frac{\delta_G - \delta_{CB}}{(1 + \delta_{CB})\Delta} (\mathbb{E}[\pi_{CB}|s_1] + \delta_{CB}\mathbb{E}[x_{CB}|s_2]) \\ & - \frac{\delta_{CB}\bar{\pi}_G - \delta_G\bar{x}_G + \gamma_G(1 + 2\delta_{CB})\bar{g}}{\Delta} \end{aligned}$$

Proposition C.1. *In the case that both central bank's targets are uncertain, greater transparency (higher \mathcal{A}_1 or higher \mathcal{A}_2) increases the variability of inflation and reduces the volatility of output whenever $\delta_G > \delta_{CB}$. The opposite results hold when $\delta_G < \delta_{CB}$. Furthermore, in any case, a change in the degree of transparency does not affect the average levels of inflation and output.*

Proof of Proposition C.1. Taking into account Equation (C1) and after some algebra, we have

$$\begin{aligned} \mathbb{E}[\pi] = & \frac{\delta_G + \gamma_G(1 + 2\delta_{CB})}{\Delta} (\bar{\pi}_{CB} + \delta_{CB}\bar{x}_{CB}) \\ & + \frac{\delta_{CB}(\delta_{CB}\bar{\pi}_G - \delta_G\bar{x}_G + \gamma_G(1 + 2\delta_{CB})\bar{g})}{\Delta} \quad \text{and} \end{aligned} \quad (C2)$$

$$\begin{aligned} \text{var}[\pi] = & \left(\frac{1}{\delta_{CB} + 1} + \frac{\delta_{CB}(\delta_G - \delta_{CB})}{\Delta(1 + \delta_{CB})} \mathcal{A}_1 \right)^2 \sigma_{\pi_{CB}}^2 \\ & + \left(\frac{\delta_{CB}}{\delta_{CB} + 1} + \frac{\delta_{CB}^2(\delta_G - \delta_{CB})}{\Delta(1 + \delta_{CB})} \mathcal{A}_2 \right)^2 \sigma_{x_{CB}}^2 + \left(\frac{\delta_{CB}(\delta_G - \delta_{CB})}{\Delta(1 + \delta_{CB})} \mathcal{A}_1 \right)^2 \sigma_{\nu_1}^2 + \left(\frac{\delta_{CB}^2(\delta_G - \delta_{CB})}{\Delta(1 + \delta_{CB})} \mathcal{A}_2 \right)^2 \sigma_{\nu_2}^2. \end{aligned}$$

Given that $\sigma_{v_1}^2 = \left(\frac{1}{\mathcal{A}_1} - 1\right)\sigma_{\pi_{CB}}^2$ and $\sigma_{v_2}^2 = \left(\frac{1}{\mathcal{A}_2} - 1\right)\sigma_{x_{CB}}^2$ and using the expression of Δ , we have

$$\begin{aligned} \text{var}[\pi] &= \frac{\sigma_{\pi_{CB}}^2 + \delta_{CB}^2\sigma_{x_{CB}}^2}{(1 + \delta_{CB})^2} \\ &+ \frac{\delta_{CB}(\delta_G - \delta_{CB})(\delta_{CB}^2 + \delta_G\delta_{CB} + 2\delta_G + 2\gamma_G(\delta_{CB} + 1)(2\delta_{CB} + 1))}{\Delta^2(1 + \delta_{CB})^2} \\ &\times \left(\mathcal{A}_1\sigma_{\pi_{CB}}^2 + \delta_{CB}^2\mathcal{A}_2\sigma_{x_{CB}}^2\right). \end{aligned} \tag{C3}$$

Concerning output, recall that $x = x_{CB} + \frac{\pi_{CB} - \pi}{\delta_{CB}}$. Therefore,

$$\mathbb{E}[x] = \bar{x}_{CB} + \frac{\bar{\pi}_{CB} - \mathbb{E}[\pi]}{\delta_{CB}} \tag{C4}$$

and, using Equation (C1) and after some algebra, it follows that

$$\text{var}[x] = \left(\frac{\Delta - \mathcal{A}_1(\delta_G - \delta_{CB})}{\Delta(1 + \delta_{CB})}\right)^2 \sigma_{\pi_{CB}}^2 + \left(\delta_{CB}\frac{\Delta - \mathcal{A}_2(\delta_G - \delta_{CB})}{\Delta(1 + \delta_{CB})}\right)^2 \sigma_{x_{CB}}^2 + \left(\frac{\delta_G - \delta_{CB}}{\Delta(1 + \delta_{CB})}\mathcal{A}_1\right)^2 \sigma_{v_1}^2 + \left(\frac{\delta_{CB}(\delta_G - \delta_{CB})}{\Delta(1 + \delta_{CB})}\mathcal{A}_2\right)^2 \sigma_{v_2}^2.$$

Given that $\sigma_{v_1}^2 = \left(\frac{1}{\mathcal{A}_1} - 1\right)\sigma_{\pi_{CB}}^2$ and $\sigma_{v_2}^2 = \left(\frac{1}{\mathcal{A}_2} - 1\right)\sigma_{x_{CB}}^2$ and using the expression of Δ , we have

$$\begin{aligned} \text{var}[x] &= \frac{\sigma_{\pi_{CB}}^2 + \delta_{CB}^2\sigma_{x_{CB}}^2}{(1 + \delta_{CB})^2} \\ &- \frac{(\delta_G - \delta_{CB})(2\delta_{CB}^2 + \delta_G + \delta_{CB} + 2\gamma_G(\delta_{CB} + 1)(2\delta_{CB} + 1))}{\Delta^2(1 + \delta_{CB})^2} \left(\mathcal{A}_1\sigma_{\pi_{CB}}^2 + \mathcal{A}_2\delta_{CB}^2\sigma_{x_{CB}}^2\right). \end{aligned} \tag{C5}$$

Note that Equations (C2) and (C4) imply that the expected inflation and the expected output depend neither on \mathcal{A}_1 nor on \mathcal{A}_2 . However, Equations (C3) and (C5) show an increase in \mathcal{A}_1 or in \mathcal{A}_2 results in an increase in the variance of inflation and a decrease in the variance of output whenever $\delta_G > \delta_{CB}$. \square