

Suppressing Chaotic Behavior in a Top Drive Drilling System by Means of Fixed-Time Synergetic Control

Abstract

In this paper, the dynamics of an induction motor drive-based oil drilling system with a top drive is considered. The system model, consisting of an eleventh-order set of differential equations, is first derived. Then, its nonlinear dynamics are numerically investigated under variations of parameter values. Using this nonlinear model, the bifurcation diagram and the corresponding Lyapunov exponent spectrum are calculated, revealing undesired subharmonic periodic regimes and chaotic behavior for certain parameter values. The fixed-time synergetic control technique (FTSCT) is employed to stabilize the system toward a desired equilibrium point. The control strategy uses a macro-variable, defined as a linear combination of the speed error and the flux error of the induction motor. The results show that the FTSCT is effective in suppressing undesired oscillations.

Keywords: Chaos, Top Drive, Drilling String, Synergetic Control, Fixed-Time Theory

1 Introduction

Oil exploration and extraction rely on specialized oil drilling systems. The two main parts of a drilling system are the mechanical section and the electrical section. The electrical part (top drive) consists of an electric motor and a gearbox, which provide the rotational motion to the mechanical section. The latter is composed of a series of special pipes connected to each other and, at the end, attached to a drilling bit. The schematic of the drilling system is shown in Fig. 1.

The mechanical section (drilling strings) is a highly nonlinear system due to the frictional impacts between the rocks and the drilling bit during the cutting operation, as well as the friction torque resulting from contact between the borehole and the pipes. This frictional force may cause complex nonlinear vibrations. Similarly, the IM and the AC drive that compose the electrical section are also strongly nonlinear. From the electrical perspective, the nonlinearity is due to the combination of two nonlinear parts: the IM and the AC inverter with a speed control governor.

Both sections of the drilling system are complex nonlinear systems and contain different parameters. Some of these parameters may vary during operation, human

intervention, or aging. Such variations can lead to complex nonlinear oscillations in the drilling system. The oscillations in the dynamic of the drilling strings have been reported in several studies [1–10]. In particular, chaotic oscillations of the drilling string speed, driven by an IFOCIM system, have been reported in [11], where variations in certain parameter values lead to chaotic behavior.

Oscillations arising from bit–rock friction and the nonlinearity of the AC drive can lead to drilling string failure, increased wear, and bit damage—they also raise costs due to increased maintenance or lost time. In 1985, 45% of all drilling system problems were related to drill string failures, with average cost of about \$106,000. Moreover, drill string failures occur in approximately 1 out of 7 drill rigs. In 1991, it was reported that Shell Expro drill string failures cost more than US \$2 million in 2 months [12].

Therefore, the complex chaotic behavior of drilling systems must be controlled and stabilized. The control of chaotic behavior in drilling systems can be addressed using different approaches. For instance, in [13], output-feedback and state-feedback controllers have been proposed to mitigate torsional oscillations in a drilling string, while in [14], a Linear Quadratic Gaussian (LQG) controller has been applied to a two degree of freedom (2-DoF) drilling system model to suppress undesired oscillations. A delay feedback controller has also been used in [15] to control a multi-DoF model of drill strings.

Torsional torque oscillations in a 2-DoF drilling string have been controlled using Sliding Mode Control (SMC) in [16], where good agreement between experimental and numerical results was observed in terms of reducing the stick–slip amplitude. An output-feedback controller combined with an observer was designed in [17] to stabilize a hydraulically driven string system. A state observer and a reference governor were proposed in [18] to eliminate nonlinear stick–slip oscillations in a 2-DoF drill string, the results show that the approach is effective in suppressing undesired oscillations. A fractional-order PID controller was employed in [19] to mitigate stick–slip motion in a 4-DoF system, and it was demonstrated that this fractional-order PID exhibits superior dynamic performance compared to a conventional PID controller. An adaptive output-feedback controller based on the Lyapunov approach was developed in [20] for suppressing torsional oscillations in a one-dimensional offshore drilling system. The effect of adding a spring to the drilling string on longitudinal torque oscillations was experimentally studied in [21], where reductions between 10% and 90% in load fluctuations were reported.

In all the above control methods, the convergence time of the system response is infinite, and the settling time depends on system parameters and initial conditions.

Synergetic Control (SC) is a control approach originating from the Russian literature [22], which exploits the internal nonlinear characteristics of a system. The design of a synergetic controller begins by defining a macro-variable $\psi(x)$, which is a function of the system’s state variables. Similar to Sliding Mode Control (SMC), the objective is to drive the system trajectory toward a manifold defined by $\psi = \{x \mid \psi(x) = 0\}$.

In the simplest form of synergetic control, $\psi(x)$ is a linear combination of the state variables. After selecting the macro-variable, its dynamic evolution is imposed according to a linear first-order differential equation: $T\dot{\psi} + \phi(\psi) = 0$, where $T > 0$ is a design parameter (time constant) that determines the convergence speed toward

the manifold $\psi = 0$. The control law is synthesized based on this evolution equation and the dynamic model of the controlled system. With a linear choice of both the macro-variable and its dynamic evolution, $\psi(x)$ converges to zero, exhibiting infinite convergence time, as is typical for linear systems [23, 24].

SC and SMC are similar in the sense that in both approaches, the system trajectories are driven to a suitable manifold $\Sigma = \{x \mid \psi(x) = 0\}$. However, SMC relies on a discontinuous control action that induces undesired chattering oscillations. In contrast to SMC, SC is chattering-free [25]. Moreover, the convergence time in an SC approach can be made finite by appropriately selecting the macro-variable as a suitable nonlinear function of the state variables but it depends on initial conditions [26]. This approach is referred to as finite-time synergetic control, while the fixed-time synergetic control technique (FTSCT) guarantees convergence to the desired manifold in a fixed time, regardless of initial conditions. These properties make the FTSCT an appropriate choice to mitigate chaotic behavior in drilling systems by ensuring that trajectories in the state space converge to the equilibrium point in an upper limit time [27].

In this paper, the FTSCT is used to suppress chaotic oscillations in complex oil drilling systems. The stability of the system under the proposed FTSCT is guaranteed based on Lyapunov stability theory. The rest of this paper is organized as follows. Section 2 describes the drilling system components. Section 3 begins by presenting the dynamic model of the IM, which consists of a fifth-order differential equation. In the same section, the 2-DoF drilling string model with frictional torque equations and the drilling string with the top drive (IFOCIM and gearbox) are modeled, as detailed in Subsection 3.3. Section 4 shows the effect of parameter changes on the dynamic behavior of a drilling system actuated by IFOCIM. The fixed time theorem is introduced and improved in Section 5. In Section 6, the fixed-time theorem is employed to improve the controller's time response. Section 7 presents the implementation of FTSCT to control the oil drilling system driven by the IFOCIM driver and stabilize the drilling system. The conclusion is provided in Section 8.

2 System Description

The drilling system under study consists of four main parts described by the dynamic equations: (1) AC drive, (2) geared top drive, (3) drilling string, and (4) mud system. The AC drive consists of two parts: a DC-AC converter (inverter) and an IFOC controller. The output of the converter is fed to the squirrel-cage IM, and simultaneously, the IM stator current and rotor speed are used as feedback signals to the controller. The speed loop is used to calculate the estimated motor flux signals applied to the controller. Additionally, the reference values of direct and quadrature currents (i_{ds} and i_{qs}) are generated by the speed and flux PI controllers [28].

The top drive system also consists of two components: an AC motor and a gearbox. The AC motor is a three-phase squirrel-cage IM, while the gearbox reduces the rotation speed. The drilling string includes the drilling pipes, which can be extended for thousands of meters, and is rotated by the top drive, while the drilling collars are

used to add additional weight on the bit for cutting and enhancing drilling string stability. The bit is responsible for the rock-cutting action. Finally, the drilling mud is used to reduce friction, provide cooling, and remove the cuttings from the borehole.

The mechanical part (drilling string) is represented by two disks (2-DoF) coupled by a steel string [29], as shown in Fig. 1. The upper disk is driven by the IM via a gearbox. In this drilling string model, there are two frictional torques: the upper disk frictional torque (T_{fu}) and the lower disk frictional torque (T_{fl}). T_{fu} is generated by the bearings of the motor and the upper disk, while T_{fl} is produced by the bearings of the lower disk and a brake applied to the lower disk (the brake represents the torque caused by friction between the rocks and the drilling bit).

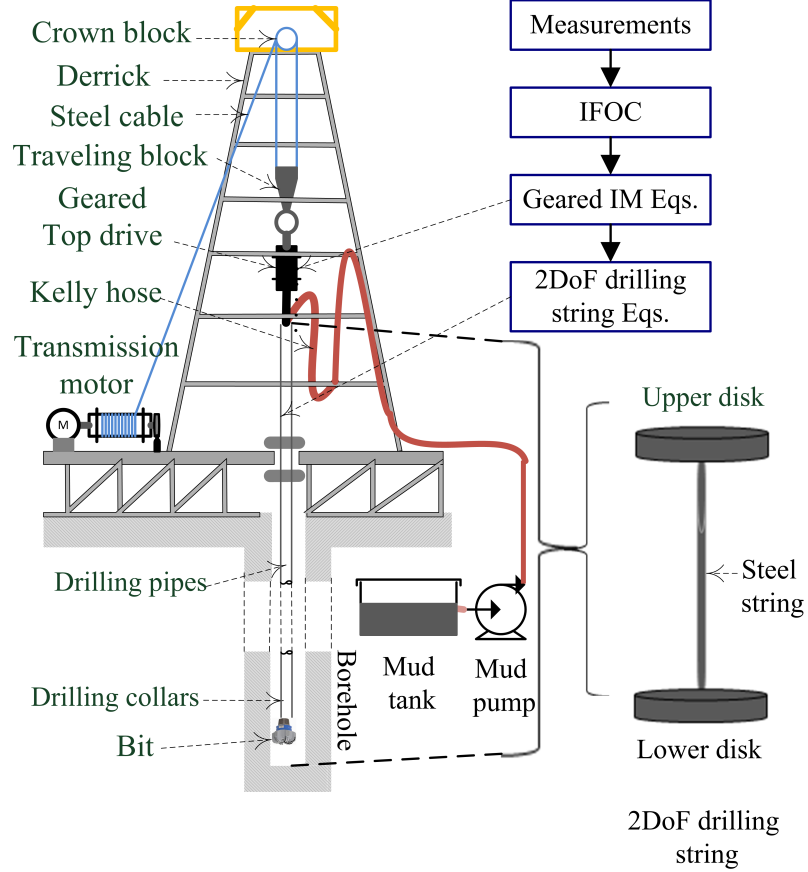


Fig. 1: Scheme of the top drive drilling system (string with IFOCIM drive system) [1].

3 Mathematical Modeling

3.1 Model of the Induction Motor

The abc-to-dq0 transformation in the synchronous reference frame of a 3-phase squirrel-cage IM is given by the following equations [30].

$$\frac{di_{ds}}{dt} = -\gamma i_{ds} + \omega_e i_{qs} + \zeta \beta \psi_{dr} + \beta \omega_r \psi_{qr} + \frac{v_{ds}}{\sigma L_s} \quad (1)$$

$$\frac{di_{qs}}{dt} = -\omega_e i_{ds} - \gamma i_{qs} - \beta \omega_r \psi_{dr} + \zeta \beta \psi_{qr} + \frac{v_{qs}}{\sigma L_s} \quad (2)$$

$$\frac{d\psi_{dr}}{dt} = \zeta L_m i_{ds} - \zeta \psi_{dr} + (\omega_e - \omega_r) \psi_{qr} \quad (3)$$

$$\frac{d\psi_{qr}}{dt} = \zeta L_m i_{qs} - (\omega_e - \omega_r) \psi_{dr} - \zeta \psi_{qr} \quad (4)$$

$$\frac{d\omega_r}{dt} = \frac{P}{2J} \left[\frac{3}{2} \frac{P}{2} \frac{L_m}{L_r} (\psi_{dr} i_{qs} - \psi_{qr} i_{ds}) - T_L - \frac{2}{P} B_m \omega_r \right] \quad (5)$$

where $v_{ds}, v_{qs}, i_{ds}, i_{qs}, \psi_{dr}$, and ψ_{qr} are the stator voltages, currents, and rotor fluxes along the direct (d) and quadrature (q) axes of the synchronous reference frame referred to the stator, respectively; ω_e is the angular synchronous speed of the motor; $\sigma = 1 - \frac{L_m^2}{L_s L_r}$, $\tau_r = \frac{L_r}{R_r}$, $\gamma = \frac{R_s}{\sigma L_s} + \frac{1 - \sigma}{\sigma \tau_r}$, $\beta = \frac{L_m}{\sigma L_s L_r}$, and $\zeta = \frac{1}{\tau_r}$. The parameters used in the model are listed in Table 1.

Table 1: System parameters.

Parameter	Definition	Units
R_s	Stator resistance of the IM	Ω
L_s	Stator inductance of the IM	H
R_r	Rotor resistance of the IM	Ω
L_r	Rotor inductance of the IM	H
L_m	Mutual inductance of the IM	H
J	Rotor inertia of the IM	$\text{N} \cdot \text{m}/\text{rad/s}$
ω_r	Rotor speed of the IM	rad/s
P	Number of poles of the IM stator	—
T_L	Load torque	$\text{N} \cdot \text{m}$
ω_{ref}	Reference speed	rad/s

Table 2: Parameters of the drilling system [31].

Parameter	Definition	Value	Unit
J_u	Upper disk moment of inertia	0.4765	$\text{kg} \cdot \text{m}^2$
J_l	Lower disk moment of inertia	0.035	$\text{kg} \cdot \text{m}^2$
b_u	Viscous friction coefficient at upper disk	2.4245	$\text{kg} \cdot \text{m}^2/(\text{rad} \cdot \text{s})$
Δb_u	Change in the viscous friction coefficient	-0.0084	$\text{kg} \cdot \text{m}^2/(\text{rad} \cdot \text{s})$
T_{su}	Upper disk Coulomb friction torque	0.37975	$\text{N} \cdot \text{m}$
ΔT_{su}	Change in frictional torque at upper disk	-0.00575	$\text{N} \cdot \text{m}$
k_θ	Torsional stiffness of the steel string	0.075	$\text{N} \cdot \text{m}/\text{rad}$
T_{cl}	Coulomb friction torque at lower disk	0.05	$\text{N} \cdot \text{m}$
T_{sl}	Static friction torque at lower disk	0.26	$\text{N} \cdot \text{m}$
b_l	Viscous friction coefficient at lower disk	0.009	$\text{kg} \cdot \text{m}^2/(\text{rad} \cdot \text{s})$
δ_{sl}	Stribeck shape parameter	1.5	—
ω_{sl}	Stribeck reference velocity	2.2	rad/s

3.2 Model of The Drilling String

The mathematical model of the mechanical part (drilling string) consists of the following equations [31]:

$$J_u \dot{\omega}_u + k_\theta (\theta_u - \theta_l) + b (\omega_u - \omega_l) + T_{fu} (\omega_u) = T_e \quad (6)$$

$$J_l \dot{\omega}_l - k_\theta (\theta_u - \theta_l) - b (\omega_u - \omega_l) + T_{fl} (\omega_l) = 0 \quad (7)$$

where $\omega_u, \omega_l, \theta_u, \theta_l, J_u, J_l$ are the rotational speed, angular position, and the inertia constant of the upper and lower disks, respectively, k_θ and b are the stiffness and damping coefficients of the torsional and lateral torques, respectively, and T_e is the electrical motor torque. The frictional torque T_{fu} acting on the upper disk is [31]:

$$T_{fu} (\omega_u) = \begin{cases} T_{cu} (\omega_u) \text{sign} (\omega_u), & \text{for } \omega_u \neq 0 \\ [-T_{su} + \Delta T_{su}, T_{su} + \Delta T_{su}], & \text{for } \omega_u = 0 \end{cases} \quad (8)$$

$$T_{cu} (\omega_u) = T_{su} + \Delta T_{su} \text{sign} (\omega_u) + b_u |\omega_u| + \Delta b_u \omega_u \quad (9)$$

The characteristic equation of the stick-slip effect for T_{fl} is [12]:

$$T_{fl} (\omega_l) = \begin{cases} T_{cl} (\omega_l) \text{sign} (\omega_l), & \text{for } \omega_l \neq 0 \\ [-T_{sl}, T_{sl}], & \text{for } \omega_l = 0 \end{cases} \quad (10)$$

$$T_{cl} (\omega_l) = T_{sl} + (T_{sl} - T_{cl}) e^{-\left| \frac{\omega_l}{\omega_{sl}} \right| \delta_{sl}} + b_l |\omega_l| \quad (11)$$

The system parameters of the mechanical part are listed in Table 2.

3.3 Model of the Top Drive Drilling System

The full-order IM model given in (1)-(5) is used together with the drilling string model given in (6) and (7). Let $x_1 = i_{ds}$, $x_2 = i_{qs}$, $x_3 = \psi_{dr}$, $x_4 = \psi_{qr}$, $x_5 = \omega_u$, $x_6 = (K_{pw} + K_{iw} \int dt)(\omega_{ref} - n_r \omega_u)$, $x_7 = \int (i_{dref} - i_{ds})dt$, $x_8 = \int (i_{qref} - i_{qs})dt$, $x_9 = \omega_l$, $x_{10} = \theta_u$, and $x_{11} = \theta_l$. From the IFOCIM scheme shown in Fig. 2, the model (1)-(5), the model of the string (6) and (7), the drilling system that is formed by a string model driven by the AC drive model can be denoted by the following system of equations [1]:

$$\begin{aligned} \dot{x}_1 = & -\gamma x_1 + \left[n_r x_5 + \frac{\alpha}{\tau_r} L_m x_6 \right] x_2 + \frac{\beta}{\tau_r} x_3 + \beta \frac{L_r}{L_m} K_{pd} \left[\frac{1}{L_m} \psi_{dref} + \frac{\tau_r}{L_m} \dot{\psi}_{dref} - x_4 x_6 - x_1 \right] \\ & + \beta n_r x_5 x_4 + \beta \frac{L_r}{L_m} K_{id} x_7 \end{aligned} \quad (12)$$

$$\dot{x}_2 = - \left[n_r x_5 + \frac{\alpha}{\tau_r} L_m x_6 \right] x_1 - \gamma x_2 - \beta n_r x_5 x_3 + \frac{\beta}{\tau_r} x_4 + \beta \frac{L_r}{L_m} K_{pq} [x_3 x_6 - x_2] + \frac{\beta L_r}{L_m} K_{iq} x_8 \quad (13)$$

$$\dot{x}_3 = \frac{L_m}{\tau_r} x_1 - \frac{1}{\tau_r} x_3 + \frac{\alpha}{\tau_r} L_m x_4 x_6 \quad (14)$$

$$\dot{x}_4 = \frac{L_m}{\tau_r} x_2 - \frac{1}{\tau_r} x_4 - \frac{\alpha}{\tau_r} L_m x_3 x_6 \quad (15)$$

$$\dot{x}_5 = -J_u^{-1} [b(x_5 - x_9) + k_\theta(x_{10} - x_{11}) + T_{fu}(x_5) - n_r c_5(x_2 x_3 - x_1 x_4)] \quad (16)$$

$$\dot{x}_6 = -n_r K_{p\omega} \dot{x}_5 + K_{i\omega} (\omega_{ref} - n_r x_5) \quad (17)$$

$$\dot{x}_7 = -x_1 - x_4 x_6 + \frac{1}{L_m} \psi_{dref} \quad (18)$$

$$\dot{x}_8 = -x_2 + x_3 x_6 \quad (19)$$

$$\dot{x}_9 = J_l^{-1} [b(x_5 - x_9) + k_\theta(x_{10} - x_{11}) - T_{fl}(x_9)] \quad (20)$$

$$\dot{x}_{10} = x_5 \quad (21)$$

$$\dot{x}_{11} = x_9 \quad (22)$$

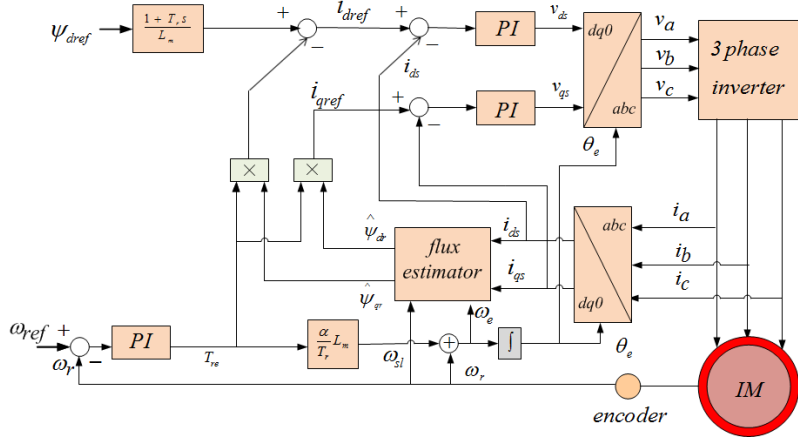
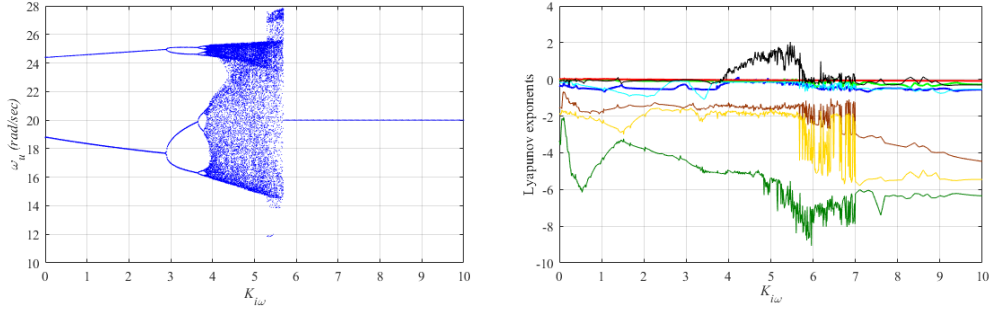


Fig. 2: IFOCIM controller scheme [30].

where $n_r = \frac{1500}{200}$ is the gearbox ratio and $c_5 = \frac{3PL_m}{4L_r}$, K_{pd} , K_{id} , K_{pq} , and K_{iq} are the proportional and integral gains for the i_{ds} , i_{qs} controllers, respectively.

4 System Nonlinear Dynamics and Chaotic Behavior

Using the previous model, the nonlinear dynamics of the induction motor drive-based drilling system were investigated numerically under changes in the system parameters, with the controller parameters: $K_{pd} = K_{pq} = 20$, $K_{id} = K_{iq} = 40$, $K_{p\omega} = 2$, $\psi_{drref} = 0.55$ Wb, $\alpha = 1.3$, and $\omega_{ref} = 150$ rad/s, while $K_{i\omega}$ was chosen as a bifurcation parameter [1]. The IM parameters are listed in Table 3. The results are characterized by the bifurcation diagram and Lyapunov exponents spectrum, as shown in Fig. 3. The bifurcation diagram shows a period-doubling route to chaotic



(a) Bifurcation diagram of the drilling system.

(b) Corresponding largest eight Lyapunov exponents spectrum.

Fig. 3: Simulation results with controller parameters: $K_{pd} = K_{pq} = 20$, $K_{id} = K_{iq} = 40$, $K_{p\omega} = 2$, $\psi_{drref} = 0.55$ Wb, $\alpha = 1.3$, and $\omega_{ref} = 150$ rad/s.

behavior as $K_{i\omega}$ changes from 0 to 5.69, after which the system behavior becomes a fixed point for $K_{i\omega} \geq 5.7$. Furthermore, the Lyapunov spectrum indicates that a periodic behavior occurs for $K_{i\omega}$ from 0 to 3.83, where the largest exponent is zero. For $K_{i\omega}$ from 3.84 to 5.69, a chaotic behavior appears with the largest exponent positive, while all exponents are negative for $K_{i\omega} \geq 5.7$, indicating that the system reaches a steady state (stable fixed point).

5 Fixed-Time Theory

Definition 3.1 [28], [33].

Consider a nonlinear system described by:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}), \quad \mathbf{x}(0) = \mathbf{x}_0, \quad (23)$$

where $\mathbf{x} \in \mathbb{R}$ and $\mathbf{f}(\mathbf{x}) \in \mathbb{R}$.

Assume that the origin is the equilibrium point of the system in (23), then it is said to be a fixed-time stable if it is stable with bounded convergence time $T(x_0)$, that is $\exists T_{max} > 0$, such that $\lim_{x_0 \rightarrow \infty} [T(x_0)] \leq T_{max}$.

Lemma 1 [29]: Consider the following system:

$$\dot{y} = -\alpha_c y^{\frac{m_c}{n_c}} - \beta_c y^{\frac{q}{p}}, \quad y(0) = y_0, \quad (24)$$

where $\alpha_c, \beta_c > 0$, and m_c, n_c, q, p are positive odd integers satisfying $m_c > n_c$ and $0 < \frac{q}{p} < 1$.

If the origin of the system in (24) is stable with convergence time $T(y_0)$, then y will converge to the origin within a fixed-time upper bound T_{max} , such that $\lim_{y_0 \rightarrow \infty} T(y_0) \leq T_{max}$, where T_{max} is given by the following expression

$$T_{max} = \frac{n_c}{\alpha_c(m_c - n_c)} + \frac{p}{\beta_c(p - q)}. \quad (25)$$

Theorem 1: [33] Consider a continuous positive definite function V and assume it satisfies the differential equation:

$$\dot{V} = -\alpha_1 V^{\delta_1} - \beta_1 V^{\delta_2}, \quad V(0) = V_0 \quad (26)$$

where $\alpha_1, \beta_1, \delta_1$, and δ_2 are all positive real numbers satisfying $\delta_1 > 1$ and $0 < \delta_2 < 1$. Then, the time required for the system in (26) to converge to the origin is denoted

Table 3: IM parameters for simulation [32].

Parameter	Value	Parameter	Value
P	4	L_r (H)	0.2235
R_s (Ω)	0.76	L_m (H)	0.2176
L_s (H)	0.2248	J (Nm/rad/sec)	0.111
R_r (Ω)	0.675	B_m (Nm/rad/sec)	7.355×10^{-4}

$T(V_0)$, and V will reach the origin within a fixed-time upper bound $T_{\max}$, such that

$$\lim_{V_0 \rightarrow \infty} T(V_0) \leq T_{\max},$$

where

$$T_{\max} = \frac{1}{\alpha_1(\delta_1 - 1)} + \frac{1}{\beta_1(1 - \delta_2)}. \quad (27)$$

Proof: Rewriting (26) in a suitable form, we obtain:

$$V^{-\delta_2} \dot{V} = -\alpha_1 V^{\delta_1 - \delta_2} - \beta_1 \quad (28)$$

Let $z = V^{1-\delta_2}$. Then, from (28):

$$\dot{z} = (1 - \delta_2) (-\alpha_1 z^{1+\epsilon} - \beta_1) \quad (29)$$

where $\epsilon = \frac{\delta_1 - 1}{1 - \delta_2}$.

The convergence time $T(z_0)$ for z from the initial value z_0 to the origin is then given by:

$$T(z_0) = \frac{1}{1 - \delta_2} \int_{z_0}^0 \frac{dz}{-\alpha_1 z^{1+\epsilon} - \beta_1} \quad (30)$$

Evaluating the integral in the limit as $V_0 \rightarrow \infty$, we have:

$$\begin{aligned} \lim_{V_0 \rightarrow \infty} T(V_0) &= \lim_{z_0 \rightarrow \infty} T(z_0) \\ &= \frac{1}{1 - \delta_2} \left(\lim_{z_0 \rightarrow \infty} \int_0^{z_0} \frac{dz}{\alpha_1 z^{1+\epsilon} + \beta_1} \right) \\ &\leq \frac{1}{1 - \delta_2} \left(\int_0^1 \frac{dz}{\beta_1} + \int_1^\infty \frac{dz}{\alpha_1 z^{1+\epsilon}} \right) \\ &= \frac{1}{\alpha_1(\delta_1 - 1)} + \frac{1}{\beta_1(1 - \delta_2)} = T_{\max}. \end{aligned} \quad (31)$$

6 Fixed-Time Synergetic Control Technique (FTSCT) to Suppress Chaos in Drilling System

The conventional SC technology is based on the concept of self-organization of the controlled systems [23]. SC is a robust control strategy that tolerates inaccuracies in system parameters and modeling errors. The controller design aims to drive the system trajectories toward a stable manifold. Once the trajectories reach the manifold, they guide the system toward the stable equilibrium at the origin. To achieve this, a macro variable ψ is defined according to the characteristics of the nonlinear system. The evolution of the macro variable is designed so that the control action ensures convergence to the stable manifold, which is described by:

$$T\dot{\psi} + \phi(\psi) = 0 \quad (32)$$

where T is a positive design constant that determines the convergence rate toward the stable manifold, and $\phi(\psi)$ is a function chosen to satisfy the following conditions:

- Invertible and differentiable,
- $\phi(0) = 0$, and
- $\phi(\psi)\psi > 0, \quad \forall \psi \neq 0$.

The steady state time of the conventional SCT can be limited by developing it. The finite time concept was used to make the controlled system reach the steady state at a predetermined time related to the initial conditions. Another time limitation method is the fixed time theory (FTT). In this method, the FTT is employed to ensure the response time of SCT is bounded within an upper limit that does not depend on the initial conditions which called FTSCCT [34–38]. In these studies [37–39], designers consider the fixed time technique to develop the controllers.

The drilling system that is controlled by SCT can be rewritten as follows:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}u(\mathbf{x}, t) \quad (33)$$

where $\mathbf{x} \in \mathbb{R}^n$ is the state variables vector, $u(\mathbf{x}, t)$ is the SCT output, and $\mathbf{g} = [0, 0, 0, 0, 0, g_6, 0, 0, 0, 0, 0]^T$, and $\mathbf{f}(\mathbf{x})$ is given as follows

$$\mathbf{f}(\mathbf{x}) = [f_1, f_2, \dots, f_{11}]^T = \begin{bmatrix} -\gamma x_1 + \left(n_r x_5 + \frac{\alpha}{\tau_r} L_m x_6\right) x_2 + \frac{\beta}{\tau_r} x_3 \\ + \beta \frac{L_r}{L_m} K_{pd} \left(\frac{1}{L_m} \psi_{dref} + \frac{\tau_r}{L_m} \dot{\psi}_{dref} - x_4 x_6 - x_1\right) \\ + \beta n_r x_5 x_4 + \beta \frac{L_r}{L_m} K_{id} x_7 \\ - \left(n_r x_5 + \frac{\alpha}{\tau_r} L_m x_6\right) x_1 - \gamma x_2 - \beta n_r x_5 x_3 + \frac{\beta}{\tau_r} x_4 \\ + \beta \frac{L_r}{L_m} K_{pq} (x_3 x_6 - x_2) + \frac{\beta L_r}{L_m} K_{iq} x_8 \\ \frac{L_m}{\tau_r} x_1 - \frac{1}{\tau_r} x_3 + \frac{\alpha}{\tau_r} L_m x_4 x_6 \\ \frac{L_m}{\tau_r} x_2 - \frac{1}{\tau_r} x_4 - \frac{\alpha}{\tau_r} L_m x_3 x_6 \\ - J_u^{-1} [b(x_5 - x_9) + k_\theta(x_{10} - x_{11}) + T_{fu}(x_5) - n_r c_5(x_2 x_3 - x_1 x_4)] \\ - n_r K_{p\omega} \dot{x}_5 + K_{i\omega}(\omega_{ref} - n_r x_5) \\ - x_1 - x_4 x_6 + \frac{1}{L_m} \psi_{dref} \\ - x_2 + x_3 x_6 \\ J_l^{-1} [b(x_5 - x_9) + k_\theta(x_{10} - x_{11}) - T_{f1}(x_9)] \\ x_5 \\ x_9 \end{bmatrix}$$

The controller is applied to the drilling system to stabilize its equilibrium point for a wide range of parameter values and hence eliminate undesired chaotic oscillations. In the IFOCIM driver that is included in the model (12)–(22), two errors can be taken into

account to be the control goal, the error of the speed (e_s) and flux (e_f), respectively :

$$e_s = \omega_{\text{ref}} - \omega_r \quad (34)$$

$$e_f = \psi_{\text{dref}} - \psi_{dr} \quad (35)$$

The desired system response in the steady state is achieved when (34) and (35) vanish. Therefore, the two errors can be combined together to form a macro variable. Defining the following transformation:

$$y_1 = (\omega_{\text{ref}} - \omega_r) - (\psi_{\text{dref}} - \psi_{dr}) \quad (36)$$

The SCT controller output is designed according to (36). The controlled drilling system is given by:

$$\dot{y}_1 = y_2 \quad (37)$$

$$\dot{y}_2 = -n_r \dot{f}_5 + \dot{f}_3 = f_s + g_s u(x, t) \quad (38)$$

where $f_s = -n_r \dot{f}_5 + f_{12}$, $f_{12} = \frac{L_m}{\tau_r} f_1 - \frac{1}{\tau_r} f_3 + \frac{\alpha}{\tau_r} x_6 f_4 + \frac{\alpha}{\tau_r} x_4 f_6$ and $g_s = \frac{\alpha}{\tau_r} x_4 g_6$.

According to the SCT, the macro variable will converge towards the stable manifold $\psi(0)$. The variable ψ can be considered as follows:

$$\psi = k y_1 + y_2 \quad (39)$$

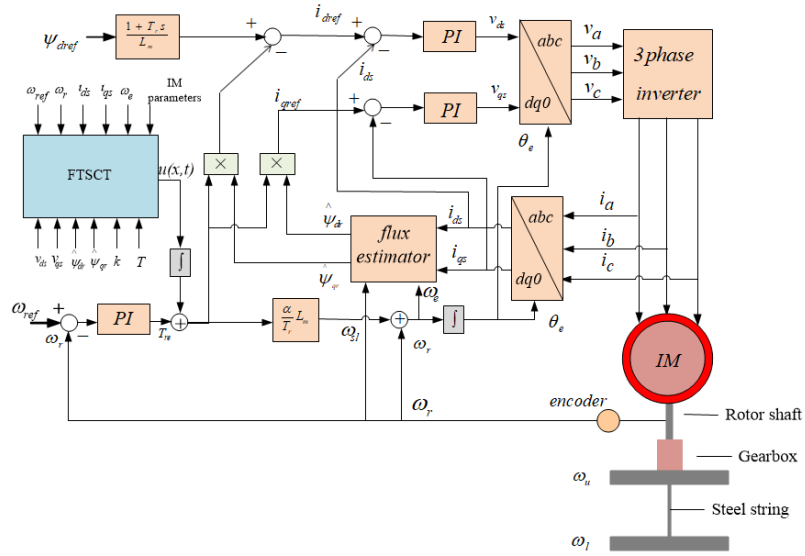


Fig. 4: FTSTC for IFOCIM drilling system.

where k is a positive constant. According to the fixed time theorem, the variable $\phi(\psi)$ is given by:

$$\phi(\psi) = (ky_1 + y_2)^{\frac{p_1}{q_1}} + (ky_1 + y_2)^{\frac{q_1}{p_1}} \quad (40)$$

By arranging (32), (33), (37), and (38), the controller output can be obtained as follows::

$$\mathbf{u}(\mathbf{x}, t) = \frac{-\tau_r}{g_6 \alpha x_4} \left[f_{12} - n_r \dot{f}_5 - k(n_r f_5 - f_3) + \frac{1}{T} \phi(\psi) \right] \quad (41)$$

Fig. 4 represents the final drilling system model scheme.

7 Simulation Results

The parameters of the IFOCIM are set as $K_{pd} = K_{id} = 20$, $K_{pq} = K_{iq} = 40$, $K_{p\omega} = 2$, $K_{i\omega} = 4$, and $\psi_{drref} = 0.55$ Wb, while the reference speed is defined as $\omega_{ref} = 150$ rad/s. The FTSCCT parameters are chosen as $k = 200$, $T = 0.85$ s, $p_1 = 5$, $q_1 = 3$, and $g_6 = 2 \times 10^{-4}$.

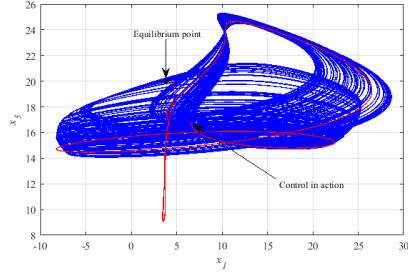
Fig. 5 shows the drilling system dynamics under FTSCCT, with the controller output applied at $t = 100$ s. Fig. 5a denotes the phase portrait projection on (x_1, x_5) . The system states transition from chaotic dynamics to the stable manifold, eventually settling at the equilibrium point. The red part of the plot denotes the system trajectory after the FTSCCT control action is applied. Fig. 5b presents the upper disk speed time response, showing tracking of the reference speed with a settling time of $t_{ss} = 2.04$ s, a rise time of $t_r = 0.2$ s, and an overshoot of $M_p = 0.55$. Fig. 5c illustrates the speed error before and after activation of the FTSCCT controller. Figures 5d, 5e, 5f, and 5g show the time responses of the dq currents and fluxes of the IM, respectively.

The functionality of the FTSCCT controller is further evaluated under sudden reference speed changes. For the same system parameters, the reference speed is stepped to five levels after stabilization of the chaotic speed, as shown in Fig. 6, where Fig. 6a demonstrates the x_5 response and Fig. 6b shows x_1 . Fig. 7a illustrates the enlarged step-change cases. Fig. 7b presents the time response when the reference speed of the motor decreases from 150 rad/s to 100 rad/s, requiring 1.5 s to reach the new steady state. In Fig. 7c and Fig. 7d, the reference speed changes from 100 rad/s to 50 rad/s and from 50 rad/s to 125 rad/s, with settling times of 2.24 s and 1.55 s, respectively. Finally, the reference speed is altered from 125 rad/s to 150 rad/s and from 150 rad/s to 75 rad/s, as shown in Fig. 7e and Fig. 7f. In each case, the output speed successfully tracks the new reference, with settling times of 2.17 s and 1.98 s, respectively. For all sudden changes, the rise time t_r ranges between 0.34 s and 0.5 s, with no overshoot. Fig. 8 shows enlarged subplots of x_1 in Fig. 6a. The results indicate that the quadrature current response is slower than the speed response, leading to a larger settling time.

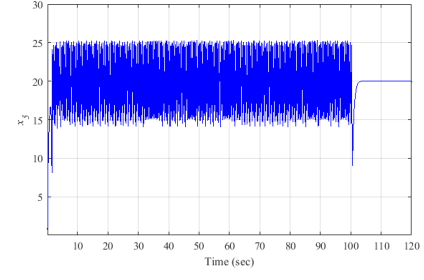
The system performance parameters under the FTSCCT controller (M_p , t_r , t_f , and t_{ss}) are listed in Table 4.

8 Conclusion

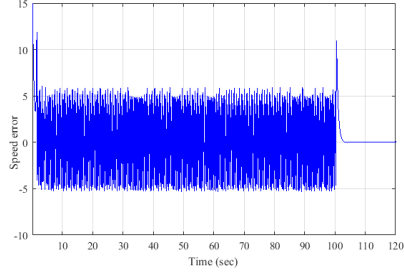
In this work, the modeling and control of a top-drive drilling system for oil exploration have been investigated. The system dynamics were shown to exhibit periodic, chaotic,



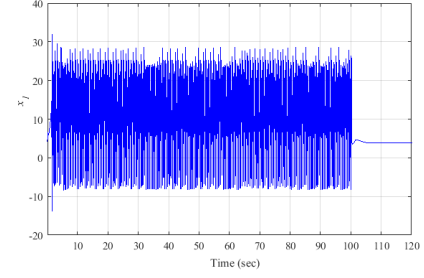
(a)



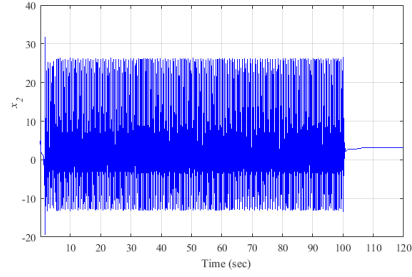
(b)



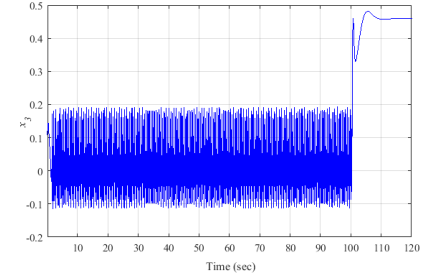
(c)



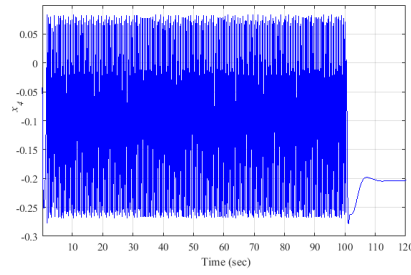
(d)



(e)



(f)



(g)

Fig. 5: Drilling system response under FTSC at $K_{pd} = K_{id} = 20$, $K_{pq} = K_{iq} = 40$, $K_{p\omega} = 2$, $K_{i\omega} = 4$, $\psi_{drref} = 0.55$ Wb, $\omega_{ref} = 150$. $k = 200$, $T =$, $p_1 =$, $q_1 =$, and $g_6 = 2 \times 10^{-4}$. (a) x_1, x_5 phase portrait, (b) x_5 time series, (c) Speed error time series, (d) x_1 time series, (e) x_2 time series, (f) x_3 time series, (g) x_4 time series.

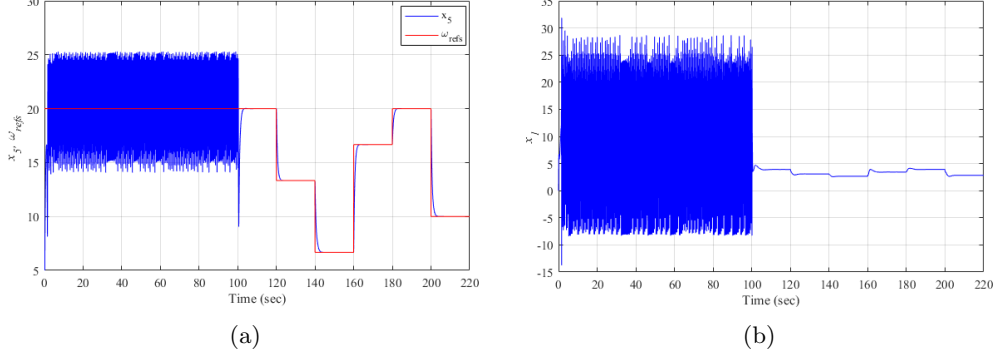


Fig. 6: Drilling system under FTSCCT with sudden change in the reference speed, the system parameters as in caption of Fig. 5, (a) x_5 time response, and (b) x_1 time response. $\omega_{refs} = \omega_{ref}/n_r$.

Table 4: System Performance.

Figure No.	M_p	t_r (s)	t_f (s)	t_{ss} (s)
(8a)	4	0.3	1.1	2.1
(8b)	0.16		0.9	3.78
(8c)	0.4		1	2.44
(8d)	0.13	1		3.74
(8e)	0.06	0.7		3.22
(8f)	0.75		1	3.52

and fixed-point behaviors, which were identified through the Lyapunov exponent spectrum. To suppress the undesired dynamics, the fixed-time synergetic control technique (FTSCT) was applied to stabilize the chaotic response of the drilling system driven by an indirect field-oriented controlled (IFOC) induction motor. The FTSCT output was formulated based on the errors in motor speed and rotor flux, ensuring robust corrective action. The two-dimensional phase portrait of the quadrature current and upper disk speed highlighted the effectiveness of the controller in directing the system trajectories toward stability. Complementary time-series responses of the upper disk speed, speed error, dq -currents, and dq -fluxes further demonstrated the controlled system behavior. The controller was additionally tested under sudden changes in the reference speed, confirming its capability to promptly recover and accurately track new commands. Overall, the simulation results validated that the proposed FTSCT can effectively suppress chaotic dynamics and guarantee stabilization of the drilling system within a fixed-time bound.

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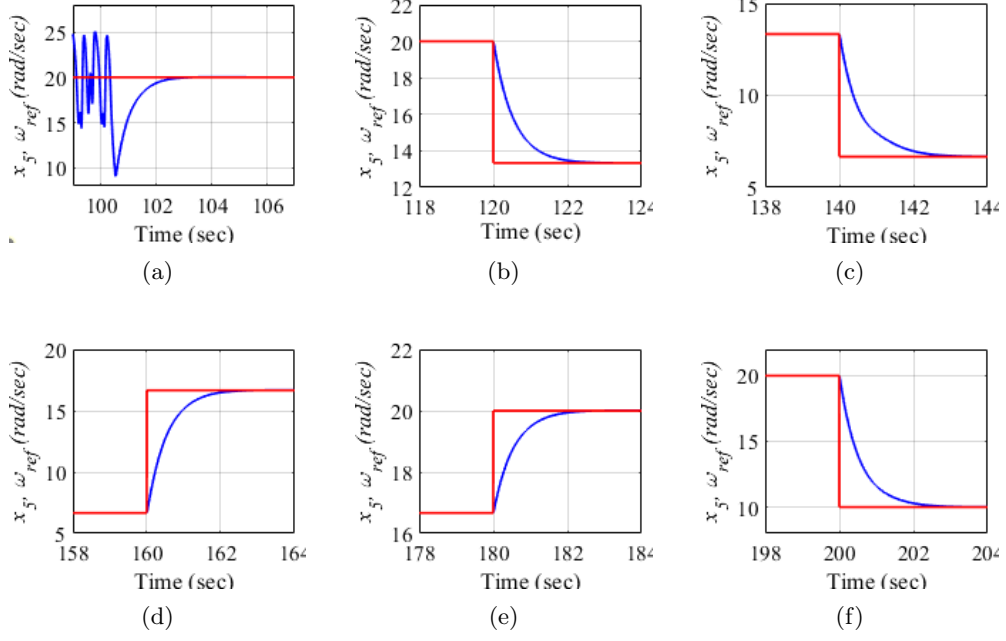


Fig. 7: Enlarged x_5 time response under FTSC with sudden change in the reference speed. The system parameters as in the caption of Fig. 5. (a) $u(t)$ is applied at $t = 100$ sec, the reference speed is changed in (b) from 150 to 100 rad/sec, (c) from 100 to 50 rad/sec, (d) from 50 to 125 rad/sec, (e) from 125 to 150 rad/sec, and (f) from 150 to 75 rad/sec, where, $x_5 = \frac{\omega_r}{n_r}$.

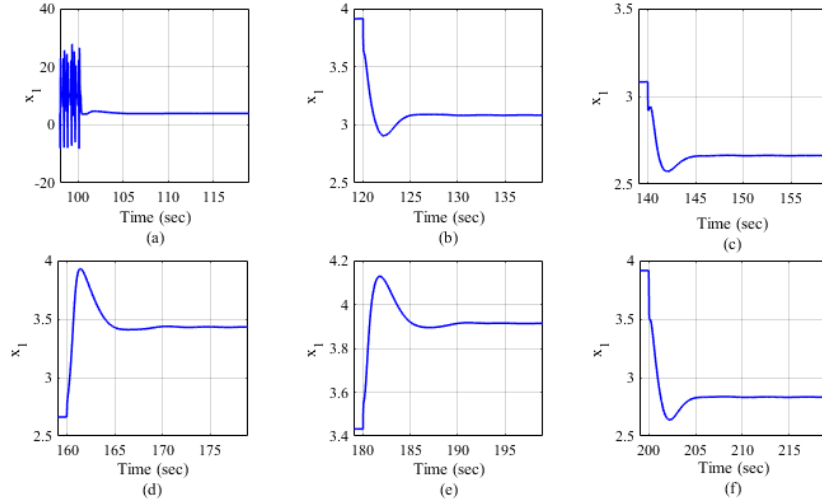


Fig. 8: Enlarged x_1 time response under FTSC with sudden change in the reference speed. The system parameters as selected in the caption of Fig. 5.

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