

Bogdanov–Takens Bifurcation in a Bidirectional DC–DC Converter Supplying a Constant Power Load

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Abstract—Over the decades, bifurcation theory has emerged as a significant area of research, providing deep insights into the complex dynamics of systems across multiple disciplines. Moreover, it serves as a foundation for devising effective control methodologies aimed at avoiding or delaying undesirable dynamical transitions. This paper deals with both local and global dynamics of a bidirectional dc-dc boost converter supplying a constant power load (CPL) with a stabilizing resistor inserted in series with the main inductor. Numerical simulations performed on the averaged model of the system show interesting bifurcation phenomena explaining its local and global dynamical behavior. In particular, it is shown that in some parametric region, the system has two coexisting equilibria, one of them being a saddle and the other one an anti-saddle. The latter can be stable or unstable. An unstable limit cycle also coexists with the stable anti-saddle equilibrium. This limit cycle disappears through a homoclinic bifurcation. Moreover, a parameter space reduction is carried out by choosing suitable bifurcation parameters, and the normal form of the Bogdanov–Takens bifurcation is obtained hence mathematically demonstrating its existence. Finally, the analytical and simulation results on the switched model are partially validated by experimental measurements from a laboratory prototype.

Index Terms—DC–DC converters, bidirectional, constant power load, homoclinic bifurcation, Bogdanov–Takens bifurcation.

I. INTRODUCTION

THE increasing demand for the use of distributed power systems has led to the adoption of a cascaded power

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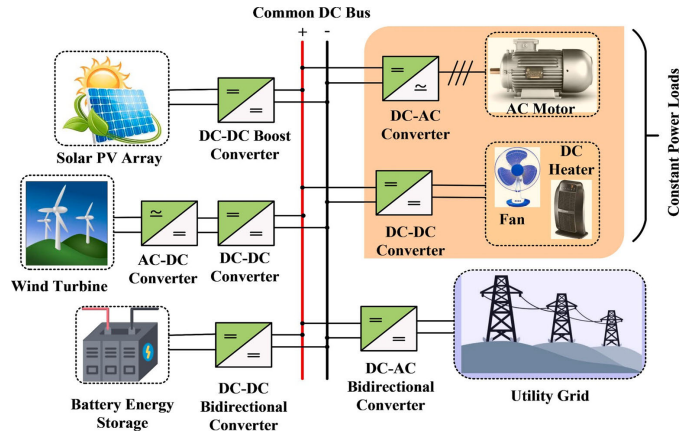


Fig. 1. Distributed power supply configurations where constant power loading may appear.

converter structure [1], [2], [3], [4] in many industrial applications such as in modern electric vehicles (EV) [5], [6], sea and undersea vehicles [7], and dc microgrids [8], [9], [10] as shown in Fig. 1. In such a structure, constant power is delivered to the downstream converter when it is tightly regulated, thus acting as a constant power load (CPL) for the upstream converter [10], [11], [12], [13], [14]. The dc-dc boost converter is one of the most commonly used topologies as a source converter in a two-stage cascaded system. For example, this converter is widely used in power conversion and motor drive applications, such as electric vehicles (EVs), which typically exhibit CPL behavior [15]. In particular, in the powertrain of an EV, this converter is bidirectional to allow energy to flow between its input and output ports during traction and braking [16]. This is also the case in dc microgrids, where bidirectionality allows for the charging and discharging of a storage battery or a supercapacitor [17], [18].

The study of the nonlinear behavior in closed-loop power converters loaded by resistive loads had been a topic of extensive research two decades earlier [19]. However, it is well established that even in open-loop, dc-dc power converters with CPL are inherently unstable [15], [20], [21] in both continuous and discontinuous conduction modes. Such instability which can be mitigated using either passive [6] or active [7], [22] damping strategies. Stability analysis of power converters supplying CPLs is most commonly conducted using a small-signal approach, wherein the system is linearized around the operating equilibrium point. This procedure yields a small-

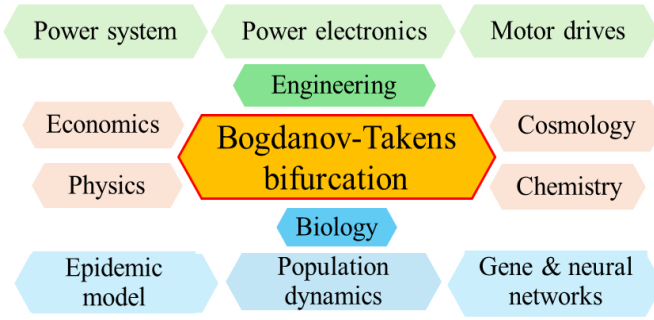


Fig. 2. The reported occurrence of BT bifurcation in different disciplines.

signal model that offers valuable insight into the local dynamic behavior of the system as well as the associated stability conditions [23], [24]. However, when the system exhibits more than one equilibrium point, obtaining the boundary between their basins of attraction is of paramount importance, and this can only be carried out using large signal techniques. In this case, global bifurcation analysis plays an important role in revealing the global dynamics and can be used to determine the parametric region that guarantees the global stability of the desired operating point.

Despite extensive research on many types of codimension-one bifurcations in dc-dc converters—such as smooth period-doubling, saddle-node, Neimark-Sacker and nonsmooth border-collision bifurcations—very few focus on codimension-two type such as Bogdanov-Takens (BT) bifurcation [25], [26], [27]. Although it is local, it unfolds global behaviors. For example, the BT bifurcation often serves as an organizing center for other complex bifurcation behaviors, such as homoclinic bifurcations, which involve trajectories that asymptotically approach and depart from saddle-type equilibria. The study of BT bifurcation offers insight into the complex dynamics of systems across disciplines [28], [29], [30], [31], [32], [33], [34], [35], [36], [37] as shown in Fig. 2.

In dc-dc converters, the BT bifurcation represents a critical codimension-two bifurcation point at which the dynamic behavior of the system may undergo significant qualitative changes. Such changes may manifest themselves in the form of undesired behaviours such as sustained periodic, quasiperiodic or even chaotic oscillations. Bifurcation analysis of closed-loop controlled bidirectional dc-dc converter with CPL has been conducted in [25], [26], and [27], where simultaneous occurrence of multiple bifurcation phenomena, including Hopf, saddle-node, and homoclinic behavior, is mapped in the two-parameter space, consistent with a codimension-two organizing center such as a BT point. Particularly, in [26], the BT point is identified using center manifold reduction and normal form theory. It was intrinsically limited to local dynamics near equilibrium points. This local framework does not address basins of attraction, which are global properties of the system. In engineering applications, basin boundaries in parameter space are crucial for evaluating robustness against disturbances and initial-condition sensitivity. Without basin-of-attraction analysis, stability predictions remain incomplete for practical design and safe operation. On the other hand, the BT bifurcation analysis was carried out in open-loop dc-dc

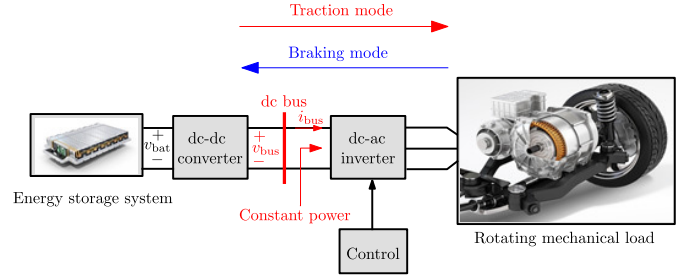


Fig. 3. EV powertrain using a multi-converter approach where the CPL behavior may appear.

converter without formal analysis and stability curves in the parameter plane [38]. In case of open-loop operation, there is less possibility of the occurrence of the BT bifurcation because there is no feedback gains to vary. The occurrence of a BT bifurcation is typically induced by the simultaneous variation of two system parameters, which may be associated with circuit parameters (e.g., input voltage, output power) or control variables (e.g., duty cycle, feedback gains).

By systematically analyzing the BT bifurcation, engineers can delineate stability boundaries and devise robust control methodologies that prevent the system from operating near bifurcation-induced transitions. Such strategies ensure consistent and reliable converter performance under varying input voltages, load disturbances, and component tolerances. In essence, effective mitigation of BT bifurcation in dc-dc converters requires detailed nonlinear analysis and the application of advanced control techniques, such as sliding mode control with washout filtering [23], [39], parameter adaptation, or other stabilization frameworks [40], [41], [42], [43], to maintain desired system behavior and prevent instability.

The aim of this paper is therefore to present an in-depth study of the bifurcation behavior of a bidirectional dc-dc converter with a CPL. Existence conditions of equilibrium points and limit cycles are determined and an analysis of their possible bifurcations is performed showing that the converter can exhibit different types of bifurcation such as Hopf and saddle-node bifurcations all emerging from a codimension-two BT bifurcation point in a suitable parameter plane. The conditions leading to these bifurcations are deduced, and their boundaries are located in this plane. The basin of attraction of the desired equilibrium point is also determined.

The remainder of the paper is organized as follows. In Section II, the system description and its mathematical modeling are given, and in Section III, the analysis of its dynamics is performed. The basin of attraction of the desired equilibrium point is determined in Section IV and a normal form of the BT bifurcation for the system is obtained in Section V. Theoretical predictions and numerical simulations are validated by experimental measurements in Section VI. Finally, conclusions are provided in Section VII.

II. SYSTEM DESCRIPTION AND MODELING

Fig. 3 illustrates a typical EV powertrain, where CPL behavior may arise. It consists of an energy storage system, for example, a battery with a voltage V_{bat} which is the input

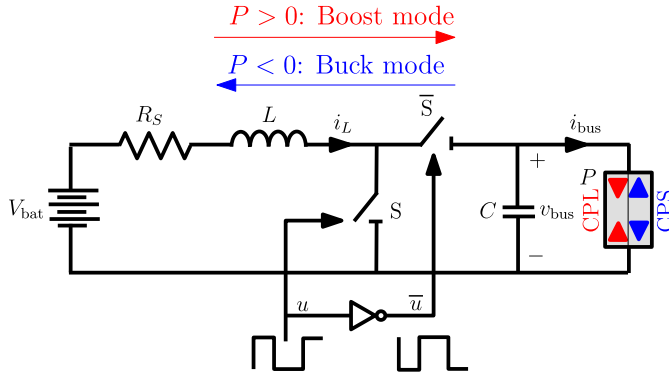


Fig. 4. Schematic circuit diagram of a bidirectional dc-dc converter loaded by a CPL in the boost mode and supplied by a CPS in the buck mode.

of a bidirectional dc-dc converter whose output voltage v_{bus} is connected to a dc-ac inverter. The latter is the drive for a three-phase ac motor. When the motor is operating at constant torque and constant speed, it draws constant mechanical power from the electrical part of the powertrain [3]. Therefore, a CPL is seen from the bidirectional dc-dc converter in the traction mode and it operates as a boost converter. In the regenerative braking mode this constant power is delivered to the battery through the bidirectional dc-dc converter. Therefore, this converter is supplied in this case by a constant power source (CPS) and it works as a buck converter.

The simplified schematic circuit diagram of a bidirectional dc-dc converter loaded by a CPL in the boost mode and supplied by a CPS in the buck mode is shown in Fig. 4. The switches S and $\bar{S}$ of the converter are driven by the complementary periodic square wave signals u and $\bar{u} = 1 - u$ both with period T_S . Their duty ratios are d and $\bar{d} = 1 - d$.

In the boost mode, the energy flows from the left port to the right port and the current i_L and the power P are positive. In the buck mode, the energy flows from the right port to the left port and the current i_L and the power P are negative. The state variables are the capacitor voltage v_{bus} and the inductor current i_L . The physical parameters are the capacitance C , the inductance L , the input voltage source V_{bat} and P is the power of the CPL/CPS characterized by a nonlinear voltage-current equation given by $i_{bus} = P/v_{bus}$. The resistor with resistance R_S could be real representing losses in the components such as the inductor [7] or the resistance of the battery, or emulated by a loss-less electronic circuit to introduce active damping [44], [45]. Therefore, the inherently unstable converter in open-loop and CCM operation, could be stabilized by suitably adjusting the value of R_S .

By applying standard circuit laws in Fig. 4, the following set of differential equations is obtained

$$\begin{aligned} \frac{dv_{bus}}{d\tau} &= -\frac{P}{Cv_{bus}} + (1-u)\frac{i_L}{C}, \\ \frac{di_L}{d\tau} &= -(1-u)\frac{v_{bus}}{L} - \frac{R_S}{L}i_L + \frac{V_{bat}}{L}. \end{aligned} \quad (1)$$

where τ represents time. The averaged model of the converter is obtained by substituting the binary signals u or $\bar{u}$ by their duty cycles d and $\bar{d}$. Let us define the following dimensionless

time, state variables and parameters

$$t = \omega_0 \tau, \quad x = \frac{v_{bus}}{V_{bat}}, \quad y = \frac{i_L Z_0}{V_{bat}}, \quad \omega_0 = \sqrt{\frac{1}{LC}}, \quad Z_0 = \sqrt{\frac{L}{C}} \quad (2)$$

Note that because $v_{bus} > 0$, $x > 0$. Let us also define the dimensionless parameters p and r as follows

$$p = \frac{PZ_0}{V_{bat}^2}, \quad r = \frac{R_S}{Z_0}. \quad (3)$$

Then, the following averaged dynamical system is obtained

$$\begin{aligned} \dot{x} &= -\frac{p}{x} + \bar{d}y, \\ \dot{y} &= -\bar{d}x - ry + 1, \end{aligned} \quad (4)$$

in which the overdot stands for differentiation with respect to the dimensionless time t . To further simplify the system (4), let us make the following change of time and state variables

$$t \rightarrow \frac{t}{r}, \quad x \rightarrow \frac{x}{d}, \quad y \rightarrow \frac{y}{r}, \quad (5)$$

hence, the following equivalent system is obtained

$$\begin{aligned} \dot{x} &= \beta \left(-\frac{\alpha}{4x} + y \right), \\ \dot{y} &= -x - y + 1, \end{aligned} \quad (6)$$

where the parameters α and β are given by

$$\alpha = 4pr, \quad \beta = \left(\frac{\bar{d}}{r} \right)^2. \quad (7)$$

Note that $\alpha > 0$ (i.e., $p > 0$) for the boost mode and $\alpha < 0$ (i.e., $p < 0$) for the buck mode. Although the dimensionless averaged system (4) has three independent parameters, $p \in \mathbb{R}$, $\bar{d} \in (0, 1)$ and $r \in \mathbb{R}^+$, only two parameters $\alpha \in \mathbb{R}$ and $\beta \in \mathbb{R}^+$ of equivalent system (6) are necessary for the analysis of the dynamical behavior. It is worth noting that the dimensionless formulation allows easier mathematical analysis. The results obtained from dimensionless parameters and state variables can be easily mapped back to the ones in terms of the original physical ones.

III. DYNAMICAL ANALYSIS OF THE SYSTEM

A. Existence and Stability of Equilibrium Points

Two equilibria are possible for system (6). These are the intersection points of the two nullclines defined by:

$$4xy - \alpha = 0, \quad x + y - 1 = 0. \quad (8)$$

Solving these two equations, the two obtained equilibria are given by:

$$\begin{aligned} \mathbf{x}_+ &= (x_+, y_+) = \left(\frac{1 + \sqrt{1 - \alpha}}{2}, \frac{1 - \sqrt{1 - \alpha}}{2} \right), \\ \mathbf{x}_- &= (x_-, y_-) = \left(\frac{1 - \sqrt{1 - \alpha}}{2}, \frac{1 + \sqrt{1 - \alpha}}{2} \right). \end{aligned} \quad (9)$$

The equilibria $\mathbf{x}_+$ and $\mathbf{x}_-$ only exist if $\alpha \leq 1$. In order to perform local stability analysis of the equilibria, the Jacobian

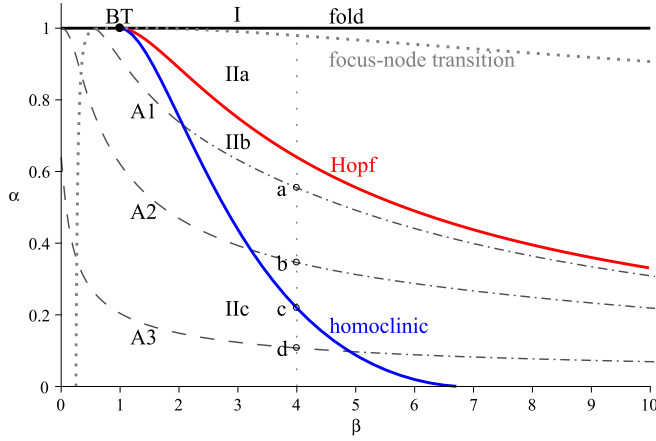


Fig. 5. Parameter plane diagram with lines for fold, Hopf and homoclinic bifurcations. The dotted line stands for the focus-node transition. Lines {A1, A2, A3} do for the crossing point of the basin of attraction with the x -axis at $\{1, 0.5, 0.2\}$ respectively. This boundary is the stable manifold of the saddle (dashed section) or the unstable limit cycle (dot-dashed section). State portraits in Fig. 6 are made using the parameter values at corresponding a-d points in this diagram.

matrix evaluated at these equilibria must be obtained. By using (6), this matrix can be expressed as follows

$$J_{\pm} = \begin{pmatrix} \frac{\beta\alpha}{4x_{\pm}^2} & \beta \\ -1 & -1 \end{pmatrix} \quad (10)$$

Its trace and determinant are given by

$$\text{trace}(J_{\pm}) = \frac{\beta\alpha}{4x_{\pm}^2} - 1, \quad \det(J_{\pm}) = (4x_{\pm}^2 - \alpha) \frac{\beta}{4x_{\pm}^2}. \quad (11)$$

For the buck mode ($\alpha < 0$), according to (11), $\det(J_{\pm}) > 0$ and $\text{trace}(J_{\pm}) < 0$, so the equilibrium points $\mathbf{x}_{\pm}$ are stable and anti-saddle. Taking into account the first equation in (8), x and y have opposite signs, therefore since in the buck mode $y < 0$ but $x > 0$, the only valid equilibrium point is the one corresponding to a negative current, i.e. $\mathbf{x}_{+}$ which is always stable and therefore no bifurcation can take place. In the following, the analysis corresponds to the boost mode ($\alpha > 0$).

For the boost mode ($\alpha > 0$), taking into account that $4x_{+}^2 - \alpha \geq 0$ and $4x_{-}^2 - \alpha \leq 0$, we conclude that $\det(J_{+}) \geq 0$ and $\det(J_{-}) \leq 0$, and so the two equilibria $\mathbf{x}_{+}$ and $\mathbf{x}_{-}$ are an anti-saddle and a saddle respectively. These two equilibria collide as well as disappear at $\alpha = 1$ in a typical saddle-node bifurcation scenario. To reveal the nature of the anti-saddle $\mathbf{x}_{+}$, the sign of the discriminant of the characteristic equation $\delta = (\text{trace}(J_{+}))^2 - 4\det(J_{+})$ must be evaluated. For $\delta < 0$, the eigenvalues of the anti-saddle are complex conjugate, so it is a focus, otherwise it is a node. The transition focus-node occurs when $\delta = 0$, at the critical value $\alpha = \alpha_T(\beta)$, given in terms of β as follows

$$\alpha_T(\beta) = \frac{4\beta(2\sqrt{\beta} - 1)}{(\beta + 2\sqrt{\beta} - 1)^2}$$

If $0 < \alpha < \alpha_T(\beta)$, the anti-saddle $\mathbf{x}_{+}$ is a focus and is a node if $\alpha_T(\beta) < \alpha < 1$. The dotted line in Fig. 5 corresponds to the focus-node transition $\alpha_T(\beta)$. Note that for $0 < \beta < 1/4$, the anti-saddle is a node in the full domain of the parameter α .

B. Bifurcation Set

In Fig. 5, some relevant lines determining different zones in the quadrant $\alpha > 0, \beta > 0$ of the plane (α, β) are shown. The black line $\alpha = \alpha_F = 1$, where a saddle node bifurcation occurs, splits this quadrant in two zones. In the zone $\alpha > 1$ (region I in Fig. 5) there exists no equilibrium point and therefore, there exists no limit cycle because in a continuous planar vector field, a limit cycle must contain at least one equilibrium point in its interior [46].

Along the line $\alpha = \alpha_F = 1$ the system has only one equilibrium point. In the zone $0 < \alpha < 1$ (region II in Fig. 5) there exist two equilibria, one of them is a saddle and the other one is an anti-saddle. This zone is subdivided in two parts by the dotted line $\alpha_T(\beta)$ at which the anti-saddle changes from a node to a focus.

Since $\det(J_{+}) \geq 0$, a Hopf bifurcation may occur under the condition $\text{trace}(J_{+}) = 4x_{+}^2 - \alpha\beta = 0$. This condition implies $\det(J_{+}) = \beta - 1 \geq 0$, and so $\beta > 1$. Consequently, system (6) undergoes a Hopf bifurcation at $\alpha = \alpha_H(\beta)$, where

$$\alpha_H(\beta) = \frac{4\beta}{(\beta + 1)^2}, \quad \text{with } \beta > 1. \quad (12)$$

In Section V, it will be shown that system (6) has a BT bifurcation by reducing this system to a normal form, in which the two nonlinear terms have the same sign, see (24). Then according to Theorem 8.4 and Theorem 8.5 in [47], the system (6) undergoes a subcritical Hopf bifurcation for $\alpha = \alpha_H(\beta)$ given rise to an unstable limit cycle.

The red line in Fig. 5 represents $\alpha_H(\beta)$. If $0 < \alpha < \alpha_H(\beta)$ (regions IIb and IIc in Fig. 5) then $\text{trace}(J_{+}) < 0$ and therefore, the anti-saddle is stable. Conversely, it is unstable if $\alpha_H(\beta) < \alpha < 1$ (region IIa in Fig. 5). Note that the three lines α_F, α_H and α_T meet at the parameter point $(\alpha, \beta) = (1, 1)$. It is also worth noting that a limit cycle is born when $\alpha = \alpha_H(\beta)$. Also, we have numerically detected that this limit cycle disappears at $\alpha = \alpha_M(\beta)$ which corresponds to a homoclinic bifurcation in which the stable and unstable manifolds of the saddle merge together. The line $\alpha = \alpha_M(\beta)$, which is drawn in blue in Fig. 5, also passes through the parameter point $(\alpha, \beta) = (1, 1)$. Thus, in the zone $\alpha_M(\beta) < \alpha < \alpha_H(\beta)$ (region IIb in Fig. 5), an unstable limit cycle coexists with the stable focus.

We have extensively used the Rosenbrock method [48] (a stiffly refinement of the classical Runge–Kutta scheme) for numerical integration of the trajectories. For each parameter point (α, β) , we select two starting points very near to the saddle point in their tangential directions of its unstable and stable manifolds. Then, we integrate the corresponding vector field (6) in forward and backward time respectively, until they intersect the nullcline ($x + y - 1 = 0$). Fixed a value of α , by following the Newton–Raphson procedure [49], we adjust β to vanish the difference between the two y -coordinates at those intersecting points.

The whole bifurcation scenario reminds us of the well-known codimension-two BT bifurcation. In Section V, it will be shown that in the neighborhood of the critical parameter point $(1, 1)$ and around the corresponding equilibrium $(\frac{1}{2}, \frac{1}{2})$, the system is topologically equivalent to an unfolding of the

normal form of the BT bifurcation where the associated Hopf bifurcation is subcritical.

C. Adequacy of the Averaged Model for Local Bifurcation Analysis

It is to be noted that the continuous-time state-space averaged model is used assuming a smooth field for its evolution. This means that the state variables represent their moving average values within a switching period, effectively replacing the switched model with a simplified averaged one [50]. Therefore, the small-amplitude switched limit cycles become equilibrium points in the averaging frame. As the observed and studied phenomena are taking place at the slow-timescale not the fast switching timescale, the averaged model being in this case able to predict these phenomena. If the converter presents bifurcations at the fast switching timescale (for instance, period-doubling bifurcation), then the use of a discrete-time model becomes necessary [51]. Since the focus of this paper is only on slow-timescale bifurcation phenomena, a discrete-time model was not necessary.

IV. HINTS ON THE BASIN OF ATTRACTION OF THE STABLE EQUILIBRIUM POINT

One important consideration in a practical design is the convergence to the desired stable equilibrium (6). A reasonable initial condition for this system is $(v_{\text{bus}}(0), i_L(0)) = (V_{\text{bat}}, 0)$. The non-zero initial condition in the capacitor voltage can be achieved by connecting a diode between the two ports of the converter [52]. Due to the concatenated change of state variables in (2) and (5), the former physical initial state is transformed to $(x(0), y(0)) = (\bar{d}, 0)$ for system (6). This fact imposes a practical design restriction that will be managed in the equivalent two parameter system (6). Obviously, the anti-saddle equilibrium must exist and be stable and also, to meet the initial condition requirement, we must ensure that the likely initial state $(\bar{d}, 0)$ be inside its basin of attraction. Then, the boundary of this basin must be taken into account, in particular the left intersection of this boundary with the x -axis, let us denote x_B , has to satisfy the condition $x_B < \bar{d}$ (recall that $0 < \bar{d} < 1$).

Two types of basin of attraction boundaries (BAB) for the equilibrium $\mathbf{x}_+$ are possible. If the unstable limit cycle exists, then it surrounds $\mathbf{x}_+$ and so it is itself the BAB. Otherwise, the saddle stable manifold (SSM) plays the role of BAB. Lines A1, A2 and A3, which are plotted in Fig. 5 and have been obtained by numerical computation, correspond to three sets of parameter (α, β) such that x_B has a fixed value, arbitrarily chosen equal to 1, 0.5 and 0.2, respectively. Note that lines A1, A2 and A3 in Fig. 5 are divided in two sections by the homoclinic line where the unstable limit cycle disappears. The two sections are represented by dashed lines on the left or by dot dashed ones on the right of the homoclinic line, where the BAB is determined by the SSM or by the unstable limit cycle respectively. These A1, A2 and A3 curves in the parameter plane have been constructed following a similar procedure to the one used in the computation of the homoclinic one. In this case, the intersection point with the x -axis is considered.

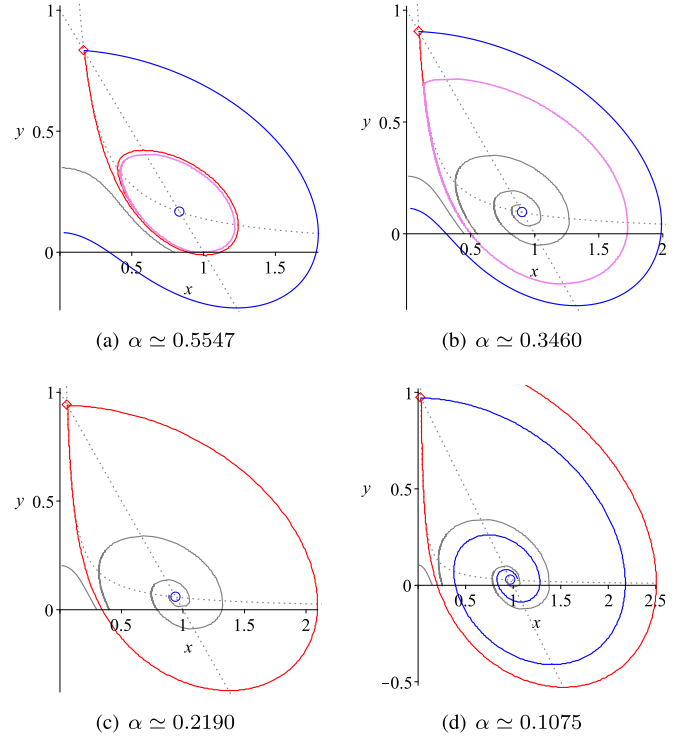


Fig. 6. State plane diagrams for system (6) with parameters $\beta = 4$ and α specified in the caption (note these parameter points appear in Fig. 5). Representative trajectories are the stable saddle manifold in red color, the unstable saddle manifold in blue color, the unstable limit cycle in violet color (this limit cycle collapses at case (c) and does not exist in case (d) and some illustrative trajectories in gray color. Also, the equilibrium points $\mathbf{x}_+$ and $\mathbf{x}_-$ have been represented using circle (blue) and diamond (red) symbols respectively. Note that these points are located at the intersection of the two nullclines (dotted lines).

Finally, in Fig. 6, the relevant orbits are represented in the dimensionless state plane (x, y) of the system (6) with parameter $\beta = 4$ and α indicated in the caption. The case c corresponds to the homoclinic bifurcation, so that the stable and the unstable manifolds of the saddle merge together. Cases a, b and d correspond to lines A1, A2 and A3 respectively, so x_B takes the values 1, 0.5 and 0.2 respectively. Note that the first case belonging to A1 (with $x_B = 1$) implies that the unstable limit cycle, if it exists, or the SSM is tangent to the abscissa and therefore, parameter values selected on this line will make the whole abscissa axis outside the basin of attraction of the desired equilibrium, making this choice impractical. In the other cases ($x_B < 1$), the value of $\bar{d}$ should be larger than x_B to ensure that the initial condition is located inside the BAB.

Now we draw the critical lines to identify the stable region in the physical parameter plane in correspondence with normalized parameters, as shown in Fig. 5 specifically for the A2 line. Keeping the parameters $V_{\text{bat}} = 10$ V, $Z_0 = 4$ Ω and $d = 0.5$ fixed, the critical lines are drawn in the parameter space of electrical power (P) versus stabilizing resistance (R_s) as shown in Fig. 7. The BT point is calculated as (2 Ω , 12.5 W). If P remains below the dot-dashed and dashed line (region IIc), then one equilibrium is stable and with the initial point $(V_{\text{bat}}, 0)$ inside its boundary basin of attraction. If P is over that line

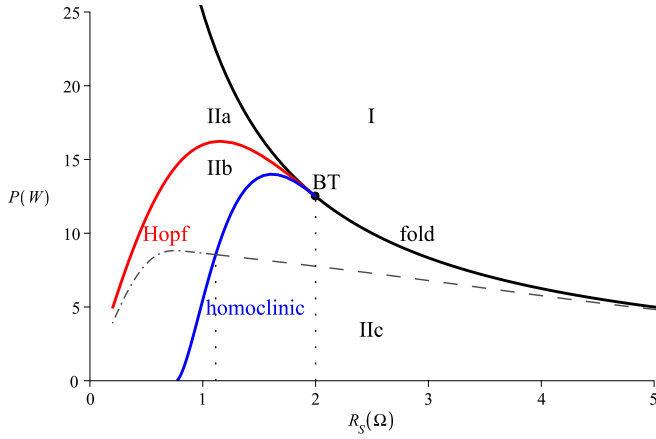


Fig. 7. Critical lines for fold, Hopf, and homoclinic bifurcations using physical parameters electrical power (P) versus resistance (R_S) in correspondence with normalized parameters in Fig. 5, using fixed parameters $V_{\text{bat}} = 10$ V, $Z_0 = 4$ Ω and $d = 0.5$ (that is specifically for A2 line). Note that the critical lines and regions have the same interpretation as given in Fig. 5.

but below the continuous blue line for Hopf condition (region IIb), then only the stability condition is met. Moreover, if P is above the continuous black line for the fold condition (region I), then there is no equilibrium.

V. APPROXIMATION TO THE NORMAL FORM OF THE BOGDANOV-TAKENS BIFURCATION

In this section, we will prove that the system (6) undergoes a BT bifurcation for $\alpha = 1$ and $\beta = 1$ at the point $(x, y) = (\frac{1}{2}, \frac{1}{2})$. The systematic calculation steps are given below.

Step 1 (Preliminary transformation and first reduction): As a first step, the origin of the coordinates is translated to that critical point by using the change of variables

$$x \longrightarrow x + \frac{1}{2}, \quad y \longrightarrow y + \frac{1}{2},$$

hence, we obtain the following set of equations

$$\begin{aligned} \dot{x} &= \beta \left(-\frac{\alpha}{4x+2} + y + \frac{1}{2} \right), \\ \dot{y} &= -x - y. \end{aligned} \quad (13)$$

and then a series expansion is used for small values of x in the first equation in (13) leads to

$$\dot{x} = \frac{\beta(1-\alpha)}{2} + \beta(\alpha x - 2\alpha x^2 + y) + \mathcal{O}(x^3) \quad (14)$$

In order to further simplify the Jacobian matrix, the following change of variables is used

$$x \longrightarrow x + y, \quad y \longrightarrow -x,$$

thus resulting in the following equation after neglecting higher order terms

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= \frac{\beta(1-\alpha)}{2} + \beta(\alpha - 1)x + (\alpha\beta - 1)y \\ &\quad - 2\alpha\beta x^2 - 2\alpha\beta y^2 - 4\alpha\beta xy. \end{aligned}$$

For the sake of clarity, we define the parameters γ , ε_1 and ε_2 as follows

$$\gamma = -2\alpha\beta, \quad \varepsilon_1 = \frac{\beta(1-\alpha)}{2}, \quad \varepsilon_2 = \alpha\beta - 1.$$

Note that α and β are close to 1, then ε_1 and ε_2 are small parameters.

Thus, we arrive at the following system

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= \varepsilon_1 - 2\varepsilon_1 x + \varepsilon_2 y + \gamma x^2 + 2\gamma xy + \gamma y^2. \end{aligned} \quad (15)$$

Next, we closely follow Kuznetsov (see Section 8.4 in [47]).

Step 2 (Parameter-dependent shift): A parameter-dependent shift of coordinates in the direction of x (variable y remains unchanged)

$$x \longrightarrow x + \zeta,$$

where ζ is a small parameter to be determined later, system (15) is transformed into

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= \varepsilon_1 - 2\zeta\varepsilon_1 + \gamma\zeta^2 + (2\gamma\zeta + \varepsilon_2)y \\ &\quad + (2\gamma\zeta - 2\varepsilon_1)x + \gamma x^2 + 2\gamma xy + \gamma y^2. \end{aligned} \quad (16)$$

Now, as $\gamma = -2\alpha\beta \neq 0$, for $\alpha = 1$ and $\beta = 1$, we choose

$$\zeta = -\frac{\varepsilon_2}{2\gamma}$$

so that the linear term in the variable y of the second equation in (16) vanishes, hence this reduces to

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= b_0 + \eta x + \gamma x^2 + 2\gamma xy + \gamma y^2, \end{aligned} \quad (17)$$

where

$$b_0 = \varepsilon_1 - 2\zeta\varepsilon_1 + \gamma\zeta^2 = \frac{(2-3\alpha)\alpha\beta^2 + 2\beta - 1}{8\alpha\beta}$$

and

$$\eta = 2\gamma\zeta - 2\varepsilon_1 = 1 - \beta$$

Step 3 (Time reparametrization and second reduction): We introduce a new time θ defined by

$$dt = (1 + \vartheta x)d\theta,$$

where ϑ is a parameter to be defined later. Note that for small $|x|$ the sense of time is preserved. Then, using the overdot to denote differentiation with respect to θ , one gets

$$\begin{aligned} \dot{x} &= y + \vartheta xy, \\ \dot{y} &= b_0 + (b_0\vartheta + \eta)x + (\eta\vartheta + \gamma)x^2 + 2\gamma xy + \gamma y^2. \end{aligned} \quad (18)$$

Now, we introduce the following new coordinates

$$u = x, \quad v = y(1 + \vartheta x)$$

whose inverse change is

$$x = u, \quad y = \frac{v}{1 + \vartheta u} = v(1 - \vartheta u + \vartheta^2 u^2 + \dots).$$

Then, from (18) neglecting higher order terms, one gets

$$\begin{aligned} \dot{u} &= v, \\ \dot{v} &= b_0 + (2b_0\vartheta + \eta)u + (b_0\vartheta^2 + 2\eta\vartheta + \gamma)u^2 \\ &\quad + 2\gamma uv + (\vartheta + \gamma)v^2. \end{aligned} \quad (19)$$

Next, we choose $\theta = -\gamma$ so that the coefficient corresponding to the term in the variable v^2 vanishes and after renaming the variables (u, v) as (x, y) , system (19) reduces to

$$\begin{aligned}\dot{x} &= y, \\ \dot{y} &= b_0 + b_1 x + \gamma xy + b_2 x^2.\end{aligned}\quad (20)$$

where b_0 and γ were defined above and b_1 and b_2 are given by

$$b_1 = \eta - 2b_0\gamma \frac{(2-3\alpha)\alpha\beta^2 + 1}{2} \quad (21)$$

$$b_2 = \gamma - 2\gamma\eta + \gamma^2 b_0 = \frac{((2-3\alpha)\beta^2\alpha - 6\beta + 3)\alpha\beta}{2}. \quad (22)$$

Step 4 (Final scaling and setting of new parameters): Finally, we simultaneously perform a scaling of variables and introduce a new time t again as follows

$$x \rightarrow \frac{b_2}{\gamma^2}x, \quad y \rightarrow \frac{b_2^2}{\gamma^3}y, \quad t \rightarrow \frac{\gamma}{b_2}\theta, \quad (23)$$

Note that the denominators are nonzero at $\alpha = \beta = 1$ ($\gamma = -2$ and $b_2 = -2$). By neglecting higher order terms, the normal form is obtained as

$$\begin{aligned}\dot{x} &= y, \\ \dot{y} &= \mu_1 + \mu_2 x + xy + x^2,\end{aligned}\quad (24)$$

where μ_1 and μ_2 are given by

$$\mu_1 = \frac{b_0\gamma^4}{b_2^3} = \frac{16((3\alpha-2)\alpha\beta^2 - 2\beta + 1)}{((3\alpha-2)\alpha\beta^2 + 6\beta - 3)^3} \quad (25)$$

$$\mu_2 = \frac{b_1\gamma^2}{b_2^2} = \frac{8((2-3\alpha)\alpha\beta^2 + 1)}{((3\alpha-2)\alpha\beta^2 + 6\beta - 3)^2}. \quad (26)$$

Note that in the unfolding of the BT normal form (24), which has been obtained as an approximation of system (6), the coefficients of terms xy and x^2 have the same sign, which implies that the Hopf bifurcation is subcritical [47]. This means that the stable focus coexists with an unstable limit cycle, which emerges at that bifurcation and collapses with a homoclinic orbit. These features agree with our results in Section III-B. Moreover, it can be easily checked that the expressions $\alpha(\beta)$ that can be obtained by applying condition $4\mu_1 = \mu_2^2$ or $\mu_1 = 0$, which corresponds to the saddle-node or Hopf bifurcation respectively for system (24), have the same series expansion up to degree 2 than its related expressions $\alpha_F(\beta)$ and $\alpha_H(\beta)$ obtained for system (6) in Section III-B.

VI. EXPERIMENTAL VALIDATION

To validate the theoretical predictions, an experimental prototype has been developed. The inductor, fabricated in the laboratory, has an inductance of $L = 160 \mu\text{H}$ and series resistance of $r_L = 100 \text{ m}\Omega$. A thin film capacitor with a capacitance $C = 10 \mu\text{F}$ and equivalent series resistance of $r_C = 10 \text{ m}\Omega$ is used. The normalized impedance and frequency are calculated as $Z_o = \sqrt{\frac{L}{C}} = 4.0 \Omega$, $\omega_o = \frac{1}{\sqrt{LC}} = 25 \text{ krad/s}$ respectively.

The schematic diagram of the experimental setup is presented in Fig. 8(a). The switching element is implemented using a metal-oxide-semiconductor field-effect transistor (MOSFET). Specifically, the power circuit is constructed

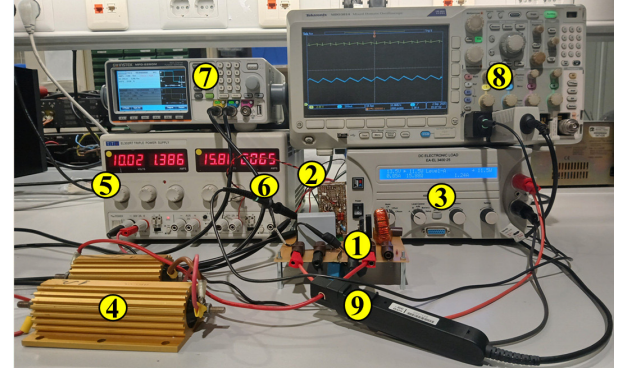
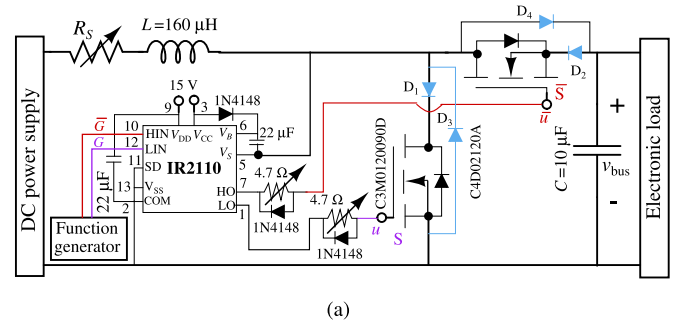


Fig. 8. (a) Schematic circuit diagram of the power circuit with IR2110 MOSFET driver for the experimental setup. (b) Photograph of the overall experimental setup. ①—boost converter power stage, inductor, capacitor, ②—MOSFET Driver IR2110, ③—Elektro-Automatik EA-EL 3400-25 electronic load, ④—compensating resistor, ⑤—⑥ power supply EL302RT for converter input and gate driver, ⑦—GW INSTEK-MFG-2260M multi-channel function generator, ⑧ Tektronix MD0314 Mixed Signal Oscilloscope, ⑨—Tektronix TCP0020 current probe.

using two silicon carbide (SiC) N-channel power MOSFETs (part number: C3M0120090D) and four SiC Schottky diodes with part number C4D02120E are used to realize the power circuit. Note that in this prototype four diodes are used in addition to MOSFET's internal diode. These diodes have been added to the boost converter to ensure bidirectionality (Fig. 4) of the prototype. In case of bidirectionality, under applying gate signals to both MOSFETs, diodes D_1 and D_2 block the conduction of internal diodes of the two MOSFETs. This methodology is used to disable the internal body diode of the MOSFET, which is often a slow recovery diode and can significantly increase switching losses. On the other hand, D_3 and D_4 allow the current to be in both directions. The parasitic elements of the switch and diode are $R_{DS(on)} = 0.12 \Omega$ and $R_D = 0.15 \Omega$ respectively. The equivalent virtual resistance is the sum of the resistance of the parasitics and the fixed resistance of value 1.0Ω . The small mismatch in the value indicated for the experimental measurements was also taken into account by the internal resistance of the voltage source and the connecting wires.

Before showing an experimental validation of some of the theoretical findings, let us first present some numerical simulations from the switched model described in (1) by using the same parameter values considered in the experimental prototype.

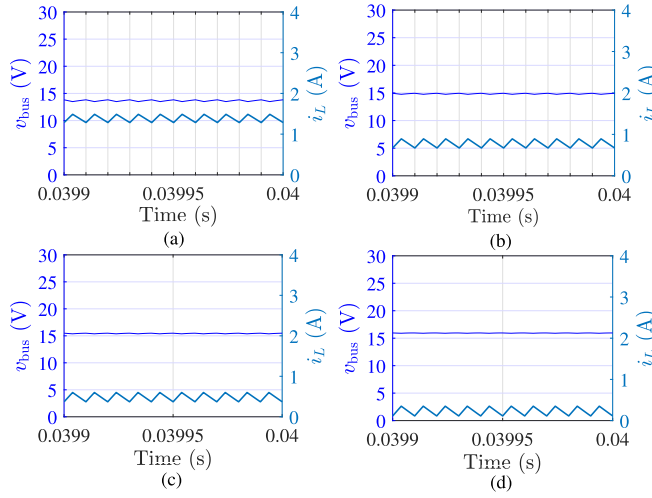


Fig. 9. Simulation waveforms of the switched model corresponding to Fig. 6 for $L = 160 \mu\text{H}$, $C = 10 \mu\text{F}$, $R_S = 1.2 \Omega$, $V_{\text{bat}} = 10 \text{ V}$, $d = 0.4$, $f_S = 1/T_S = 100 \text{ kHz}$. Therefore, $Z_o = \sqrt{L/C} = 4.0 \Omega$, $\omega_o = 1/\sqrt{LC} = 25 \text{ krad/s}$, $T = \omega_o T_S = 0.25$, $r = R_S/Z_o = 0.3$. For fixed value of $\beta = 4$, $\alpha = 4pr$ varies as: (a) $\alpha = 0.5547$ ($P \approx 11.55 \text{ W}$), (b) $\alpha = 0.3460$ ($P \approx 7.08 \text{ W}$), and (c) $\alpha = 0.2190$ ($P \approx 4.56 \text{ W}$), (d) $\alpha = 0.1075$ ($P \approx 2.24 \text{ W}$).

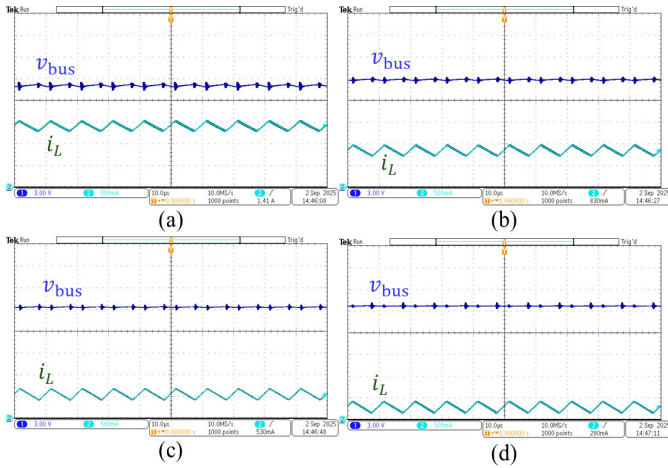


Fig. 10. Experimental measurements corresponding to similar simulation results shown in Fig. 9: (a) $\alpha = 0.5547$ ($P \approx 11.55 \text{ W}$), (b) $\alpha = 0.3460$ ($P \approx 7.08 \text{ W}$), and (c) $\alpha = 0.2190$ ($P \approx 4.56 \text{ W}$), (d) $\alpha = 0.1075$ ($P \approx 2.24 \text{ W}$).

Fig. 9 and Fig. 10 show the output capacitor voltage and inductor current waveforms from numerical simulations and experimental measurements. In these figures, the values of r , d , consequently also the value of β , are kept fixed and the value of α decreased. This parameter was changed using the physical parameter P representing the power of the CPL/CPS. It can be observed that in both simulated and experimental waveforms, the average value of the inductor current decreases when α decreases. It can also be seen that the simulation and experimental results are in good agreement with the theoretical analysis illustrated in Fig. 6.

To show the nature of the equilibrium point, an 1 V step change is applied on the input voltage at $t = 0.02 \text{ s}$. After applying this perturbation, the system waveforms exhibit damped oscillations corresponding to a focus equilibrium point which is reached by the system in steady-state operation as

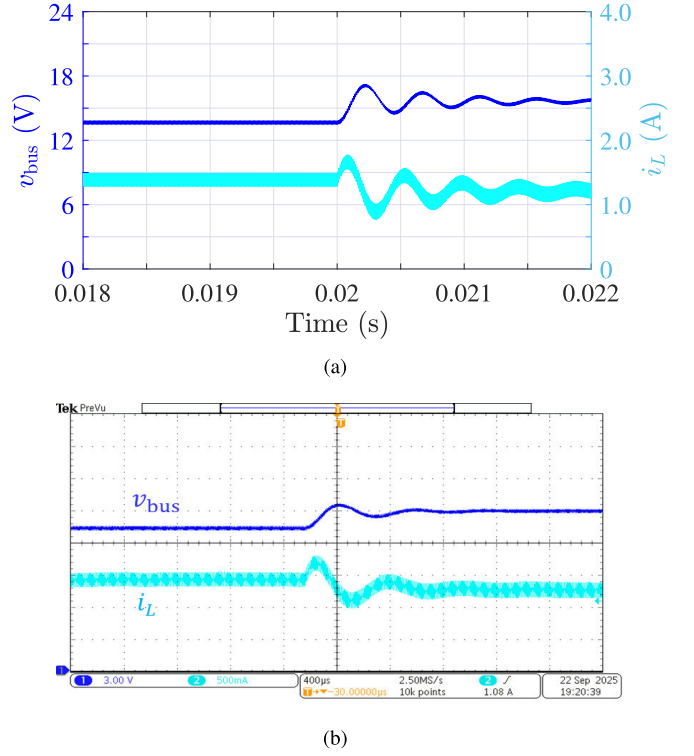


Fig. 11. Waveforms before and after $t = 0.02 \text{ s}$ when a step change of 1 V on input voltage is applied to the system at $\beta = 4$ and $\alpha = 0.5547$ ($P \approx 11.55 \text{ W}$) (a) numerical simulation from the switched model (b) experimental measurement.

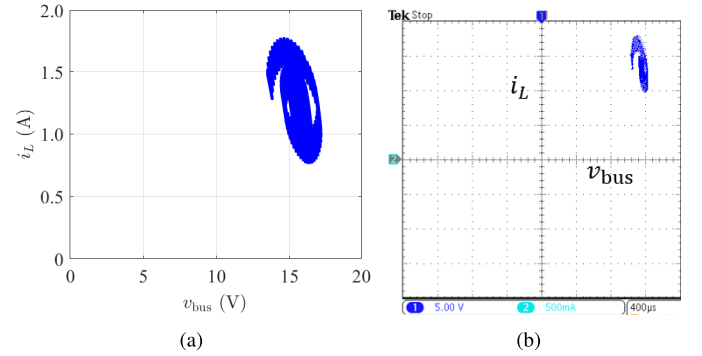


Fig. 12. State plane diagram corresponding to Fig. 11 at $\beta = 4$ and $\alpha = 0.5547$ ($P \approx 11.55 \text{ W}$) (a) numerical simulation (b) experimental measurement. The system trajectory shows a spiral behavior corresponding to a focus equilibrium point of the averaged model.

shown in Fig. 11. In the state plane diagram (Fig. 12) it shows spiraling in behavior. This occurs when the eigenvalues of the corresponding Jacobian matrix are complex conjugates with negative real parts, indicating that the system is both oscillatory and dissipative. For $\beta = 4$ and $\alpha = 0.5547$, theoretical calculation from (6) and mapping back to the original physical parameters and variables using transformations (2) and (5), the eigenvalues are $-759 \pm 13400j$, where $j = \sqrt{-1}$. This means that steady-state will theoretically be reached within 5.3 ms (2% criteria) after a small disturbance in any physical parameter of the system takes place.

Table I shows the comparison between theoretical and experimental results for one case. Note that the damped

TABLE I
COMPARISON OF THE THEORETICAL ANALYSIS AND EXPERIMENTAL MEASUREMENTS OF FIG. 10(A) AND FIG. 11

	Theoretical	Experimental
Damped oscillation frequency	13.4 krad/s	13.09 krad/s
Steady-state voltage	13.89 V	13.80 V
Steady-state current	1.38 A	1.41 A

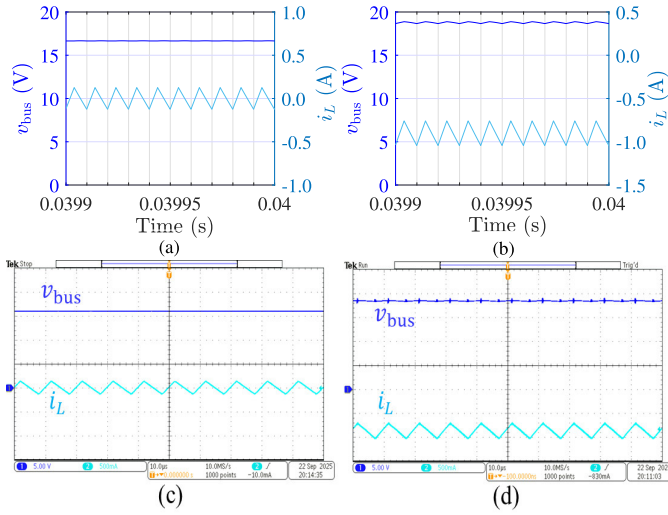


Fig. 13. Simulation waveforms from the switched model and the corresponding experimental results at $\beta=4$ and varying α . (a) and (c) $\alpha=0$ ($P \approx 0$ W), (b) and (d) $\alpha=-0.48$ ($P \approx -10$ W).

oscillation frequency is calculated from Fig. 11(b) where one complete oscillation after disturbance takes $480 \mu\text{s}$. The small discrepancies are due to the unmodeled parasitics and nonidealities of the experimental prototype.

Finally, the experimental prototype was tested with a negative value of the power P which means that the CPL is replaced by a CPS on the load side. A constant voltage electronic load is connected on the source side. Two cases $P = 0$ W and $P = -10$ W were considered to show the bidirectionality of the system and its stability as shown in Fig. 13. A clear agreement between numerical simulation and experimental measurement can be observed.

VII. CONCLUSION

The analysis presented in this study reveals the occurrence of multiple bifurcation phenomena in an open loop dc-dc bidirectional converter supplying a CPL, with a stabilizing emulated resistor incorporated in series with the main inductor. Comprehensive numerical simulations based on the averaged model of the system have been performed to characterize these bifurcations, providing insight into both the local and global dynamical behaviors of the converter. In particular, the presence of a BT bifurcation has been rigorously demonstrated under specific parameter variations.

The detailed bifurcation analysis not only captures the intricate nonlinear dynamics of the system but also enables the precise identification of parametric regions that ensure global stability at the desired operating point. From a converter design perspective, the obtained bifurcation maps provide

practical guidelines for the selection of circuit parameters, allowing designers to avoid operating regions associated with undesirable behaviors such as oscillations, coexistence of attractors, or abrupt transitions to instability. In this sense, the codimension-two bifurcation framework goes beyond conventional single-parameter analyses by explicitly revealing the interaction between key system parameters and their combined effect on stability.

Moreover, the identification of stable and unstable regions in the parameter space supports the definition of safe operating boundaries, which can be directly exploited during the design and tuning stages to enhance robustness against parameter uncertainties and load variations. These results are particularly relevant for applications involving bidirectional power flow and tightly regulated dc buses, such as electric vehicle powertrains, energy storage interfaces, and dc microgrids, where CPL-induced instabilities pose significant challenges to reliable operation.

The analytical and simulation results are partially validated by experimental measurements carried out on a laboratory prototype, showing close agreement with the theoretical predictions and confirming the practical relevance of the proposed analysis. Overall, the bifurcation phenomena identified in this work provide a critical understanding of the global behavior of bidirectional dc-dc converters with nonlinear loads and demonstrate that codimension-two bifurcation analysis constitutes not only a powerful theoretical tool but also a valuable design-oriented methodology. This study therefore paves the way for future research on a broader class of bidirectional power converters interfacing nonlinear loads and sources, with an emphasis on stability-oriented design.

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